Introduction to Metrology: SI unit system and measurement standards, traceability, calibration and measurement uncertainty

Experimental Design
7.2.2013

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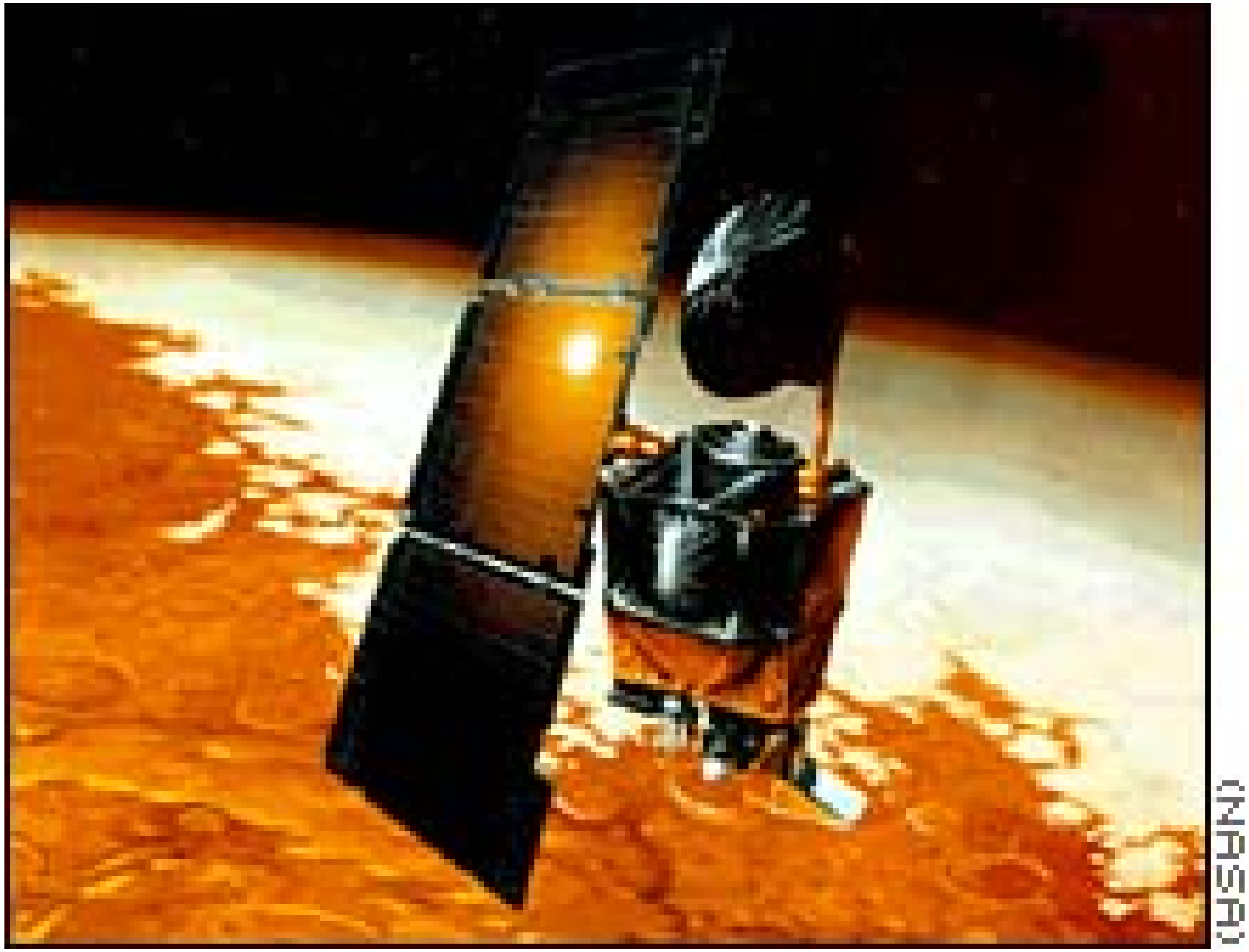
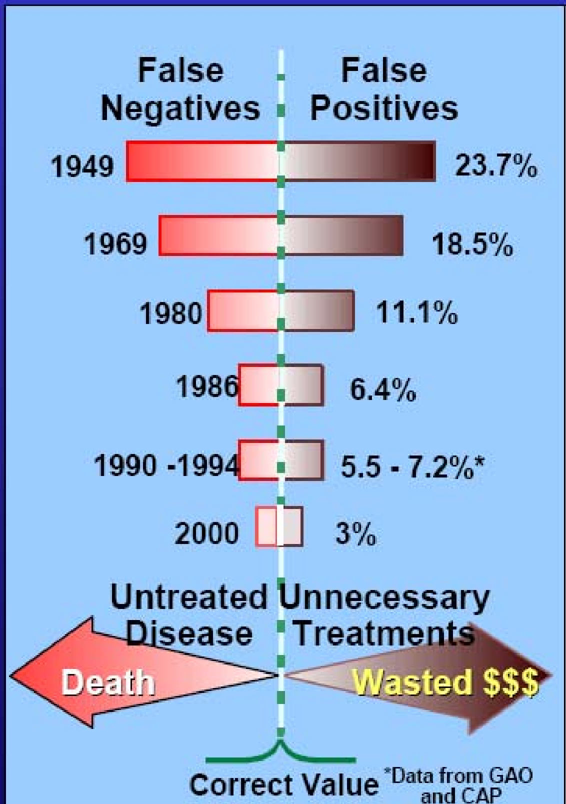
Mars Climate Orbiter



Impact of metrology: NIST SRM case

Cholesterol Measurements

Improved Cholesterol Measurement Accuracy Saves Health Care Dollars



NIST Contributions to National Reference System for Cholesterol

1967 – Pure Cholesterol SRM *(SRM 911)*

1980 – Cholesterol in Serum Definitive Method

1981 – First Cholesterol in Human Serum SRM *(SRM 09)*

1988 – New Suite of Cholesterol in Serum SRMs at MedicalDecision Points

1997 – New Suite of Fresh-Frozen Serum SRMs designed to address clinical analyzer commutability issues; Total-, HDL-, and LDL-Cholesterol and Triglyceride Values

Improvement in precision since 1968 has been estimated to save $100M/yr in treatment costs

Outline

1. Traceability in measurements
   - 1.1 Basic concepts
   - 1.2 Why we need traceability?
   - 1.3 Infrastructure ensuring availability of traceable measurements
2. SI – the international system of units
   - 2.1 System of units: from trade to science
   - 2.2 Base and derived units
   - 2.3 Measurement standards and traceability
3. Measurement uncertainty – part 1: Introduction
   - 3.1 Terminology
   - 3.2 Importance of the measurement uncertainty
4. Measurement uncertainty – part 2: Methods
   - 4.1 Calculating uncertainty
   - 4.2 Calculations step by step
   - 4.3 Uncertainty calculation in practice

1 Traceability in measurements

1.1 Basic concepts

1.2 Why we need traceability?

1.3 Infrastructure ensuring availability of traceable measurements

1.1 Basic concepts

Terms

QUANTITY

- Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference
  (A reference can be a measurement unit, a measurement procedure, a reference material, or a combination of such.)
- Quantity can be a general quantity (e.g. length) or particular quantity (e.g. wavelength of Sodium D line)

MEASURAND

- Quantity intended to be measured

ESTIMATE (of the measurand); called also MEASURED QUANTITY VALUE

- measured value of a quantity measured value
- quantity value representing a measurement result

MEASUREMENT ERROR

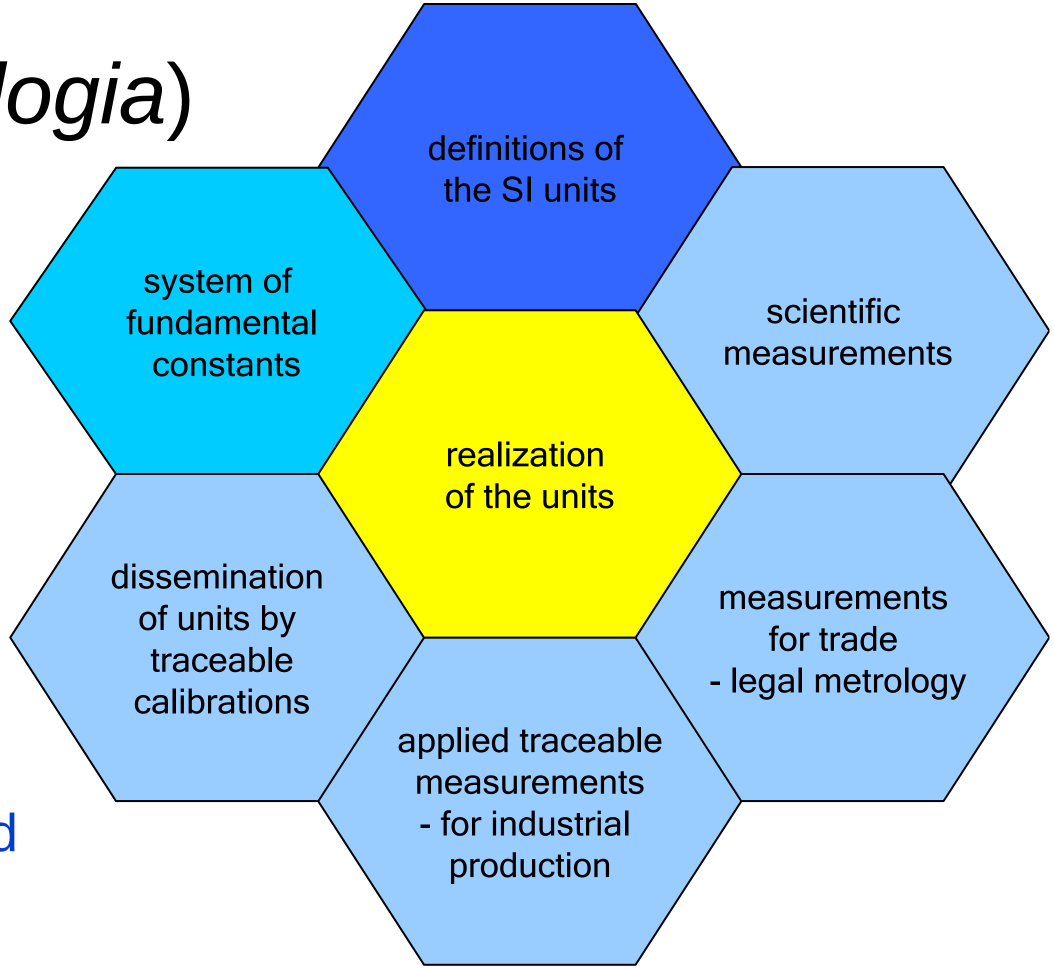
- measured quantity value minus a reference quantity value

Metrology (*metrologia*)

metrology is:

science of measurement and its application
[1.1, 1.2]

Metrology includes all theoretical and practical aspects of measurement, whatever the **measurement uncertainty** and field of application.

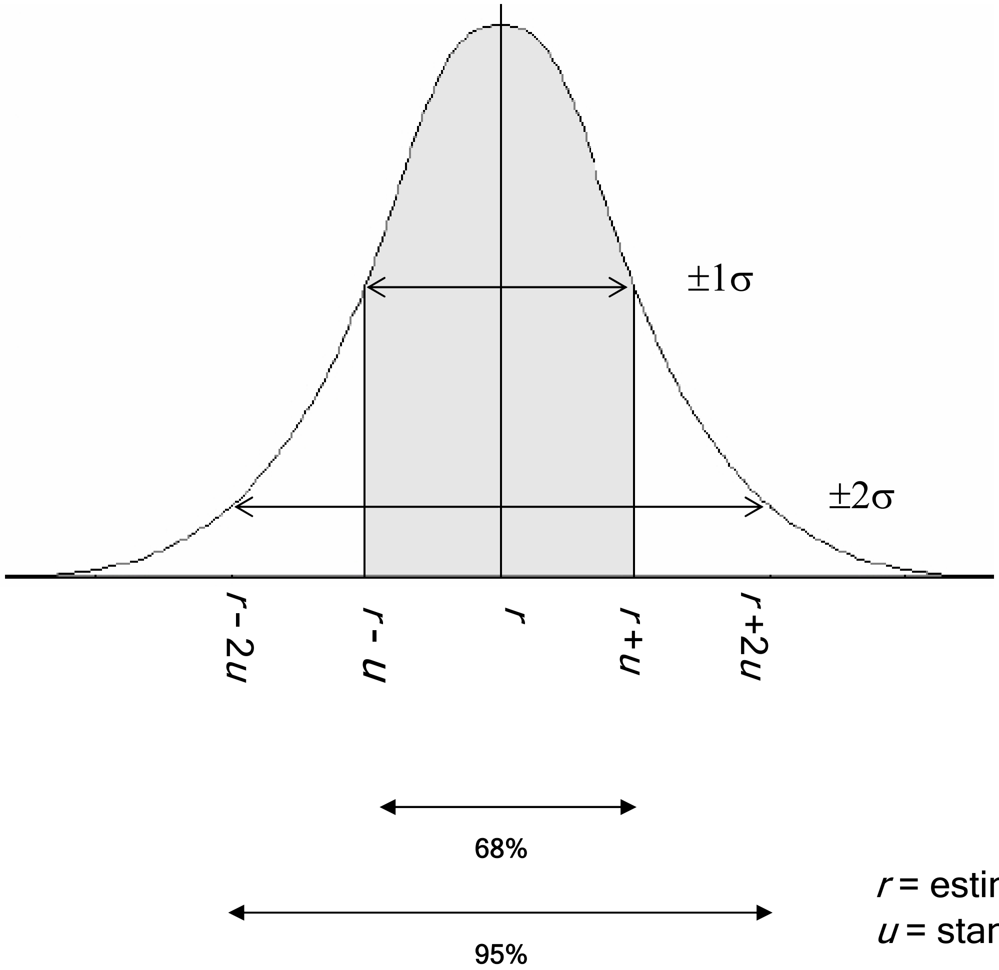


Metrology (*metrologia*)

Usually the concept of metrology is limited to:

- Science of measurement studying the relationship between measurands and their estimates (measured quantity values)

Uncertainty of measurement (*epävarmuus*)



non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

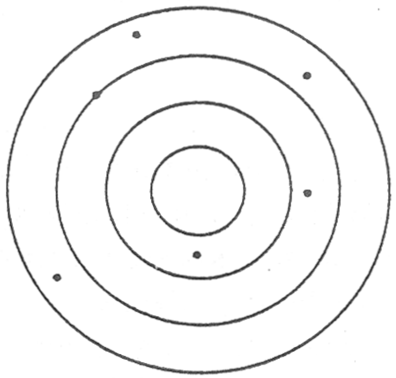
[1.1, 1.2]

r = estimate = mean value
u = standard uncertainty

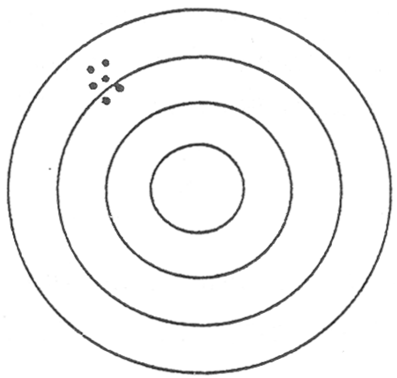
Facts about Measurement Results

- Usually a measurement disturbs its target affecting thus the measurement result.
- A measurement result is always a combination of many factors. (reading or output signal of the measuring device is only one of them)
- We never know all the factors.
- We never know the “true value” of the factors (we can only estimate).
- The combined effect of unknown factors and inaccurate knowledge on the known factors is described quantitatively with the measurement uncertainty.
- “Absolute certainty is the privilege of uneducated minds ... or fanatics.”

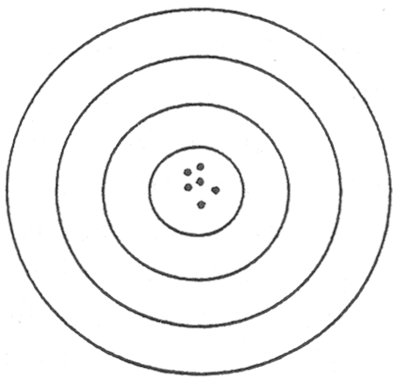
Accuracy and Precision



Low precision - low accuracy



High precision - low accuracy



High precision - high accuracy

accuracy (*tarkkuus*)

closeness of agreement between a measured quantity value and a true quantity value of a measurand

precision (*täsmällisyys*)

closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions

[1.1, 1.2]

Calibration (*kalibrointi*)

operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication. [1.1, 1.2]

The result of a calibration can be given as

- correction value(s) at discrete measurement point(s)
- calibration equation (curve)
- re-calculated constant(s)
- statement that the error is in given specifications.

A calibration result includes always the uncertainty.
(also the probability level should be given)

Conditions of calibration should be stated when reporting calibration results.

Adjustment (*viritys*)

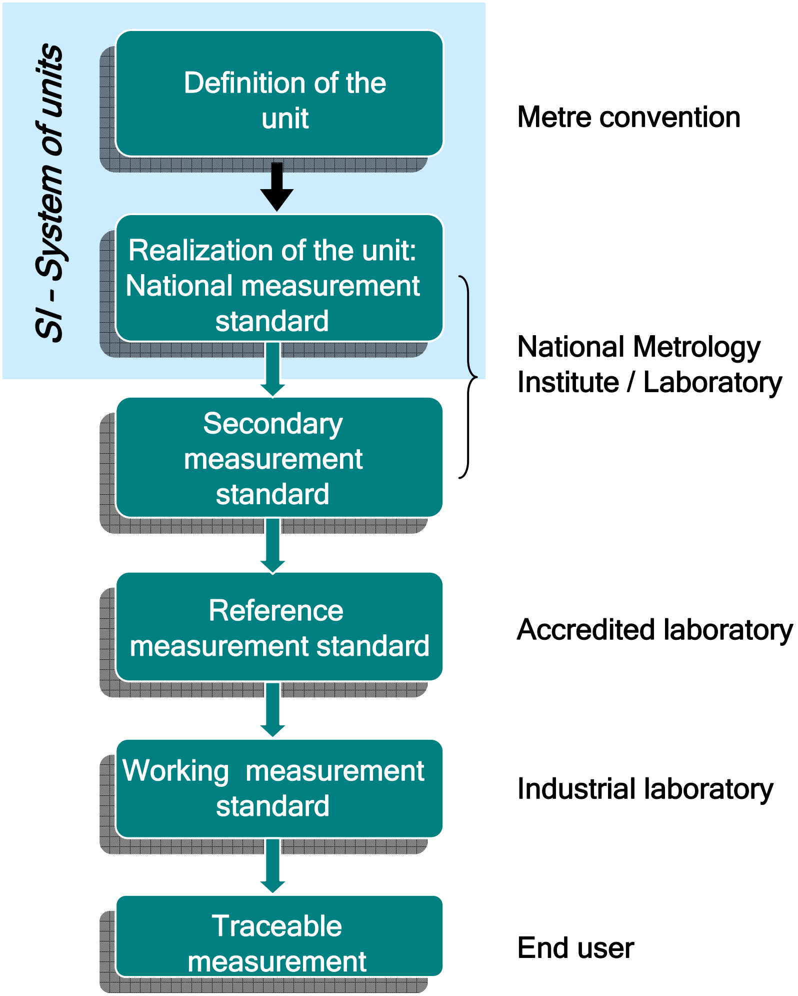
set of operations carried out on a measuring system so that it provides prescribed indications corresponding to given values of a quantity to be measured [1.1, 1.2]

NOTE 1: Types of adjustment of a measuring system include zero adjustment of a measuring system, offset adjustment, and span adjustment (sometimes called gain adjustment).

NOTE 2: Adjustment of a measuring system should not be confused with calibration, which is a prerequisite for adjustment.

NOTE 3: After an adjustment of a measuring system, the measuring system must usually be recalibrated.

Traceability (*jäljitettävyys*)



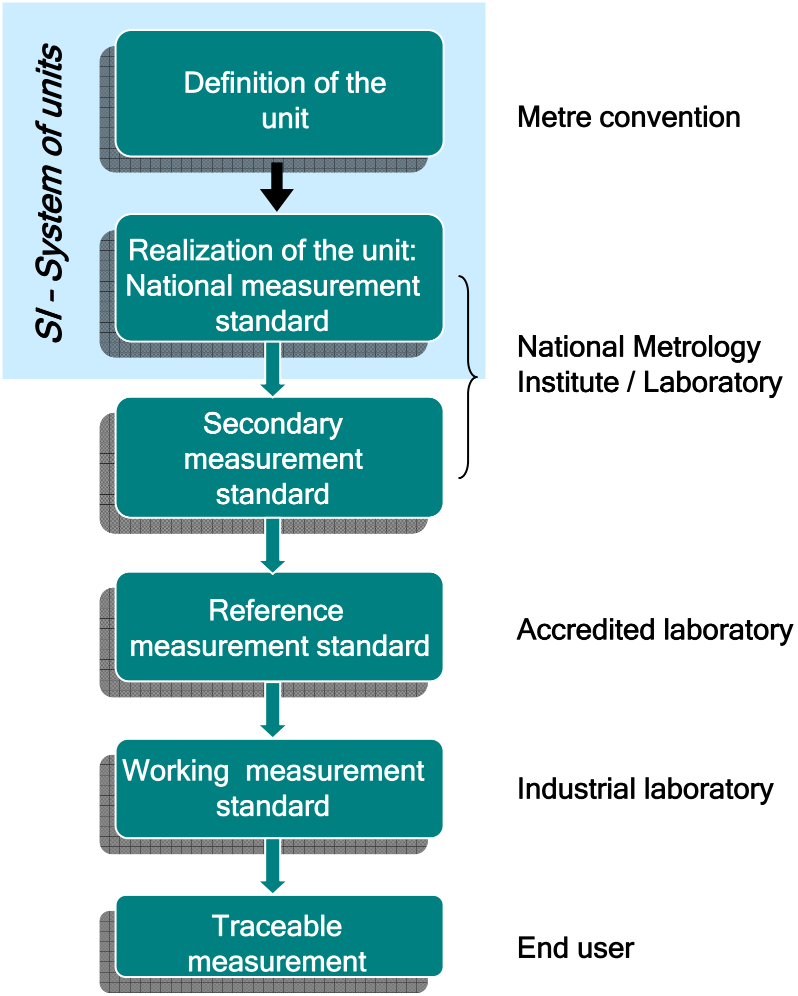
Example of traceability chain

(Metrological traceability)
property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty

Traceability chain:
sequence of measurement standards and calibrations that is used to relate a measurement result to a reference

[1.1, 1.2]

Traceability (*jäljitettävyys*)



Example of traceability chain

Traceability ⇒ Evidence that your unit is of the same size as the internationally accepted one.

Measurement standard (*mittanormaali*)

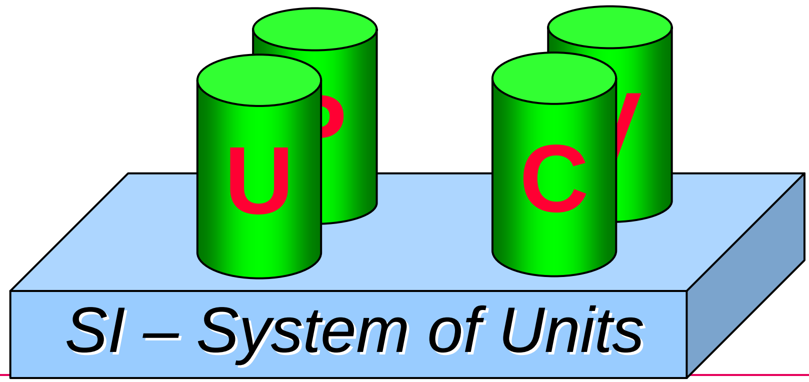
- realization of a given quantity, with stated quantity value and associated measurement uncertainty, used as a reference
- Measurement standard can be an instrument, a measuring system, material measure or a reference material.

[1.2]

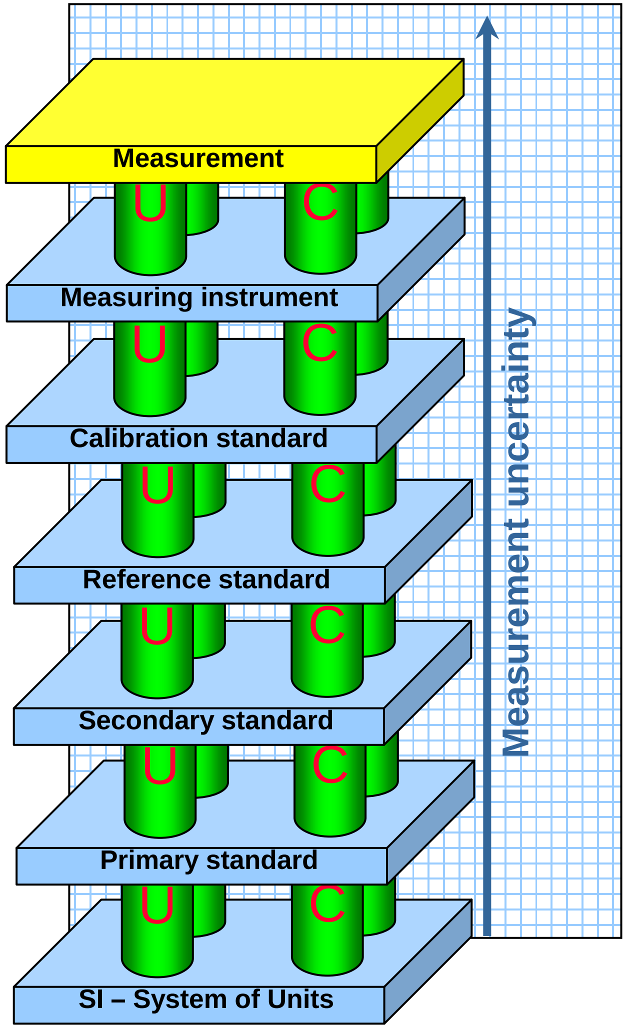
Characteristics of Unbroken Traceability

For each calibration of the chain:

- Uncertainty estimation
- Documented and generally acknowledged procedures, documented results
- Competence
- Calibration is valid for the application.
  (interval of calibrations, conditions etc.)



Traceability Tower



- The tower is collapsed if any part of it is missing or incomplete

  i.e.

  there is no traceability unless all levels include all the characteristics of traceability

- At any level the measurement uncertainty can´t be smaller than levels below.

Typical problems in traceability

No calibration

Calibration interval too long

No evidence on the traceability in calibration

Calibration was carried out in conditions too different from the application

Incompelete traceability chain

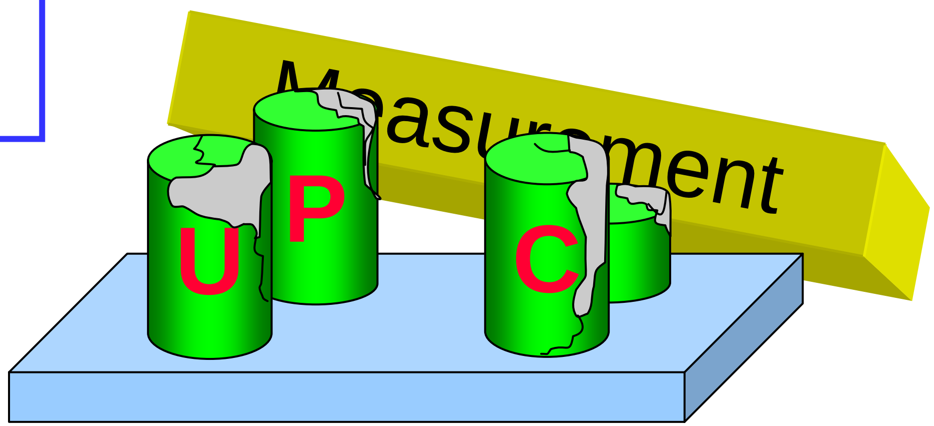
=

no traceability

Calibration results have not been used

Calibration covers only a part of the measuring system

There is no evidence on the traceability of the refence materials



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1.2 Why we need traceability?

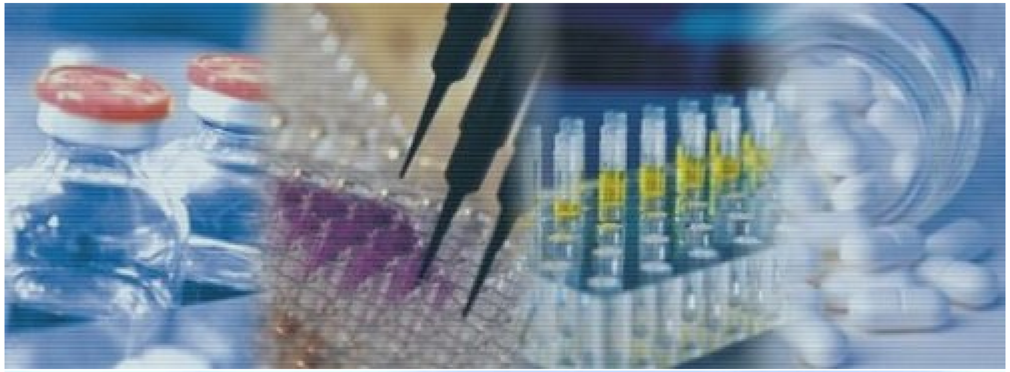
Traceability – why?

- Modern industrial production is not possible if subcontractors have different scales
- Scientific technical research can not necessarily be repeated if it is done without traceably calibrated instruments
- Traceability is the answer to the questions:
  - From where can I find a reliable measurement standard?
  - How can I convince others of the reliability of my measurement standard?

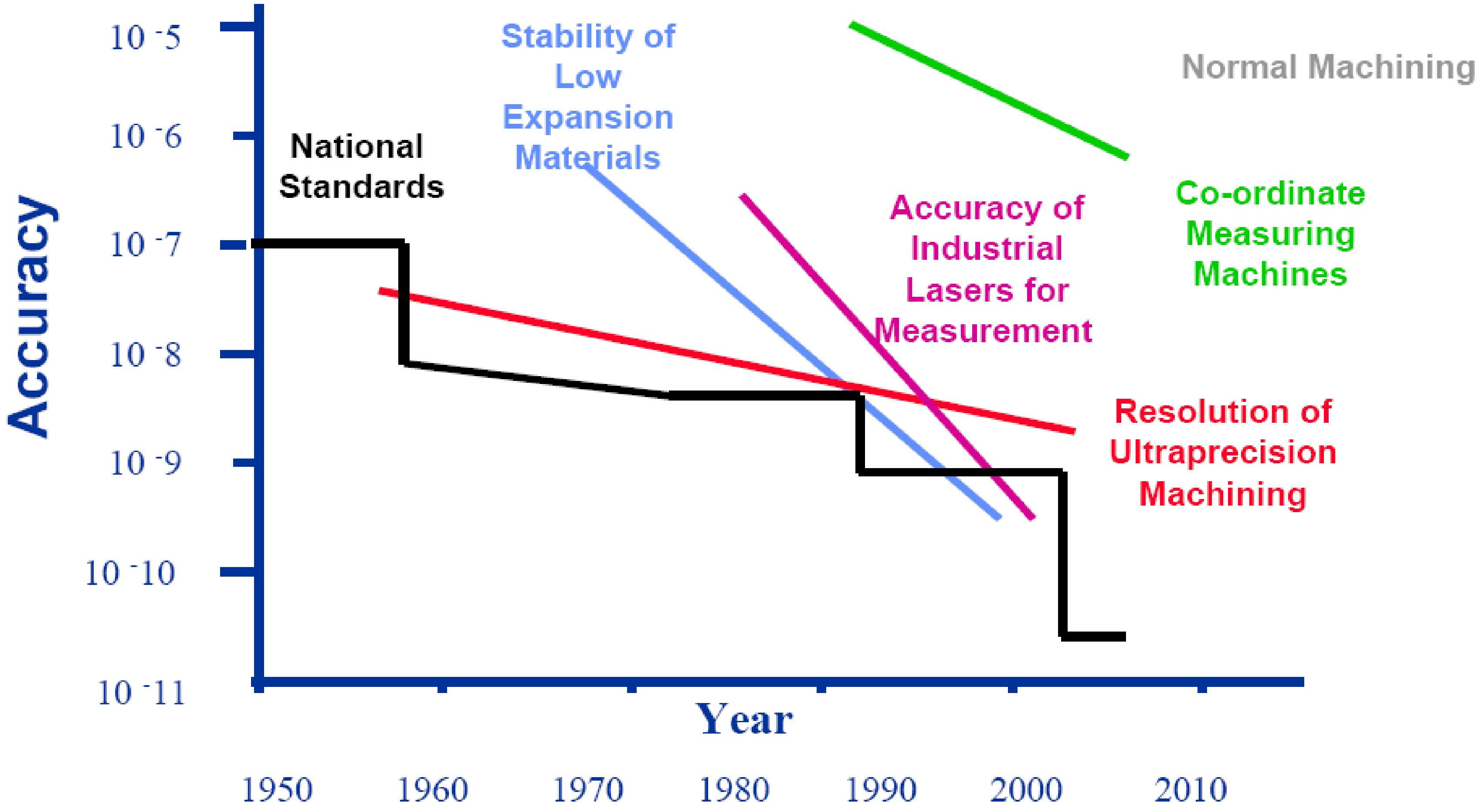
Traceability – why?

Examples:

- Pharmaceutics
- Telecommunication
- Aviation
- Space technology
- Safety: Nuclear power plant



Accuracy of Length Standards and Commercial Capability 1950 - 2010



Source: DTI UK

1.3 Infrastructure ensuring availability of traceable measurements

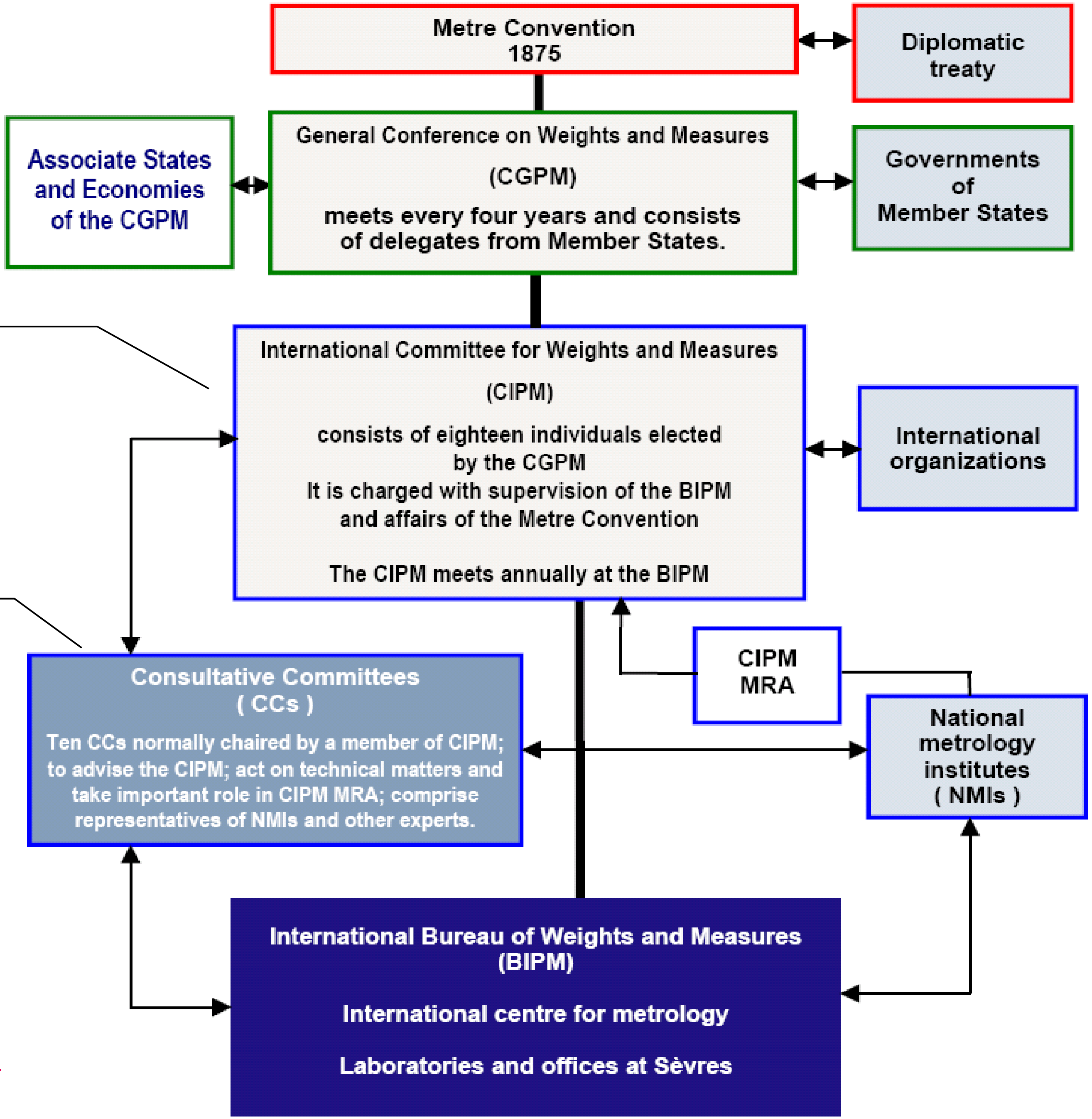
Metrology Infrastructure

- for ensuring global availability of traceable measurements
- foundation: The Metre Convention – a diplomatic treaty first signed in 1875 by representatives of 17 nations
- three levels:
  - Global
  - Regional
  - National

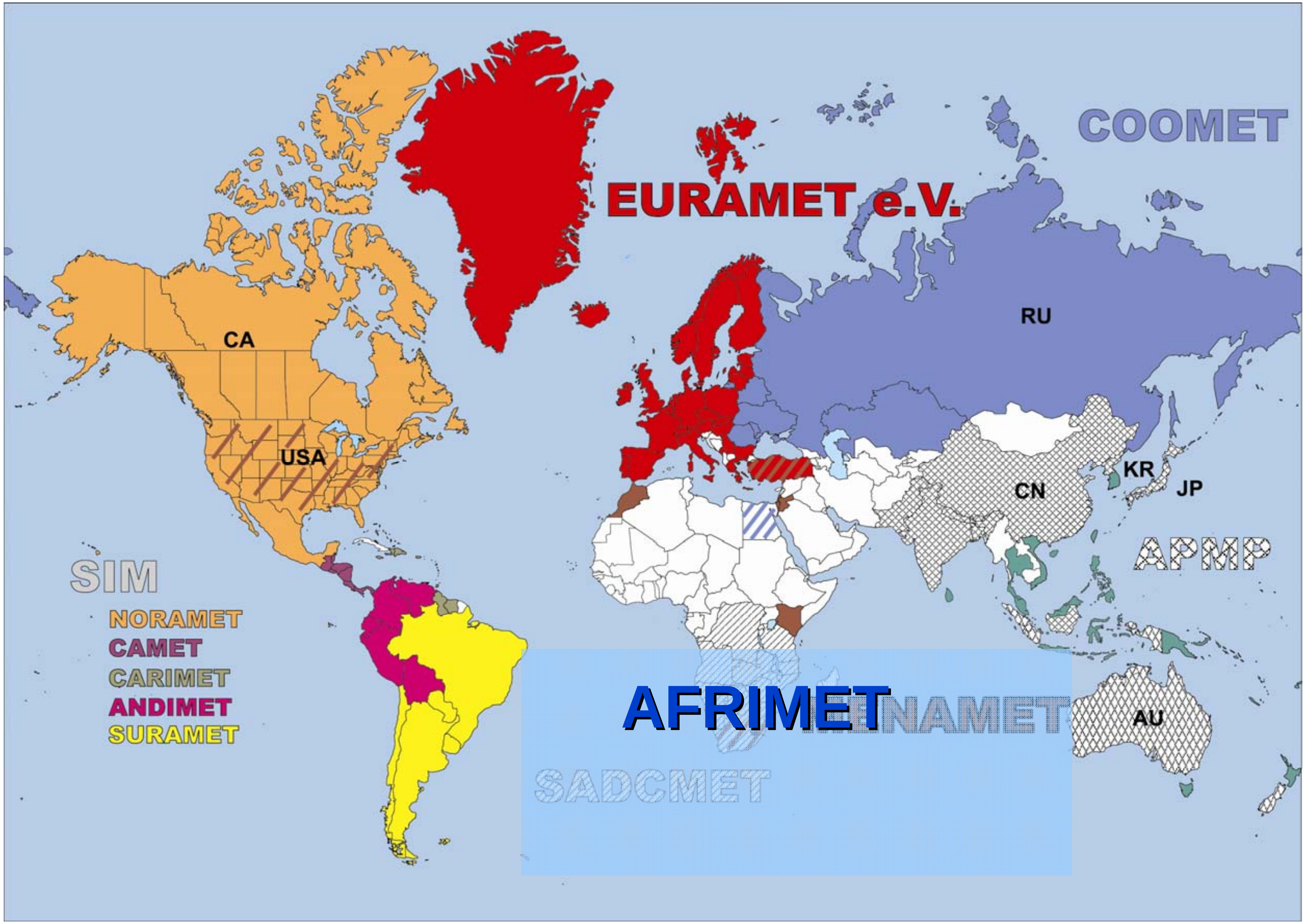
Global:

Finnish members:
1953-1971 Yrjö Väisälä
1992-1998 Olli Lounasmaa

MIKES is a member of:
- CC for Length,
- CC for Electricity and Magnetism
- CC for Thermometry
- CC for Photometry and Radiometry



Regional:



EURAMET e.V.
COOMET
RU
CA
USA
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ANDIMET
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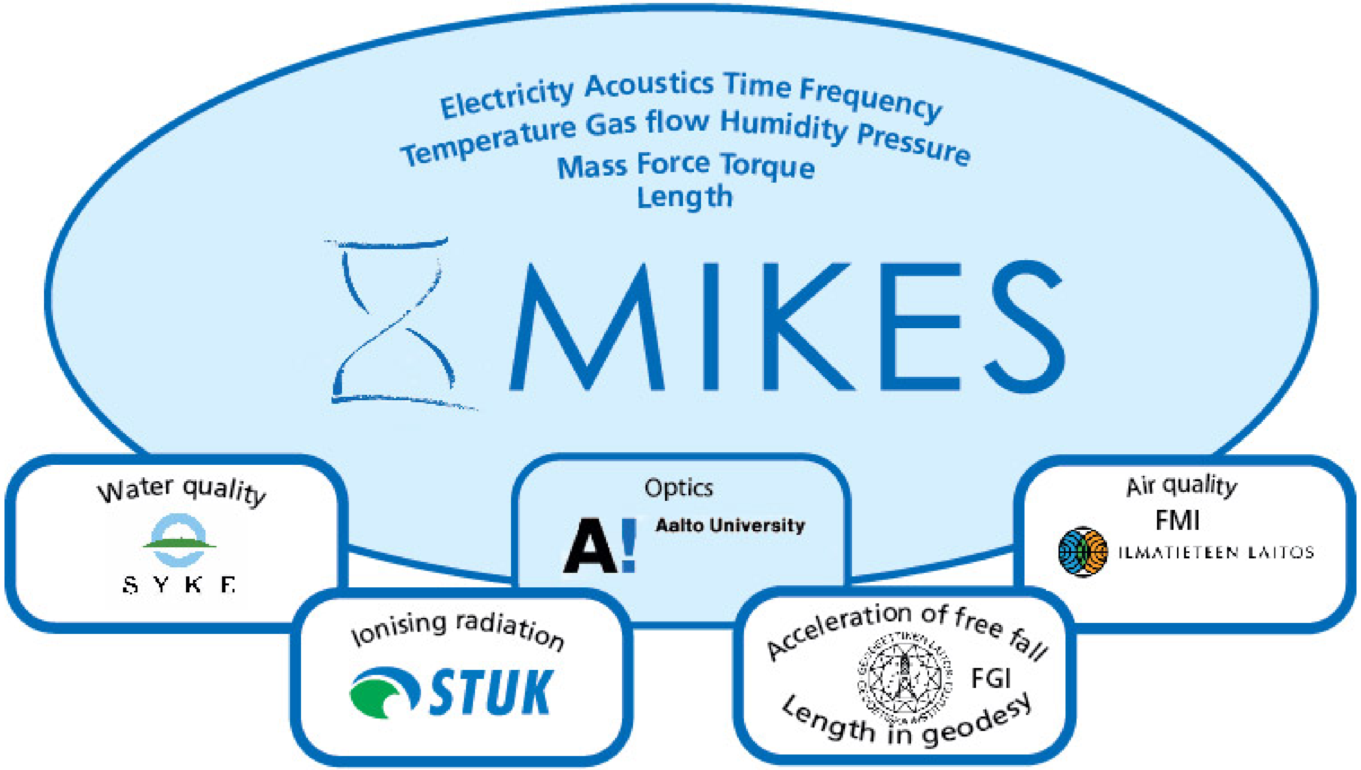
AFRIMET
NAMET
AU
SADCMET

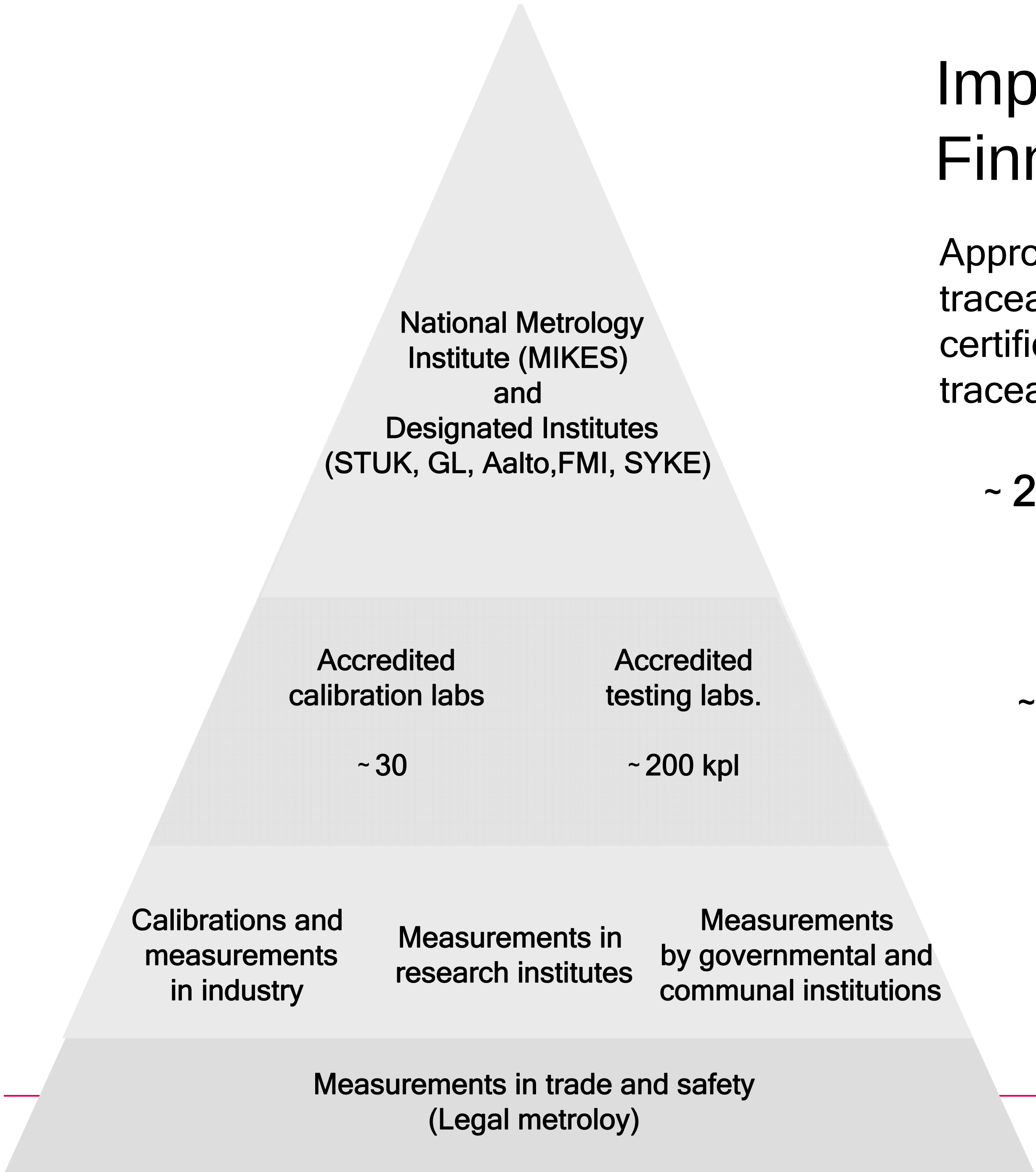
National:



Electricity Acoustics Time Frequency
Temperature Gas flow Humidity Pressure
Mass Force Torque
Length
MIKES
Water quality
SYKE
Optics
Aalto University
Air quality
FMI
ILMATIETEEN LAITOS
Ionising radiation
STUK
Acceleration of free fall
FGI
Length in geodesy

Impact of metrology: Finnish numbers

Approximate amount of traceable calibration and testing certificates issued annually and traceable instruments/measurements



~ 2000

~ 20 000

~ 2 000 000

~ 2 000 000 000 (00?)

Reconnaissance mutuelle

des étalons nationaux de mesure
et des certificats d'étalonnage et de mesurage
émis par les laboratoires nationaux de métrologie

Paris, le 14 octobre 1999

Mutual recognition

of national measurement standards
and of calibration and measurement certificates
issued by national metrology institutes

Paris, 14 October 1999

Comité international des poids et mesures

Bureau international des poids et mesures	Organisation intergouvernementale de la Convention du Mètre

CIPM MRA objectives:

- to establish the degree of equivalence of measurement standards maintained by NMIs;
- to provide for the mutual recognition of calibration and measurement certificates issued by NMIs; thereby to
- to provide governments and other parties with a secure technical foundation for wider agreements related to international trade, commerce and regulatory affairs

MRA has been signed by 67 NMIs from 45 states and 20 associate states

MRA includes database of NMIs reviewed calibration services with corresonding uncertainties (CMCs)

MIKES has several 100s of different CMC entries

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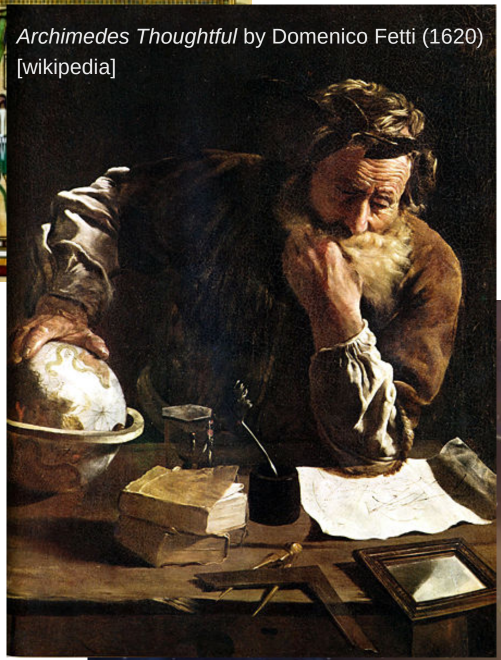
2 SI – the international system of units

2.1 System of units: from trade to science
2.2 Base and derived units
2.3 Unit realisations and measurement standards

2.1 A system of units: from trade to science

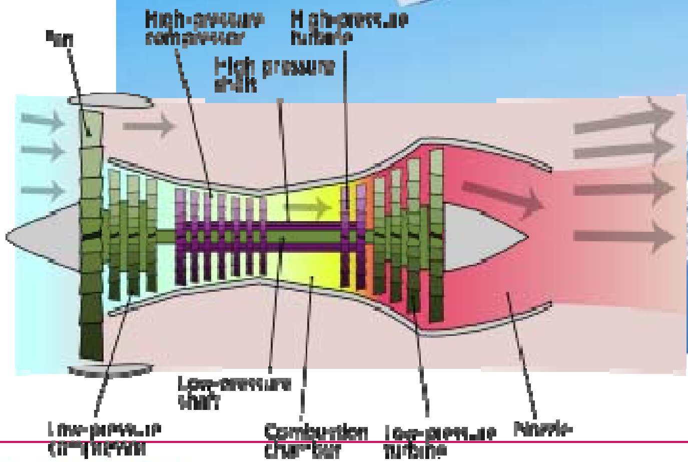


Archimedes Thoughtful by Domenico Fetti (1620) [wikipedia]

SI – system for characterizing physical and chemical phenomena

speed:	$v = \frac{\Delta x}{\Delta t}$
force:	$F = ma = m\frac{\Delta x}{(\Delta t)^2}$
energy:	$E = F\Delta x = m\frac{(\Delta x)^2}{(\Delta t)^2}$
power:	$P = \frac{F}{\Delta t} = \frac{m(\Delta x)^2}{(\Delta t)^3}$
pressure:	$p = \frac{F}{A} = \frac{m\Delta x}{(\Delta t)^2 l^2}$
density:	$\rho = \frac{m}{V} = \frac{m}{l^3}$

SI – system for characterizing physical and chemical phenomena



for real gas: $p_g V = n_g R T Z_g(p,T)$

rotating speed: $\omega = \frac{n}{\Delta t}$

thermal expansion: $\Delta l = l \alpha \Delta T$

frequency: $f = \frac{n}{\Delta t} = \frac{1}{\tau}$

resistance: $R = \frac{U}{I}$

Electr. power: $P = U \cdot I$

illuminance: $E_v = \frac{F}{A} = \frac{I_v \Omega}{A}$

$$C_{12}H_{26}(l) + {}^{37}/_{2}\ O_2(g) \rightarrow 12\ CO_2(g) + 13\ H_2O(g)$$

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2.2 Base and derived units

SI system – properties

- based on a set of 7 base units
  - by convention these are assumed to be independent
- other units are formed as products of powers from base units (derived units)
- some derived units have special names like Newton
- system is coherent i.e.

  equations between the numerical values of quantities take exactly the same form as the equations between the quantities themselves
- decimal system is used

SI system – Base units

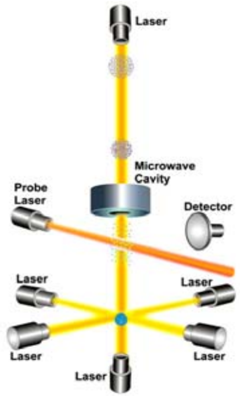
Base quantity		SI base unit	
Name	Symbol	Name	Symbol
length	l, x, r, etc.	metre	m
mass	m	kilogram	kg
time, duration	t	second	s
electric current	I, i	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	I_v	candela	cd

[2.1]

Definitions of base units

- **Kilogram**:
  The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

- **Metre:**
  The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.



- **Second:**
  The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Definitions of base units

- **Kelvin**:
  The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- **Mole:**
  The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.
  - When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

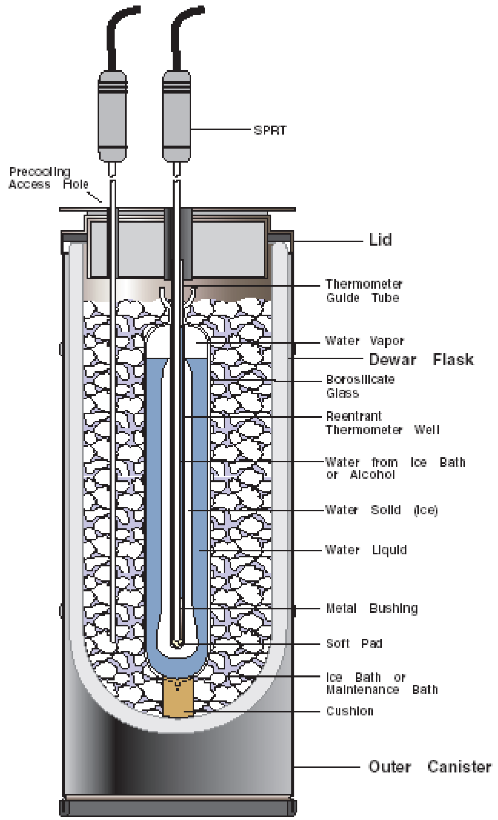
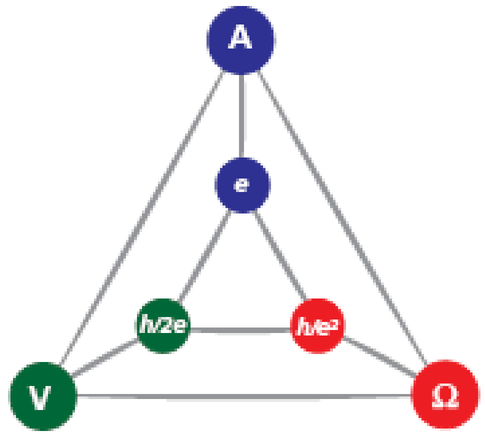


Figure 4 Water Triple Point Cell in an Ice Bath Dewar

Definitions of base units



- **Ampere**:
  The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
  - NOTE 1: Direct realisation of the SI ampere is very difficult in practice
  - NOTE 2: $N = kg \cdot m \cdot s^{-2} \Rightarrow$ Depends on the prototype of kg
- **Candela:**
  The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency
  540 x 1012 hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Need for redefining SI base units

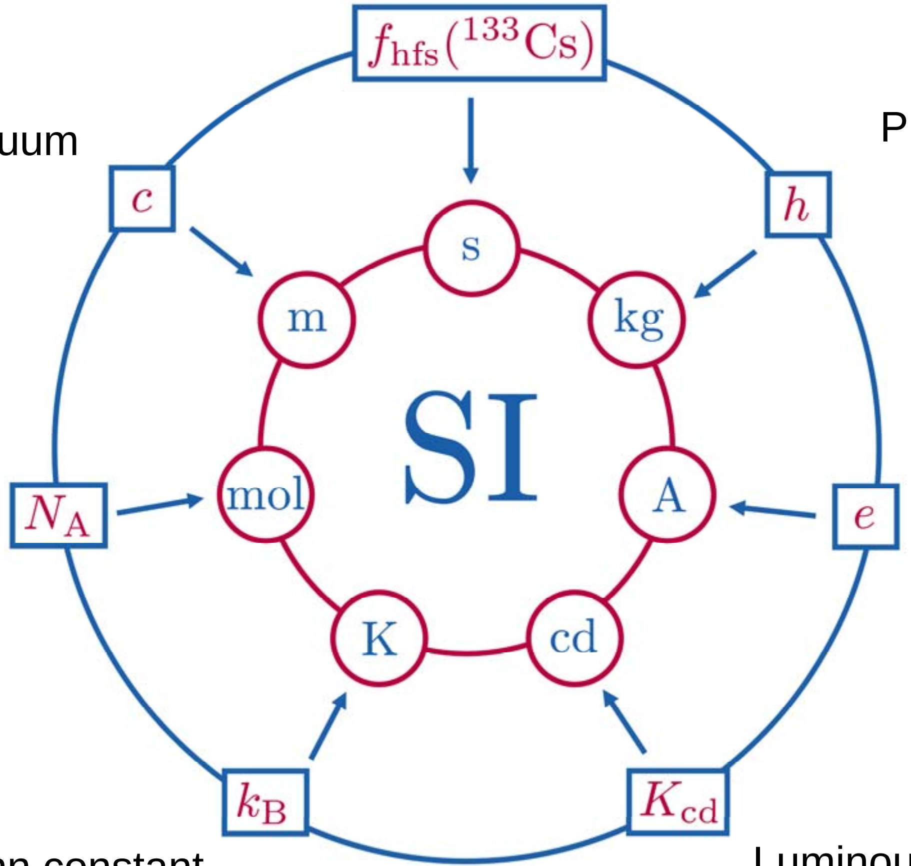
- Drawbacks of the current SI are:
  - Kilogram is defined with an artefact
  - The definition of kelvin is based on material properties
  - The definition of ampere limits the achievable accuracy and is not currently use (quantum standards of ohm and voltage are used instead through the Ohm's law)
- Because of these, drift, non-uniqueness and limitations in accuracy cannot completely be prevented.

SI base units in future

Ground state hyperfine splitting frequency of the caesium 133 atom

Speed of light in vacuum

Planck constant



Avogadro constant

Elementary charge

Boltzmann constant

Luminous efficacy

SI system – Derived units

Derived quantity		SI coherent derived unit	
Name	Symbol	Name	Symbol
area	A	square metre	m^2
volume	V	cubic metre	m^3
speed, velocity	v	metre per second	m/s
acceleration	a	metre per second squared	m/s^2
wavenumber	σ, $\tilde{\nu}$	reciprocal metre	m^{-1}
density, mass density	ρ	kilogram per cubic metre	kg/m^3
surface density	ρ_A	kilogram per square metre	kg/m^2
specific volume	v	cubic metre per kilogram	m^3/kg
current density	j	ampere per square metre	A/m^2
magnetic field strength	H	ampere per metre	A/m
amount concentration [(a)], concentration	c	mole per cubic metre	mol/m^3
mass concentration	ρ, γ	kilogram per cubic metre	kg/m^3
luminance	L_v	candela per square metre	cd/m^2
refractive index [(b)]	n	one	1
relative permeability [(b)]	μ_r	one	1

(*a*) In the field of clinical chemistry this quantity is also called substance concentration.

(*b*) These are dimensionless quantities, or quantities of dimension one, and the symbol "1" for the unit (the number "one") is generally omitted in specifying the values of dimensionless quantities.

[2.1]

SI system – Derived units with special names

	SI coherent derived unit [a]			
Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
plane angle	radian [b]	rad	1 [b]	m/m
solid angle	steradian [b]	sr [c]	1 [b]	m^2/m^2
frequency	hertz [d]	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, amount of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiant flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge, amount of electricity	coulomb	C		s A
electric potential difference, electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$

[2.1]

SI system – Derived units with special names 2

	SI coherent derived unit [a]			
Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}\ A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Celsius temperature	degree Celsius [e]	°C		K
luminous flux	lumen	lm	cd sr [c]	cd
illuminance	lux	lx	lm/m^2	m^{-2} cd
activity referred to a radionuclide [f]	becquerel [d]	Bq		s^{-1}
absorbed dose, specific energy (imparted), kerma	gray	Gy	J/kg	$m^2\ s^{-2}$
dose equivalent, ambient dose equivalent, directional dose equivalent, personal dose equivalent	sievert [g]	Sv	J/kg	$m^2\ s^{-2}$
catalytic activity	katal	kat		s^{-1} mol

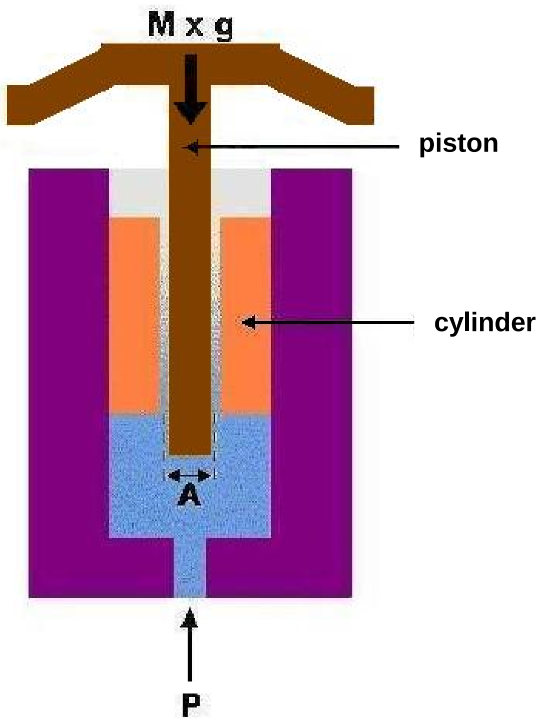
[2.1]

Realisation of a derived unit

- The realisation forms a link between the measurements of a derived quantity and the SI base units.
- Characteristics of the realisation:
  - Based on the definition of the quantity
  - Measurements in terms of SI units (base or derived but not of the same quantity) with traceability to national/international measurement standards
  - Full uncertainty analysis
  - Comparisons to other realisations of the same quantity
- National Standards Laboratories maintain realisations (or secondary standards) for a set of derived quantities

Realisation of the pascal

- Unit with a special name: 1 Pa = 1 m^{-1} kg s^{-2}



Definition of pressure: $$p = \frac{F}{A}$$

Realisation of the unit:

Force lifting the piston F_1:

$$F_1 = pA$$

Force lowering the piston F_2:

$$F_2 = mg$$

In equilibrium:

$$F_1 = F_2 \Rightarrow p = \frac{mg}{A}$$

Pressure balance

Realisation of the unit of the dew-point temperature (K or °C)

- Same unit as of the temperature but different meaning:

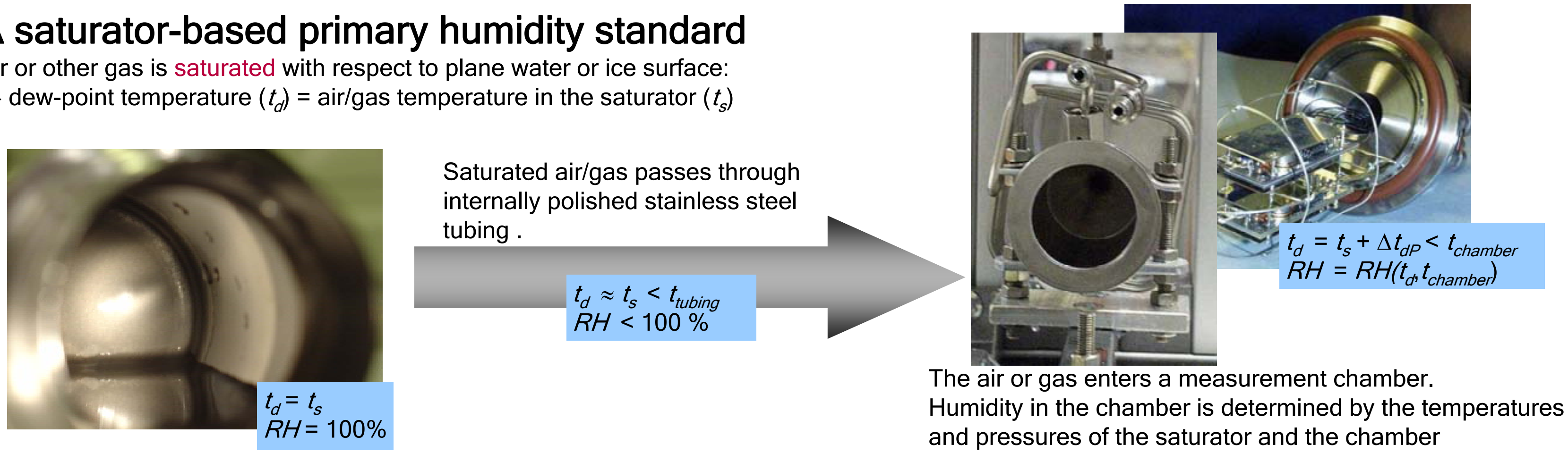
Dew-point temperature (t_d):

The temperature at which the gas would be saturated with respect to a plane surface of pure liquid water or solid ice at the same gas pressure.

A saturator-based primary humidity standard

Air or other gas is saturated with respect to plane water or ice surface:

⇒ dew-point temperature (t_d) = air/gas temperature in the saturator (t_s)



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2.3 Measurement standards and traceability

Traceability in chemical measurements

SI unit: mole

The mole was added to SI in 1971.

↓

Chemical primary methods

- Isotope Dilution Mass Spectrometry
- Gravimetry
- Titrimetry
- Coulometry
- Freezing Point Depression (Differential Scanning Calorimetry)

↓

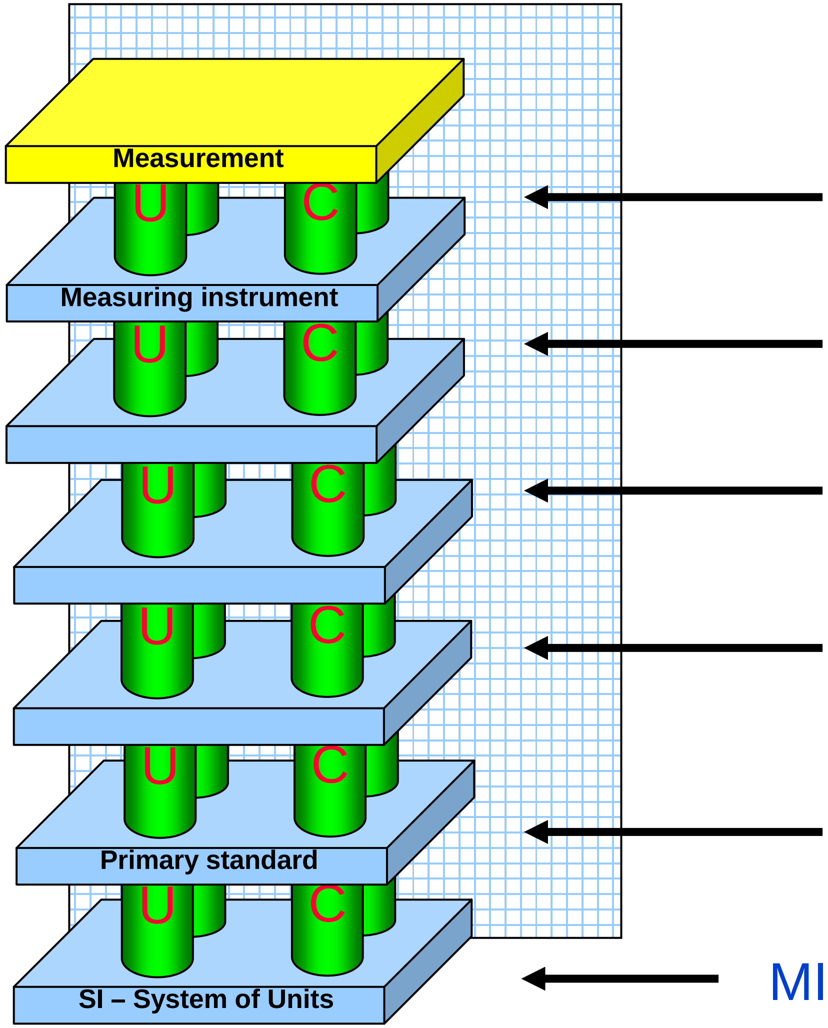
Certified reference materials

ISO Guides 30 – 35, 1989 - 2000

↓

Validated measurement methods and reference materials in laboratories

Traceability to altitude measurements in a military aircraft



Altimeter
±10 m
The pressure simulator of the brigade
±30 Pa ~ ±2.6 m
K013 pressure working standard
±20 Pa ~ ±1.7 m
K013 pressure reference standard
±10 Pa ~ ±0.9 m
MIKES Primary Pressure Standard
MIKES Primary Mass and Length Standards

3 Measurement uncertainty – part 1: Introduction

3.1 Terminology

3.2 Importance of the measurement uncertainty

3.1 Terminology

Terms

MEASUREMENT REPEATABILITY

- measurement precision under a set of repeatability (similar) conditions of measurement

MEASUREMENT REPRODUCIBILITY

- measurement precision under reproducibility (changed) conditions of measurement

RANDOM ERROR

- Component of measurement error that in replicate measurements varies in an unpredictable manner

SYSTEMATIC ERROR

- Component of measurement error that in replicate measurements remains constant or varies in a predictable manner

[1.1, 1.2]

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3.2 Importance of the measurement uncertainty

Measurement result and its uncertainty

- Estimated quality of a result is expressed as the uncertainty
- The uncertainty is an essential part of a measurement result:

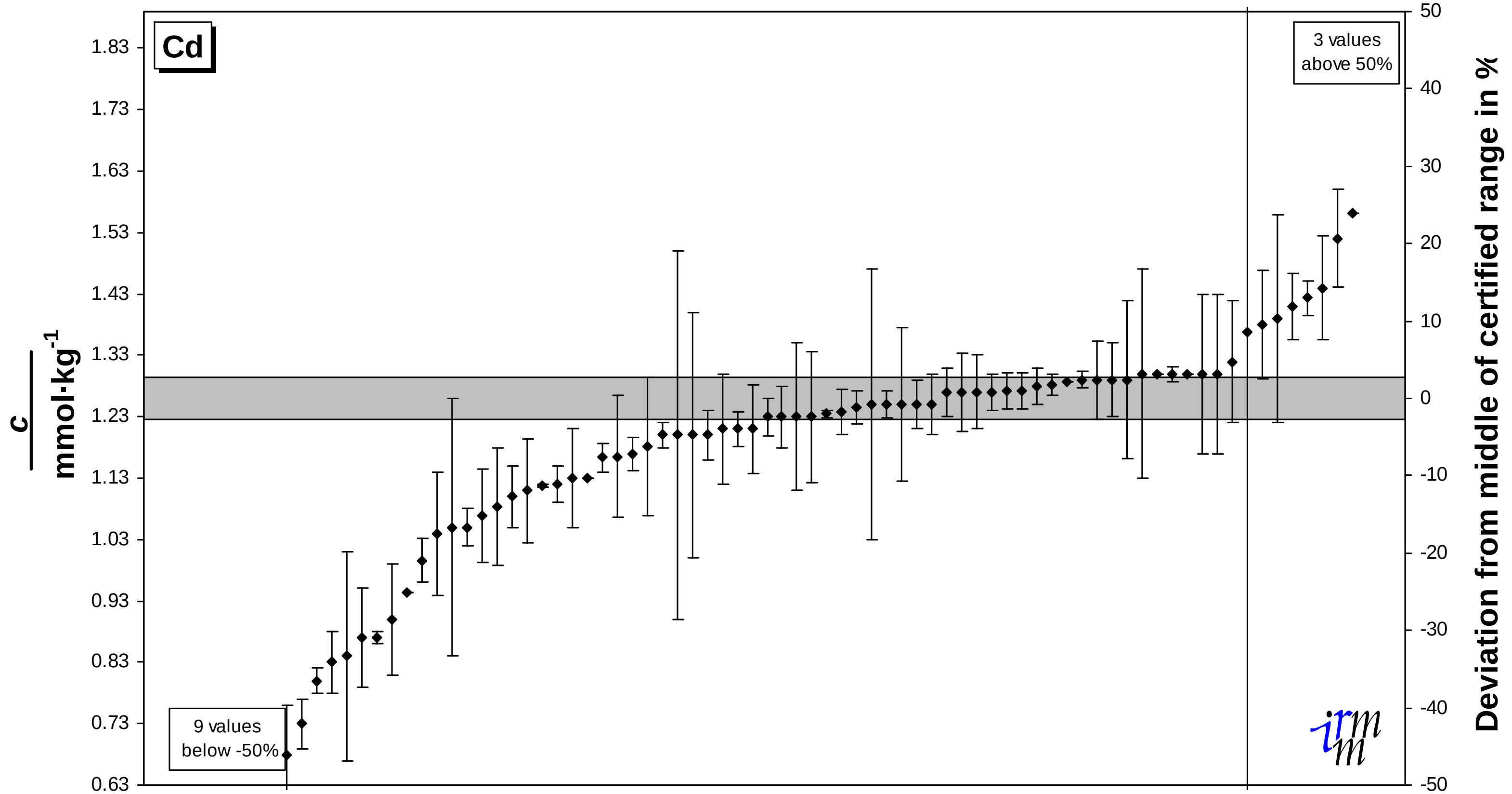
  Measurement result = Estimate ± uncertainty

- The uncertainty gives the limits of the range in which the “true” value of the measurand is estimated to be at a given probability.
- Too often the uncertainty is not presented explicitly with the estimate
  ⇒ confusion, incorrect conclusions, non-equal users etc.
- If the uncertainty is not taken into account, incorrect conclusions are drawn and the number of unsatisfactory products increases.
- When you know the uncertainty in a measurement, then you can judge its fitness for purpose.

Importance of uncertainty of measurements

- Measurements are never absolutely accurate, but there is always some uncertainty in the measured values.
- Measurement result without uncertainty estimate is meaningless.
- Estimation of measurement uncertainties is one of the most important parts of practical metrology.
- A proper uncertainty budget indicates what parts of the measurement should be developed to decrease the overall uncertainty most effectively.

Certified range [$U = k \cdot u_c$ (k=2)]: 1.226 - 1.294 mmol·kg^{-1}



Results from all participants.

[Mittaustulosten epävarmuus, MIKES 26.6.2005]

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4 Measurement uncertainty – part 2: Methods

4.1 Calculating uncertainty

Calculating a measurement result

- A measurement result is calculated from input data. In addition to the measurement values, the data often include information from earlier measurements, specifications, calibration certificates etc.
- The calculation method is described with an equation (or a set of equations) called measurement model (mathematical model)
- The model is used for both calculation of the estimate and the uncertainty of the results.
- The model should include all factors (input quantities) affecting significantly the estimate and/or the uncertainty.
- The model is never complete; approximations are needed.

6 steps to evaluating uncertainty

1) Measurement model:
   List essential input quantities (i.e. parameters x_i having a significant effect on the result) and build up a mathematical model (function) showing how they are related to the final result: $y = f(x_{1,}, \ldots, x_i)$

2) Standard uncertainty:
   Estimate the *standard uncertainty* of each input quantity (x_i)

3) Using the model in uncertainty calculations:
   Determine the uncertainty due to standard uncertainty of each input quantity (x_i).

4) Correlation:
   Determine correlation between the input quantities (if relevant)

5) Calculate the *combined standard uncertainty*

6) Calculate the *expanded uncertainty.*

4.2 Calculations step by step

Step 1: Measurement model

Step 2: Standard uncertainty

Type A evaluation of standard uncertainty
Type B evaluation of standard uncertainty

Step 3:Using the model in uncertainty calculations

Step 4: Correlation

Step 5: Combining the uncertainty components

Step 6: Expanded uncertainty

Step 1: Measurement model

Equation which describes the measurement:

Measurand ⟶ $Y = f(X_1, X_2, \ldots, X_n)$ ⟵ Input quantities X_i

- The model should include:

  Measurement results, corrections, reference values, influence quantities...
- The magnitude of a correction can be zero but it can still have uncertainty
- The values and uncertainties of the input quantities should be determined

Measurement result (y) is:

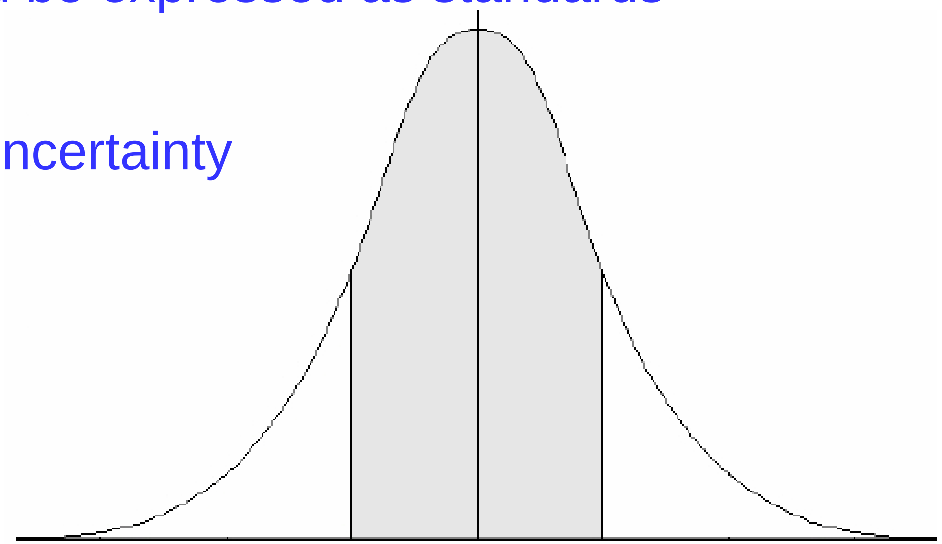
$$y = f(x_1, x_2, \ldots, x_n)$$

- x_i is the value (estimate) of the input quantity X_i

Example : $t_x = t_{ind} + \delta t_{Cal} + \delta t_D + \delta t_{resol} + \delta t_G$

Step 2: Standard uncertainty

- All uncertainty components should be comparable ⇒ standard uncertainty u_{i}
- The variance of the sum of non-correlating random variables is the sum of their variances
- standard uncertainty is the square root of variance
- all uncertainty components should be expressed as standards uncertainties
- For normal distribution standard uncertainty corresponds to about 68% confidence level



Two methods for estimating the standard uncertainty of an input quantity

- **Type A**:
  - Evaluated from a number of observations (usually > 10)
- **Tyyppi B**:
  - Evaluated from a single (or a small number of) data value(s)
  - Often taken from data reported earlier or by others

Type A evaluation of standard uncertainty

- Uncertainty is evaluated by statistical analysis of a series of observations q_i
  - The spread of the results is assumed to be random
  - An estimate for the value of the quantity is the arithmetic mean $\bar{q}$

$$\bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i$$

  - An estimate for the variance of the probability distribution is $s(x)^2$:
  - $s(q)$ is termed the experimental standard deviation

$$s^2(q) = \frac{1}{n-1}\sum_{i=1}^{n}\left(q_i - \bar{q}\right)^2$$

  - An estimate for the variance of the mean $s^2(\bar{q})$ (the experimental variance of the mean) is:

$$s^2(\bar{q}) = \frac{s^2(q)}{n}$$

  - The standard uncertainty of q equals the experimental standard deviation of the mean:

$$u(q) = s(\bar{q}) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(q_i - \bar{q})^2}$$

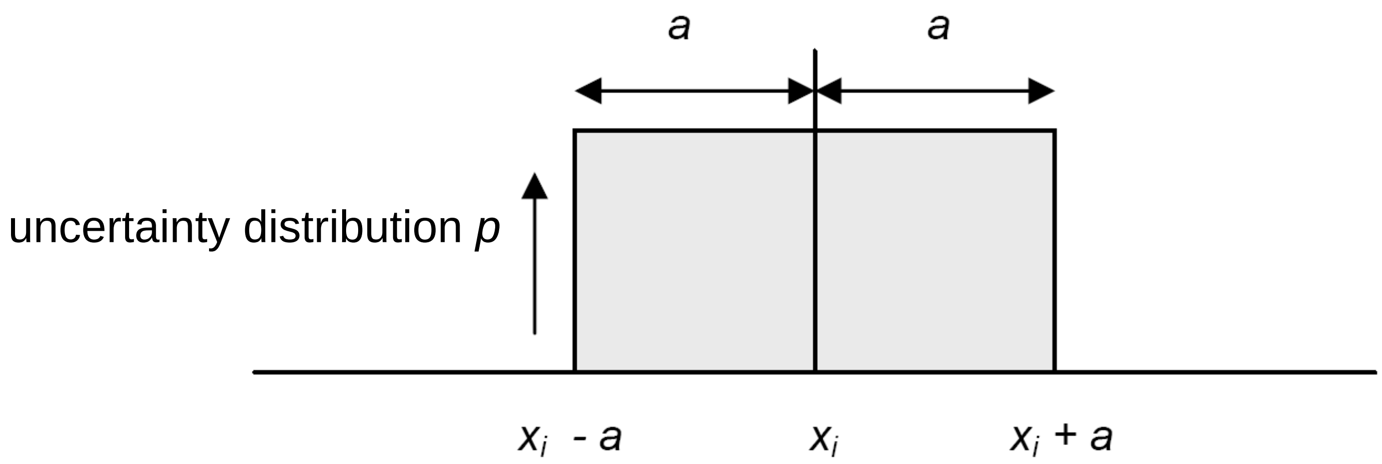
If type A measurement uncertainty is based on few measurements the estimation of $u(x)$ is not reliable and normal distribution can not be assumed.

(unless other information on the distribution is available)

Type B evaluation of standard uncertainty

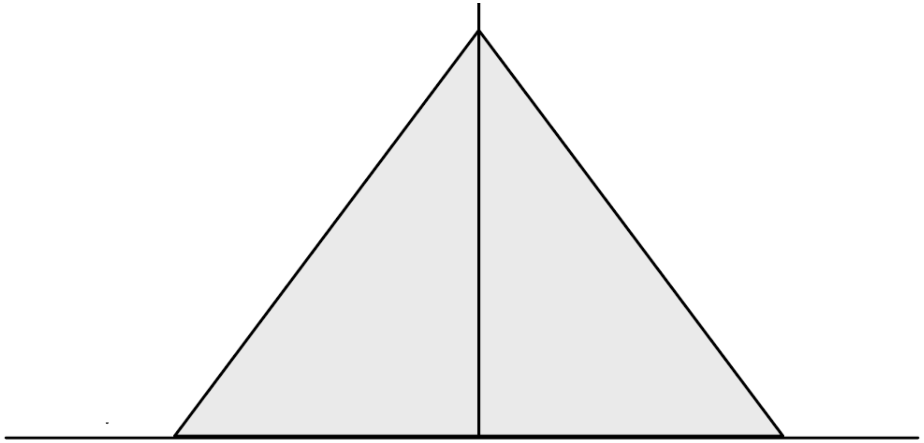
- To be applied for estimates of input quantities that has not been obtained from repeated measurements
- Typical examples :
  - uncertainties of values and drifts of reference standards
  - uncertainties of environmental quantities
  - uncertainties from specifications of instrument
  - uncertainties from literature values
  - uncertainty due to the method or calculation
  - uncertainty due to staff
  - uncertainties from calibration certificates

Rectangular distribution



- All values in the range x_i - a ...x_i + a have equal probability
- Standard uncertainty $$u(x_i) = \frac{a}{\sqrt{3}} \approx 0{,}577a$$
- Examples: specifications, resolution
- Applied if only limiting values are known

Triangular distribution

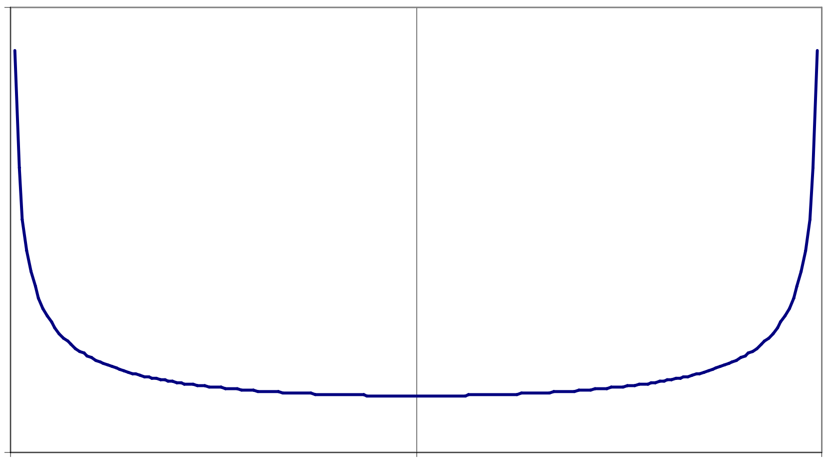


- Example: convolution of two rectangular distribution

- Standard uncertainty:

$$u(x_i) = \frac{a}{\sqrt{6}} \approx 0{,}408\,a$$

U-shape distribution



$x_i - a$ $x_i + a$

- Example: sinusoidal variation between limits ± a

- Standard uncertainty:

$$u(x_i) = \frac{a}{\sqrt{2}} \approx 0{,}707\, a$$

Step 3:Using the model in uncertainty calculations

- The contribution of $u(x_i)$ to the uncertainty of y is determined by the sensitivity coefficient c_i
- The sensitivity coefficient can be determined
  - from partial derivative of $f(X_1, X_2,\ldots,X_n)$ with X_i
    i.e. $c_i = \partial f / \partial X_i$ (at $x_1, x_2,\ldots,x_n$)
  - by numerical methods $c_i = \Delta y / \Delta x_i$
  - experimentally by changing xi by Δx_i and determining Δy ;
    $c_i = \Delta y / \Delta x_i$
- The contribution of $u(x_i)$ to the uncertainty of y is:

$$u_i(y) = c_i\, u(x_i)$$

Step4: Correlation

The covariance $u(x_i,x_j)$ of two random variables is a measure of their mutual dependence.

If $X_i = F(Q_l)$ and $X_j = G(Q_l)$ depend on the same quantities Q_l $(l=1..n)$ then

$$u(x_i,x_j)=\sum_i \frac{\partial F}{\partial q_l}\frac{\partial G}{\partial q_l}u^2(q_l)$$

- Correlation coefficient : $$r(x_i,x_j)=\frac{u(x_i,x_j)}{u(x_i)u(x_j)}$$

The covariance can increase or decrease uncertainty.

If the correlation coefficient is r=1 the components will be added in a linear way.

Step 5: Combining the uncertainty components

- Uncorrelated input quantities:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} u_i^2(y_i)} = \sqrt{\sum_{i=1}^{N} \left(\frac{\delta f}{\delta x_i}\right)^2 u^2(x_i)}$$

- Correlated input quantities:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\delta f}{\delta x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\delta f}{\delta x_i} \frac{\delta f}{\delta x_j} u(x_i, x_j)}$$

$u_c(y)$ = combined standard uncertainty; $u(x_i,x_j)$ = covariance

Step 6: Expanded uncertainty

Often the result of the measurement is reported with a higher level of confidence than given by the standard uncertainty.

- Expanded uncertainty U is the standard uncertainty multiplied by a coverage factor k:

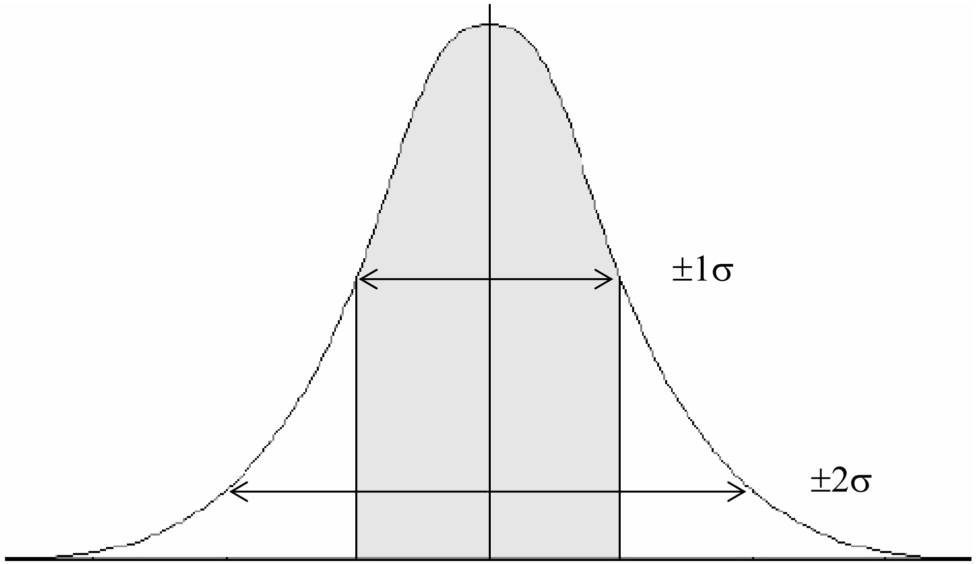
$$U = k u_c(y)$$

- In calibration it is recommended to report 95 % level of confidence.
- For normal distribution this corresponds to k=2 (approximately).

Normal distribution:

Coverage probability *p*	Coverage factor *k*
68,27 %	1,00
90 %	1,65
95 %	1,96
95,45%	2,00
99 %	2,58
99,73%	3,00

Normal distribution



Measurement result is approximately normally distributed if

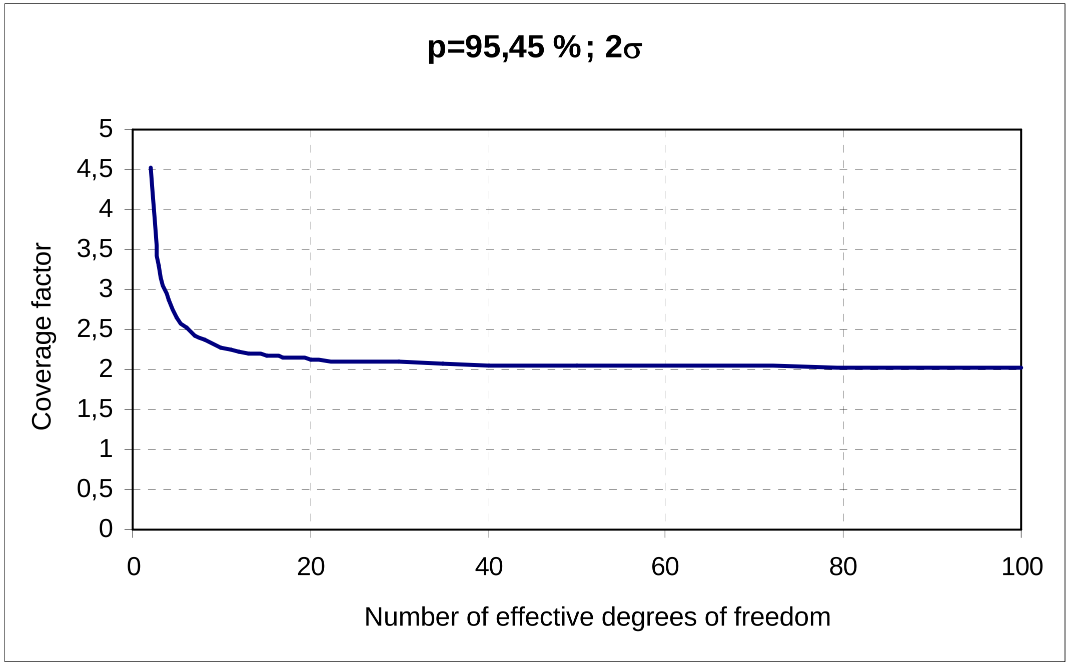
- it is a combination of several random variables (independent of distribution)
- none of the (non-normally distributed) components is dominating.

Degrees of freedom and the coverage factor

- For a combined standard uncertainty, we can calculate the effective number of degrees of freedom (ν_{eff}) :

$$\nu_{eff} = \frac{u^4(y)}{\sum_{i=1}^{N} \frac{u^4(x_i)}{\nu_i}}$$

- The figure shows that we need a coverage factor larger than 2 to obtain 95 % confidence level if the number of degrees of freedom is small.



4.2 Uncertainty calculation in practice

Example: Measurement of SO_2 content

- An analyzer with electrical current signal output was used for measuring SO_2 content in exhaust gas.
- The signal was measured with a DMM and the total error in the current measurement was estimated to be within ±0,1 mA.
- The arithmetic mean of the 15 recorded DMM readings is 9,59mA and the corresponding standard deviation is 0,49 mA.
- An accredited laboratory has determined the calibration function for the analyzer:

$$f_c(I_m) = -7{,}64\ \mathrm{mg/m^3} + 3{,}25\,\frac{\mathrm{mg/m^3}}{\mathrm{mA}} \cdot I_m$$

  The reported expanded uncertainty (k=2) is 6 $\mathrm{mg/m^3}$
- When comparing two last calibrations, we can conclude that the drift of the analyzer is less than 5 $\mathrm{mg/m^3}$/year (calibr. interval is 1 year)

Example: Measurement of SO_2 content - continuing

- The measurement result is calculated as follows:

$$C_{SO2} = f_c(I_m + \delta I_m) + \delta f_c + \delta_{Drift}$$

$$= -7{,}64\ \mathrm{mg/m^3} + 3{,}25\,\frac{\mathrm{mg/m^3}}{\mathrm{mA}} \cdot (I_m + \delta I_m) + \delta f_c + \delta_{Drift}$$

- The variables can be assumed independent on each other; therefore we can calculate the uncertainty:

$$u(C_{SO2}) = \sqrt{[c_1 u(I_m)]^2 + [c_2 u(\delta I_m)]^2 + [c_3 u(\delta f_c)]^2 + [c_4 u(\delta_{Drift})]^2}$$

Example: Measurement of SO_2 content - continuing

- The sensitivity coefficients are:

$$c_1 = \frac{\partial C_{SO2}}{\partial I_m} = 3{,}25 \frac{\mathrm{mg/m^3}}{\mathrm{mA}}$$

$$c_2 = \frac{\partial C_{SO2}}{\partial \delta I_m} = c_1$$

$$c_3 = \frac{\partial C_{SO2}}{\partial \delta f_c} = c_4 = \frac{\partial C_{SO2}}{\partial \delta_{Drift}} = 1$$

- Thus:

$$u(C_{SO2}) = \sqrt{c_1^{\,2}[u^2(I_m) + u^2(\delta I_m)] + u^2(\delta f_c) + u^2(\delta_{Drift})}$$

Example: Measurement of SO_2 content - continuing

- Standard uncertainties of the components:

$$u(I_m) = 0{,}49\ \text{mA}$$

type A, normal distribution

$$u(\delta I_m) = \frac{0{,}1\ \text{mA}}{\sqrt{3}} = 0{,}06\ \text{mA}$$

type B, rectangular distribution

$$u(\delta f_c) = \frac{6\ \text{mg/m}^3}{2} = 3\ \text{mg/m}^3$$

type B, normal distribution

$$u(\delta_{Drift}) = \frac{5\ \text{mg/m}^3}{\sqrt{3}} = 2{,}9\ \text{mg/m}^3$$

type B, rectangular distribution

Example: Measurement of SO_2 content - continuing

- Estimate for the SO_2 content:

$$C_{SO2} = -7{,}64\ \mathrm{mg/m^3} + 3{,}25\,\frac{\mathrm{mg/m^3}}{\mathrm{mA}} \cdot (9{,}59\ \mathrm{mA} + 0\ \mathrm{mA}) + 0\ \mathrm{mg/m^3} + 0\ \mathrm{mg/m^3}$$

$$= 23{,}5\ \mathrm{mg/m^3}$$

- and its combined standard uncertainty:

$$u(C_{SO2}) = \sqrt{\left(3{,}25\,\frac{\mathrm{mg/m^3}}{\mathrm{mA}}\right)^2 \left[(0{,}49\ \mathrm{mA})^2 + (0{,}06\ \mathrm{mA})^2\right] + (3\ \mathrm{mg/m^3})^2 + (2{,}9\ \mathrm{mg/m^3})^2}$$

$$= 4{,}5\ \mathrm{mg/m^3}$$

Example: Measurement of SO_2 content - continuing

- Thus the expanded uncertainty at about 95 % confidence level (k = 2) is:

$$U = k \times u = 2 \times 4{,}5\ \text{mg/m}^3 = 9{,}0\ \ \text{mg/m}^3$$

- As a conclusion, the measurement result is :

$$C_{SO2} = (24 \pm 9)\ \text{mg/m}^3$$

References and literature

[1.1] ISO/IEC Guide 99-12:2007, *International Vocabulary of Metrology — Basic and General Concepts and Associated Terms*, VIM

[1.2] *International vocabulary of metrology — Basic and general concepts and associated terms (VIM),* 3rd ed., JCGM 200:2008
(*can be downloaded at http://www.bipm.org/en/publications/guides/*)

[2.1] http://www.bipm.org/en/si/si_brochure/

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Literature about estimating uncertainty:

- *Guide to the Expression of Uncertainty in Measurement.* International Organization for Standardization, Geneva, Swizerland. ISBN 92-67-10188-9, First Edition, 1993.
- JCGM 100:2008, *Evaluation of measurement data – Guide to the expression of uncertainty in measurement*, First edition, JCGM 2008 (*http://www.bipm.org/en/publications/guides/*)
- European cooperation for Accreditation, *EA-4/02 Expression of the Uncertainty of Measurement in Calibration*, 1999. (*http://www.european-accreditation.org/n1/doc/ea-4-02.pdf*)
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MIKES has published several guides in Finnish on the uncertainty estimations in different fields.