

WHEN IS PRESCHOOL CHILDREN'S PLAY MATHEMATICAL?

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One of Bishop's 6 mathematical activities is play. It is deemed as mathematical because of its relationship to modelling, hypothetical thinking and abstraction, all of which can be seen in preschool children's play. In this paper, we explore the question about when young children's play can be labelled as mathematical. This exploration contrasts different definitions of play with what is known about mathematicians' academic play and how mathematics education researchers have described young children's play. From this theoretical discussion, we draw out the similarities between these types of play and discuss what makes play mathematical. We use these features to analyse a small episode of children playing to discuss if and how this could be considered to be mathematical.

INTRODUCTION

Mathematics and play are often combined, especially in discussing young children's engagement in mathematical tasks in preschools (see for example, (Ginsburg, 2006; Sarama & Clements, 2009; Lange, Meaney, Riesbeck, & Wernberg, 2014)). In these discussions, mathematics and play are connected in three different ways. In regards to young children, often play is considered as a vehicle for learning, while for mathematicians play is described as a necessary component of their creativity in problem solving. The third relationship is that which considers play as a mathematical activity. In this paper, we compare these different perspectives in order to identify the features of play that can be considered mathematical. This is important because often what young children are engaged in is recognised as play but dismissed as not being mathematical unless it includes obvious mathematical content, such as number. Although mathematical processes such as problem solving are deemed as important by mathematicians and mathematics educators alike, there seems to be scant support for categorising young children's actions as mathematical processes. In this paper, we discuss when young children's play should be considered mathematical.

Many people have identified features belonging to play (see for example Huizinga, 1976; Bruner, 1975; Ugurel & Morali, 2010). Incorporating features of other researchers, Fromberg (1999) defined young children's play as:

Symbolic, in that it represents reality with an "as if" or "what if" attitude

Meaningful, in that it connects or relates experiences

Active, in that children are doing things

Pleasurable, even when children are engaged seriously in activity

Voluntary and intrinsically motivated, whether the motive is curiosity, mastery, affiliation, or something else

Rule-governed, whether implicitly or explicitly expressed

Episodic, characterized by emerging and shifting goals that children develop spontaneously and flexibly. (p. 28)

Features such as these can be seen in the Swedish preschool curriculum, in which play is considered the foundation for children's learning, including the learning of mathematics:

Play is important for the child's development and learning. Conscious use of play to promote the development and learning of each individual child should always be present in preschool activities. Play and enjoyment in learning in all its various forms stimulate the imagination, insight, communication and the ability to think symbolically, as well as the ability to co-operate and solve problems. (Skolverket, 2011, p. 6)

Having play as the vehicle for learning affects many aspects of the interactions between children and between children and the teacher. For example, from examining an activity where preschool children explored glass jars, we found that although the teacher could offer suggestions about activities, the children did not have to adopt them and could suggest alternatives (Lange et al., 2014). The importance of children's ability to control their environment in a play situation has been acknowledged by others—"I suggest that the success of the physical manipulations, and ultimate mathematical conceptualisations, is very much dependent upon the successful self-regulation of the social context" (Macmillan, 1995, p. 123).

Although the features of play that are connected to mathematics learning are often undefined, the mathematics of young children is generally equated with school mathematics topics. For example, Vogel (2014) stated:

The conception of the mathematical situations of play and exploration provides that the arrangement has its root in one of the following five mathematical domains: number and operations, geometry and spatial thinking, measurements, patterns and algebraic thinking or data and probability (including combinatorics) (p. 224)

In discussing everyday mathematics that occurs in young children's play, Ginsburg (2006) drew on the work of John Dewey, to provide a less extensive list of counting, measuring and rhythmic sequencing. Even when a mathematical process, such as argumentation (Perry & Dockett, 1998), is discussed in regard to play, generally it is seen only from the perspective of what it contributes to children's learning of mathematical content, particularly school mathematical content. For example, van Oers's (2014) discussion of a play-based curriculum sees it as providing possibilities for children to become aware of "quantitative and spatial dimensions of reality" (p. 115) within problem solving situations—"Mathematics emerges in children's development, not as an elaboration of implicit mathematics in play, but as an

attribution from outside of mathematical meanings to children's actions or utterances" (p. 114).

The focus on mathematical content is somewhat surprising given that the attributes of play seem, at least at first glance, to be more closely connected to mathematical processes than to content. However, the lack of definitions of play in many of the articles that promote play as an approach to mathematical learning may explain this anomaly.

There has also been some work on mathematicians' views about the connection between mathematics and play. Bergen (Bergen, 2009) in discussing the childhood memories of adults working in the mathematics, science and technology industries found that many of them had spent time involved in construction play. She suggested that:

The "worlds" children construct, either with concrete materials such as blocks or interlocking pieces or with virtual-reality simulation games, give them the imaginative experiences and the interest in "seeing what might happen" to prepare them to create new worlds of design in later work experiences. (p. 419)

Creativity and imagination, rather than content, seem to have been the impetus for mathematical understanding. This is supported by other studies, which looked at the long-term, mathematical achievement implications of making constructions with Lego (Wolfgang, Stannard, & Jones, 2003). In their longitudinal study of the impact of block play on school mathematics achievement, Wolfgang, Stannard and Jones (2001) found that the complexity and adaptiveness of children's block play in preschool correlated with their mathematics achievement in high school. The more complexity in their building play in preschool, the more likely the children were to have higher mathematical achievement in high school. Similarly, Moranyi, Devine, Nobes and Szucs (2013) found that ten-year-old children who displayed higher mathematics performance also had better ability to reason logically about belief-inconsistent fantasy content. For example, they were better able to deduce that the mouse was bigger than an elephant from the two statements, *the elephant is smaller than the dog* and *the dog is smaller than the mouse*.

This research seems to present an alternative conception of the relationship between play and mathematics than the previous one in which play was considered as a vehicle for learning mathematical content. Play itself is seen as a factor for developing mathematical creativity and imagination. From interviews with research mathematicians, Bloom suggested that the way that most children were introduced to mathematics, through precision and accuracy, actually stifled their development (Brandt, 1985). He suggested that a playful approach, as recommended by Alfred North Whitehead (1959), would be a better way to encourage children to respond to mathematics as mathematicians had.

In contrast to the earlier reviewed research which focused on mathematical content in relationship to learning through play, Holton, Ahmed, Williams and Hill (Holton,

Ahmed, Williams, & Hill, 2001) defined mathematical play as “that part of the process used to solve mathematical problems, which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion” (p. 403). Thus, rather than being a vehicle for learning mathematics, Holton et al. (2001) situate play as a necessary component for doing mathematics. They identified six criteria that they saw as essential components of mathematical play:

- (1) it is a solver-centred activity with the solver in charge of the process;
- (2) it uses the solver’s current knowledge;
- (3) it develops links between the solver’s current schemata while the play is occurring;
- (4) it will, via 3, reinforce current knowledge;
- (5) it will, via 3, assist future problem solving/mathematical activity as it enhances future access to knowledge;
- (6) it is irrespective of age. (Holton et al., 2001, p. 404)

These criteria have some resemblance to Fromberg’s (1999) attributes of play. As well, in her review of the literature she mentioned almost all of Holton et al.’s (2001) list. For example, Fromberg’s identification of play as needing to be meaningful is similar to Holton et al.’s (2001) discussion of using the solver’s current knowledge. She also acknowledged that it was valuable that children could control the intensity of their play, such as in play fighting situations, as well as using current knowledge for exploration of past or future experiences.

Nevertheless, there are differences between the lists. In Fromberg’s (1999) review, she emphasised the social aspects of play, “for young children, play is a way to strengthen worthwhile, meaningful learning and co-operation with others rather than merely acquiring facts alone” (p. 45). In Holton et al.’s (2001) mathematical play, the focus is on the individual solver of problems and the important role of social interactions in research mathematicians’ problem solving is not emphasised. In Meaney (Meaney, 2005), an exchange between two mathematicians showed how the ways that they interacted allowed them to put forward and discuss the merits of different ideas in what Holton et al. (2001) would label as mathematical play. It therefore seems somewhat problematic to have a definition of mathematical play that focuses only on the role of the individual problem solver.

Mathematicians, as belonging to a shared social situation, continuously negotiate what can and cannot be mathematics. For example, theoretical computer science was initially an area within mathematics but was not considered “mathematical enough”. In recent times, there have been indications that other areas of applied mathematics is also being pushed out of mathematical departments (Osgood, 1998; Garfunkel & Young, 1990).

The final approach to considering the relationship between play and mathematics is that of Bishop (1988) who considered playing to be one of six mathematical activities that all cultures engaged in, with both adults and children as possible participants. He considered the activities to be processes that lead to the development of mathematics. For him playing provided an answer to how mathematics is done. This is in contrast to a focus on mathematical topics as products, such is nominated in a curriculum like “number, measurement, geometry and language/logic” (p. 23). Playing is the social procedures and rules of performance, “the ‘as if’ of imagined and hypothetical behaviour” (p. 24). Consequently, he described the features of play as:

- to imagine something—which is the basis for thinking hypothetically and beginning to think abstractly
- to model—which means abstracting certain features from reality
- to formalise and ritualise rules, procedures and criteria
- to predict, guess, estimate, assume what could happen
- to explore numbers, shapes, dimensions, positions and arguments (i.e. engage in the other five mathematical activities in playful ways)

Although Bishop’s six activities have been used in regard to research in to preschool mathematics (Macmillan, 1995; Macmillan, 1998; Flottorp, 2011; Johansson, Lange, Meaney, Riesbeck, & Wernberg, 2012; Helenius, Johansson, Lange, Meaney, Riesbeck, & Wernberg, 2014 forthcoming), there has been little discussion about playing as a mathematical activity. In Macmillan’s (1998) research, play is seen as the situation in which children participate, with no explicit mentioning of playing as a mathematical activity. The closest that the discussion came was in descriptions of episodes which involved a “play on words” (p. 60) and where the children negotiated and regulated the play situation. In her 1995 article, Macmillan summarised Bishop’s (1988) description but did not operationalise it in regard to her data. Although Johansson et al. (2012) and Helenius et al. (2014 forthcoming) identify and describe examples of playing as a mathematical activity, the discussion of what counted as playing relied on Bishop’s own definition. Consequently, in this research playing as a mathematical activity was equated with hypothetical thinking, modelling and abstraction. Although there seems to be some overlap with aspects of the two previously discussed approaches, playing as a mathematical activity has been used less in empirical research.

In the next section, we synthesise the features connected to the different approaches that link mathematics to play, before using those features to analyse an interaction between a group of 6 year old children.

THE FEATURES OF PLAY WHICH ARE MATHEMATICAL

In order to determine the features that could contribute to play being considered mathematical, we identified what is common across the approaches. As is outlined in the next sections, we group the common features as: creative, participatory and rule

negotiation. Often, in putting the groups together, we started from the features that Bishop's provided for play as a mathematical activity as it seemed to show more explicitly the links between the features of play and the features of mathematics.

Our expectation is that in play, both children's and adults', these groups of features are inter-related and it is the inter-relationship that contributes to mathematical ideas being developed, perhaps for the first time by mathematicians or as a reproduction of culturally-valued mathematics by children (see Figure 1). Our argument is that unless aspects of all three components are in evidence then the activity is neither playful nor mathematical.

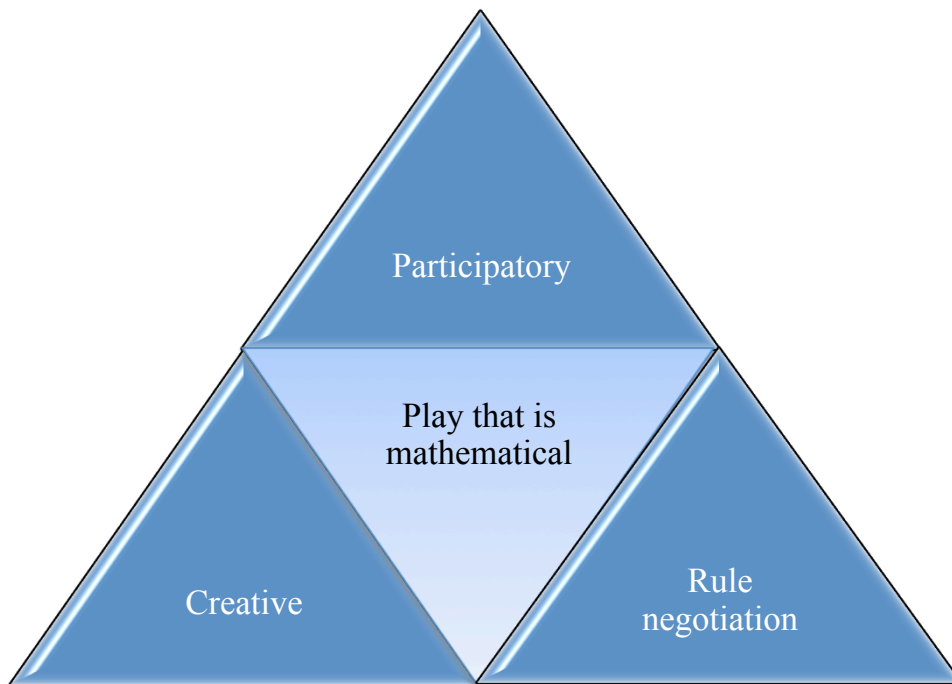


Figure 1: The interrelationship between groups of features that contribute to play being considered mathematical

Before discussing each group of features, we describe how the inter-relationship is connected to the historical development of mathematical ideas. Although mathematics can be understood philosophically in many ways (Ernest, 1991), one approach is to see it as a humanistic enterprise, intelligible only in a social context (Hersh, 1997). Creating new mathematical ideas, such as the introduction of negative numbers, requires old mathematical truths to be abandoned or reinterpreted through rule negotiation. It took several hundred years of social negotiations among mathematicians before negative numbers were accepted as a creative solution to certain kinds of mathematical problems. By participating in these negotiations, mathematicians eventually agreed on which rules needed changing and which rules could stay the same. Imre Lakatos' book *Proof and Refutations* (1976) describes how mathematics as a discipline can keep its internal coherence despite regularly and repeatedly dealing with new "rule breaking" entities. Part of this comes about

because mathematicians accept what it means to participate in both local debates but also in the wider community of practising mathematicians, as determined at the time.

Mathematics is a complicated activity that can be described in many ways. Therefore, we are not suggesting that viewing play as being mathematical is a better way to understand mathematics in general. Our point is that many elements of doing mathematics could be described as play and that in deliberating whether young children's play should be considered mathematical it is relevant to consider how this is related to what mathematicians do. For example, as discussed in the following section, young children's play often includes creatively posing problems and finding solutions that involve rule negotiation.

Creative

In the earlier discussion, it seems that both playing and doing mathematics include being creative and thus a creative element is necessary for play to be considered mathematical. Fromberg (1999) talks about play's symbolic nature in that it is a representation of reality. Posing problems and finding solutions, which were recognised by Bergen (Bergen, 2009) as linking children's play to the work of scientist, mathematicians and engineers, can be considered expressions of the "as .. if", "what ... if" aspects identified by Fromberg (1999). Similarly, Bishop (1988) suggested that hypothetical thinking grows out a requirement for play to be distanced from reality. Play models reality by referencing to it, but not including all aspects of it. This allows certain issues to be explored imaginatively, without the constraints that reality might require. Holton et al. (2001) acknowledged that it is the solver of the problem who is in charge of the solution process and thus can determine the features that should be considered as integral for solving it. Fromberg's (1999) definition of play included that it was voluntary and intrinsically motivated, often because children were curious about something and this encouraged them to continue their play. When play occurs over a long period of time, participants can change the focus or problems that are being explored as other ideas become more interesting.

Therefore, an operational description of what creative aspects make play mathematical should include these features:

- Play models a situation that includes some, but not all aspects of reality
- The participants determine or accept the altered reality through playing
- Playing involves posing and solving problems that they set themselves

Participatory

Mathematics has long been recognised as a cultural activity and thus as constructed by groups of people (Bishop, 1988). Participating in mathematics involves posing or solving a problem that others would recognise as mathematics. When solving an individual mathematical problem, participants cannot do anything that they like, instead they must agree to abide by the rules of mathematics (Holton et al., 2001). However, young children are unlikely to know the rules of mathematics so for play to

count as mathematical, there must be abiding by group negotiated rules, but these may not necessarily be about mathematical content knowledge per se.

In discussing playing as a mathematical activity, Bishop indicated that participation involves agreeing to suspend normal reality in order to take on the specific reality of a particular play situation. However, because the play situation models some aspect of reality, even if done in an imaginative way, there is also a recognition that participation is both at the local level of the immediate situation but also at the societal level which determines the rules and values that affect immersion in reality. As participants move backwards and forwards between the two levels, play situations are adapted when new problems become of interest. The acceptance of play situations as allowing modelling of the participation that occurs in real situations contributes to participants predicting, guessing, estimating, and making assumptions about what could happen within its altered reality. Thus, participation ties the imaginative, creative aspects of play to reality. Playing being Martians living on Mars can only be done by basing that play on what is known about being humans on earth.

For play to be considered a mathematical activity, either implicitly or explicitly, participants need to acknowledge that their participation occurs both within the play situation and as part of the wider societal reality. Therefore, it should include the following features:

- Participants showing an awareness that their participation depends on others recognising that they are acting acceptably
- That participation is at both the local and societal level

Rule Negotiation

Fromberg (1999) suggested that play was rule-governed, although as was the case with the earlier example from Perry and Dockett (1998) of the girl putting a case that she was the mother, the rules were implicit rather than explicit. As discussed in the previous section, Bishop (1988) acknowledged that during play participants have to agree to the suspending of some aspects of reality. If participants do not agree to this rule, then play cannot occur. Nonetheless, rules can be changed as the play situation develops but only if all participants agree to the changes, although possibly under threat of being excluded from the play. Rule negotiation is an essential component of play.

Although school students often regard mathematics as just a set of rules (Wong, Marton, Wong, & Lam, 2002), the rules have been agreed to as a result of consensus amongst mathematicians, as was described previously. Indeed as van Oers (2001) stated:

It is not the link with meaningful problem situations as such that defines the nature of 'real' mathematics, but the observance of particular rules, the use of particular concepts and tools, the engagement with certain values that define whether one is doing mathematics or not. (p. 71-72)

Like Fromberg (1999), Bishop (1988) identified rules as being a component of play. He considered that when play became incorporated into games, there is a formalising and ritualising of rules, procedures and criteria. This also contributes to the use of strategies, as the focus shifts to winning the game. Often strategies require logical thinking, similar to that used in mathematics, if a player wants to maximise their chances of winning (Holton et al., 2001).

For play to be considered mathematical, it must include all or most of the following:

- Participants must abide by the implicit or explicit rules of the play
- For rules to change, there needs to be negotiation by participants
- Negotiating the rules contributes to forming the boundaries of the play situation and thus what aspects of reality can be suspended and what aspects are modelled in what ways.

METHODOLOGY

To ascertain if the criteria in the previous sections contribute anything meaningful to determining if a play situation could be labelled mathematical, it is important to use them with some empirical material. Therefore, we have chosen a short video to analyse which is just over three and a half minutes long, in which a group of six-year olds, attending a Swedish preschool class engage in free play. Children do not begin school in Sweden until they are around seven years old, but the year before they attend preschool class which is usually situated in the school that they will attend. The preschool class is considered a bridge between preschool and school. The videos were collected as part of wider project investigating what mathematics is in preschool, with preschool class providing a contrast to this.

The videos were collected from the same preschool class over several months but this was the only video that included free play. The extract was chosen because although the children discuss numbers as part of the buying and selling in their game, it is not immediately clear that this example would fulfil all the criteria that we outlined. Analysing such a situation using our criteria would indicate whether the criteria worked in the complex environments in which play generally occurs with young children.

In the extract four children, three boys and a girl, play with different Lego constructions. The situation is somewhat chaotic as the children leave and come back and move in and out of different storylines. Stills from the video are provided with the transcripts in order to clarify what was occurring. The transcripts are provided in the original Swedish with an English translation.

In the following sections, we discuss extracts of the video to explore which can be classified as mathematical and what this tells us about the play situation.

TO BE OR NOT TO BE MATHEMATICAL

Rather than considering situations as either being mathematical or not being mathematical, it is more valuable to consider whether an interaction was more or less

mathematical. This allows the analysis to consider how different combinations of criteria appear in the play and what this tells us about the play situation.

Buying a popsicle



Figure 2: Negotiating to buy a popsicle

The video begins with Teo wanting to buy a pretend popsicle (piggelin), something which would be quite cheap with some pretend money (kroner bills). Figure 2 shows Teo (left) showing Tom (right) the money that he has.

Teo:	Får man köpa nåt här?	Can I buy something here?
Patrik:	Nej.	No.
Teo:	Men får jag ändå köpa nåt?	But can I still buy something?
Tom:	Men var är hundralapparna?	But where are the hundred kroner bills?
Teo:	A men jag har bara såna här pengar, men kan jag, kan jag få köpa något?	Ah, but I only have this kind of money, but can I, can I buy anything?
Tom:	Ja	Yes
Teo:	Jag vill köpa piggelinen.	I want to buy the popsicle.
Tom:	Den kostar alla dom.	It costs all of those.
Teo:	Nä inte alla mina pengar.	No, not all my money.
Tom:	Jo, den kostar allt det.	Yes, it costs all that.
Teo:	Nä!	No!

In this episode, the children are being creative in regard to modelling the reality that they want to explore. One of the Lego pieces, a blue plastic pole comes to take on the role of a popsicle. The pretend money is also allowed to take the role of real money, although Tom queries why there are only 3 and 4 kroner bills and not hundred kroner bills. This suggest that Tom does not see these smaller bills as being sufficient for the kind of buying and selling activity that he wants to model. Nonetheless, this situation fulfils the criteria of being mathematical, in that in models a real situation but does

not try to include all aspects of a real buying and selling situation. Although Patrik initially does not accept that Teo can buy anything, Tom's continual engagement with Teo suggest that these two boys at least have accepted the conditions of this being a buying and selling exchange. The problem of what does a popsicle costs is one that the children set up and try to resolve themselves.

In regard to the criteria for participation, it seems that Patrik withdraws from this game because he did not want to sell any of the Lego constructions. Teo, however, was able to continue in this play because of Tom's willingness to interact under the conditions invoked by Teo's desire to buy something with his money. Teo's querying of the need to give Tom all of his kroner bills draws on his understanding of the real world, in which popsicles rarely cost all of the money that his parents might have in their wallets. Thus he moves between the local play situation and his understanding of the real world.

The querying by Tom of the kind of money that Teo has as well as Teo's querying of Tom's definition that the popsicle costs all of Teo's money shows some negotiation about what rules should apply in this situation. However, the basic premises that a blue plastic Lego pole could stand for a popsicle and that it could be bought with pretend kroner bills are not queried. Therefore, the boundaries of the play situation remain in place.

Klara's chocolate

Klara returned to the group with a small brown plastic block. She then showed it to the boys (see Figure 3) and said "Have you had the chocolate?" (Har ni haft chokladen?). Tom said, "No," before returning to his conversation with Teo. A few turns later, Klara tried again to attract the boys' attention by saying "Check this out a small chocolate" (Kolla in detta då en liten choklad) but is not successful in having the boys take up this alternative possibility for the play. She made one last attempt to gain the boy's interest before dropping this discussion.



Figure 3: Klara showing her chocolate to Tom

Although Klara seemed to have been just as creative as Tom and Teo in turning a plastic block into a chocolate, she did not get an opportunity to present a problem

because this situation was not accepted by the others. Therefore, there was no joint participation and no rule negotiation, so the situation did not fulfil the criteria of being mathematical– or even play.

The car

Having been unsuccessful in changing the direction of the play by introducing the chocolate, Klara tried to enter the buying and selling play situation by offering first a trade and then by supporting Patrik’s offer to buy the car. Although there was some interest in her offers, they were eventually rejected as not being appropriate. Figure 4 showed Klara and Patrik’s interest in the car.



Figure 4: Klara and Patrik showing strong interest in the car

Patrik:	Bilen.	The car.
Klara:	Byter ni den här bilen mot alla de här och mitt bygge?	Do you want to trade this car against all of these and my construction?
Patrik:	Det här är inte vårt.	This is not ours.
Klara:	Jo den här delen är min.	Yes, this part is mine.
Tom:	Oh, kolla vad häftigt! Om man snurrar på denna, så snurrar de här däcken.	Oh, check this out, so cool! If you spin this one, the wheels spin.
Klara:	Kolla!	Check this!
Patrik:	Den här vill jag köpa.	I want to buy this.
Tom:	Men var är pengarna då?	But where is the money then?
Klara:	Här i ...	Here in ...
Tom:	Nä där är dom inte!	No, they are not there.

In this episode, the children do not seem to be able to enter the virtual reality of the buying and selling situation that Tom and Teo had been in. Ownership of the

constructions, determined in the real world by who had built them, thwarts Klara's efforts to make a trade. She returned to the facts about what had been her contribution to the construction to support her claim that she could do the trade. Patrik also had no success in convincing Tom that he had a legitimate right to enter into the buying and selling play situation. Although the money was only play money, Teo held on to it and Patrik had nothing else which was accepted to be an appropriate model for real money. Thus, in spite of the fact that there was some joint participation, there was no seamless merging between the local and societal levels of the modelling. Tom positioned himself as the arbitrator of what was acceptable for the play situation and judged that Patrik and Klara's suggestions could be ignored. Thus, this situation could also not be accepted as mathematical—or play.

Successful negotiation

Towards the end of the video, Teo re-entered the discussion with another attempt to buy the popsicle. In the meantime, the teacher has given him some more kroner bills, although still only worth 3 and 4 kroner each. At one point, he copied Klara's trading attempt by picking up a small blue Lego block, labelling it as a popsicle and then trying to use it to exchange for the pretend popsicle that he wanted from the start. Eventually, Tom accepted the deal but took Teo's money to count out the 40 000 kroner that he said it would cost.



Figure 5: Teo with his popsicle trade and Tom counting out the 40 000 kroner

Teo:	Men, ok, jag vill köpa piggelinen för de här.	But, ok, I want to buy the popsicle for these.
Tom:	Nej, nej, nej.	No, no, no.
Teo:	Här så får jag piggelinen	Here so I will get the popsicle.
Tom:	Varför måste du ha piggelinen, då dör du ju	Why do you have to have the popsicle, then you will die.
	(ohörbart)	Inaudible
Tom:	Vad köper du?	What do you buy?

Teo:	Piggelinen.	The popsicle.
Patrik:	Men då blir ni också sjuka.	But then you will get sick as well.
Teo:	Men kan jag betala med den här piggelinen	But can I pay with this popsicle?
Tom:	Det där är fyrtiotusen, I så fall får du betala, vänta de här också.	That is forty thousand, in that case you will have to pay, wait those too.
Teo:	Men	But
Tom:	Jo, väldigt mycket kostar det alltihop.	Yes, it cost very much all of that.
Teo:	Ni är så elaka varför måste det kosta så mycket?	You are so mean, why does it have to cost that much?
Tom:	Den kostar inte alls mycket den kostar bara en sån där.	It doesn't cost that much only one of these.

As with the earlier episode, Tom and Teo built up the play situation in which Teo wanted to buy a popsicle, first with the money he had and when this initially did not work, to do a trade. Unlike the situation with Klara and Patrik, Tom seemed to accept that as possible. Although initially he rejected the sale and trade offers, a reference to the real world, by Teo labelling him as “mean”, meant that he did accept Teo’s money as being equivalent of the 40 000 kroner he had demanded as payment. Thus the situation was creative and accepted by the two main participants. Although Tom attempted to change the play situation by suggesting that Teo would get sick if he ate the popsicle, something that Patrik also supported, Teo ignored this suggestion. Thus, this offer to renegotiate the play situation was rejected by one of the main characters, Teo. Unlike the previous episode, this rejection did not affect the possibilities to continue playing; instead it strengthened the boundaries of what was and what was not acceptable for this play situation. The criteria outlined in the previous section are all met within this episode, at least for Teo and Tom, suggesting that for this episode of play could be considered mathematical.

CONCLUSION

In this paper, we present the case that play sometimes has the characteristics which could deem it to be mathematical. This is not to say that mathematics is play, rather that some of the characteristics of playful situations allow those situations to be classified as mathematical. This is important because mathematics education in preschool, if mentioned at all, concentrates on content knowledge. Generally, play is not recognised as having anything in itself which could add to children’s mathematical understandings. For example, Lee and Ginsburg (2009) wrote:

Children do indeed learn some mathematics on their own from free play. However, it

does not afford the extensive and explicit examination of mathematical ideas that can be provided only with adult guidance. ... Early mathematics is broad in scope and there is no guarantee that much of it will emerge in free play. In addition, free play does not usually help children to mathematise; to interpret their experiences in explicitly mathematical forms and understand the relations between the two. (p. 6)

Based on research with mathematicians, we suggest that the criteria that we have identified about what makes play mathematical indicates that playing can support children to develop the mathematical process understandings. The longitudinal studies on block play described earlier (Wolfgang, Stannard, & Jones, 2001; 2003) showed that some kinds of play done while in preschool could support mathematical achievement in school. However these studies did not clarify exactly what it was about play which was likely to support mathematicians to solve new mathematical problems or children to achieve in mathematics at school, except to identify the block play as complex and adaptive. Therefore, it seemed important to identify criteria that would allow play situations to be classified as mathematical. Our approach of having an integrated set of three kinds of criteria—creative, participatory and rule negotiation—includes the features that were common to Fromberg's (1999) definition of play, research on playful aspects of the work of mathematicians (Holton et al., 2001) and Bishop's (1988) description of playing as a mathematical activity.

Having stated this, like Lee and Ginsburg (2009), we acknowledge the importance of the role of the teacher in children's play. The examples that we provide of 6 year olds playing together suggests that not all children may have the same opportunities to engage in play that is mathematical. It will depend on the social relationships within the group, amongst other things. Therefore, the teacher has a role in the play so that all children are encouraged to be creative, participatory and contribute to the negotiation of rules. Notwithstanding that more research is needed both to test out the criteria for what makes play mathematical and of a longitudinal nature, we suggest that a teacher's active participation in the play could contribute to children learning more about mathematics. We hypothesise that playing in this way would actually contribute to the mathematising valued in Lee and Ginsburg's quote above and lend itself to the mathematical content knowledge being learnt in more meaningful ways.

As indicated in the previous paragraph, we acknowledge that identifying these criteria are only the start of a research programme in to better understanding the contribution that play which is mathematical makes to children's later understandings of mathematics. The criteria for what makes play mathematical needs to be tested with other kinds of play as well as considering how the teacher's role in the play could contribute to more equitable learning opportunities. It is also important to consider how to conduct longitudinal studies to ascertain whether there are benefits for children from engaging in this kind of play in their later lives. With the schoolification of preschools (Alcock & Haggerty, 2013; Garnier, 2012; Sofou & Tsafos, 2010), there is a real need to know whether the reduction of play in preschools will be detrimental to children's mathematical learning, rather than

valuable as politicians and policy makers suggest. We do not want to be “throwing the baby out with the bathwater”, so that increasing the amount of formal mathematics education that children receive in preschool actually decreases their interest and actual learning of mathematics.

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