# Notes 01
# Introduction to Power Electronics

Marc T. Thompson, Ph.D.
Thompson Consulting, Inc.
9 Jacob Gates Road
Harvard, MA  01451
Phone: (978) 456-7722
Fax:  (240) 414-2655
Email: marctt@thompsonrd.com
Web: http://www.thompsonrd.com

# Introduction to Power Electronics

- Power electronics relates to the control and flow of electrical energy
- Control is done using electronic switches, capacitors, magnetics, and control systems
- Scope of power electronics: milliWatts $\Rightarrow$ gigaWatts
- Power electronics is a growing field due to the improvement in switching technologies and the need for more and more efficient switching circuits

# Summary

- History/scope of power electronics
- Some interesting PE-related projects
- Circuit concepts important to power electronics
- Some tools for approximate analysis of power electronics systems
- DC/DC converters --- first-cut analysis
- Key design challenges in DC/DC converter design
- Basic system concepts

# Scope of Power Electronics

| Power Level (Watts) | System |
|---|---|
| 0.1-10 | • Battery-operated equipment<br>• Flashes/strobes |
| 10-100 | • Satellite power systems<br>• Typical offline flyback supply |
| 100 – 1kW | • Computer power supply<br>• Blender |
| 1 – 10 kW | • Hot tub |
| 10 – 100 kW | • Electric car<br>• Eddy current braking |
| 100 kW –1 MW | • Bus<br>• micro-SMES |
| 1 MW – 10 MW | • SMES |
| 10 MW – 100 MW | • Magnetic aircraft launch<br>• Big locomotives |
| 100 MW – 1 GW | • Power plant |
| > 1 GW | • Sandy Pond substation (2.2 GW) |

# Scope of Power Electronics
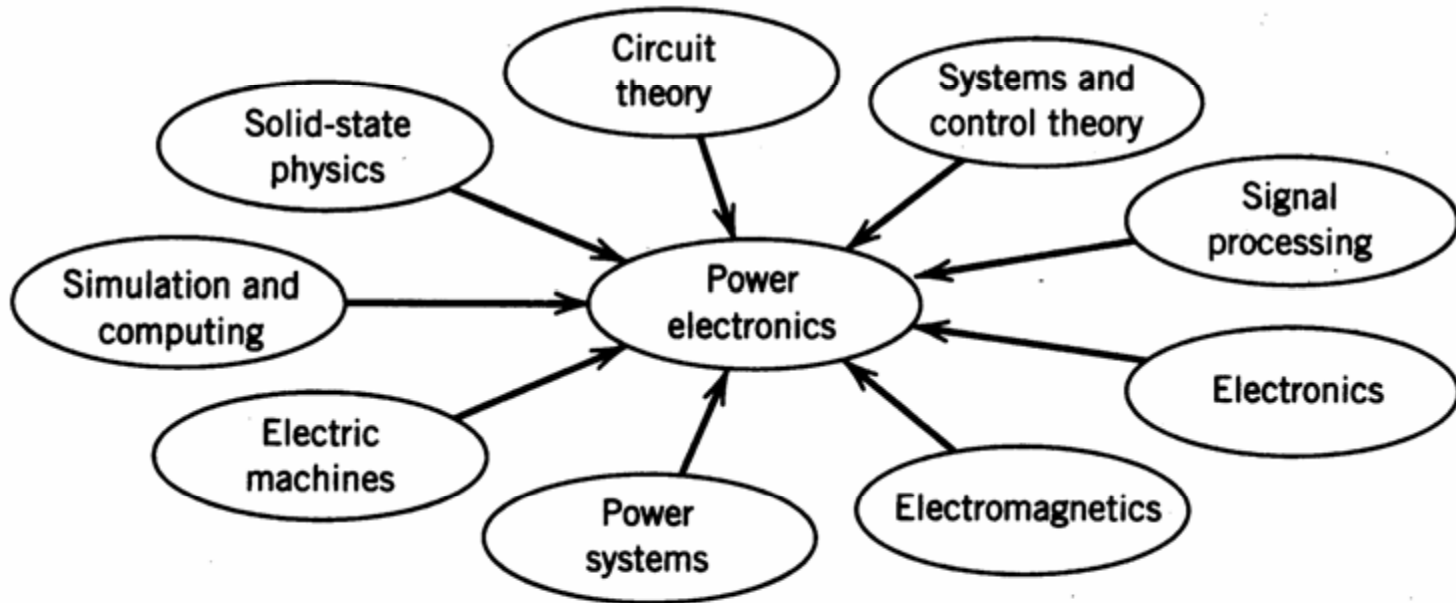


**Figure 1-10**  Interdisciplinary nature of power electronics.

# Areas of Application of Power Electronics

- High frequency power conversion
  - DC/DC, inverters
- Low frequency power conversion
  - Line rectifiers
- Distributed power systems
- Power devices

- Power Transmission
  - HVDC
  - HVAC
- Power quality
  - Power factor correction
  - Harmonic reduction
    - Passive filtering
    - Active filtering

# Some Applications

- Heating and lighting control
- Induction heating
- Fluorescent lamp ballasts
  - Passive
  - Active
- Motor drives
- Battery chargers
- Electric vehicles
  - Motors
  - Regenerative braking
- Switching power supplies
- Spacecraft power systems
  - Battery powered
  - Flywheel powered

- Uninterruptible power supplies (UPS)
- Electric power transmission
- Automotive electronics
  - Electronic ignitions
  - Alternators
- Energy storage
  - Flywheels
  - Capacitors
  - SMES
- Power conditioning for alternative power sources
  - Solar cells
  - Fuel cells
  - Wind turbines

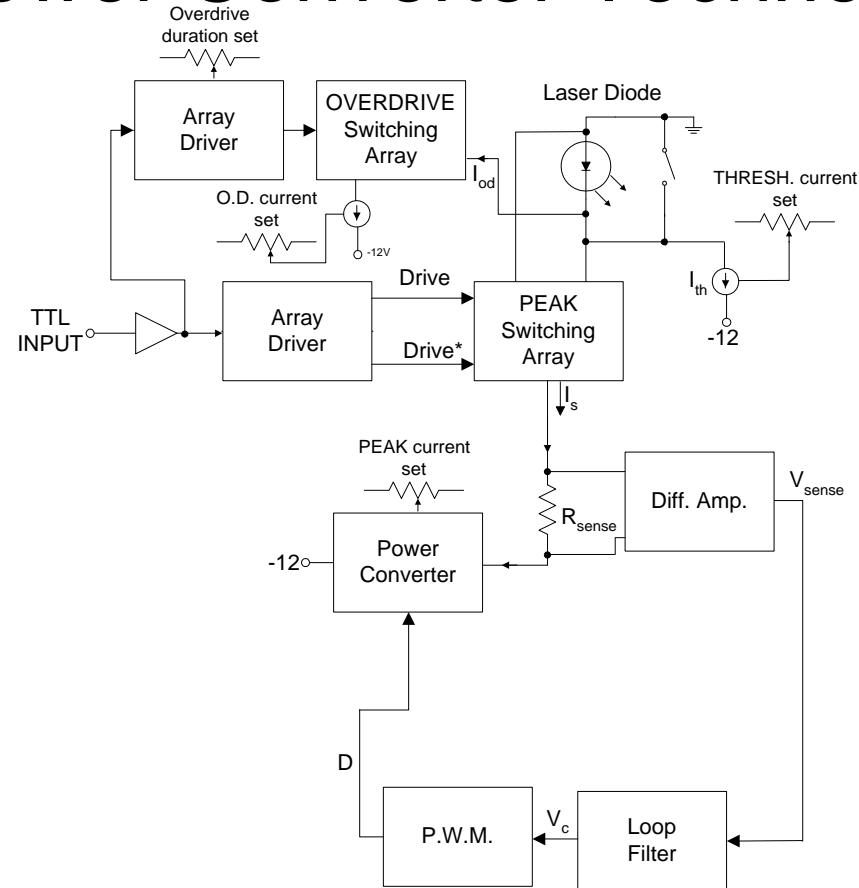# Some Power Electronics-Related Projects Worked on at TCI (Harvard Labs)

- High speed lens actuator
- Laser diode pulsers
- Levitated flywheel
- Maglev
- Permanent magnet brakes
- Switching power supplies
- Magnetic analysis
- Laser driver pulsers
- 50 kW inverter switch
- Transcutaneous (through-skin) non-contact power supply

# Lens Actuator



z

r

Iron

Coil

Back iron

Permanent
Nd-Fe-Bo
Magnet

Lens

Air gap

Introduction to Power Electronics
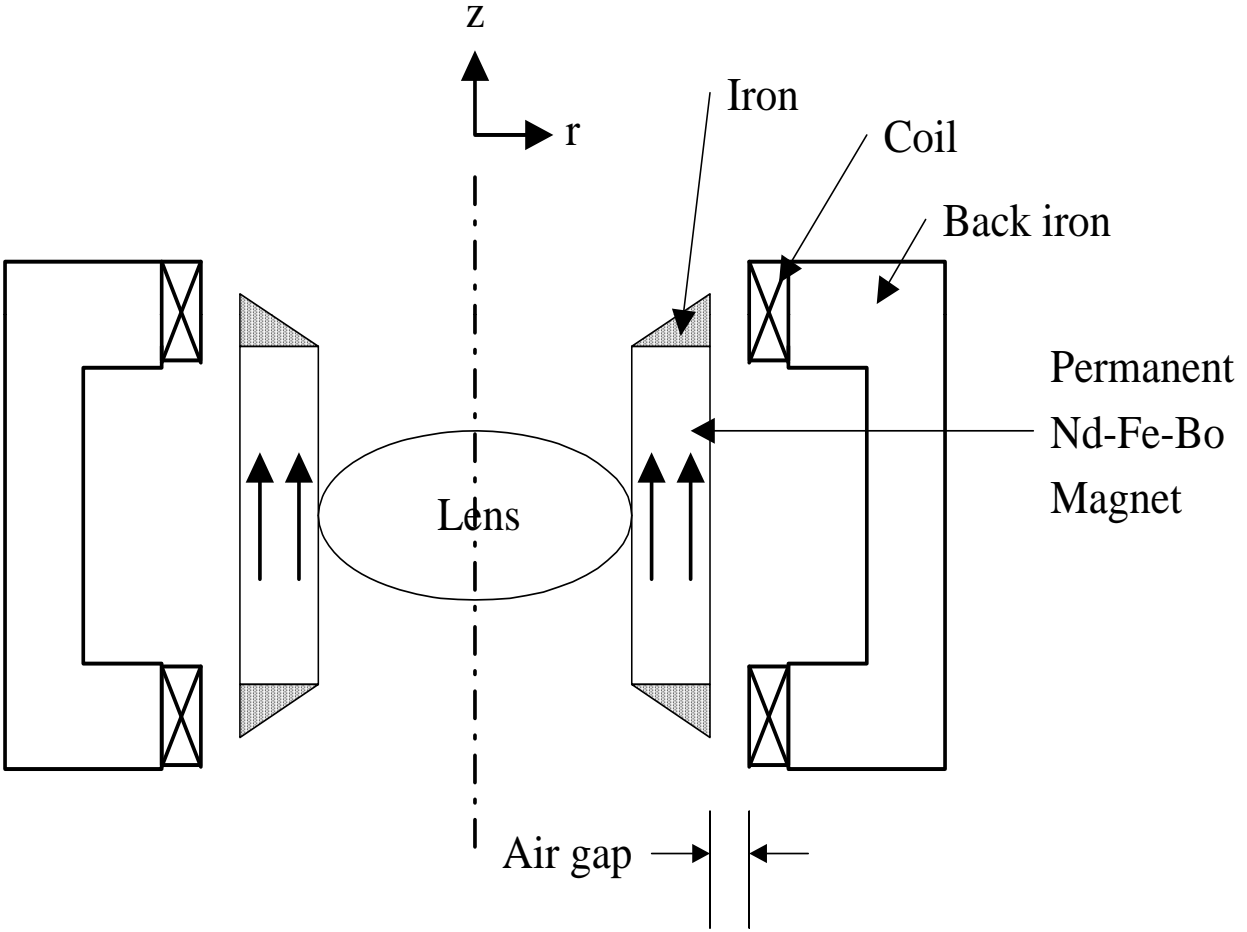
# High Power Laser Diode Driver Based on Power Converter Technology
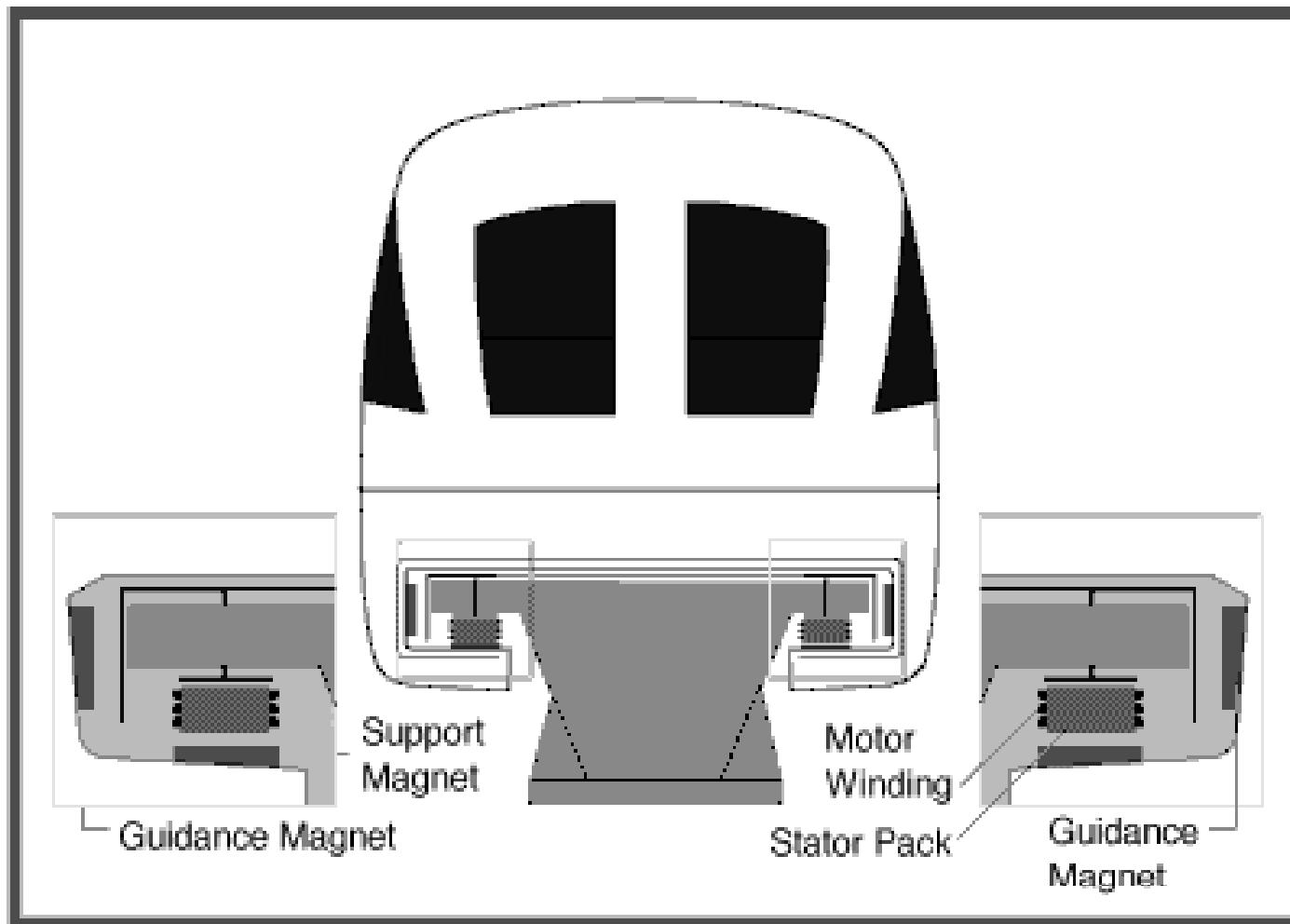


See:
1. B. Santarelli and M. Thompson, U.S. Patent #5,123,023, "Laser Driver with Plural Feedback Loops," issued June 16, 1992
2. M. Thompson, U.S. Patent #5,444,728, "Laser Driver Circuit," issued August 22, 1995
3. W. T. Plummer, M. Thompson, D. S. Goodman and P. P. Clark, U.S. Patent #6,061,372, "Two-Level Semiconductor Laser Driver," issued May 9, 2000
4. Marc T. Thompson and Martin F. Schlecht, "Laser Diode Driver Based on Power Converter Technology," *IEEE Transactions on Power Electronics*, vol. 12, no. 1, Jan. 1997, pp. 46-52

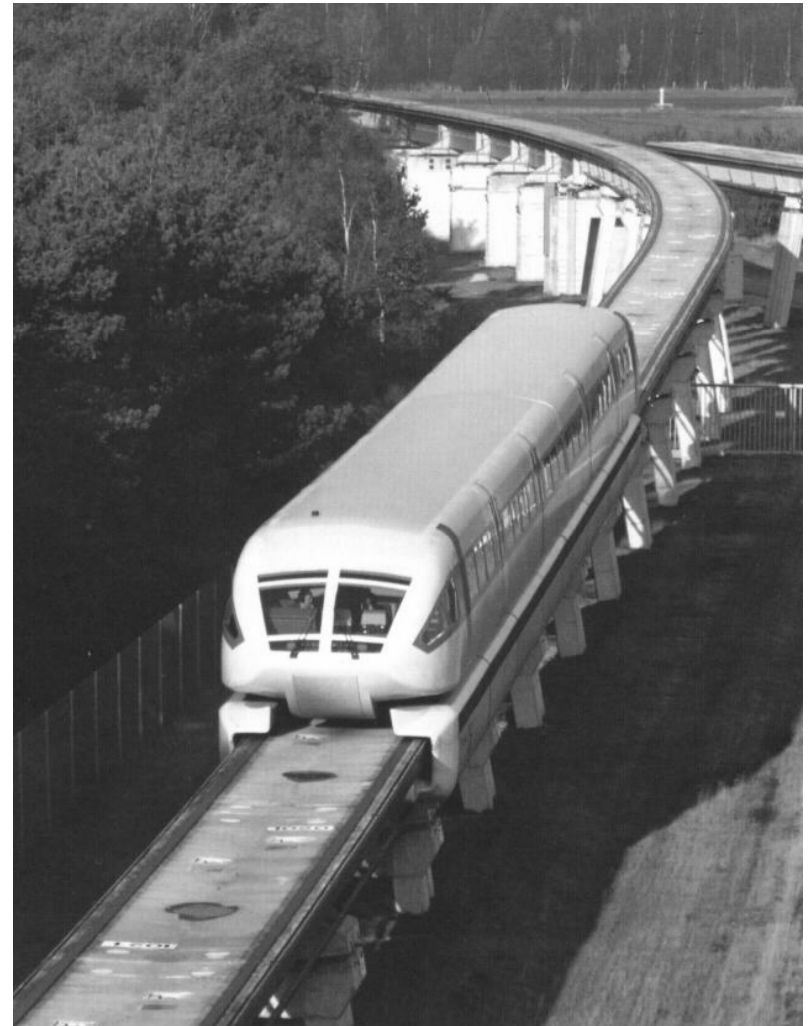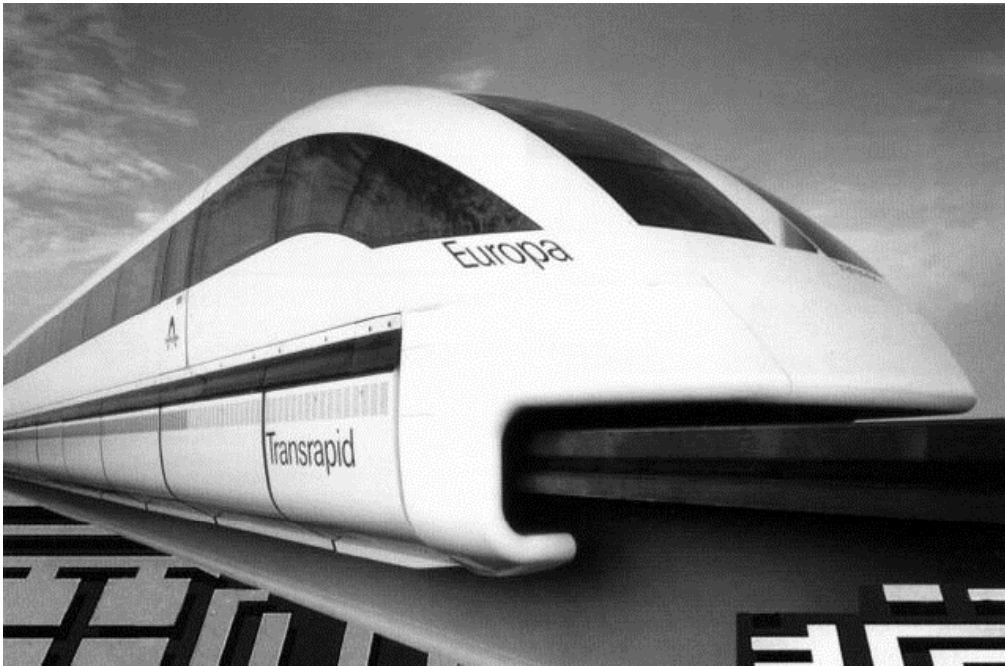# Magnetically-Levitated Flywheel Energy Storage

- For NASA; P = 100W, energy storage = 100 W-hrs

Guidance and Suspension

Flywheel (Rotating)

Stator Winding

S N

N S

N S

N S

N S

S N

N S

S N

S N

S N

S N

S N

N S

z

r
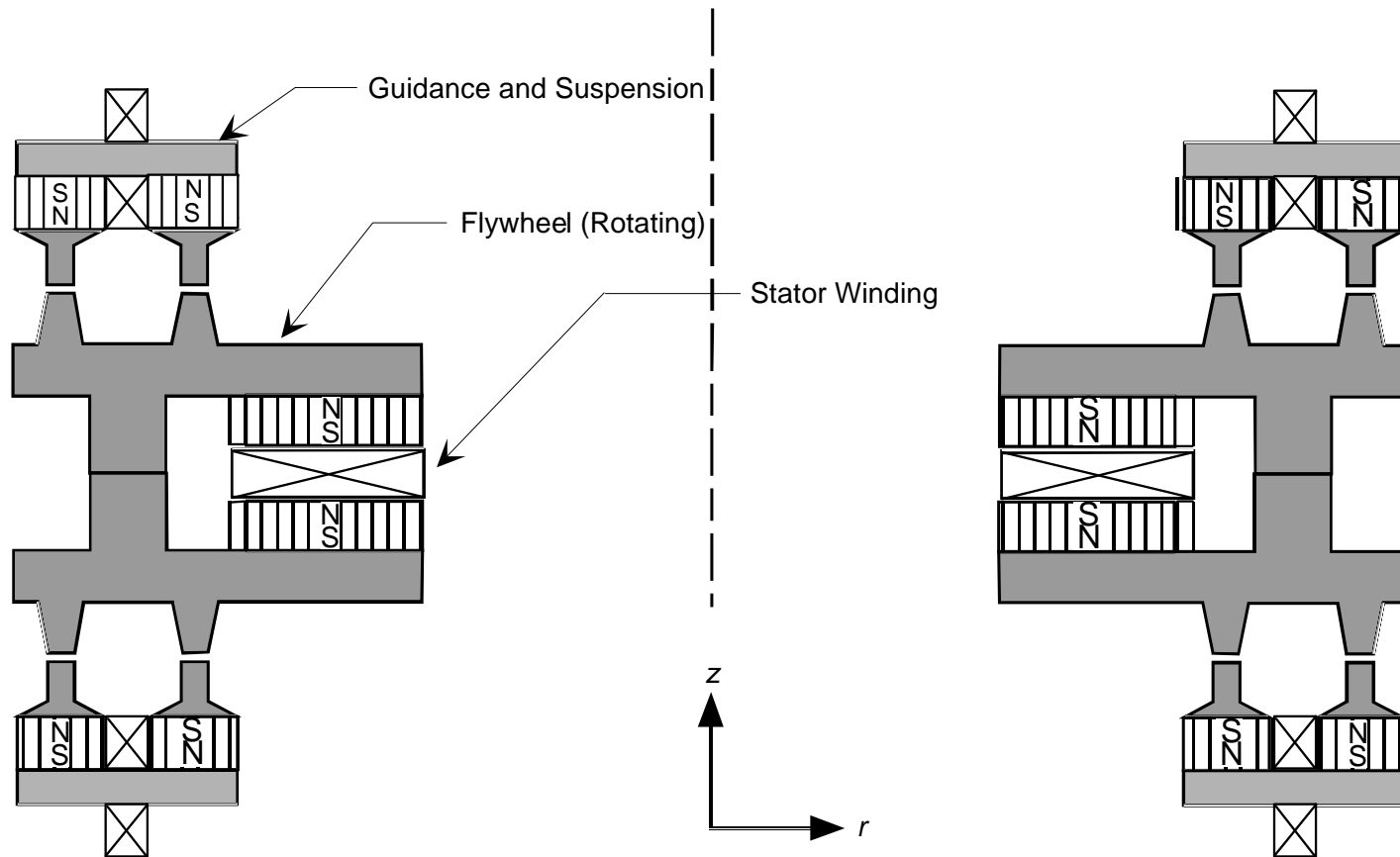
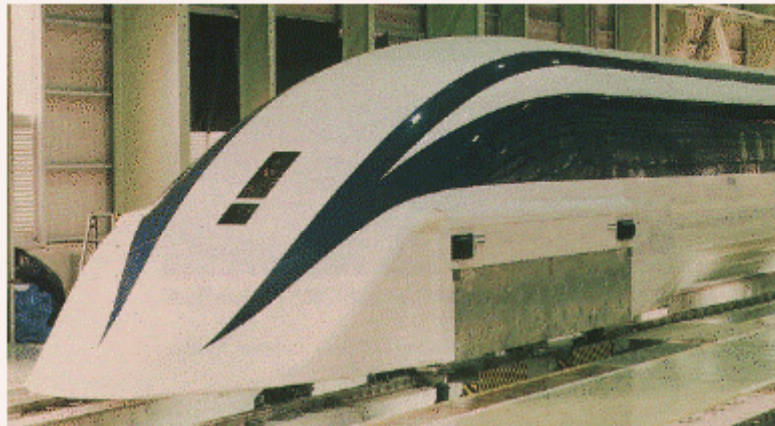# Electromagnetic Suspension --- Maglev
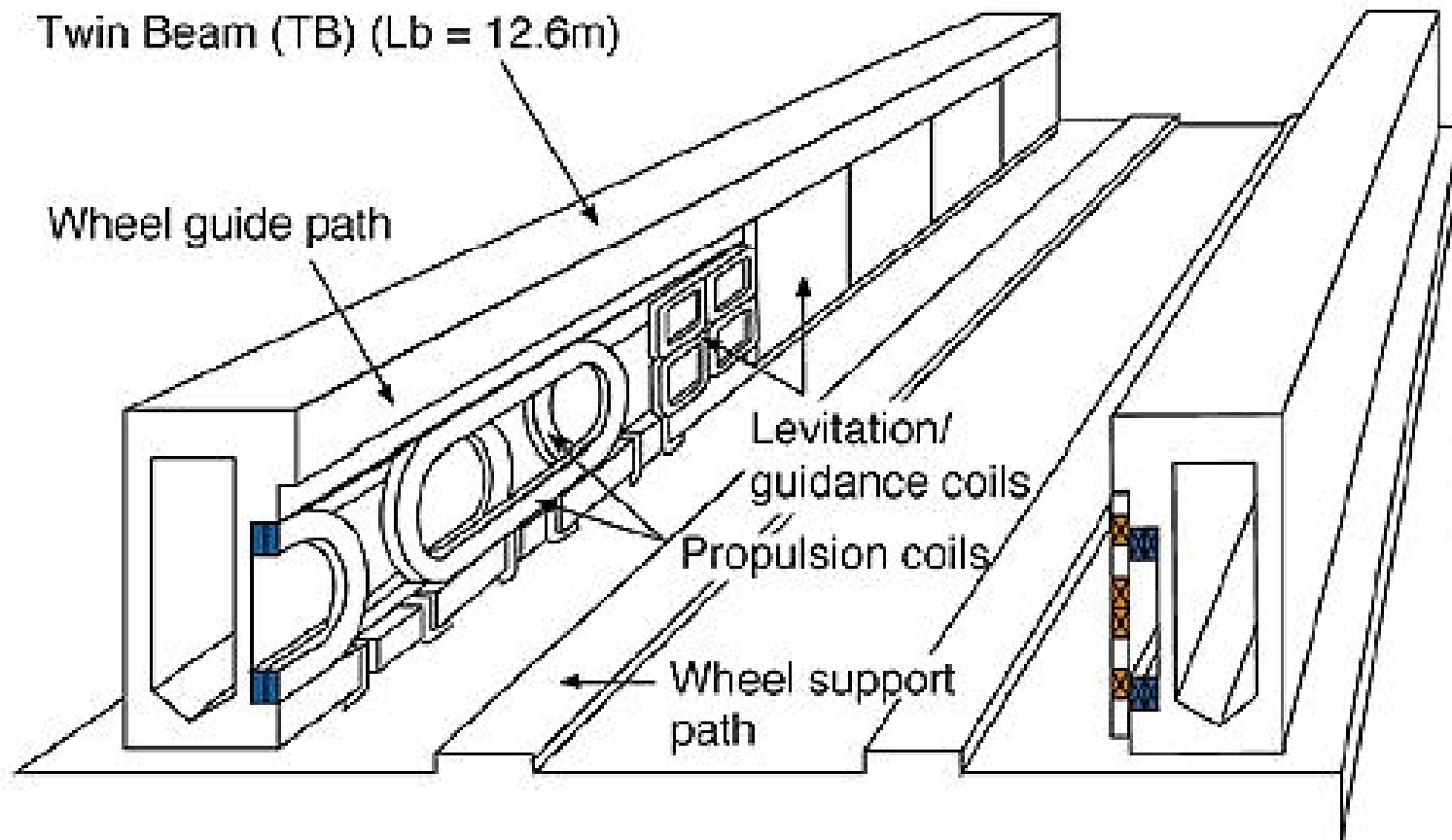
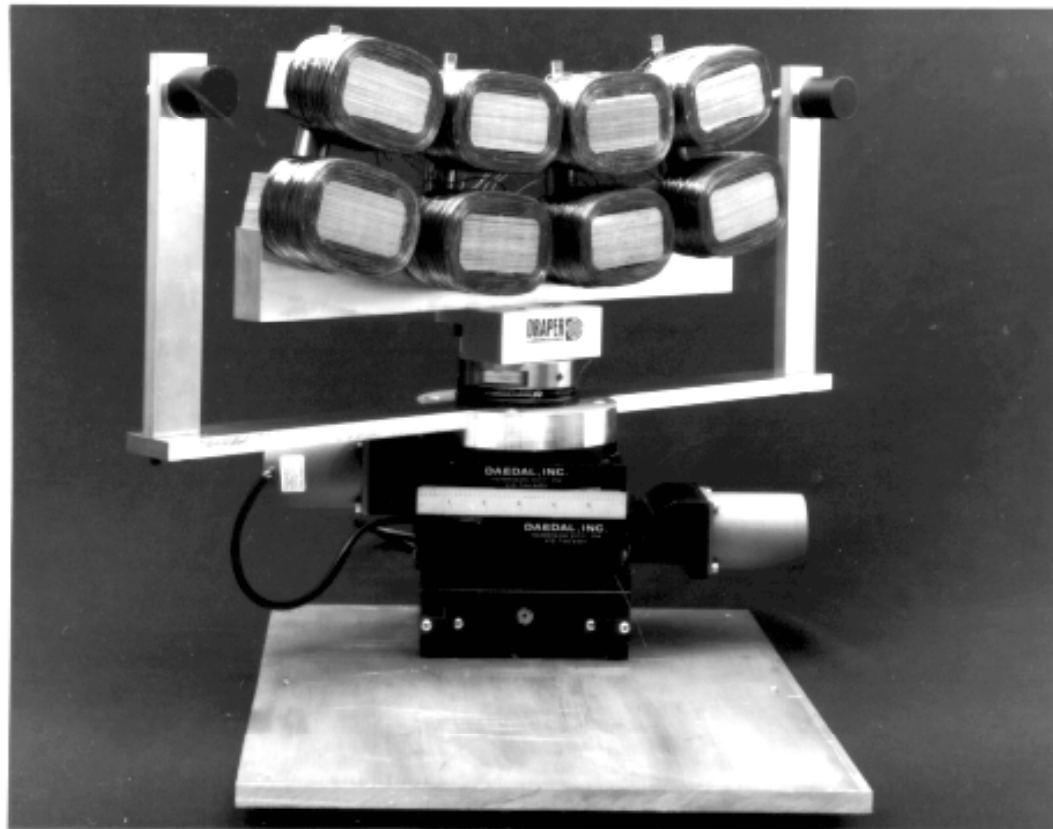# Maglev - German Transrapid

# Maglev - Japanese EDS



Aero-wedge Style

Double Cusp Style

# Japanese EDS Guideway

Twin Beam (TB) (Lb = 12.6m)

Wheel guide path

Levitation/ guidance coils

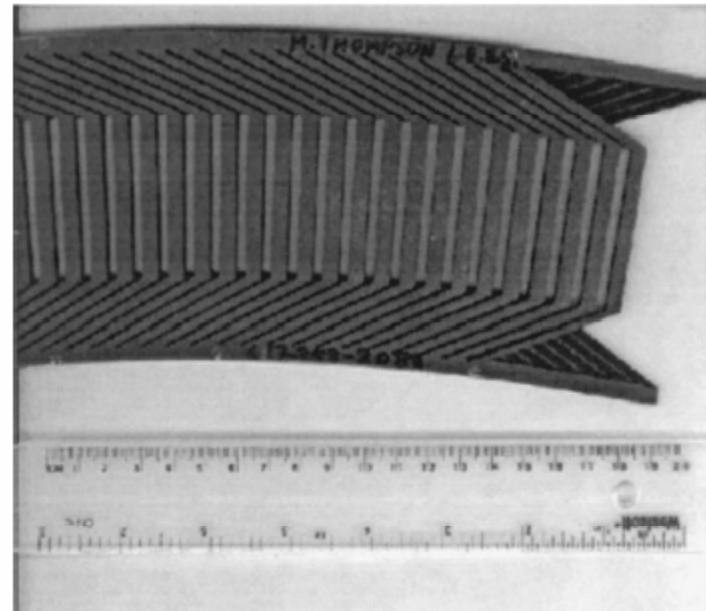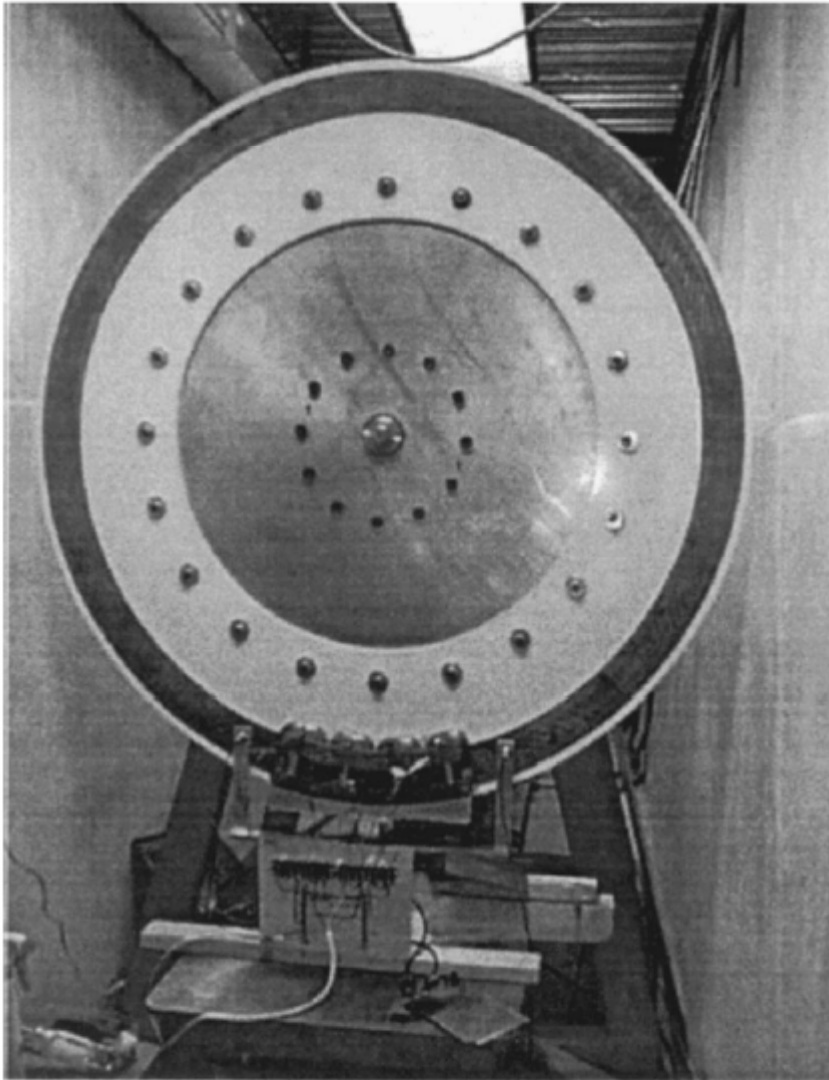Propulsion coils

Wheel support path

# MIT Maglev Suspension Magnet



Reference:  M. T. Thompson, R. D. Thornton and A. Kondoleon, "Flux-canceling electrodynamic maglev suspension: Part I. Test fixture design and modeling," *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999 pp. 1956-1963
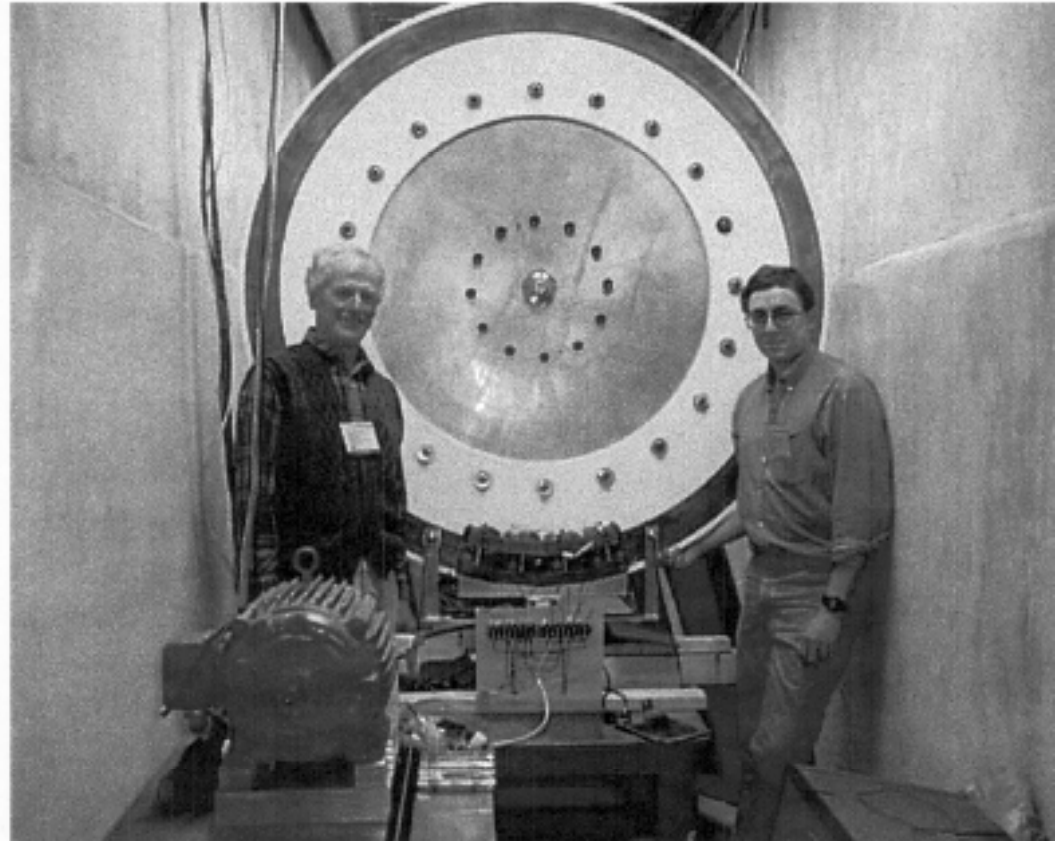
# MIT Maglev Test Fixture





M. T. Thompson, R. D. Thornton and A. Kondoleon, "Flux-canceling electrodynamic maglev suspension: Part I. Test fixture design and modeling," *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999 pp. 1956-1963

# MIT Maglev Test Fixture

- 2 meter diameter test wheel
- Max. speed 1000 RPM (84 m/s)
- For testing "flux canceling" HTSC Maglev
- Sidewall levitation



M. T. Thompson, R. D. Thornton and A. Kondoleon, "Flux-canceling electrodynamic maglev suspension: Part I. Test fixture design and modeling," *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999 pp. 1956-1963

# Permanent Magnet Brakes

- For roller coasters
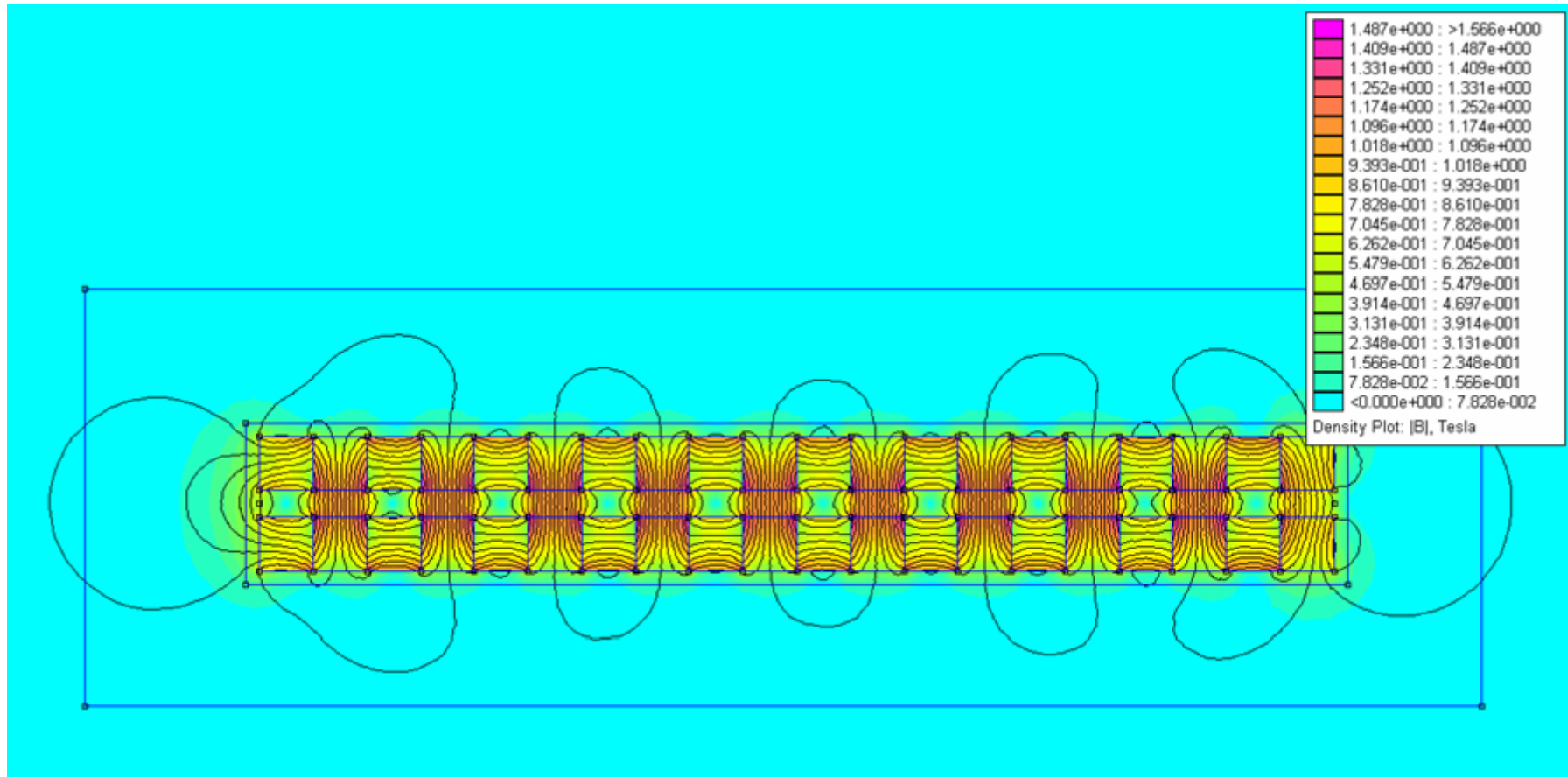- Braking force > 10,000 Newtons per meter of brake





Reference: http://www.magnetarcorp.com
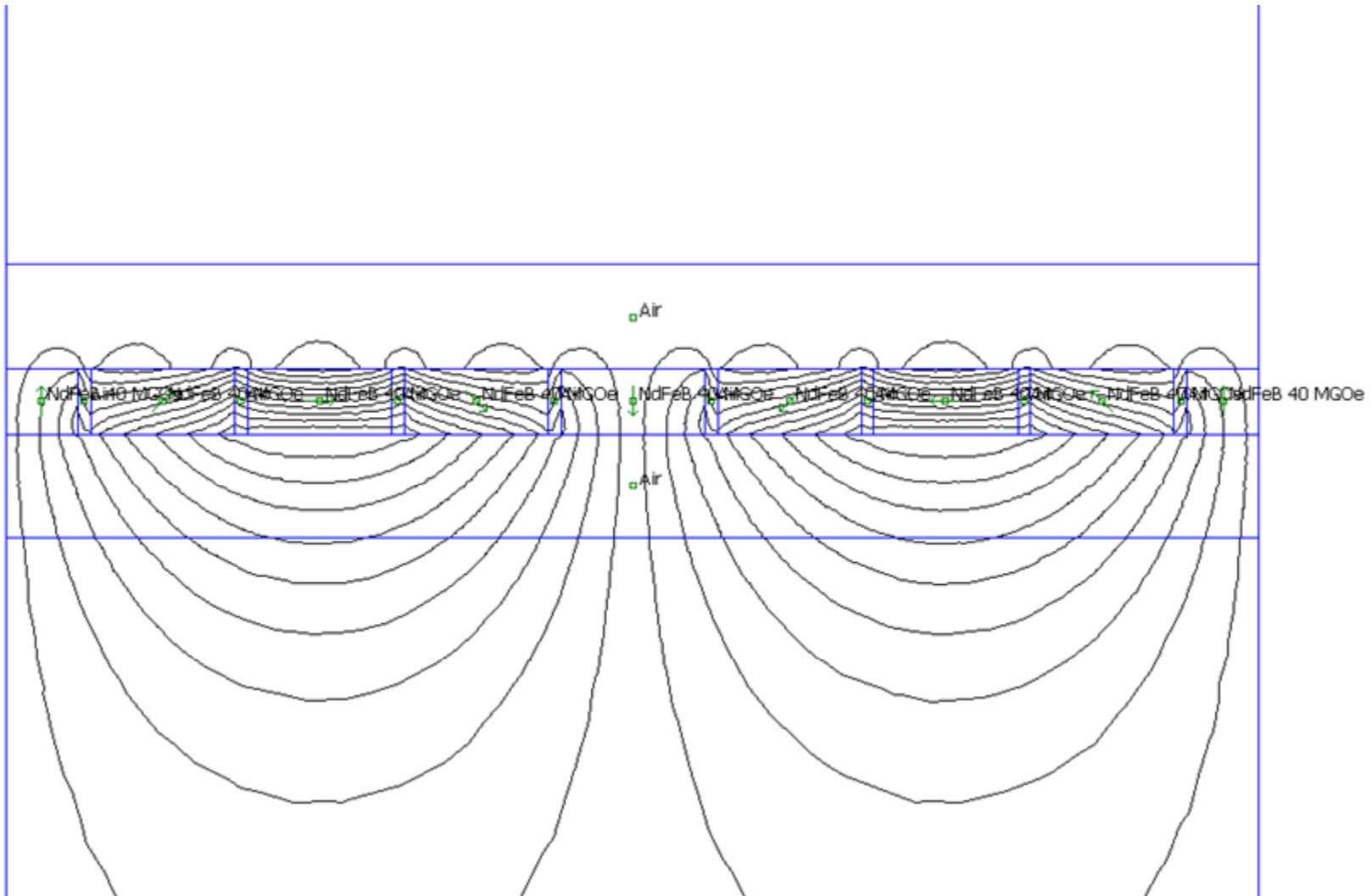
# Halbach Permanent Magnet Array

- Special PM arrangement allows strong side (bottom) and weak side (top) fields
- Applicable to magnetic suspensions (Maglev), linear motors, and induction brakes
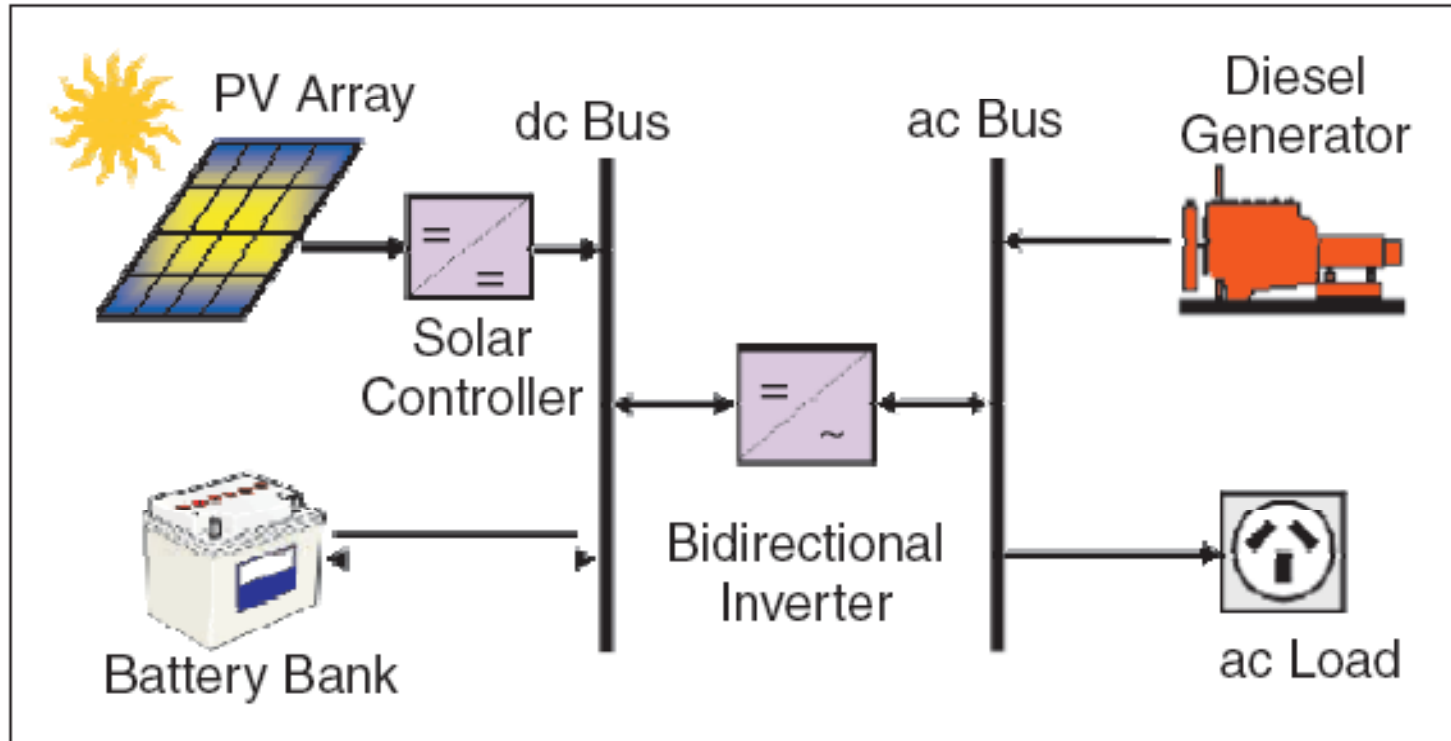
# Halbach Permanent Magnet Array

# Linear Motor Design and Analysis
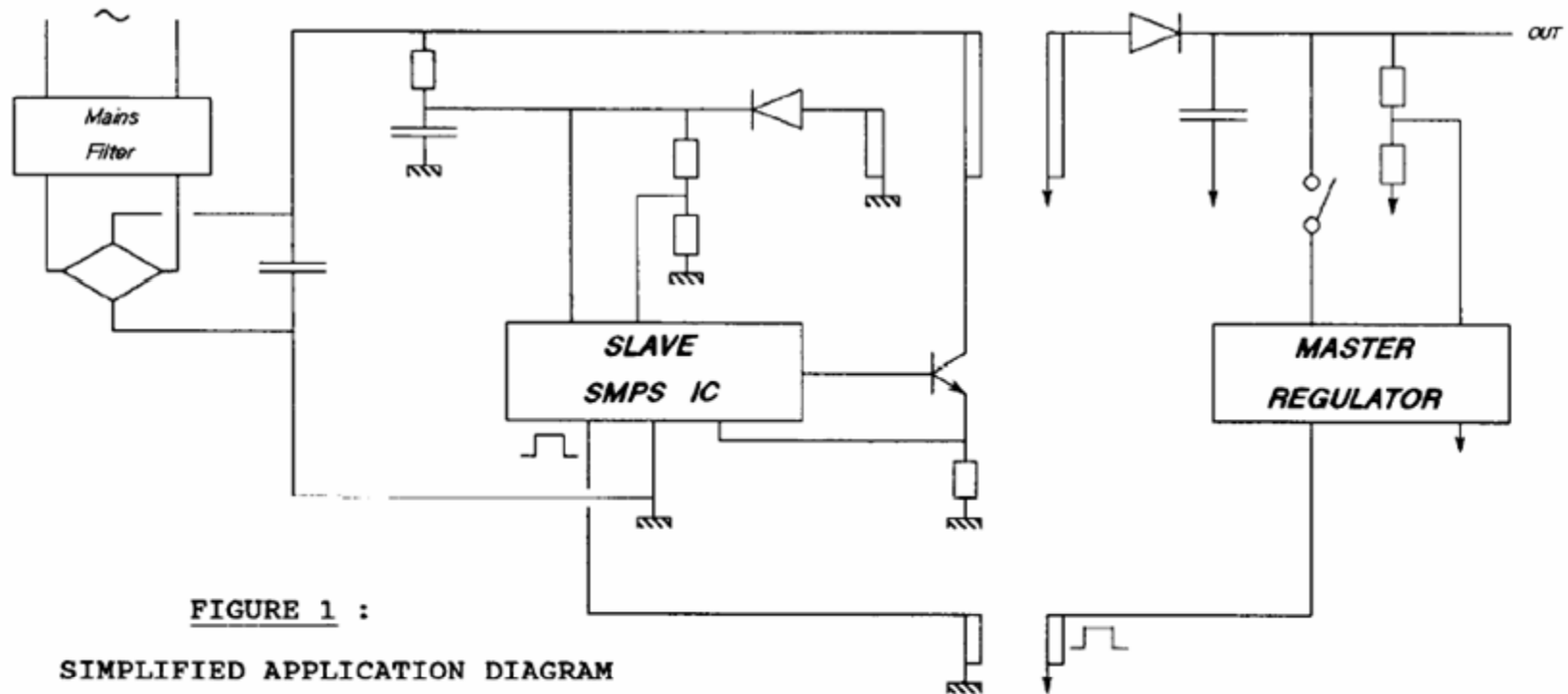
# Variac Failure Analysis

# Photovoltaics



Fig. 1. Typical hybrid system layout.

Reference: S. Druyea, S. Islam and W. Lawrance, "A battery management system for stand-alone photovoltaic energy systems," *IEEE Industry Applications Magazine*, vol. 7, no. 3, May-June 2001, pp. 67-72

# Offline Flyback Power Supply



FIGURE 1 :

SIMPLIFIED APPLICATION DIAGRAM

Reference: P. Maige, "A universal power supply integrated circuit for TV and monitor applications," *IEEE Transactions on Consumer Electronics*, vol. 36, no. 1, Feb. 1990, pp. 10-17

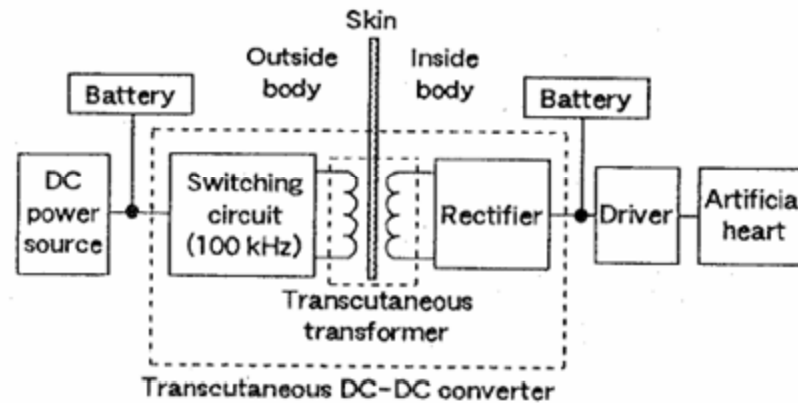# Transcutaneous Energy Transmission

Fig. 1. Transcutaneous energy transmission system for an implantable artificial heart.
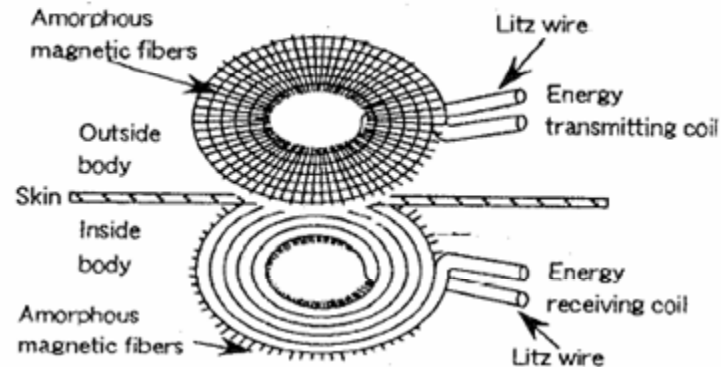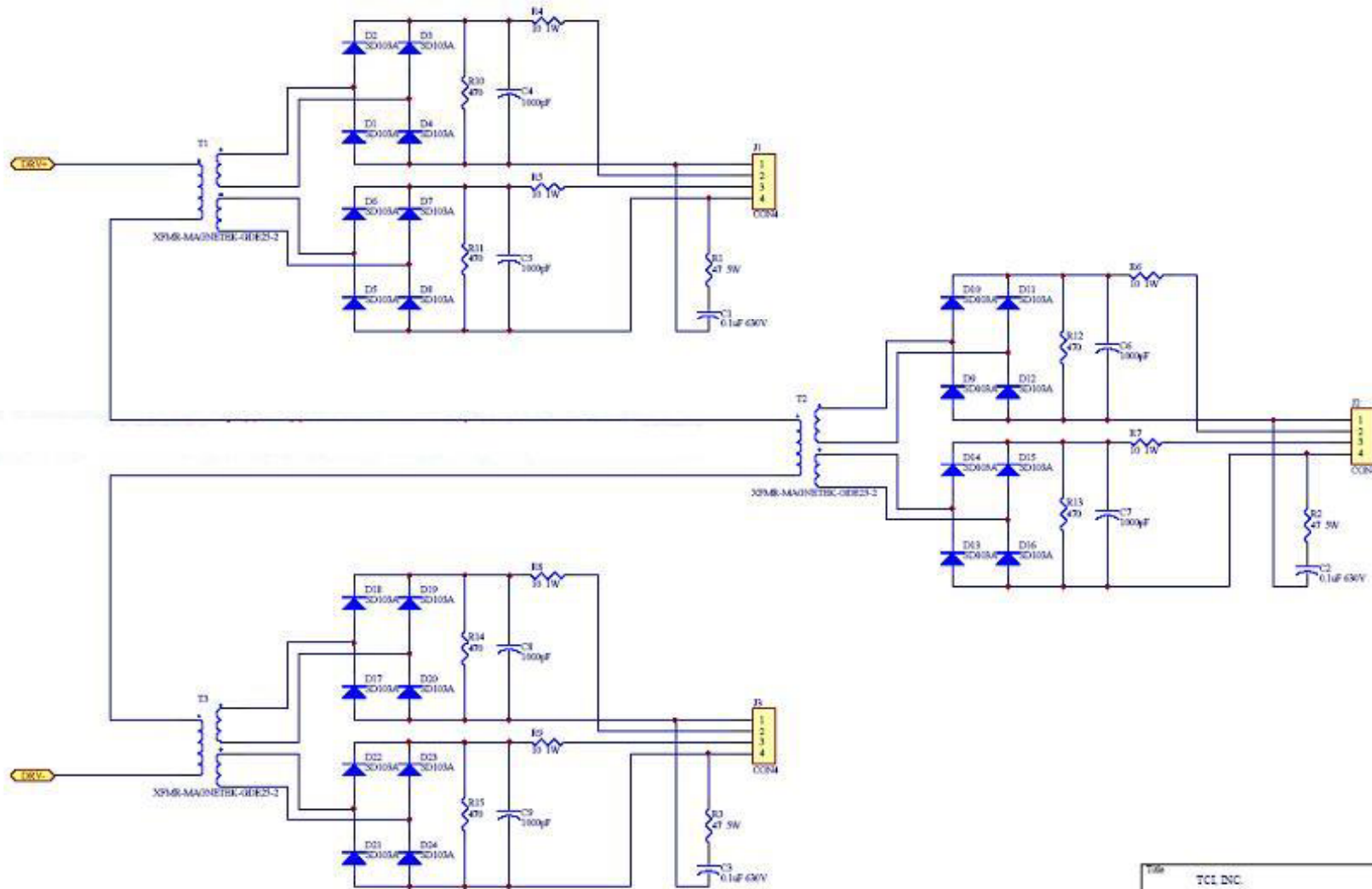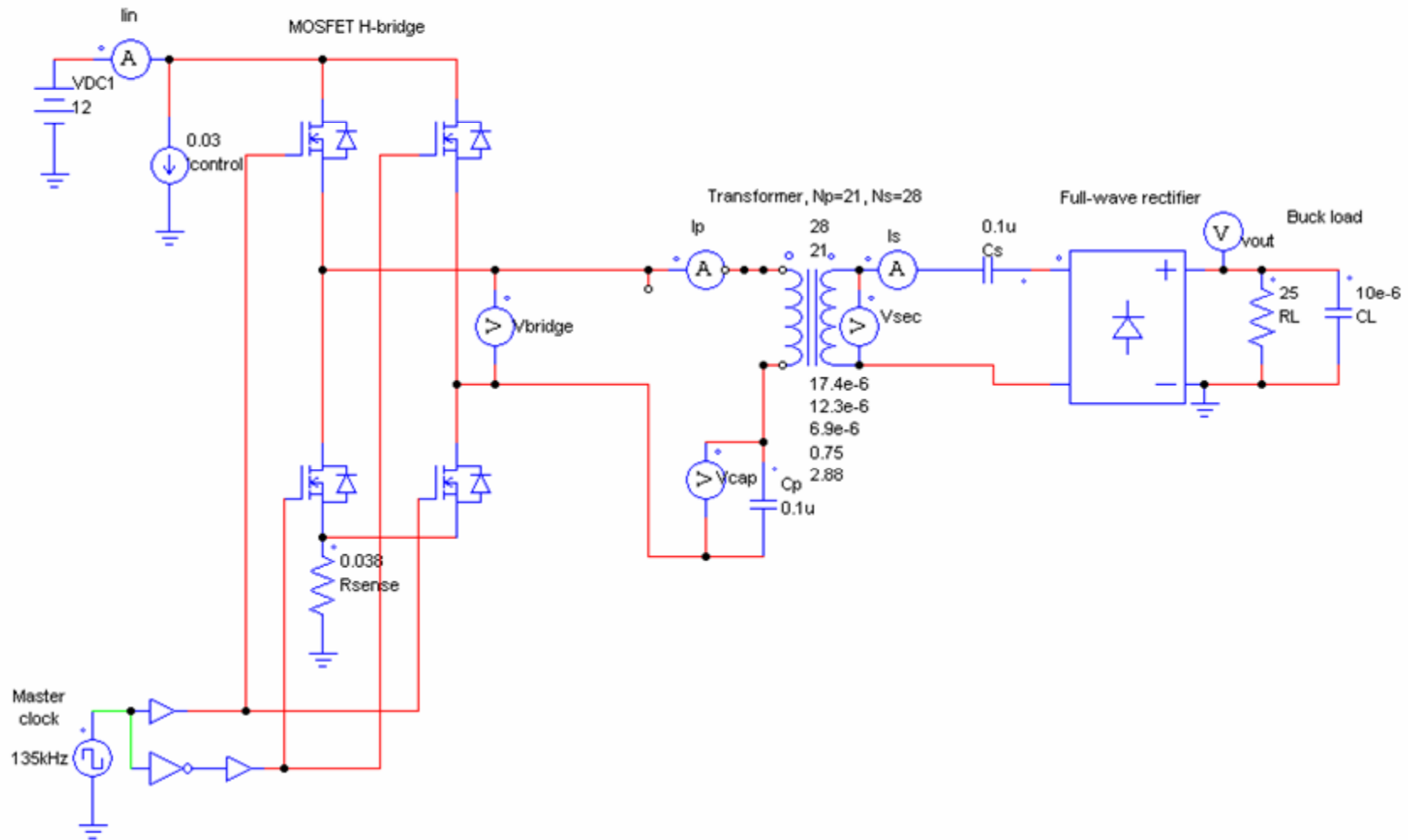
Fig. 2. Transcutaneous transformer.

Reference:  H. Matsuki, Y. Yamakata, N. Chubachi, S.-I. Nitta and H. Hashimoto, "Transcutaneous DC-DC converter for totally implantable artificial heart using synchronous rectifier," *IEEE Transactions on Magnetics*, vol. 32 , no. 5, Sept. 1996, pp. 5118 - 5120
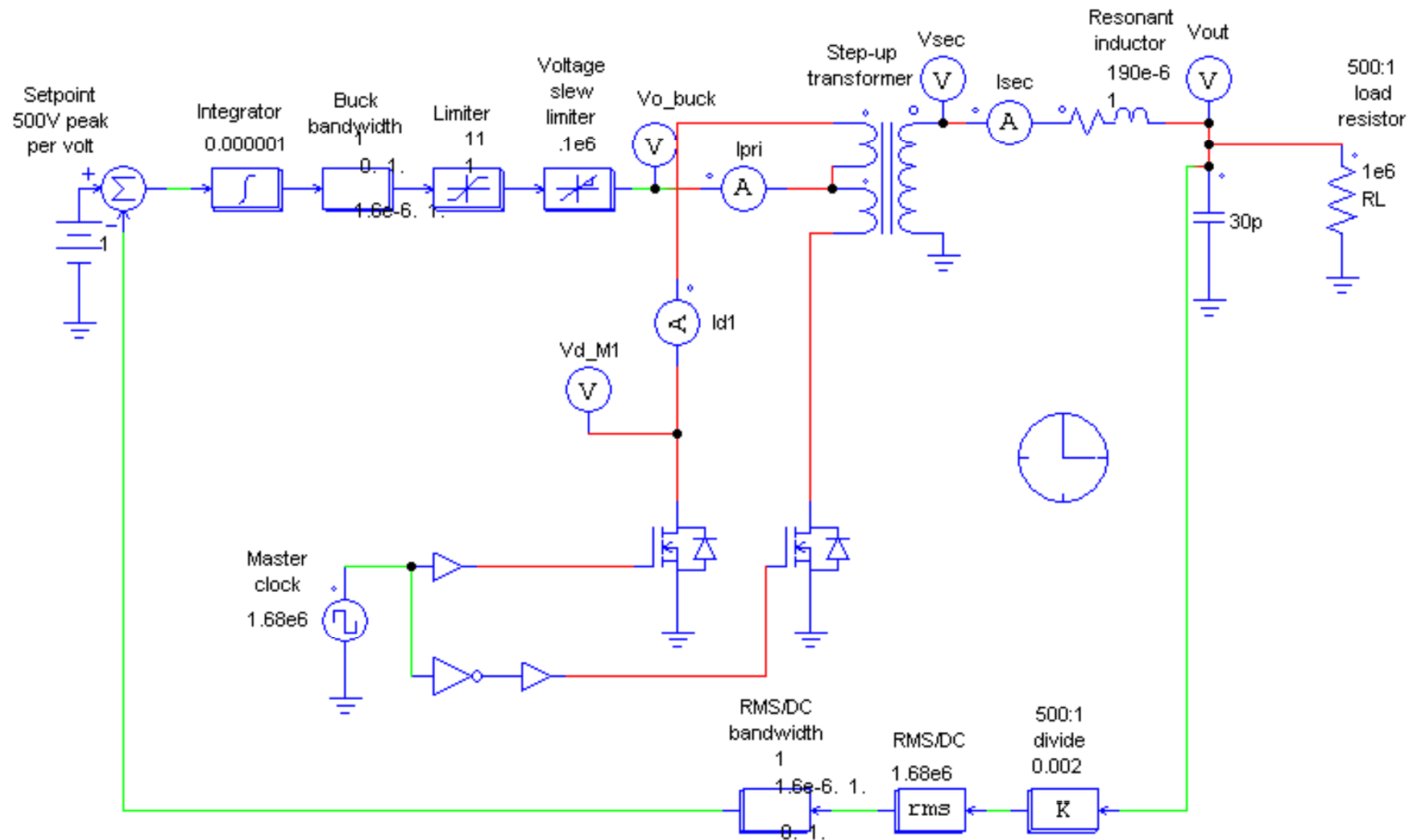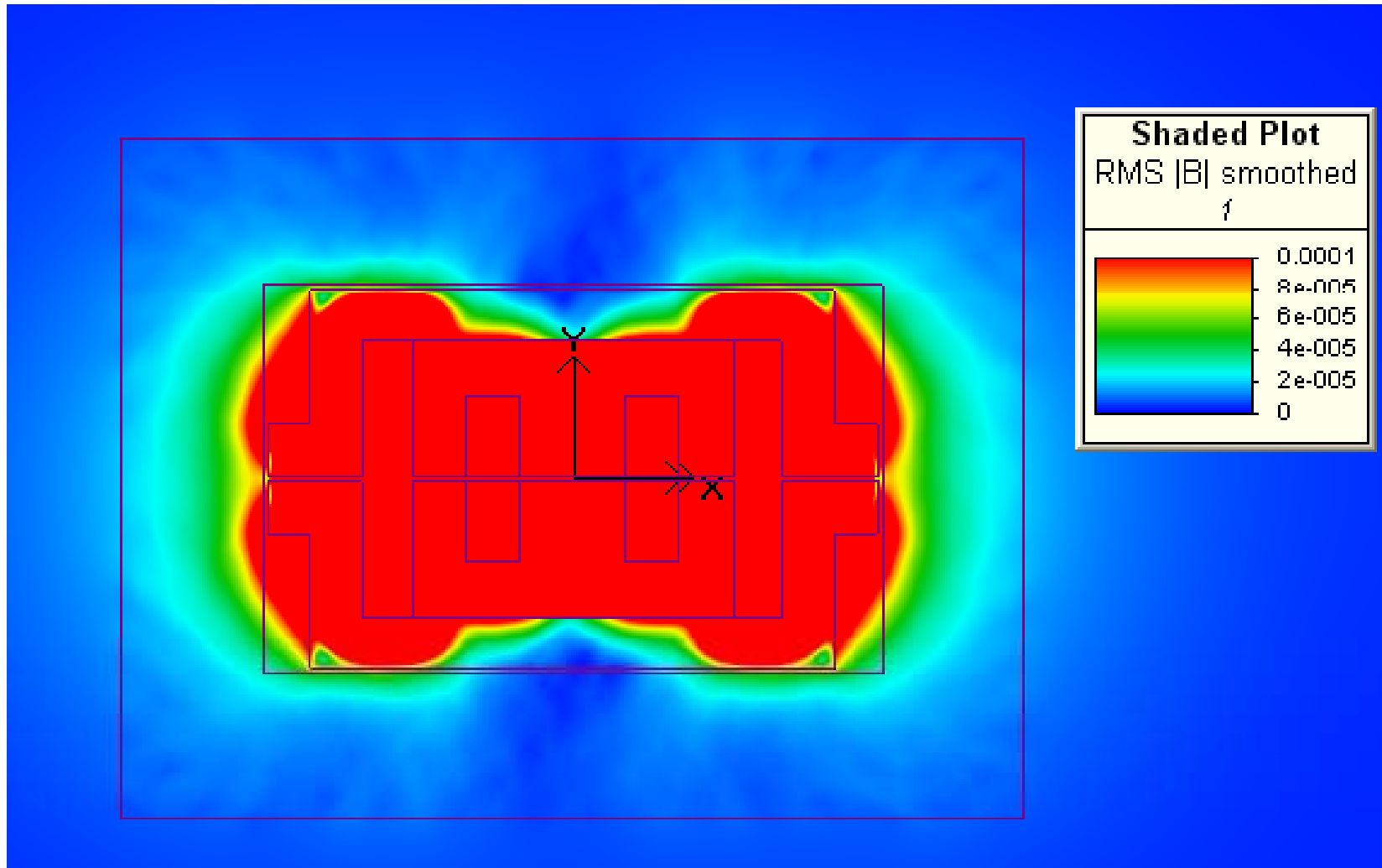
# 50 KW Inverter Switch

# Non-Contact Battery Charger

# High Voltage RF Supply

# 60 Hz Transformer Shielding Study

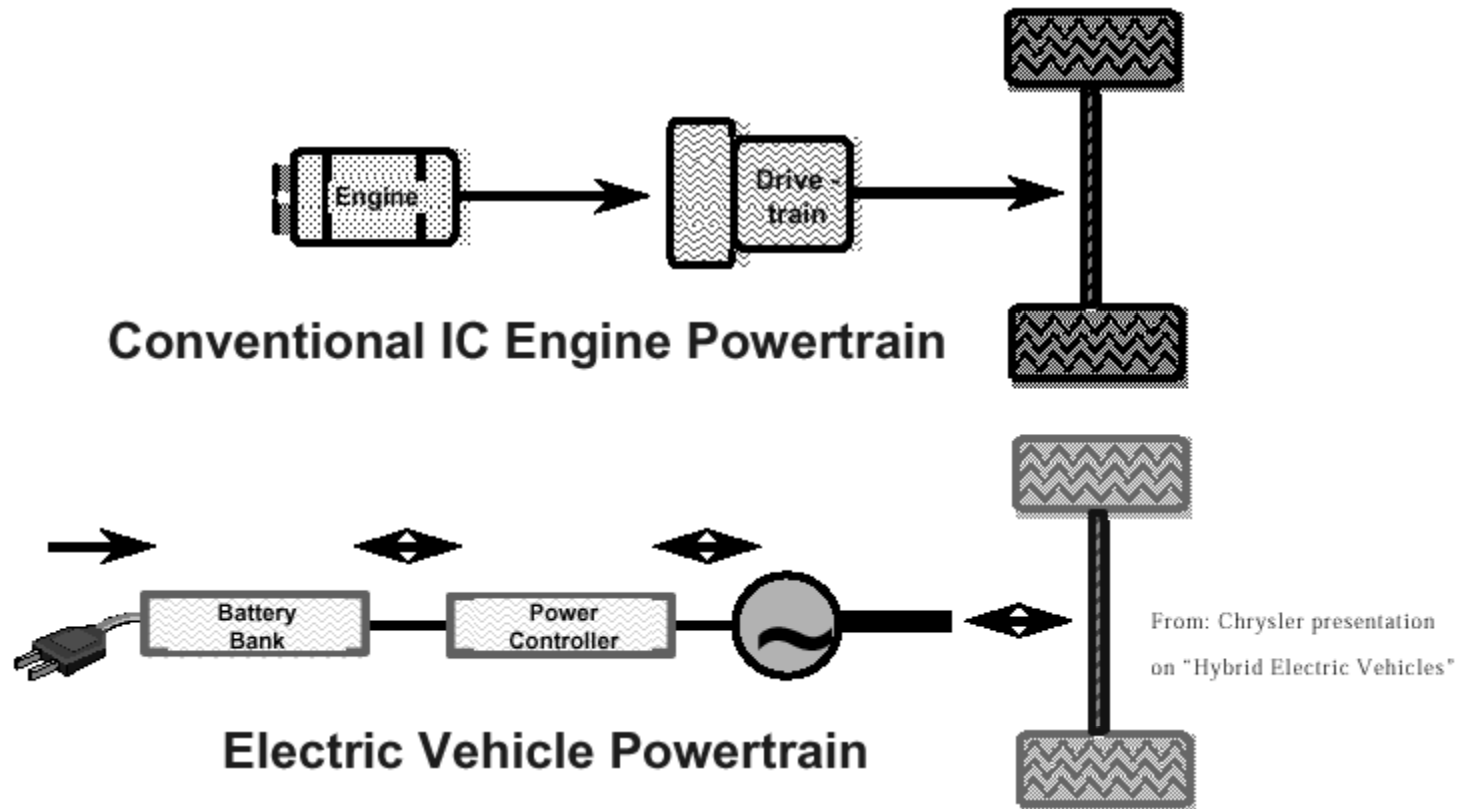# "Intuitive Analog Circuit Design"

Introduction to Power Electronics
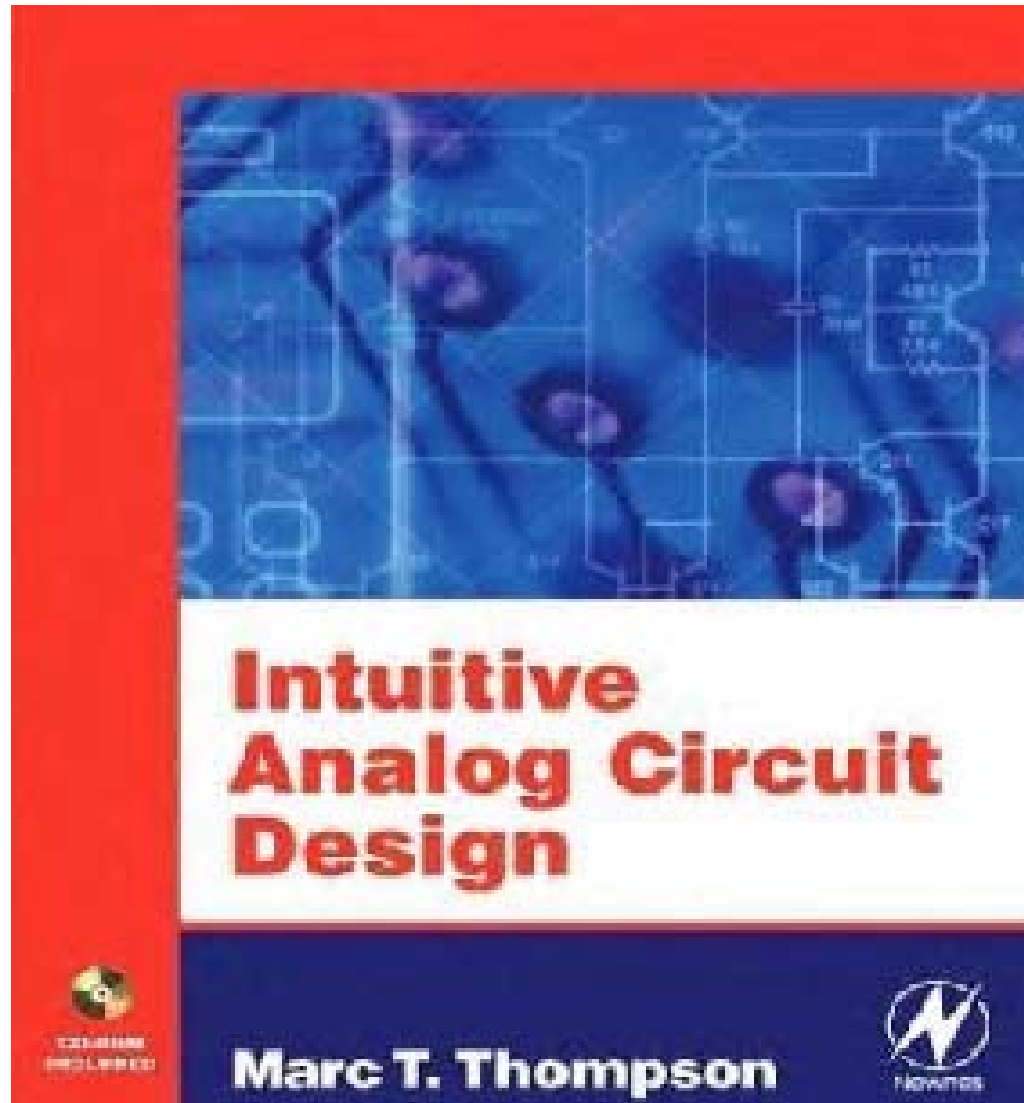
# "Power Quality in Electrical Systems"
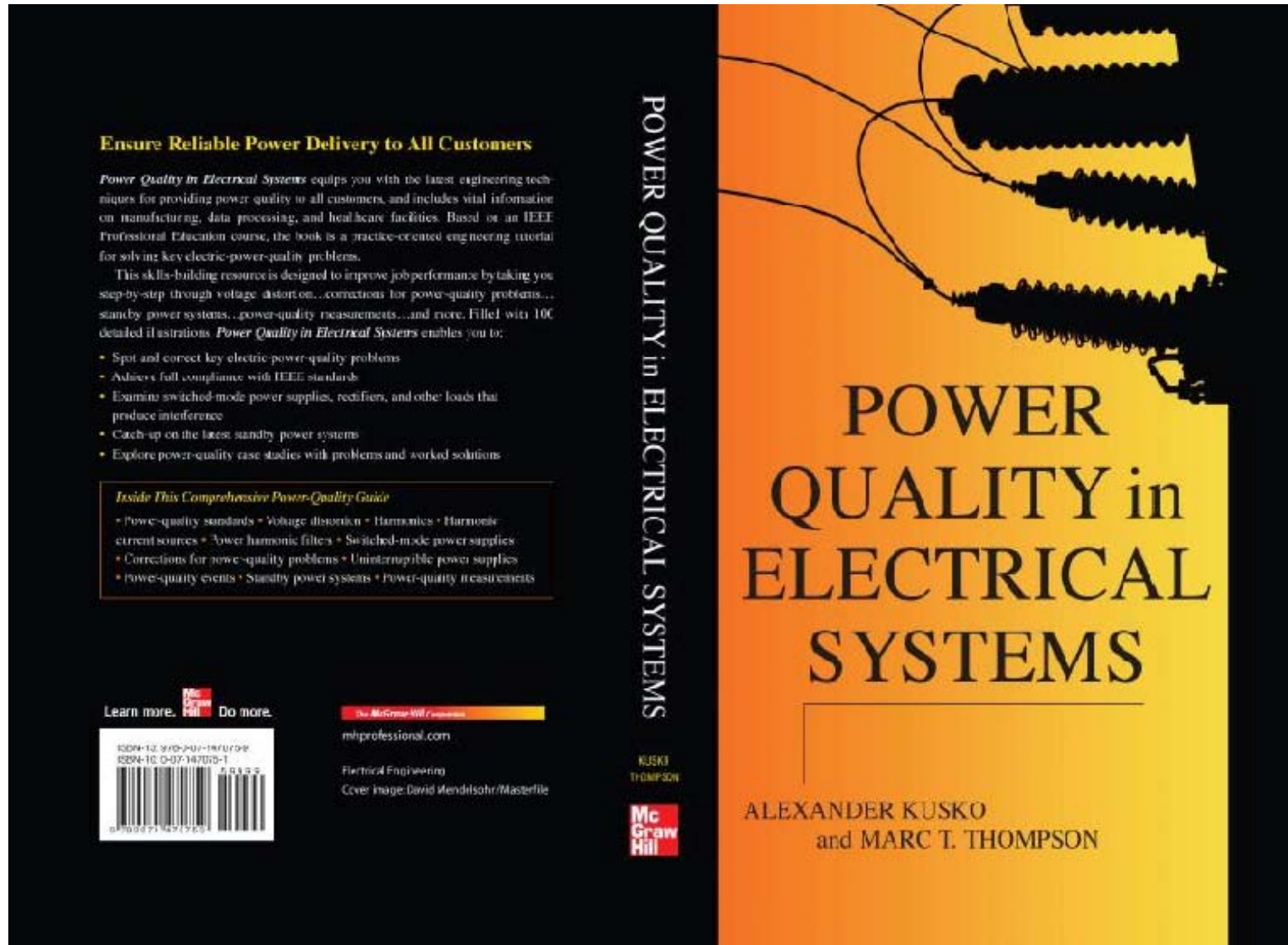
# Some Other Interesting Power-Electronics Related Systems

# Conventional vs. Electric Car

**Conventional IC Engine Powertrain**

Engine → Drive-train →

**Electric Vehicle Powertrain**

Battery Bank — Power Controller —

From: Chrysler presentation on "Hybrid Electric Vehicles"

# High Voltage DC (HVDC) Transmission



[1] A set of three quadruple thyristor-based valves feeds each of the three dc poles [the third is barely seen at the right] in this valve hall. The equipment serves India's 910-km, 1500-MW high-voltage dc link. First operated in 1991, the link ties coal-based power plants in Rihand, in the eastern part of India, to Delhi's fast-growing farther west.

PHOTO:

[2] A 600-MW back-to-back HVDC tie between the Welsh and Monticello power plants connects the Texas electric power system to the Eastern U.S. system. The different frequency controls of the two systems make it impossible to connect them otherwise. Both converters (for ac to dc or vice versa) are at the same location, with a dc busbar connecting them. A large outdoor area is occupied by harmonic filters, capacitors, switching gear, and other equipment.

PHOTO: SIEMENS ENERGY & AUTOMATION INC.

# Mass Spectrometer



Reference: http://www.cameca.fr/doc_en_pdf/oral_sims14_schuhmacher_ims1270improvements.pdf

# Some Disciplines Encompassed in the Field of Power Electronics

- Analog circuits
  - High speed (MOSFET switching, etc.)
  - High power
  - PC board layout
  - Filters
    - EMI
- Control theory
- Magnetics
  - Inductor design
  - Transformer design
- Power systems
  - Transmission lines
  - Line filtering

- Machines/motors
- Simulation
  - SPICE, Matlab, etc.
- Device physics
  - How to make a better MOSFET, IGBT, etc.
- Thermal/cooling
  - How to design a heat sink
  - Thermal interfaces
  - Thermal modeling

# Selected History of Power Switching Devices

- 1831 --- Transformer action demonstrated by Michael Faraday
- 1880s: modern transformer invented



Photo of Faraday's original transformer (courtesy MIT Burndy Library).

Reference: J. W. Coltman, "The Transformer (historical overview)," *IEEE Industry Applications Magazine*, vol. 8, no. 1, Jan.-Feb. 2002, pp. 8-15
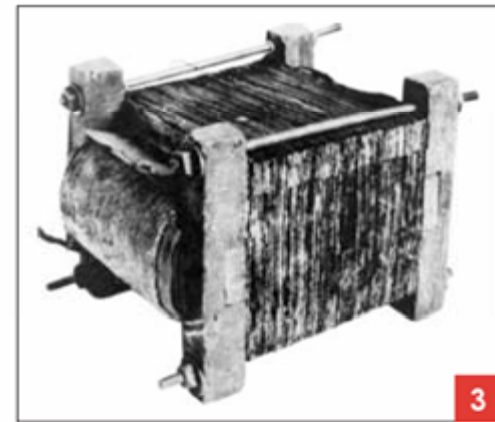


This Stanley transformer from the first ac power station in Great Barrington, Massachusetts, dates from 1885. The transformer is about a foot long; copper windings wrapped with cotton protrude between wood endpieces at the left. The middle arm of E-shaped iron laminations was slid into the prewound coil in alternating directions. The ends of the other two arms are visible as dense regions at the top and bottom of the laminations.

# Selected History of Power Switching Devices

- Early 1900s: vacuum tube
  - Lee DeForest --- triode, 1906
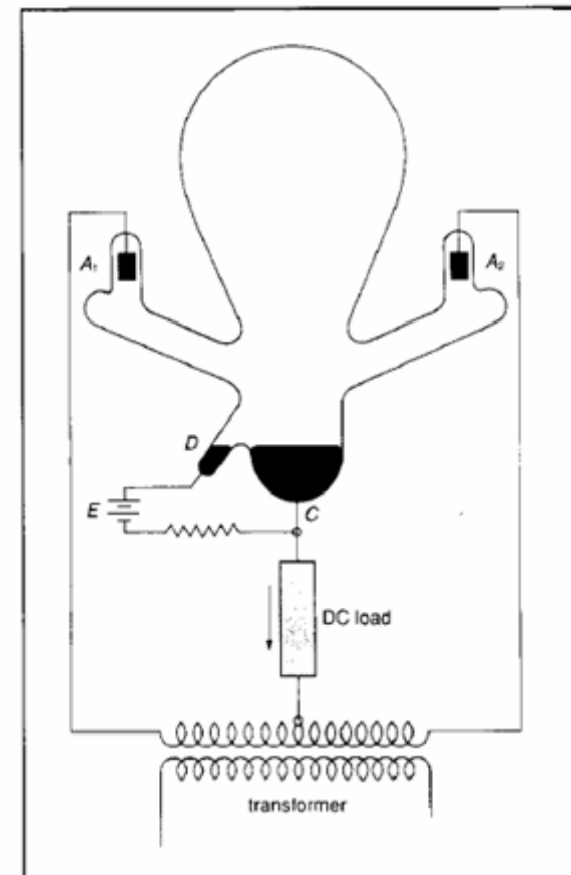- 1920-1940: mercury arc tubes to convert 50Hz, 2000V to 3000VDC for railway

Fig. 1   Single-phase full-wave mercury-arc rectifier, 1910

Reference: M. C. Duffy, "The mercury-arc rectifier and supply to electric railways," *IEEE Engineering Science and Education Journal*, vol. 4, no. 4, August 1995, pp. 183-192

# Selected History of Power Switching Devices

- 1930s: selenium rectifiers
- 1948 - Silicon Transistor (BJT) introduced (Bell Labs)
- 1950s - semiconductor power diodes begin replacing vacuum tubes
- 1956 - GE introduces Silicon-Controlled Rectifier (SCR)



Fig. 7. Cross section of an early (1957–60) GE Si p-n-p-n SCR, with Ga diffused p-type regions) into both sides of a ~250 μm n-type Si wafer and with an n-type emitter regrown on top from a Au (+Sb) alloy.

Reference: N. Holonyak, Jr., "The Silicon p-n-p-n Switch and Controlled Rectifier (Thyristor)," *IEEE Transactions on Power Electronics,* vol. 16, no. 1, January 2001, pp. 8-16

# Selected History of Power Switching Devices

- 1960s - switching speed of BJTs allow DC/DC converters possible in 10-20 kHz range
- 1960 - Metal Oxide Semiconductor Field-Effect Transistor (MOSFET) for integrated circuits
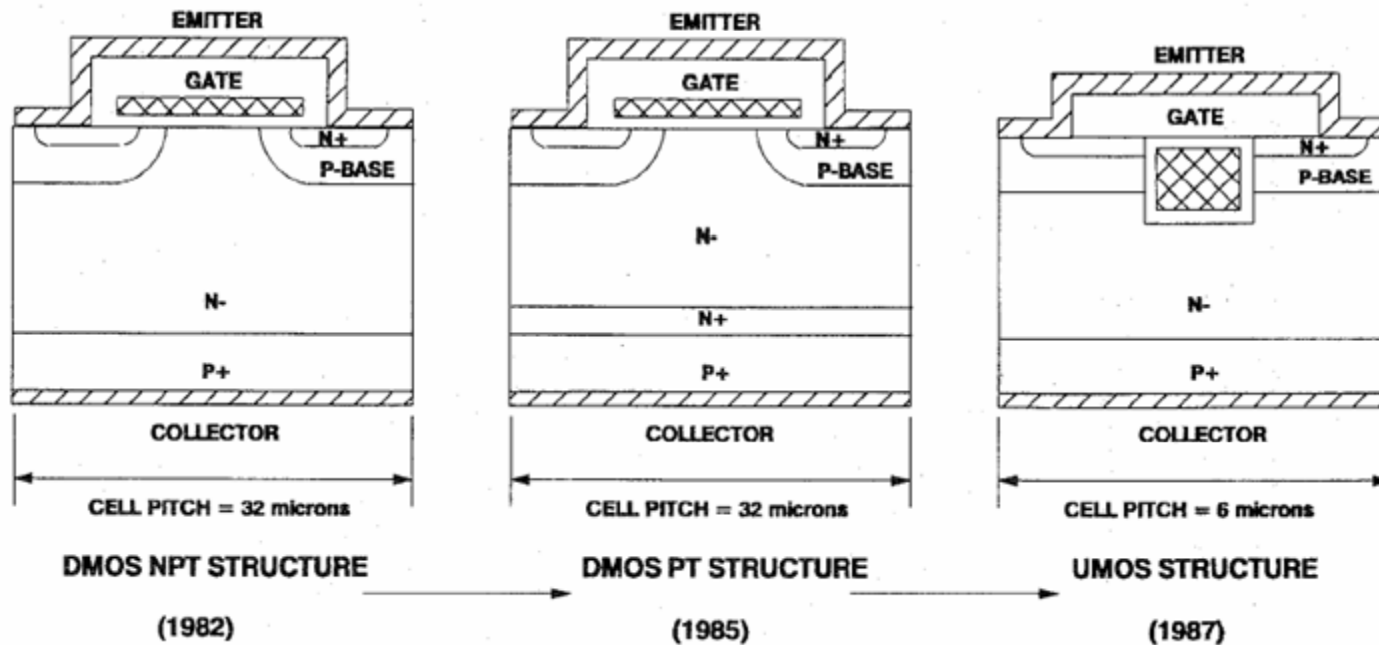- 1976 - power MOSFET becomes commercially available, allows > 100 kHz operation
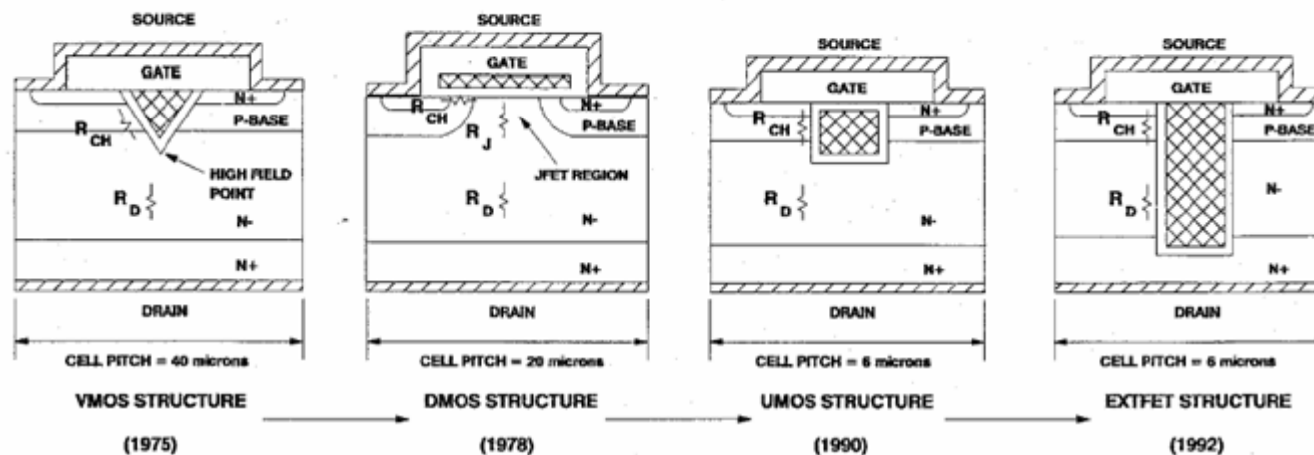


Fig. 8. Evolution of power MOSFET structures.

Reference: B. J. Baliga, "Trends in Power Semiconductor Devices," *IEEE Transactions on Electron Devices,* vol. 43, no. 10, October 1996, pp. 1717-1731

# Selected History of Power Switching Devices

- 1982 - Insulated Gate Bipolar Transistor (IGBT) introduced

Fig. 10. Evolution of IGBT structures.

Reference: B. J. Baliga, "Trends in Power Semiconductor Devices," *IEEE Transactions on Electron Devices,* vol. 43, no. 10, October 1996, pp. 1717-1731

# Review of Basic Circuit Concepts

• Some background in circuits
  • Laplace notation
  • First-order and second-order systems
  • Resonant circuits, damping ratio, Q

• Reference for this material: M. T. Thompson, _Intuitive Analog Circuit Design_, Elsevier, 2006 (course book for ECE529) and _Power Quality in Electrical Systems_, McGraw-Hill, 2007 by A. Kusko and M. Thompson
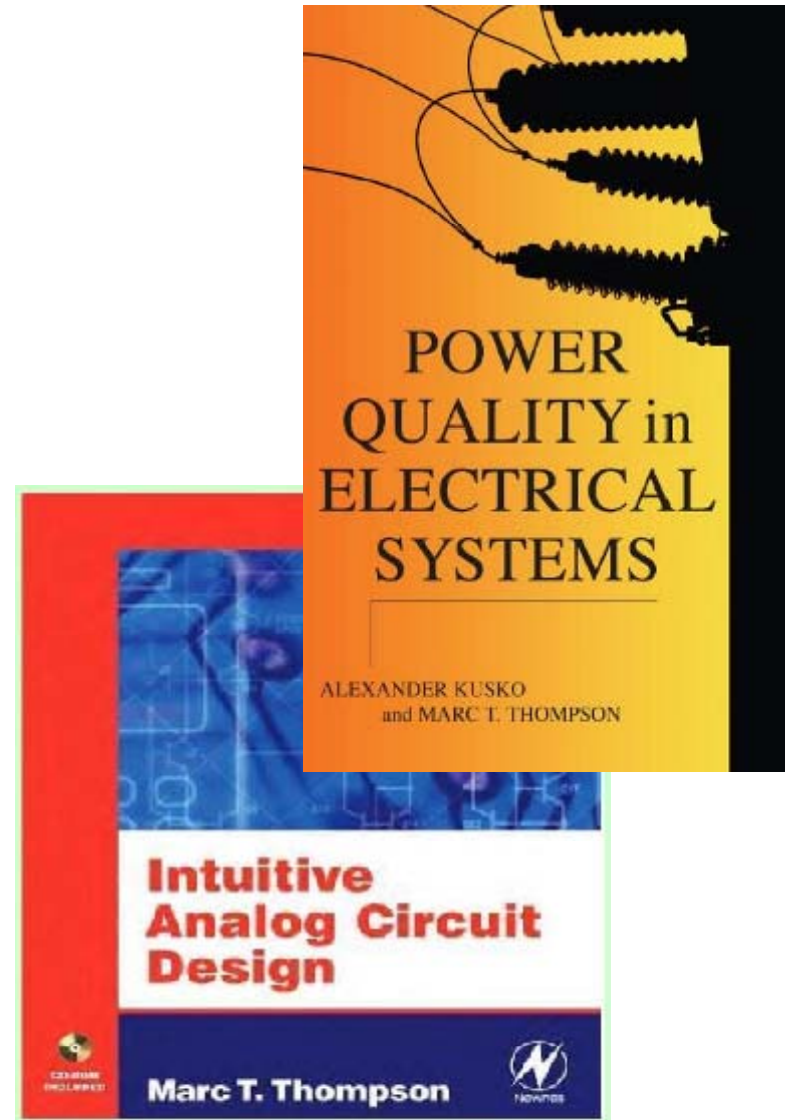
# Laplace Notation

- Basic idea: Laplace transform converts differential equation to algebraic equation
- Generally, method is used in sinusoidal steady state after all startup transients have died out

| Circuit domain | Laplace ($s$) domain |
|---|---|
| Resistance, R | R |
| Inductance L | $Ls$ |
| Capacitance C | $\dfrac{1}{Cs}$ |
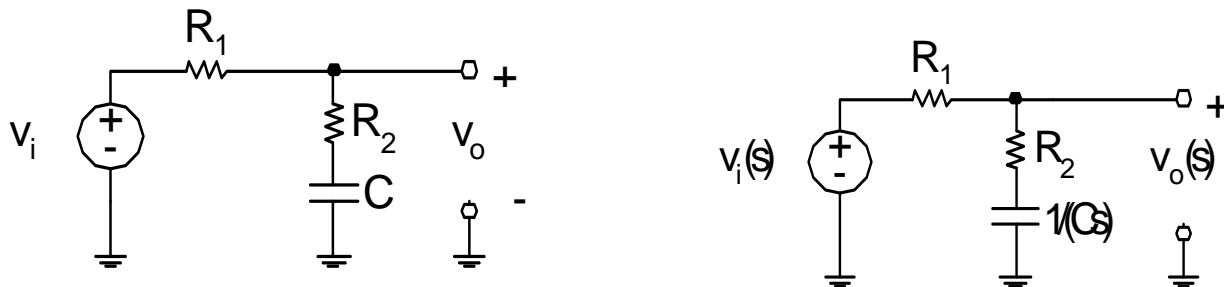
# System Function

- Find "transfer function" H(s) by solving Laplace transformed circuit
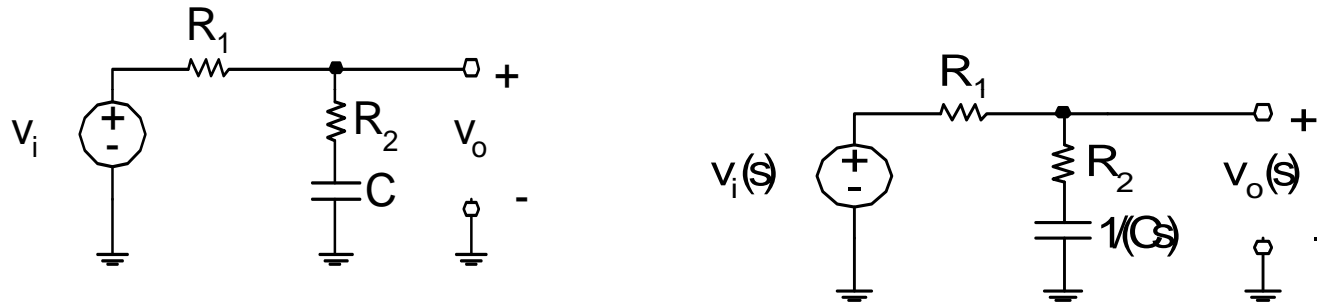


$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{R_2 + \dfrac{1}{Cs}}{R_1 + R_2 + \dfrac{1}{Cs}} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

# First-Order Systems

$$v_o(t) = V(1 - e^{\frac{-t}{\tau}})$$

$$i_r(t) = \frac{V}{R} e^{\frac{-t}{\tau}}$$

$$\tau = RC \quad \text{Time constant}$$

$$\tau_R = 2.2\tau \quad \text{Risetime}$$

$$\omega_h = \frac{1}{\tau}$$

$$\text{Bandwidth} \qquad \tau_R = \frac{0.35}{f_h}$$

$$f_h = \frac{\omega_h}{2\pi}$$
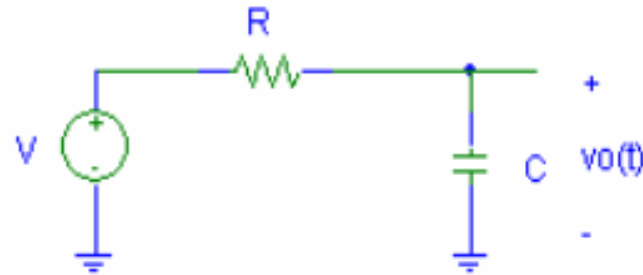
# First-Order Step and Frequency Response
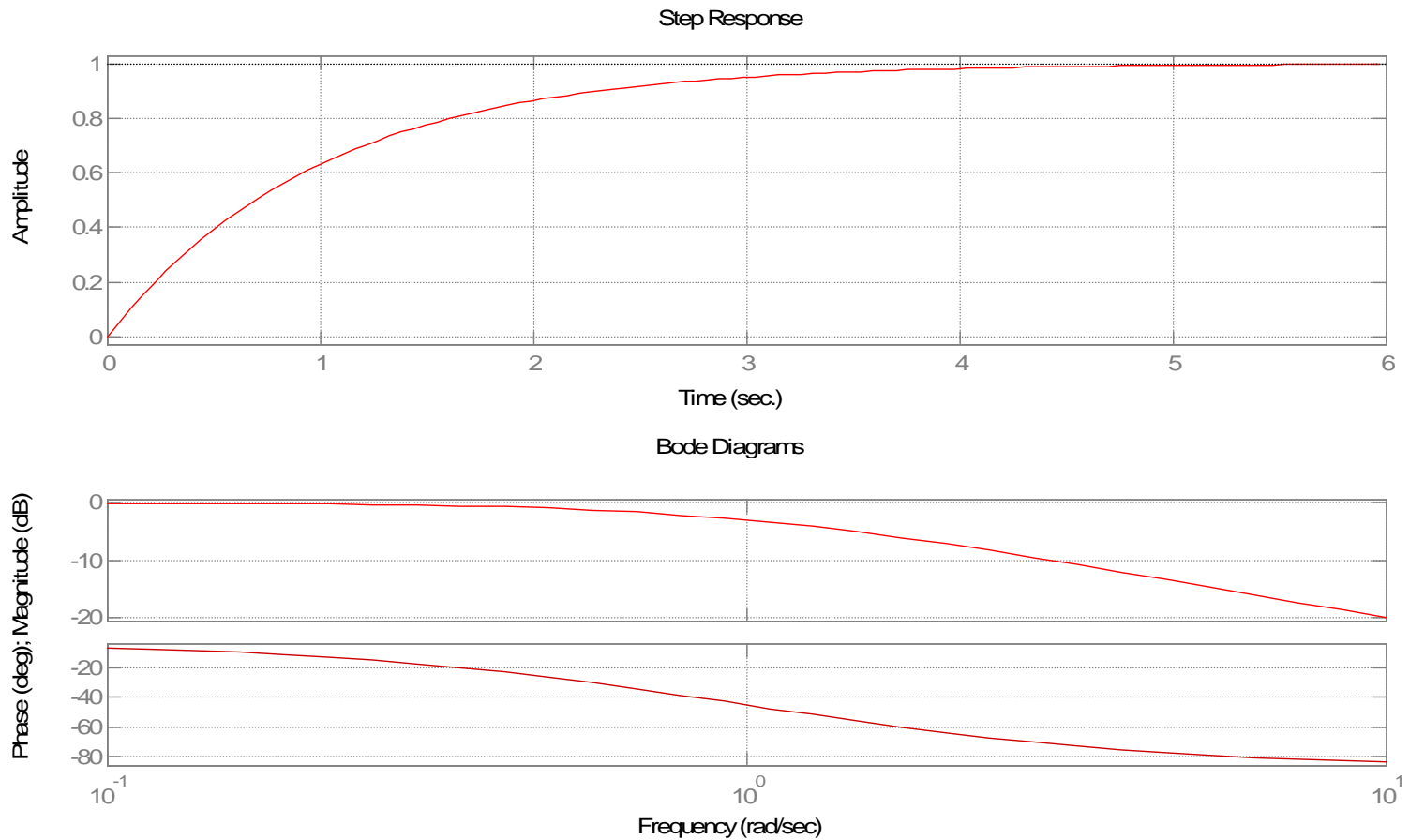
# Review of Second-Order Systems
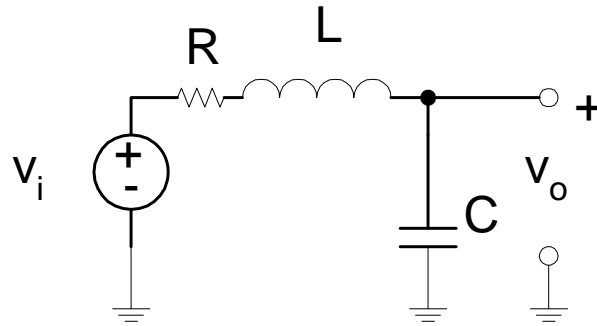


$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{\dfrac{s^2}{\omega_n^2} + \dfrac{2\zeta s}{\omega_n} + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Natural frequency $\quad \omega_n = \dfrac{1}{\sqrt{LC}}$

Damping ratio $\quad \varsigma = \dfrac{\omega_n RC}{2} = \dfrac{1}{2}\dfrac{R}{\sqrt{\dfrac{L}{C}}} = \dfrac{1}{2}\dfrac{R}{Z_o}$

# Second-Order System Frequency Response

$$H(j\omega) = \frac{1}{\dfrac{2j\varsigma\omega}{\omega_n} + \left(1 - \dfrac{\omega^2}{\omega_n^2}\right)}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{\left(\dfrac{2\varsigma\omega}{\omega_n}\right)^2 + \left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\frac{\left(\dfrac{2\varsigma\omega}{\omega_n}\right)}{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)} = -\tan^{-1}\left(\frac{2\varsigma\omega\omega_n}{\omega_n^2 - \omega^2}\right)$$

# Second-Order System Frequency Response

- Plots show varying damping ratio

Frequency response for natural frequency = 1 and various damping ratios
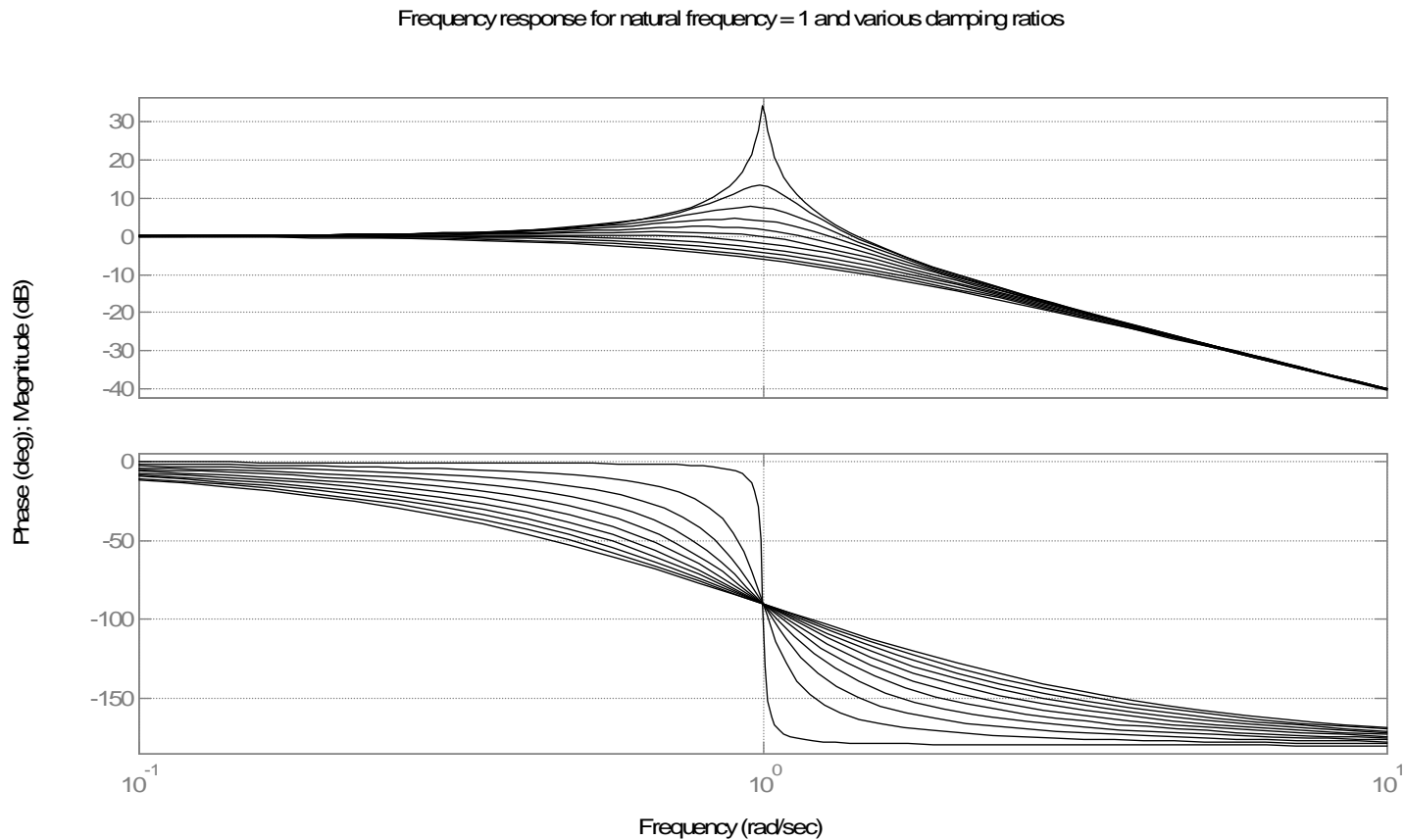
# Second-Order System Frequency Response at Natural Frequency

• Now, what happens if we excite this system exactly at the natural frequency, or $\omega = \omega_n$? The response is:

$$\left| H(s) \right|_{\omega=\omega_n} = \frac{1}{2\varsigma}$$

$$\angle H(s)_{\omega=\omega_n} = -\frac{\pi}{2}$$

# Relationship Between Damping Ratio and "Quality Factor" Q

• A second order system can also be characterized by its "Quality Factor" or "Q"

$$\left.|H(s)|\right|_{\omega=\omega_n} = \frac{1}{2\varsigma} = Q$$

# Second-Order System Step Response

- Shown for varying values of damping ratio



**Step Response**
Step response for natural frequency = 1 and various damping ratios
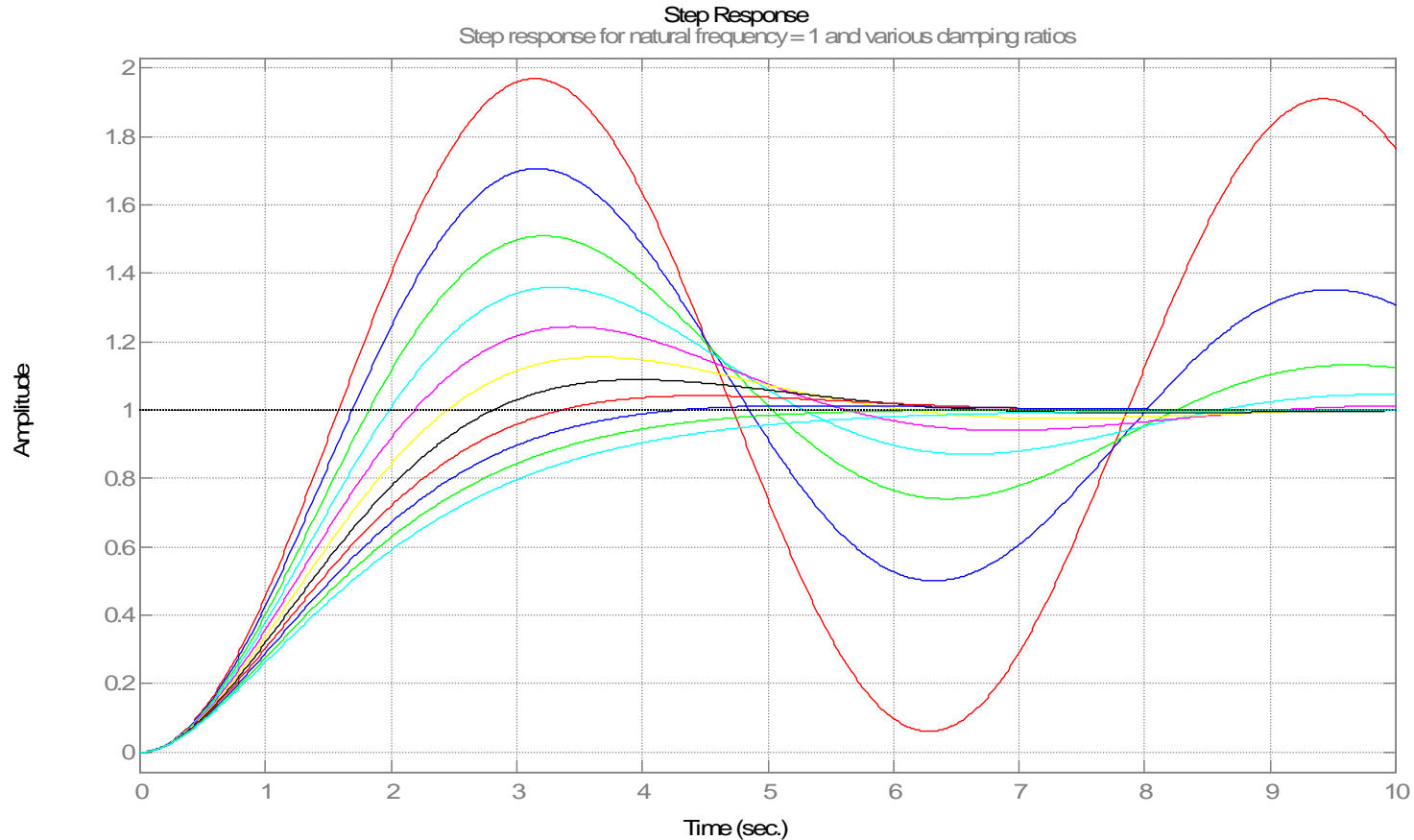
# Second-Order Mechanical System

• Electromechanical modeling
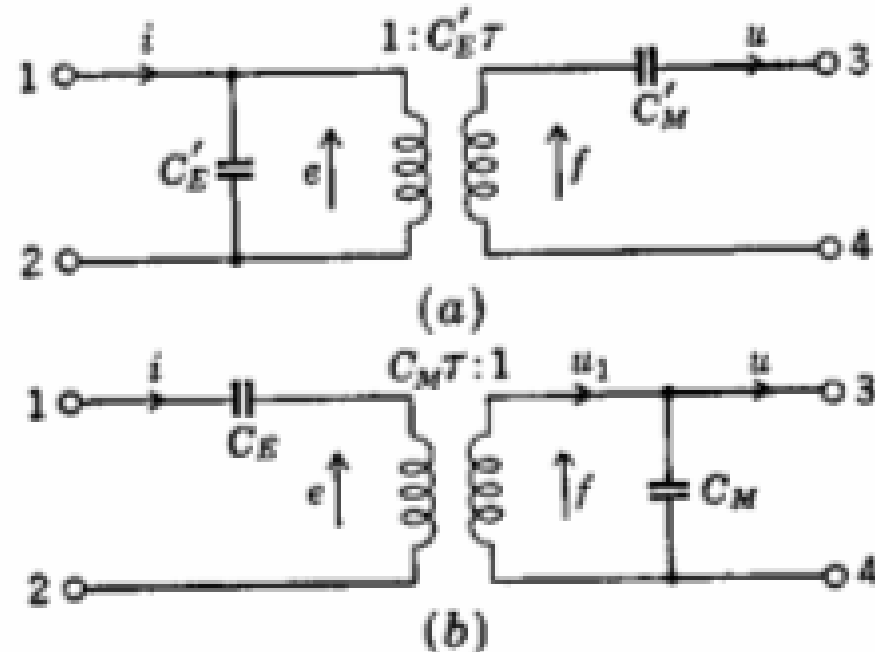


FIG. 3.37. Two forms of analogous symbols for piezoelectric transducers. The mechanical sides are of the impedance type.

Reference:  Leo Beranek, *Acoustics*, Acoustical Society of America, 1954

# Pole Location Variation with Damping

Very underdamped

$j\omega$

$\times + j\omega_n$

$\sigma$

$\times - j\omega_n$
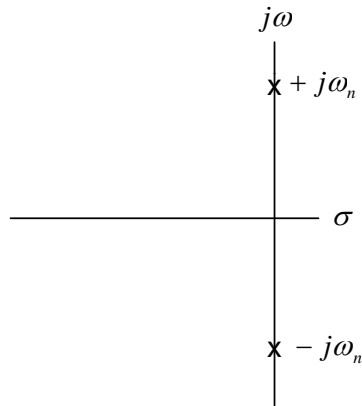
Zero damping
$\zeta = 0$

Critically damped

$j\omega$

$-\omega_n$

$\times$

$\sigma$

2 poles
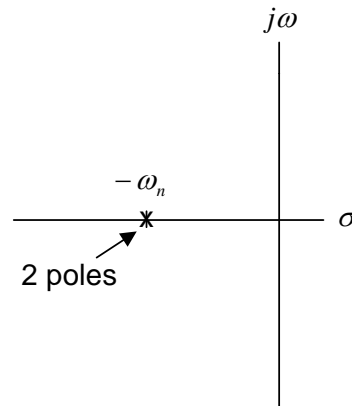
Critical damping
$\zeta = 1$

Overdamped

$j\omega$

$\times$     $\times$     $\sigma$
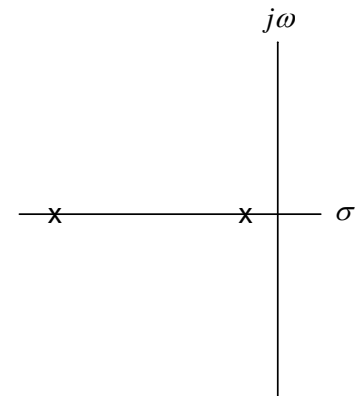
Overdamped
$\zeta \gg 1$

# Undamped Resonant Circuit
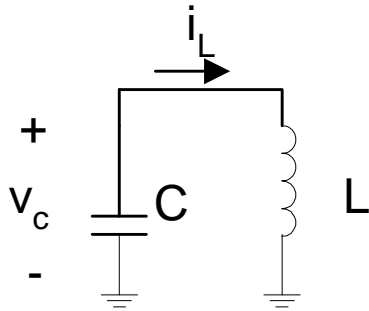
$$\frac{di_L}{dt} = \frac{v_c}{L} \qquad \frac{dv_c}{dt} = \frac{-i_L}{C}$$

$$\frac{d^2 v_c}{dt^2} = -\frac{1}{C}\frac{di_L}{dt} = -\frac{v_c}{LC}$$

Now, we can find the resonant frequency by guessing that the voltage *v(t)* is sinusoidal with *v(t)* = $V_o$*sinωt*.  Putting this into the equation for capacitor voltage results in:

$$-\omega^2 \sin(\omega t) = -\frac{1}{LC}\sin(\omega t)$$

This means that the resonant frequency is the standard (as expected) resonance:

$$\omega_r^2 = \frac{1}{LC}$$

# Energy Methods
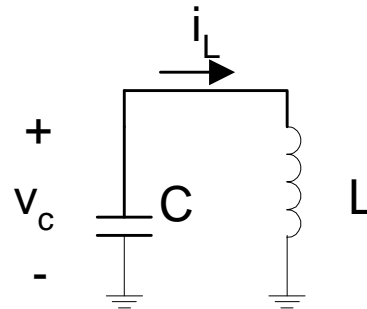


By using energy methods we can find the ratio of maximum capacitor voltage to maximum inductor current.  Assuming that the capacitor is initially charged to $V_o$ volts, and remembering that capacitor stored energy $E_c = \frac{1}{2}CV^2$ and inductor stored energy is $E_L = \frac{1}{2}LI^2$, we can write the following:

$$\frac{1}{2}CV_o^2 = \frac{1}{2}LI_o^2$$

# Energy Methods
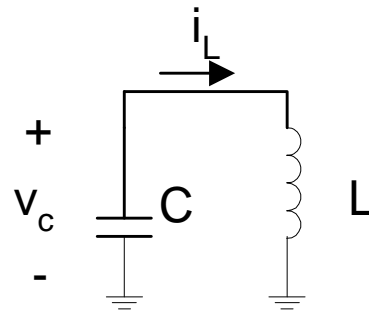


What does this mean about the magnitude of the inductor current ?  Well, we can solve for the ratio of $V_o/I_o$ resulting in:

$$\frac{V_o}{I_o} = \sqrt{\frac{L}{C}} \equiv Z_o$$

The term "$Z_o$" is defined as the characteristic impedance of a resonant circuit.  Let's assume that we have an inductor-capacitor circuit with C = 1 microFarad and L = 1 microHenry.  This means that the resonant frequency is $10^6$ radians/second (or 166.7 kHz) and that the characteristic impedance is 1 Ohm.

# Simulation
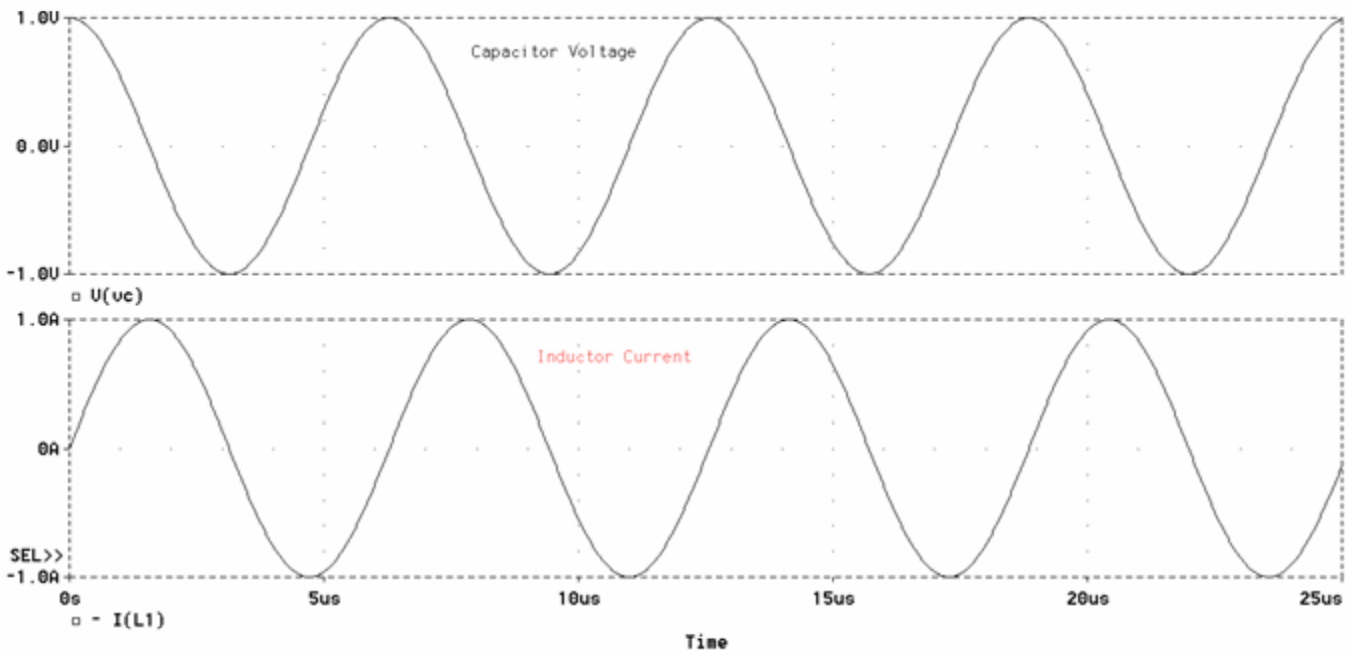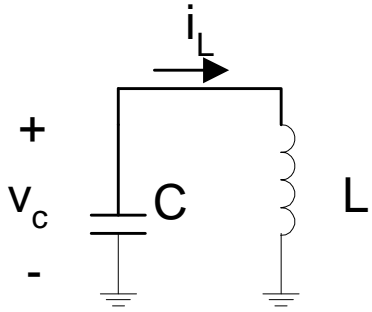
# Typical Resonant Circuit

• Model of a MOSFET gate drive circuit
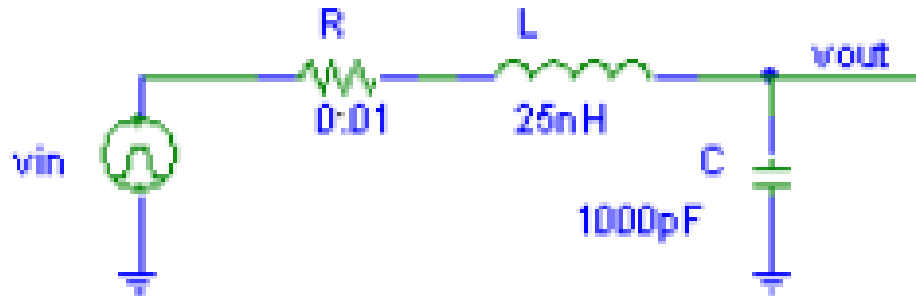


$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{\dfrac{s^2}{\omega_n^2} + \dfrac{2\zeta s}{\omega_n} + 1}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\varsigma = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\dfrac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o}$$

# Resonant Circuit --- Underdamped

• With "small" resistor, circuit is underdamped
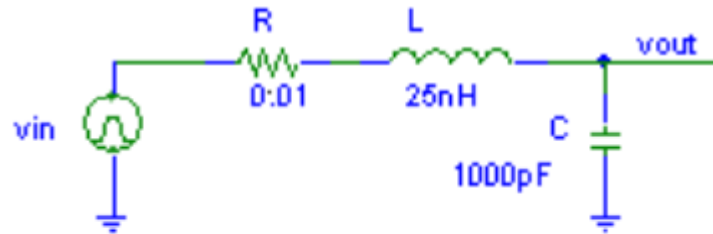


$$\omega_n = \frac{1}{\sqrt{LC}} = 200 \; Mrad/\sec$$

$$f_n \; 31.8 \; MHz$$

$$Z_o = \sqrt{\frac{L}{C}} = 5\Omega$$

$$\varsigma = \frac{\omega_n RC}{2} = \frac{1}{2}\frac{R}{\sqrt{\dfrac{L}{C}}} = \frac{1}{2}\frac{R}{Z_o} = 0.001$$

# Resonant Circuit --- Underdamped Results

• This circuit is very underdamped, so we expect step response to oscillate at around 31.8 MHz
• Expect peaky frequency response with peak near 31.8 MHz

$$\omega_n = \frac{1}{\sqrt{LC}} = 200\ Mrad/\sec$$

$$f_n\ \ 31.8\ MHz$$

$$Z_o = \sqrt{\frac{L}{C}} = 5\Omega$$

$$\varsigma = \frac{\omega_n RC}{2} = \frac{1}{2}\frac{R}{\sqrt{\dfrac{L}{C}}} = \frac{1}{2}\frac{R}{Z_o} = 0.001$$

# Resonant Circuit --- Underdamped Results, Step Response

- Rings at around 31.8 MHz
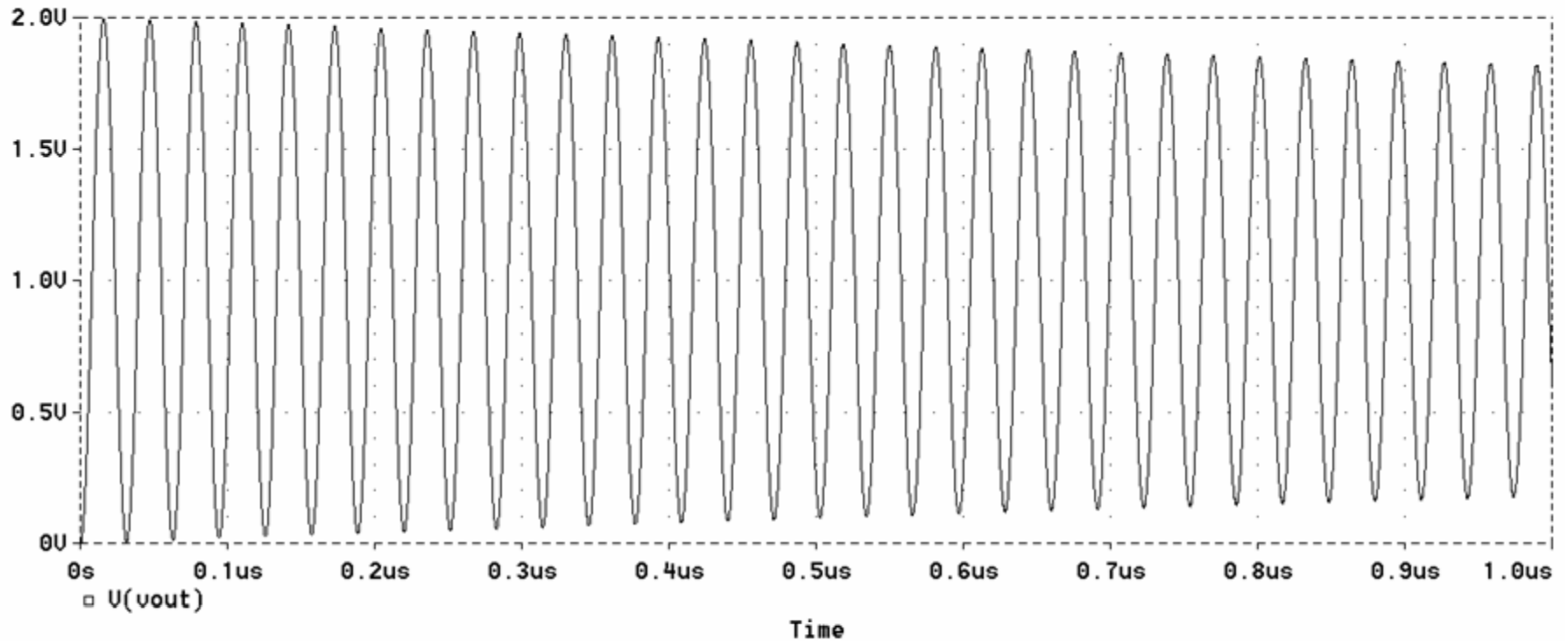
# Resonant Circuit --- Underdamped Results, Frequency Response

- Frequency response peaks at 31.8 MHz
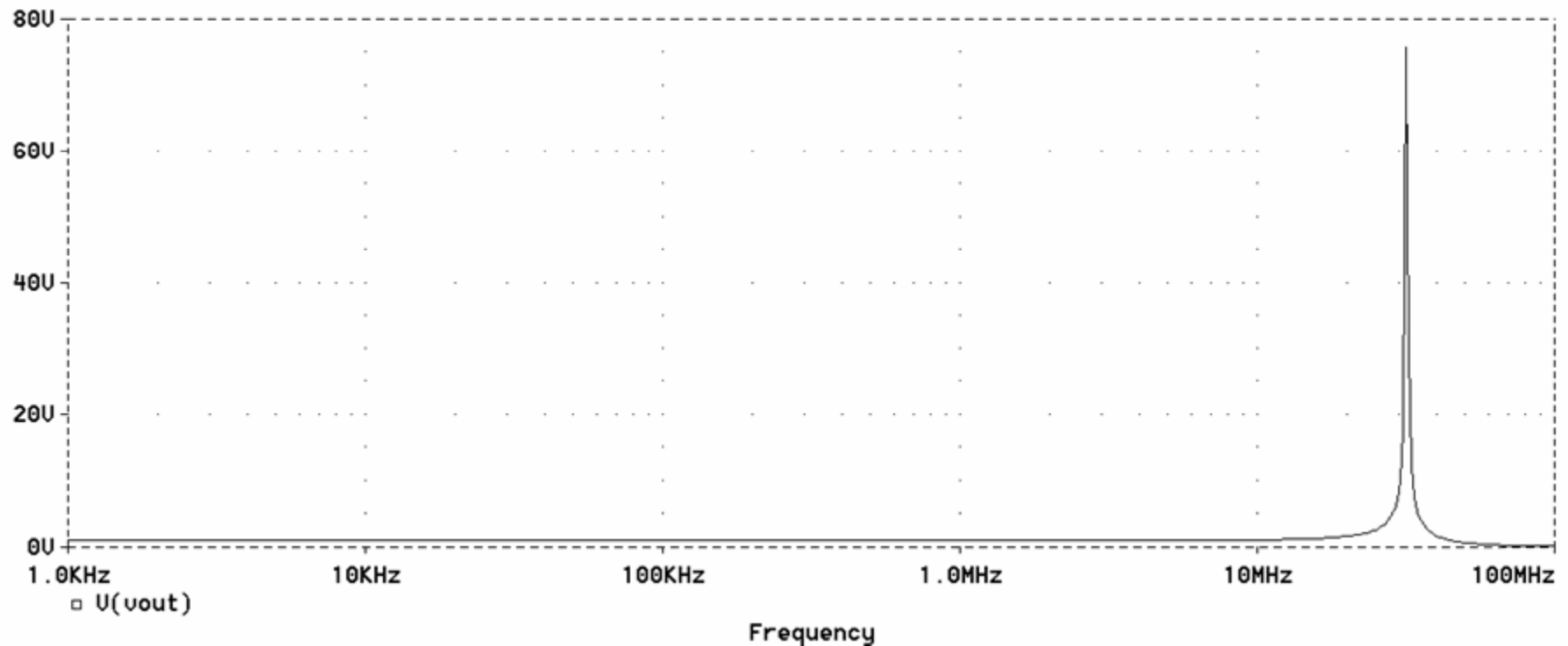
# Resonant Circuit --- Critical Damping

• Now, let's employ "critical damping" by increasing value of resistor to 10 Ohms
• This is also a typical MOSFET gate drive damping resistor value
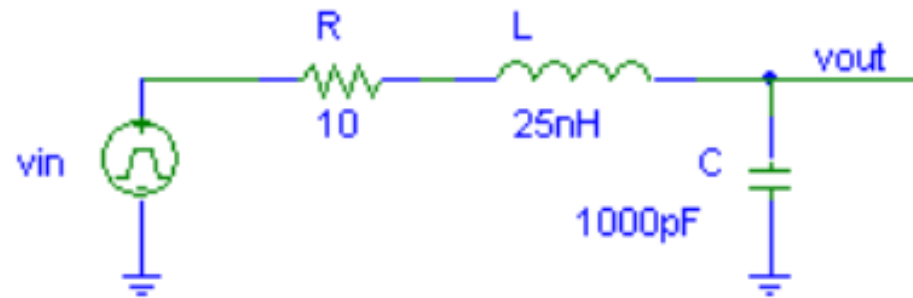
# Critical Damping, Step Response

• Note that response is still relatively fast (< 100 ns response time) but with no overshoot
• If we make R larger, the risetime slows down
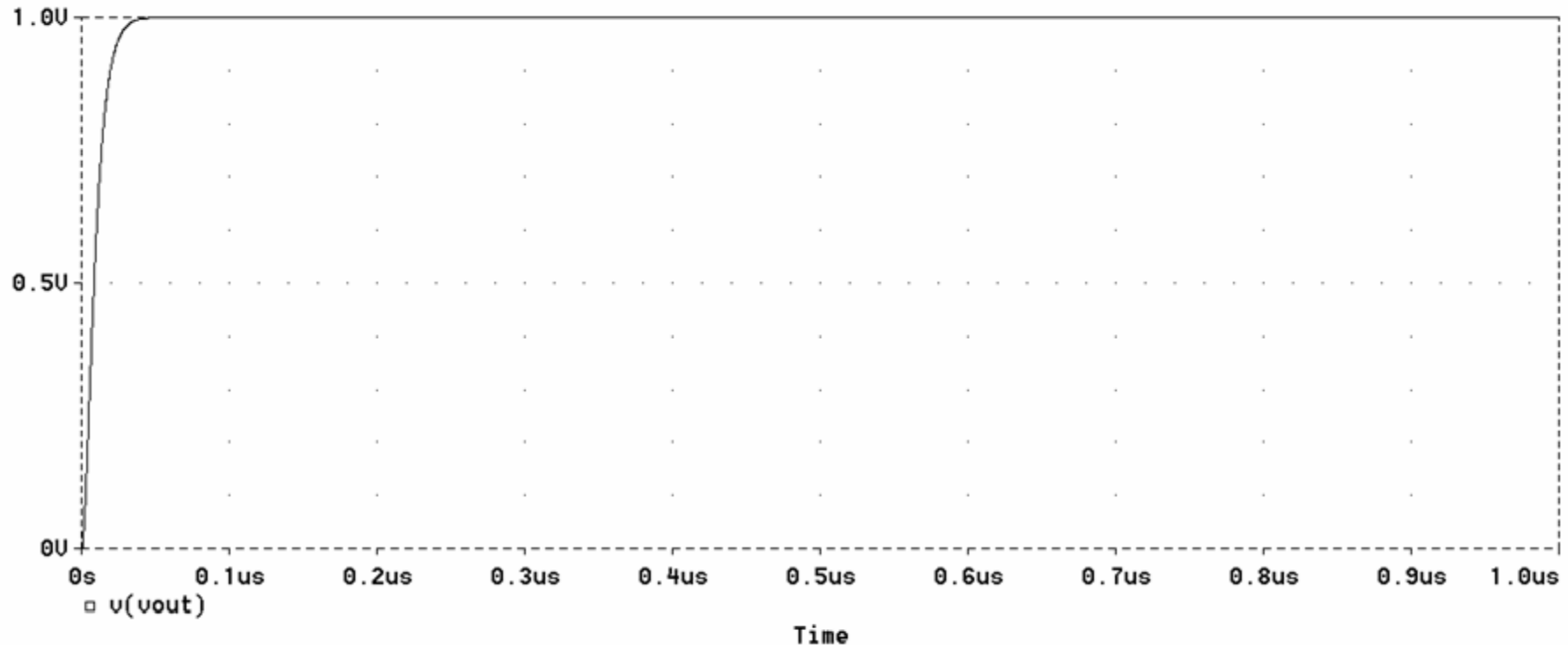
# Critical Damping, Frequency Response

• No overshoot in the transient response corresponds to no peaking in the frequency response
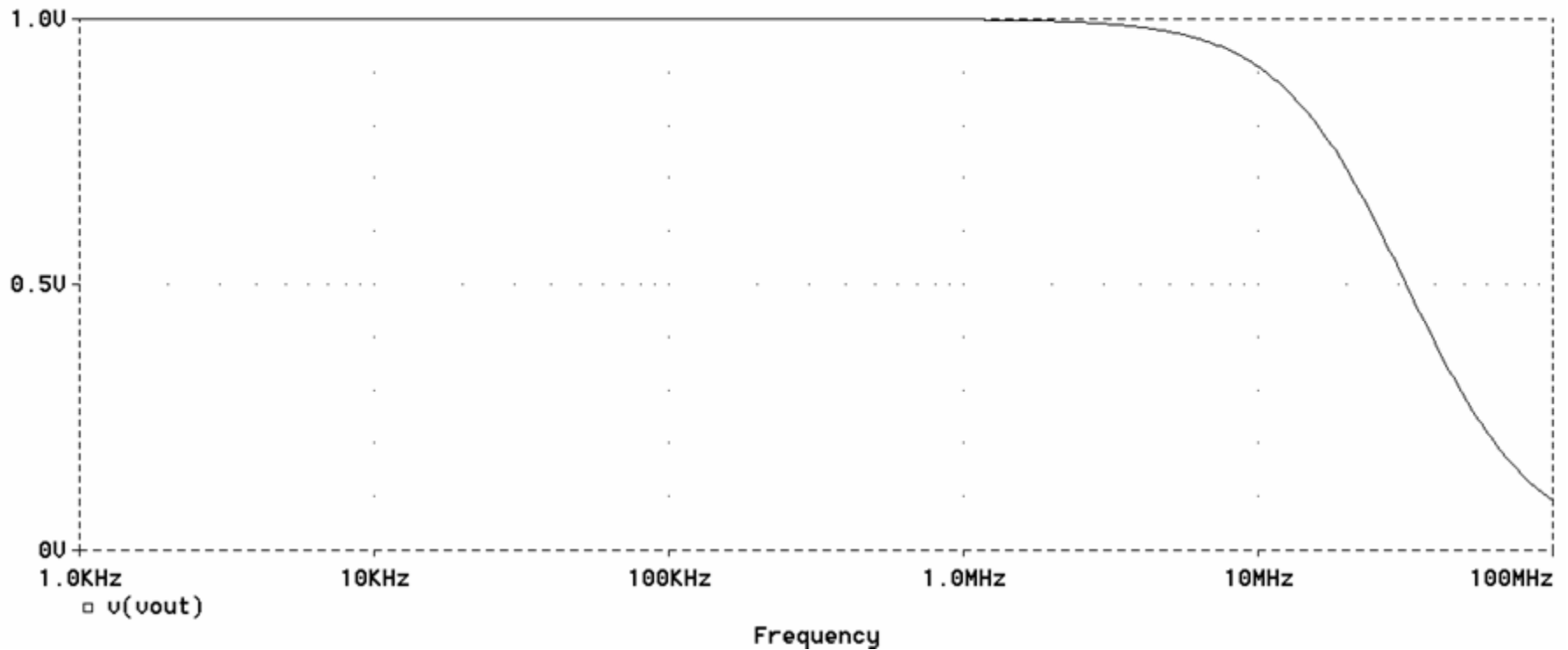
# Circuit Concepts

- Power
  - Reactive power
  - Power quality
  - Power factor
- Root Mean Square (RMS)
- Harmonics
  - Harmonic distortion

# Sinewaves

- A sinewave can be expressed as v(t) = $V_{pk}\sin(\omega t)$
- $V_{pk}$ = peak voltage
- $\omega$ = radian frequency (rad/sec)
- $\omega = 2\pi f$ where f is in Hz
- $V_{RMS} = V_{pk}/sqrt(2)$ = 120V for sinewave with peaks at ±170V
- More on RMS later



120V RMS 60 Hz sinewave

Time [sec.]
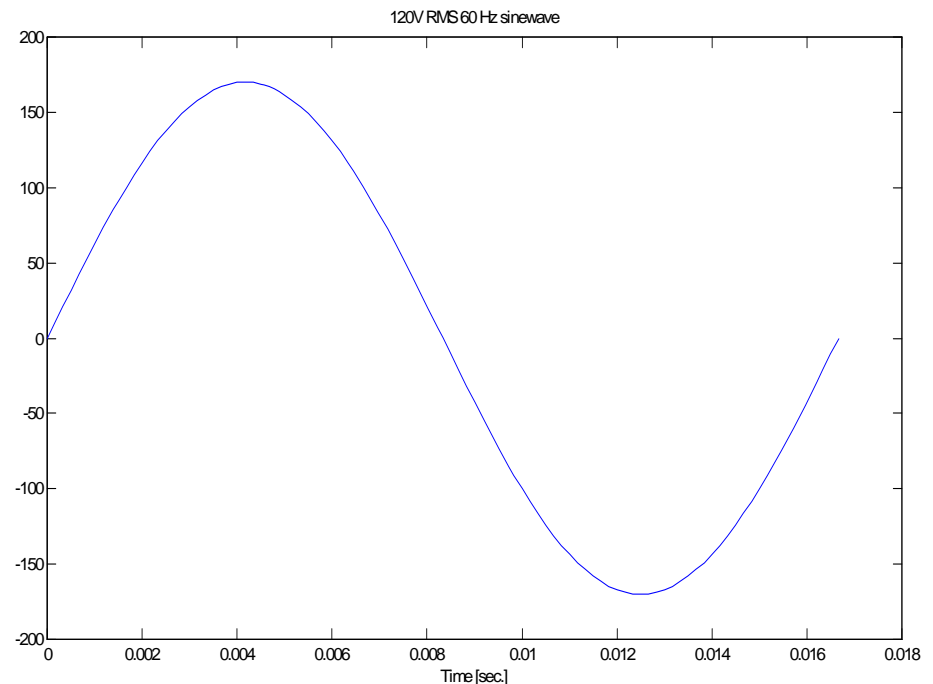
# Sinewave with Resistive Load

• v(t) and i(t) are in phase and have the same shape;
i.e. no harmonics in current

- Time representation
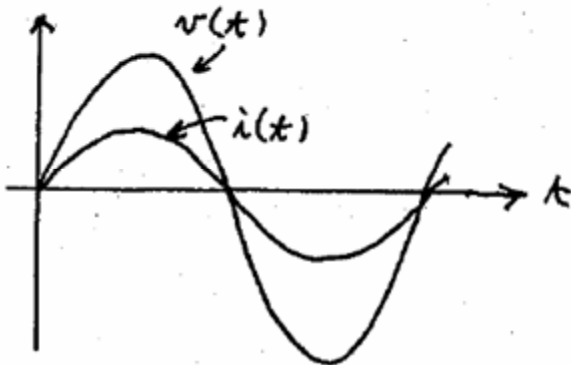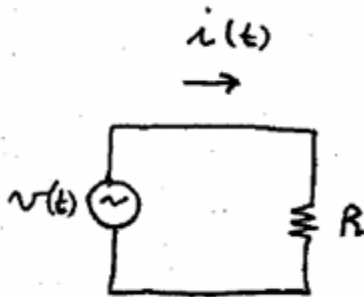


-Phasor representation
-In this case, V and I
have the same phase
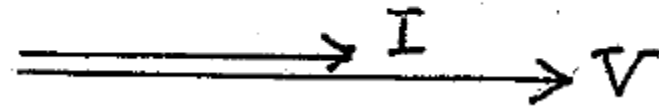
# Sinewave with Inductive Load

- For an inductor, remember that v = Ldi/dt
- So, i(t) lags v(t) by 90$^o$ in an inductor
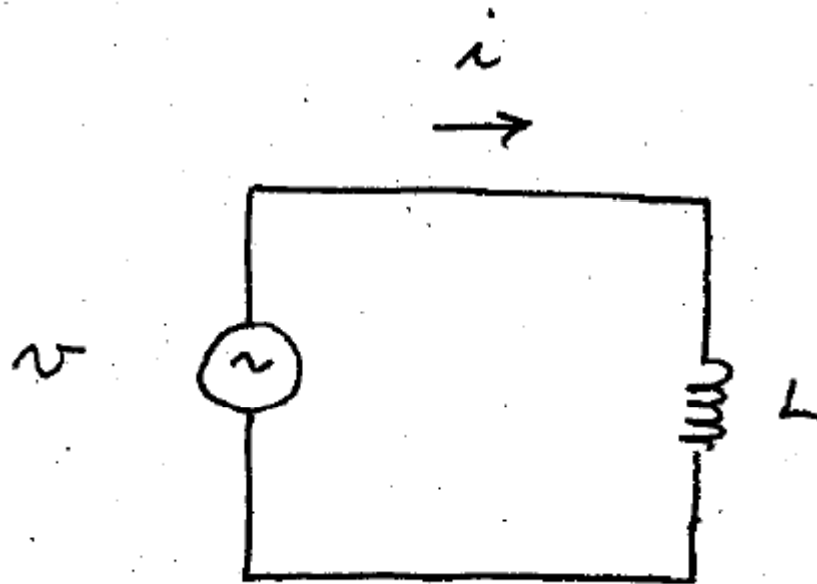
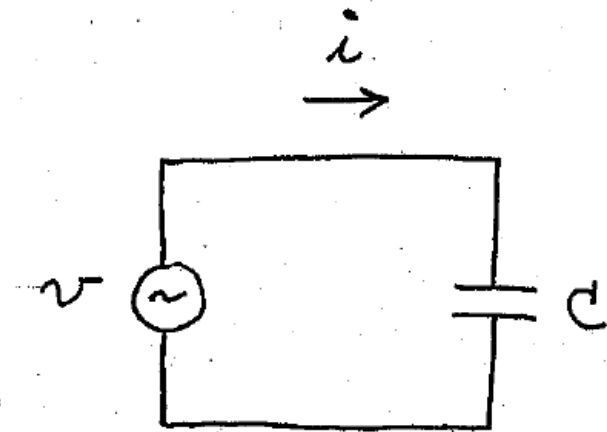# Sinewave with Inductive Load --- PSIM
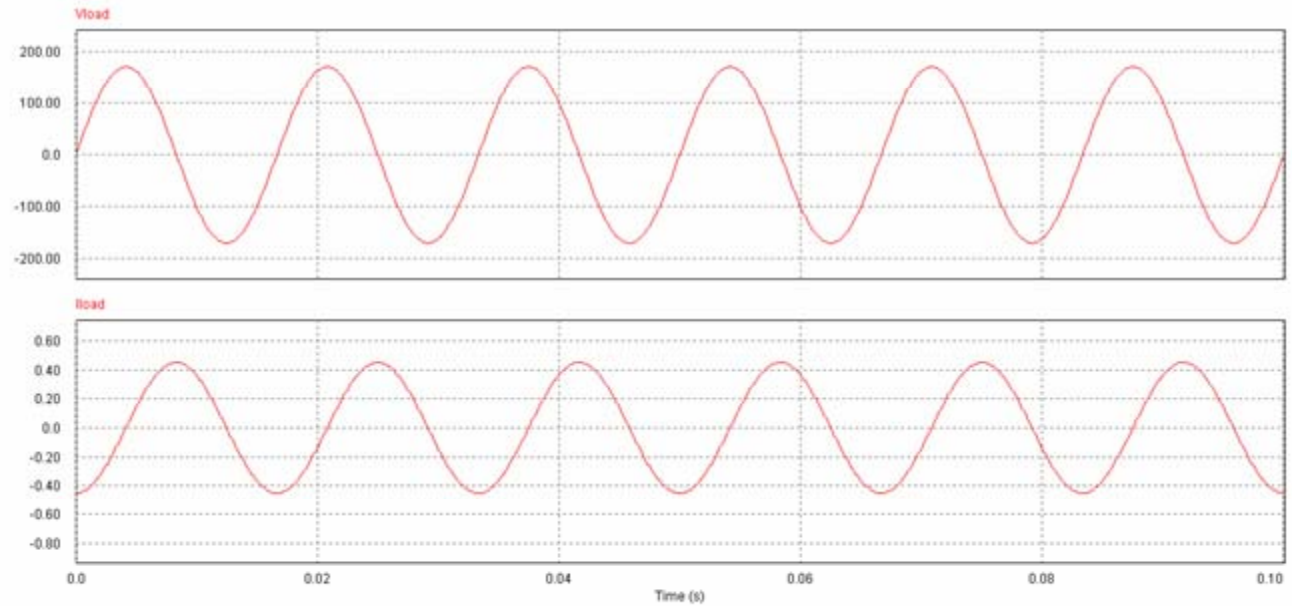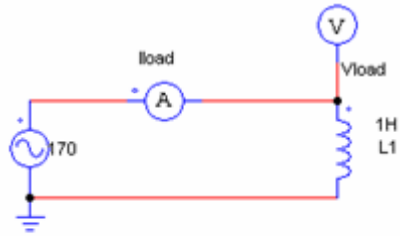
# Sinewave with L/R Load

• Phase shift (also called angle) between v and i is somewhere between $0^o$ and $-90^o$

$$\angle = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$
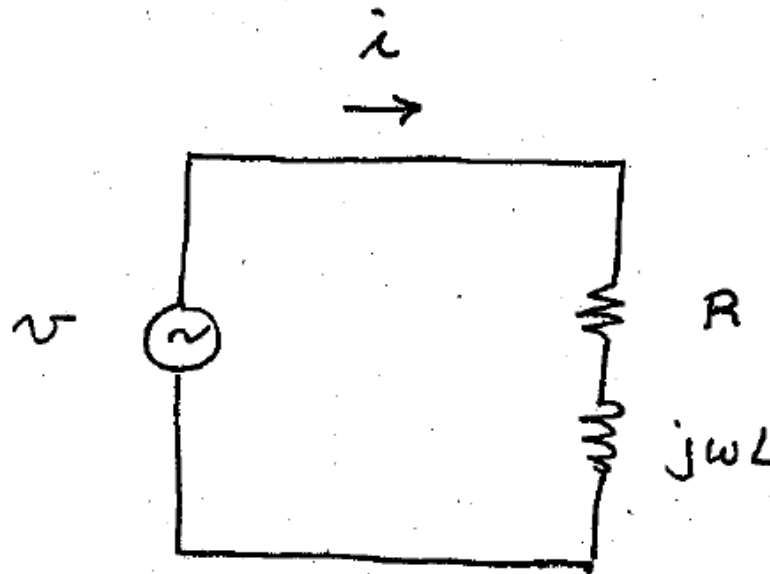
# Sinewave with Capacitive Load

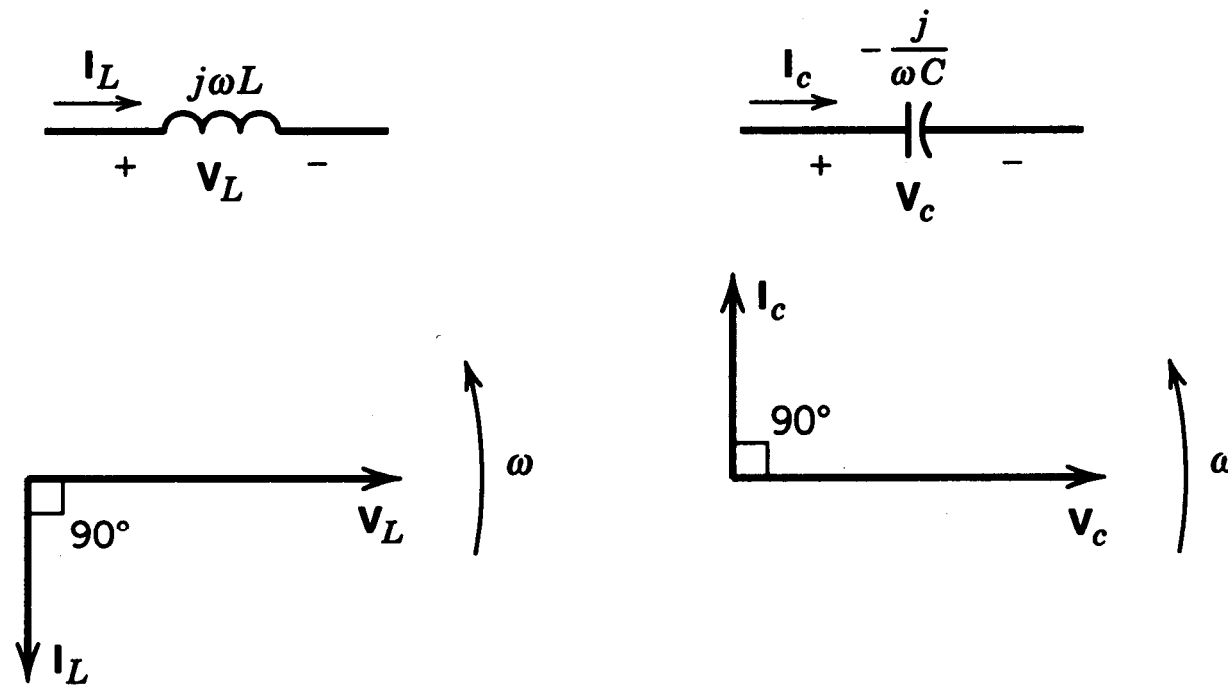- Remember that i = Cdv/dt for a capacitor
- Current leads voltage by $+90^o$



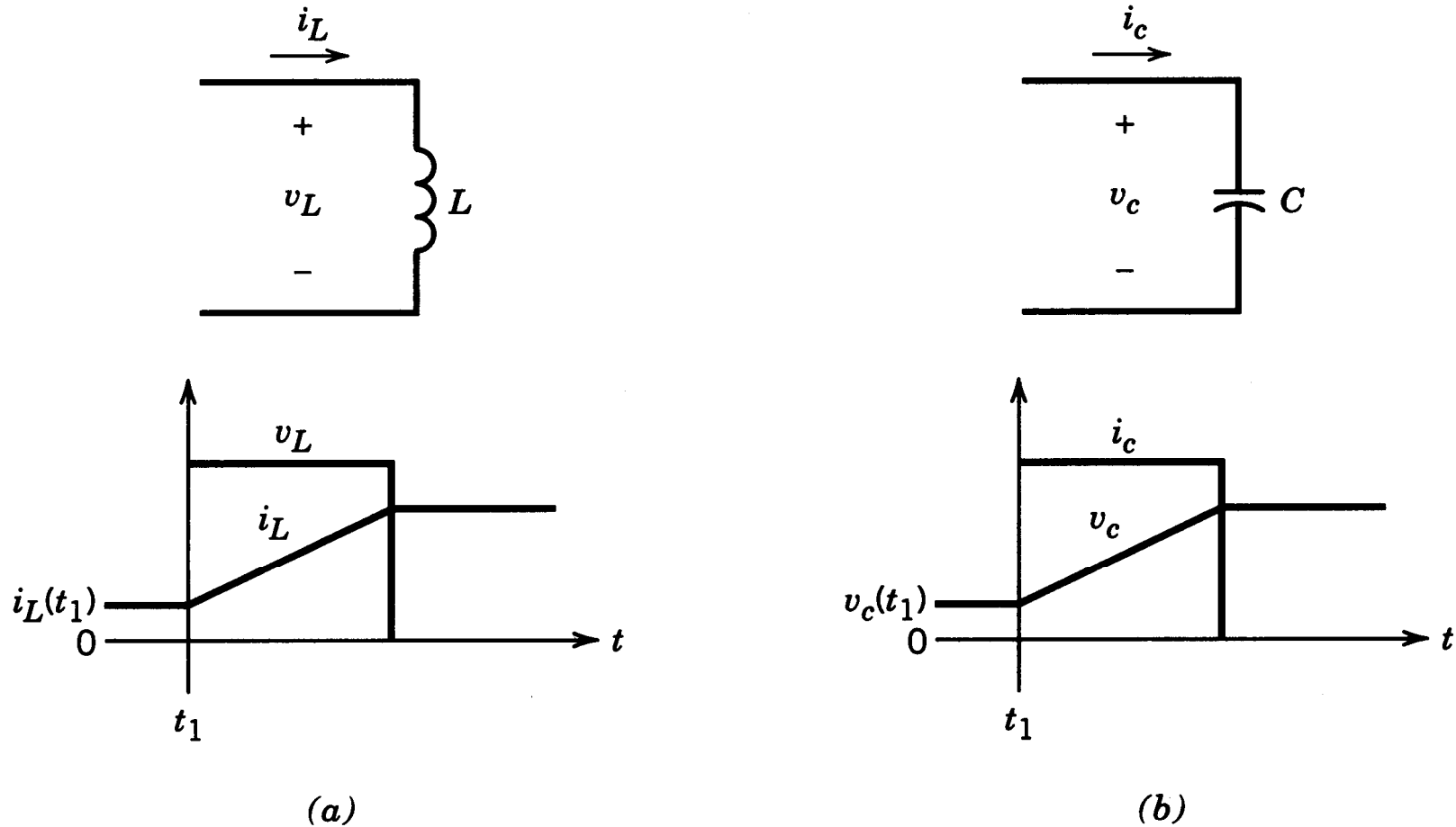## - Phasor representation
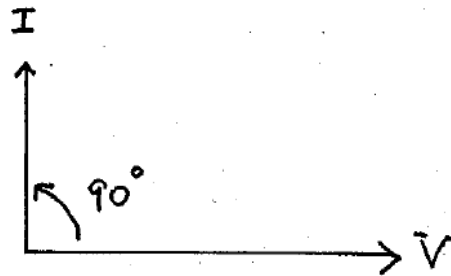
# Phasor Representation of L and C



**Figure 3-6** Phasor representation.

In inductor, current lags voltage by 90 degrees

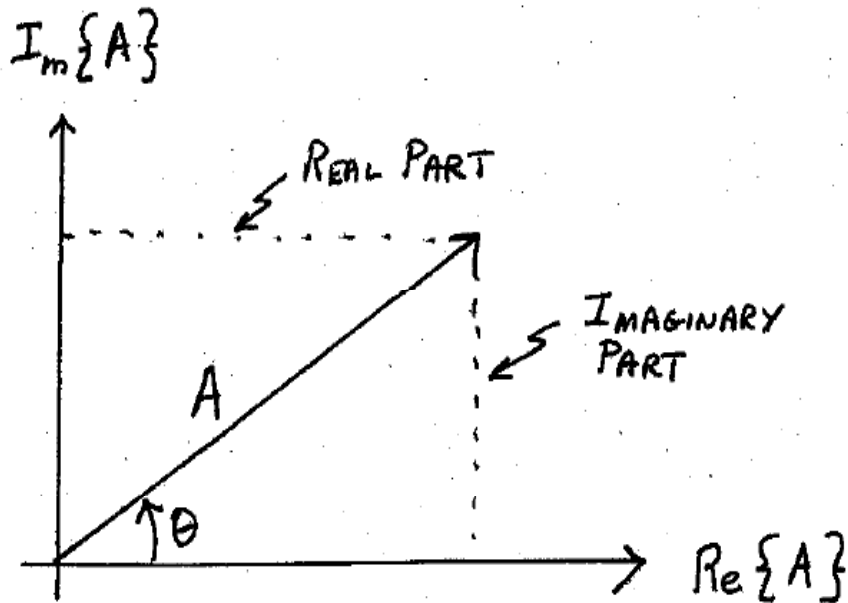In capacitor, voltage lags current by 90 degrees

# Response of *L* and *C* to pulses



Figure 3-7 Inductor and capacitor response.

# Review of Complex Numbers

• In "rectangular" form, a complex number is written in terms of real and imaginary components
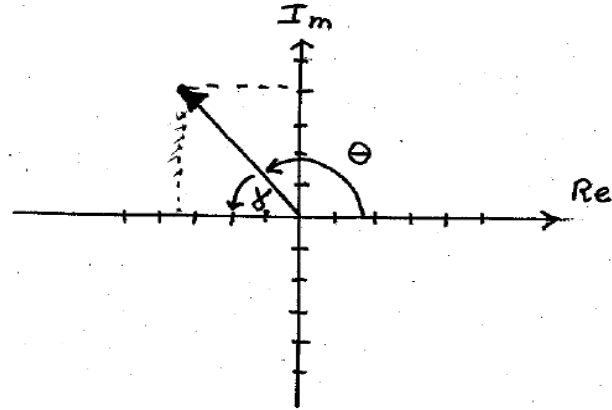
• A = Re(A) + j×Im(A)

**- Angle**

$$\theta = \tan^{-1}\left(\frac{\text{Im}(A)}{\text{Re}(A)}\right)$$

**- Magnitude of A**

$$|A| = \sqrt{\left(\text{Re}(A)\right)^2 + \left(\text{Im}(A)\right)^2}$$

# Find Polar Form

- Assume that current I = -3.5 + j(4.2)



$$|I| = \sqrt{(-3.5)^2 + (4.2)^2} = 5.5A$$
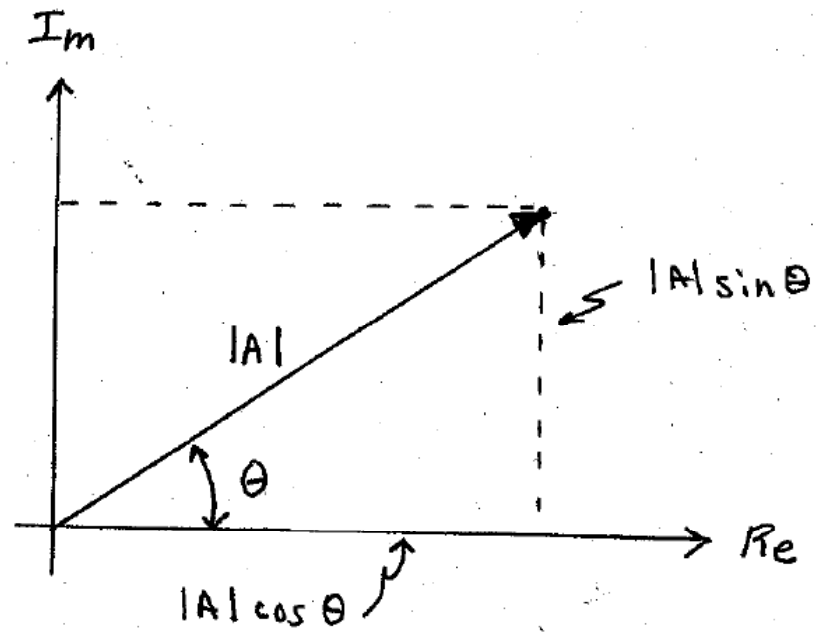
$$\theta = 180^o - \gamma$$

$$\gamma = \tan^{-1}\left(\frac{4.2}{3.5}\right) = 50.2^o$$

$$\theta = 180^o - 50.2^o = 129.8^o$$

$$\therefore 5.5A\angle 129.8^o$$

# Converting from Polar to Rectangular Form



$$\mathrm{Re}\{A\} = |A|\cos(\theta)$$

$$\mathrm{Im}\{A\} = |A|\sin(\theta)$$

# Power

- "Power" has many shapes and forms
  - Real power
  - Reactive power
    - Reactive power does <u>not</u> do real work
  - Instantaneous power

$$p(t) = v(t)i(t)$$

  - Peak instantaneous power
  - Average power

$$\langle p(t) \rangle = \frac{1}{T} \int_0^T v(t)i(t)dt$$
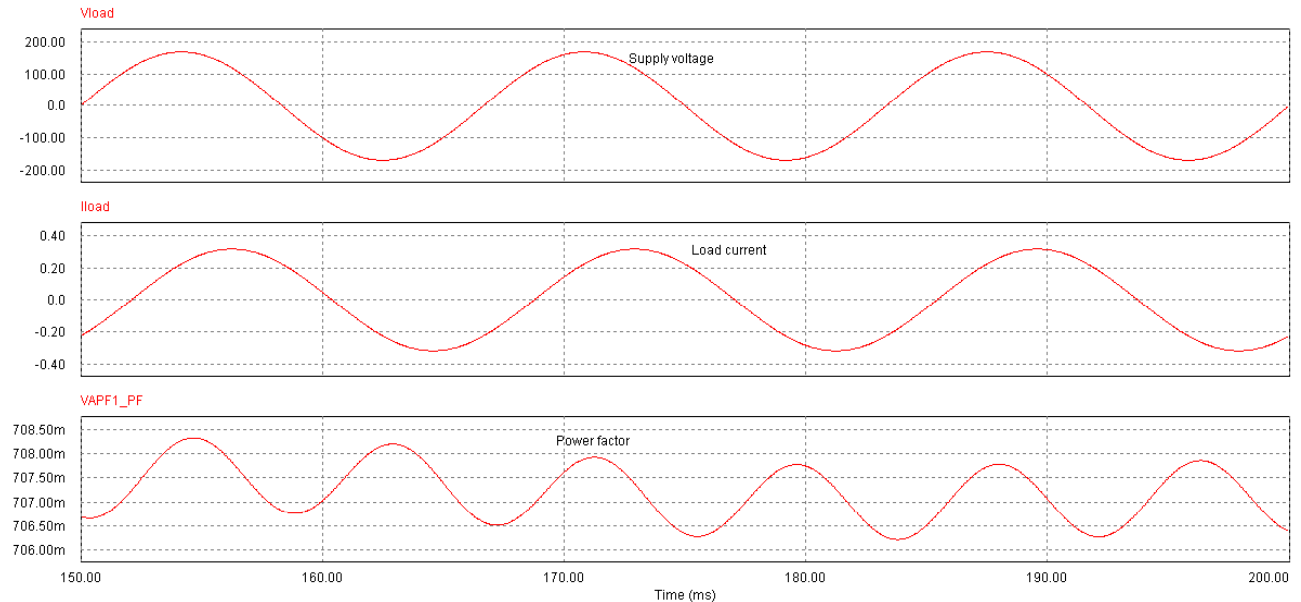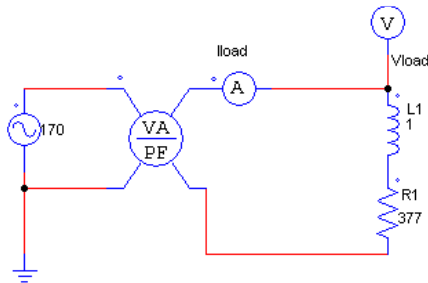
# Power Factor

- Ratio of delivered power to the product of RMS voltage and RMS current

$$PF = \frac{<P>}{V_{RMS} I_{RMS}}$$

- Power factor always <= 1
- With pure sine wave and resistive load, PF = 1
- With pure sine wave and purely reactive load, PF = 0
- Whenever PF < 1 the circuit carries currents or voltages that do not perform useful work
- The more "spikey" a waveform is the worse is its PF
  – Diode rectifiers have poor power factor
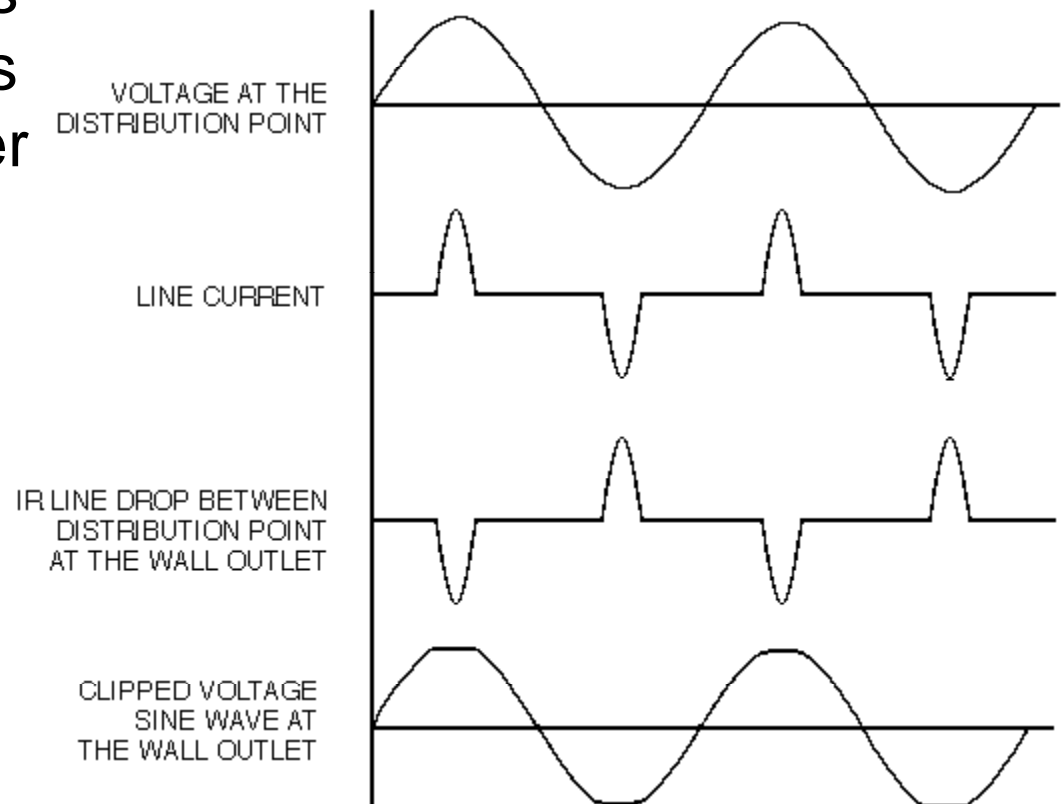- Power factor can be helped by "power factor correction"

# Causes of Low Power Factor--- L/R Load

- Power angle is $\theta = \tan^{-1}(\omega L/R)$
- For L = 1H, R = 377 Ohms, $\theta = 45^o$ and PF = $\cos(45^o)$ = 0.707

# Causes of Low Power Factor --- Non-linear Load

- Nonlinear loads include:
  - Variable-speed drives
  - Frequency converters
  - Uninterruptable power supplies (UPS)
  - Saturated magnetic circuits
  - Dimmer switches
  - Televisions
  - Fluorescent lamps
  - Welding sets
  - Arc furnaces
  - Semiconductors
  - Battery chargers

VOLTAGE AT THE DISTRIBUTION POINT

LINE CURRENT

IR LINE DROP BETWEEN DISTRIBUTION POINT AT THE WALL OUTLET

CLIPPED VOLTAGE SINE WAVE AT THE WALL OUTLET

# Half Wave Rectifier with RC Load

• In applications where cost is a major consideration, a capacitive filter may be used.
• If RC >> 1/f then this operates like a peak detector and the output voltage $<v_{out}>$ is approximately the peak of the input voltage
• Diode is only ON for a short time near the sinewave peaks
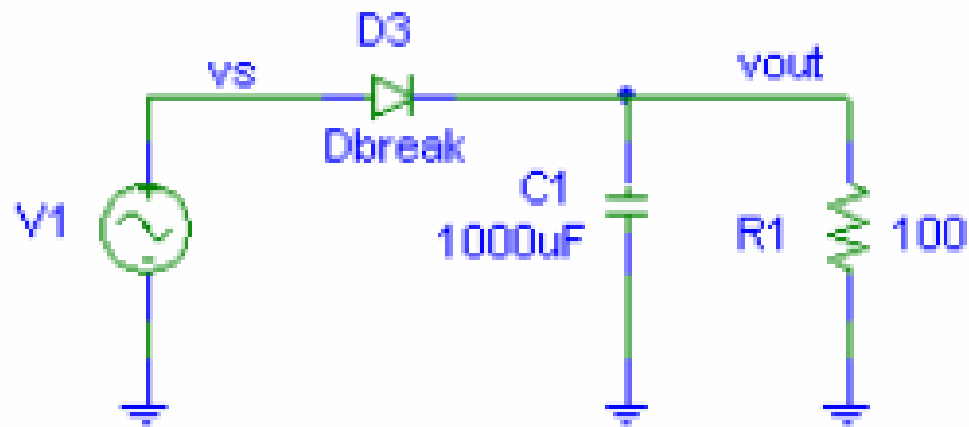
# Half Wave Rectifier with RC Load

- Note poor power factor due to peaky input line current
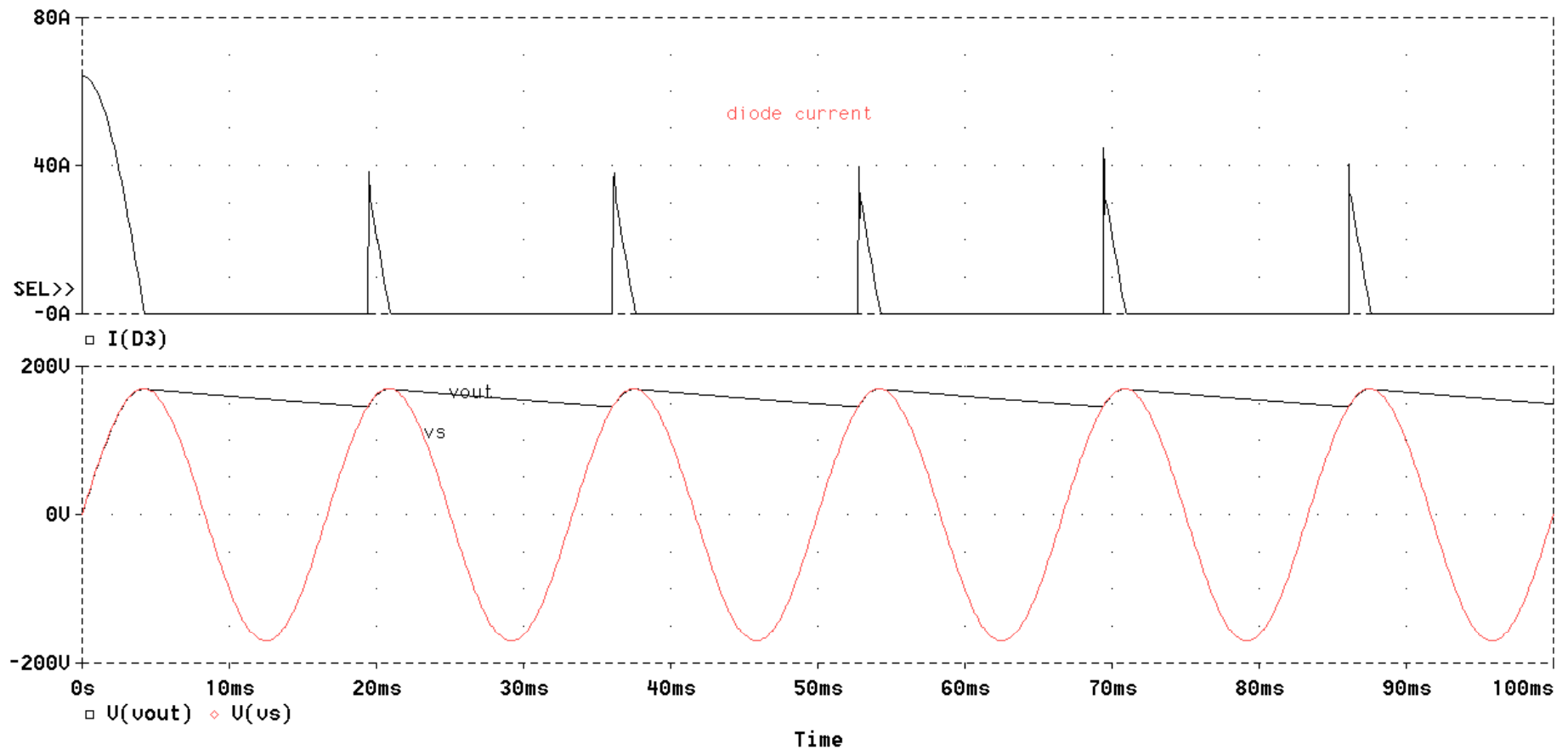
# Unity Power Factor --- Resistive Load

- Example: purely resistive load
  - Voltage and currents in phase

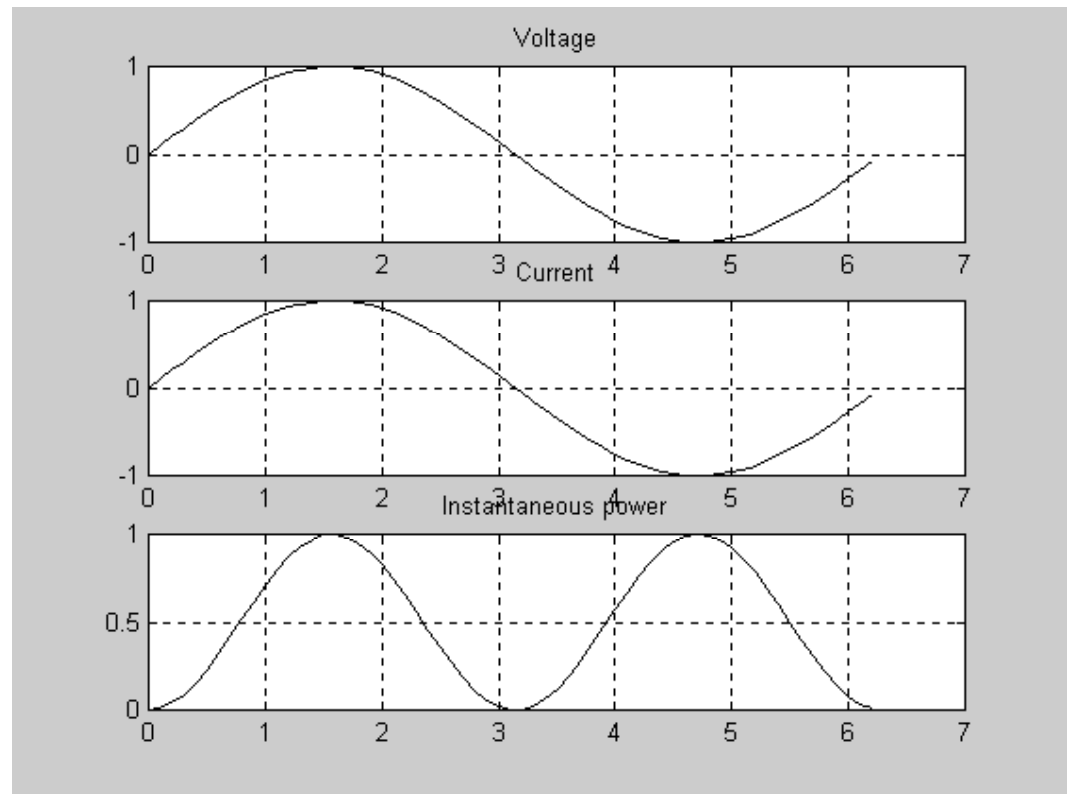$$v(t) = V \sin \omega t$$

$$i(t) = \frac{V}{R} \sin \omega t$$

$$p(t) = v(t)i(t) = \frac{V^2}{R} \sin^2 \omega t$$

$$< p(t) >= \frac{V^2}{2R}$$

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

$$I_{RMS} = \frac{V}{R\sqrt{2}}$$

$$PF = \frac{< p(t) >}{V_{RMS} I_{RMS}} = \frac{\dfrac{V^2}{2R}}{\left(\dfrac{V}{\sqrt{2}}\right)\left(\dfrac{V}{R\sqrt{2}}\right)} = 1$$

# Causes of Low Power Factor --- Reactive Load

- Example: purely inductive load
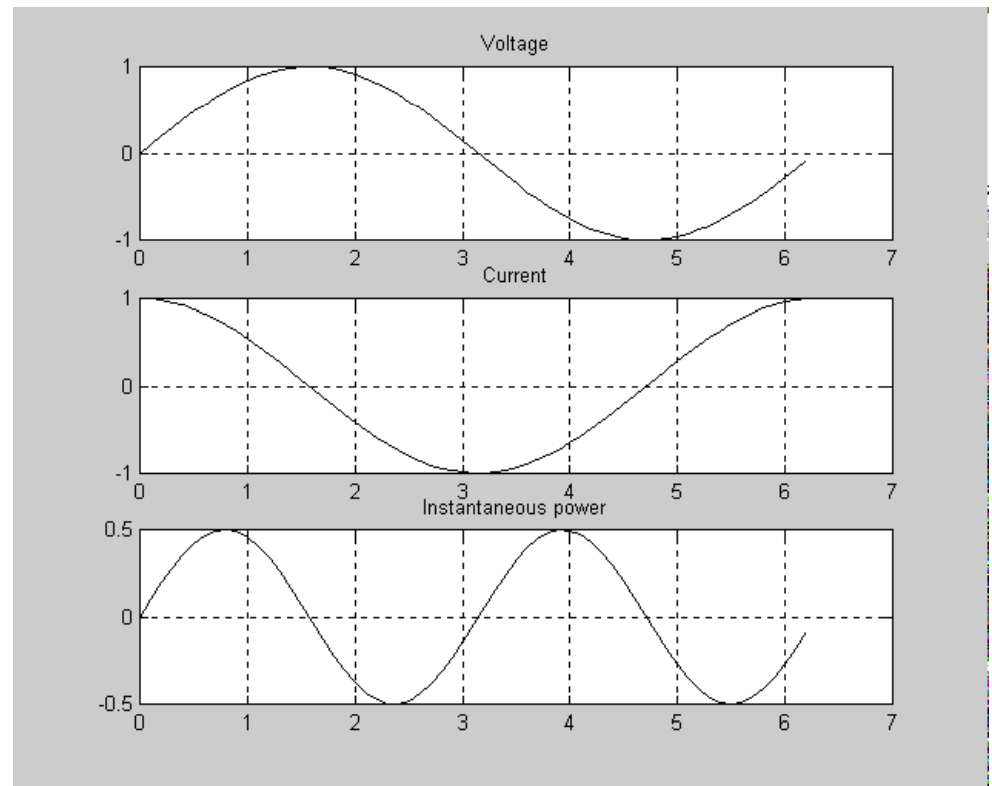  - Voltage and currents 90º out of phase

$$v(t) = V \sin \omega t$$

$$i(t) = \frac{V}{\omega L} \cos \omega t$$

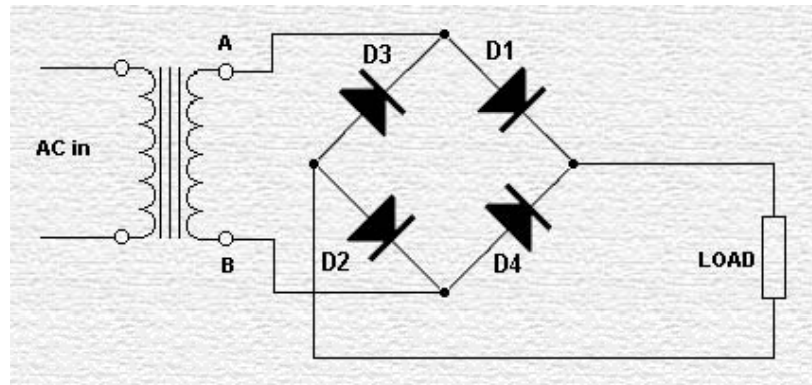$$p(t) = v(t) i(t) = \frac{V^2}{\omega L} \sin \omega t \cos \omega t$$

$$< p(t) >= 0$$

- For purely reactive load, PF=0

# Why is Power Factor Important?

- Consider peak-detector full-wave rectifier



- Typical power factor $k_p$ = 0.6
- What is maximum power you can deliver to load ?
  - $V_{AC}$ x current x $k_p$ x rectifier efficiency
  - (120)(15)(0.6)(0.98) = 1058 Watts
- Assume you replace this simple rectifier by power electronics module with 99% power factor and 93% efficiency:
  - (120)(15)(0.99)(0.93) = 1657 Watts

# Power Factor Correction

- Typical toaster can draw 1400W from a 120VAC/15A line
- Typical offline switching converter can draw <1000W because it has poor power factor
- High power factor results in:
  - Reduced electric utility bills
  - Increased system capacity
  - Improved voltage
  - Reduced heat losses
- Methods of power factor correction
  - Passive
    - Add capacitors across an inductive load to resonate
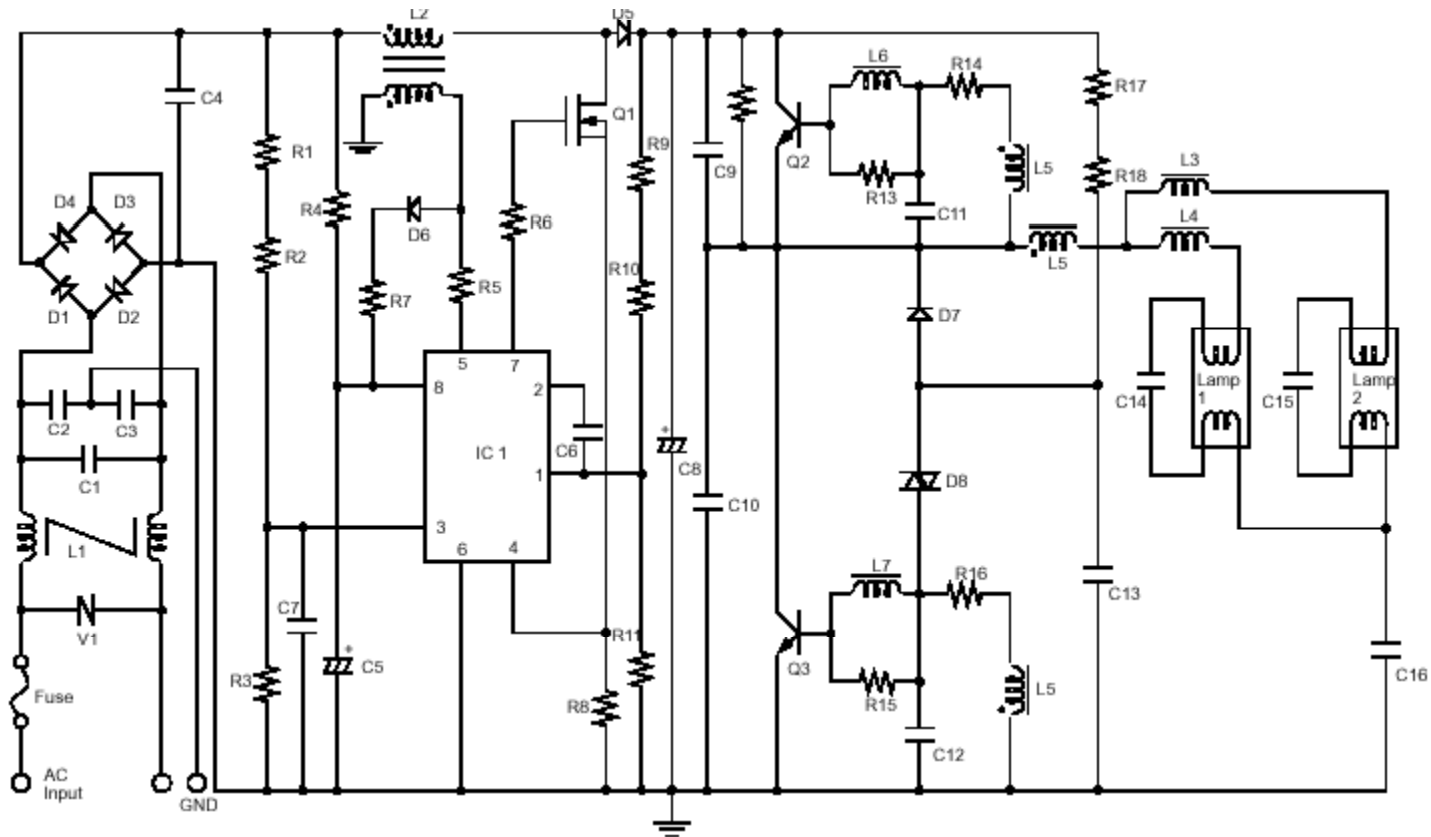    - Add inductance in a capacitor circuit
  - Active

# Power Factor Correction --- Passive

- Switch capacitors in and out as needed as load changes

# Power Factor Correction --- Active

- Fluorescent lamp ballast application

# Root Mean Square (RMS)

- Used for description of periodic, often multi-harmonic, waveforms
- Square root of the average over a cycle (mean) of the square of a waveform

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

- RMS current of any waveshape will dissipate the same amount of heat in a resistor as a DC current of the same value
  - DC waveform: $V_{rms} = V_{DC}$
  - Symmetrical square wave:
    - $I_{RMS} = I_{pk}$
  - Pure sine wave
    - $I_{RMS} = 0.707 I_{pk}$
- Example: 120 VRMS line voltage has peaks of $\pm169.7$ V

# Intuitive Description of RMS

- The RMS value of a sinusoidal or other periodic waveform dissipates the same amount of power in a resistive load as does a battery of the same RMS value

- So, 120VRMS into a resistive load dissipates as much power in the load as does a 120V battery
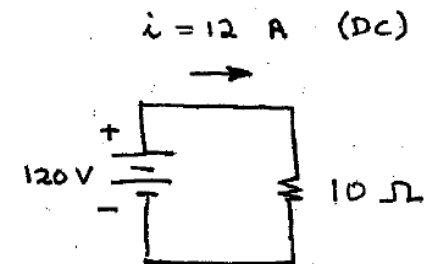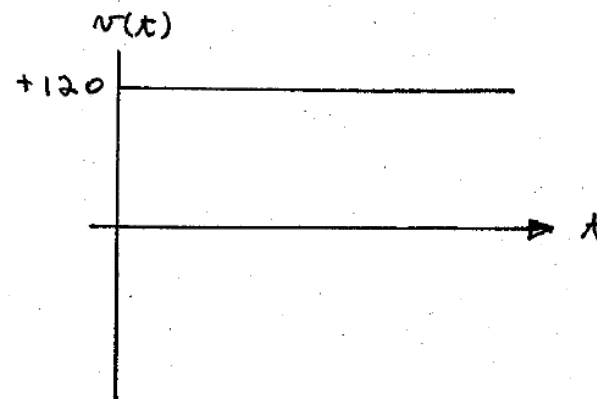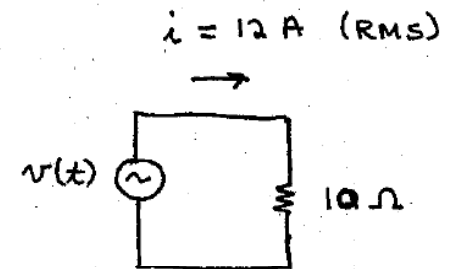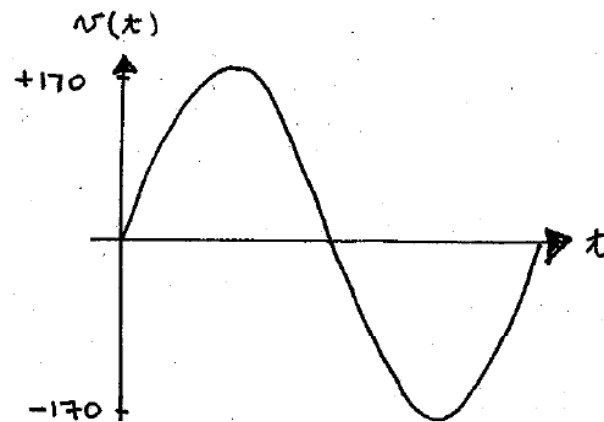
# RMS Value of Various Waveforms

• Following are a bunch of waveforms typically found in power electronics, power systems, and motors, and their corresponding RMS values
• Reference:  R. W. Erickson and D. Maksimovic, _Fundamentals of Power Electronics_, 2$^{nd}$ edition, Kluwer, 2001
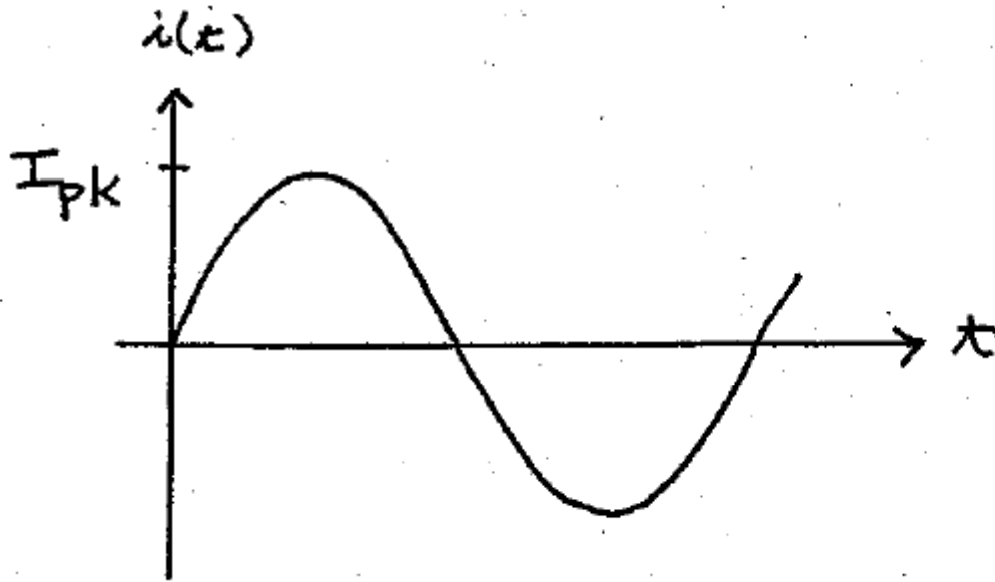
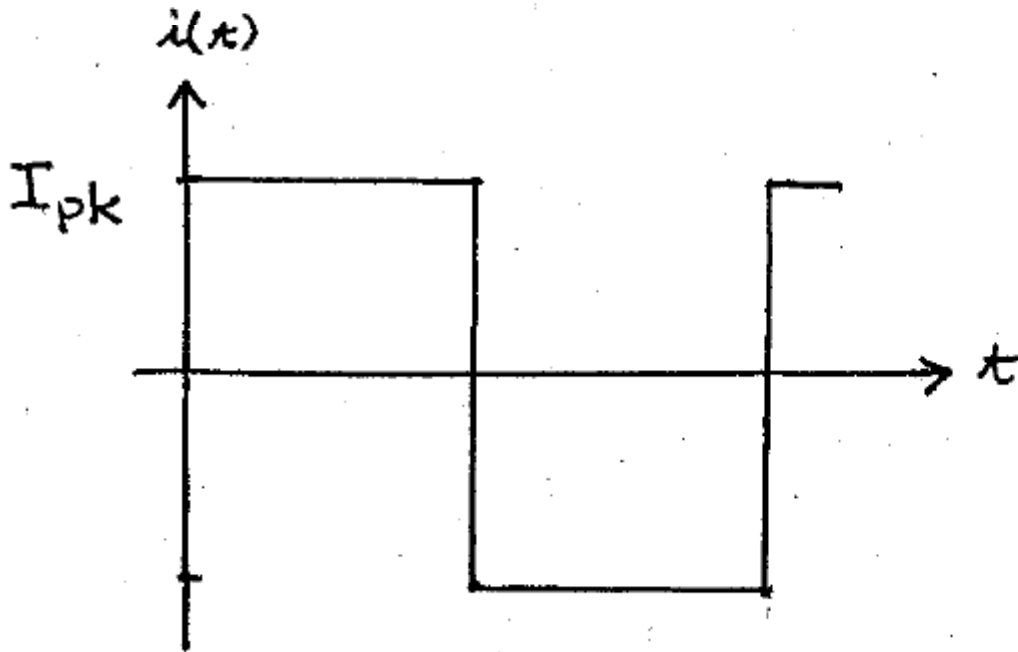# DC Voltage

• Battery
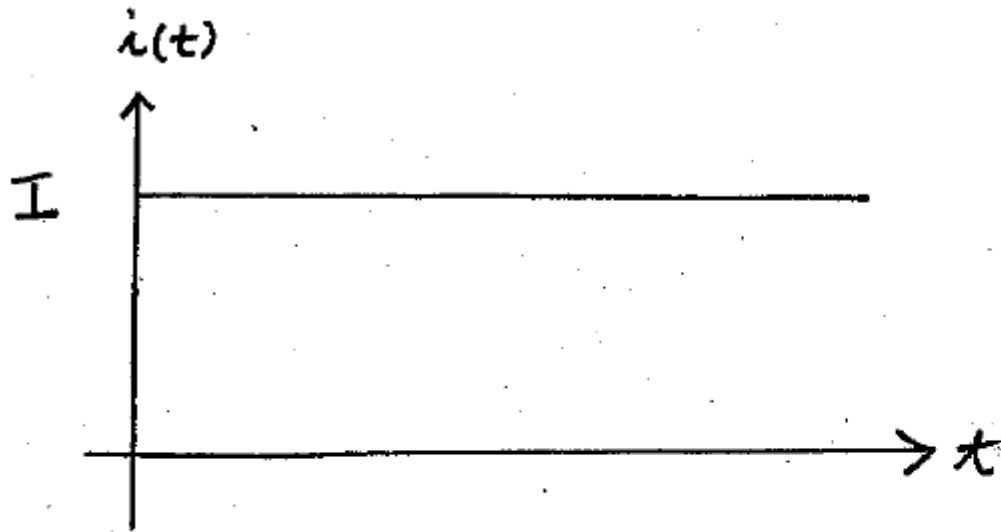


$$RMS = I$$

# Sinewave

- AC line
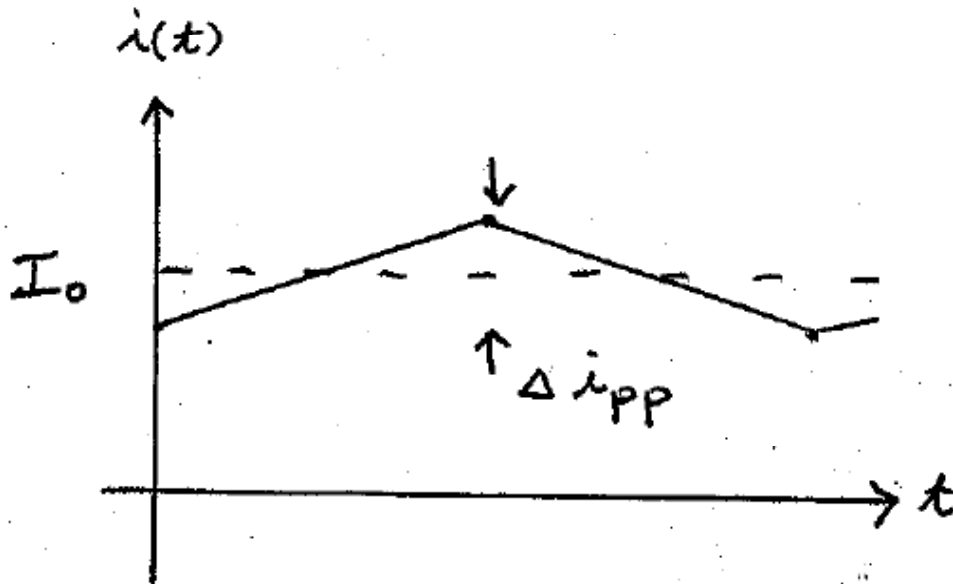


$$RMS = \frac{I_{pk}}{\sqrt{2}}$$

# Square Wave

• This type of waveform can be put out by a square wave converter or full-bridge converter
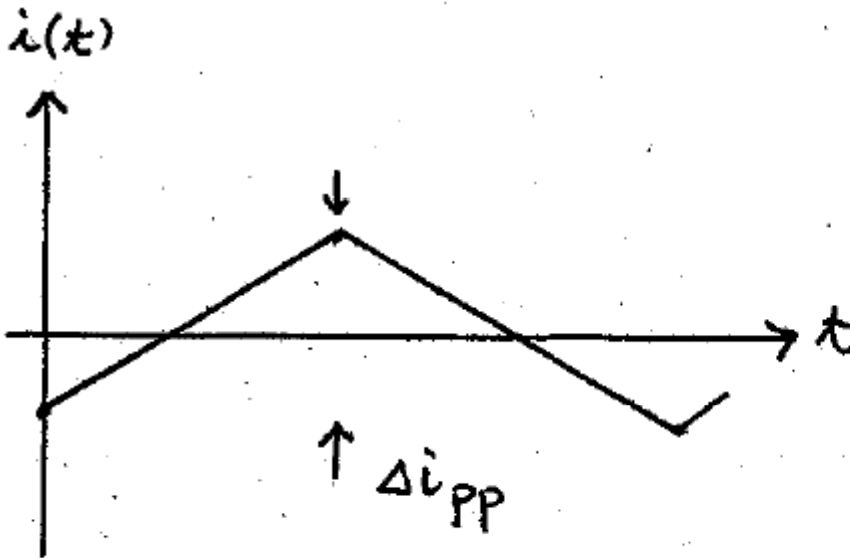


$Rms = I_{pk}$

# DC with Ripple

- Buck converter inductor current (DC value + ripple)

$$RMS = I_o \sqrt{1 + \left(\tfrac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2 I_o}\right)^2}$$
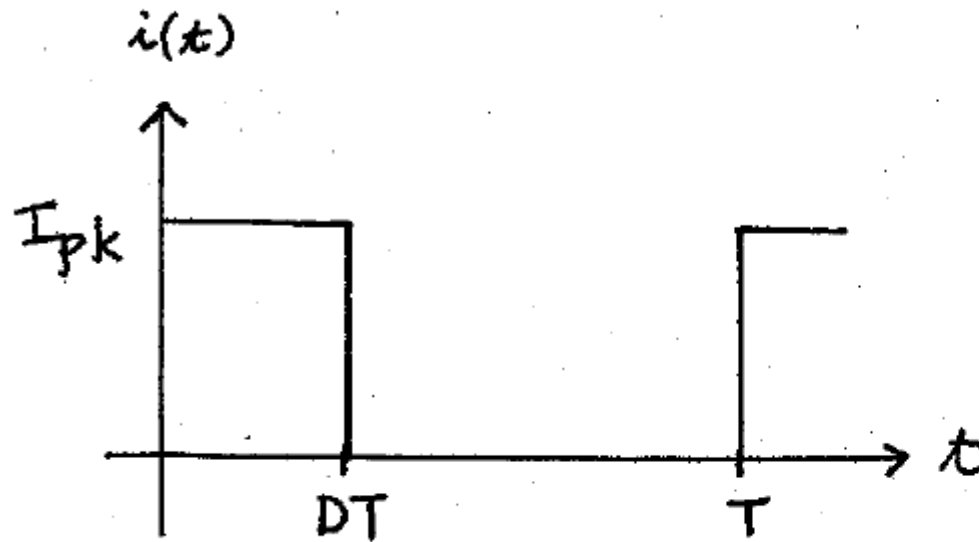
# Triangular Ripple

- Capacitor ripple current in some converters (no DC value)



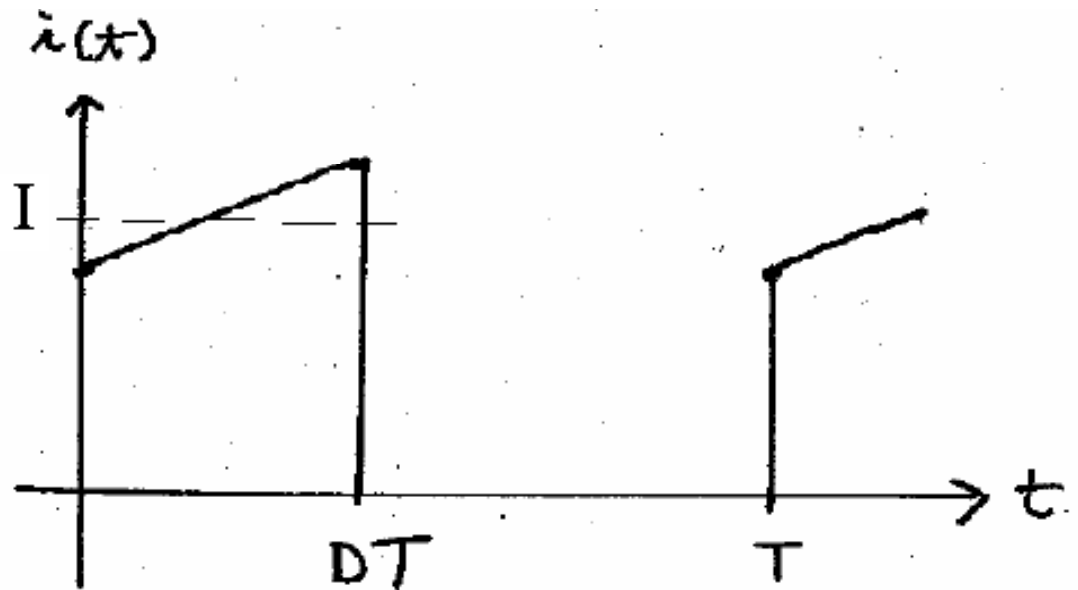$$RMS = \frac{\Delta i_{PP}}{2\sqrt{3}}$$

# Pulsating Waveform

• Buck converter input switch current (assuming small ripple)
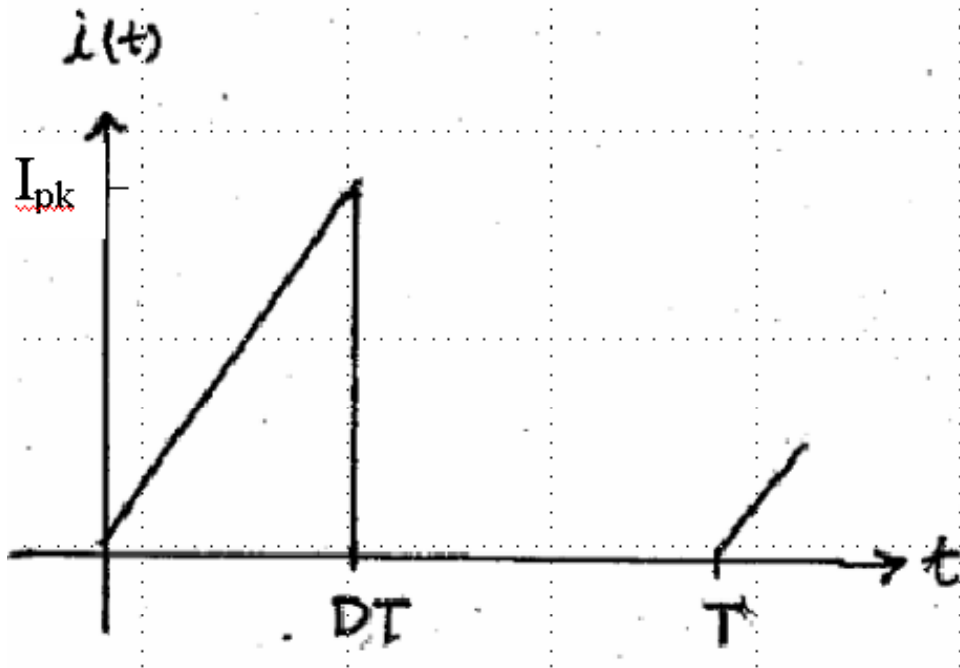


$$RMS = I_{pk}\sqrt{D}$$

$$0 < D < 1$$

# Pulsating with Ripple

- i.e. buck converter switch current
- We can use this result to get RMS value of buck diode current

$i(t)$

$I$

$DT$ $T$ $t$

$$RMS = I\sqrt{D}\sqrt{1 + \left(\tfrac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2I}\right)^2}$$

# Triangular



$$RMS = I_{pk} \sqrt{\frac{D}{3}}$$

# Piecewise Calculation

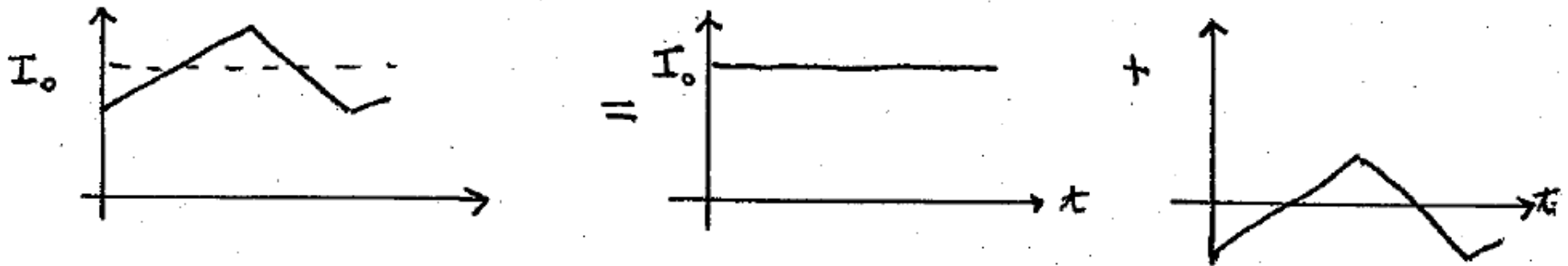- This works if the different components are at different frequencies

$$\text{For } I = I_1 + I_2 + I_3 \ldots$$

then

$$I_{RMS} = \sqrt{I_{1,RMS}^2 + I_{2,RMS}^2 + \cdots}$$

# Piecewise Calculation --- Example

- What is RMS value of DC + ripple (shown before)?

$$RMS = \sqrt{I_o^2 + \left(\frac{\Delta i_{pp}}{2\sqrt{3}}\right)^2} = I_o\sqrt{1 + \left(\frac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2I_o}\right)^2}$$

# Harmonics

- Harmonics are created by nonlinear circuits
    - Rectifiers
        - Half-wave rectifier has first harmonic at 60 Hz
        - Full-wave has first harmonic at 120 Hz
    - Switching DC/DC converters
        - DC/DC operating at 100 kHz generally creates harmonics at DC, 100 kHz, 200 kHz, 300 kHz, etc.
- Line harmonics can be treated by line filters
    - Passive
    - Active

# Total Harmonic Distortion

- Total harmonic distortion (THD)
  - Ratio of the RMS value of all the nonfundamental frequency terms to the RMS value of the fundamental

$$THD = \sqrt{\frac{\sum_{n\neq 1} I_{n,RMS}^2}{I_{1,RMS}^2}} = \sqrt{\frac{I_{RMS}^2 - I_{1,RMS}^2}{I_{1,RMS}^2}}$$
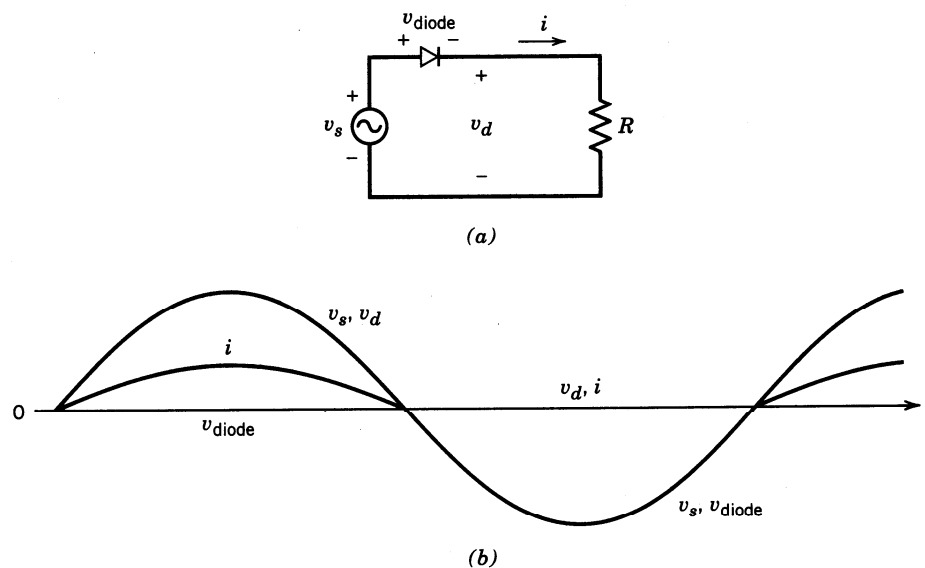
- Symmetrical square wave: THD = 48.3%

$$I_{RMS} = 1$$

$$I_1 = \frac{4}{\sqrt{2}\pi}$$

$$THD = \sqrt{\frac{1 - \left(\frac{4}{\sqrt{2}\pi}\right)^2}{\left(\frac{4}{\sqrt{2}\pi}\right)^2}} = 0.483$$

- Symmetrical triangle wave: THD = 12.1%

# Half-Wave Rectifier, Resistive Load

- Simplest, cheapest rectifier
- Line current has DC component; this current appears in neutral
- High harmonic content, Power factor = 0.7



(a)

(b)

**Figure 5-2**  Basic rectifier with a load resistance.

$$P.F. = \frac{P_{avg}}{V_{RMS} I_{RMS}}$$

# Half Wave Rectifier with Resistive Load --- Power Factor and Average Output Voltage

Average output voltage:

$$< v_d >= \frac{1}{2\pi} \int_0^\pi V_{pk} \sin(\omega t) d(\omega t) = \frac{V_{pk}}{2\pi} \left[ -\cos(\omega t) \right]_{\omega t=0}^{\omega t=\pi} = \frac{V_{pk}}{\pi}$$

Power factor calculation:

$$< P >= \frac{1}{2} \left( \frac{I_{pk}}{\sqrt{2}} \right)^2 R = \frac{I_{pk}^2}{4} R$$

$$V_{RMS} = \frac{V_{pk}}{\sqrt{2}} = \frac{R I_{pk}}{\sqrt{2}}$$

$$I_{RMS} = \frac{I_{pk}}{\sqrt{2}} \frac{1}{\sqrt{2}}$$

$$PF = \frac{< P >}{V_{RMS} I_{RMS}} = \frac{\dfrac{I_{pk}^2}{4} R}{\left( \dfrac{I_{pk}}{R\sqrt{2}} \right) \left( \dfrac{I_{pk}}{\sqrt{2}} \dfrac{1}{\sqrt{2}} \right)} = 0.707$$

# Half-Wave Rectifier, Resistive Load --- Spectrum of Load Voltage
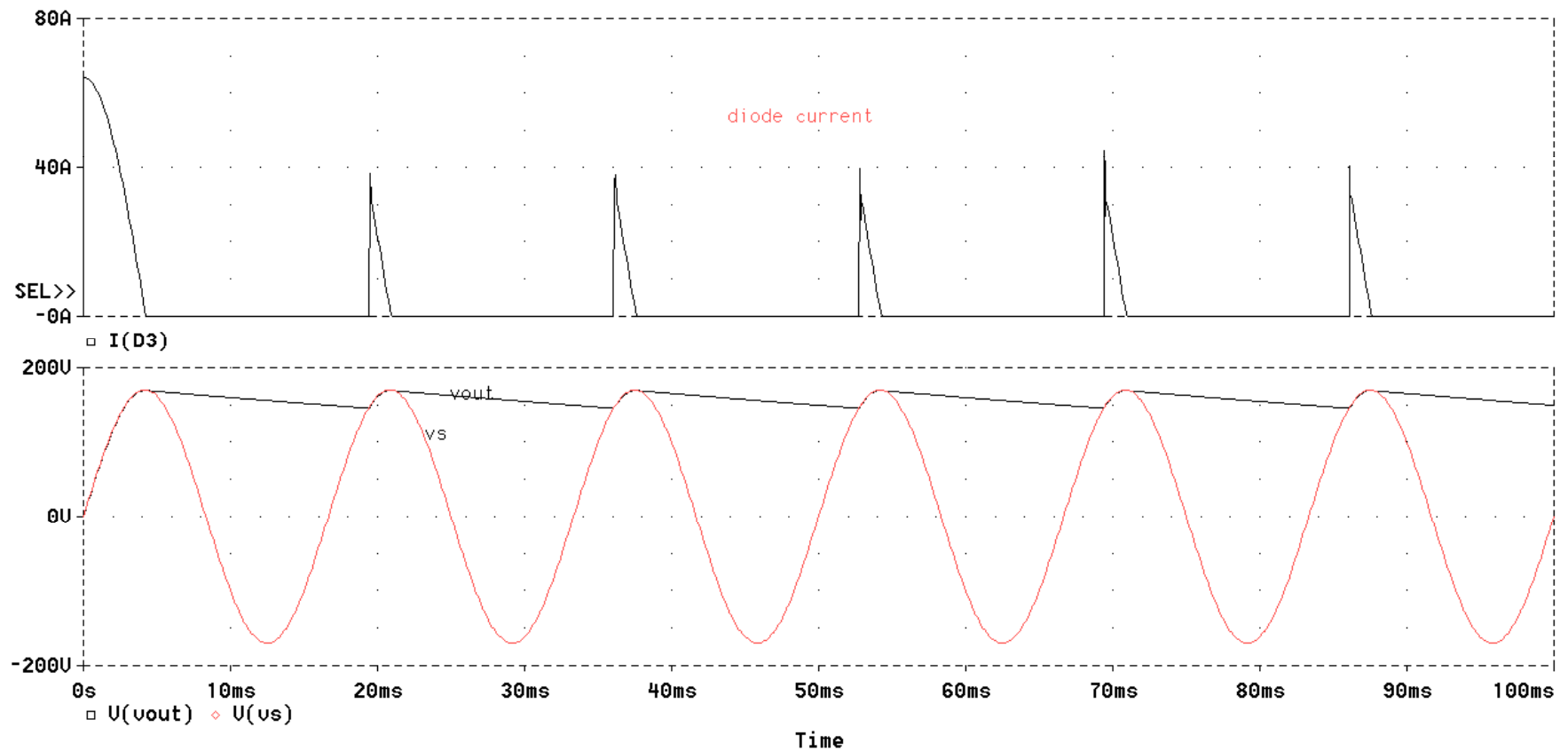


Spectrum of output of half-wave rectifier
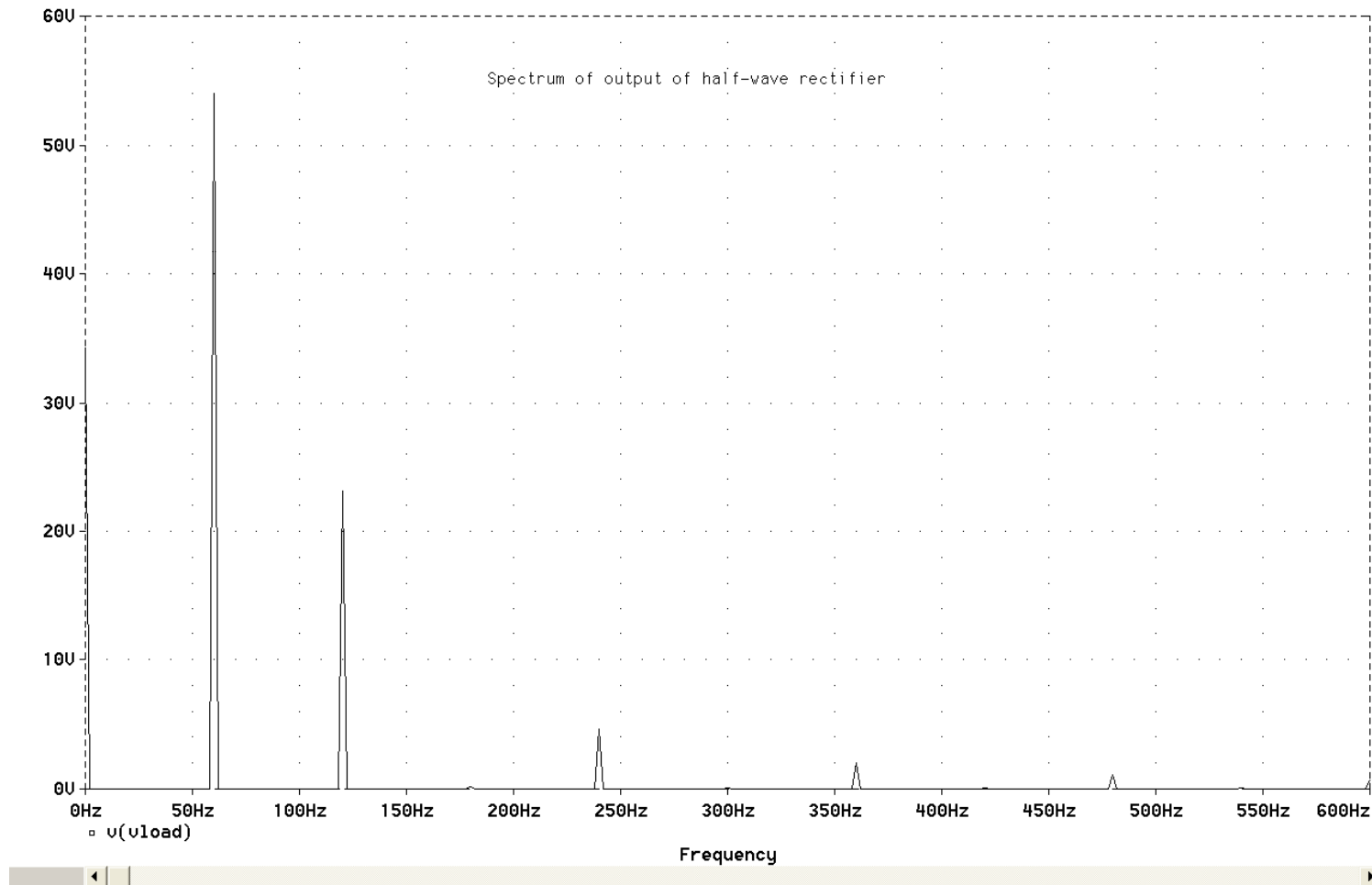
# Half Wave Rectifier with RC Load

- More practical rectifier
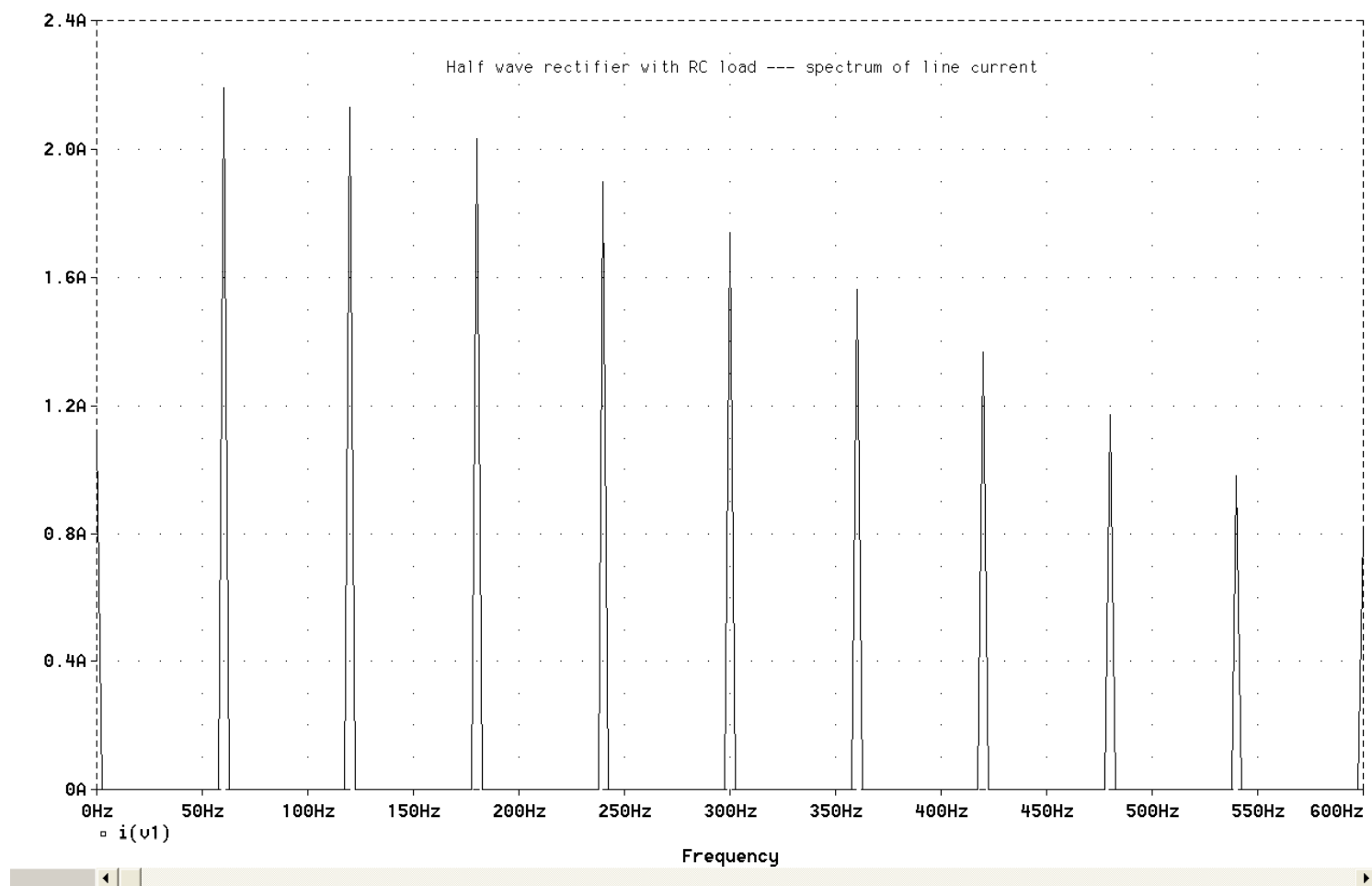- For large RC, this behaves like a peak detector
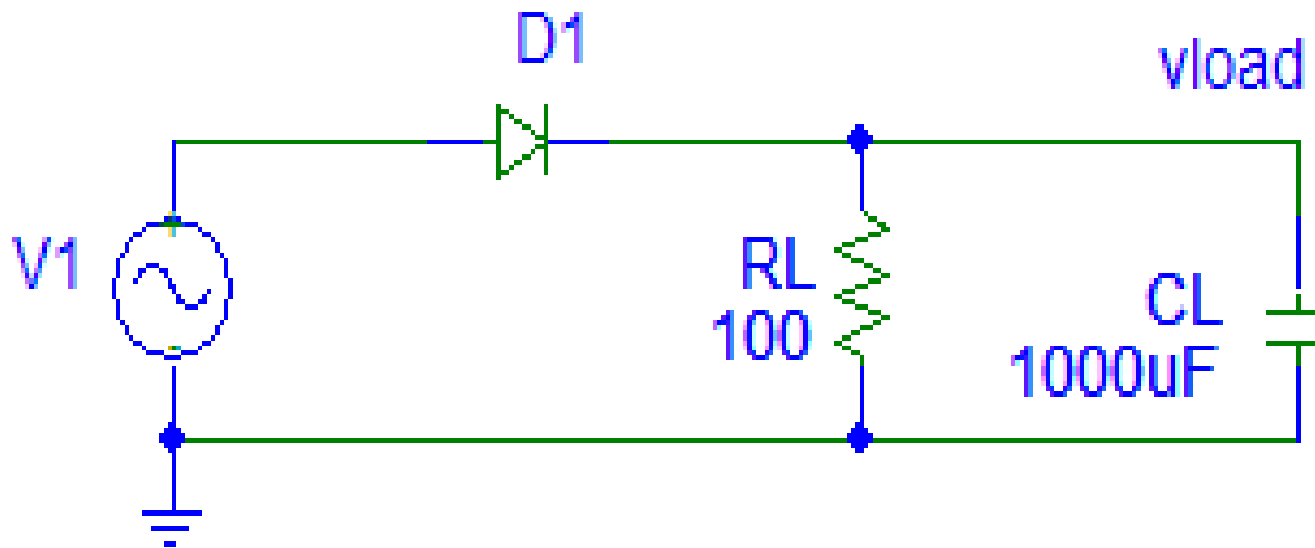
# Half Wave Rectifier with RC Load

- Note poor power factor due to peaky line current
- Note DC component of line current

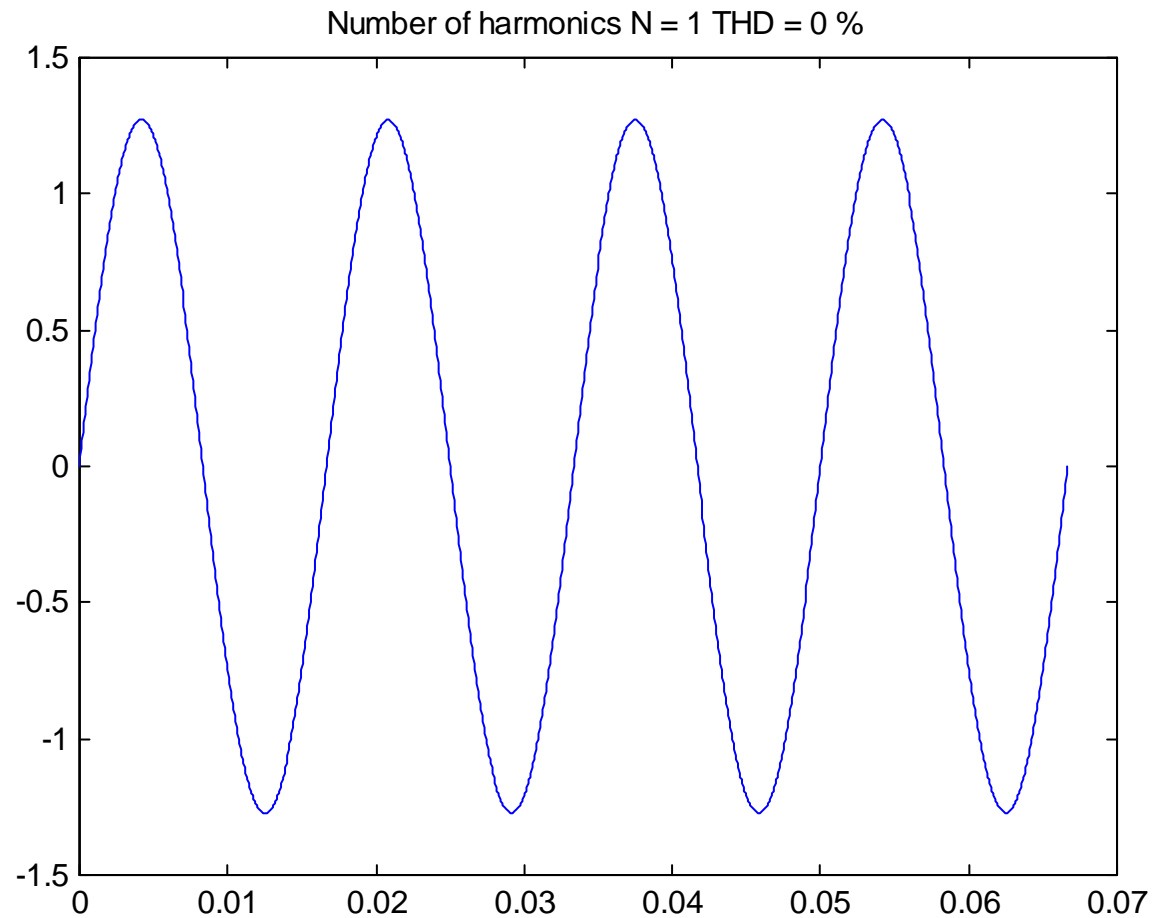# Half Wave Rectifier with RC Load --- Spectrum of Line Current



Half wave rectifier with RC load --- spectrum of line current

# Crest Factor

- Another term sometimes used in power engineering
- Ratio of peak value to RMS value
- For a sinewave, crest factor = 1.4
  - Peak = 1; RMS = 0.707
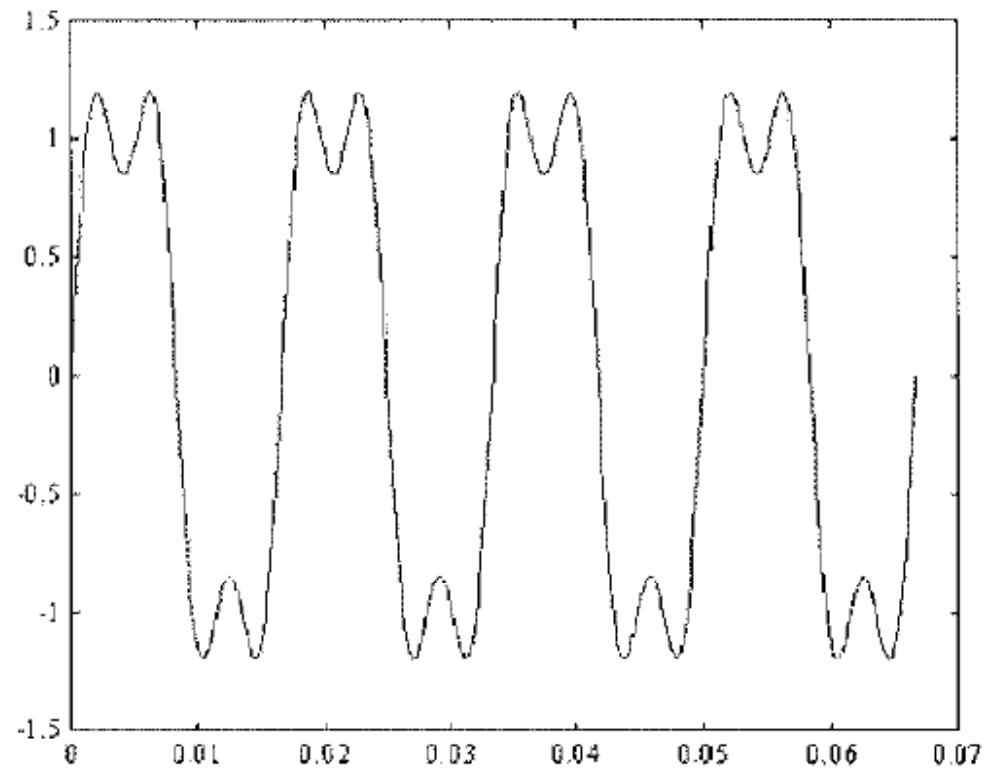- For a square wave, crest factor = 1
  - Peak = 1; RMS = 1
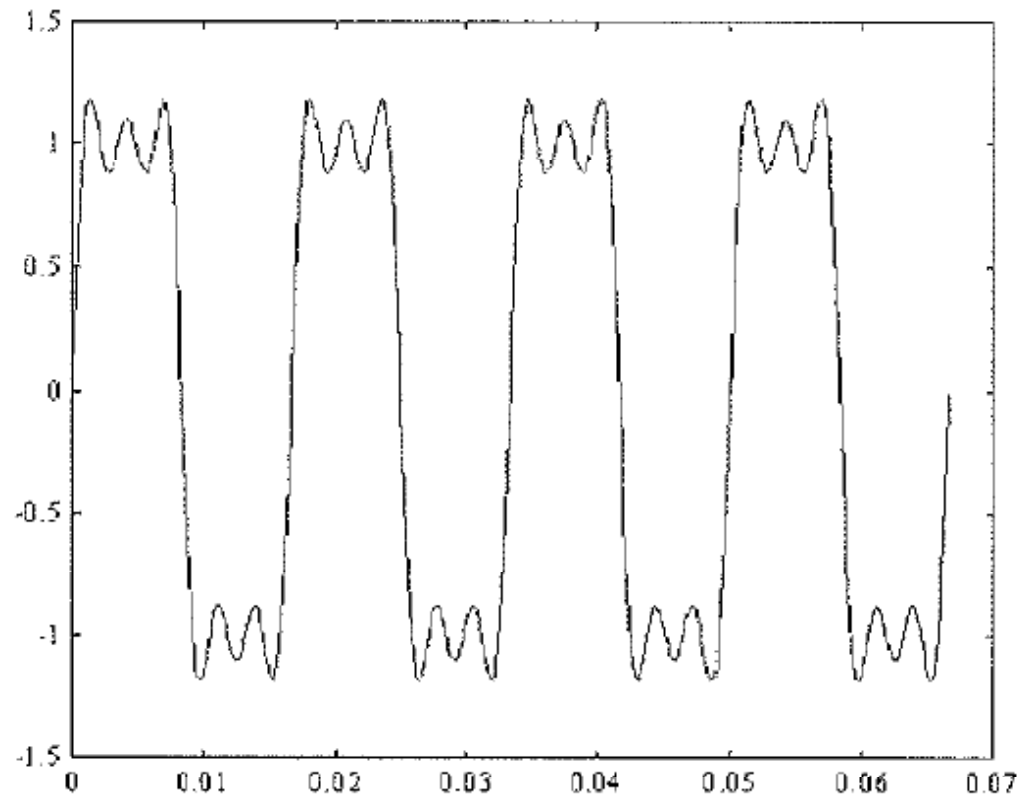
# Harmonics and THD - Sinewave

- THD = 0%

Number of harmonics N = 1 THD = 0 %

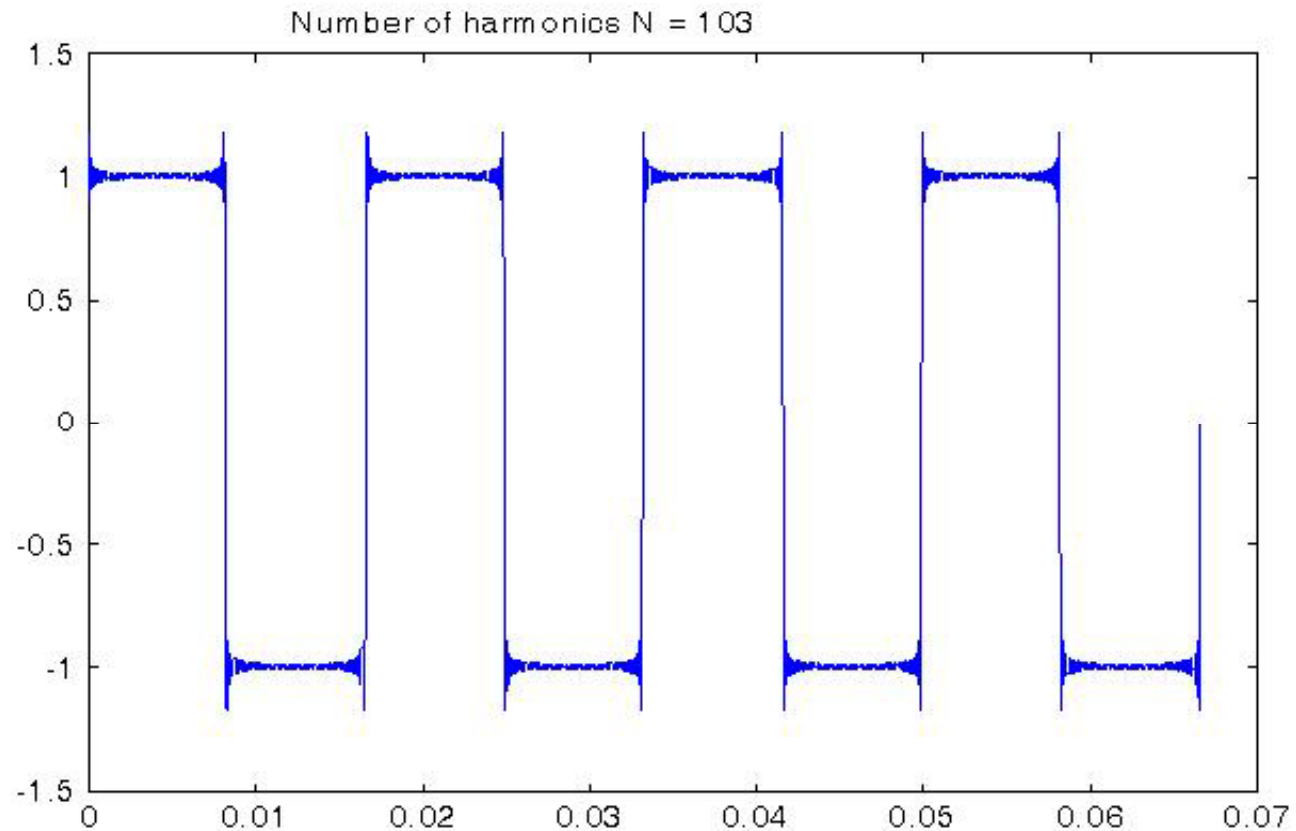# Harmonics and THD - Sinewave + 3rd Harmonic
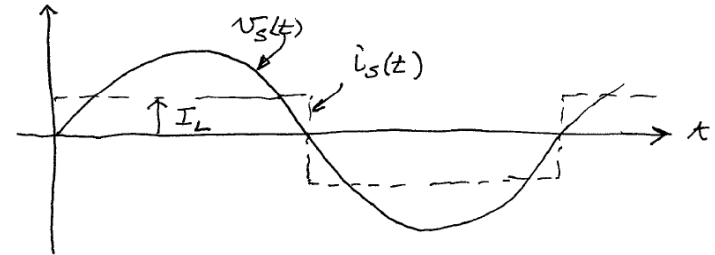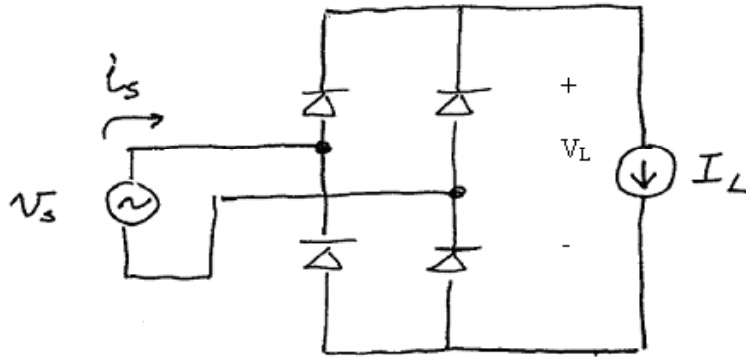
THD = 33.3%

# Harmonics and THD --- Sinewave + 3rd + 5th Harmonic

- THD = 38.9%

# Harmonics --- Up to N = 103

- THD = 48%



Number of harmonics N = 103
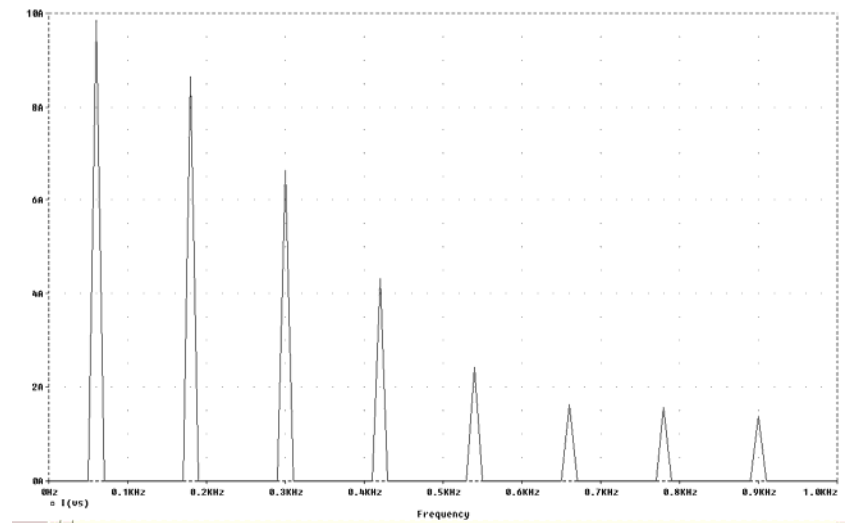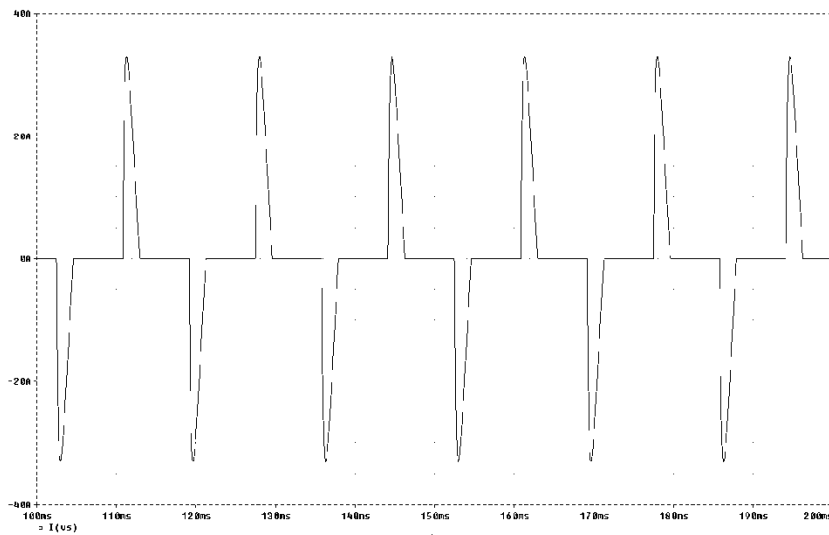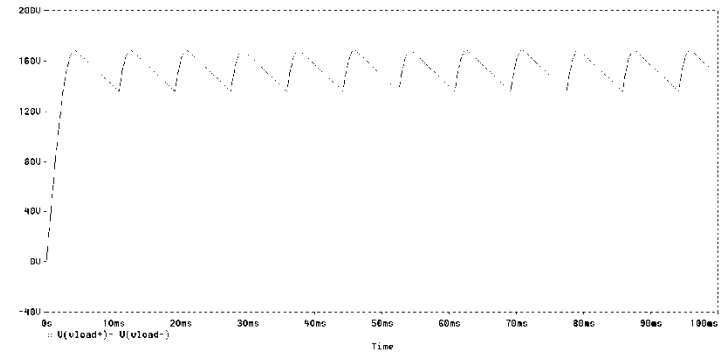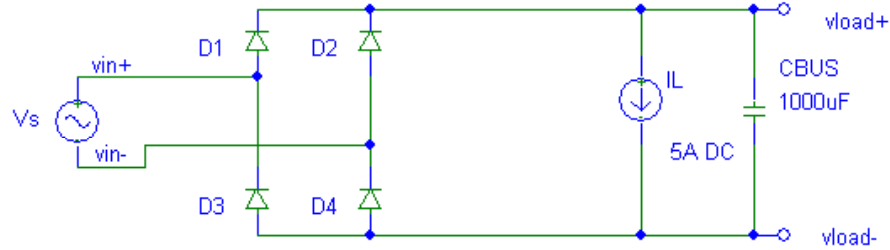
# Full-Wave Rectifier (Single Phase)
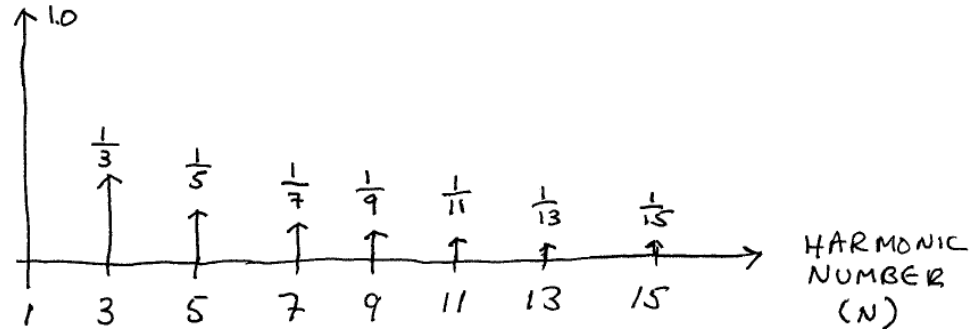
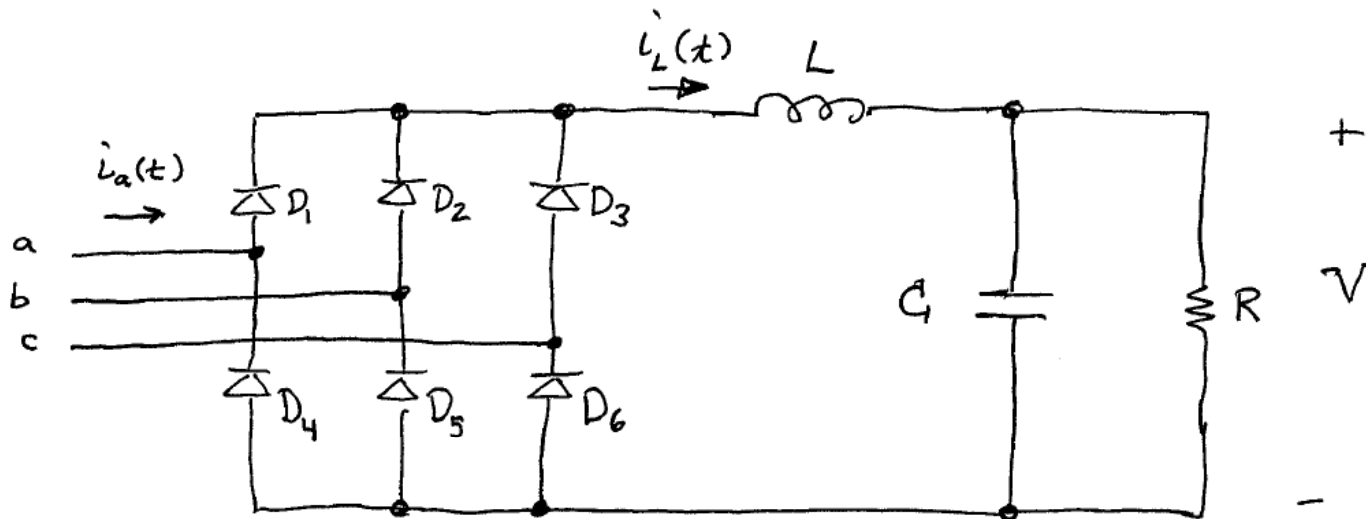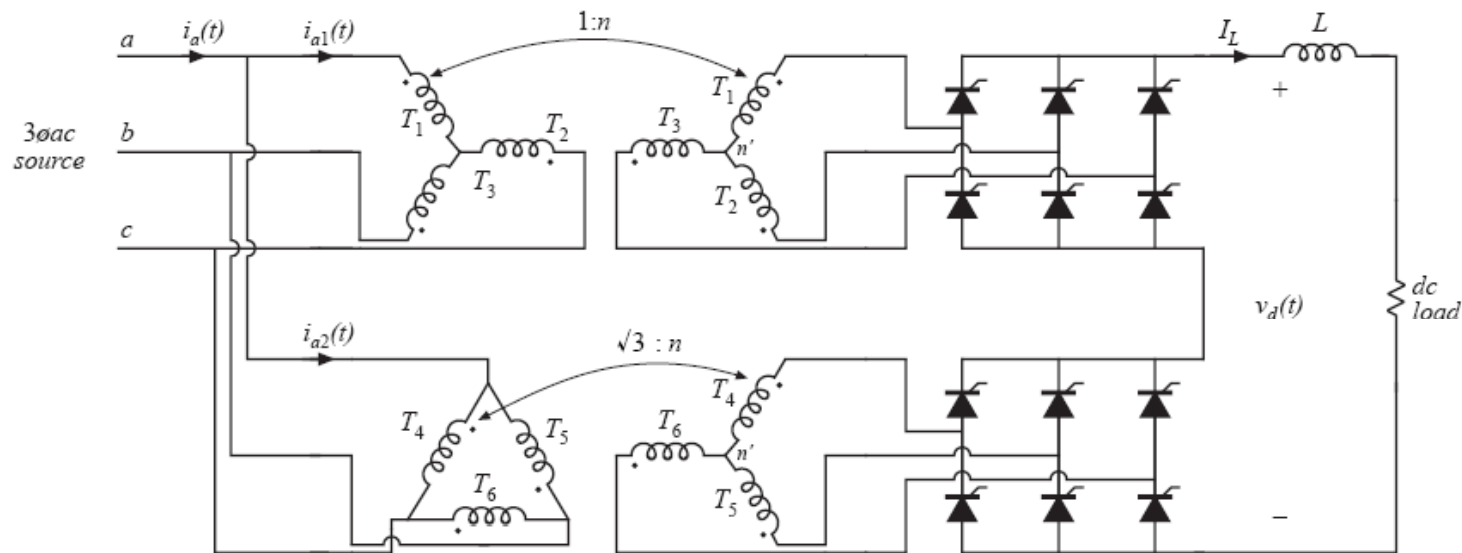# Full-Wave Rectifier with Capacitor Filter

# 6-Pulse (3-Phase) Rectifier

- Typically used for higher-power applications where 3-phase power is available

# 12-Pulse Rectifier

- Two paralleled 6-pulse rectifiers
- 5th and 7th harmonics are eliminated
- Only harmonics are the 11th, 13th, 23rd, 25th …



Reference: R. W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*, 2d edition

# Techniques for Analysis of Power Electronics Circuits

- Power electronics systems are often switching, nonlinear, and with other transients.  A variety of techniques have been developed to help <u>approximately</u> analyze these circuits
  - Assumed states
  - Small ripple assumption
  - Periodic steady state
- After getting approximate answers, often circuit simulation is used

# Assumed States

- In a circuit with diodes, etc. or other nonlinear elements, how do you figure out what is happening ?
- <u>Guess</u>….and then check your guess
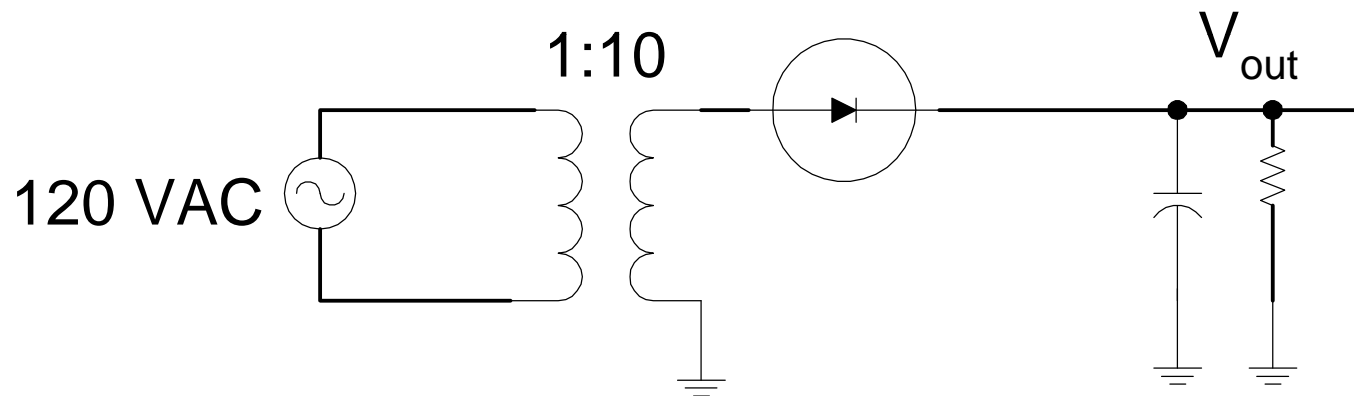
# Small Ripple Assumption

- In power electronic circuits, generally our interest is in the average value of voltages and current if the ripple is small compared to the nominal operating point.
- In DC/DC converters, often our goal is to regulate the average value of the output voltage $v_o$.  State-space averaging is a circuit approach to analyzing the local average behavior of circuit elements.  In this method, we make use of a running average, or:

$$\bar{v}(t) = \frac{1}{T} \int_{t-T}^{t} v(\tau)d\tau$$

# Periodic Steady State

- In the periodic steady state assumption, we assume that all startup transients have died out and that from period-to-period the inductor currents and capacitor voltages return to the same value.
- In other words, for one part of the cycle the inductor current ripples UP; for the second part of the cycle, the inductor current ripples DOWN.
- Can calculate converter dependence on switching by using volt-second balance.

# Motivation for DC/DC Converter: Offline <u>Linear</u> +5V, 50 Watt Regulator
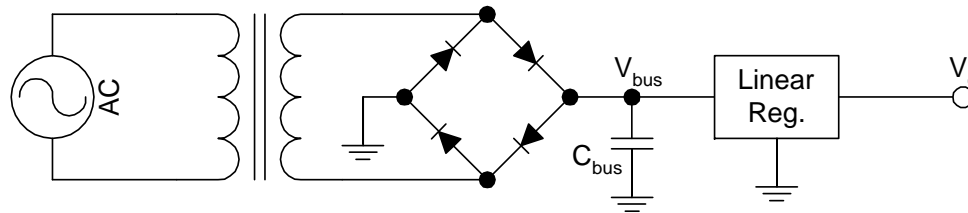


- Must accommodate:
  - Variation in line voltage
    - Typically 10%
  - Drop in rectifier, transformer
    - Rectifier 1-2V total
    - Transformer drop depends on load current
  - Ripple in bus voltage

$$\Delta V_{bus} \approx \frac{I_L}{120 C_{bus}}$$

  - Dropout voltage of regulator
    - Typically 0.25-1V

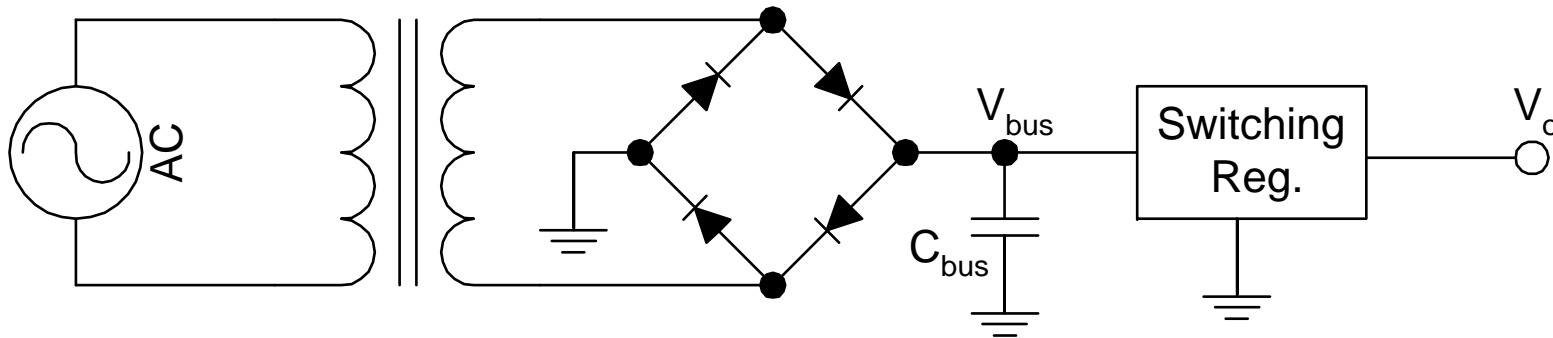- In order to maintain regulation:

$$V_{bus} > 5V + V_{dropout}$$

- Regulator power dissipation:

$$P \approx \left[ <V_{bus}> - V_o \right] I_o$$
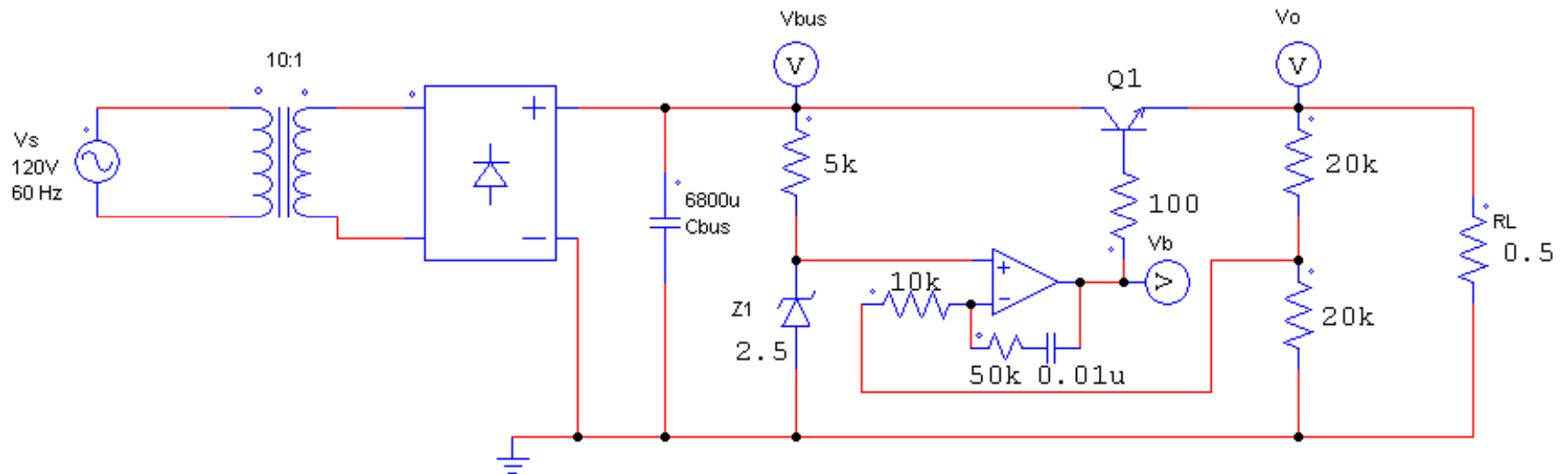
- For $<V_{bus}>$ = 7V and $I_o$ = 10A, P = 20 Watts !

# Offline Switching +5V, 50 Watt Regulator

- If switching regulator is 90% efficient, $P_{reg} = $ 5.6 Watts (ignoring losses in diode bridge and transformer)
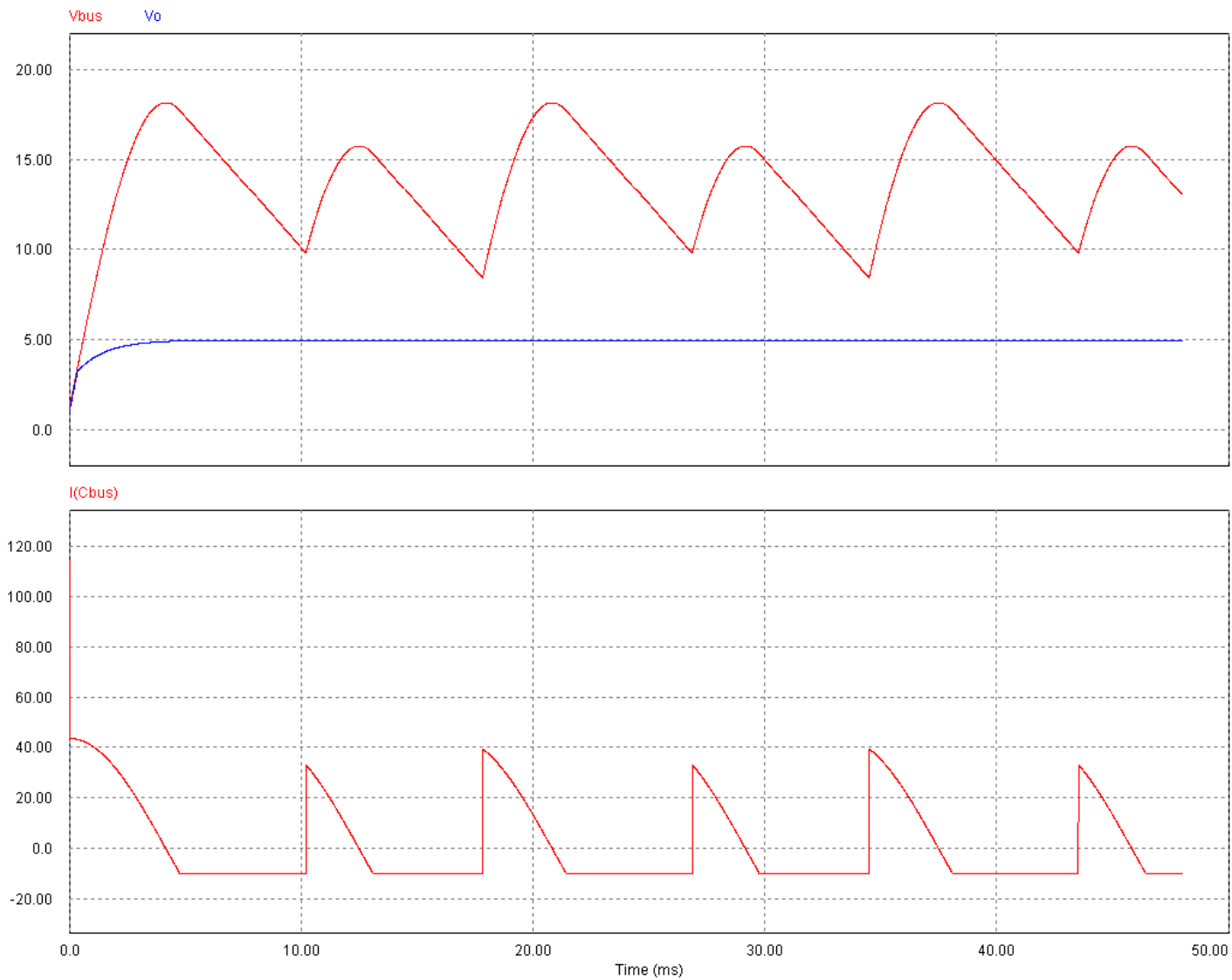- Other switching topologies can do better
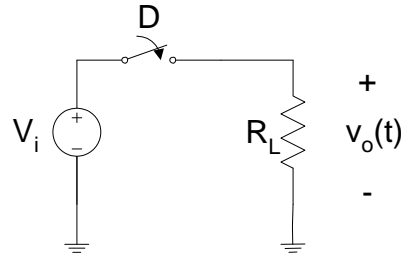
# PSIM Simulation

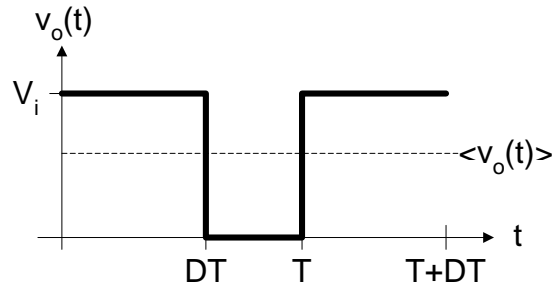

BJT Linear Regulator

PSIM file: BJT linear regulator.sch

# PSIM Simulation

# Switcher Implementation



- Switch turns on and off with switching frequency $f_{sw}$
- D is "duty cycle," or fraction of switching cycle that switch is closed



- Average value of output $<v_o(t)> = Dv_i$
  - Can provide real-time control of $<v_o(t)>$ by varying duty cycle
- Unfortunately, output is has very high ripple at switching frequency $f_{sw}$
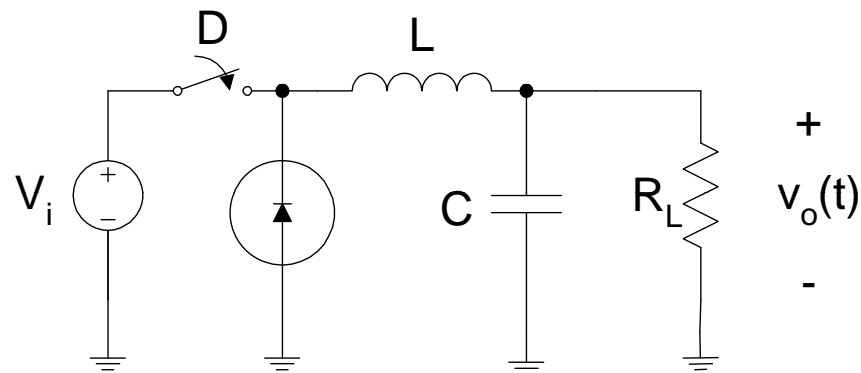
# Switcher Design Issues

- Lowpass filter provides effective ripple reduction in $v_o(t)$ if LC >> $1/f_{sw}$
- Unfortunately, this circuit has a fatal flaw.....
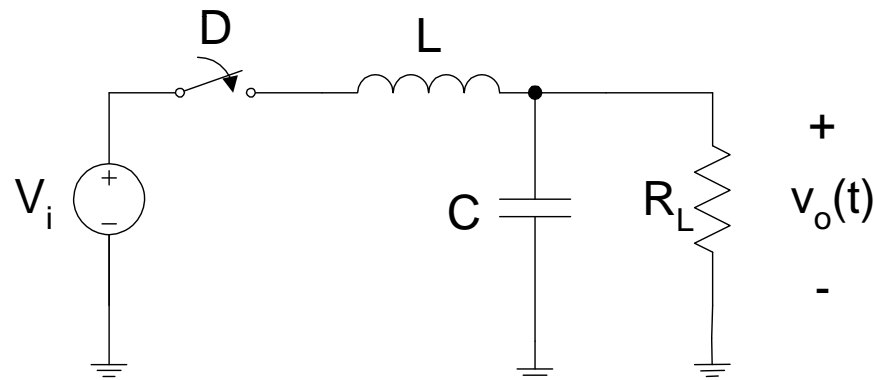
# Buck Converter

- Add diode to allow continuous inductor current flow when switch is open



- This is a common circuit for voltage step-down applications
- Examples of buck converter given later

# Types of Converters

- Can have DC or AC inputs and outputs
- AC $\Rightarrow$ DC
  - Rectifier
- DC $\Rightarrow$ DC
  - Designed to convert one DC voltage to another DC voltage
  - Buck, boost, flyback, buck/boost, SEPIC, Cuk, etc.
- DC $\Rightarrow$ AC
  - Inverter
- AC $\Rightarrow$ AC
  - Light dimmers
  - Cycloconverters

# Ideal Power Converter

- Converts voltages and currents without dissipating power
  - Efficiency = 100%

$$\varepsilon = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}}$$

- Efficiency is very important, especially at high power levels
- High efficiency results in smaller size (due to cooling requirements)
- Example: 100 kW converter
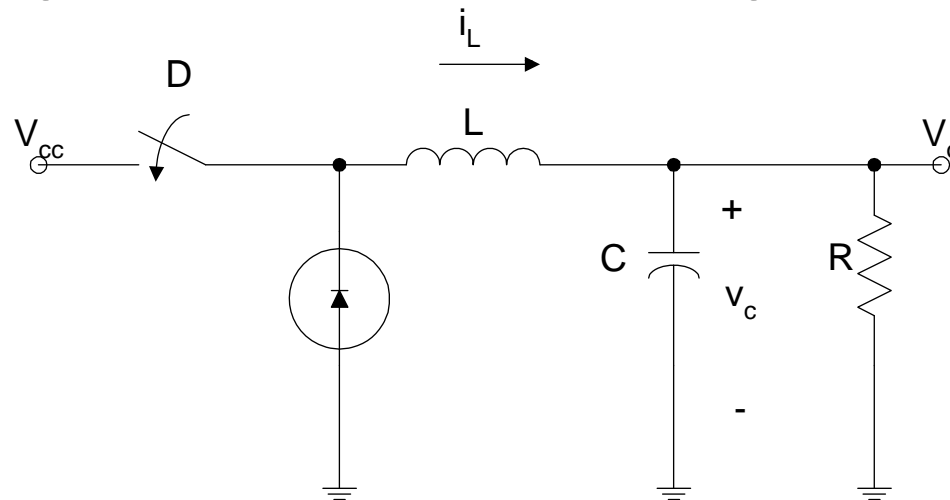  - 90% efficient dissipates 11.1 kW
  - 99% efficient dissipates 1010 W
  - 99.9% efficient dissipates 100 W

$$P_{diss} = P_{out}\left(\frac{1}{\varepsilon} - 1\right)$$

# Buck Converter
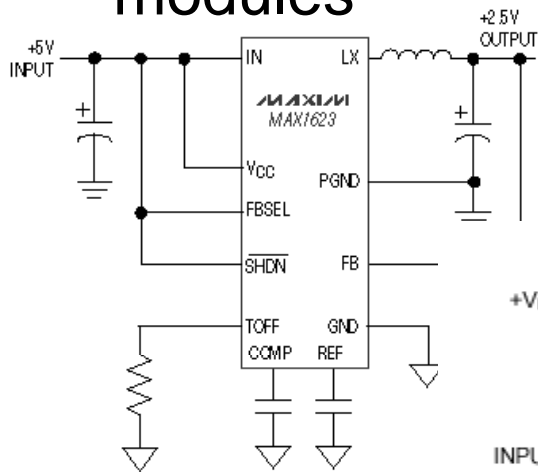
- Also called "down converter"
- Designed to convert a higher DC voltage to a lower DC voltage
- Output voltage controlled by modifying switching "duty ratio" D



- We'll figure out the details of how this works in later weeks

# Possible Implementations

- Many companies make buck controller chips (where you supply external components) as well as complete modules
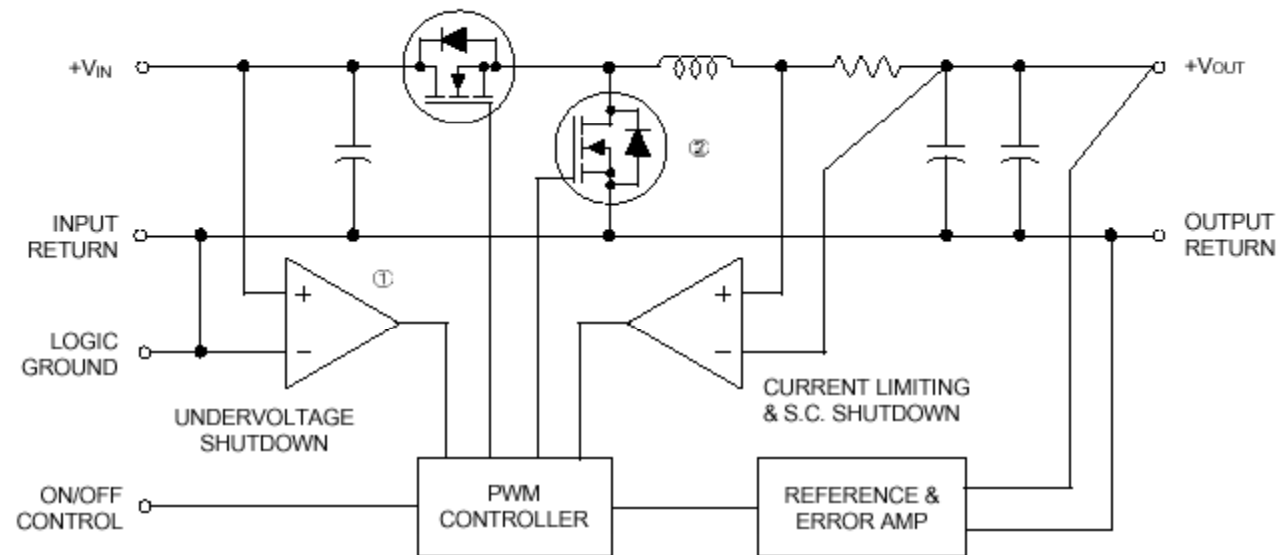
# Real-World Buck Converter Issues

- Real-world buck converter has losses in:
  - MOSFET
    - Conduction loss
    - Switching loss
  - Inductor
    - ESR
  - Capacitor
    - ESR
  - Diode
    - Diode ON voltage

# Converter Loss Mechanisms

- Input rectifier
- Input filtering
  - EMI filtering
  - Capacitor ESR
- Transformer
  - DC winding loss
  - AC winding loss
    - Skin effect
    - Proximity effect
  - Core loss
    - Hysteresis
    - Eddy currents
- Output filter
  - Capacitor ESR

- Control system
  - Controller
  - Current sensing device
- Switch
  - MOSFET conduction loss
  - MOSFET switching loss
  - Avalanche loss
  - Gate driving loss
  - Clamp/snubber
- Diode
  - Conduction loss
  - Reverse recovery
  - Reverse current