VECTOR QUANTITIES and KINEMATICS

Define a vector as ***a physical quantity that has both magnitude and direction*** and give examples

displacement **velocity** **acceleration** **force**

→ **straight line to represent magnitude**

arrow to represent direction

Define a scalar quantity as ***a physical quantity that has magnitude only*** and give examples

distance **speed** **time** **energy**

Kinematics – **the study of objects that are in motion**

KINEMATICS

Definitions

- distance as *the length of path travelled.* and know that distance is a scalar quantity
- displacement as *a change in position. (vector quantity)*
- speed as *the rate of change of distance.* (scalar quantity)

$$speed = \frac{\Delta d}{\Delta t}$$

- velocity as *the rate of change of position* or *the rate of displacement* or *the rate of change of displacement.* (vector quantity)

$$v = \frac{\Delta x}{\Delta t}$$

- acceleration as the rate of change of velocity

$$a = \frac{\Delta v}{\Delta t}$$

Distinguish between average velocity and instantaneous velocity

Average velocity is the change in displacement of an object over a certain period of time

x = 0 **Δt = 10s** **x = 50m**

$$v\ (avg) = \frac{\Delta x}{\Delta t} = \frac{50 - 0}{10} = \mathbf{5}\ \text{m.s}^{-1} \text{ to the right}$$

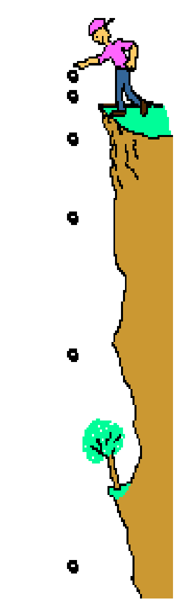
Instantaneous velocity is the velocity of an object at an instant in time

x = 0 **x = 50m**

the velocity at this exact point

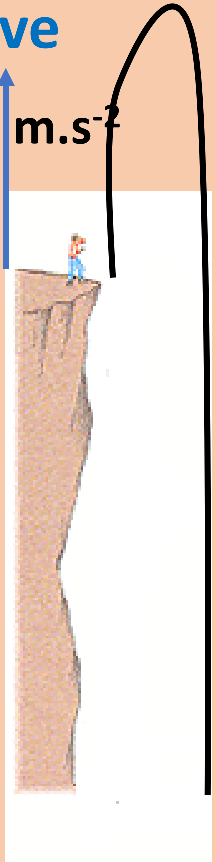
Vertical Projectile motion

- All free-falling objects (on Earth) accelerate downwards at a rate of 9,8 m.s^{-2}
- Gravitational acceleration g = 9,8 m.s^{-2} (The acceleration for any object moving under the sole influence of gravity).
- To accelerate at 9,8 m.s^{-2} means to change the velocity by 9,8 m.s^{-1} each second.



initial direction is downwards = +ve

g = +9,8 m.s^{-2}



+ve

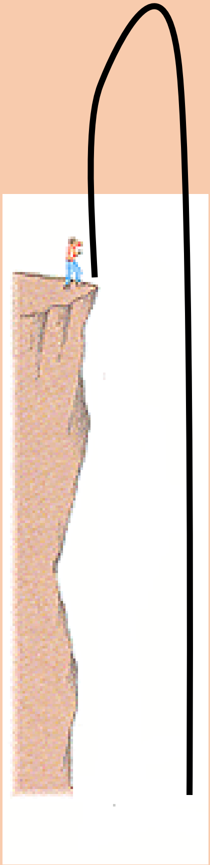
g = -9,8 m.s^{-2}

-ve

Initial direction is up

g = -9,8 m.s^{-2}

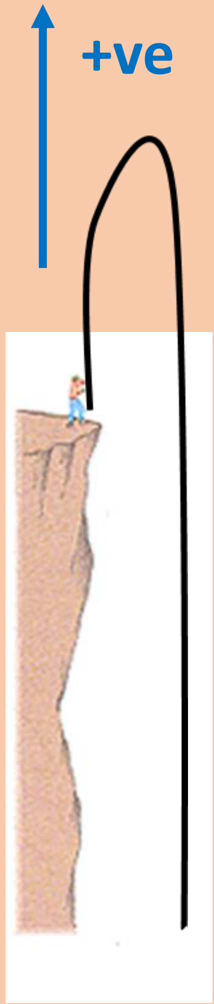
Worked example



A boy standing on the edge of a cliff throws a stone vertically into the air at 25 $m.s^{-1}$. The stone hits the ground at the base of the cliff 8s later.

1. Calculate the time taken to reach its maximum height
2. How long does take for the stone to reach his hand again?
3. Calculate the maximum height reached by the stone from its point of release.
4. Calculate the velocity of the stone at the moment it hits the ground below.

MEMORANDUM (Worked example)



1. $v_i = +25\ m.s^{-1}$
$v_f = 0\ m.s^{-1}$
$g = -0,8\ m.s^{-2}$
$\Delta x =$
$\Delta t =\ ?$

$v_f = v_i + g\Delta t$
$0 = 25 + (-9,8)\Delta t$
$$\Delta t = \frac{-25}{-9,8}$$
$\Delta t = 2,55s$

2. Symmetrical motion :

$t_{total} = t_{up} + t_{down}$
$= 2,55 + 2,55$
$t_{total} = 5,10s$

3. $v_f^2 = v_i^2 + 2g\Delta x$
$0^2 = 25^2 + 2(-9,8)\Delta x$
$0 = 625 - 19,6\Delta x$
$\Delta X = 31,89m$

4. $Vi = +25\ m.s^{-1}$
$Vf =\ ?$
$g = -9,8$
$\Delta x =$
$\Delta t = 8s$

$v_f = v_i + g\Delta t$
$= 25 + (-9,8)(8)$
$= -53.4$
$Vf = 53\ m.s^{-1}$ down

Practice Examples

Example 1

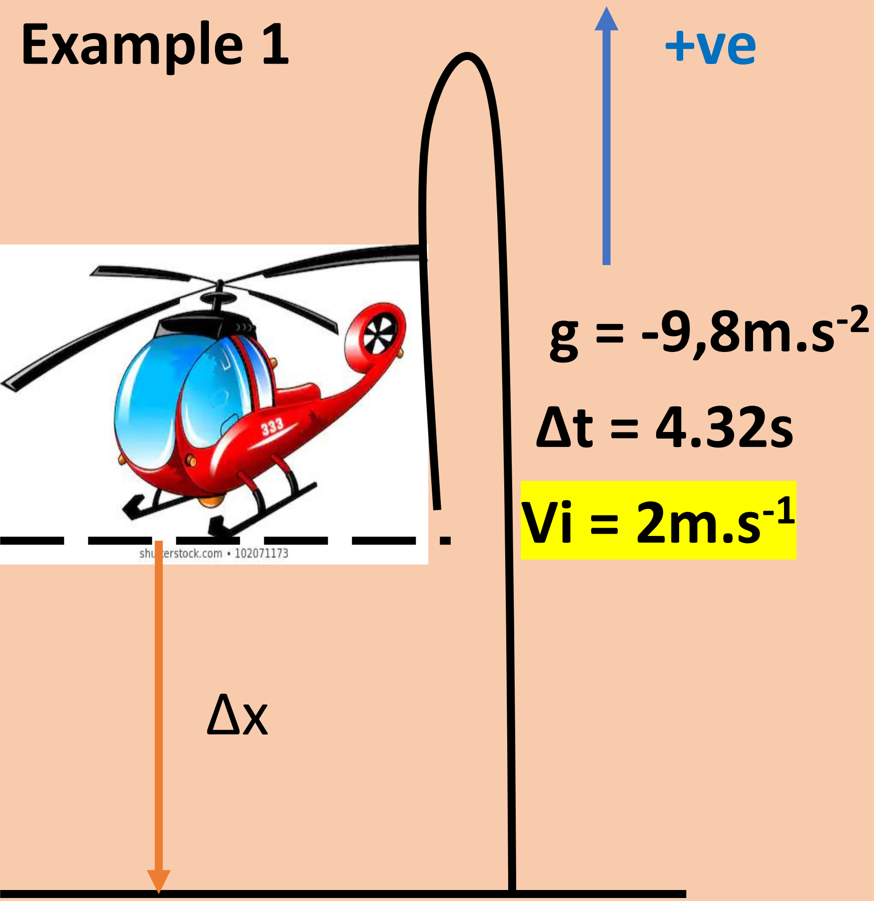
A helicopter is flying vertically upwards at a constant speed of 2 $m \cdot s^{-1}$ when a stuntman steps out of an open door. The stuntman lands on a trampoline 4,32 s later.

1.1 Calculate the height of the helicopter above the trampoline as the stuntman jumped.

1.2 Calculate the velocity of the stuntman when he reaches the trampoline.

MEMORANDUM

Example 1



$$s = ut + \frac{1}{2}at^2$$

$$s = 2(4,32) + \frac{1}{2}(-9,8)(4,32)^2$$

$$s = -82,81\text{ m}$$

$\therefore$ **Helicopter 82,81 m above trampoline**

$$v = u + at$$

$$v = 2 + (-9,8)(4,32)$$

$$v = -40,34$$

$\mathbf{v = 40,34\ m \cdot s^{-1}}$ **down**

OR

$$v^2 = u^2 + 2as$$

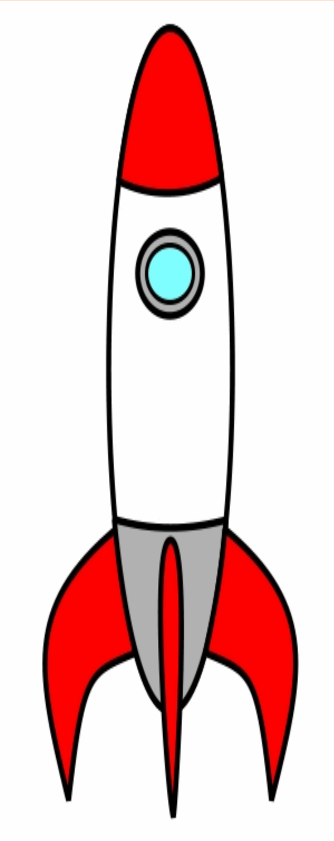
$$v^2 = 2^2 + 2(-9,8)(-82,81)$$

$$v = 40,34$$

$\mathbf{v = 40,34\ m \cdot s^{-1}}$ **down**

Practice Examples

Example 2

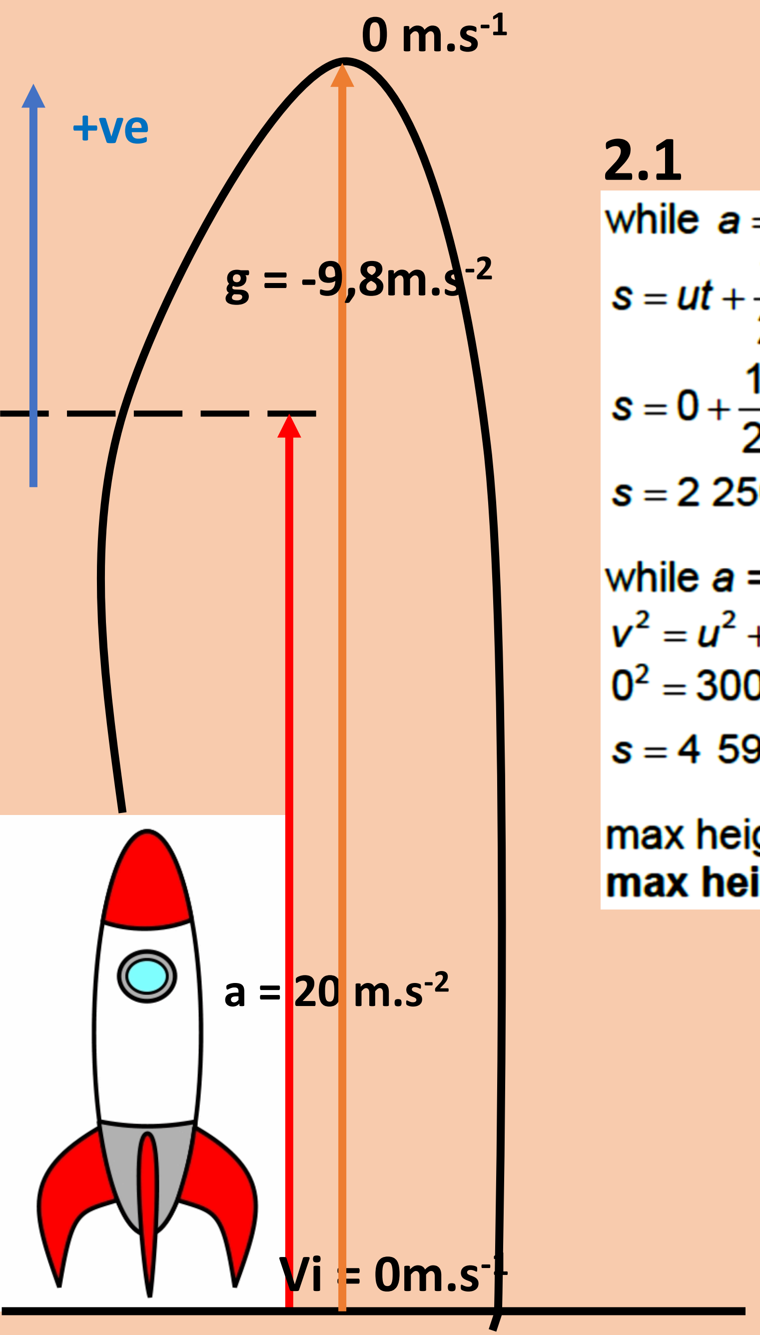


A model rocket is fired vertically upwards from rest at ground level. The rocket accelerates upwards at 20 $m \cdot s^{-2}$ for 15 s and then runs out of fuel. Air resistance is negligible.

Calculate the rocket's maximum height above the ground. (6)

Calculate the time the rocket is in the air from launch to crashing on the ground. (5)

At which position will the rocket experience a maximum velocity? (2)



2.1

while $a = 20\ m \cdot s^{-2}$

$$s = ut + \frac{1}{2}at^2 \qquad v = u + at$$

$$s = 0 + \frac{1}{2}20(15)^2 \qquad v = 0 + 20(15)$$

$$s = 2\ 250\ m \qquad v = 300\ m \cdot s^{-1}$$

while $a = g$

$$v^2 = u^2 + 2as$$

$$0^2 = 300^2 + 2(-9,8)s$$

$$s = 4\ 591,84\ m$$

max height = 2 250 +4 591,84

max height = 6 841,84 m

2.2

Time to max height after rocket runs out of fuel

$$s = \frac{u+v}{2}t$$

$$4\ 591,84 = \frac{300+0}{2}t$$

$$t = 30,61\ s$$

Time to reach the ground from max height

$$s = ut + \frac{1}{2}at^2$$

$$-6841,84 = 0 + \frac{1}{2}(-9,8)t^2$$

$$t = 37,37\ s$$

total $t = 15 + 30,61 + 37,37$

total $t = 82,98$ s

$$s = ut + \frac{1}{2}at^2$$

$$-2\ 250 = 300t + \frac{1}{2}(-9,8)t^2$$

$$t = 67,98\ s$$

total t = 15 +67,98

total $t = 82,98$ s

2.3

as it hits the ground

Representing Kinematics

The Graphs of Motion

<u>Velocity vs time</u> (Δv vs Δt)

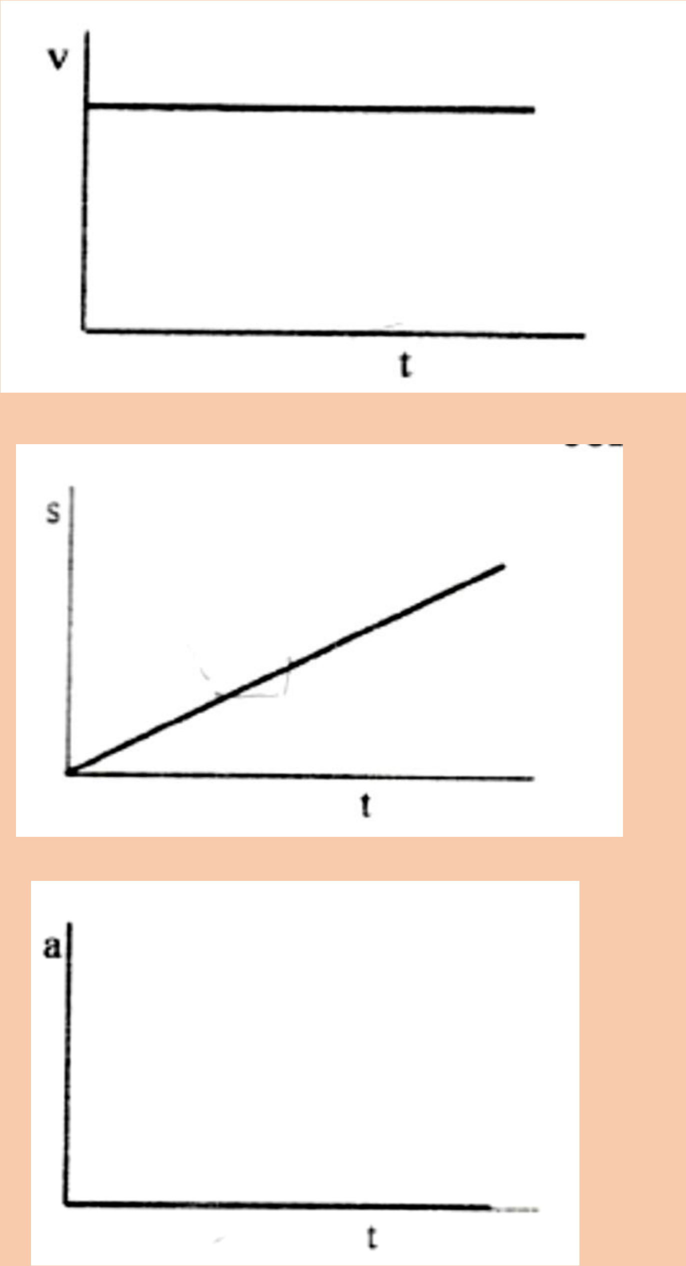
- Gradient of a Δv vs Δt graph represents ACCELERATION of an object
- Area under a Δv vs Δt graph represents DISPLACEMENT of an object

<u>Position vs time</u>

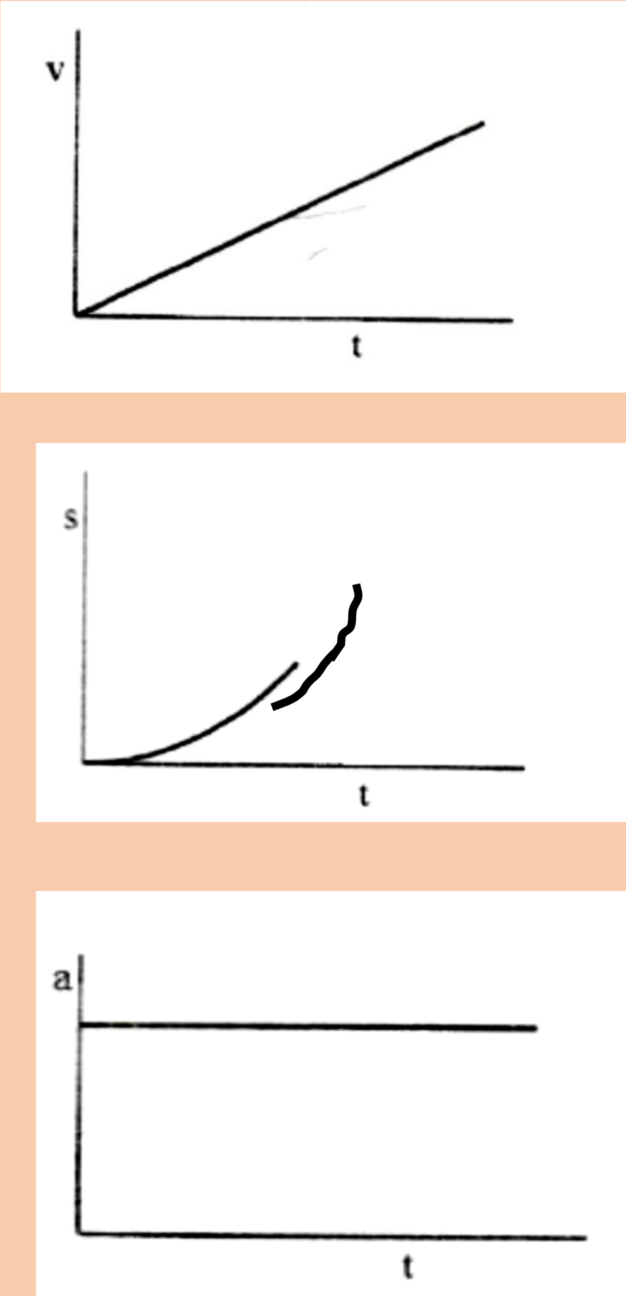
- Gradient of a Δx vs Δt graph represents VELOCITY of an object

<u>Acceleration vs time</u>

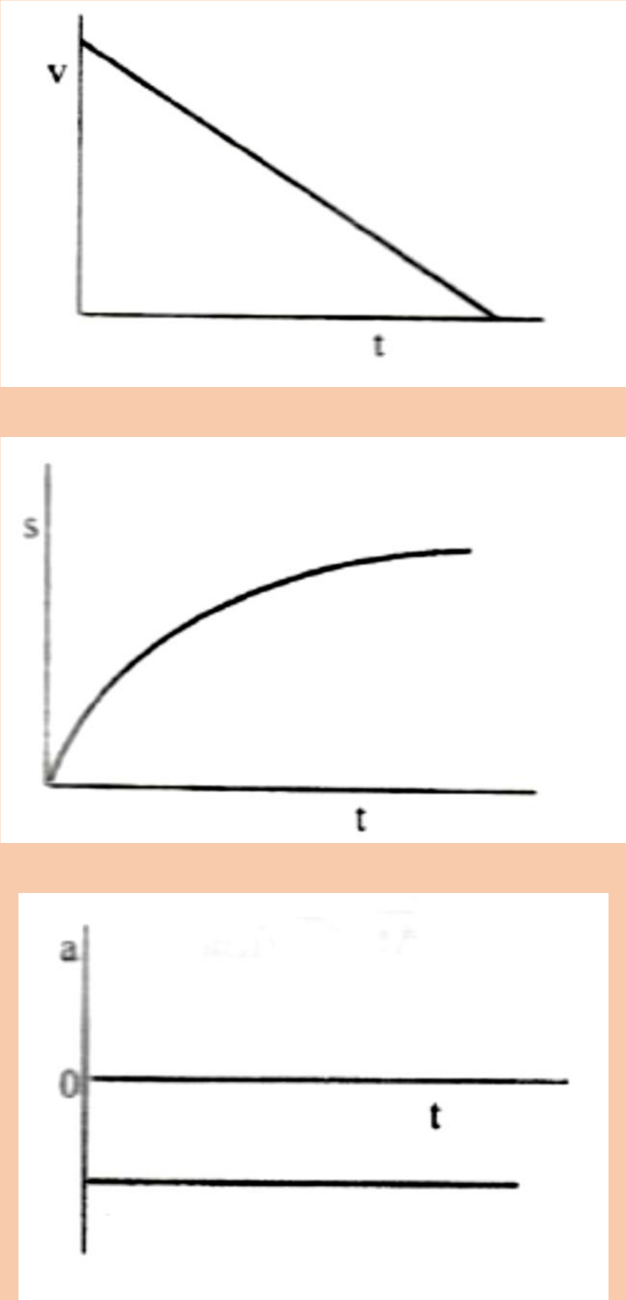
Constant velocity



Increasing velocity (+ve acc)

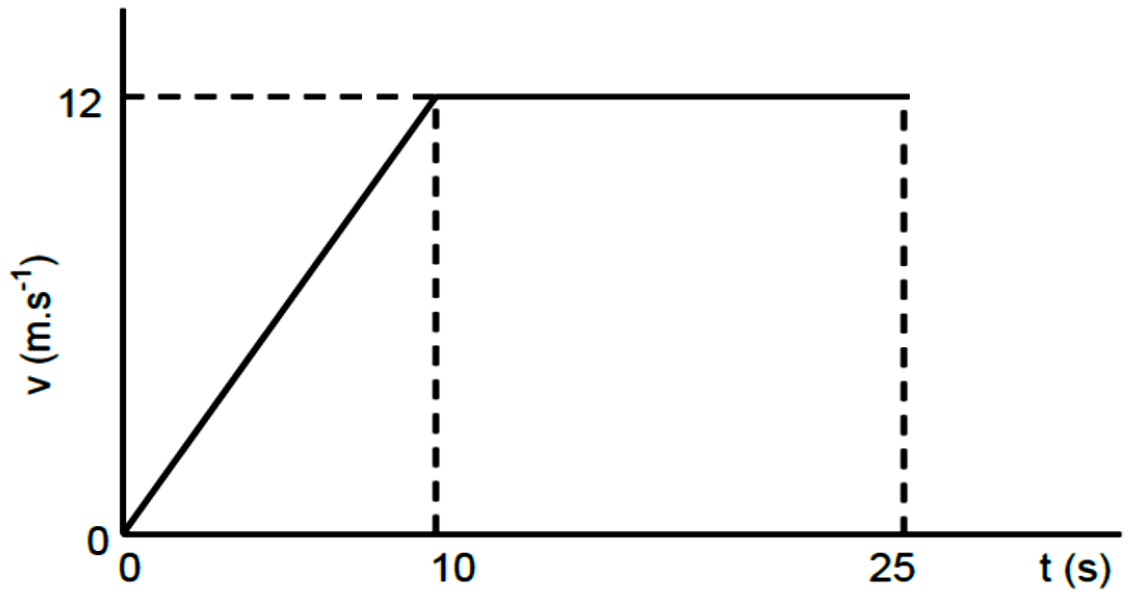


Decreasing velocity (-ve acc)



Worked Example 1

Tsholofelo runs **North** along a straight track. A velocity vs time graph of his motion is shown below.



2.1.1 Define *velocity*. (2)

2.1.2 Determine Tsholofelo's instantaneous velocity at 25 s. (2)

2.1.3 Calculate the magnitude of Tsholofelo's average velocity for 25 s. (6)

2.1.4 Define *acceleration*. (2)

2.1.5 Calculate the magnitude of Tsholofelo's acceleration for the first 10 s of his run. (3)

Memorandum

Worked example 1

2.1.1 Velocity is the rate of change of position **OR** the rate of displacement **OR** the rate of change of displacement.

2.1.2 **$12\ m \cdot s^{-1}$ North**

2.1.3 $s = \text{area under v-t graph}$

$$s = \frac{1}{2}(10)(12) + (25-10)(12)$$

$$s = 240\ m$$

$$\text{average } v = \frac{\text{total } s}{\text{total } t}$$

$$\text{average } v = \frac{240}{25}$$

$$\textbf{average v} = \mathbf{9{,}6\ m \cdot s^{-1}}$$

2.1.4 Acceleration is the rate of change of velocity.

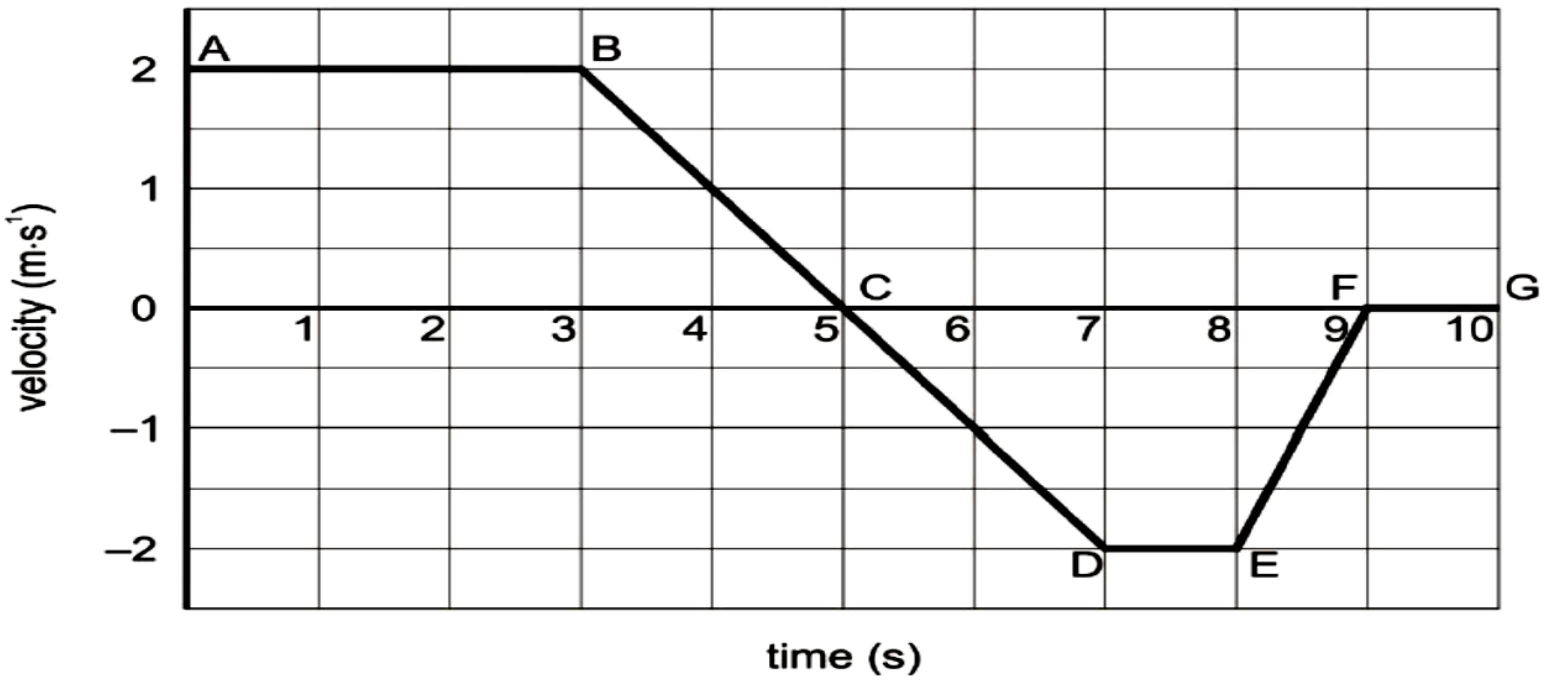
2.1.5 $a = \text{slope of v-t graph OR } \frac{\Delta v}{\Delta t}$

$$a = \frac{12-0}{10-0}$$

$$\mathbf{a = 1{,}2\ m \cdot s^{-2}}$$

Practice Example 1

A cyclist is riding **north** at 2 $m \cdot s^{-1}$ along a long straight road. A velocity vs time graph of the motion of the cyclist for 10 s is shown below.



2.1 Define *velocity*. (2)

2.2 During which time interval(s) is the cyclist's velocity constant? (3)

2.3 During which time interval(s) is the cyclist slowing down? (2)

2.4 During which time interval(s) is the cyclist travelling south? (2)

2.5 Define *acceleration*. (2)

2.6 Calculate the acceleration of the cyclist between 3 s and 7 s. (4)

2.7 Sketch a position vs time graph for the cyclist on the axes provided **on the answer sheet**. Values are not required but you must use the labels A–G. The cyclist's position is zero at $t = 0$ s. (6)

Memorandum
Practice example 1

2.1 Velocity is the rate of change of position OR the rate of displacement OR the rate of change of displacement.

2.2 AB (0 – 3 s); DE (7 – 8 s); FG (9 – 10 s)

2.3 BC (3 – 5 s); EF (8 – 9 s)

2.4 CF (5 – 9 s)

2.5 Acceleration is the rate of change of velocity.

2.6

a = slope of v-t graph OR $\frac{\Delta v}{\Delta t}$

$a = \frac{-2-2}{7-3}$

a = – 1

a = 1 m·s^{-2} South

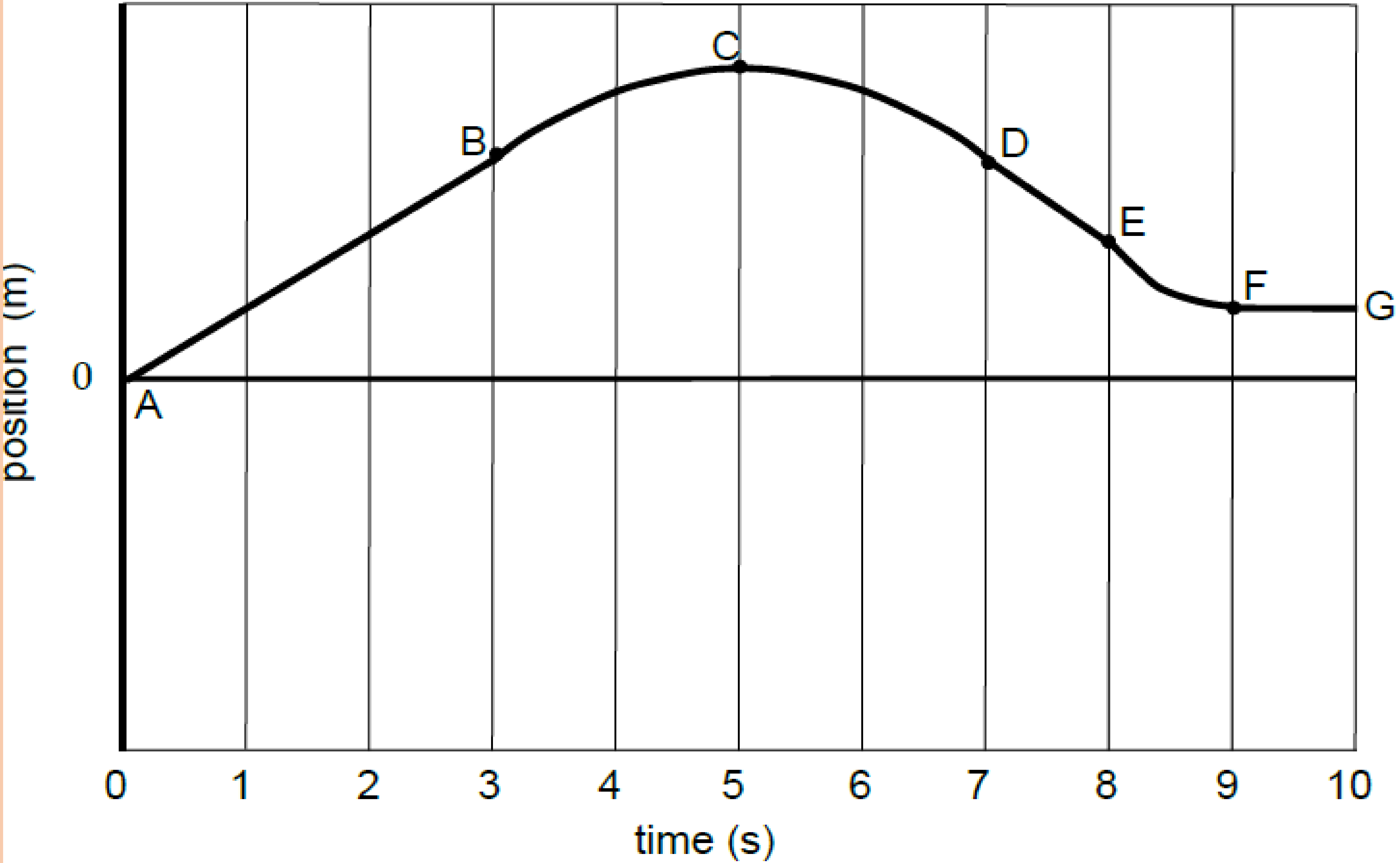
OR $v = u + at$

–2 = 2 + a(4)

a = – 1

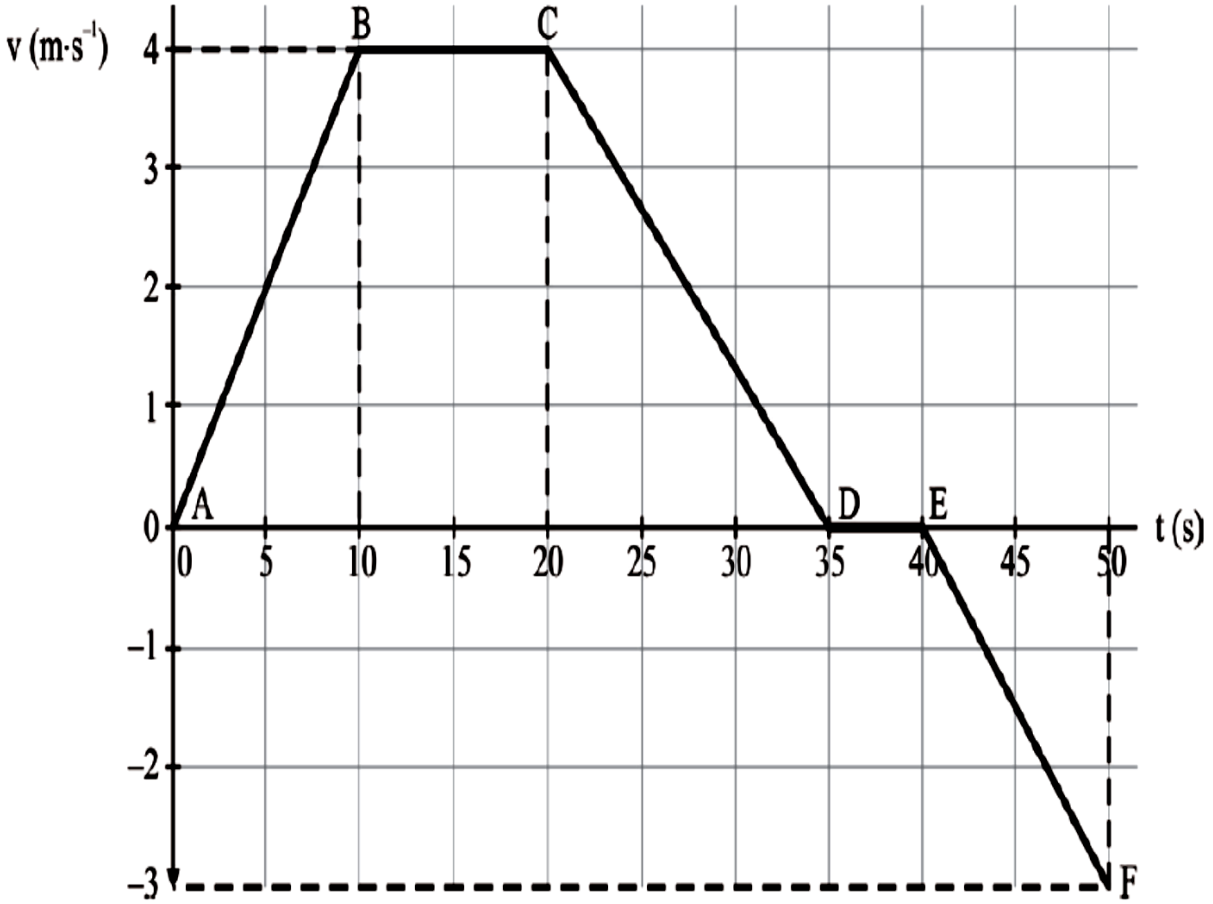
a = 1 m·s^{-2} South

2.7



Practice Example 2

2.1 A model car starts from rest and initially travels **east**. A velocity vs time graph of the motion is shown below.



2.1.1 Use the graph of motion to determine the acceleration of the car between 20 s and 35 s of the motion. (4)

2.1.2 During which time interval/s is the speed of the car increasing? (2)

2.1.3 Define *displacement*. (2)

2.1.4 Use the graph of motion to determine the displacement of the car after 50 s. (4)

2.1.5 Calculating the distance travelled by this car from the graph would give you a value greater than the calculated displacement. Is the distance travelled for any object always greater than its displacement? Explain your answer. (2)

2.1.6 Sketch a position vs time graph for the car from 0 s to 50 s on the axes provided **in the Answer Booklet**. Values are not required but you must use the labels A – F. (5)

Memorandum
Practice example 2

2.1.1 a = slope of v-t graph OR $\frac{\Delta v}{\Delta t}$

$$a = \frac{0-4}{35-20}$$

a = -0,27

a = 0,27 m·s^{-2} West

OR v = u + at

0 = 4 +a(15)

a = -0,27

a = 0,27 m·s^{-2} West

2.1.2 **0-10 s** (A-B) and **40-50 s** (E-F)

2.1.3 *displacement is a change in position*

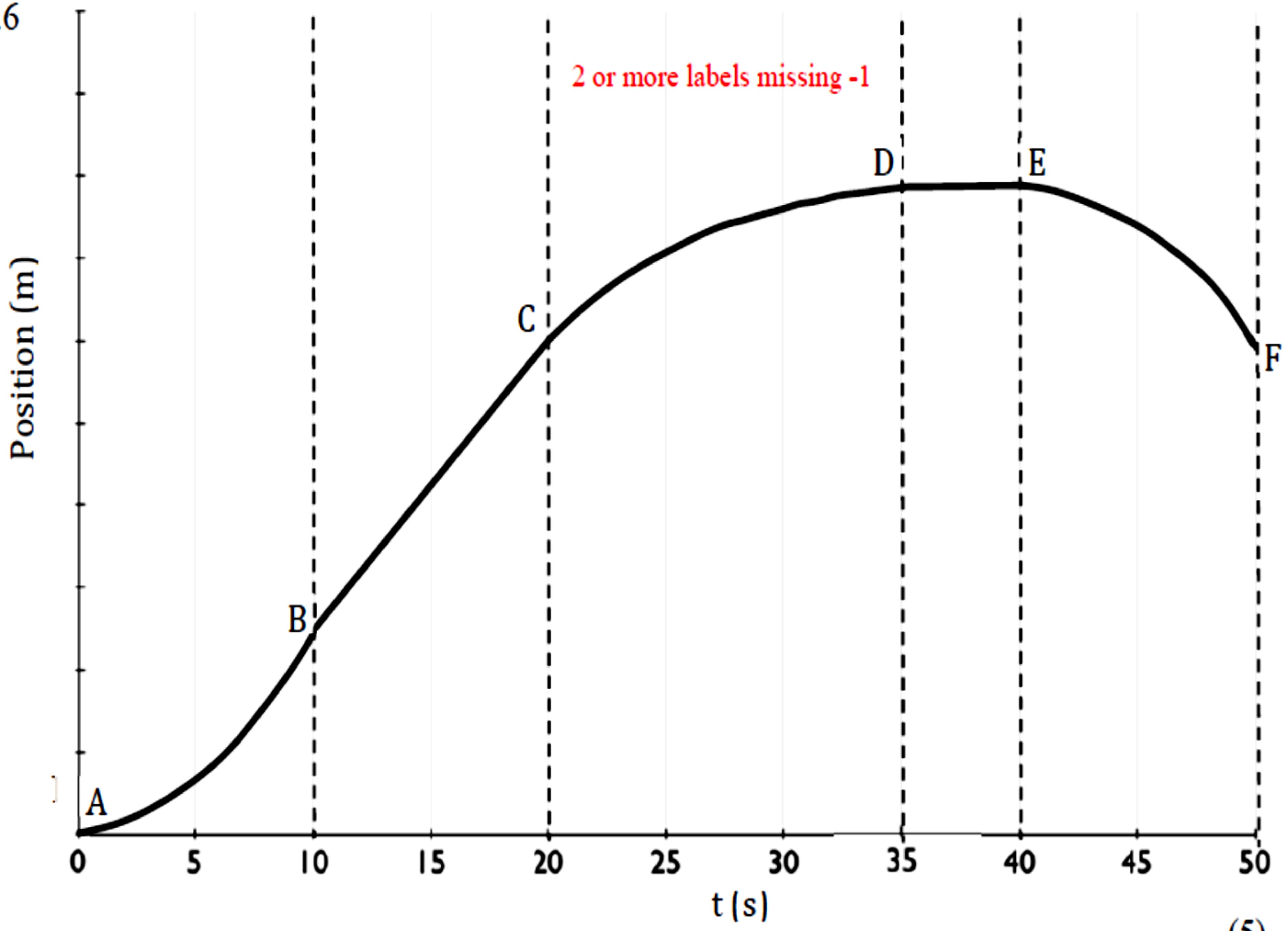
2.1.4 s = area under v-t graph

$$s = \frac{1}{2}(10)(4)+(10)(4)+\frac{1}{2}(15)(4)-\frac{1}{2}(10)(3)$$

s = 75 m East

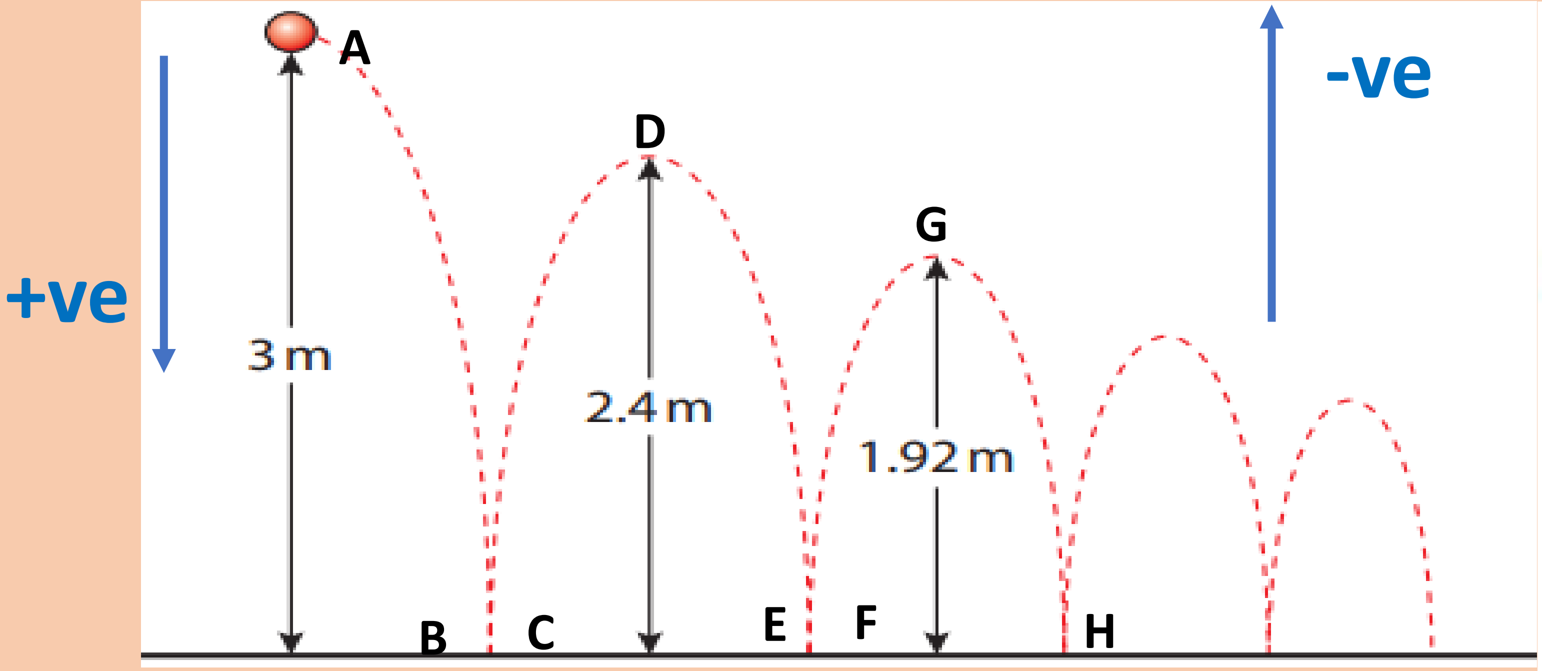
2.1.5 No, it will not always be true. Displacement and distance can be equal only if the object travels in a straight line in one direction.

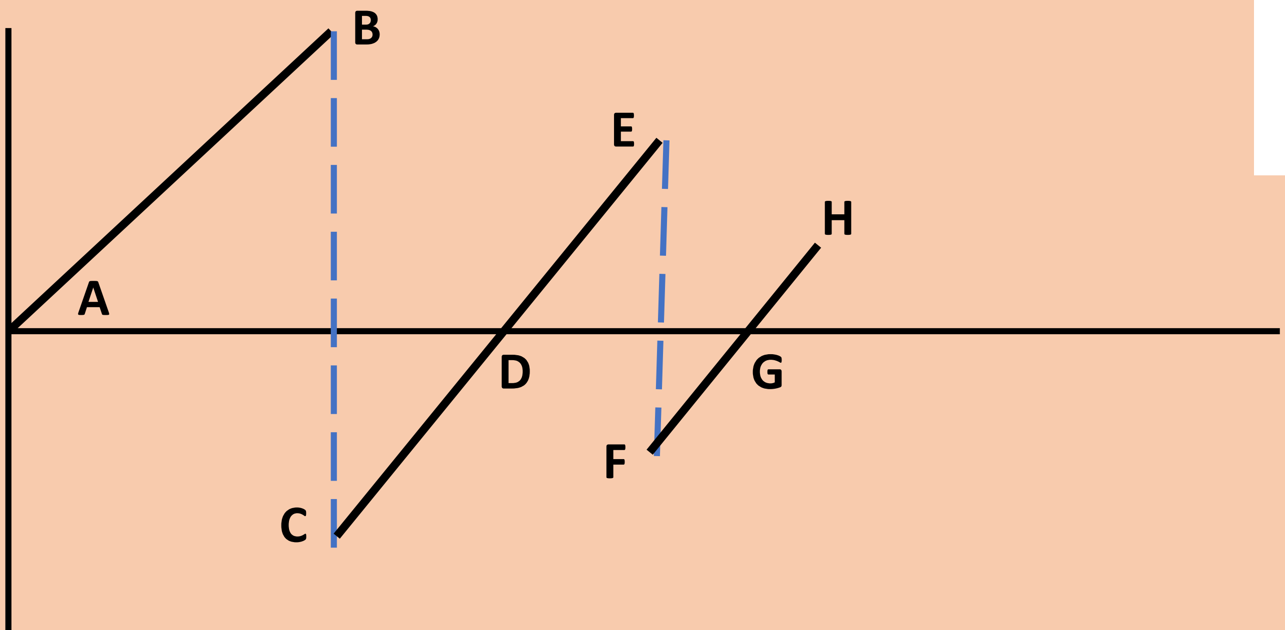
2.1.6


2 or more labels missing -1
A
B
C
D
E
F
Position (m)
0
5
10
15
20
25
30
35
40
45
50
t (s)

(5)

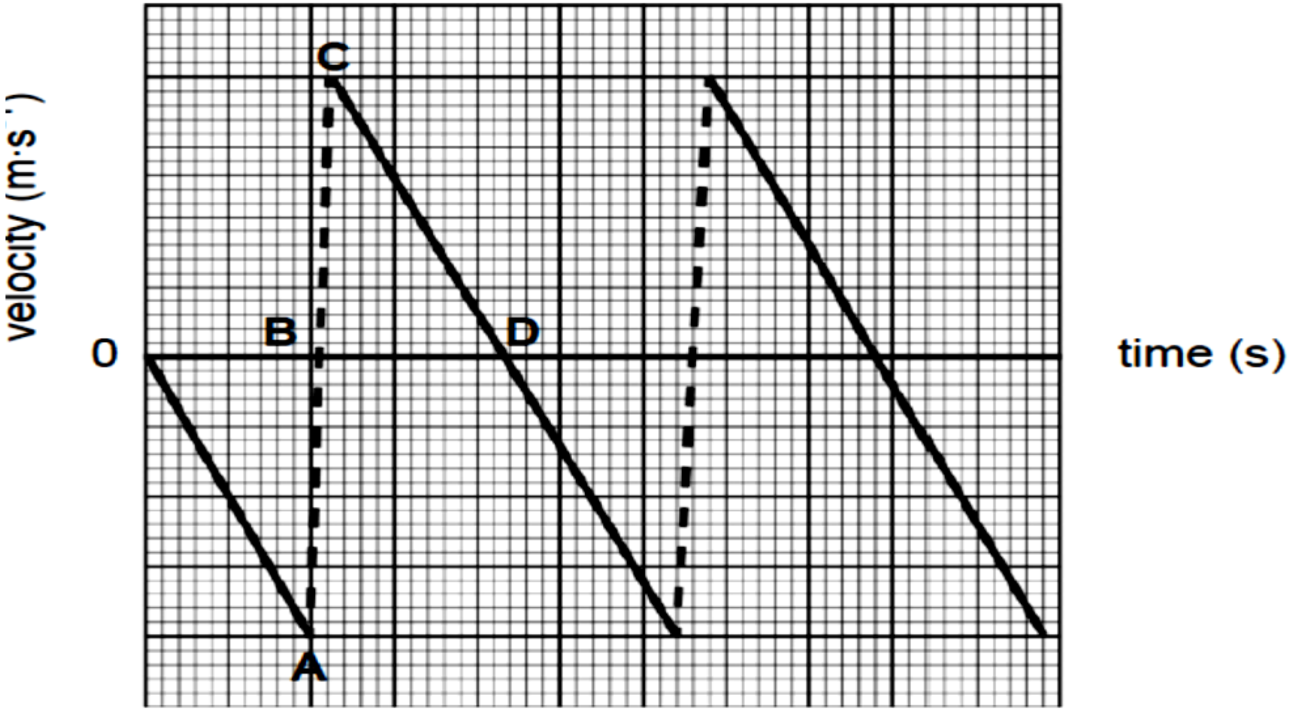
The Bouncing Ball





Worked Example

A ball is dropped from a certain height above the ground and bounces a few times as it hits the ground. The velocity-time graph below describes the motion of the ball from the time it is dropped. Ignore the effects of air friction.



In relation to the velocity-time graph for the bouncing ball shown above, which statement below is correct?

A Down is taken as the positive direction and the ball is at the highest position of its first bounce at C.

B Down is taken as the negative direction and the ball is at the highest position of its first bounce at D.

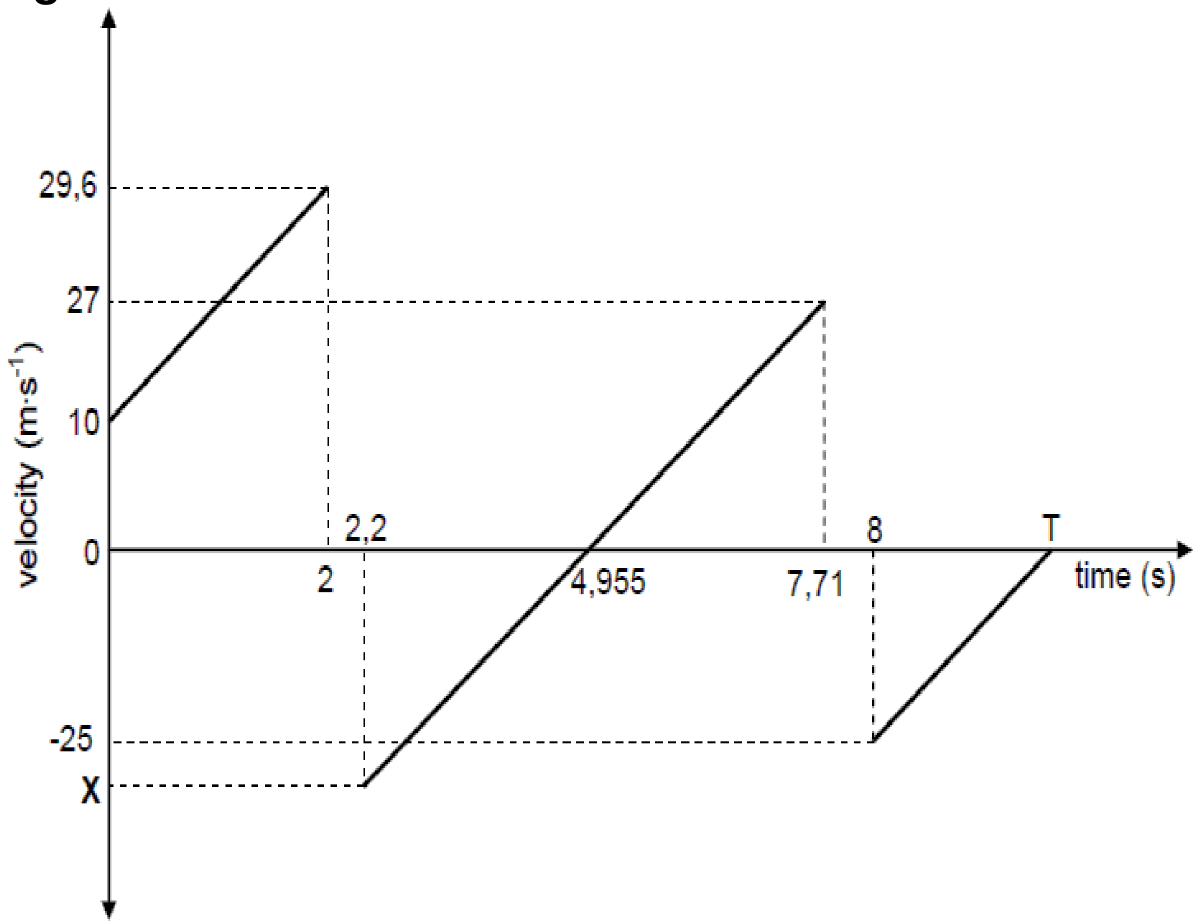
C Down is taken as the negative direction and the ball is at the highest position of its first bounce at C.

D Down is taken as the positive direction and the ball is at the highest position of its first bounce at D.

PRACTICE EXAMPLE 1

A ball is thrown vertically downwards from the top of a building and bounces a few times as it hits the ground. The velocity-time graph below describes the motion of the ball from the time it is thrown to a certain time T.

Take downwards as the positive direction The graph is NOT drawn to scale. The effects of air friction are ignored.



3.1 Write down the:

3.1.1 time that the ball is in contact with the ground at the first bounce (1)

3.1.2 time at which the ball reaches its maximum height after the first bounce (1)

3.1.3 value of X shown on the graph (1)

3.2 Is the collision of the ball with the ground *elastic* or *inelastic*? Give a reason for your answer using information in the graph. No calculation needed. (2)

3.3 Draw a displacement versus time sketch graph of the motion of the ball up to 7,71s. Choose the ground as zero reference. You do not need to indicate any displacement values on the vertical axis but you must indicate the relevant time values on the horizontal axis. (4)

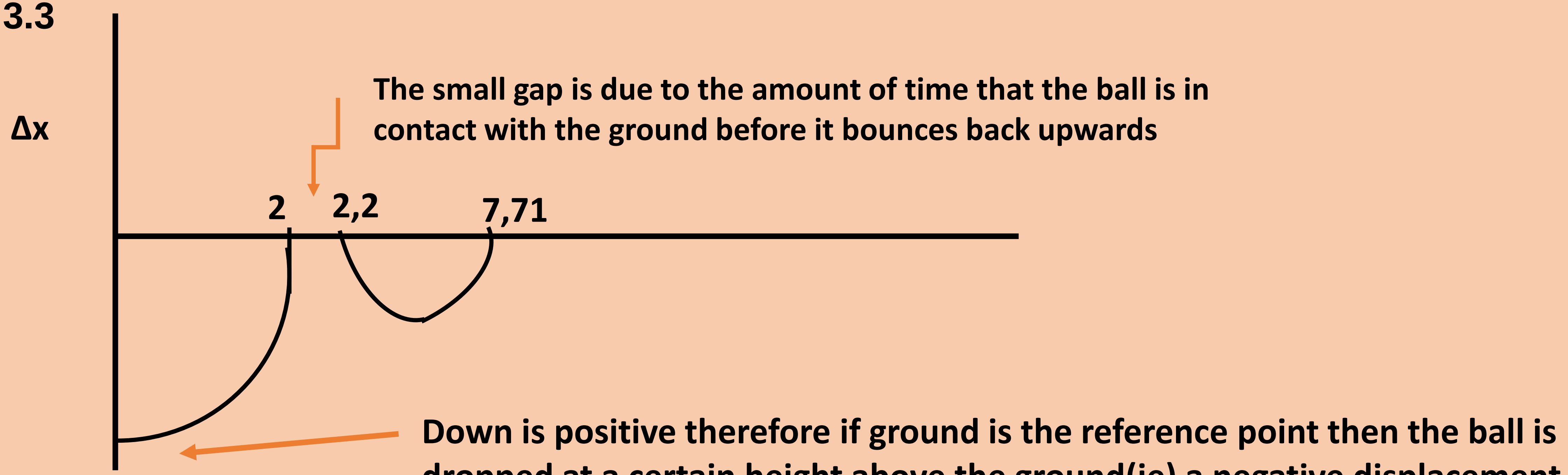
3.1.1 0,2 s ✓ (1)

3.1.2 4,955 s ✓ (1)

3.1.3 - 27 ($m.s^{-1}$) ✓ (1)

3.2 Inelastic✓ The speeds at which it strikes and leaves the ground are not the same/The kinetic energies will not be the same ✓ (2)

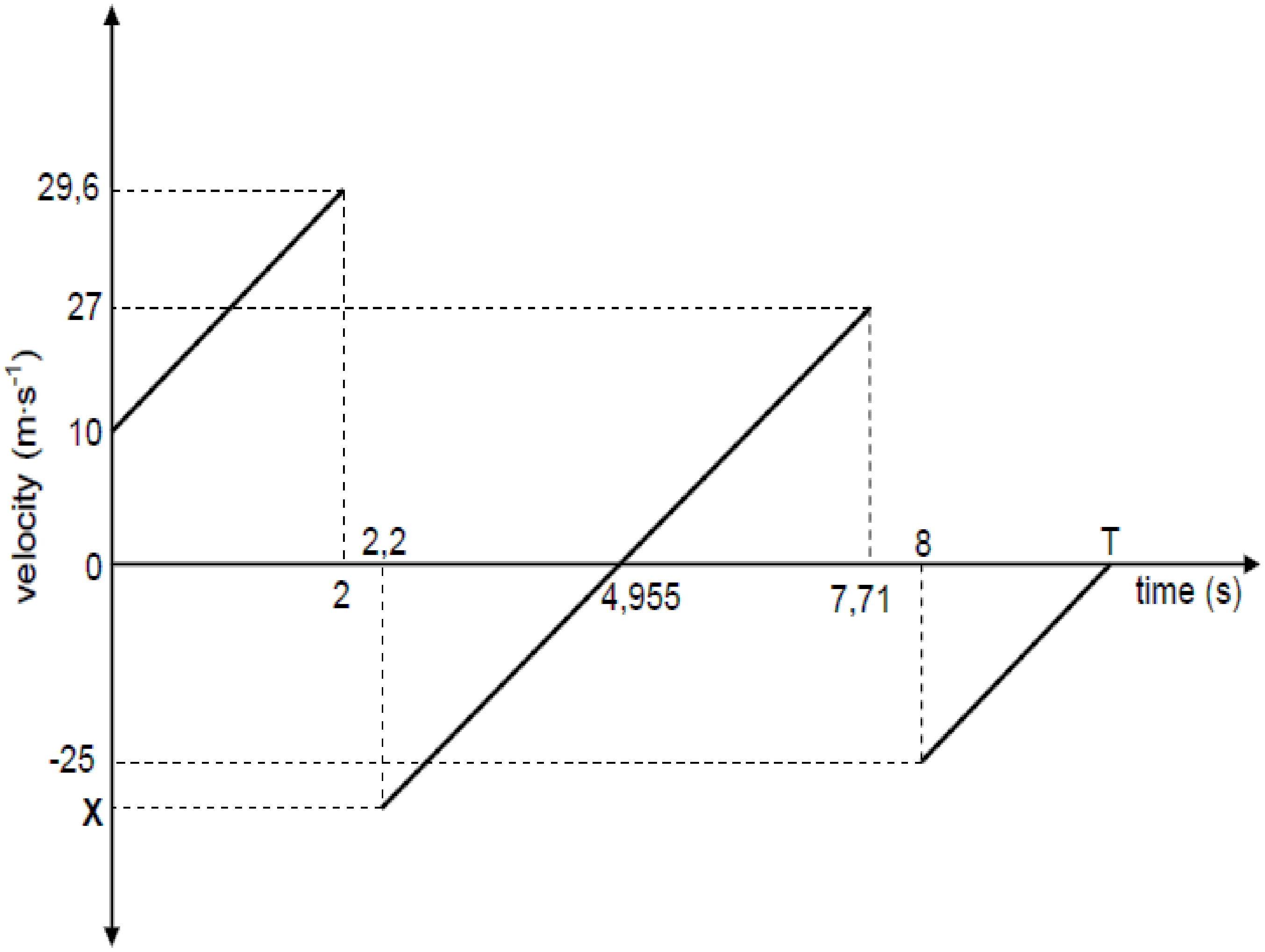
3.3



PRACTICE EXAMPLE 2

A ball is thrown vertically downwards from the top of a building and bounces a few times as it hits the ground. The velocity-time graph below describes the motion of the ball from the time it is thrown to a certain time T.

Take downwards as the positive direction and the ground as zero reference. The graph is NOT drawn to scale. The effects of air friction are ignored.



3.1 Define *velocity*. (2)

3.2 Write down the speed with which the ball is thrown downwards. (1)

3.3 ALL parts of the graph have the same gradient. Give a reason for this. (2)

3.4 Calculate the:

3.4.1 height from which the ball is throw(3)

3.4.2 time (T) shown on the graph (4)

3.1 Velocity is *the rate of change of position* or *the rate of displacement* or *the rate of change of displacement* ✓✓ (2)

3.2 10 $m \cdot s^{-1}$ ✓ (1)

3.3 The gradient represents the acceleration due to gravity (*g*) *which is constant* for free fall. ✓✓ (2)

3.4.1 Vi = 10 $m.s^{-1}$ s = ut + ½at² ✓ or $s = \frac{(u+v)}{2}t = \frac{(29.6+10)2}{2} = 39.6m$

Vf = = (10)(2) + ½ (9,8)(2²) ✓

g = 9,8 $m.s^{-2}$ = 39,6 m ✓

Δx =

Δt = 2s

(3)

3.4.2 Vi = -25 $m.s^{-1}$ v= u + at

Vf = 0 $m.s^{-1}$ 0 = -25 + (9,8)t

g = +9,8 $m.s^{-2}$ t = 2,55 s Total time T = 8 +2,55 = 10,55 s ✓ (4)

Δx =

Δt = ?