Chapter 1

Foundations of Engineering Economy

1-1 Introduction

The need for engineering economy is primarily motivated by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of **making decisions**. These decisions involve the fundamental elements of **cash flows of money, time, and interest rates.** This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

1-2 Why Engineering Economy and the Time Value of Money are Important

Decisions are made routinely to choose one alternative over another by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money, called **capital** or **capital funds,** which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of **adding value** as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design, analyze, and synthesize. The factors upon which a decision is based are commonly a combination of economic and noneconomic elements. Engineering economy deals with the economic factors. By definition,

Engineering economy involves formulating, estimating, and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives.

Because the formulas and techniques used in engineering economics are applicable to all types of money matters, they are equally useful in business and government, as well as for individuals.

To be financially literate is very important as an engineer and as a person. Unfortunately many people do not have the fundamental understanding of concepts such as financial risk and diversification, inflation, numeracy, and compound interest. You will learn and apply these basic concepts, and more, through the study of engineering economy

Other terms that mean the same as engineering economy are engineering economic analysis, capital allocation study, economic analysis, and similar descriptors.

People make decisions; computers, mathematics, concepts, and guidelines assist people in their decision-making process. Since most decisions affect what will be done, the time frame of engineering economy is primarily the **future.** Therefore, the numbers used in engineering economy are **best estimates of what is expected to occur.** The estimates and the decision usually involve four essential elements:

Cash flows

Times of occurrence of cash flows

Interest rates for time value of money

Measure of worth for selecting an alternative

Since the estimates of cash flow amounts and timing are about the future, they will be somewhat different than what is actually observed, due to changing circumstances and unplanned events. In short, the variation between an amount or time estimated now and that observed in the future is caused by the stochastic (random) nature of all economic events. **Sensitivity analysis** is utilized to determine how a decision might change according to varying estimates, especially those expected to vary widely.

The criterion used to select an alternative in engineering economy for a specific set of estimates is called a **measure of worth.** The measures developed and used in this text are

Present worth (PW)	Future worth (FW)	Annual worth (AW)
Rate of return (ROR)	Benefit/cost (B/C)	Capitalized cost (CC)
Payback period	Profitability index	Economic value added (EVA)

All these measures of worth account for the fact that money makes money over time. This is the concept of the **time value of money.**

It is a well-known fact that money **makes** money. The time value of money explains the change in the amount of money **over time** for funds that are owned (invested) or owed (borrowed). **This is the most important concept in engineering economy.**

The time value of money is very obvious in the world of economics. If we decide to invest capital (money) in a project today, we inherently expect to have more money in the future than we invested. If we borrow money today, in one form or another, we expect to return the original amount plus some additional amount of money. An engineering economic analysis can be performed on future estimated amounts or on past cash flows to determine if a specific measure of worth, e.g., rate of return, was achieved. Engineering economics is applied in an extremely wide variety of situations. Samples are:

- Equipment purchases and leases
- Chemical processes
- Cyber security
- Construction projects
- Airport design and operations
- Sales and marketing projects
- Transportation systems of all types
- Product design
- Wireless and remote communication and control
- Manufacturing processes
- Safety systems
- Hospital and healthcare operations
- Quality assurance
- Government services for residents and businesses

In short, any activity that has money associated with it−which is just about everything− is a reasonable topic for an engineering economy study.

EXAMPLE 1.1

Cyber security is an increasingly costly dimension of doing business for many retailers and their customers who use credit and debit cards. A 2014 data breach of U.S.-based

Home Depot involved some 56 million cardholders. Just to investigate and cover the immediate direct costs of this identity theft amounted to an estimated $62,000,000, of which $27,000,000 was recovered by insurance company payments. This does not include indirect costs, such as, lost future business, costs to banks, and cost to replace cards. If a cyber security vendor had proposed in 2006 that a $10,000,000 investment in a malware detection system could guard the company's computer and payment systems from such a breach, would it have kept up with the rate of inflation estimated at 4% per year?

Solution

As a result of this data breach, Home Depot experienced a direct out-of-pocket cost of $35,000,000 after insurance payments. From learning to the engineering economy, you will learn how to determine the future equivalent of money at a specific rate. In this case, the estimate of $10,000,000 after 8 years (from 2006 to 2014) at an inflation rate of 4% is equivalent to $13,686,000.

The 2014 equivalent cost of $13.686 million is significantly less than the out-of-pocket loss of $35 million. The conclusion is that the company should have spent $10 million in 2006.

Besides, there may be future breaches that the installed system will detect and eliminate. This is an extremely simple analysis; yet, it demonstrates that at a very elementary level, it is possible to determine whether an expenditure at one point in time is economically worthwhile at some time in the future. In this situation, we validated that a previous expenditure (malware detection system) should have been made to overcome an unexpected expenditure (cost of the data breach) at a current time, 2014 here.

1.3 Performing an Engineering Economy Study

An engineering economy study involves many elements: problem identification, definition of the objective, cash flow estimation, financial analysis, and decision making. Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

1. Identify and understand the problem; identify the objective of the project.

2. Collect relevant, available data and define viable solution alternatives.

3. Make realistic cash flow estimates.

4. Identify an economic measure of worth criterion for decision making.

5. Evaluate each alternative; consider noneconomic factors; use sensitivity analysis as needed.

6. Select the best alternative.

7. Implement the solution and monitor the results.

Technically, the last step is not part of the economy study, but it is, of course, a step needed to meet the project objective.
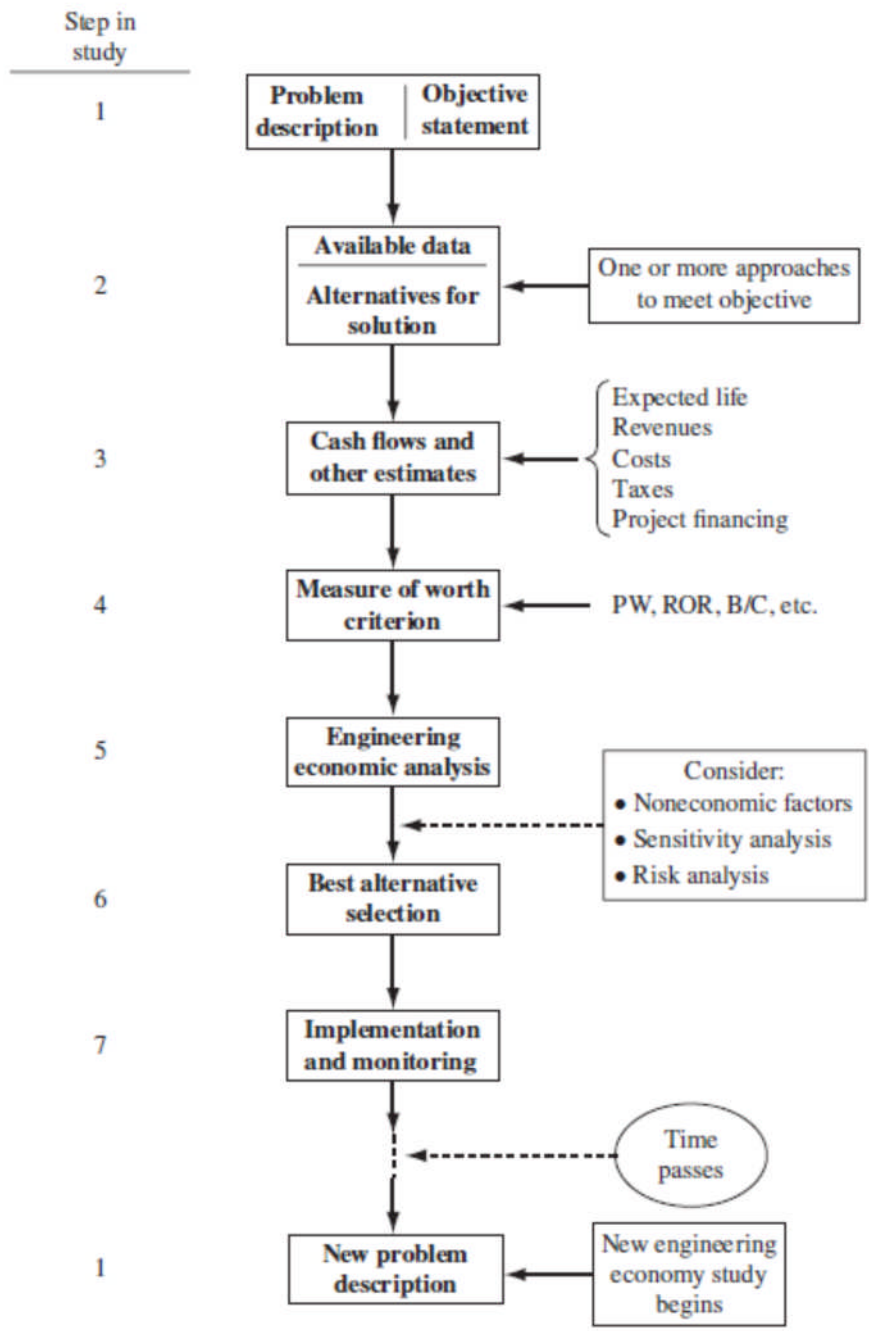
Figure 1–1
Steps in an engineering economy study.

Fig. 1 Steps in an engineering economy study.

1.4 Interest Rate and Rate of Return

Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest−interest paid and interest earned. These are illustrated in Fig.2.

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Interest is **paid** when a person or organization borrowed money (obtained a loan) and repays a larger amount over time. Interest is earned when a person or organization saved, invested, or lent money and obtains a return of a larger amount over time. The numerical values and formulas used are the same for both perspectives, but the interpretations are different.

Interest paid on borrowed funds (a loan) is determined using the original amount, also called the principal,

$$\textbf{Interest = amount owed now} - \textbf{principal} \qquad (1)$$

When interest paid over a specific time unit is expressed as a percentage of the principal, the result is called the interest rate.

$$\textbf{Interest rate (\%)} = \frac{\textbf{interest acucrrd per time unit}}{\textbf{principal}} \times 100\% \qquad (2)$$

The time unit of the rate is called the **interest period.** By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as 1% per month.

Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.
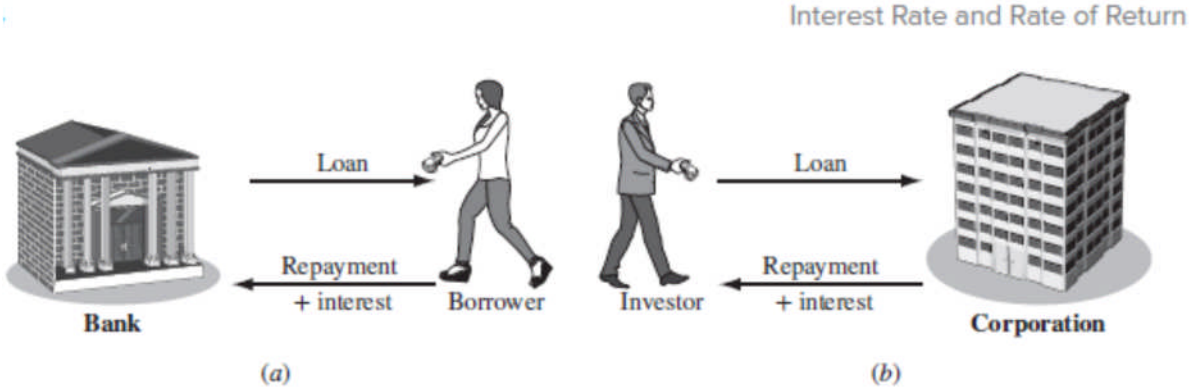

Fig. 1–2 (*a*) Interest paid over time to lender. (*b*) Interest earned over time by investor.

EXAMPLE 1.2

An employee at LaserKinetics.com borrows $10,000 on May 1 and must repay a total of $10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

Solution

The perspective here is that of the borrower since $10,700 repays a loan. Apply Equation

$$\text{Interest} = \text{amount owed now} - \text{principal}$$

To determine the interest paid.

Interest paid = $10,700 − 10,000 = $700

Then determine the interest rate paid for 1 year.

$$\textbf{Interest rate (\%)} = \frac{\textit{interest acucrrd per time unit}}{\textit{principal}} \times 100\%$$

Percent Interest rate (%) = $\frac{700}{10000} \times 100\% =$ 7% per year

EXAMPLE 1.3

Stereophonics, Inc. plans to borrow $20,000 from a bank for 1 year at 9% interest for new recording equipment. (*a*) Compute the interest and the total amount due after 1 year. (*b*) Construct a column graph that shows the original loan amount and total amount due after 1 year used to compute the loan interest rate of 9% per year.

Solution

(*a*) Compute the total interest accrued by solving Equation [2] for interest accrued.

Interest = $20,000(0.09) = $1800

The total amount due is the sum of principal and interest.

Total due = $20,000 + 1800 = $21,800

(*b*) Figure 3 shows the values used in Equation [2]: $1800 interest, $20,000 original loan principal, 1-year interest period.
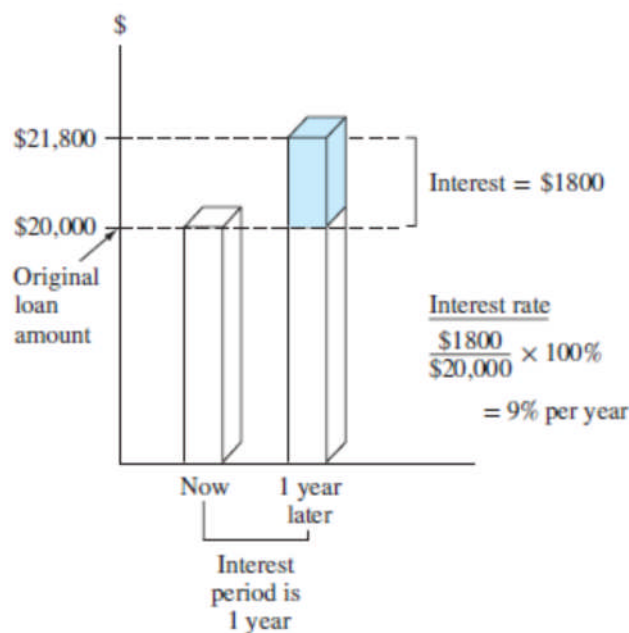

Fig. 3 Values used to compute an interest rate of 9% per year. Example 1.3.

Comment

Note that in part (a), the total amount due may also be computed as

Total due = principal (1 + interest rate) = $20,000(1.09) = $21,800

Later we will use this method to determine future amounts for times longer than one interest period.

From the perspective of a saver, a lender, or an investor, **interest earned** (Fig.2*b*) is the final amount minus the initial amount, or principal.

$$\textbf{Interest earned = total amount now − principal} \qquad (3)$$

Interest earned over a specific period of time is expressed as a percentage of the original amount and is called **rate of return (ROR).**

$$\textbf{rate of return} = \frac{\textbf{Interest } \textit{accrued per unit time}}{\textit{principal}} \times \textbf{100}\% \qquad (4)$$

The time unit for rate of return is called the **interest period,** just as for the borrower's perspective.

Again, the most common period is 1 year.

The term **return on investment (ROI)** is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs.

The numerical values in Equations [2] and [4] are the same, but the term interest rate paid is more appropriate for the borrower's perspective, while the term rate of return earned applies for the investor's perspective.

EXAMPLE 1.5

(a) Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.

(b) Calculate the amount of interest earned during this time period.

Solution

(*a*) The total amount accrued ($1000) is the sum of the original deposit and the earned interest.

If X is the original deposit,

$$\text{Total accrued} = \text{deposit} + \text{deposit (interest rate)}$$

$$\$1000 = X + X\,(0.05) = X\,(1 + 0.05) = 1.05X$$

The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

(*b*) Apply Equation [3] to determine the interest earned.

$$\text{Interest} = \$1000 - 952.38 = \$47.62$$

1.5 Terminology and Symbols

The equations and procedures of engineering economy utilize the following terms and symbols.

Sample units are indicated.

P = value or amount of money at a time designated as the present or time 0. Also P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); monetary units, such as dollars

F = value or amount of money at some future time. Also F is called future worth (FW) and future value (FV); dollars

A = series of consecutive, equal, end-of-period amounts of money. Also A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, euros per month

n = number of interest periods; years, months, days

i = interest rate per time period; percent per year, percent per month

t = time, stated in periods; years, months, days

The symbols P and F represent one-time occurrences: A occurs with the same value in each interest period for a specified number of periods. It should be clear that a present value P represents a single sum of money at some time prior to a future value F or prior to the first occurrence of an equivalent series amount A.

It is important to note that the symbol A always represents a uniform amount (i.e., the same amount each period) that extends through *consecutive* interest periods. Both conditions must exist before the series can be represented by A.

The interest rate i is expressed in percent per interest period, for example, 12% per year. Unless stated otherwise, assume that the rate applies throughout the entire n years or interest periods.

The decimal equivalent for i is always used in formulas and equations in engineering economy computations.

All engineering economy problems involve the element of time expressed as n and interest rate i. In general, every problem will involve at least four of the symbols P, F, A, n, and i, with at least three of them estimated or known.

EXAMPLE 1.6

Last year Jane's grandmother offered to put enough money into a savings account to generate $5000 in interest this year to help pay Jane's expenses at college. (a) Identify the symbols, and (b) calculate the amount that had to be deposited exactly 1 year ago to earn $5000 in interest now, if the rate of return is 6% per year.

Solution

(a) Symbols P (last year is -1) and F (this year) are needed.

$P =?$

$i = 6\%$ per year

$n = 1$ year

$F = P + \text{interest} =? + \$5000$

(b) Let F = total amount now and P = original amount. We know that $F - P = \$5000$ is accrued interest. Now we can determine P. Refer to Equations [1] through [4].

$F = P + Pi$

The $5000 interest can be expressed as

$\text{Interest} = F - P = (P + Pi) - P = Pi$

$\$5000 = P (0.06)$

$P = \dfrac{\$5000}{0.06} = \$83,333.33$

1.6 Cash Flows: Estimation and Diagramming

As mentioned in earlier sections, cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period. For example, a payment of $10,000 once every year in December for 5 years is a series of 5 outgoing cash flows. And an estimated receipt of $500 every month for 2 years is a series of 24 incoming cash flows. Engineering economy bases its computations on the timing, size, and direction of cash flows.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A **plus sign** indicates a cash inflow.

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business Cash flow activity. A **negative or minus sign** indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Some examples of cash flow estimates are shown here. As you scan these, consider how the cash inflow or outflow may be estimated most accurately.

Cash Inflow Estimates

Income: +$150,000 per year from sales of solar-powered watches

Savings: +$24,500 tax savings from capital loss on equipment salvage

Receipt: +$750,000 received on large business loan plus accrued interest

Savings: +$150,000 per year saved by installing more efficient air conditioning

Revenue: +$50,000 to +$75,000 per month in sales for extended battery life iPhones

Cash Outflow Estimates

Operating costs: _$230,000 per year annual operating costs for software services

First cost: _$800,000 next year to purchase replacement earthmoving equipment

Expense: _$20,000 per year for loan interest payment to bank

Initial cost: _$1 to _$1.2 million in capital expenditures for a water recycling unit

All of these are **point estimates,** that is, *single-value estimates* for cash flow elements of an alternative, except for the last revenue and cost estimates listed above. They provide a **range estimate,** because the persons estimating the revenue and cost do not have enough knowledge or experience with the systems to be more accurate. For the initial chapters, we will utilize point estimates. Once all cash inflows and outflows are estimated (or determined for a completed project), the **net cash flow** for each time period is calculated.

Net cash flow = cash inflows - cash outflows   **(5)**

NCF = *R* - *D*   **(6)**

Where NCF is net cash flow, *R* is receipts, and *D* is disbursements

At the beginning of this section, the *timing, size, and direction of cash flows* were mentioned as important. Because cash flows may take place at any time during an

interest period, as a matter of convention, all cash flows are assumed to occur at the end of an interest period.

The **cash flow diagram** is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on the y axis with a time scale on the x axis. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time $t = 0$ is the present, and $t = 1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Fig.4 is set up for 5 years. Since the end-of-year convention places cash flows at the ends of years, the "1" marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, you will probably avoid errors if you make a neat diagram to approximate scale for both time and relative cash flow magnitudes.

The direction of the arrows on the diagram is important to differentiate income from outgo. A vertical arrow pointing up indicates a positive cash flow. Conversely, a down-pointing arrow indicates a negative cash flow. **We will use a bold, colored arrow to indicate what is unknown and to be determined.**

For example, if a future value F is to be determined in year 5, a wide, colored arrow with $F =$? is shown in year 5. The interest rate is also indicated on the diagram.

Figure 5 illustrates a cash inflow at the end of year 1, equal cash outflows at the end of years 2 and 3, an interest rate of 4% per year, and the unknown future value F after 5 years. The arrow for the unknown value is generally drawn in the opposite direction from the other cash flows; however, the engineering economy computations will determine the actual sign on the F value.
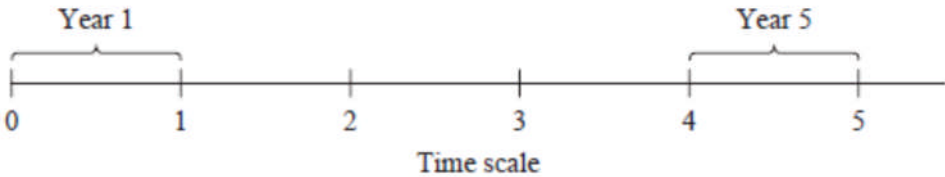

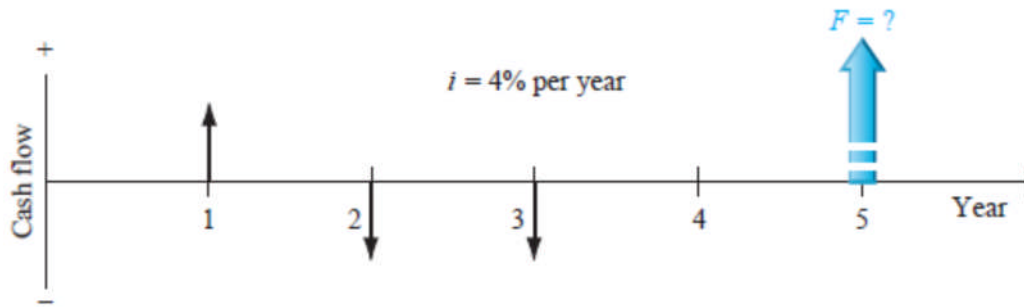**Fig. 4 A typical cash flow time scale for 5 years.**

Fig. 5 Example of positive and negative cash flows.

Before the diagramming of cash flows, a perspective or vantage point must be determined so that + or − signs can be assigned and the economic analysis performed correctly. Assume you borrow $8500 from a bank today to purchase an $8000 used car for cash next week, and you plan to spend the remaining $500 on a new paint job for the car two weeks from now. There are several perspectives possible when developing the cash flow diagram—those of the borrower (that's you), the banker, the car dealer, or the paint shop owner. The cash flow signs and amounts for these perspectives are as follows.

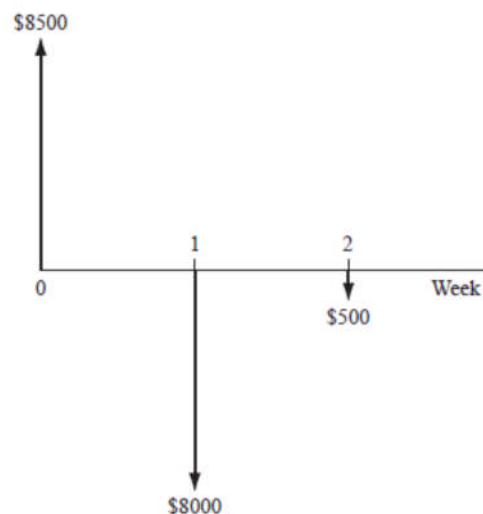| Perspective	Activity	Cash flow with Sign, $	Time, week
You	Borrow	+8500	0
	Buy car	−8000	1
	Paint job	−500	2
Banker	Lender	−8500	0
Car dealer	Car sale	+8000	1
Painter	Paint job	+500	2

Fig. 6 Cash flows from perspective of borrower for loan and purchases.

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EXAMPLE 1.7

Each year Exxon-Mobil expends large amounts of funds for mechanical safety features throughout its worldwide operations. Carla Ramos, a lead engineer for Mexico and Central American operations, plans expenditures of $1 million *now* and each of the next 4 years just for the improvement of field-based pressure-release valves. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

Solution

Figure 7 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown F value (positive cash flow equivalent) at exactly the same time as the fifth expenditure. Since the expenditures start immediately, the first $1 million is shown at time 0, not time 1. Therefore, the last negative cash flow occurs at the end of the fourth year, when F also occurs. To make this diagram have a full 5 years on the time scale, the addition of the year -1 completes the diagram. This addition demonstrates that year 0 is the end-of-period point for the year =1.
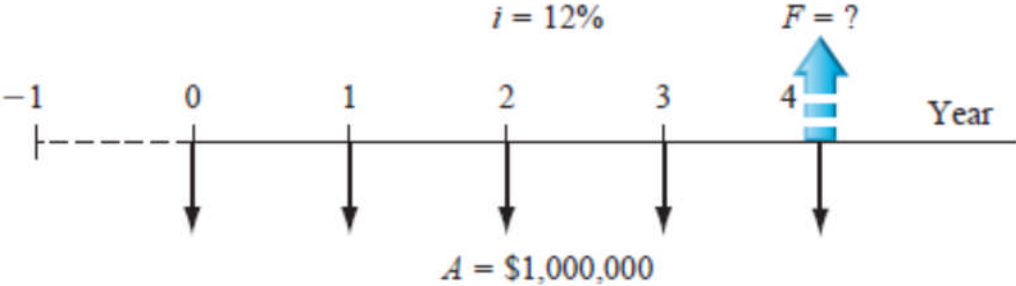

Fig. 7 Cash flow diagram

EXAMPLE 1.8

An electrical engineer wants to deposit an amount P now such that she can withdraw an equal annual amount of $A_1 = \$2000$ per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of $A_2 = \$3000$ per year for the following 3 years. How would the cash flow diagram appear if $i = 8.5\%$ per year?

Solution

The cash flows are shown in Fig. 8. The negative cash outflow P occurs now. The withdrawals (positive cash inflow) for the $A1$ series occur at the end of years 1 through 5, and A_2 occurs in years 6 through 8.
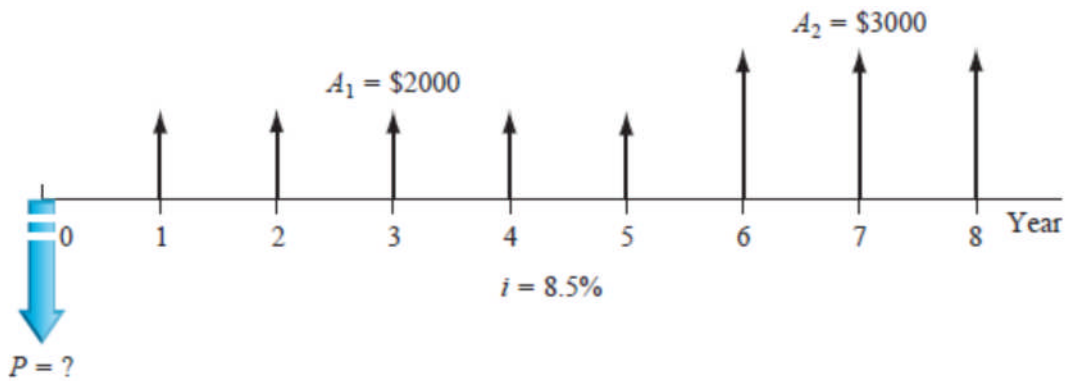
Fig. 8 Cash flow diagram with two different *A* series.

EXAMPLE 1.9

A rental company spent $2500 on a new air compressor 7 years ago. The annual rental income from the compressor has been $750. The $100 spent on maintenance the first year has increased each year by $25. The company plans to sell the compressor at the end of next year for $150. Construct the cash flow diagram from the company's perspective and indicate where the present worth now is located.

Solution

Let now be time $t = 0$. The incomes and costs for years -7 through 1 (next year) are tabulated below with net cash flow computed using Equation (5). The net cash flows (one negative, eight positive) are diagrammed in Fig. 9. Present worth P is located at year 0.

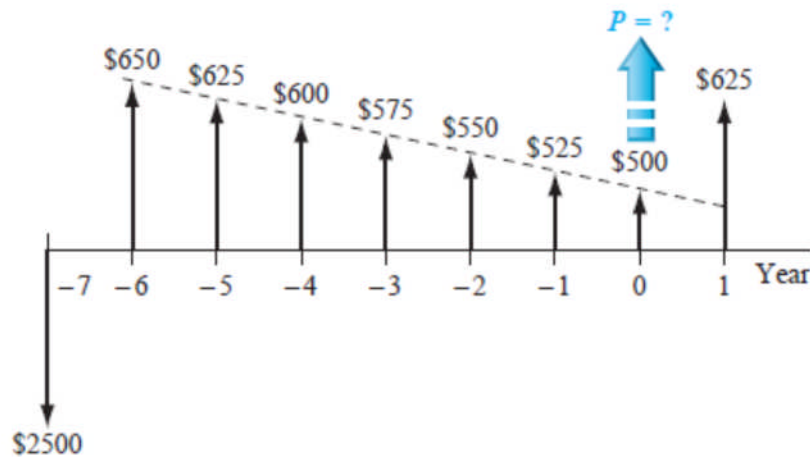| End of Year	Income	Cost	Net Cash Flow
−7	$ 0	$2500	$−2500
−6	750	100	650
−5	750	125	625
−4	750	150	600
−3	750	175	575
−2	750	200	550
−1	750	225	525
0	750	250	500
1	750 + 150	275	625

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Fig. 9 Cash flow diagram.

1.7 Economic Equivalence

Economic equivalence is a fundamental concept upon which engineering economy computations are based.

Economic equivalence is a combination of **interest rate** and **time value of money** to determine the different amounts of money at different points in time that are equal in economic value.

As an illustration, if the interest rate is 6% per year, $100 today (present time) is equivalent to $106 one year from today.

Amount accrued = 100 + 100(0.06) = 100(1 + 0.06) = $106

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of $100 now is equivalent to $100/1.06 = $94.34 one year ago at an interest rate of 6% per year. From these illustrations, we can state the following: $94.34 last year, $100 now, and $106 one year from now are equivalent at an interest rate of 6% per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1-year interest periods.

$$\frac{\$6}{\$100} \times 100\% = 6\% \text{ per year}$$

and

$$\frac{\$5.66}{\$94.34} \times 100\% = 6\% \text{ per year}$$

The cash flow diagram in Fig.10 indicates the amount of interest needed each year to make these three different amounts equivalent at 6% per year.
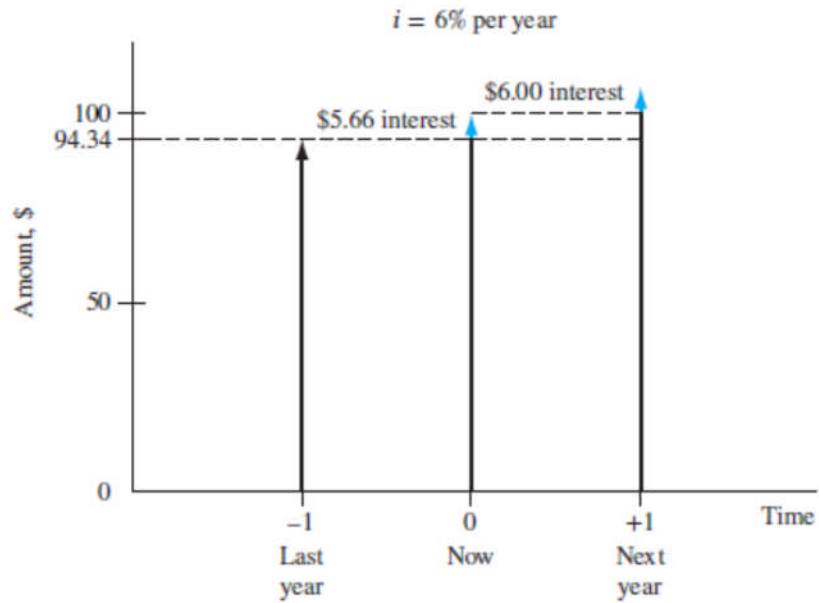
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$i = 6\%$ per year

Fig. 10 Equivalence of money at 6% per year interest.

1.8 Simple and Compound Interest

The terms *interest, interest period,* and *interest rate* (introduced in Section 1.4) are useful in calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms *simple interest* and *compound interest* become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

Simple interest = (principal) (number of periods) (interest rate)   **(7)**

I = Pni

Where *I* is the amount of interest earned or paid and the interest rate *i* is expressed in decimal form.

EXAMPLE 1.10

GreenTree Financing lent an engineering company $100,000 to retrofit an environmentally unfriendly building. The loan is for 3 years at 10% per year simple interest. How much money will the firm repay at the end of 3 years?

Solution

The interest for each of the 3 years is

Interest per year = $100,000(0.10) = $10,000

Total interest for 3 years from Equation (7) is

Total interest = $100,000(3) (0.10) = $30,000

The amount due after 3 years is

Total due = $100,000 + 30,000 = $130,000

The interest accrued in the first year and in the second year does not earn interest. The interest due each year is $10,000 calculated only on the $100,000 loan principal.

In most financial and economic analyses, we use **compound interest** calculations.

For *compound interest,* the interest accrued for each interest period is calculated on the **Principal plus the total amount of interest accumulated in all previous periods.** Thus, compound interest means interest on top of interest. Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as

Compound interest = (principal + all accrued interest) (interest rate) **(8)**

In mathematical terms, the interest I_t for time period t may be calculated using the relation.

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j \right)(i)$$

(9)

EXAMPLE 1.11

Assume an engineering company borrows $100,000 at 10% per year compound interest and will pay the principal and all the interest after 3 years. Compute the annual interest and total amount due after 3 years. Graph the interest and total owed for each year, and compare with the previous example that involved simple interest.

Solution

To include compounding of interest, the annual interest and total owed each year are calculated by Equation (8).

Interest, year 1: 100,000(0.10) = $10,000

Total due, year 1: 100,000 + 10,000 = $110,000

Interest, year 2: 110,000(0.10) = $11,000

Total due, year 2: 110,000 + 11,000 = $121,000

Interest, year 3: 121,000(0.10) = $12,100

Total due, year 3: 121,000 + 12,100 = $133,100

The repayment plan requires no payment until year 3 when all interest and the principal, a total of $133,100, are due. Figure 1-11 uses a cash flow diagram format to compare end-of-year (*a*) simple and (*b*) compound interest and total amounts owed. The differences due to compounding are clear. An extra $133,100 – 130,000 = $3100 in interest is due for the compounded interest loan.

Note that while simple interest due each year is constant, the compounded interest due grows geometrically. Due to this geometric growth of compound interest, the difference between simple and compound interest accumulation increases rapidly as the time frame increases.

For example, if the loan is for 10 years, not 3, the extra paid for compounding interest may be calculated to be $59,374.

(*a*)   (*b*)

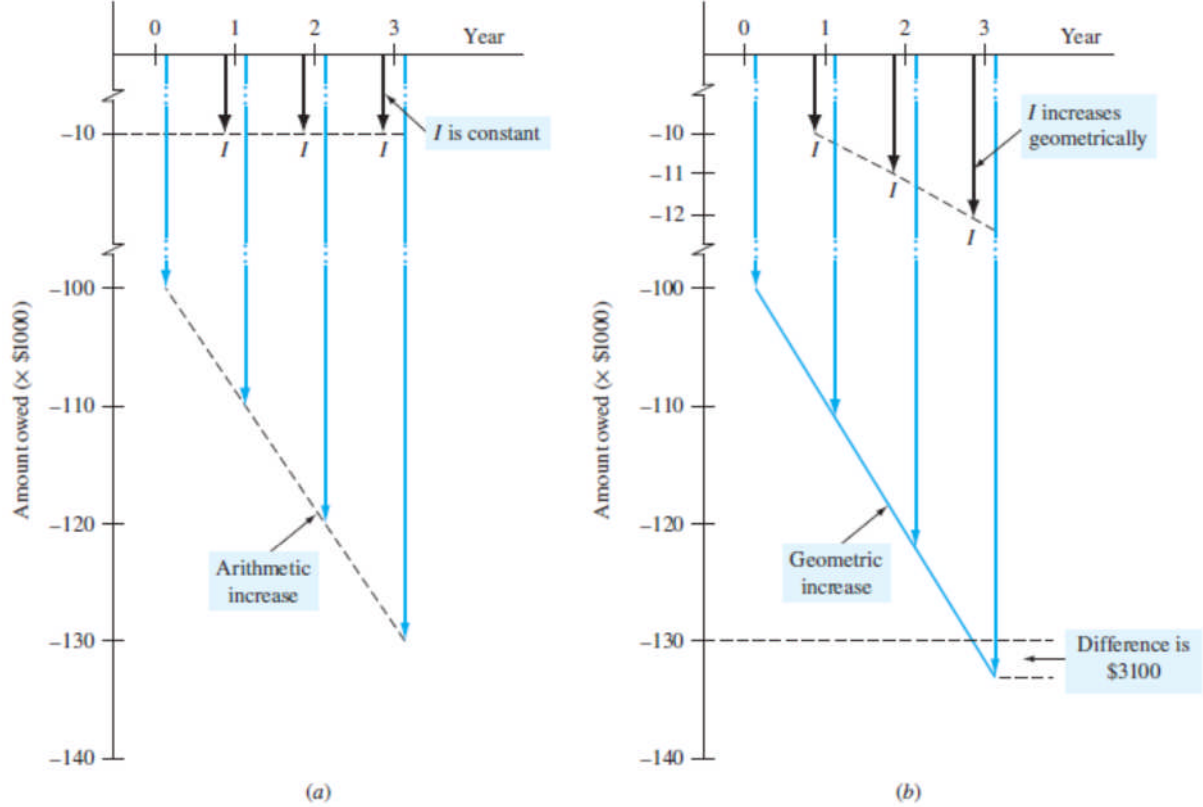**Fig.11** Interest *I* owed and total amount owed for (*a*) simple interest (Example 1.10) and (*b*) compound interest (Example 1.11).

A more efficient way to calculate the total amount due after a number of years in Example 1.11 is to utilize the fact that compound interest increases geometrically. This allows us to skip the year by year computation of interest. In this case, the **total amount due at the end of each year** is

Year 1: $100,000(1.10)1 = $110,000

Year 2: $100,000(1.10)2 = $121,000

Year 3: $100,000(1.10)3 = $133,100

This allows future totals owed to be calculated directly without intermediate steps. The general form of the equation is

Total due after *n* years = principal (1 + interest rate) $^{n \text{ years}}$ **(10)**

$$= P (1 + i)^n$$

Where i is expressed in decimal form. The total due after n years is the same as the future worth F, defined in Section 1.5. Equation (10) was applied above to obtain the $133,100 due after 3 years. This fundamental relation will be used many times in the upcoming chapters.

We can combine the concepts of interest rate, compound interest, and equivalence to demonstrate that different loan repayment plans may be equivalent, but differ substantially in amounts paid from one year to another and in the total repayment amount. This also shows that there are many ways to take into account the time value of money.

EXAMPLE 1.12

Table 1‑1 details four different loan repayment plans described below. Each plan repays a $5000 loan in 5 years at 8% per year compound interest.

- **Plan 1: Pay all at end.** No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principal and all accrued interest.

- **Plan 2: Pay interest annually, principal repaid at end.** The accrued interest is paid each year, and the entire principal is repaid at the end of year 5.

- **Plan 3: Pay interest and portion of principal annually.** The accrued interest and one-fifth of the principal (or $1000) are repaid each year. The outstanding loan balance decreases each year, so the interest (column 2) for each year decreases.

- **Plan 4: Pay equal amount of interest and principal.** Equal payments are made each year with a portion going toward principal repayment and the remainder covering the accrued interest. Since the loan balance decreases at a rate slower than that in plan 3 due to the equal end-of-year payments, the interest decreases, but at a slower rate.

(*a*) Make a statement about the *equivalence* of each plan at 8% compound interest.

(*b*) Develop an 8% per year *simple* interest repayment plan for this loan using the same approach as plan 2. Comment on the total amounts repaid for the two plans.

Solution

(*a*) The amounts of the annual payments are different for each repayment schedule, and the total amounts repaid for most plans are different, even though each repayment plan requires exactly 5 years. The difference in the total amounts repaid can be explained by the time value of money and by the partial repayment of principal prior to year 5.

A loan of $5000 at time 0 made at 8% per year compound interest is equivalent to each of the following:

Plan 1 $7346.64 at the end of year 5

Plan 2 $400 per year for 4 years and $5400 at the end of year 5

Plan 3 Decreasing payments of interest and partial principal in years 1 ($1400) through 5 ($1080)

Plan 4 $1252.28 per year for 5 years

An engineering economy study typically uses plan 4; interest is compounded, and a constant amount is paid each period. This amount covers accrued interest and a partial amount of principal repayment.

TABLE 1–1	Different Repayment Schedules Over 5 Years for $5000 at 8% Per Year Compound Interest			
(1) End of Year	**(2)** Interest Owed for Year	**(3)** Total Owed at End of Year	**(4)** End-of-Year Payment	**(5)** Total Owed After Payment
Plan 1: Pay All at End				
0				$5000.00
1	$400.00	$5400.00	—	5400.00
2	432.00	5832.00	—	5832.00
3	466.56	6298.56	—	6298.56
4	503.88	6802.44	—	6802.44
5	544.20	7346.64	$−7346.64	
Total			$−7346.64	
Plan 2: Pay Interest Annually; Principal Repaid at End				
0				$5000.00
1	$400.00	$5400.00	$−400.00	5000.00
2	400.00	5400.00	−400.00	5000.00
3	400.00	5400.00	−400.00	5000.00
4	400.00	5400.00	−400.00	5000.00
5	400.00	5400.00	−5400.00	
Total			$−7000.00	
Plan 3: Pay Interest and Portion of Principal Annually				
0				$5000.00
1	$400.00	$5400.00	$−1400.00	4000.00
2	320.00	4320.00	−1320.00	3000.00
3	240.00	3240.00	−1240.00	2000.00
4	160.00	2160.00	−1160.00	1000.00
5	80.00	1080.00	−1080.00	
Total			$−6200.00	
Plan 4: Pay Equal Annual Amount of Interest and Principal				
0				$5000.00
1	$400.00	$5400.00	$−1252.28	4147.72
2	331.82	4479.54	−1252.28	3227.25
3	258.18	3485.43	−1252.28	2233.15
4	178.65	2411.80	−1252.28	1159.52
5	92.76	1252.28	−1252.28	
Total			$−6261.40	

(*b*) The repayment schedule for 8% per year simple interest is detailed in Table 1‑2. Since the annual accrued interest of $400 is paid each year and the principal of $5000 is repaid in year 5, the schedule is exactly the same as that for 8% per year compound interest, and the total amount repaid is the same at $7000. In this unusual case, simple

and compound interest result in the same total repayment amount. Any deviation from this schedule will cause the two plans and amounts to differ.

TABLE 1–2	A 5-Year Repayment Schedule of $5000 at 8% per Year Simple Interest			
End of Year	Interest Owed for Year	Total Owed at End of Year	End-of-Year Payment	Total Owed After Payment
0				$5000
1	$400	$5400	$–400	5000
2	400	5400	–400	5000
3	400	5400	–400	5000
4	400	5400	–400	5000
5	400	5400	–5400	0
Total			$–7000	

Problems

1- RKI Instruments borrowed $3,500,000 from a private equity firm for expansion of its manufacturing facility for making carbon monoxide monitors/ controllers. The company repaid the loan after 1 year with a single payment of $3,885,000. What was the interest rate on the loan?

2- Emerson Processing borrowed $900,000 for installing energy-efficient lighting and safety equipment in its La Grange manufacturing facility. The terms of the loan were such that the company could pay interest only at the end of each year for up to 5 years, after which the company would have to pay the entire amount due. If the interest rate on the loan was 12% per year and the company paid only the interest for 4 years, determine the following:

 (a) The amount of each of the four interest Payments

  (b) The amount of the final payment at the end of year 5

3- Which of the following 1-year investments has the highest rate of return?

(*a*) $12,500 that yields $1125 in interest,

(*b*) $56,000 that yields $6160 in interest, or

(*c*) $95,000 that yields $7600 in interest .

4- A new engineering graduate who started a consulting business borrowed money for 1 year to furnish the office. The amount of the loan was $23,800, and it had an interest rate of 10% per year. However, because the new graduate had not built up a credit history, the bank made him buy loan-default insurance that cost 5% of the loan amount. In addition, the bank charged a loan setup fee of $300. What was the effective interest rate the engineer paid for the loan?

5- When the inflation rate is expected to be 8% per year, what is the market interest rate likely to be?

6- Many credit unions use semiannual interest periods to pay interest on customer savings accounts. For a credit union that uses June 30 and December 31 as its semiannual interest periods, determine the end of period amounts that will be recorded for the deposits shown in the table.

Month	Deposit, $
Jan	50
Feb	70
Mar	—
Apr	120
May	20
June	—
July	150
Aug	90
Sept	—
Oct	—
Nov	40
Dec	110

7- Construct a cash flow diagram for the following cash flows: $25,000 outflow at time 0, $9000 per year inflow in years 1 through 5 at an interest rate of 10% per year, and an unknown future amount in year 5.

8- Construct a cash flow diagram to find the present worth in year 0 at an interest rate of 15% per year for the following situation.

Year	Cash Flow, $
0	−19,000
1–4	+8,100

9- Construct a cash flow diagram that represents the amount of money that will be accumulated in 15 years from an investment of $40,000 now at an interest rate of 8% per year.

10- At an interest rate of 15% per year, an investment of $100,000 one year ago is equivalent to how much now?

11- During a recession, the price of goods and services goes down because of low demand. A company that makes Ethernet adapters is planning to expand its production facility at a cost of $1,000,000 one year from now. However, a contractor who needs work has offered to do the job for $790,000 if the company will do the expansion now instead of 1year from now. If the interest rate is 15% per year ‹how much of a discount is the company getting?

12- As a principal in the consulting firm where you have worked for 20 years, you have accumulated 5000shares of company stock. One year ago, each share of stock was

worth $40. The company has offered to buy back your shares for $225,000. At what interest rate would the firm's offer be equivalent to the worth of the stock last year?

13- If a company sets aside $1,000,000 now into a contingency fund, how much will the company have in 2 years, if it does not use any of the money and the account grows at a rate of 10% per year?

14- Iselt Welding has extra funds to invest for future capital expansion. If the selected investment pays simple interest, what interest rate would be required for the amount to grow from $60,000 to $90,000 in 5 years?

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