The accounting system as an algebraic automaton: Research Articles

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The Accounting System as an Algebraic Automaton

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This article aims to present a mathematical model to describe the accounting system. In effect, in the literature on accounting we can find several mathematical models trying to introduce the accounting mechanics but in this article we use the concept of algebraic automaton, which has been successfully used in other economic fields, like finance. Thus, from this new point of view we can construct all definitions and accounting techniques as a particular case of the concept of automaton.

1. INTRODUCTION

In Chambers,[1] accounting is defined as a method of monetary calculation designed to provide a continuous source of financial information as a guide to future action in markets. In the search for the modern foundations of the accounting, Ijiri[2,3] and Mattessich[4] began with mathematical approaches. Sterling[5] emphasizes the theory of the decision. Mock[6] introduces the elements of the formal theory of the measure. Gonedes and Dopuch[7] and Beaver[8] put emphasis on the framework of capital markets, in the same way as in modern finances. Demski and Feltham[9] were the first ones to apply the approach of economic information, extended by Christensen and Demski.[10] Hilton[11] examines the implications of the probabilistic choice models that incorporate mathematical psychology in the account information theory. Watts and Zimmerman[12] present a positive theory of accounting. Sunders[13] offers a theory of accounting based on contract theory. The advances made by these authors increased the stature and the respectability of accounting.[14]

Mattessich[15] distinguishes between economic and accounting models. Moreover, with the help of some ratios of balance, he gives an example in which he

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combines accounting models with economic ones and it indicates the application of this approach to a prospective area that can still be considered as belonging to accountants. We cannot deny that the systematic application of mathematical models to certain branches of economic science have opened new perspectives, not without enriching our vision. Finally, he mentions the increasing importance that the application of matrices has in accounting. Matrices not only facilitate the formulation of accounting axioms, theorems, and their proofs, but rather they have found a use as models solving several problems in costs accounting.

According to Ijiri[16] matrix representation of accounting entries has gained in popularity both in practice and in the classroom, partly due to the use of the spreadsheet. Now some of these software packages have extended their capacity to include operations even with three-dimensional matrices. Using an example, he examines the meaning of the dimensionality of the arrays used in accounting as the number of dimensions increases from zero (scalars), to one (vectors), to two (matrices), and to three (three-dimensional arrays). And this is to relate the dimensionality of arrays with an underlying accounting model that can increase also from the null entry, to single entry, to double entry, and to triple entry.

In this article we will try to find a group of axioms and structures extracted from conventional accounting in a way so that the set of axioms and measure rules so developed are not only necessary but also sufficient in order to explain most of the principles and practices in conventional accounting. Unfortunately, conventional accounting is a collection of many different principles and accounting practices. Moreover, in some cases, they are mutually inconsistent and therefore the systematic theories cannot describe all of them. However, many axioms and measure rules can be necessary to cover all the principles and practices in conventional accounting. Thus, our findings have been to directly approach conventional accounting by means of a group of axioms and relatively simple measure rules.

To do this, Mattessich[17] gives the following simple recommendations:

(1) Unify the norms and empirical propositions in a simple theoretical frame but, simultaneously, treat the general empirical assumptions from purpose-orientated hypotheses.
(2) Treat academic accounting as an applied science in which the conditional-normative reasoning should be very central.
(3) Reject the false pride that the expression "applied science" seems to engender.

In the following section the main concepts of Algebraic Automata Theory are introduced. Section 3 states the concept of message as the input to be used in the automaton. Section 4 continues defining the concept of accounting. In Section 5, the accounting system is described. Finally, Section 6 concludes and summarizes.

2. SEMIAUTOMATA AND AUTOMATA

Nowadays, the Algebraic Automata Theory has been applied in several fields: biology, psychology, biochemistry, and sociology. Moreover, it has been applied to economics through systems theory[18] and, more recently, in finance.[19] Hence, the aim of this article is to introduce a mathematical approach to accounting concepts with the use of advanced tools in the algebraic field, more specifically, the well-known concept of automaton. The Algebraic Theory of Automata has its origins in the research of Turing,[20] Shannon,[21] and McCulloch and Pitts[22] (see also Refs. 23–27).

2.1. Definition (Semiautomaton)

A *semiautomaton* is a triple

$$\mathcal{S} = (Z, A, \delta)$$

consisting of two nonempty sets Z and A and a function

$$\delta : Z \times A \to Z$$

where Z is called the *set of states*, A the *input alphabet*, and δ the *next-state function* of $\mathcal{S}$.

2.2. Definition (Automaton)

An *automaton* is a quintuple

$$\mathcal{A} = (Z, A, B, \delta, \lambda)$$

where

$$(Z, A, \delta)$$

is a semiautomaton, B is a nonempty set called *output alphabet*, and

$$\lambda : Z \times A \to B$$

is the *output function*.

If $z \in Z$ and $a \in A$, then we interpret $\delta(z, a) \in Z$ as the next state into which z is transformed by the input a. $\lambda(z, a) \in B$ is the output of z resulting from the input a. Thus if the automaton is in state z and receives input a, then it changes to state $\delta(z, a)$ with output $\lambda(z, a)$.

2.3. Description of Automata (by Graphs)

We depict z as a "disc" in the plane and we draw an arrow labeled a from z to $z' = \delta(z, a)$. In case of an automaton we denote the arrow also by $\lambda(z, a)$. This graph is called the *state graph* (Figure 1).

Let us consider the set $\bar{A} = F_A$ of the words (the empty word included) that we can write with the elements (letters) of a set A. We define in $\bar{A}$ an operation $\top$ in the following way. Let $\bar{a} = a_1 a_2 \ldots a_p$, $\bar{b} = b_1 b_2 \ldots b_q$:

$$\bar{a} \top \bar{b} = a_1 a_2 \ldots a_p b_1 b_2 \ldots b_q$$

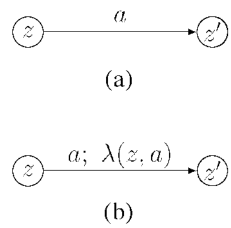

Figure 1. (a) Case of a semiautomaton. (b) Case of an automaton.

Obviously, $\top$ is an internal operation in $\bar{A}$, is associative, and the empty word Λ is the identity element, because

$$\bar{a} \top \Lambda = \Lambda \top \bar{a} = \bar{a}, \text{ for all } \bar{a} \in \bar{A}$$

The monoid $\bar{A}$ is called the *free monoid on A*.[28]

In our study of automata we extend the input set A to the free monoid $\bar{A} = F_A$, with Λ as identity. We also extend δ and λ from $Z \times A$ to $Z \times \bar{A}$, by defining for $z \in Z$ and $a_1, a_2, \ldots, a_r \in A$:

$$\begin{aligned}
\bar{\delta}(z, \Lambda) &= z \\
\bar{\delta}(z, a_1) &= \delta(z, a_1) \\
\bar{\delta}(z, a_1 a_2) &= \delta(\bar{\delta}(z, a_1), a_2) \\
&\vdots \\
\bar{\delta}(z, a_1 a_2 \ldots a_r) &= \delta(\bar{\delta}(z, a_1 a_2 \ldots a_{r-1}), a_r)
\end{aligned}$$

and

$$\begin{aligned}
\bar{\lambda}(z, \Lambda) &= \Lambda \\
\bar{\lambda}(z, a_1) &= \lambda(z, a_1) \\
\bar{\lambda}(z, a_1 a_2) &= \lambda(z, a_1)\bar{\lambda}(\delta(z, a_1), a_2) \\
&\vdots \\
\bar{\lambda}(z, a_1 a_2 \ldots a_r) &= \lambda(z, a_1)\bar{\lambda}(\delta(z, a_1), a_2 a_3 \ldots a_r)
\end{aligned}$$

In this way we obtain functions $\bar{\delta} : Z \times \bar{A} \to Z$ and $\bar{\lambda} : Z \times \bar{A} \to \bar{B}$. The semiautomaton $\mathcal{S} = (Z, A, \delta)$ (respectively, the automaton $\mathcal{A} = (Z, A, B, \delta, \lambda)$) is thus generalized to the new semiautomaton $\bar{\mathcal{S}} = (Z, \bar{A}, \bar{\delta})$ (respectively, automaton $\bar{\mathcal{A}} = (Z, \bar{A}, \bar{B}, \bar{\delta}, \bar{\lambda})$). We can describe the action of $\mathcal{S}$ and $\mathcal{A}$ in the following way: Let $z \in Z$ and $a_1, a_2, \ldots, a_n \in A$:

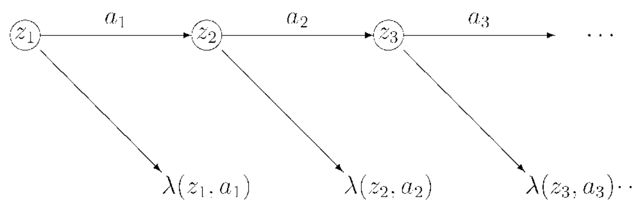

Figure 2. Input and output sequences.

$$z_1 = z$$
$$z_2 = \delta(z_1, a_1)$$
$$z_3 = \bar{\delta}(z_1, a_1 a_2) = \bar{\delta}(\delta(z_1, a_1), a_2) = \delta(z_2, a_2)$$
$$z_4 = \delta(z_3, a_3), \ \ldots$$

If the automaton (resp. semiautomaton) is in state z and an input sequence $a_1 a_2 \ldots a_r \in \bar{A}$ operates, then the states are changed from $z = z_1$ to z_2, from z_2 to $z_3, \ldots$ until the final state z_{r+1} is obtained. As a result the output sequence is

$$\lambda(z_1, a_1)\lambda(z_2, a_2)\ldots\lambda(z_r, a_r)$$

See Figure 2.

Let $\mathcal{S} = (Z, A, \delta)$ be a semiautomaton. Let us consider $\bar{\mathcal{S}} = (Z, \bar{A}, \bar{\delta})$.

2.4. Notation

For all $\bar{a} \in \bar{A}$, let $f_{\bar{a}} : Z \to Z/\ z \mapsto f_{\bar{a}}(z) = \bar{\delta}(z, \bar{a})$.

2.5. Theorem

$M_{\mathcal{S}} = (\{f_{\bar{a}} /\ \bar{a} \in \bar{A}\}, \circ)$ is a monoid.

Proof. See Ref. 27.

2.6. Equivalence Relation on $\bar{A}$

For $\bar{a} \in \bar{A}$, let $f_{\bar{a}} : Z \to Z,\ z \mapsto \bar{\delta}(z, \bar{a})$.

$$\bar{a}_1 \equiv \bar{a}_2 \Leftrightarrow f_{\bar{a}_1} = f_{\bar{a}_2} \Leftrightarrow \text{for all } z \in Z, f_{\bar{a}_1}(z) = f_{\bar{a}_2}(z)$$
$$\Leftrightarrow \text{for all } z \in Z,\ \bar{\delta}(z, \bar{a}_1) = \bar{\delta}(z, \bar{a}_2)$$

2.7. Equivalence Relation on Z

Let $\mathcal{A} = (Z, A, B, \delta, \lambda)$ be an automaton and $z, z' \in Z$. Then

$$z \sim z' \quad \text{if for all } \bar{a} \in \bar{A},\ \bar{\lambda}(z, \bar{a}) = \bar{\lambda}(z', \bar{a})$$

3. THE CONCEPT OF ACCOUNT

3.1. Introduction

There is no doubt that every economic fact acts in some way on the shareholders' equity of any company. So, it is necessary to develop a translation of the economic phenomena into a set of values corresponding to the affected net worth. Thus, we have the scheme shown in Figure 3.

To fix our ideas, let us consider a company starting its lucrative activity with \$240 in cash, \$450 in stocks, and \$310 in office equipment. Let us state that, in addition to these three patrimonial elements, the company will acquire machinery for its activity. Thus, its patrimonial situation can be represented through the following 4th:

$$(a_1, a_2, a_3, a_4)$$

where a_1 represents cash, a_2 stocks, a_3 office equipment, and a_4 machinery. Therefore, in this case, the patrimonial situation of the company at that moment will be

$$(240, 450, 310, 0)$$

However, if we acquire stocks at \$260, equipment at \$340, and machinery at \$410, paying \$125 in cash and accounts payable for the rest, the new patrimonial situation of the company would be

$$(115, 710, 650, 410)$$

with the inconvenience over the previous 4th that the different origins of the financing have not been reflected. Thus, it is necessary to amplify the generic 4th:

$$(a_1, a_2, a_3, a_4)$$

to a 6th:

$$(a_1, a_2, a_3, a_4, a_5, a_6)$$

where a_5 represents equity and a_6 company debts.

Noticing the dualities investment/finance, rights/debts, and so forth, negative signs for a_5 and a_6 can be accepted; so, the situation of the company would now be

$$(115, 710, 650, 410, -1{,}000, -885)$$

Figure 3. An economic fact influencing a company.

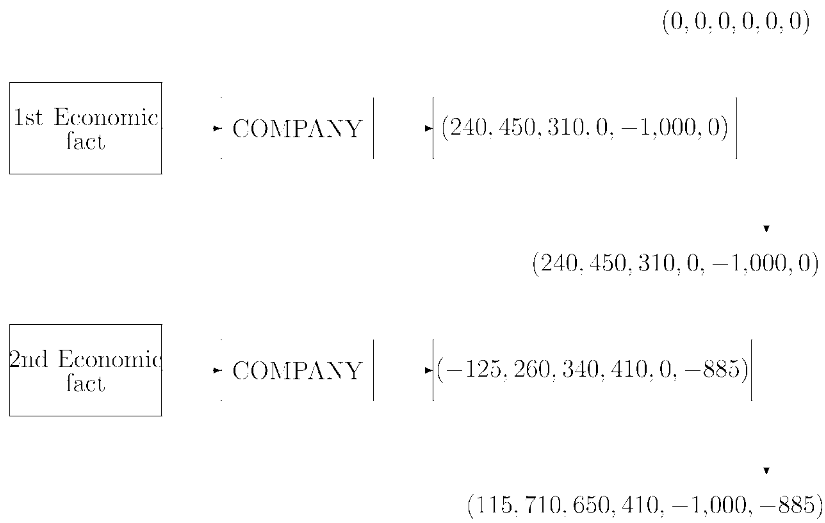

Figure 4. Example of two consecutive economic facts affecting a company.

This is shown graphically in Figure 4, with the first economic fact being the setting up of the company and the second fact the sequence of the previously described purchases. This example justifies the following definition.

3.2. Definition (Message)

From now on, any element of $\mathcal{Z} \times \Re^n$

$$(t, (a_1, a_2, \ldots, a_n)) \in \mathcal{Z} \times \Re^n$$

such that

$$a_1 + a_2 + \cdots + a_n = 0$$

will be called a *message of n-dimension* or the *decodification of an n-message*. The first component of the previous ordered pair will be called the *message expiration* and the second component the *movement of the message*, with each a_i $(i = 1, 2, \ldots, n)$ being the movement of the ith component.

Then, we can observe the dynamic behavior of the message and how the set of movements with respect to a fixed expiration message t_0 gives rise to a hyperplane in $\Re^n$. Moreover, we have to highlight the discrete characteristic given to the variable time. This does not suppose a lack of generality, as this is the traditional way of doing it.

4. DEFINITION (ACCOUNT)

Each component of $\Re^n$ in the concept of an n-dimension message will be called *account*, in such a way that, if $a_i > 0$, the movement of the ith account is said to be *debit*, if $a_i < 0$ *credit*, and if $a_i = 0$, we will say that *there is no movement* in this account.

Observe that, in the previous example, the patrimonial situation of the company could have been written in the following way:

$$((240)(-125),(450)(260),(310)(340),410,-1{,}000,-885)$$

where the information included in the 6th would be higher as each component is split into movement sequences. This example justifies the following definition. A *state of the* ith account is an element in $\overline{\mathcal{Z}\times\Re}$, for example, a sequence of elements in $\mathcal{Z}\times\Re$,

$$\bar{z}_i = z_{1i}\,z_{2i}\ldots z_{mi} \in \overline{\mathcal{Z}\times\Re}$$

such that

$$t_{1i} \leq t_{2i} \leq \cdots \leq t_{mi}, \quad \sum_{i=1}^{n}\sum_{j=1}^{m_i} a_{ji} = 0$$

where

$$z_{ji} = (t_{ji}, a_{ji}); \quad j = 1,2,\ldots,m$$

The representation of each state $\bar{z}_i$ is shown in Figure 5, where $j_1, j_2, \ldots, j_m$ a reordination of

$$1,2,\ldots,m$$

such that

$$j_1 < j_2 < \cdots < j_k$$

$$a_{j_1 i}, a_{j_2 i}, \ldots, a_{j_k i} > 0$$

$$j_{k+1} < j_{k+2} < \cdots < j_m$$

i-th account

$(t_{j_1 i}, a_{j_1 i})$	$(t_{j_{k+1} i}, a_{j_{k+1} i})$
$(t_{j_2 i}, a_{j_2 i})$	$(t_{j_{k+2} i}, a_{j_{k+2} i})$
$\vdots$	$\vdots$
$(t_{j_k i}, a_{j_k i})$	$(t_{j_m i}, a_{j_m i})$

Figure 5. Representation of state $\bar{z}_i = z_{1i}\,z_{2i}\ldots z_{mi}$, $z_{ji} = (t_{ji}, a_{ji})$, $j = 1,2,\ldots,m$.

and

$$a_{j_{k+1}i}, a_{j_{k+2}i}, \ldots, a_{j_m i} < 0$$

In the example described by Butterworth,[29] the economic facts affecting the company become messages or transactions, for example:

(1) Sell capital stock for cash.
(2) Pay dividends.
(3) Sell merchandise for cash.
(4) Dispose of inventory.
(5) Buy inventory or credit.
(6) Pay accounts payable.
(7) Acquire fixed assets for cash.
(8) Depreciate fixed assets.

These transactions are decoded through a matrix in which each column s_j indicates the effect of a transaction of type j upon the 15 accounts of the system:

$$\mathcal{S} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 0 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ . & . & . & . & . & . & . & . \\ 1 & -1 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where 1 is assigned to all accounts through which there are flows toward the account, the -1 to all accounts to which there are flows away from the account, and 0 to all others. Thus, for the transaction (5) "purchase inventory on credit," one has

$$\begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

indicating that there exists a flow from Assets (more specifically, Inventory) toward Equities (concretely, Liabilities, and more specifically, Accounts Payable).

Observe that

- The columns at the bottom of matrix $\mathcal{S}$ are elements of the hyperplane

$$x_1 + x_2 + \cdots + x_9 = 0$$

  in $\Re^9$.
- The sequence of transactions (1), (2), ..., (8) is a message:

$$a_1 a_2 \ldots a_9$$

5. DESCRIPTION OF ACCOUNTING

From now on, we will try to describe accounting as an automaton. Then, we will have to define the main elements that come into that concept. In the first place, and as we have already defined, the set of states of the ith account is a subset of $\bar{\bar{\mathcal{Z}}} \times \Re$ satisfying certain conditions. That is the reason why the set of states of n accounts will be a subset

$$Z \subseteq (\bar{\bar{\mathcal{Z}}} \times \Re)^n$$

which verifies the previously stated conditions, component by component.

In the second place, the set of inputs A is $\mathcal{Z} \times H_n$, with H_n being the hyperplane of $\Re^n$ such that

$$x_1 + x_2 + \cdots + x_n = 0$$

In this way, the input function is

$$\delta : Z \times A \to Z$$

defined by

$$((\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n), (t, (a_1, a_2, \ldots, a_n))) \mapsto (\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n)$$

if $t < t_{j_i}$, for some $i = 1, 2, \ldots, n$, that is to say,

$$((\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n), (t, (a_1, a_2, \ldots, a_n))) \mapsto (\bar{z}_1(t, a_1), \bar{z}_2(t, a_2), \ldots, \bar{z}_n(t, a_n))$$

with $\bar{z}_i(t, a_i)$ being the concatenation of $\bar{z}_i$ and (t, a_i), only in the cases in which $a_i \neq 0$.

Let us see how the function δ acts. To do this, suppose that $n = 3$ and that the states $\bar{z}_1$, $\bar{z}_2$, and $\bar{z}_3$ are as shown in Figure 6.

Let us suppose that the input $(t_5, (0, -5, 5)) \in \mathcal{Z} \times H_3 = A \subseteq \mathcal{Z} \times \Re^3$ acts on these states, with $t_5 > t_3, t_4$. The resulting state is shown in Figure 7.

In the third place, and following with the description of the automaton, the output set B is $\mathcal{Z} \times H_{r+1}$, with H_{r+1} being the hyperplane of $\Re^{r+1}$ such that

$$x_1 + x_2 + \cdots + x_{r+1} = 0$$

with $r < n$. Thus, the output function is

$$\lambda : Z \times A \to B$$

defined by

$$((\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n), (t, (a_1, a_2, \ldots, a_n)))$$

$$\mapsto \left(t, \left(a_1 + \sum_{j=1}^{m_1} a_{j1}, a_2 + \sum_{j=1}^{m_2} a_{j2}, \ldots, a_r + \sum_{j=1}^{m_r} a_{jr}, \sum_{i=r+1}^{n} \left(a_i + \sum_{j=1}^{m_i} a_{ji} \right) \right) \right)$$

The element $\lambda((\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n), (t, (a_1, a_2, \ldots, a_n)))$ will be called the *balance sheet at moment t*, and the automaton,

$$\mathcal{C} = (Z, A, B, \delta, \lambda)$$

Account 1		Account 2		Account 3	
$(t_1, 4)$	$(t_3, 1)$		$(t_1, 1)$	$(t_3, 1)$	$(t_1, 3)$
$(t_2, 6)$			$(t_2, 6)$		$(t_4, 2)$
$(t_4, 9)$			$(t_4, 7)$		

Figure 6. Representation of initial states $\bar{z}_1$, $\bar{z}_2$, and $\bar{z}_3$.

Account 1	
$(t_1, 4)$	$(t_3, 1)$
$(t_2, 6)$	
$(t_4, 9)$	

Account 2	
	$(t_1, 1)$
	$(t_2, 6)$
	$(t_4, 7)$
	$(t_5, 5)$

Account 3	
$(t_3, 1)$	$(t_1, 3)$
$(t_5, 5)$	$(t_4, 2)$

Figure 7. Representation of final states $\bar{z}_1$, $\bar{z}_2$, and $\bar{z}_3$.

where Z, A, B, δ, and λ are the sets and functions previously described, will be called the *system account*.

Assigning a name to each component of the set of states Z, the set of all these denominations is called the *account plan*.

The ith component of

$$\lambda((\bar{z}_1, \bar{z}_2, \dots, \bar{z}_n), (t, (a_1, a_2, \dots, a_n)))$$

will be called the *due of the ith plan account at moment t*. In other words, we will say that the accounts, $1, 2, \dots, r+1$ are *with due*.

The $r+1$ accounts that appear in the output set will be called *asset accounts and liabilities accounts*, depending on the sign (positive or negative) of their due.

The $n - r - 1$ accounts that do not appear in the output set will be called *profit and loss accounts*, depending on the sign (positive or negative) of their due:

$$a_i + \sum_{j=1}^{m_i} a_{ji}$$

The $(r + 1)$th account that appears in the set of outputs will be called the *profit and loss account*.

The input set A can be extended to the free monoid $\bar{A} = F_A$ as was described in Section 2 of this article; the set of statements can be a finite sequence of messages applying each input over the state resulting from the action on the previous message.

In this way, if over an initial state

$$(t_0, (\bar{z}_1^0, \bar{z}_2^0, \dots, \bar{z}_{r+1}^0))$$

acts a sequence of inputs (messages)

$$(t_1, (a_{11}, a_{21}, \dots, a_{n1})), \dots, (t_m, (a_{1m}, a_{2m}, \dots, a_{nm}))$$

such that $t_1 < \cdots < t_m$ (this does not suppose a lack of generality), we will say that

$$\lambda(\delta(\bar{z}_1^0, \bar{z}_2^0, \dots, \bar{z}_{r+1}^0), (t_1, (a_{11}, a_{21}, \dots, a_{n1})) \dots (t_m, (a_{1m}, a_{2m}, \dots, a_{nm})))$$

is the *final balance sheet at* t_m of a certain accounting whose *initial balance sheet* is

$$(t_0, (\bar{z}_1^0, \bar{z}_2^0, \ldots, \bar{z}_{r+1}^0))$$

during the period of time $[t_0, t_m]$.

Usually, the application of the output function λ is not made all the times a message is received, but just only at some moments of the life of the company: end of the year, end of quarter, and so on. The process of calculating the image through λ will be called the *regularization*.

We must observe that the task of the accountant is the decodification of messages, for example, translation of any message into an $(n+1)$th, where the first component is the expiration and the second is the sequence of movements of the n accounts belonging to every account plan.

In Butterworth,[29] the accounting system can be considered as a function over the space of states of the world:

$$b^{k+1} = \theta(\bar{x}^k, x^k)$$

where

- $\bar{x}^k$ represents all knowledge existing prior to the current period,
- x^k is the state of the world in the kth interval of time,
- b^k is the vector of account balances at the start of the kth period, and
- the accounting function θ expresses the relationship between the history of states of the world and those account balances.

The accounting function can be considered as the iterative composition on an aggregation function σ and a measurement function τ in the following way:

$$\begin{aligned} b^{k+1} &= \theta(\bar{x}^k, x^k) = \sigma[\theta(\bar{x}^{k-1}, x^{k-1}), \tau(x^k)] \\ &= \sigma[\sigma(\theta(\bar{x}^{k-2}, x^{k-2}), \tau(x^{k-1})), \tau(x^k)] = \ldots \end{aligned}$$

Hence

$$b^{k+1} = \sigma(b^k, \tau(x^k)) = \sigma[\sigma(b^{k-1}, \tau(x^{k-1})), \tau(x^k)]$$

Observe the similarity between the following two expressions:

$$\begin{aligned} \bar{\lambda}(0, 012\ldots k) &= \lambda(\bar{\lambda}(0, 012\ldots(k-1)), k) \\ &= \sigma(\bar{\lambda}(0, 012\ldots(k-1)), \tau(k)) \end{aligned}$$

In Willet[30] the axiomatic covers the case where a resource, active at some time in the past, has not been used in production during a given interval.

Let $\bar{a}$ and $\bar{b}$ be two messages. We will say that $\bar{a}$ and $\bar{b}$ are *equivalent* and we will write

$$\bar{a} \equiv \bar{b}$$

if $f_{\bar{a}} = f_{\bar{b}}$.

In other words, if

$$\bar{a} = (t_1, (a_{11}, a_{21}, \ldots, a_{n1}))(t_2, (a_{12}, a_{22}, \ldots, a_{n2}))\ldots(t_r, (a_{1r}, a_{2r}, \ldots, a_{nr}))$$

and

$$\bar{b} = (t'_1, (b_{11}, b_{21}, \ldots, b_{n1}))(t'_2, (b_{12}, b_{22}, \ldots, b_{n2}))\ldots(t'_s, (b_{1s}, b_{2s}, \ldots, b_{ns}))$$

then

$$\bar{a} \equiv \bar{b} \quad \text{if} \sum_{i=1}^{r} a_{ji} = \sum_{i=1}^{s} b_{ji}; \quad j = 1, \ldots, n$$

Thus, two composed messages are equivalent when, coming from the same state, the balance sheet at t_r and at t'_s, resulting from the messages $\bar{a}$ and $\bar{b}$, respectively, are the same.

Let $(\bar{z}_1^0, \bar{z}_2^0, \ldots, \bar{z}_n^0)$ and $(\bar{z}_1^1, \bar{z}_2^1, \ldots, \bar{z}_n^1)$ be two states. We will say that those states are *equivalent* and we will write

$$(\bar{z}_1^0, \bar{z}_2^0, \ldots, \bar{z}_n^0) \sim (\bar{z}_1^1, \bar{z}_2^1, \ldots, \bar{z}_n^1)$$

if for all $(t, (a_1, a_2, \ldots, a_n)) \in \mathcal{Z} \times \Re^n$,

$$\begin{aligned} &\lambda((\bar{z}_1^0, \bar{z}_2^0, \ldots, \bar{z}_n^0), (t, (a_1, a_2, \ldots, a_n))) \\ &\quad = \lambda((\bar{z}_1^1, \bar{z}_2^1, \ldots, \bar{z}_n^1), (t, (a_1, a_2, \ldots, a_n))) \end{aligned}$$

In particular, taking $(t, (a_1, a_2, \ldots, a_n)) = (t, (0, 0, \ldots, 0))$, we would have

$$\begin{gathered} \sum_{j=1}^{m_1} a_{j1} = \sum_{j=1}^{m'_1} a'_{j1}, \quad \sum_{j=1}^{m_2} a_{j2} = \sum_{j=1}^{m'_2} a'_{j2} \\ \vdots \\ \sum_{j=1}^{m_r} a_{jr} = \sum_{j=1}^{m'_r} a'_{jr}, \quad \sum_{i=r+1}^{n} \sum_{j=1}^{m_i} a_{ji} = \sum_{i=r+1}^{n} \sum_{j=1}^{m'_i} a'_{ji} \end{gathered}$$

If the states are simple, we would have

$$z_1^0 = z_1^1, \quad z_2^0 = z_2^1, \ldots, z_r^0 = z_r^1, \quad \sum_{i=r+1}^{n} z_i^0 = \sum_{i=r+1}^{n} z_i^1$$

that is, they have the same dues in the assets, liabilities, and profit and loss accounts.

6. CONCLUSION

The main objective of accounting theory is to provide a basis for the prediction and explanation of accounting and economic facts.[31] Up to now, we only have some approaches which help us to understand the whole accounting process:

(1) the pragmatic or informal approach, and
(2) theoretical approaches (deductive, inductive, sociological, economic, and eclectic).[31]

These statements were defended by the American Accounting Association Committee in 1977 with its "Statement of Accounting Theory and Theory Acceptance" (SATTA).[32]

It is clear that the underlying elements of an accounting theory respond to mathematical logic. As an example, Ijiri[33] introduced a vectorial formulation of accounting. In the same way, Mattessich[34] applied system theory to accounting, and, finally, Willett[30,35] and Butterworth[29] proposed an axiomatic theory of accounting measurement.

Thus, the accounting process can be defined as an automaton, that is to say, a mechanism that transforms states through the action of inputs, giving rise to specific outputs. Hence, through this accounting-mathematical model double-entry bookkeeping is formulated.

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