CHAPTER 1

INTRODUCTION TO ENGINEERING ECONOMICS

1.0 ENGINEERING ECONOMICS

Economics is defined as the study of allocation of scarce resources among unlimited ends (or wants).

Our wants are unlimited or at least increasing ever and to satisfy all these wants, we need unlimited supply of productive resources which could provide necessary goods and services to the community. However, resources are scarce i.e. limited in supply and obtained at some cost. In other words, resources are scarce in relation to its needs Therefore, scarce resources should be used wisely judiciously and more effectively at optimum level, minimizing the cost and maximizing profit and benefit without compromising the quality of product or service.

All engineering decisions involve number of feasible alternatives or options. These feasible alternatives must be properly evaluated before implementing them. If there is no alternative, there is no need of economic study.

Mission of engineers is to transform the resources of nature for the benefit of the human race. Engineers translate an idea into reality. However an idea may be technically excellent incorporating sound design, latest technology but if it does not convert into real product or service that is affordable and fit for purposes satisfying needs and requirements of its end users, clients, target group, beneficiary group, then it is not worthwhile to invest in such ventures. The products or services generated should use optimized utilization of various resources so that cost of production is not high, affordable to users and compete with similar product and services of competitors in the market.

Engineering economy involves the systematic evaluation of the economic merits of proposed solutions to engineering problems. To be economically acceptable (i.e. affordable), solutions to engineering problems must be demonstrate a positive balance of long-term benefits over long-term costs,… (Accreditation board for Engineering and Technology)..

1.1 ORIGIN OF ENGINEERING ECONOMY

Development of Engineering Economy as a separate field of study is relatively recent. It has no well recorded past history. It does not mean that, historically, costs are overlooked in engineering decisions. Ultimate economy is primary concern to the engineer.

The Economic Theory of Railway Location, 2nd ed. New York: John Wiley & Sons, 1987 written by Arthur M. Wellington, a civil engineer, pioneered engineering interest in economic evaluation. His interest was railway in USA.

A text book ***Principles of Engineering Economy***, New York: The Ronald Press Company, 1930, was published by Eugene Grant. He is considered as the father of engineering economy.

Current developments are pushing to encompass new methods of risk, sensitivity, resource conservation and effective utilization of public funds and so on.

1.2 PRINCIPLES OF ENGINEERING ECONOMY

The development, study and application of any discipline must begin with a basic foundation. Engineering economy involves set of principles. In engineering economic analysis, experience has shown that most errors can be traced to some violation or lack of adherence to the basic principles.

PRINCIPLE 1 - DEVELOP THE ALTERNATIVES: The choice (decision) is among alternatives. The alternatives need to be identified. A decision involves making a choice among alternatives. Developing and defining alternatives depends upon engineer's creativity and innovation.

PRINCIPLE 2 - FOUCUS ON THE DIFFERENCE: Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision. If all prospective outcomes of the feasible alternatives were exactly the same, obviously, only the differences in the future outcomes of the alternatives are important. Outcomes that are common to all alternatives can be disregarded in the comparison and decision. For example, if two apartments were with same purchase price or rental price, decision on selection of alternatives would depend on other factors such as location and annual operating and maintenance expenses.

PRINCIPLE 3 - USE A CONSISTENT VIEWPOINT: The prospective outcomes of the alternatives, economic and other, should be consistently developed from a defined viewpoint (perspective). Often perspective of decision maker is owner's point of view. For the success of the engineering projects viewpoint may be looked upon from the various perspective e.g. donor, financer, beneficiary group & stakeholders. However, viewpoint must be consistent throughout the analysis.

PRINCIPLE 4 - USE A COMMON UNIT OF MEASURE: Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of the alternatives. For economic consequences, a monetary units such as dollars or rupees is the common measure.

PRINCIPLE 5 - CONSIDER ALL RELEVANT CRITERIA: Selection of preferred alternative (decision making) requires the use of a criterion (or several criteria). The decision process should consider both the outcomes enumerated in the monetary unit and those expressed in some other unit of measurement or made explicit in a descriptive manner. Apart from the long term financial interest of owner, needs of stakeholders should be considered.

PRINCIPLE 6 - MAKE UNCERTAINTY EXPLICIT: Uncertainty is inherent in projecting (or estimating) the future outcomes of the alternatives ad should be recognized in their analysis and comparison. The magnitude & impact of future impact of any course of action are uncertain or probability of occurrence changes from the planned one. Thus dealing with uncertainty is important aspect of engineering economic analysis.

PRINCIPLE 7 - REVISIT YOUR DECISIONS: Improved decision making results from an adaptive process; to the extent practicable, the initial projected outcomes of the selected alternative should be subsequently compared with actual results achieved. If results significantly different from the initial estimates, appropriate feedback to the decision making process should occur.

1.3 ROLE OF ENGINEERS ON ECONOMIC DECISION

We will restrict our focus to various economic decisions related to engineering projects, ventures, undertakings.

Engineering is involved in every detail of a product's production, from the conceptual design to the shipping. Engineering decisions accounts majority of (say 85%) of product cost. Engineers must consider the effective use of capital assets such buildings, plants and workshops, machine and equipments.. One of the engineer's primary task is to plan for acquisition of equipment (capital expenditure decision).With the acquisition of any fixed capital, we need to estimate or predict the cash flows and profits that asset will generate during its service period and make decision whether the investment would be justified.

Engineers play a role in effective utilization of assets. They utilize same technique for engineering economic decision. Judicious, effective and wise of poor predication or estimation or projection of performance of investment into future is a challenging and risky job which can be rewarding or disastrous.

Engineers are called upon to translate an idea into reality. Constant flow of innovative and creative ideas for generating new products as per ever changing needs of its clients in dynamic environment and market conditions affect growth and development of firm, also make competitive. Based on past experience, and research and development, investments decisions are made to make existing product better or produce them at a competitive price. Engineers must understand how their investment decisions affect overall position of the company and its future growth and prospectus.

The steps/procedure in the engineering economic decision making are:

- Identification of problem and prospects
- Develop feasible & relevant alternatives
- Determine appropriate selection criteria.
- Analysis, comparison of various alternatives
- Evaluate & recommend the alternative
- Select the best alternative
- Implementation of the selected alternative
- Monitoring and controlling

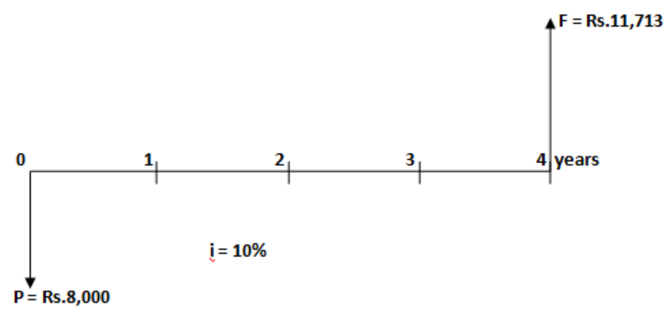
1.4 CASH FLOW DIAGRAM

Cash flow: The actual rupees or dollar coming into or out of the treasure of a firm. A cash flow occurs when money is transferred from one organization or individual to other. Thus, cash flow represents the economic effects of an alternative in terms of money spend or received.

Cash Inflow or Positive Cash Flow: Actual rupee or dollar coming into firm. i.e. receipts or incomes.

Cash Outflow or Negative Cash Flow: Actual rupee or dollar paid out by a firm. i.e. expenditures or payment.

Net Cash Flow: Difference between total cash inflows (receipts) and the total cash outflows for a specified period of time. e.g. one year.



Cash Flow Diagram

- Horizontal line in a cash flow diagram is a time scale with progression of time moving from left to right. The period (or year) labels are applied to the intervals of time rather than points on the tie scale. The end of the period 2 is coincident with beginning of period 3.
- The arrows signify cash flows. Cash outflows are represented by downward arrows and Cash inflows are represented by upward arrows.
- Cash flow diagram is dependent on point of view. If a lender lends Rs. 25,000. (it is cash outflows for him) and at the end of 4 years, receives compound interest plus his principal Rs. 11,713 at 10% interest rate per annum (it is cash inflow for him).
- If the directions of arrows are reversed, diagram would be from borrower's point of view.

CHAPTER 2

INTEREST
AND
TIME VALUE OF MONEY

2.1. TIME VALUE OF MONEY

2.2 SIMLPLE INEREST

2.3. COMPOUND INTEREST

2.4 ECONOMIC EQUIVALENCE

2.5 DEVELOPMENT INTEREST FORMULA

2.1 TIME VALUE OF MONEY

The relationship between money and time leads to the concept of time value of money. A rupee or dollar in hand is worth more than a rupee or dollar received 'N' years from now.

Money has time value because the purchasing power of money as well as the earning power of money changes with time. During inflation, purchasing power of money decreases over time. Money can earn an interest for a period of time. Interest represents the earning power of money. Therefore, both purchasing power and earning power of money should be considered while taking into account the time value of money.

Engineering economic investment studies involve huge capital for longer period of time. So, the effect of time value of money should be considered in the analysis.

2.2 SIMLPLE INEREST

When the interest earned or charged is directly proportional to the initial investment or principal amount (P), the interest rate (i), and number of interest period (N), the interest (I) and the interest rate is said to be simple interest and simple interest rate.

$$I = P*N*i$$

2.3 COMPOUND INTEREST

When the interest charge for any interest period (a year) is based on the remaining principal amount plus any accumulated interest charges up the beginning of that period, the interest is said to the compound.

2.3.1 NOMINAL INTEREST RATE

In general, interest charged or earned on the principal amount is quoted as ' i % compounded annually or i % per year'. Very often, the interest period or time between successive compounding, is less than year. It has become customary to quote interest rates an annual basis, followed by the compound period if different from one year in length. For example, if the interest rate is 6% per six month, it is customary to quote this rate as '12% compounded semi-annually. The basic annual interest rate, 12% in this case, is known as nominal interest rate and denoted by 'r'.

2.3.2 EFFECTIVE INTEREST RATE

The actual or exact rate of interest rate earned on the principal during one year is known as effective interest rate and dented by 'i'. The effective interest rate is always expressed on annual basis or per annum.

The relationship between effective interest rate 'i' and nominal interest rate 'r' is

$$i = \left(1 + \frac{r}{M}\right)^{M} - 1$$

Where M is number of compounding periods per year.

When M > 1, then i > r

the effective interest rate is useful for describing the compounding effect of interest earned on interest within one year.

2.3.3 CONTINUOUS COMOUNDING

As a limit, interest may be considered an infinite number of times per year - i.e. continuously, Under these conditions, the effective interest for continuous compounding is derived from equation.

If $\frac{M}{r} = p$,

$\frac{r}{M} = \frac{1}{p}$,

M= rp

$$\left(1 + \frac{r}{M}\right)^{M}$$

$$= \left(1 + \frac{1}{p}\right)^{rp}$$

$$= \left[\left(1 + \frac{1}{p}\right)^{p}\right]^{r}$$

$$= e^{r}$$

$$i = \lim_{M\to\infty} \left(1 + \frac{r}{M}\right)^{M} - 1 = \lim_{M\to\infty} \left[\left(1 + \frac{r}{M}\right)^{\frac{M}{r}}\right]^{r} - 1 = e^{r} - 1$$

$$i = e^{r} - 1$$

$$e^{r} = 1 + i$$

$$e^{rN} = (1 + i)^{N}$$

2.3.3.1 CONTINUOUS COMPOUNDING FORMULA FOR DISCRETE CASH FLOWS

Discrete cash flows assume the cash flows occur a discrete intervals (e.g. once a year), but continuous compounding assumes compounding is continuous throughout the interval.(M = ∞)

Substitute $e^r = 1 + i$

F = P $(e^r)^N$ = e^{rN} **F = P * (F/P, r%, N)**

F = A $\frac{e^{rN}-1}{e^r - 1}$ **F = A * (F/A, r%, N)**

P = A $\frac{e^{rN}-1}{(e^r - 1)*e^{rN}}$ **P = A * (P/A, r%, N)**

r% denotes nominal rate continuous compounding

Example:

What will be FW at the end of 5 years of cash flow at the rate of Rs. 500 per year for 5 years with interest compounded continuously at nominal annual rate of 8%.

F = A * (F/ A, r%, N) F = A * (F/A, 8%, 5)

F = A $\frac{e^{rN}-1}{e^r - 1}$ F = 500 $\frac{e^{0.08*5}-1}{e^{0.08} - 1}$ = 2952.58

2.3.3.2 CONTINUOUS COMPOUNDING FORMULA FOR CONTINUOUS CASH FLOWS

Continuous cash flow means a series of cash flow s occurring at infinitesimally short interval of time. It may have annuity having an infinite number of short time

F = $\hat{A}$ $\frac{e^{rN}-1}{r}$ **F = $\hat{A}$ * (F/ $\hat{A}$, r%, N)**

P = $\hat{A}$ $\frac{e^{rN}-1}{r*e^{rN}}$ **P = $\hat{A}$ * (P/ $\hat{A}$, r%, N)**

Example:

What will be FW at the end of 5 years of a uniform continuous cash flow at the rate of Rs. 500 per year for 5 years with interest compounded continuously at nominal annual rate of 8%.

F = $\hat{A}$ * (F/ $\hat{A}$, r%, N) F = $\hat{A}$ * (F/ $\hat{A}$, 8%, 5)

F = $\hat{A}$ $\frac{e^{rN}-1}{r}$ F = 500 $\frac{e^{0.08*5}-1}{0.08}$ = 3074

2.4 ECONOMIC EQUIVALENCE

Two things are said to be equivalent when they have the same effect. Economic equivalence refers to the fact that a cash flow - whether single payment or series of payments - can be converted to an equivalent cash flow at any point in time.

GENERAL PRINCIPAL

PRINCIPLE 1 :- Equivalence calculations made to compare alternatives require common time basis.

When selecting a pint in time at which to compare the value of alternative cash flows we commonly use either the present time & calculate present worth (PW) of the cash flow, or the present time & calculate future worth (FW) of the cash flow. The choice of time depends on the circumstances surrounding a particular decision, or it may be chosen for convenience.

PRINCIPLE 2 : - Equivalence depends on interest rate.

The equivalence between cash flows is a function of the magnitude and timing of individual cash flows and the interest rate or rates that operate on those flows. This principle is easy to grasp in relation to principle 1.

PRINCIPLE 3 :- Equivalence calculations may require converting multiple cash flow to a single cash flow.

Convert the given cash flows of alternatives consisting various type of cash flows to a particular type of cash flow. Different alternatives consist of various types of cash flow according to nature of work. For comparison, convert them into one particular type of cash flow.

PRINCIPLE 4 :- Equivalence is maintained regardless of point of view.

Cash flow diagram are drawn with different point of view as mentioned in previous chapter. However, as long as we use the same interest rate in equivalence calculations, equivalence can be maintained regardless of point of view.

2.5 DEVELOPMENT INTEREST FORMULA

A better understanding of the conversion process is achieved by the development of the interest formulas. Based on equivalence concept and notations used, a series of interest formulas developed for use in more complex comparisons of cash flows.

2.5.1 THE FIVE TYPES OF CASH FLOWS

Interest formulas can be classified into five categories.

2.5.1.1 SINGLE CASH FLOW

A present sum P invested now for N interest periods at interest rate i% per period.

Its future worth F would be **$F = P * (1 + i)^N$**

The factor **$(1 + i)^N$** is termed as **Single Payment Compound Amount Factor**.

Factor Notation

We may also express that factor in functional notation as

(F/P, i, N), which is read as "Find F, given P, i, and N."

It is expressed as

$F = P * (1 + i)^N = P * (F/P, i, N)$

Example:

If you invest Rs.10,000 now for 10 years at 10% per annum, how much would it be worth at the end of 10 years?

$F = P * (1 + i)^N = 10,000 * (1 + 0.1)^{10} =$ Rs.25,937

Alternately, $F = P * (F/P, i, N) = 10,000 * (P/F, 10, 10) = 10,000 * (2.5937) =$ Rs.25,937

Present Worth Factor

Finding the present worth of a future sum is simply the reverse of compounding and is known as the **discounting process.**

$P = F * (1 + i)^{-N} = F * (P/F, i, N)$

Factor Notation

The factor **$(1 + i)^{-N}$** is known as the single payment present worth factor and is designate as **(P/F, i, N).** The interest rate **i** is also known as **discount rate** and **the P/F, i, N factor** is termed as **discounting factor.**

Example:

An investor wants to purchase a land that will worth Rs. 100,000,000 in 6 years. If the land value increases 8% each year, how much should invest now?

$P = F * (1 + i)^{-N}$ = Rs. 100,000,000 * $(1 + 0.0.8)^{-10}$
= Rs. 60,000,302

Alternately,

P = P * (P/F, i, N) = 100,000,000 *(P/F, 8%, 6)
= 100,000,000 * (0.6302) = Rs.60,000,302

2.5.1.2 UNEVEN PAYMENT SERIES

Future worth of any uneven series of payments can be calculated by the future worth of each individual payment and summing the results.

Example:

Find FW & PW of the following cashflow:

End of Year (EOY)	0	1	2	3	4
Cash Flow	-15,000	+10,000	+5000	0	+3000

End of Year (EOY)	Cash Flow	FW Factor	FW(10%)
0	-15,000	1.1^4 = 1.4641	-21,962
1	10,000	1.1^3 = 1.3310	13,310
2	5000	1.1^2 = 1.2100	6,050
3	0	1.1^1 = 1.1000	0
4	3000	1.1^0 = 1.0000	3,000
		∑ FW(10%)	398

Present worth of any uneven series of payments can be calculated by the present worth of each individual payment and summing the results.

End of Year (EOY)	Cash Flow	PW Factor	PW(10%)
0	-15,000	1.1^{-0} = 1.0000	-15000
1	10,000	1.1^{-1} = 0.9091	9091
2	5000	1.1^{-2} = 0.8264	4132
3	0	1.1^{-3} = 0.7513	0
4	3000	1.1^{-4} = 0.6830	2049
		∑ PW(10%)	272

DISCRETE CASH FLOWS AND DISCRETE COMPOUNDING				
To Find:	Given:	Factor	Factor Name	Factor Notation
For single cash flows:				
F	P	$(1 + i)^{N}$	Single Payment Compound Amount Factor	(F/P, i, N)
P	F	$(1 + i)^{-N}$	Single Payment Present Worth Factor	(P/F, i, N)

2.5.1.3 UNIFORM (EQUAL) SERIES AT REGULAR INTERVALS.

If an amount **A** is invested at the end of each periods for N interest periods at interest rate i% per period,
Its future worth F would be

$$\mathbf{F = A\ \frac{(1+i)^{N}-1}{i}}$$

F = A * (F/A, i, N)

The factor $\frac{(1+i)^{N}-1}{i}$ is termed as **equal payment series compound amount factor or uniform series compound factor**.

For uniform form series (annuity) formula, an amount A starts from at the end of 1st period onwards at the end of each period for N periods with i% interest period.

Example:

If you with to withdraw Rs. 10,000 at the end of each year at an interest rate of 10% per annum for 4 years. How much amount should you deposit now?

Cash flow at the end of each Year (A)	Number of years (N)	Interest rate (i)	PW Factor	PW (10%)
RS. 10,000	4	10%	$\frac{(1+0.1)^{4}-1}{0.1*(1+0.1)^{4}}$ = 3.1699	10,000* 3.1699 = 31699

Alternately,

End of Year (EOY)	Cash Flow	PW Factor	PW(10%)
1	10,000	1.1^{-1} = 0.9091	9091
2	10,000	1.1^{-2} = 0.8264	8264
3	10,000	1.1^{-3} = 0.7513	7513
4	10,000	1.1^{-4} = 0.6830	6830
		∑ PW(10%) =	31698

2.5.1.4 LINEAR GRADIENT SERIES

If an amount **A** is invested or paid at the end of interest period 1 changes (increases or decreases) by a constant amount **G** at the end of each periods for N interest periods at interest rate i% per period,

Its future worth F would be

$$\mathbf{F} = \frac{\mathbf{G}}{\mathbf{i}}\left[\left\{\frac{(\mathbf{1+i})^{N}-\mathbf{1}}{\mathbf{i}} - \boldsymbol{N}\right\}\right]$$

F = $\frac{G}{i}$ *** {(F/A, i, N) - N }**

F = $\frac{G}{i}$ *** (F/A, i, N) –** $\frac{NG}{i}$

F = G * (F/G, i, N)

The factor **(F/G, i, N)** is termed as **gradient series compound amount factor.**

For uniform linear gradient, G starts from at the end of 2nd period onwards at the end of each period for N periods with i% interest period.

Discrete cash flow means cash flows spaced at the end of each interval / period.

Discrete compounding means that interest is compounded at the end each finite period.

For example: a month, a year.

F = P (1 + i)^{N}

F = $A\frac{(1+i)^N-1}{i}$

P = A $\frac{(1+i)^N-1}{i(1+i)^N}$

F = $\frac{G}{i}\left[\left\{\frac{(1+i)^N-1}{i}-N\right\}\right]$

$F = P\,(1+i)^N$	**F = P * (F/P, i, N)**
$F = A\,\frac{(1+i)^N-1}{i}$	**F = A * (F/A, i, N)**
$F = G\left[\frac{1}{i}\left\{\frac{(1+i)^N-1}{i}-N\right\}\right]$	**F = G *(F/G, i, N)**

Combining

$$F = P\,(1+i)^N = A\,\frac{(1+i)^N-1}{i} = G\left[\frac{1}{i}\left\{\frac{(1+i)^N-1}{i}-N\right\}\right]$$

F = P * (F / P , i , N) = A * (F / A , i , N) = G * (F / G , i , N)

F/P,i,N **= (1 + i)^{N}**

F/A,i,N **=** $\frac{(1+i)^N-1}{i}$ **=** $\left\{\frac{F/P,\ i,\ N)-1}{i}\right\}$

F/G,i,N **=** $\left[\frac{1}{i}\left\{\frac{(1+i)^N-1}{i}-N\right\}\right]$ **=** $\left[\frac{1}{i}\ \{F/A, \mathrm{i}, \mathrm{N}\ -N\}\right]$

i%	N	F/P,i,N	F/A,i,N	P/A,i,N	F/G,i,N	A/G,i,N	P/G,i,N
		(1+i)^{N}	$\frac{\frac{F}{P}-1}{i}$	$\frac{F/A}{F/P}$	$\frac{\frac{F}{A}-N}{i}$	$\frac{F/G}{F/A}$	$\frac{F/G}{F/P}$
10	10	2.5937	15.9374	6.1446	59.3742	3.7255	22.8913
12	10	3.1058	17.5487	5.6502	145.4061	3.5847	46.8176

MARR = i = 10% compounded annually												
N	**F/P,i,N**	**F/A,i,N**	**P/A,i,N**	**F/G,i,N**	**A/G,i,N**	**P/G,i,N**	**P/F,i,N**	**A/F,i,N**	**A/P,i,N**	**G/F,i,N**	**G/A,i,N**	**G/P,i,N**
1	1.1000	1.0000	0.9091	0.0000	0.0000		0.9091	1.0000	1.1000			
2	1.2100	2.1000	1.7355	1.0000	0.4762	0.8264	0.8264	0.4762	0.5762	1.0000	2.1000	1.2100
3	1.3310	3.3100	2.4869	3.1000	0.9366	2.3291	0.7513	0.3021	0.4021	0.3226	1.0677	0.4294
4	1.4641	4.6410	3.1699	6.4100	1.3812	4.3781	0.6830	0.2155	0.3155	0.1560	0.7240	0.2284
5	1.6105	6.1051	3.7908	11.0510	1.8101	6.8618	0.6209	0.1638	0.2638	0.0905	0.5524	0.1457
6	1.7716	7.7156	4.3553	17.1561	2.2236	9.6842	0.5645	0.1296	0.2296	0.0583	0.4497	0.1033
7	1.9487	9.4872	4.8684	24.8717	2.6216	12.7631	0.5132	0.1054	0.2054	0.0402	0.3814	0.0784
8	2.1436	11.4359	5.3349	34.3589	3.0045	16.0287	0.4665	0.0874	0.1874	0.0291	0.3328	0.0624
9	2.3579	13.5795	5.7590	45.7948	3.3724	19.4215	0.4241	0.0736	0.1736	0.0218	0.2965	0.0515
	2.5937	15.9374	6.1446	59.3742	3.7255	22.8913	0.3855	0.0627	0.1627	0.0168	0.2684	0.0437

Example:

Expenses at the end of year one is Rs. 1000 and increases by Rs. 250 thereafter for four more years. If interest is 12%, how much should have you now to cover the expenses.

To apply linear gradient formula, first cash flow must occur at the end of 2nd period. G occurs at the end of period 2 through N periods.

In order to apply linear gradient formula, we can divide above cash flow into two cash flow series:

Annual cash flow with A
= Rs. 1000 + Gradient cash flow with G = 250

Cash flow	N	i%	PW Factor	PW (10%)
A = 1000	5	12	$\frac{(1+0.12)^5-1}{0.12*(1+0.12)^5}$ = 3.6048	1000 * 3. 6048 = 3605
G = 250	5	12	$\left[\frac{1}{0.12*(1+0.12)^5}\left\{\frac{(1+0.12)^5-1}{0.12}-5\right\}\right]$ = 6.397	250 * 6.397 = 1599
			∑ PW(10%) =	5204

PW(12%) = 1000 (P/A,12%,5) + 250 (P/G,12%,5)
= 3605 + 1599 = 5204

2.5.1.5 GEOMETRIC GRADIENT SERIES

If an amount $\mathbf{A_1}$ is invested or paid at the end of interest period 1 changes (increases or decreases) by a constant percentage **(g%)** at the end of each periods for N interest periods at interest rate i% per period,
if i ≠ g

Its present worth P would be

$$P = \frac{A_1}{(1+i)} + \frac{A_1(1+g)^1}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \frac{A_1(1+g)^3}{(1+i)^4} + \cdots \frac{A_1(1+g)^{N-2}}{(1+i)^{N-1}} + \frac{A_1(1+g)^{N-1}}{(1+i)^N} \ldots.(1)$$

Multiplying both side by $\frac{(1+g)}{(1+i)}$

$$P\frac{(1+g)}{(1+i)} = \frac{A_1(1+g)^1}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \frac{A_1(1+g)^3}{(1+i)^4} + \cdots + \frac{A_1(1+g)^{N-1}}{(1+i)^N} + \frac{A_1(1+g)^N}{(1+i)^{N+1}} \ldots.(2)$$

Subtracting (2) by (1)

$$\mathbf{P - P\frac{(1+g)}{(1+i)} = \frac{A_1}{(1+i)} - \frac{A_1(1+g)^N}{(1+i)^{N+1}}} \qquad \ldots.(3)$$

$$\mathbf{P = \frac{A_1}{(i-g)}\left[1 - \frac{(1+g)^N}{(1+i)^N}\right]}$$

P = $\mathbf{A_1}$ * (P/ $\mathbf{A_1}$, g, i, N)

The factor **(P/ $\mathbf{A_1}$, i, g, N)** is termed as **geometric gradient series present worth factor.**

if i = g $\qquad$ **P = N *** $\mathbf{\frac{A_1}{(1+i)}}$

Let $\qquad \mathbf{i_{CR} = i_{g'} = \frac{(1+i)}{(1+g)} - 1}$

$$\mathbf{1 + i_{CR} = \frac{(1+i)}{(1+g)}}$$

$$\mathbf{i_{CR} = i_{g'} = \frac{(1+i)}{(1+g)} - 1 = \frac{(i-g)}{(1+g)}}$$

$$\mathbf{(i - g) = i_{CR}(1 + g)}$$

i_{CR} denotes convenience rate.

$$\mathbf{P = \frac{A_1}{(i-g)}\left[1 - \frac{(1+g)^N}{(1+i)^N}\right]}$$

$$P = \frac{A_1}{i_{CR}(1+g)}\left[1 - \frac{1}{\frac{(1+i)^N}{(1+g)^N}}\right]$$

$$= \frac{A_1}{i_{CR}(1+g)}\left[1 - \frac{1}{(1+i_{CR})^N}\right]$$

$$= \frac{A_1}{(1+g)}\left[\frac{1}{i_{CR}}\,\frac{(1+i_{CR})^N - 1}{(1+i_{CR})^N}\right]$$

$$P = \frac{A_1}{(1+g)}\left[\frac{(1+i_{CR})^N - 1}{i_{CR}(1+i_{CR})^N}\right]$$

$$P = \frac{A_1}{(1+g)}\ (P/A, i_{CR}, N)$$

After finding P, we can find F or A or G as usual

$$F = P(1+i)^N$$

if i ≠ g $$F = \frac{A_1}{(i-g)}\left[1 - \frac{(1+g)^N}{(1+i)^N}\right] * (1+i)^N$$

$$F = \frac{A_1}{(i-g)}\left[(1+i)^N - (1+g)^N\right]$$

$$F = A_1\,(F/A_1, g, i, N)$$

if i = g $$F = N\frac{A_1}{(1+i)}(1+i)^N = NA_1(1+i)^{N-1}$$

The factor **$(F/A_1, i, N)$** is termed as **geometric gradient series compound amount factor.**

Example:

If fuel consumption at the end of year one is $2000 & increases at 8% per year thereafter for next three years. What is its equivalent PW & FW at i= 5%?

Cash flow	N	i%	g%	PW Factor $\frac{1}{(i-g)}\left[1 - \frac{(1+g)^N}{(1+i)^N}\right]$	PW (10%)
A_1= 2000	4	5	8	$\frac{1}{(0.05-0.08)}\left[1 - \frac{(1+0.08)^4}{(1+0.05i)^4}\right]$ = 3.9759	2000 * 3. 9759 = 7952

if i ≠ g

$$PW(12\%) = \frac{A_1}{(i-g)}\left[1 - \frac{(1+g)^N}{(1+i)^N}\right]$$

$$= \frac{2000}{(0.05-0.08)}\left[1 - \frac{(1+0.08)^4}{(1+0.05i)^4}\right] = 2000 * 3.9759 = 7952$$

Alternatively,

$$i_{CR} = i_{g'} = \frac{(1+i)}{(1+g)} - 1 = \frac{(i-g)}{(1+g)} = \frac{(0.05-0.08)}{(1+0.08)} = -0.027777777$$

$$P = \frac{A_1}{(1+g)}\ (P/A, i_{CR}, N) = \frac{2000}{(1+0.08)}\left[\frac{(1+i_{CR})^N - 1}{i_{CR}(1+i_{CR})^N}\right]$$

$$= 2000 * 3.9759 = 7952$$

In Practice, combination of one or more above series occurs because different costs and revenues follow different cash flow pattern.

2.5.1.6 IRREGULAR OR MIXED SERIES

Example:

A project consists of following cash flow:

- Initial investment (I) = Rs 10,000 (Single cash flow)
- Revenue (R) at the end of year one is Rs. 2000 and increases by Rs. 500 thereafter for four more years.
  (Annual cash flow with A = Rs. 2000
  + Linear Gradient cash flow with G = 500)
- If fuel consumption at the end of year one is Rs. 1000 & increases at 8% per year thereafter for next three years.
  (Geometric gradient g = 8%)
- Salvage Value (S) = Rs. 1500 (Single cash flow)
- Useful Life (N) = 10 years
- MARR = I = 12%,

What is the equivalent present worth?

$PW(12\%) = A_1\ (P/A,g\%,i\%,4) =$
$2000\ ((P/A,8\%,5\%,4)$

BASIC METHODOLOGIES OF ENGINEERING ECONOMIC ANALYSIS

3.1 DETERMINATION MINIMUM ATTRACTIVE RATE OF RETURN (MARR)

3.2 PAYBACK PERIOD METHOD

3.3 EQUIVALENT WORTH (EW) METHOD

3.4 RATE OF RETURN METHOD

3.5 PUBLIC SECTOR ECONOMIC ANLYSIS (BENEFIT COST RATION METHOD)

3.6 INTRODUCTION TO LIFECYCLE COSTING

3.7 INTRODUCTION TO FINANCIAL & ECONOMIC ANALYSIS

CHAPTER 3

3.1 DETERMINATION MINIMUM ATTRACTIVE OR ACCEPTABLE RATE OF RETURN (MARR)

MARR is determined by taking into numerous considerations. Among them are:

- The amount of money available for investment, and the source and cost of these funds (i.e. equity funds and borrowed funds).
- The number of good projects available for investment and their purpose (i.e., whether they sustain present operations and are essential, or expand on present operations and are elective).
- The amount of perceived risk that is associated with investment opportunities available to the firm, and the projected cost of administering projects over short planning horizons versus long planning horizons.
- The type of organization involved (i.e., government, public utility, or competitive industry).

3.2 PAYBACK PERIOD METHOD

Payback period is defined as the number of years required to recover the initial investment. It focus on liquidity i.e. how fast an initial investment can be recovered (easy recovery). It is not measure of profitability. It does not consider cash flows of entire life of project. i.e. ignores cash flow information after payback period.

1. SIMPLE PAYBACK PERIOD

Simple Payback Period is the payback period which ignores the time value of money. i.e. i = 0. It does not consider the time value of money.

A. EQUAL OR UNIFORM CASH FLOW

Simple Payback Period = $\frac{\text{Initial Investment}}{\text{Annual net cash flow}}$

If Calculated Payback Period < Standard Payback Period, Accept the project,

If Calculated Payback Period > Standard Payback Period,
Reject the Project

Example:

Initial Investment = 10,000. Annual cash inflow = 5,000. Annual cash outflow = 3,000.

Simple Payback Period = $\frac{\text{Initial Investment}}{\text{Annual net cash flow}}$

Simple Payback Period = $\frac{10{,}000}{5{,}000-3{,}000}$ = 5 years

Suppose required Standard Payback Period = 4 years.

Since Calculated Payback Period > Standard Payback Period,
Reject the Project.

B. UNEQUAL OR UNEVEN CASH FLOW

EOY	Net Cash Flow	Cumulated Net Cash Flow
0	-10,000	-10,000
1	+2,000	-8,000
2	+3,000	-5,000
3	+4,000	-1,000
4	+5,000	+4,000
5	+6,000	+10,000
6	+7,000	+17,000

Simple Payback Period = 3 + (1000 / 5000) = 3.2 Years
Suppose required Standard Payback Period = 4 years.
Since Calculated Simple Payback Period < Standard Payback Period,
Accept the project.

2. Discounted Payback Period

Simple Payback Period ignores the time value of money. i.e. i = 0. It does not consider the time value of money. To remedy this defect of simple payback period, time value of money is considered in the Discounted Payback period. Cash flows are discounted at certain MARR and determine the number of years required to recover the initial investment.

If MARR= i =10%, evaluate discounted payback period

EOY	Net Cash flow	PW Factor for i = 10%	PW of Cash flow at i = 10%	Cumulative PW Cash flow
0	-10,000	$(1+0.1)^{-0}$	-10,000	-10,000
1	+2,000	$(1+0.1)^{-1}$	+1,818	-8,182
2	+3,000	$(1+0.1)^{-2}$	+2,479	-5,703
3	+4,000	$(1+0.1)^{-3}$	+3,005	-2,698
4	+5,000	$(1+0.1)^{-4}$	+3,415	+0,717
5	+6,000	$(1+0.1)^{-5}$	+3,725	+4,443
6	+7,000	$(1+0.1)^{-6}$	+3,951	+8,394

Discounted Payback Period = 3 + (1000 / 5000) = 3.79 Years
Suppose required Standard Payback Period = 4 years.
Since Calculated Discounted Payback Period < Standard Payback Period,
Accept the project.

MERITS OF ADVANTAGES OF PAYBACK PERIOD

- simple to understand
- easy to calculate
- inexpensive to use
- focus on liquidity i.e. how fast an initial investment can be recovered (easy recovery)
- easy and crude way to tackle/cope with riskiness of investment.
- based on cash flow information

DEMERITS OF DISADVANTAGES OF PAYBACK PERIOD

- simple payback period ignores the time value of money. use discounted payback period to take into account the time value of money.
- does not consider cash flows of entire life of project. i.e. ignores cash flow information after payback period
- is not measure of profitability.
- no rational basis to set/determine a maximum/minimum acceptable standard payback period. It is generally, a subjective decision.
- fails to consider the pattern of cash flow. i.e. timing and magnitude

	Alternative		
EOY	A	B	C
0	-10,000	-10,000	-10,000
1	2,500	4,000	1,000
2	2,500	3,000	2,000
3	2,500	2,000	3,000
4	2,500	1,000	4,000
5	2,500	2,500	2,500
Simple Payback Period	4 Years	4 Years	4 Years
Discounted Payback Period at MARR = 10%	> 5 Years	4.08 Years	> 5 Years

Due to magnitude and timing pattern difference of cashflow discounted payback period is different although simple payback period is same regardless of cash flow pattern.

3.3 EQUIVALENT WORTH (EW) METHOD

Equivalent worth methods convert all cash flows into equivalent present, future, or annual amounts at the MARR. If a single project is under consideration,

If EW ≥ 0; Accept the project,
If EW < 0, Rejected the project.

3.3.1 FUTUTE WORTH (FW) METHOD

Future worth criterion has become popular because a primary objective of all time value of money is to maximize the future wealth of the owners of the firm. i.e. how much it worth at the end of given number of years.
FW methods convert all cash flows into equivalent future amounts at the MARR. All cash inflows and outflows are compounded forward to a reference point called the future, at the interest period rate MARR.

$$FW = P_0 (1 + i)^{N-0} + P_1 (1 + i)^{N-1} + P_2 (1 + i)^{N-2} + \ldots + P_k (1 + i)^{N-k} + \ldots + P_N (1 + i)^{N-N}$$

Where i = effective interest rate, k = future cash flow at the end of period, N = number of compounding period.
If a single project is under consideration,

If FW ≥ 0; Accept the project,
If FW < 0, Rejected the project

3.3.2 PRESENT WORTH (PW) METHOD

PW methods convert all cash flows into equivalent present amounts at the MARR. All cash inflows and outflows are discounted to the base or beginning point in time at the interest period rate MARR.
Present worth of project is a measure of how much fund will have to be put aside now to provide all future expenditures during the project period. It is assumed that such fund placed in reserve earns interest rate equal to MARR.

$$PW = F_0 (1 + i)^{0} + F_1 (1 + i)^{-1} + F_2 (1 + i)^{-2} + \ldots + F_k (1 + i)^{-k} + \ldots + F_N (1 + i)^{-N}$$

If a single project is under consideration,

If PW ≥ 0; The project is economically justified. Therefore, accept the project,
If PW < 0, The project is economically not justified. Therefore, reject the project.

Higher the interest rate and further the future cash flow occurs, lower is its PW.

3.3.3 ANNUAL WORTH (AW) METHOD

Annual worth of a project is uniform series of amount that is equivalent to the cash inflows and outflows occur during the project duration. AW methods convert all cash flows into equivalent annual amounts at the MARR. If a single project is under consideration,

If **AW ≥ 0**; Accept the project,
If AW < 0, Rejected the project.

Capital Recovery Cost (CR)

Capital Recovery Cost (CR) of a project is the equivalent uniform cost of the capital invested. It covers both depreciation and interest on invested capital (MARR). It can be calculate by either of the following formulas.

CR = I (A/P, i, N) - S (A/F, i, N)
CR = (I - S) (A/P, i, N) + S (i)
CR = (I - S) (A/F, i, N) + I (i)
Alternately,

AW = R - E - CR

where R = Annual equivalents receipts or savings,
E = annual equivalent expenses,
CR = Capital Recovery Cost.

CR = R - E
i.e. Capital to be recovered per year = net annual cash flow.
No gain/No loss.
CR < R - E
i.e. Capital to be recovered per year < net annual cash flow.
There is gain**.**
CR > R - E
i.e. Capital to be recovered per year > net annual cash flow.
There is loss.

Example:

The initial investment is Rs. 25,000, and salvage value is 10% of the initial investment at the end of its useful life 10 years. Annual revenue and expenses are Rs 14,000 and Rs. 10,000 respectively. Evaluate the investment proposal by EW (FW/PW/AW) methods. MARR = 10%.

Given:
I = Rs. 25,000. S = 10% * 25,000 = Rs. 2,500. i = 10%.
A.R. = Rs. 14,000. A.E. = Rs. 10,000. N = 10 years.

FW (i%) = - I * (F/P, i, N) + A * (F/A, i, N) + S
AW (i%) = - I * (A/P, i, N) + A + S * (A/F, i, N)
PW (i%) = - I + A * (P/A, i, N) + S * (P/F, i, N)
Calculate the required factors for N = 10 and MARR = 10%.
Present them in tabular form.

(F/P, i%, N) = $(1 + i)^N = (1 + 0.10)^{10}$ = 2.5937
(F/A, i%, N) = [(1/i) * {$(1 + i)^N$ - 1}] = [(1/i) * {F/P - 1}]
= [(1/0.10) * {2.5937 - 1}] = 15.9374
(P/A, i%, N) = [(1/i) * {F/P - 1}] / $(1 + i)^N$
= (F/A) / (F/P) = 15.9374/2.5937 = 6.1446

N	MARR	(F/P, i%, N)	(F/A, i%, N)	(P/A, i%, N)
10	10%	2.5937	15.9374	6.1446

FW (i%) = - I * (F/P, i, N) + A * (F/A, i, N) + S
FW (10%) = - 25000 * (F/P, 10%, 10) + (14,000 - 10,000) * (F/A, 10%, 10) + 2,500
FW (10%) = - 25,000 * 2.5937 + 4,000* 15.9374 + 2,500
FW (10%) = - 64,842.5 + 63,749.6 + 2,500
FW (10%) = + Rs.1,407.1

AW (i%) = - I * (A/P, i, N) + A + S * (A/F, i, N)
AW (10%) = - 25,000 * (A/P, 10%, 10) + (14,000 - 10,000) + 2,500 * (A/F, 10%, 10)
AW (10%) = - 25,000/6.1446 + 4,000 + 2,500/15.9374
AW (10%) = - 4068.61 + 4,000 + 156.86
AW (10%) = + Rs.88.25

PW (i%) = - I + A * (P/A, i, N) + S * (P/F, i, N)
PW (10%) = - 25,000 + (14,000 - 10,000)* (P/A, 10%, 10) + 2,500 * (P/F, 10%, 10)
PW (10%) = - 25,000 + 4,000 * 6.1446 + 2,500/2.5937
PW (10%) = - 25000 + 24578.4 + $963.87
PW (10%) = + Rs.542.27
Convert answer AW(10%) & PW(10%) to FW(10%) and check answer with calculated FW(10%)
FW(10%) = AW(10%) * (F/A, 10%, 10)
FW(10%) = 88.25 * 15.9374 = 1406.47

FW(10%) = PW(10%) * (F/P, 10%, 10)
FW(10%) = 542.27 * 2.5937 = 1406.48

3.4 RATE OF RETURN METHOD

If the return on investment is expressed in terms of rate of return or percentage, even common layman can readily understand. There would be little danger of misinterpreting rate of return figures because interest rate is well understood throughout the world. Rate of return should be at least equal to or greater than MARR to accept a proposed project, otherwise rejected. So, this method is widely used in practice.

3.4.1 INTERNAL RATE OF RETURN METHOD

In this method, that interest is found out that equates the equivalent worth of an all cash inflows (receipts or savings) and cash outflows (investment and expenditure). In other word IRR is the interest rate at which given cash flow becomes zero. IRR can be found out by any of EW (PW or FW or AW) Method.

If I = Initial Investment, S = Salvage Value, R = annual Revenue, and E = annual expenses N = Study Period and i = MARR and i*= IRR.

IRR using FW formulation:

FW of +ve cash flows (receipts) - FW of -ve cash flows (disbursement) = 0

[R (F/A, i*%, N) + S] - [I (F/P, i*%, N) + E (F/A, i*%, N)] = 0

Alternately, FW of disbursement = FW of receipts

I (F/P, i*%, N) + E (F/A, i*%, N) = R (F/A, i*%, N) + S

Or I (F/P, i*%, N) - S = R (F/A, i*%, N) - E (F/A, i*%, N)

I (F/P, i*%, N) - S = (R - E) (F/A, i*%, N)

IRR using PW formulation:

PW of cash inflows (receipts) - PW of cash outflows (disbursement) = 0

[R (P/A, i*%, N) + S (P/F, i*%, N)] - [I + E (P/A, i*%, N)] = 0

Alternately, PW of disbursement = PW of receipts

I + E (P/A, i*%, N) = R (P/A, i*%, N) + S (P/F, i*%, N)

Or I - S (P/F, i*%, N) = R (P/A, i*%, N) - E (P/A, i*%, N)

I - S (P/F, i*%, N) = (R - E) (P/A, i*%, N)

IRR using AW formulation:

AW of receipts - AW of disbursement = 0

[R + S (A/F, i*%, N)] - [I (A/P, i*%, N) + E] = 0

Alternately, AW of disbursement = AW of receipts

I (A/P, i*%, N) + E = R + S (A/F, i*%, N)

Or I (A/P, i*%, N) - S (A/F, i*%, N) = R - E

For any investment, IRR is positive only if sum of positive cash flows exceeds sum of negative cash flows. therefore, both cash inflows and cash outflows should be present in the cash flow pattern.
Once IRR is computed, it is compared with MARR.

If IRR ≥ MARR, Accept the project,

If IRR < MARR, Rejected the project

Example:

Evaluate IRR for the following proposal using the gradient formula.
Take MARR = 15%.

EOY	0	1	2	3	4	5
Cash inflow	0	500	500	500	500	500
Cash outflow	-1,000	100	140	180	220	260

Convert the above cash flow into suitable equivalent cash flow so that gradient formula can be used to solve the problem.

EOY	0	1	2	3	4	5
Cash inflow	0	400	400	400	400	400
Cash outflow	-1,000	0	40	80	120	160

N	MARR	F/P	F/A	F/G	FW
5	15%	2.0114	6.7424	11.6159	(+)221
5	25%	3.0518	8.2070	12.8281	(-)282
5	19.39%	2.4257	7.3528	12.1343	(+)30.05
5	19.80%	2.4677	7.4124	12.1838	(+)9.95
5	20.20%	2.5091	7.4709	12.2323	(-)10.05
5	20.066%	2,4952	7.4513	12.2160	(-)3.32
5	20%	2.4883	7.4416	12.2080	0.00

Let i* = IRR

FW(i*%) = - 1,000(F/P, i*%, 5) + 400(F/A, i*%, 5)- 40(F/G, i*%, 5)= 0

FW(15%) = - 1,000(F/P, 15%, 5) + 400(F/A, 15%, 5)- 40(F/G, 15%, 5)

= - 1,000 (2.0114) + 400 (6.7424) - 40 (11.6159)

= - 2011 + 2697 - 465 = +221

FW(25%) = - 1,000(F/P, 25%, 5)+400(F/A, 25%, 5) - 40 (F/G, 25%, 5)

= - 1,000 (3.0518) + 400 (8.2070) - 40 (12.8281)

= - 3052 + 3283 - 513 = - 282

By linear interpolation $(X-X_1)/ (X_2-X_1) = (Y-Y_1)/ (Y_2-Y_1)$

Or (X-15)/ (25-15) = (0-221)/ (-282-221)

IRR = i*% = X = 15 + (-221/-503) * (25 - 15) = 19.39%

Check whether at FW(19.39%) is equal to zero or not.
FW(19.39%) =
- 1,000 (F/P, 19.39%, 5) + 400 (F/A, 19.39%, 5) - 40 (F/G, 19.39%, 5)
= - 1,000 (2.4257) + 400 (7.3528) - 40 (12.1343)
= - 2425.72 + 2941.13 - 485.37 = + 30.05
FW(19.39%) ≠ 0.
This is due to linear interpretation assumption of non-linear phenomenon.
FW(19.39%) is positive. Try some more small increments.
FW(19.80%) =
- 1,000 (F/P, 19.80%, 5) + 400 (F/A, 19.80%, 5) - 40(F/G, 19.80%, 5)
= - 1,000 (2.4677) + 400 (7.4124) - 40 (12.1838)
= - 2467.6530 + 2964.956 - 487.351 = + 19.95
FW(20.2%) =
- 1,000 (F/P, 20.2%, 5) + 400 (F/A, 20.2%, 5) - 40(F/G, 20.2%, 5)
= - 1,000 (2.5091) + 400 (7.4709) - 40 (12.2323)
= -2509.1252 + 2988.367 - 489.29 = - 10.05
By linear interpolation $(X-X_1)/(X_2-X_1) = (Y-Y_1)/(Y_2-Y_1)$
Or (X - 19.8)/ (20.2 - 19.8) = (0 - 19.95)/(-10.05 - 19.95)
X = 19.8 + (-19.9513/-30.0002) * (20.2 - 19.8) = 20.066%
Check whether at FW(20.066%) is equal to zero or not.
FW(20.066%) =
- 1,000(F/P,20.066%,5) + 400(F/A,20.066%,5) -40 (F/G,20.066%,5)
= - 1,000 (2.4952) + 400 (7.4513) - 40 (12.2160) = -3.32 ≠ 0.
This is due to linear interpretation assumption of non-linear phenomenon. Try at FW(20.00%)
FW(20%)
= - 1,000 (F/P, 20%, 5) + 400 (F/A, 20%, 5) - 40 (F/G, 20%, 5)
= - 1,000 (2.4883) + 400 (7.4416) - 40 (12.2080)
= -2488.3200 + 2976.64 - 488.32 = 0.0000
Since, at FW(20%) = 0, Therefore, i*% = IRR = 20%.

3.4.1.1 UNRECOVERED INVESTMENT BALANCE (UIB)

IRR is that interest rate that causes unrecovered investment balance to exactly equal to zero at the end of study period (N). UIB at the beginning of the year shows how much of the original investment is still to be recovered as a function of time. Net annual cash flow (Receipts less Expenses) indicates annual returns or how much annually recovered. Interest on UIB at the beginning of the year represents profit on the beginning of year unrecovered investment balance or opportunity cost of interest.

IRR is rate of return calculated on the beginning-of-year unrecovered investment balance. IRR is not an annual average rate of return based on the initial investment or first cost.

EOY	UIB at the beginning of the year	Interest on (ii) @ IRR = 20%	Total UIB including interest	Net annual cash flow	UIB at the end of the year
	(i)	(ii)	(iii) = (i) + (ii)	(iv)	(v) = (iii) + (iv)
1	-1000	-200	-1200	+400	-800
2	-800	-160	-960	+360	-600
3	-600	-120	-720	+320	-400
4	-400	-80	-480	+280	-200
5	-200	-40	-240	+240	0

- **3.4.1.2 DRAWBACKS OF IRR**

- The IRR method assumes that the recovered funds, if not consumed at the end of the year, are reinvested at IRR rather than at MARR. Greater the IRR, at much higher rate of return, it may not be practically possible to reinvest net cash proceeds from the project within the firm.
- IRR may not be uniquely defined. There is possibility of multiple rate of return, in case of non-simple investment (i.e. cash flow stream of a project has more than one changes in sign). IRR = 25% & 400% for the following cashflow:

EOY	0	1	2
Casflow	-1,600	+10,000	-10,000

- When choosing among mutually exclusive projects, IRR may be misleading:

a. substantial different cash outlays

Project	0	1	IRR	PW(12%)
P	-10000	+20,000	100%	+7858
Q	-50,000	+75,000	50%	+16968

b. different project lives

Project	0	1	2	3	4	5	IRR	PW(12%)
P	-10000	+12,000	-	-	-	-	20%	+909
Q	-10,000	-	-	-	-	20114	15%	+2489

c. different timing of cashflows

Project	Cashflow				IRR	PW	PW	PW	PW
	0	1	2	3	%	5%	10%	20%	30%
P	-1680	1,400	700	140	23	409	276	-53	-125
Q	-1680	140	840	1510	17	520	276	-106	-388

Q is better at less than 10%, P is better at greater than 10%. Incremental analysis should be conducted (See Chapter 4).

3.4.2 EXTERNAL OR MODIFIED RATE OF RETURN METHOD

The ERR method takes into account the external reinvestment rate (Ɛ) at which net cash flows generated (or required) by the project over its life can be reinvested (or borrowed) outside the firm.

(∑ PW of negative net cash flows at Ɛ%) * $(1+ERR)^N$

= (∑ FW of positive net cash flows at Ɛ%).

$$(1+ERR)^N = \frac{\sum \text{FW of positive net cash flows at } \mathcal{E}\%}{\sum \text{PW of negative net cash flows at } \mathcal{E}\%}$$

Positive net cash flows = excess of receipts over expenses in period k.
Negative net cash flows = excess of expenses over receipts in period k.

Example:

Evaluate ERR for the following proposal using the gradient formula.
Take MARR = 15% & Ɛ = 16%.

EOY	0	1	2	3	4	5
Cash inflow	0	500	500	500	500	500
Cash outflow	1,000	100	140	180	220	260
Net Cash flow	-1,000	+400	+360	+320	+280	+240

(∑ PW of negative net cash flows at Ɛ%) * $(1+ERR)^N$

= (∑ FW of positive net cash flows at Ɛ%).

(1000) * $(1+ERR)^5$ = [400* (F/P, 16%, 4) + 360* (F/P, 16%, 3) + 320* (F/P, 16%, 2) + 280*(F/P, 16%, 1) + 240* (F/P, 16%, 0)

(1000) * $(1+ERR)^5$ = $(400 * 1.16^4 + 360 * 1.16^3 + 320 * 1.16^2 + 280 * 1.16^1 + 240 * 1.16^0)$

(1000) * $(1+ERR)^5$ = 724+562+431+325+240 =2282

$(1+ERR)^5$ = 2282/1000 = 2.282

ERR = $(2.282)^{(1/5)}$ - 1 = 1.1794 - 1 = 0.1794

ERR = 17.94% > (MARR = 15%)

Therefore, Accept the project.

In above problem, Evaluate ERR, if MARR = 20%.
Since ERR = 17.94% which is less than 20%,
Reject the project.

3.5 PUBLIC SECTOR ECONOMIC ANLYSIS (BENEFIT COST RATIO METHOD)

Private enterprises evaluate its activities in terms of profitability. Public sector evaluates public activities in terms of the general welfare of public. Therefore, the economic evaluation of public activities proceeds on the basic element that the purpose of government is to serve its citizens. Public projects are financed and operated by agencies. Sources of finance are generally the taxes collected from its citizens. Multipurpose projects are common. Effects of politics are frequent. Conflict of purposes and conflict of interests are quite common. Measurement of efficiency of public projects is very difficult. In many cases decisions are made by elected officials whose tenure of office is very uncertain. As a result, immediate cost and benefits may be stressed, to the detriment of long-range economy.

Basically, an engineering public projects have multiple benefits. Public projects are evaluated by equivalent worth of annual costs or by benefit cost ratio (BCR). The benefit cost ratio (BCR or B/C ratio) can be defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs. the equivalent worth utilized is customarily present worth (PW) or annual worth(AW), but it can also be future worth(FW). The BCR is also referred to as the savings-investment ratio (SIR).

If a single project is under consideration,

If BCR ≥ 1; Accept the project,

If BCR < 1, Reject the project.

Two commonly used BCR are:
Conventional BCR and Modified BCR.

BCR using AW formulation:
Conventional BCR = AW(B) / [AW (I) - AW(S) + AW(O&M)]
Modified BCR = [AW(B) - AW(O&M)]/ [AW (I) - AW(S)]

BCR using PW formulation:
Conventional BCR = PW(B) / [PW (I) - PW(S) + PW(O&M)]
Modified BCR = [PW(B) - PW(O&M)]/ [FW(I) - PW(S)]

BCR using AW formulation:
Conventional BCR = FW(B) / [FW (I) - FW(S) + FW(O&M)]
Modified BCR = [FW(B) - FW(O&M)]/ [FW (I) - FW(S)]

Example:

Evaluate BCR for the following proposal using the gradient formula.
Take MARR = 15% & SV = 10% * initial investment

EOY	0	1	2	3	4	5
Cash inflow	0	500	500	500	500	500
Cash outflow	1,000	100	140	180	220	260

N	MARR	F/P	F/A	P/A	F/G	A/G
5	15%	2.0114	6.7424	3.3521	11.6159	1.7228

Using Annual Worth Method

AW (I) = 1,000 (A/P, 15%, 5) = 1000 /3.3521 = 298.32

AW(S) = 10% * 1000 (A/F, 15%, 5) = 100 /6.7424 = 14.83

AW(B) = 500

AW(O&M) = 100 + 40 (A/G, 15%, 5) = 100 + 40 * 1.7228

= 100 + 68.91 = 168.91

Conventional BCR = AW(B) / [AW (I) - AW(S) + AW(O&M)]

= 500 / (298.32 – 14.83 + 168.91) = 500/452.40

= 1.1052 > 1 Therefore, Accept the project.

Modified BCR = [AW(B) - AW(O&M)]/ [AW (I) - AW(S)]

= (500 - 168.91) / (298.32 – 14.83)

= 1.1679 > 1 Therefore, Accept the project.

Using Annual Worth Method

FW (I) = 1,000 (F/P, 15%, 5) = 1000 * 2.0114 = 2011.40

FW(S) = 10% * 1000 = 100

FW(B) = 500 (F/A, 15%, 5) = 500 * 6.7424 = 3371.2

FW(O&M) = 100 (F/A, 15%, 5) + 40 (F/G, 15%, 5)100 * 6.7424 + 40 * 11.6159 = 674.24 + 464.64 = 1138.88

Conventional BCR = FW(B) / [FW (I) - FW(S) + FW(O&M)]

= 3371.2 / (2011.4 - 100 + 1138.88) = 3371.2/3050.28

= 1.1052 > 1 Therefore, Accept the project.

Modified BCR = [FW(B) - FW(O&M)]/ [FW (I) - FW(S)]

= (3371.2 - 1138.88) / (2011.40 - 100) = 2232.32 / 1911.40

= 1.1679 > 1 Therefore, Accept the project.

3.6 INTRODUCTION TO LIFE CYCLE COSTING

Life cycle cost is all costs, both non-recurring &recurring that occurs over the life cycle, related to a product, structure, system or service. It is applied to alternatives with cost estimates over the entire system life span. It means that costs from the very early stage of project (initiation) through final stage (phase-out & disposal) are estimate. To understand how life cycle cost analysis works, we must understand the phases & stages.

1. **Acquisition Stage/Phase:** Costs of all activities in planning stage prior to the construction & operation, delivery of product & service.
    - **Need Assessment Stage:** includes determination of user/ customer/ beneficiary needs/requirements, assessing them relative to the anticipated system, & preparation of the system requirements documentation.
    - **Preliminary Design Stage:** includes feasibility study, conceptual, & early stage plans, final go no go decision is probably made here.
    - **Detailed Design Stage:** includes construction or/& production planning, resource acquisition, detailed plans for resources – capital, human, facilities, information system, marketing etc.
2. **Construction & Operation Stage/Phase:** Costs of all activities need to execute or implement plan to function or real actual work takes place. Construction, production, delivery of end items or services & their operation & customer use occurs.
    - **Construction Stage:** includes costs for purchase, fabrication, erection, assembly, installation, construction, trial runs, testing, training, preparation, implementation of system etc.
    - **Operation & Usage Stage:** operating costs required for production, manufacture, use of system to generate product/service, to keep it going include personnel consumable supplies, overhead, maintenances, facilities & services.
    - **Phase-out, Termination, Disposal Stage:** covers cost for clear, transition to new-system, removal/ recycling of old system.

3.7 INTRODUCTION TO FINANCIAL & ECONOMIC ANALYSIS

Main objective of individual firm or a company in investing on project is to earn maximum possible returns for the investment. Promotes are solely interest in wealth maximization & tend to evaluate only commercial (financial) profitability of a project. Some projects that may not offer attractive financial profitability but such projects are undertaken since they have social implications. Such projects are public projects (e.g. road, bridge, irrigation, hydro-power projects etc.) for which socio-economic consideration play a significant part rather than financial profitability. Such project re analyzed for their socio-economic benefits (public welfare).

Financial Analysis	Economic Analysis
Objective: To determine financial feasibility i.e. whether someone is willing to pay for a project & capability to raise the necessary funds.	Objective: To determine if a project represents the best use of resources over the analysis period. i.e. project is justified socio-economic objectives.
Perspective: Evaluation is from the perspective of parties expected to pay their allocated costs	Perspective: Evaluation is from the perspective of many parties i.e. investors, beneficiaries, govt., environment, communities etc.
Cost & Benefits: Consider only controlled price (market price) fixed by government in monetary units.	Cost & Benefits: All tangible/ intangible, primary/secondary/ tertiary effects to society/economy as a whole is taken into consideration at shadow price (modified/adjusted market price to reflect real price/value.
Tax & Subsidy: Relevant	Tax & Subsidy: Not relevant
Inflation: Project income, capital & annual operation costs are estimated in inflated rupees or dollar considering inflation	Inflation: Project benefit, capital & annual operation costs are estimated in base year (constant) rupees or dollar without taking into considering inflation.

CHAPTER 4

COMPARATIVE ANALYSIS OF ALTERNATIVES

4.1 COMPARISON OF EXCLUSIVE ALTERNATIVES HAVING SAME USEFUL LIFE

4.1.1. PAYBACK PERIOD & EQUIVALENT WORTH (EW) METHOD

4.1.2 RATE OF RETURN METHOD & BENEFIT COST RATIO METHOD

4.2. COMPARISON OF EXCLUSIVE ALTERNATIVES HAVING DIFFERENT OR UNEQUAL USEFUL LIFE

4.2.1. REPEATABILITY ASSUMPTION

4.2.2. COTERMINATION ASSUMPTION

4.2.3. CAPITALIZED WORTH (CW) METHOD

4.3. COMPARING MUTUALLY EXCLUSIVE, CONTINGENT AND INDEPENDENT PROJECT IN COMBINATION

4.0 Comparative Analysis of Alternatives

Five basic methods discussed in Chapter 3 provide a basis for economic comparison of alternatives for an engineering project. The problem of deciding which mutually exclusive alternative should be selected is made easier if we adopt this rule based on Principle 2 in Chapter1: *The alternative that requires the minimum investment of capital and produces satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits.*

Purpose: to obtain at least MARR for each Rupee or Dollar invested.

4.1 Comparison of Exclusive Alternatives Having Same Useful Life

When the useful life of alternatives are equal to the selected study period adjustments to the cash flows are not required. Such alternatives are evaluated using payback period, equivalent worth method, rate of return method & BCR method .
First payback period & equivalent worth method is discussed & then rate of return & BCR method will be explained.

4.1.1. Payback Period & Equivalent Worth (EW) Method

Consider the following mutually exclusive alternatives, each having useful lives of 10years, the salvage values are 0. Which alternative should be chosen if required payback period = 5 years & required MARR = 10%

a) Simple Payback period

Investments considered	Alternatives (Rs.'000)					
	A	B	C	D	E	F
Initial investment *(I)*	900	1500	2500	4000	5000	7000
Annual Benefit *(B)*	250	376	500	1025	1225	1525
Annual O&M *(O&M)*	100	100	100	100	100	100

Simple Payback period = $\frac{Initial\ investment}{Net\ annual\ revenue}$

$=\frac{Initial\ investment}{Annual\ Benefit - Anuual\ O\&M}=\frac{I}{B - O\&M}$

Simple Payback period for Alternative A = $\frac{900}{250-100}$= 6years > Required payback period (5 years)
Similarly calculate the Simple payback for all other alternatives

	Alternatives (Rs.'000)					
	A	B	C	D	E	F
Simple Payback period	6 Years	5.4 Years	6.2 Years	4.3 Years	4.4 Years	4.9 Years
Is Alternative Acceptable	NO	NO	NO	YES	YES	YES

b. Equivalent Worth (EW) method

Calculate the Equivalent worth (EW) i.e. Present Worth (PW) or Future Worth (FW) or Annual Worth (AW) based on the total investment at i = MARR. Select the alternative having the greatest positive equivalent worth. In cost only alternatives, select alternative having the greatest negative equivalent worth.

PW(i%) = - I + (B – O&M)(P/A,i%,N)

PW(10%) of alternative A = - 900 + (250 – 100)(P/A,10%,10)

= 21.69 > 0. Hence, O.K.

Similarly, calculate PW(10%) for all the other remaining alternatives

	Alternatives (Rs.'000)					
	A	B	C	D	E	F
PW (10%)	22	196	-42	1684	1913	1756
Is Alternative acceptable	YES	YES	NO	YES	YES	YES

Order of Preference or Preference ranking → E > F > D > B > A

The sign " > " is read as preferred to. For example alternative E is preferred to alternative F.

Similarly we can calculate FW & select the best alternative.

	Alternatives (Rs.'000)					
	A	B	C	D	E	F
FW (10%)	56	508	-109	4367	4961	4555
Is Alternative acceptable	YES	YES	NO	YES	YES	YES

Similarly we can calculate AW & select the best alternative.

	Alternatives (Rs.'000)					
	A	B	C	D	E	F
AW(10%)	3.5	31.9	-6.8	274	311	286
Is Alternative acceptable	YES	YES	NO	YES	YES	YES

Order of preference is same for all three methods.

4.1.2 Rate of Return Method & Benefit Cost Ratio Method

For comparison of mutually exclusive alternatives using Rate of return method & Benefit Cost Ratio Method, apply incremental analysis procedure to avoid incorrect ranking.

Procedure for incremental analysis:

1) Calculate IRR or ERR or BCR for each alternative & screen out unfeasible alternatives from the analysis.
2) Arrange the feasible alternatives based on increasing initial investment.
3) Choose the feasible alternative having least initial investment as the base alternative.
4) Incremental analysis is performed between base alternative & alternative with the next higher initial investment. Analysis is aimed to check whether it is worthwhile to increase investment from base alternative to next higher initial investment. Calculate the incremental cash flow & calculate incremental IRR or ERR or BCR as the case may be. If incremental IRR≥ MARR or ERR≥MARR or BCR≥1, then increment of investment to next higher initial investment is justified. Otherwise, return to base alternative.
5) Repeat & select the best alternative

a) Using IRR method

Let us take again above problem.

Calculate the individual IRR with PW formulation for each alternative.

$$PW(i\%) = - I + (B – O\&M)(P/A,i\%,N) = 0$$

For alternative A,

$PW(i_A\%) = - 900 + (250-100)\ (P/A,i_A\%,10) = 0$

$IRR_A = i_A\% = 10.6\%$. Alternative C is not feasible, since required MARR is

	Alternatives					
	A	B	C	D	E	F
Initial Investment (I)	900	1500	2500	4000	5000	7000
Net Annual Benefit (B)	150	276	400	925	1125	1425
IRR (%)	10.6	13.0	9.6	19.1	18.3	15.6
Is Alternative acceptable	YES	YES	NO	YES	YES	YES

10%. All other alternatives are feasible.

Now perform incremental analysis:

Increment Considered	A	Δ (B – A)	Δ (D – B)	Δ (E – D)	Δ (F – E)
Δ initial investment	- 900	- 600	- 2500	- 1000	- 2000
Δ net annual revenue	150	126	649	200	300
Incremental IRR (%)	10.6	16.4	22.6	15.1	8.1
Is Alternative Acceptable	YES	YES	YES	YES	NO

Select the alternative E

Three errors commonly made in this type of analysis are:

i) Choose the mutually exclusive alternative with the highest overall IRR on total cashflow.

i.e. **Alternative D has IRR = 19.1% > MARR**

ii) Choose the mutually exclusive alternative with the highest IRR on an incremental initial investment.

i.e. **Δ (D – B) has IRR = 22.6% > MARR. Alternative D appears to be best.**

iii) Choose the mutually exclusive alternative with the largest initial investment that has IRR ≥ MARR.

i.e. **Alternative F has IRR = 15.61% > MARR**

It is already shown that alternative E is the best alternative.

- In above problem, if Salvage value is 10 % of initial investment, which alternative would you choose?

Calculate the individual IRR with PW formulation for each alternative.

PW(i%) = - I + (B – O&M)(P/A,i%,N) + S (P/F,i%,N) = 0

For alternative A,

PW(i_A%) = - 900 + (250-100)(P/A,i_A%,10) + 10%*900(P/F, i_A %,N)= 0

i_A = IRR_A = 11.38%

Since required MARR is 10%. All alternatives are feasible.

	Alternatives					
	A	B	C	D	E	F
Initial Investment (I)	900	1500	2500	4000	5000	7000
Net Annual Benefit (B)	150	276	400	925	1125	1425
Salvage Value (S)	90	150	250	400	500	700
IRR (%)	11.38	13.67	10.48	19.59	18.83	16.17
Is Alternative acceptable	YES	YES	YES	YES	YES	YES

Now perform incremental analysis:

Increment Considered	A	**Δ (B – A)**	**Δ (C – B)**	**Δ (D – B)**	**Δ (E – D)**	**Δ (F – E)**
Δ Initial Investment	- 900	- 600	- 1000	- 2500	- 1000	- 2000
Δ Net Annual Benefit	150	126	124	649	200	300
Δ Salvage Value (S)	90	60	100	250	100	200
Incremental IRR (%)	11.38	16.97	5.36	22.96	15.7	9.1
Is Alternative acceptable	YES	YES	NO	YES	YES	NO

Select the alternative E

b) Using BCR method

Let us take again above problem.

Calculate the individual Modified BCR with PW formulation for each alternative.

Modified BCR = $\frac{AW(B) - AW(O\&M)}{AW(I) - AW\ (S)}$

For alternative A,

AW(I) = 900 (A/P, 10%,10) = 146.47

Modified BCR = $\frac{AW(B) - AW(O\&M)}{AW(I) - AW\ (S)} = \frac{250 - 100}{146.47 - 0}$ = 1.02 > 1 .

Similarly, calculate the Modified BCR for all other remaining alternatives.

Investments Considered	Alternatives (Rs.'000)					
	A	B	C	D	E	F
Initial investment *(I)*	900	1500	2500	4000	5000	7000
AW (10%) of I	146	244	407	651	814	1139
AW (10%) of B	250	376	500	1025	1225	1525
AW (10%) of O&M	100	100	100	100	100	100
Modified BCR	1.02	1.13	0.98	1.42	1.38	1.25
Is Alternative acceptable	YES	YES	NO	YES	YES	YES

Alternative C is not feasible, since BCR < 1.

All other alternatives are feasible. Now perform incremental analysis:

Select the alternative E

Three errors commonly made in this type of analysis are:

Increment Considered	A	Δ (B – A)	Δ (D – B)	Δ (E – D)	Δ (F – E)
Δ initial investment	- 900	- 600	- 2500	- 1000	- 2000
Δ net annual revenue	150	126	649	200	300
Modified BCR	1.02	1.29	1.60	1.23	0.92
Is Alternative acceptable	YES	YES	YES	YES	NO

i) Choose the mutually exclusive alternative with the highest BCR on total cashflow. i.e. **Alternative D has BCR = 1.42 > 1**

ii) Choose the mutually exclusive alternative with the highest BCR on an incremental initial investment.

i.e. **Δ (D – B) has IRR = 1.60 > 1.**

Alternative D appears to be best.

iii) Choose the mutually exclusive alternative with the largest initial investment that has BCR ≥ 1. i.e. **Alternative F has IRR = 1.25 > 1.**

It is already shown that alternative E is the best alternative.

Both IRR & BCR method gives same result.

c) Using ERR method

Let us take again above problem using **MARR =10%** and external reinvestment rate **Ɛ = 10%.**

First of all, convert given cash flow to net cash flow.

Calculate the individual ERR for each alternative.

I (F/P, ERR%,N) = (B – O&M)(F/A, Ɛ %,N)

For alternative A,

Net annual benefit = (B – O&M) = (250-100) = 150

900 (F/P, ERR%,10) = 150 (F/A, 10 %,10)

ERR_A = 12.96%% ~ 13% > MARR. Similarly, calculate other ERR.

	Alternatives					
	A	**B**	**C**	**D**	**E**	**F**
Initial investment (I)	900	1500	2500	4000	5000	7000
Net Annual Benefit (B)	150	276	400	925	1125	1425
ERR (%)	10.3	11.3	5.8	13.9	13.6	12.5
Is Alternative acceptable	YES	YES	No	YES	YES	YES

Alternatives 'C' is not feasible, since required MARR is 10%. All other alternatives are feasible. Now perform incremental analysis:

Increment Considered	**A**	**Δ (B – A)**	**Δ (B – D)**	**Δ (E – D)**	Δ (F – E)
Δ initial investment	-900	-600	- 2500	- 1000	- 2000
Δ net annual revenue	150	126	649	200	300
Incremental ERR	10.26%	12.84%	15.25%	12.29%	9.10%
Is Alternative acceptable	YES	YES	YES	YES	NO

Select the alternative E

Three errors commonly made in this type of analysis are:

i) Choose the mutually exclusive alternative with the highest ERR on total cashflow. i.e. **Alternative D has ERR = 13.9% > MARR**

ii) Choose the mutually exclusive alternative with the highest ERR on an incremental initial investment.

i.e. **Δ (D – B) has ERR = 15.25% > MARR. Alternative D appears to be best.**

iii) Choose the mutually exclusive alternative with the largest initial investment that has ERR ≥ MARR.

i.e. **Alternative F has ERR = 12.5% > MARR.**

It is already shown that alternative E is the best alternative.

ERR, IRR & BCR method gives same result.

Let us take again above problem
using **MARR = Ɛ = 15%**

Calculate the individual ERR for each alternative.

I (F/P, ERR%,N) = (B – O&M)(F/A, Ɛ %,N)

For alternative A,

Net annual benefit = (B – O&M) = (250-100) = 150

900 (F/P, ERR%,10) = 150 (F/A, 15 %,10)

ERR_A = 12.96% ~ 13% < MARR. Hence, not feasible.

Similarly, calculate ERR for other alternatives.

	Alternatives					
	A	B	C	D	E	F
ERR (%)	13	14	12	17	16	15
Is Alternative acceptable	NO	NO	NO	YES	YES	YES

Alternatives A, B & C is not feasible, since required MARR is 15%.
All other alternatives are feasible.
Now perform incremental analysis:

Increment Considered	D	Δ (E – D)	Δ (F – E)
Δ initial Investment	- 4000	- 1000	- 2000
Δ net annual Revenue	925	200	300
Incremental ERR	17%	15%	11%
Is Alternative Acceptable	YES	YES	NO

It is not worthwhile to increase investment to alternative F, therefore, select the alternative E.

4.2. Comparison of Exclusive alternatives having Different or Unequal Useful Life

Different mutually exclusive alternatives have different useful lives. Further required analysis period do not match with the lives of alternatives. In such cases, following method are used to solve problem:

i) Repeatability Assumption
ii) Coterminated Assumption
iii) Capitalized Worth Method

4.2.1 Repeatability Assumption

a) The analysis period over which alternatives are being compared is either indefinitely long or equal to common multiple of the lives of the alternatives,
b) The economic consequences that are estimated to happen in an alternatives life span will happen in all succeeding life spans (identical replacement).

Example:

Select the best project using equivalent worth methods. MARR = 10%. Market value at the end of useful life of each project is 0. Use repeatability assumption.

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1020
Useful life	4 years	6 years

Least common multiple (LCM) of the useful lives of project A & project B = 12 years.
Identical replacement of project A at the end of year 4 and year 8 occurs

A1	A2	A3	
4	8	12	years

(i.e. project A is repeated 3 times).
Identical replacement of project B at the end of year 6 occurs

B1	B2	
6	12	years

(i.e. project B is repeated 2 times).

Equivalent Worth Method

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1020
Useful life	4 years	6 years

a) **FW Method**

FW of Project A(10%)

= - 3500 (F/P,10%,12) - 3500 (F/P,10%,8) - 3500 (F/P,10%,4)

+ (1900 - 645) (F/A,10%,12)

= - 23611 + 26837 = 3226

FW of Project B(10%)

= - 5000 (F/P,10%,12) - 5000 (F/P,10%,6)

+ (2500 - 1020) (F/A,10%,12)

= 7099.6

FW of Project B(10%) > FW of Project A(10%)

Therefore, Select the Project B

b) PW Method

PW of Project A(10%)

= - 3500 - 3500 [(P/F,10%,4) +(P/F,10%,8)]

+ (1900 - 645) (P/A,10%,12)

= 1028

PW of Project A(10%)

= - 5000 - 5000 (P/F,10%,6)

+ (2500 - 1020) (P/A,10%,12)

= 2262

PW of Project B(10%) > PW of Project A(10%)

Therefore, Select the Project B

c) AW Method

AW of Project A (10%)

= - 3500 (A/P,10%,4) + (1900 - 645) = 151

AW of Project B (10%)

= - 5000 (A/P,10%,6)+ (2500 - 1020) = 332

AW of Project B(10%) > AW of Project A(10%)

Therefore, Select the Project B.

Future Worth, Present Worth & Annual Worth method, All of them lead to same result i.e. selection of Project B.

Rate of Return Method

Example:

IRR Method

Select the best project using IRR method if MARR = 10%.

Market value at the end of useful life of each project is 0.

Use repeatability assumption.

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1383
Useful life	4 years	8 years

LCM of the useful lives of projects A & B = 12 years.

Identical replacement of project A at the end of year 4 and year 8 occurs

A1	A2	A3	
4	8	12	Years

(i.e. project A is repeated 3 times).

Identical replacement of project B at the end of year 6 occurs

B1	B2	
6	12	Years

(i.e. project B is repeated 2 times).

PW of Project A (i*%) = 0

PW of Project A (i_A*%)

= - 3500 + (1900-645) (P/A, i_A*%,8) - 3500(P/F, i_A*%,4)= 0

IRR of Project A = i_A* = 16.2% > MARR

PW of Project B (i*%) = 0

PW of Project B (i*%)

= - 5000 + (2500-1383) (P/A,i*%,8) = 0

IRR of Project B = i_B* = 15.1% > MARR. Both Projects A & B are acceptable. Now, perform incremental analysis.

	Project Δ (B – A)
Δ Initial Investment	**-1500**
Δ Investment at the end of Year 4	**+3500***
Δ Annual Benefit	**+600**
Δ Annual O&M	**-738**

* When Initial Investment increases from project A to project B, this cost is avoided, which is gain.

PW of Incremental cash flow ($i_{(B-A)}$*%) = 0

PW of incremental cash flow ($i_{(B-A)}$*%) = - 1500 + (600-738) (P/A, (i_{B-A}*%,8) 3500(P/F, i_A *%,4)= 0

IRR of incremental cash flow = i_{B-A}* = 12.7% > MARR

Hence increment of investment from Project A to Project B is worthwhile. Therefore select Project B.

4.2.2 Co-termination Assumption

The repeatability assumption has limited use in engineering practice, because actual situation seldom meet both condition assumed in repeatability method. Co-terminated assumption involves the use of finite analysis period for all feasible alternatives. This is the approach most frequently used in engineering practice. Often, one or more of the useful lives will be shorter or longer than the selected study period. When this is the case, cash flow adjustments based on additional assumptions need to be used so all the alternatives are compared over the same study period. The following guidelines apply to this situation:

1. **Required Study period < Useful life:** The most common technique is to truncate the alternative at the end of the study period using an estimated market value. This assumes that the disposable assets will be sold at the end of the study period at that value. Market value is determined using following formula:
   Market Value of truncated Project at the end of n years
   = PW of remaining capital recovery cost at the end of n years
   + PW of salvage value at the end of useful life (N) at the end of n^{th} years.
2. **Required Study period > Useful life**

a. Cost alternatives: Another potential course of action is to repeat part of the useful life of the original, and then use an estimated value to truncate it at the end of the study period.

Because each cost alternative has to provide the same level of service over the study period, contracting for the service or leasing the needed equipment for the remaining years may be appropriate.

b. Investment alternative: the assumption used is that all cash flows will be reinvested in other opportunities available to the firm at the MARR to the end of the study period. A convenient method is to calculate the FW of each mutually exclusive alternative at the end of the study period. The PW can also be used for investment alternatives since the FW at the end of the study period, say N, of each alternative is its PW times a common constant (F/P,i%,N), where i% = MARR. However, often it is not possible practically reinvest at MARR for remaining years to the end of the study period. So though calculation is easier, but limitation is that it is not practically useful or reinvested at external reinvestment rate i.e. prevailing market interest rate **(Ɛ%).** This is more realistic than reinvesting at MARR.

1. Required Study period < Useful life:

The most common technique is to truncate the alternative at the end of the study period using an estimated market value. This assumes that the disposable assets will be sold at the end of the study period at that value.

Example:

Select the best project using PW methods if MARR = 10%.
Market value at the end of useful life of each project is zero.
Use co-terminated assumption. Required study period is 4 Years.

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1383
Useful life	4years	8 years

In this case, Co-terminate at the end of 4 years.
Required study period < Project B's Useful life
PW of Project A (10%)

= - 3500 + (1900-645) (P/A,10%,4) = 478

Market Value of truncated Project at the end of n years

= PW of remaining capital recovery cost at the end of n years
+ PW of salvage value at the end of useful life (N) at the end of n^{th} years.

Market value of Project B at the end of 4years

= [5000(A/P,10%,8)] (P/A,10%,4) = 2971

PW of Project B (10%)

= - 5000 + (2500-1383) (P/A,10%,4) + 2971 (P/F,10%,4)= 570

Both Projects A & B are acceptable.
PW of Project B (10%) > PW of Project A (10%)
Select Project B

Solve above problem, using EW (PW, FW, AW) method assuming repeatability hold true and if required study period required is
a) 2 years, b) 6 years, c) 10 years with.

Solve the above problem, using rate of return method (IRR, ERR) method & BCR method assuming the repeatability hold true and co-terminate at the end of a) 3 years, b) 5 years, c) 12 years.

Example:

2. Study period > Useful life

Select the best project using IRR methods (PW formulation).
Market value at the end of useful life of each project is zero.
Use co-terminated at the end of 8 Years. MARR = 10%.

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1383
Useful life	4 years	8 years

In this case, Required study period > Useful life for Project A
Assuming as in repeatability assumption, identical replacement of project A:

A1	A2
4	8 years

PW of Project A (i^*%) = 0

PW of Project A (i_A *%) = - 3500 - 3500(P/F, i_A *%,4)
+ (1900-645) (P/A, i_A *%,8) = 0

IRR of Project A = i_A^* = 16.2% > MARR

PW of Project B (i^*%) = 0

PW of Project B (i^*%) = - 5000 + (2500-1383) (P/A,i^*%,8) = 0

IRR of Project B = i_B^* = 15.1% > MARR. Both Projects A & B are acceptable. Now, perform incremental analysis.

	Project **Δ** (B – A)
Δ Initial Investment	-1500
Δ Investment at the end of Year 4	+3500*
Δ Annual Benefit	+600
Δ Annual O&M	-738

* If shift to project B, this cost is avoided, which is gain.

PW of incremental cash flow = PW of ($i_{\Delta(B-A)}$%) = 0

PW of ($i_{\Delta(B-A)}$%)

= - 1500 + (600-738) (P/A, ($i_{\Delta(B-A)}$%,8) 3500(P/F, $i_{\Delta(B-A)}$ %,4)= 0

IRR of incremental cash flow = $i_{\Delta(B-A)}^*$ = 12.7% > MARR

Hence increment of investment from Project A to Project B is worthwhile. Therefore select Project B.
Therefore, co-terminating at the end of 8 years is same as repeatability method (LCM=8 YEARS)

Example:

Select the best project using equivalent worth methods. MARR = 10%.
Market value at the end of useful life of each project is 0.
Use co-terminated assumption. Required study period is 8 Years.

	Project A	Project B
Initial Investment	3500	5000
Annual Benefit	1900	2500
Annual O&M	645	1383
Useful life	4 years	8 years

In this case, Required Study period > Useful life.

a. Assuming as in repeatability assumption, identical replacement of project A:

A1	A2

4 8 years

PW of Project A (10%)
= - 3500 - 3500 (P/F, 10%,4) + (1900-645) (P/A,10%,8)
= - 3500 - 2390.6 + 6695.3 = 804.7

Project B has useful life of 8 years, so no adjustment is necessary.

B1

8 years

PW of Project B (10%)
= - 5000 + (2500-1383) (P/A,10%,8)
= - 5000 + 5959 = 959

PW of Project B(10%) > PW of Project A(10%)
Therefore, Select the Project B

b. Calculate the FW of each alternative at the end of its own useful life & reinvest at MARR to the end of the study period.

FW of Project A (10%)
= [- 3500 (F/P,10%,4) + (1900-645) (F/A,10%,4)] (F/P,10%,4)
= (-5124.35+ 5824.455)*4.641 = 3249

FW of Project B (10%)
= - 5000 (F/P,10%,8)+ (2500-1383) (F/A,10%,8)
= - 10718 + 15816 = 5098

FW of Project B(10%) > FW of Project A(10%)
Therefore, Select the Project B.

c) Calculate the FW of each alternative at the end of its own useful life & reinvest at external reinvest rate i.e. prevailing market rate (**Ɛ %**) to the end of the study period.

FW of Project A (10%)

Example:

d) Contracting for the service or leasing the needed equipment for the remaining years.

	Model	
	A	B
First Cost	\$184,000	\$242,000
Annual Expenses	\$30,000	\$26,700
Useful Life	5 years	7 years
Salvage Value at the of useful life	\$17,000	\$21,000
MARR	15%	
Required Study Period	8 years	
Cost of lease per year for remaining years after useful life	\$104,000 3 years	\$134,000 1 year

Each model will provide same level of service.

Evaluate using ERR if external reinvestment rate **Ɛ = 15%**.

EOY	Model		Incremental Cash flow
	A	B	Δ (B-A)
0	-184,000	-242,000	-5,800
1	-30,000	-26,700	+3,300
2	-30,000	-26,700	+3,300
3	-30,000	-26,700	+3,300
4	-30,000	-26,700	+3,300
5	-30,000 +17000	-26,700	-13,700
6	-104,000	-26,700	+77,300
7	-104,000	-26,700 +2100	+98,300
8	-104,000	-134,000	30,000

Calculate Coefficient / factors when i = 15%

N	1	2	4	5	8
F/P, 15%,N	1.1500	1.3225	1.7490	2.0114	3.0590
F/A, 15%,N			4.9934		

5,800+13,700(P/F, 15%,5)+30,000(P/F, 15%,8)}(1+**Δ** ERR)^8

={3300(F/A, 15%,4)}(F/P, 15%,4)

+77,300(F/P, 15%,2)+98,300(F/P, 15%,1)

{5,800+13,700/2.0114+30,000/3.0590}(1+**Δ** ERR)^8

={3300(4.9934)}(1.7490)+77,300(1.3225)+98,300(1.1500)

74,619(1+**Δ** ERR)^8=242,098

Δ ERR = 15.98%> (MARR=15%)

Hence increment to Model B is justified. Select B.

4.3 Capitalized Worth (CW) Method

CW is the present worth (PW) of all receipts &/or expenses over an infinite length of time. If only expenses are considered, it is called Capitalized Cost (CC). This method is used for comparing mutually exclusive alternatives when study period of needed service is indefinitely long or when the common multiples of the lives is very long & repeatability assumption is applicable. CW (or CC) is calculated in the same way as in a present worth (PW), where N equals infinity.

We know that $(P/A,i\%,N) = \frac{(1+i)^N - 1}{i(1+i)^N}$

The limit of (P/A,i%,N) as N approaches infinity is

$(P/A,i\%,\infty) = \frac{1}{i}$

If AW be the Annual Worth of an investment

$CW(i\%) = AW\ (P/A,i\%,\infty)$

$CW(i\%) = \frac{AW(i\%)}{i}$

Practical definition (approximation) of infinity/forever is dependent on interest rate & study period (N)

Interest rate (i)	Study Period (N)	(P/A,i%,N)	(P/A,i%,∞) $= \frac{1}{i}$
8%	130	12.49943	12.5
10%	100	9.99927	10
15%	80	6.66657	6.67
20%	50	4.99945	5
25%	35	3.99638	4
50%	20	1.99398	2

Example:

Consider the following mutually exclusive alternatives. Which alternative should be chosen if chosen alternative is required for infinite period? Take MARR = 10%.

Investments considered	Alternatives (Rs.'000)	
	A	B
Initial investment *(I)*	\$15,000	\$30,000
Annual Expenses *(E)*	\$2,000	\$1,000
Salvage Value *(S)*	0	\$3,000
Useful life *(N)*	10 Years	60 Years

AW(i%) = I (A/P,i%,N) + A + S(A/F,i%,N)

Calculate Coefficient / factors when i = 10%

N	F/A, 10%,N	P/A, 10%,N
10		6.1446
60	3034.8163	9.9672

AW(10%) = I (A/P,10%,N) + A + S(A/F,10%,N)

AW_A(10%) = 15,000 (A/P,10%,10) + 2,000 + 0(A/F,10%,10)

= 15,000/6.1446 + 2,000 + 0 = \$4441.17

AW_B(10%) = 30,000 (A/P,10%,60) + 1,000 + 3,000(A/F,10%,60)

= 30,000/9.9672 + 1,000 +3,000/3034.8163 =\$4010.87

CC (i%) = $\frac{\text{AW(i\%)}}{i}$

CC_A (10%) = $\frac{4441.17}{0.1}$ = \$44411.7

CC_B (10%)= $\frac{4010.87}{0.10}$ = \$40,108.7

Since CC_A (10%) > CC_B (10%), reject alternative A & select alternative B.

Example:

How much should you deposit now so that he can with draw Rs. 3,000 per month forever plus Rs 50,000 in every five year for infinite times if interest rate is 12% annually.

$$\mathbf{i} = \left(\mathbf{1} + \frac{r}{M}\right)^{M} - \mathbf{1}$$

Interest per period (r/M) = $(1+i)^{1/M}$ - 1

Interest per month

= $(1+0.12)^{1/12}$ – 1 = 1.009488793 – 1 = 0.009488793

Interest per each 5 year

= $(1+0.12)^5$ – 1 = 1.75241683 – 1 = 0.75241683

CW = $\frac{A}{i}$ = $\frac{3,000}{0.009488793} + \frac{50,000}{0.752416830}$ = 316,162 + 66,459 = 382,621

4.4. Comparing Mutually Exclusive, Contingent and Independent Project in Combination

In practical life, some projects are mutually exclusive, while some projects are independent of each other & some projects are contingent/dependent on other or even combination of all three categories of projects. So far we dealt only mutually exclusive projects only.

Three major categories of projects (investment opportunities) are:

Mutually exclusive: at most only one best project among feasible projects will be chosen.

Independent: choice of a project independent of the choice of any other project in the group. So any number of projects or all or none of the projects may be selected.

Contingent: The choice of a project is conditional on the choice of one or more other projects.

Example:

Some of the projects are mutually exclusive. Projects A1 & A2 are independent of B1 & B2. A1 & A2 are mutually exclusive and independent of set B. B1 & B2 are mutually exclusive and dependent (contingent) on the acceptance of A2. Project C is contingent on the acceptance B1.

Using the PW method, determine what combination of projects is best if the capital to be invested is

a) unlimited, and b) limited to $ 48,000.

Each project has useful life of 4 years. Take MARR = 10% per year.

	Projects ($)				
	A1	A2	B1	B2	C
First Cost	- 50,000	- 30,000	- 14,000	- 15,000	-10,000
Annual Benefit	20,000	12,000	4,000	5,000	6,000

PW(i%) = - I + A (P/A,i%,N)

PW(10%) = - I + A (P/A,10%,4)

PW(10%)of Project A1 = - 50,000 + 20,000 (P/A,10%,4) = 13,400

Similarly, calculate the PW of other remaining projects.

Projects	First Cost	Annual Benefit	PW(10%)
A1	-50,000	20,000	13,400
A2	-30,000	12,000	8,000
B1	-14,000	4,000	-1,300
B2	-15,000	5,000	900
C	-10,000	6,000	9,000

Mutually Exclusive Combination	First Cost	Annual Benefit	PW (10%)
None	0	0	0
A1	-50,000	20,000	13,400
A2	-30,000	12,000	8,000
A2, B1	-44,000	16,000	6,700
A2, B2	-45,000	17,000	8,900
A2, B1, C	-54,000	22,000	15,700

a) If capital is unlimited,
then combination **A2, B1, C** is best
with Highest PW of **$ 15,700**.

b) If capital is limited to $ 48,000,
then combination A1 & combination A2, B1, C
are not feasible due to insufficient first cost.
Out of remaining 3 combinations,
combination **A2 , B2** is best
with Highest PW of **$ 8,900**.

Example:

Given the following independent projects, determine which should be chosen using the AW method. MARR = 10%, and there is no limitation of fund availability. Each project has useful life of 5 years.

Project	First Cost	Net Annual Cash flow	Salvage Value
X	10,000	2300	10,000
Y	12,000	2800	0
Z	15,000	4067	0

AW(10%) = - I (A/P,i%,N) + A + S (A/F,i%,N)
AW(10%) of Project X
= - 10,000 (A/P,10%,5) + 2300 + 10,000 (A/F,10%,5) = 1300
AW(10%) of Project X
= - 12,000 (A/P,10%,5) + 2800 + 0 (A/F,10%,5) = - 366
AW(10%) of Project X
= - 15,000 (A/P,10%,5) + 4067 + 0 (A/F,10%,5) = 110
Select the projects X & Z.

CHAPTER 5

REPLACEMENT ANALYSIS

5.1 BASIC CONCEPT & TERMINOLOGY

To facilitate the discussion of the principles involved in replacement analysis, it is necessary to introduce two important terms commonly used by practitioners involved in replacement analysis.

Defender is an existing asset being considered for replacement.

Challenger is the asset proposed to be the replacement.

The challenger, being new asset, may have high capital cost, low operating cost. The defender, being old existing asset, may have high operation and maintenance cost, physical impairment, obsolete.

When to replace the old one (defender) by new one (challenger)?

Shall we replace the defender now or keep it for one or more years before replacing it?

Sunk Cost is a past cost that cannot be altered by future action and is therefore irrelevant. Sunk costs are money that is gone, and no present action can recover them. They represent past actions - the results of decisions make in the past.

Reasons for replacement:

- obsolescence - due to new technology
- depletion - gradual loss of market value
- deterioration due to ageing - more maintenance and operating cost
- physical impairment
- Inadequacy
- rapid technological change

5.1.2 Approaches for comparing defender and challenger

There are basically two approaches for comparing defender and challenger.

- **Cash flow Approach**
  Proceeding from sale of the old machine is treated as down payment toward purchasing the new machine.Net present worth and annual worth method is used for the comparison
- **Opportunity Cost Approach**
  If you decide to keep the old machine, this potential sales receipt is foregone. The opportunity cost approach views the net proceeding from sale of the old machine is treated as the investment required to keep the old machine. This approach is more commonly practiced in replacement analysis

Example:

Machine A was purchased 2 years ago for \$ 20,000. Its market value now is \$ 10,000. It was estimated to have a life of 5 years and a salvage value of \$ 2,500 at the end of its life. Its operating expenses have been found to be \$ 8,000 per year.

Machine B costs \$ 15,000. Its estimated life is 3 years and its salvage value at the end of its life at \$ 5,500. Operating costs are estimated at \$ 6,000 per year. Suppose that the firm need either machine (old or new) for only three years. Decide whether replacement is justified now. Take MARR = 12%.

a) **Cash flow Approach**

Proceeding from sale of the old machine is treated as down payment toward purchasing the new machine.

AW Method

$AW_{defender}(12\%)$

$= -\$\ 8,000 + \$\ 2,500\ (A/F,12\%,3) = -\$\ 7,259.10$

$AW_{challenger}(12\%)$

$= (\$10,000 - \$\ 15,000) * (A/P,12\%,3) - 6,000$
$+\ 5,500\ (A/F,12\%,3) = -\$\ 6,451.79$

$AW_{challenger}(12\%) > AW_{defender}(12\%)$

Therefore, Replace the defender now.

PW Method

$PW_{defender}(12\%)$

= - \$ 8,000 (P/A,12%,3) + \$ 2,500 (P/F,12%,3) = - \$ 17,434.90

$PW_{challenger}$ (12%)

= (\$10,000 - \$ 15,000) - 6,000 (P/A,12%,3)

+ 5,500 (P/F,12%,3) = - \$ 15,495.30

$PW_{challenger}(12\%) > PW_{defender}(12\%)$

Therefore, Replace the defender now.

b) Opportunity Cost Approach

The net proceeding from sale of the old machine is treated as the investment required to keep the old machine.

AW Method

$AW_{defender}(12\%)$

= - \$ 10,000 (A/P,12%,3) - \$ 8,000 + \$ 2,500 (A/F,12%,3)

= - \$ 11,422.64

$AW_{challenger}(12\%)$

= - \$ 15,000 (A/P,12%,3) - \$ 6,000 + \$ 5,500 (A/F,12%,3)

= - \$ 10,615.33

$AW_{challenger}(12\%) > AW_{defender}(12\%)$

Therefore, Replace the defender now.

PW Method

$PW_{defender}(12\%)$

= - \$ 10,000 - \$ 8,000 (P/A,12%,3) + \$ 2,500 (P/F,12%,3) = - \$ 27,434.90

$PW_{challenger}$ (12%)

= - \$ 15,000 - \$ 6,000 (P/A,12%,3) + \$ 5,500 (P/F,12%,3) = - \$ 25,495.30

$PW_{challenger}(12\%) > PW_{defender}(12\%)$

Therefore, Replace the defender now.

5.2 Economic Service Life

Economic Service Life of an asset is defined as the period of useful life that minimizes the annual equivalent costs of owning and operating the asset. We need to find the value of **N** that minimizes **AEC** as expressed in Eq. (5.1). If CR(i) is decreasing function of N and is an increasing function of N, as is often the case, AEC will be a convex function of N with a unique minimum point.

AEC(i) = CR(i) + OC(i) (5.1)

Capital (Ownership) Cost

Annual equivalent of capital cost which is called Capital Recovery (CR) Cost over the period of N years can be calculate with

CR(i) = I * (A/P, i%, N) + S_N * (A/F, i%, N)

CR(i) = (I - S_N) (A/P, i%, N) + S_N(i%)

CR(i) = (I - S_N) (A/F, i%, N) + I (i%)

Generally, as an asset becomes older, its salvage value becomes smaller. As long as the salvage value is less than the initial cost, the capital recovery cost is a decreasing function of N. In other words, the longer we keep an asset, the lower the capital recovery cost becomes. if the salvage value is equal to the initial cost no matter how long the asset is kept, the capital recovery cost is constant.

Operating Cost

OC(i) = ($\sum OC_n$ (P/F, i%, n)) * (A/P, i%, N)

OC(i) represents the annual equivalent of the operating costs over a life span of N years.

OC_n represents the total operating cost of an asset in year n of the ownership period.

As long as the annual operating costs increase with the age of the equipment, OC(i) is an increasing function of the life of the asset. If the annual operating costs are same from year to year, OC(i) is constant and equal to the annual operating costs no matter how long be asset is kept.

Example:

First Cost = \$ 3000. Salvage Value at the end of year 1 = \$ 1500 which decrease by \$ 500 each year, Operating Cost at the end of year 1 = \$ 1000 which increase by \$ 700 each year. MARR = 12%. N = 4 years. Find the economic service life of this new machine.

Calculate SV

SV_N = 1500 - G (N -1)	
SV_1 = 1500 - 500 (1 - 1)	= 1500
SV_2 = 1500 - 500 (2 - 1)	= 1000
SV_3 = 1500 - 500 (3 - 1)	= 500
SV_4 = 1500 - 500 (4 - 1)	= 0

Calculate OC

OC_N = 1000 + 700 (N - 1)	
OC_1 = 1000 + 700 (1 - 1)	= 1000
OC_2 = 1000 + 700 (2 - 1)	= 1700
OC_3 = 1000 + 700 (3 - 1)	= 2400
OC_4 = 1000 + 700 (4 - 1)	= 3100

N	F/ P	F/A	A/F	F/G	A/G
1	1.1200	1.0000	1.0000	0.0000	0.0000
2	1.2540	2.1200	0.4717	1.0000	0.4717
3	1.4050	3.3740	0.2964	3.1667	0.9234
4	1.5740	4.7790	0.2092	6.4917	1.3589

CR (12%) = (I - S) * (A / F , i %, N) + I (i%)

- **n = 1 : One year replacement cycle.**
  In this case, the machine is bought, used for one year, and sold at the end of year 1.

$CR_{n=1}$ (12%) =(3000 -1500)*(A/F,12%,1)+ I (i%)
=(1500)*((0.12)/(1.12^1-1)) + 3000(12%) = 1500*1.000 + 360= 1860

- **n = 2 :Two year replacement cycle.**
  In this case, the machine is bought, used for two year, and sold at the end of year 2.

$CR_{n=2}$ (12%) = (3000 - 1000)*(A/F, 12%, 2) + I (i%)
= (2000)*((0.12)/(1.12^2-1)) + 3000(12%) = 2000*0.4717 + 360= 1303

- **n = 3 : One year replacement cycle.**
  In this case, the machine is bought, used for three year, and sold at the end of year 3.

$CR_{n=3}$ (12%) = (3000 - 500)*(A/F, 12%, 3) + I (i%)
= (2500)*((0.12)/(1.12^3-1)) + 3000(12%) = 2500*0.2964 + 360 = 1101

- **n = 4 : One year replacement cycle.**
  In this case, the machine is bought, used for four year, and sold at the end of year 4.

$CR_{n=4}$ (12%) = (3000 - 0) * (A/F , 12%, 4) + I (i%)
= (3000)*((0.12)/(1.12^4-1)) + 3000(12%) = 3000 * 0.2092 + 360
= 987

OC_N (12%) = OC1 + G (A/G, i %, N)

OC_N (12%) = 1000 +700 (A/G 12%, N)

$OC_{n=1}$ (12%) = 1000 +700 (A/G 12%, 1) = 1000 +700 * 0.0000 = 1000

$OC_{n=2}$ (12%) = 1000 +700 (A/G 12%, 2) = 1000 +700 * 0.4717 = 1330

$OC_{n=3}$ (12%) = 1000 +700 (A/G 12%, 3) = 1000 +700 * 0.9234 = 1647

$OC_{n=4}$ (12%) = 1000 +700 (A/G 12%, 4) = 1000 +700 * 1.3589 = 1951

EOY	Market Value	Operating Cost	CR (12%)	OC (12%)	AEC (12%)
1	1500	1000	1860	1000	2860
2	1000	1700	1303	1330	**2633**
3	500	2400	1101	1647	2748
4	0	3100	987	1951	2938
N* = 2 years			**AEC* = 2633** **Minimum**		

Minimum AEC = 2,633 occurs at year 2.

Therefore Economic service life = 2 years

Example:

Suppose a company has a forklift but considering purchasing a new electric lift truck that would cost$ 18,000 and operating cost of $ 3,000 in the first year. For the remaining years, OC increases by 15% over the previous year's OC. Similarly, the salvage value declines each year by 20% from the previous year's salvage value. The lift truck has a maximum life of eight years. MARR = 12% before tax. Find the economic service life of the asset.

CR (12%) = (I - S) * (A / F , i %, n) + I (i%)

OC (12%)n = 2200 (P/A1, 15%, 12%, n)(A/P, 12%, n)

EOY	Market Value	Operating Cost	CR (12%)	OC (12%)	AEC (12%)
0	$18,000				
1	$14,400	$3,000	$5,760	$3,000	$8,760
2	$11,520	$3,450	$5,217	$3,212	$8,429
3	$9,216	$3,968	$4,763	$3,436	$8,199
4	$7,373	$4,563	$4,384	$3,672	$8,055
5	$5,894	$5,247	$4,065	$3,920	$7,985
6	$4,719	$6,034	$3,797	$4,180	**$7,977**
7	$3,775	$6,939	$3,570	$4,454	$8,024
8	$3,020	$7,980	$3,378	$4,740	$8,118
N* = 6 years			**AEC* = 7,977** **Minimum**		

Minimum AEC = 7,977 occurs at year 6.

Therefore Economic service life = 6 years

5.3 Replacement Analysis When the Required Service Is Long

Economic life of the defender is defined as the number of years of service which minimizes the annual equivalent cost, that is not necessarily the optimal time to replace the defender. The correct replacement time depends on the data on the challenger as well as on the data on the defender. It is required to decide whether now is the time to replace the defender or if not now, when is the optimal time to replace the defender.

5.3.1 Required Assumptions and Decision Frame Works:

Required Assumptions and Decision Frame Woks:

- **Planning Horizon (study period)** is the service period required by the defender and sequence of future challengers.
  - Infinite Planning Horizon: is used when the activity under consideration will be terminated is unable to predict.
  - Finite Planning Horizon: is used in situations, it may be clear that the project will have a definite and predictable duration.
- **Technology**

Predictions of technological patterns over the planning horizon refer to the development of types of challengers that may replace those under study. A number of possibilities predicting purchase cost, salvage value, and operating cost that are dictated by the efficiency of a new machine over the life of an existing asset.

- No Change in Technology: If all future machines will be the same as those in service, we are implicitly saying that no technological progress in the area will occur.
- Recognition of Technological change: we may explicitly recognize the possibility of machine becoming available in the future that will be significantly more efficient, reliable, or productive than those currently on the market. Clearly, if the best available machine gets better and better over time, we would certainly investigate the possibility of delaying an asset's replacement for a couple of years.

- **Relevant Cash Flow Information**

Many varieties of predictions can be used. Sometimes, revenue is constant, but costs increases, while salvage value decreases over the life of a machine. In other situations, revenue is varying.
The specific situation will determine whether replacement analysis is directed toward cost optimization (with constant revenue) or profit maximization (with varying revenue).

j_0		j_1					j_2			
Defender		**Challenger Type I**					**Challenger Type II**			
Infinite planning horizon with repeated identical replacements (j_0,2), (j,3)$_{\text{infinite times}}$										
J_0		J			j			J		
Defender		**Challenger**			**Challenger**			**Challenger**		
1	2	3	4	5	6	7	8	9	10	11

Planning Horizon (years)

Under the infinite planning horizon, the service is required for a very long time. Either we continue to use the defender to provide the service or we replace the defender with the available challenger for the same service requirement.

5.3.2 Replacement Analysis under Infinite Planning Horizon

Example:

Old machine A (Defender) can sell it now for $ 5000. If old machine is repaired now, it can be used for another six years. It will require an immediate $1,200 overhaul to restore it to operable condition. Future market values are expected to decline by 25% each year over the previous year's value. Operating costs are estimated at $ 2,000 during the first year and these are expected to increase by $ 1,500 per year thereafter. New machine B (Challenger) costs $ 10,000. Its estimated life is 8 year. Operating costs are estimated at $ 2,200 in the first year and will increase by 20% each year. Decide when the defender should be replaced. MV at the end of First year = $ 6,000, which decrease by 15% each year.

Take MARR = 15%.

N	F/A	P/A	F/G
1	1.00000	0.86956	0.00000
2	2.15000	1.62570	1.00000
3	3.47250	2.28322	3.15000
4	4.99338	2.85497	6.62250
5	6.74238	3.35215	11.61588
6	8.75374	3.78448	18.35826
7	11.06680	4.16042	
8	13.72682	4.48732	

Finite planning horizon of 10 years (j_0,2),(j_1,5). (j_2,3)

When, i = 15% and g = 20%.Calculate Coefficient / factors for P/A_1

N	P/A_1	
1	0.86957	$$P = \frac{A_1}{(i - g)}\left[1 - \frac{(1 + g)^N}{(1 + i)^N}\right]$$
2	1.77694	
3	2.72376	
4	3.71175	
5	4.74270	
6	5.81847	
7	6.94101	
8	8.11236	

CR(15%)n = (5000 - S) * (A/F,15 %,n) + 5000 (15%)

OC(15%)n=2000+1,500(F/G,i %,n)(A/F,i %,n)+1200(A/P,15%,n)

Defender (Machine "X")					
EOY	Market Value	Operating Cost	CR (15%)	OC (15%)	AEC (15%)
0	5,000	1,200			
1	3,750	2,000	2000	3380	5380
2	2,813	3,500	1767	3436	**5203**
3	2,109	5,000	1582	3886	5469
4	1,582	6,500	1435	4410	5844
5	1,187	8,000	1316	4942	6258
6	890	9,500	1220	5463	6682
AEC*D (15%) = 5203			**N*$_{D}$ = 2 years.**		

CR(15%)n = (10000 - S) * (A/F, 15 %,n) + 10000 (15%)

OC (15%)n = 2200 (P/A1,20%,15%, n) (A/P,15%,n)

Challenger (Machine "Y")					
EOY	Market Value	Operating Cost	CR (15%)	OC (15%)	AEC (15%)
0	10,000				
1	6,000	2,200	5500	2200	7700
2	5,100	2640	3779	2405	6184
3	4,335	3168	3131	2624	5756
4	3,685	3802	2765	2860	**5625**
5	3,132	4562	2519	3113	5631
6	2,662	5474	2338	3382	5721
7	2,263	6569	2199	3670	5869
8	1,923	7883	2088	3977	6066
AEC*C (15%) =5625			**N*$_{C}$ = 4 years.**		

Since AEC_D*= \$ 5203 < AEC_C* = \$ 5625.

The defender should not be replaced for now.

The defender should be used for at least N_D* = 2 years.

When should the defender be replaced?

Marginal analysis: Incremental cost or Marginal cost of keeping or operating the defender for just one more year beyond its economic service life. In other words, we want to see whether the cost of extending the use of the defender for an additional year exceeds the savings resulting from delaying the purchase of the challenger.

Calculate the cost of keeping and using the defender for the third year from today. i.e. what is the cost of selling the defender at the end of year 2, using it for the third year, and replacing it at the end of year?

The following cash flows are related to this question:

● Opportunity cost at the end of year 2 = the market value then = \$2813

● Operating Cost for the third year = \$5,000

● Salvage value of the defender at the end of year = \$2,109

The cost of using the defender for one more year from the end of its economic life

= 2813 * 1.15 + 5000 - 2109 = 6126. Since AEC_C* = 5625,

It is more expensive to keep the defender for the third year than to replace it with the challenger. Therefore, replace the defender at the end of year 2. If this one -year cost is still smaller than AEC_C*, we need to calculate the cost of using the defender for the fourth year and then compare that cost of using the defender for the AEC_C* of the challenger.

Should the defender be replaced now?

5.3.3 Replacement Analysis under the Finite Planning Horizon (PW Approach)

If the planning period is finite, comparison based on the AEC method over a defender's economic service life does not generally apply.
The procedure for such a problem with a finite planning horizon is to establish all 'reasonable' replacement patterns and then use the PW value for the planning period to select the most economical pattern.

Example:

Consider the defender and the challenge in above example. Suppose that the firm has a contract to perform a given service, using the current defender or the challenger for the next eight years. After the contract work, neither the defender nor the challenger will be retained. What will be the best replacement strategy?
Given: AEC values for both the defender and the challenger over eight years.
Planning horizon = 8 years, and MARR = 15%

EOY	Annual Equivalent Cost **AEC (15%)**	
n	Defender (Machine "X")	Challenger (Machine "Y")
1	5380	7700
2	**5203**	6184
3	5469	5756
4	5844	**5625**
5	6258	5631
6	6682	5721
7		5869
8		6066

Calculate necessary factor for MARR = 15%		
N	P/F	P/A
1	0.8696	0.8696
2	0.7562	1.6257
3	0.6575	2.2832
4	0.5718	2.8550
5	0.4972	3.3522
6	0.4323	3.7845
7	0.3759	4.1604
8	0.3269	4.4873

Many replacement scenario options would fulfill eight-year planning horizon.

These options are listed, and the present equivalent cost for each option is calculated, as follows:

Some Likely Pattern under a Finite Planning Horizon of Eight years.

Option 1	$(j_0,0),(j,4),(j,4)$				4				4
Option 2	$(j_0,1),(j,4),(j,3)$			1			4		3
Option 3	$(j_0,2),(j,4),(j,2)$			2			4		2
Option 4	$(j_0,3),(j,5)$				3				5
Option 5	$(j_0,3),(j,4),(j,1)$			3			4		1
Option 6	$(j_0,4),(j,4)$				4				4
	Years in Service 0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

Planning Horizon = 8 years

Option	Pattern	PW (15%)	AEC	P/A, i%,n	P/F, i%,n	PW (15%)
1	$(j_0,0)$, $(j,4)$, $(j,4)$	0 + 5625(P/A, 15%, 8)	0 5625	- 4.4873	- -	\$25,241
2	$(j_0,1)$, $(j,4)$, $(j,3)$	5380 (P/F, 15%, 1) + 5625(P/A, 15%, 4) (P/F, 15%, 1) + 5756(P/A, 15%, 3) (P/F, 15%, 5)	5380 5625 5756	- 2.8550 2.2832	0.8696 0.8696 0.4972	\$25,177
3	$(j_0,2)$, $(j,4)$, $(j,2)$	5203(P/A, 15%, 2) + 5625(P/A, 15%, 4) (P/F, 15%, 2) + 6184(P/A, 15%, 2) (P/F, 15%, 6)	5203 5625 6184	1.6257 2.8545 1.6257	- 0.7562 0.4323	\$24,948
4	**$(j_0,3)$, $(j,5)$**	5469(P/A, 15%, 3) + 5631(P/A, 15%, 5) (P/F, 15%, 3)	5469 5631	2.2832 3.3522	- 0.6575	**\$24,898**
5	$(j_0,3)$, $(j,4)$, $(j,1)$	5469(P/A, 15%, 3) + 5625(P/A, 15%, 4) (P/F, 15%, 3) + 7700(P/F, 15%, 8)	5469 5625 7700	2.2832 2.8550 -	- 0.6575 0.3269	\$25,563
6	$(j_0,4)$, $(j,4)$	5844(P/A, 15%, 4) + 5625(P/A, 15%, 4) (P/F, 15%, 4)	5844 5625	2.8550 2.8550	- 0.5718	\$25,866

Option 1: $(j_0,0),(j,4),(j,4)$ = PW (15%) = 5625(P/A, 15%, 8)
Option 2: $(j_0,1),(j,4),(j,3)$ = PW (15%) = 5380 (P/F, 15%, 1)
+ 5625(P/A, 15%, 4) (P/F, 15%, 1) + 5756(P/A, 15%, 3) (P/F, 15%, 5)
Option 3: $(j_0,2),(j,4),(j,2)$ = PW (15%)= 5203(P/A, 15%, 2)
+ 5625(P/A, 15%, 4) (P/F, 15%, 2) + 6184(P/A, 15%, 2) (P/F, 15%, 6)
Option 4: $(j_0,3),(j,5)$= PW (15%) = 5469(P/A, 15%, 3) + 5631(P/A, 15%, 5) (P/F, 15%, 3) … … …

Best replacement strategy: Option 4 with minimum cost
Retain the defender for 3 years and purchase the challenger, and keep it for 5 years.

CHAPTER 6

RISK ANLYSIS

6.1 Concept of Certainty, Risk and Uncertainty

Everything changes in dynamic environment. Nothing remains constant. Due to this, various risk & uncertainties need to consider in our engineering economic analysis. In this chapter, various useful methods are discussed taking in account the probability of occurrence.

CERTAINTY

Certainty is defined as state of knowledge in which decision maker knows in advance the specific outcome to which each alternative will invariably lead. i.e. decision maker has perfect knowledge of the environment and the result of whatever decision he might make i.e. High degree of confidence on all estimated quantities, revenues, costs. This degree of confidence is sometimes called ASSUMED CERTAINTY or DECISION UNDER CERTAINTY. This is rather misleading term, in that there is rarely a case in which estimated quantities can be assumed as certain. In all situations, there is doubt as to the ultimate results that will be obtained from an investment.

Decision under Certainty → to those decision problems in which there may have several possible outcomes whose **probability of occurrence can be almost perfectly (100%) known**.

RISK

Risk is defined as a state of knowledge in which alternative leads to one of a set of specific outcomes with each outcome occurring with a probability that is known objectively to the decision maker. Under condition of risk, the decision maker possesses some objective knowledge of environment and is able to predict objectively the probability of possible state of nature and outcome (or payoff) each contemplated strategy.

Decision under Risk → to those decision problems in which there may have several possible outcomes whose **probability of occurrence can be estimated**.

UNCERTAINTY

Uncertainty is defined as a state in which one or more alternatives result in a set of possible outcomes where probabilities are either unknown or not meaningful. Unlike risk, uncertainty is a subjective phenomenon.

Decision under Uncertainty → decision problem characterized by several unknown future outcomes for which **probabilities of occurrence cannot be estimated**.

6.2 Origin/Sources Of Project Risks

The factors that affect uncertainty are many and varied. Four major sources of uncertainty are:

- Inaccuracy of estimates under the study:
  If exact information is available, the resulting accuracy of estimates should be good. If little information is available, the accuracy may be high or low, depending on the manner (or basis) in which estimated values are obtained. Are they sound scientific estimates or merely wild guesses?
  If they are based on a considerable amount of past experiences or have been determined by adequate market survey, a fair degree of reliance may be placed on them. If they are wild guess work, it contain a sizable element of uncertainty.
  Frequently, annual income and expenses contain more error are discovered to be most sensitive elements in the study. Saving in operating expenses involves less uncertainty because based on considerable experience and past history.
- Type of business (or projects or undertaking or ventures) involved in relation to future health of economy.
  Some lines of business such as mining are notoriously less stable and high degree of risk than other business such as large retail food stores. However, it cannot be said that investment in any retail food store always involves less uncertainty than in mining. Uncertainty depends on nature and history business. No past history is usually rather uncertain.
- Type of physical plant and equipment involved.
  General plants and equipments have definite economic lives and residual value whereas special type plant and equipment have little known economic lives and residual value.
- Length of the assumed study period.
  A long study period naturally decreases the probability of all the factors turning out as estimated because lengthier the study period, all else being equal, always increases the uncertainty in an investment.

6.3 Methods of Describing Project Risk

6.3.1 Sensitivity Analysis

Sensitivity analysis is favored when several vary simultaneously as single parameter under study is varied. Thus it is helpful to determine how sensitive the situation is to the several parameters so that proper weight and consideration may be assigned to them. Sensitivity, in general, means the relative magnitude of change in the measure of merit (such as EW, RR, BCR) caused by one or more changes in estimated study parameters.

Example:

Investigate the PW of the following project of a machine over a range of ± 30% in a) Initial Investment (I), b) Net Annual Revenue (R-E), c) Salvage Value (S), d) MARR (i), f) useful life (N). Initial Investment (I) = RS. 10,000; Annual Revenue (R) = Rs.4,000; Annual Expenses (E) = Rs.2,000; Salvage Value (S) = Rs. 1,000; MARR (i) = 10%; f) useful life (N) = 10 years.

Prime Equation

PW (10%)

= - 10,000 + (4,000 – 2,000)(P/A,10%,10) + 1,000(P/F,10%,10)

= - 10,000 + 2000 (6.1446) + 1,000 (0.3855)

= - 10,000 + 12289.2 + 385.5 = 2675

a) **When initial investment (I) varies with increment of 10% up to ± 30%,**

PW (p%) = - 10,000 x (1±p%) + (4,000 – 2,000)(P/A,10%,10)
+ 1,000(P/F,10%,10)

PW (10%) = - 10,000 x (1±10%) + (4,000 – 2,000)(P/A,10%,10)
+ 1,000(P/F,10%,10)

= [- 10,000 x (1) + (4,000 – 2,000)(P/A,10%,10)
+ 1,000(P/F,10%,10)] + {- 10,000 x (±10%)}

= 2,675 + {- 10,000 x (±10%)}

Initial investment (I) increases or decreases by

{-10,000 x (±10%)] i.e. { - 10,000 x (± 0.1)} = ± 1000

PW (10%) at + 10%

= - 10,000 x 1.1 + (4,000 – 2,000)(P/A,10%,10)
+ 1,000(P/F,10%,10)

= - 10,000 x 1.0 + 2000 (6.1446) + 1,000 (0.3855) + { (-10,000)}

= - 10,000 x 1.1 + 12289.2 + 385.5

= 2675 + { - 10,000 x (+ 0.1)} = 2675 + { - 1,000}= + 1,675

PW (10%) at + 20% = 2675 + { - 10,000 x (+ 0.2)} = + 675

PW (10%) at + 30% = 2675 + { - 10,000 x (+ 0.3)} = - 325

PW (10%) at - 10% = 2675 + { - 10,000 x (- 0.1)} = + 3,675

PW (10%) at - 20% = 2675 + { - 10,000 x (- 0.2)} = + 4,675

PW (10%) at - 30% = 2675 + { - 10,000 x (- 0.3)} = + 5,675

	- 30%	- 20%	- 10%	0%	+ 10%	+ 20%	+ 30%	± 30%
Δ I	- 7,000	- 8,000	- 9,000	- 10,000	- 11,000	- 12,000	- 13,000	
PW	+ 5,675	+ 4,675	+ 3,675	+ 2,675	+ 1,675	+ 675	- 325	Δ 6,000

b) When net annual revenue (R-E) varies with increment of 10% up to ± 30%,

PW (p%) = - 10,000 + (4,000 – 2,000) x (1±p%) (P/A,10%,10)
+ 1,000(P/F,10%,10)

PW (p%) = - 10,000 + (4,000 – 2,000) x (1) (P/A,10%,10)
+ 1,000(P/F,10%,10) + (4,000 – 2,000) x (±p%) (P/A,10%,10)

Net annual benefit (R – E) increases or decreases by
+ (4,000 – 2,000) x (±p%) (P/A,10%,10)
= + 2,000 x 10% x 6.1446 = i.e. + 2000 x 0.1 x 6.1446
PW (10%) at + 10%
= [- 10,000 + (4,000 – 2,000) x 1.0 (P/A,10%,10)
+ 1,000(P/F,10%,10)]+ (4,000 – 2,000) x 10% (P/A,10%,10)
= 2675 + {2000 x (+ 0.1) x 6.1446)} = + 6,361
PW(10%) at + 20% = 2675 + {2000 x (+ 0.2) x 6.1446)} = + 5,133
PW(10%) at + 30% = 2675 + {2000 x (+ 0.3) x 6.1446)} = + 3,904
PW(10%) at - 10% = 2675 + {2000 x (- 0.1) x 6.1446)} = + 1,446
PW(10%) at - 20% = 2675 + {2000 x (- 0.2) x 6.1446)} = + 217
PW(10%) at - 30% = 2675 + {2000 x (- 0.3) x 6.1446)} = - 1,012

	- 30%	- 20%	- 10%	0%	+ 10%	+ 20%	+ 30%	± 30%
Δ R –E	+ 1,400	+ 1,600	+ 1,800	+ 2,000	+ 2,200	+ 2,400	+ 2,600	
PW	- 1,012	+ 217	+ 1,446	+ 2,675	+ 3,904	+ 5,133	+ 6,361	Δ 7,367

c) When Salvage Value (S) varies with increment of 10% up to ± 30%,

PW (10%) = - 10,000 + (4,000 – 2,000)(P/A,10%,10)
+ 1,000 x (1±10%) (P/F,10%,10)

PW (10%) = - 10,000 + (4,000 – 2,000)(P/A,10%,10)
+ 1,000(P/F,10%,10) + {1,000 x (±10%)(P/F,10%,10)}

S increases or decreases by {1,000 x (±p%)(P/F,10%,10)}
= {-1,000 x (±10%)(0.3855)} = i.e. { - 1,000 x (± 0.1)(0.3855)}
PW (10%) at + 10%
= - 10,000+(4,000–2,000)(P/A,10%,10)+1,000x1.1 (P/F,10%,10)
={-10,000+2000(6.1446)+1,000x1.0(0.3855)}
+{1,000x(+0.1) (0.3855)}
= 2675 + {1,000 x (+ 0.1) (0.3855)} = + 2,713

PW (10%) at + 20% = 2675 + {1,000 x (+ 0.2) (0.3855)} = + 2,752
PW (10%) at + 30% = 2675 + {1,000 x (+ 0.3) (0.3855)} = + 2,790
PW (10%) at - 10% = 2675 + {1,000 x (- 0.1) (0.3855)} = + 2,636
PW (10%) at - 20% = 2675 + {1,000 x (- 0.2) (0.3855)} = + 2,598
PW (10%) at - 30% = 2675 + {1,000 x (- 0.3) (0.3855)} = + 2,559

	- 30%	- 20%	- 10%	0%	+ 10%	+ 20%	+ 30%	± 30%
Δ S	+ 700	+ 800	+ 900	+ 1,000	+ 1,100	+ 1,200	+ 1,300	
PW	+ 2,559	+ 2,598	+ 2,636	+ 2,675	+ 2,713	+ 2,752	+ 2,790	Δ 231

d) When MARR varies with the increment of 10%, up to ± 30%,

PW (p%) = - 10,000 + (4,000 – 2,000){(P/A,10%(1±p%),10}
+ 1,000 x {(P/F,10%(1±p%),10}

PW (10%) = - 10,000 + (4,000 – 2,000){(P/A,10%(1±10%),10}
+ 1,000 x {(P/F,10%(1±10%),10}

PW (10%) at + 10%
= - 10,000 + (4,000 – 2,000)(P/A,11%,10) + 1,000 (P/F,11%,10)
= - 10,000 + 2000 (5.8892) + 1,000 0.3522) = 2,131

PW (10%) at + 20%
= - 10,000 + (4,000 – 2,000)(P/A,12%,10) + 1,000 (P/F,12%,10)
= 1,622

PW (10%) at + 30%
= - 10,000 + (4,000 – 2,000)(P/A,13%,10) + 1,000 (P/F,13%,10)
= 1,147

PW (10%) at - 10%
= - 10,000 + (4,000 – 2,000)(P/A,9%,10) + 1,000 (P/F,9%,10)
= 3,258

PW (10%) at - 20%
= - 10,000 + (4,000 – 2,000)(P/A,8%,10) + 1,000 (P/F,8%,10)
= 3,883

PW (10%) at - 30%
= - 10,000 + (4,000 – 2,000)(P/A,7%,10) + 1,000 (P/F,7%,10)
= 4,556

	- 30%	- 20%	- 10%	0%	+ 10%	+ 20%	+ 30%	± 30%
Δ MARR	+ 7%	+ 8%	+ 9%	+ 10%	+ 11%	+ 12%	+ 13%	
PW	+ 4,556	+ 3,883	+ 3,258	+ 2,675	+ 2,131	+ 1,622	+ 1,147	Δ 3,409

e) When Useful Life (N) varies with increment of 10%, up to ± 30%,

PW (10%) = - 10,000 + (4,000 – 2,000){(P/A,10%,10(1±p%)} + 1,000 x {(P/F,10%,10(1±p%)}

PW (10%) = - 10,000 + (4,000 – 2,000){(P/A,10%,10(1±10%)} + 1,000 x {(P/F,10%,10(1±10%)}

PW (10%) at + 10% = - 10,000 + (4,000 – 2,000)(P/A,10%,11) + 1,000 x (P/F,10%,11) = + 3,713

PW (10%) at + 20% = - 10,000 + (4,000 – 2,000)(P/A,10%,12) + 1,000 x (P/F,10%,12) = + 3,946

PW (10%) at + 30% = - 10,000 + (4,000 – 2,000)(P/A,10%,13) + 1,000 x (P/F,10%,13) = + 4,496

PW (10%) at - 10% = - 10,000 + (4,000 – 2,000)(P/A,10%,9) + 1,000 x (P/F,10%,9) = + 1,942

PW (10%) at - 20% = - 10,000 + (4,000 – 2,000)(P/A,10%,8) + 1,000 x (P/F,10%,8) = + 1,136

PW (10%) at - 30% = - 10,000 + (4,000 – 2,000)(P/A,10%,7) + 1,000 x (P/F,10%,7) = + 250

	- 30%	- 20%	- 10%	0%	+ 10%	+ 20%	+ 30%	± 30%
Δ N	+ 7	+ 8	+ 9	+ 10	+ 11	+ 12	+ 13	
PW	+ 250	+ 1,136	+ 1,942	+ 2,675	+ 3,713	+ 3,946	+ 4,496	Δ 4,246

Summary:

			± 30%	Sensitive
A	Initial Investment	ΔI	Δ 6,000	
B	Net Annual Revenue	Δ(R-E)	Δ 7,367	Most
C	Salvage Value	ΔS	Δ231	Least
D	MARR	ΔMARR	Δ 3,409	
E	Useful Life	N	Δ 4,246	

Order of sensitivity: (R-E) > I > N > MARR > S

Net Annual Revenue (R-E) is most sensitive.

Salvage Value (S) is least sensitive.

6.3.2 Breakeven Analysis

Breakeven analysis is commonly utilized when the selection among alternatives is heavily dependent on a single parameter, such as capacity utilization.

Costs can be classified into two major categories: fixed and variable.

Fixed Cost: Costs that remain constant regardless of the level of activity or output is zero or 100% are known as fixed costs, in short-run studies. However, fixed cost does not remain constant or fixed in the long run.

Variable Cost: Costs that are generally proportional to output are called variable costs. When there is no output, variable cost is zero.

Total Cost = Fixed Cost + Variable Cost

Revenues results from sales of output.

Profits represent the difference between revenue and total costs. A profit (or loss) figure is a yardstick of success.

When the selection between two alternatives is heavily dependent on a single factor, that value is known as the breakeven point, at which two alternatives are indifferent. Then, if the best estimate of the actual outcome of the common factor is higher or lower than the breakeven point, and assumed certain, the best alternative becomes apparent.

In mathematical terms, we have

$EW_A = f_1(y)$ and $EW_B = f_2(y)$

EW_A = an equivalent worth calculation for the net cash flow of Alternative A

EW_B = an equivalent worth calculation for the net cash flow of alternative B

y = a common factor of interest affecting the equivalent worth values of Alternative A and Alternative B

Therefore, the breakeven point between Alternative A and Alternative B is the value of factor y for which the two equivalent worth (EW) values are equal. That is, $EW_A = EW_B$, or $f_1(y) = f_2(y)$ which may be solved for y.

Examples of common factors for which breakeven analyses are useful:

- Annual revenue and expenses
- Rate of return Market (or Salvage) value
- Equipment life
- Capacity utilization

At breakeven point,
Total Sales (TS) = Total Cost (TC)
Total Sales(TS)=Total Fixed Cost(TFC)+Total Variable Cost (TVC)
TS = TFC + TVC
sp * X' = TFC + vc * X'
sp * X' - vc * X' = TFC
X' * (sp - vc) = TFC

$$\mathbf{X' = \frac{TFC}{(sp - vc)}}$$

Where X' is the breakeven quantity of production,
sp is the selling price per unit,
vc is the variable cost per unit

At the breakeven point there will be neither profit nor loss. If the output quantity (X) for the time period is greater than X', a profit will result & if less than X', a loss will incur. Obviously, to reach a breakeven position, the selling price has to be greater than the variable cost per unit.

Example: TFC = Rs 60 million; cv = Rs. 50 per unit; Sp = Rs. 80 per unit;
Output production = 4,000 units. Find BEP in units & also BEP in value.
If sp increases by 25%, what would be the effect on profit or loss?

					(in Rs.'000)
					When sp increases by 25%
Net Sales	4,000 units	@ Rs. 80	Rs. 320	@ Rs. 100	Rs. 400
Variable Cost	4.000 units	@ Rs. 50	Rs. 200	@ Rs. 50	Rs. 200
Contribution Margin			Rs. 120		Rs. 200
Total Fixed Cost			Rs. 60		Rs. 60
Profit/Loss			Rs. 60		Rs. 140
BEP units (X')	X'= $\frac{\text{TFC}}{(\text{sp} - \text{vc})}$	X'= $\frac{60{,}000}{(80 - 60)}$	2,000 units	X' = $\frac{60{,}000}{(100 - 50)}$	1,200 units
BEP value	X' * sp	2,000 * 80	Rs. 160	1,200 * 100	Rs. 120
Profit/Loss	(sp-cv)*(X-X')	(80 - 50)* (4000-2000)	Rs. 60	(100-50)* (4000-1200)	Rs. 140
Increase in profit				= 140 – 60	Rs. 80
% Increase in profit				= $\frac{80}{60}$ * 100	133%
					When sp increases by 25% its effect on profit is → Profit increases by 133%

Example:

	Motor A	Motor B
Initial Investment	Rs. 12,50,000	Rs. 16,00,000
Horse Power	100hp	
Efficiency (η)	74%	92%
Useful Life	10 Years	
MARR	15%	
Maintenance Cost /year	Rs. 50,000	Rs. 25,000
Electrical Cost	Rs. 5 per KW hr	
Tax & Insurance Cost/year	1% of Initial Investment	

a) How many hours per year would the motors have to be operated so that annual cost will equal?

b) If the motor have to be operated 800hours/year, which motor will you select?

Let y be the number of hours of operation at the breakeven point. Calculate AW (15%)		
	Motor A	Motor B
Capital Recovery (CR) Cost = I (A/P,15%,10)	Rs. 249,065	Rs.318,803
Operating cost of Power = 100 hp*0.746*y* 5/efficiency	504 y	405 y
Maintenance Cost /year	Rs. 50,000	Rs. 25,000
Tax & Insurance Cost/year = 1% of Initial Investment	Rs. 12,500	Rs. 16,000
Total Fixed Cost (TFC)	Rs. 311,565	Rs. 359,803
Total Variable Cost (TVC)	504 y	405 y
Total Cost (TC)	Rs. 311,565 + 504 y	Rs. 359,803 + 405 y
At Break Even Point	Rs. 311,565 + 504 y = Rs. 359,803 + 405 y	
	y = 487 hrs/year	
Total Variable Cost (TVC) at 800 hours/ year	Rs. 403,200	Rs. 324,000
Total Cost (TC) at 800 hours / year	Rs. 714,765	Rs. 683,803
Since, Total cost of Motor B < Total cost of Motor A, Select Motor B		

a) The motors have to be operated at 487 hours per year, so that annual cost will equal.

b) If the motor have to be operated 800hours/year, Select Motor B.

6.3.3 Scenario Analysis (Optimistic – Most Likely - Pessimistic Estimation)

There are scenarios where the cash flow will be different from expectations; higher than expected in some cases and lower than expected in some other cases. In scenario analysis, we estimate expected cash flows and asset value under scenarios, with the intent of getting a better sense of the effect of risk on value. Estimation of one or more factors is done in a favorable (Optimistic) direction & in an unfavorable (pessimistic) direction to investigate the effect of these changes on study results.

Example:

Consider the example below:

Optimistic, most likely & pessimistic estimates are given for a proposed project. Salvage Value (S) at the end of useful life = 0.

MARR = 8%. Perform Scenario Analysis.

	Estimation Condition (in Rs.'000)		
	Optimistic (O)	Most Likely (M)	Pessimistic (P)
Capital Investment	-150	-150	-150
Annual Revenues	110	70	50
Annual Expenses	-20	-43	-57
Useful Life	18 Years	10 Years	8 Years

Sample Calculation

	Optimistic (O)	Most Likely (M)	Pessimistic (P)
CR cost = I (A/P,8%,N)	-16	-22	-26
Annual Revenues	110	70	50
Annual Expenses	-20	-43	-57
AW (8%)	+74	+5	-33

AE (0)	*-20*
L (M)	*-22*
AR (O)	*+110*
AW (8%)	*68*

AE (M)	*-43*
L (O)	*-16*
AR (O)	*+110*
AW (8%)	*51*

AE (P)	*-57*
L (M)	*-22*
AR (O)	*+110*
AW (8%)	*31*

	Annual Expenses (AE)								
	O			M			P		
	Life (L)			Life (L)			Life (L)		
Annual Revenue (AR)	O	M	P	O	M	P	O	M	P
O	74	*68*	64	*51*	45	*41*	37	*31*	27
M	34	28	24	11	5	1	-3	-9	-13
P	14	8	4	-9	-15	-19	-23	-29	-33

From above table, it is apparent that 4 combinations result in AW> 50,000, while 9 combination results in AW<0.

6.4 Probability Concept of Economic Analysis

Variability is a recognized factor in most engineering and management activities. The properties of materials vary over time; seemingly identical machines exhibit diverse operating characteristics. Environmental factors are constant and economic conditions change irregularly. Risk analysis contributes to a more complete economic evaluation when there are significant risks involved that can be represented by the assignment of meaningful probabilities.

A formal evaluation of risk is feasible when the likelihood of possible futures can be estimated and when associated outcomes from courses of action can be identified. The first step is to determine categories of future states that affect the alternatives being compared. For example: low, average, high. After the future states are identified and bounded cans flow outcomes can be estimated by assuming that each state, in turn, is sure to occur. The next step in risk analysis is to determine the probability that each state will actually occur. The source may be objective or subjective.

Consider an investment that requires an initial cost of Rs. 25,000 and is expected to produce annual revenues of Rs. 8,000 for 5 years. Take MARR = 10%.

PW (10%) = -25,000 + 8,000 (P/A,10%,5) = 5325

Let the interest and annual revenues are two independent random variables defined by the following discrete probability distributions:

Interest rate

I, %	P (I = i)
12	0.10
10	0.70
7	0.20
∑ P (I = i)	1.00

Annual revenues

X, Rs.	P (X = x)
5,000	0.05
8,000	0.85
10,000	0.10
∑ P (X = x)	1.00

Expected Value

Expected value is a standard measure for economic comparison involving risk. It incorporated the effect of risk on potential outcomes by means of weighted average. Outcomes are weighted according to their probability of occurrence, and the sum of the products of all outcomes multiplied by their respective probabilities is the expected value.

$$EV\ (X) = \sum_{i=1}^{n} \mathrm{P}(\mathrm{X} = \mathrm{xi})\ \mathrm{xi}$$

EV for interest rate = 12% x 0.10 + 10% x 0.70 + 7% x 0.20 = 9.6%

EV for interest rate = 5,000 x 0.05 + 8,000 x 0.85 + 10,000 x 0.10 = 8,050

Measure of Variance

Useful quantitative measures of variability for a random variable are its variance and standard deviation. The variance, denoted by Var (X) or σ_x^2 is a measure of dispersion or spread about expected value.

$$\mathrm{Var}\ (\mathrm{X}) = \sum_{i=1}^{n} \mathrm{P}(\mathrm{X} = \mathrm{xi})\ [(\mathrm{xi} - \mathrm{EV}(\mathrm{X})]2$$

$\sigma_x = \sqrt{[\mathrm{Var}\ (X)]}$

Var (I) = 0.10 (12% - 9.6%)^2 + 0.70 (10% - 9.6%)^2 + 0.20 (7% - 9.6%)^2 = 2.04×10^{-4}

$\sigma_{I\,=}$ $\sqrt{[\mathrm{Var}\ (I)]} = \sqrt{[2.04 \times 10^{-4}]} = 0.014283$

Var(R) = 0.05 (5,000 – 8,050)^2 + 0.85 (8,000 – 8,050)^2 + 0.10 (10,000 – 8,050)^2 = 847500

$\Sigma_{R\,=}$ $\sqrt{[\mathrm{Var}\ (R)]} = \sqrt{[847500]} = 920.6$

Coefficient of Variation

$$\text{Coefficient of Variation} = \frac{Standard\ deviation}{Expected\ Value}$$

Example:

Proposal	Demand		
	P(low) = 0.2	P(average) = 0.6	P(high) = 0.2
A	Rs. 900	Rs. 1,000	Rs. 1,100
B	Rs. 400	Rs. 1,000	Rs. 1,600
C	Rs. 980,000	Rs. 1,000,000	Rs. 1,020,000

Proposal	Expected value	Standard deviation	Variance	Coefficient of Variation
A	1000	63.25	4,000	0.63
B	1000	379.47	144,000	0.379
C	1,000,000	12,649	160,000,000	0.0126

A direct comparison of Var (A) = 4,000 & Var (B) = 144,000 indicates obvious greater variability of Proposal B.
Proposal A is less risky.
A direct comparison of Var (A) & Var (B) with Var (C) indicates obvious even much more greater variability of Proposal C.
Proposal c is riskier because variance is much larger.

This erroneous impression is erased by calculating the coefficient of variation.
Comparing the coefficient of variation of Proposal C (0.0126) with Proposal A (0.63) & Proposal B (0.379) makes it apparent that proposal C is subject to less variability.
The standard deviation & variance can be a misleading indicator of risk when alternatives differ in size/scale.

6.5 Decision Tree or Sequential Investment Decisions

A decision tree is a graphic device that shows a sequence of strategic decisions and the expected consequences under each possible set of circumstances. The construction and analysis of a decision tree is appropriate whenever a sequential series of conditional decisions must be made under conditions of risk. By conditional decision, we mean a decision that depends upon circumstances or options that will occur at a later time.

Construction of the decision tree begins with the first or earliest decision and proceeds forward in time through a series of subsequent events and decisions. At each decision or event the tree branches out to show each possible course of action, until finally all logical consequences and the resulting payoffs are depicted. Fig. … is an example of a decision tree. The exhibit describes a problem faced by a firm that must be decide whether a firm must decide whether to sped Rs 350,000 to market a new product or to invest the money elsewhere for a 10% per annum return. Taking the sequence of events from left to right, the first decision (symbolized by a square is whether or not to market the product. If the product isn't marketed, the payoff will be Rs 35,000 from the alternative investment.

If the firm markets the product, the next event (a non-controllable situation, symbolized by a large circle) may be the entry of a competitor into the market. The probability of competition (0.8) and the probability of no competition (0.20) are parentheses beside the appropriate branches.

It is important to note that in the construction of a decision tree, the branches out of squares represent strategies and the branches out of large circles represent states of nature. Since the decision maker has full control over which strategy is chosen, the branches out of squares do not have probabilities. But the decision maker has no control over states of nature. Therefore, the branches out of large circles have probabilities and the probabilities for all branches coming from any one circle must add up to 1.0. In this example, the probabilities of competition (0.80) and no completion (0.20) add up to 1.0, since one or the other must happen.

If there is no competition the only remaining decision is whether to charge high, medium or low price. The three branches are drawn and labeled (high medium, low) and the payoff for each is noted at the end of each branch. If there is competition, the same three branches are appropriate.

However, each branch divides again to reflect the competitor's options to price high medium or low. The competitor's options are states of nature, so they proceed out of a circle. Each of these final branches is marked with a probability and the payoff is noted at the end of each one once again, the probabilities add up to 1.0 for each circle, since the competitor is certain to charge either a high, medium or low price.

The decision tree thus depicts in graphic form the expectation that the price a competitor charges depends upon the price the firm sets. At the same time, the fire's consequent profits depend upon what price the competitor charges. Since each decision depends upon the evaluation of events taking place at a later time, the analysis of a decision tree begins at the end of the sequence and works backward. Fig. … depicts the analysis for our example.

Beginning in the upper right of the exhibit, the analyst calculates the expected value if the firm's price is high and there is competition.

The expected value is (150 x 0.4) + (-50 x 0.5) + (-250 x 0.1) = 10. Similarly, expected values are calculated.

Competition	Firm's price					
	High		Medium		Low	
Competitor's price	Probability	Payoff ('000)	Probability	Payoff ('000)	Probability	Payoff ('000)
High	0.4	150	0.1	200	0.1	150
Medium	0.5	-50	0.6	100	0.2	50
Low	0.1	-250	0.3	-100	0.7	-50
Expected Value		10		**50**		-10
		Not Optimal		**Highest**		Not Optimal

No Competition	Firm's price		
	650	450	250
	Highest	Not Optimal	Not Optimal

Market	Competition (Probability)	Payoff ('000)	Do not market	Payoff ('000)	
Competition	0.8	50			Decision: Market the Product
No Competition	0.2	650			
Expected Value		**170**		35	
		Highest		Not Optimal	

This expected value is noted in or above the event circle. The expected values of medium and low prices are computed and noted in a similar manner. Since the medium price gives the highest expected value, that value is noted in the decision box, and the other two branches are slashed to indicate they are non-optimal.

In the alternative state of no competition, the only question is whether to charge a high, medium or low price. The payoffs indicate that a high price is optimal, and the other two branches are marked out.

At the first event point (introduction of a competitive product) the expected value is (50 x 0.8) + (650 x 0.2) = 170. The firm is now ready to make a decision. If it does not market, it gets Rs. 35,000. If it does market its product, there is an expected return of Rs 170,000. Clearly, then, the firm should enter the market.

The diagram also gives clear indication of the most profitable pricing strategy. The product should be initially marketed at a high price. If competition develops and there is an 80% probability that it will– the price should then be lowered to a medium price in order to maximize the expected return.

CHAPTER 7

Depreciation

7.1 Concept and Terminology

Depreciation is the decrease in value of assets with the passage of time. An asset has value because one can expect to receive future monetary benefits through its possession and use. The benefits are in the form of future cash flows resulting from the use of the asset to produce salable goods and services and the ultimate sale of the asset. Therefore depreciation represents and estimates of decrease in an assets value because its ability to produce future cash flows will, most likely decrease over time.

7.2 Basic Methods of Depreciation

The following methods of depreciation are discussed:

- Straight Line (SL) Method
- Declining/Diminishing Balance (db) Method
- Sinking Fund Method
- Sum of the Year Digit (SOYD) Method
- MACRS Method

7.2.1 Straight Line (SL) Method

It assumes that the loss in value is directly proportional to the age of the asset. The annual depreciation is fixed (or constant or uniform or equal) amount throughout the lifetime of the asset such that the accumulated sum at the end of the life is exactly equal to purchase price or value of the asset.

Basis or Cost Basis or Unadjusted Cost Basis: The initial or original cost of acquiring an asset (purchase price plus sales tax) including transportation cost and other normal costs of making the asset serviceable for its intended use.

Example:

Cost Basis (B) = Rs. 7000 Salvage Value (S) = Rs 2000
Useful life (N) = 5 years
Annual Depreciation Rate = 1/N = 1/5 = 0.2

EOY	Depreciation Charge during year n	Depreciation Reserve Accumulated by the year n	Book Value at the end of year n
	DC(n)	D*n	BV(N)
0			BV (O)
1	(1/N) * (I - S)	(1/N) * (I - S)	BV (O) - DC (1)
2	(1/N) * (I - S)	(2/N) * (I - S)	BV (1) - DC (2)
3	(1/N) * (I - S)	(3/N) * (I - S)	BV (2) - DC (3)
4	(1/N) * (I - S)	(4/N) * (I - S)	BV (3) - DC (4)
5	(1/N) * (I - S)	(5/N) * (I - S)	BV (4) - DC (5)

1	Rs.1000	Rs.1000	Rs.6000
2	Rs.1000	Rs.2000	Rs.5000
3	Rs.1000	Rs.3000	Rs.4000
4	Rs.1000	Rs.4000	Rs.3000
5	Rs.1000	Rs.5000	Rs.2000

Annual Depreciation Charge:

DC(n) = (1/N) * (B-S)

= (1/5)*(7000-2000)

= Rs.1000

Depreciation Reserve accumulated by the year n

D^*_n = n/N (B-S)

Depreciation Reserve accumulated by the year 3

D^*_3 = (3/5) * (7000-2000)

= Rs. 3000

Alternately,

Depreciation Reserve accumulated by the year n

D^*_n = DC(n) * Year n

D^*_3 = 1000 * 3 = 3000

Book Value at the end of year n:

BV(n) = B -D^*_n

Book Value at the end of year n

BV(3) = B -D^*_3 = 7000 - 3000

= RS. 4000

Alternately,

BV(n) = BV(n-1) - DC(n)

BV(3) = BV(3-1) - DC(3) = 5000 - 1000

= Rs. 4000

EOY	DC(n)	D*n	BV(N)
0			Rs.7000

7.2.2. Declining Balance (DB) Method

It is assumed in DB method that annual depreciation charge is fixed or constant percentage of book value at the beginning of the each year. i.e. loss in value at an early accelerated (faster) than latter portion of its service life. In this method, R = 2/N , when e.g. 200% declining balance is being used (i.e. rate allowed is twice as great as would be under straight line method and hence, termed also as double-declining-balance (DDB) method. For other circumstance, R = 1.5/N, when 150% declining balance is used (1.5 times SL method).

Example:

(a) When $BV_N = S$

Cost Basis (B) = Rs. 7000 Salvage Value (S) = Rs. 544

Useful life (N) = 5 years

Annual Depreciation Rate

R = 2 * (1/N) = 2 * (1/5) = 2 * (0.2) = 0.4 = 40%

Annual Depreciation Charge

$DC(n) = BV(0) * (1 - R)^{n-1} * (R)$

Annual Depreciation Charge

$DC(3) = 7000 * (1 - 0.4)^{3-1} * (0.4)$

$= 7000 * (0.6)^2 * (0.4) = 1008$

Alternately,

Annual Depreciation Charge

DC(n) = BV(n-1) * R

DC(3) = BV(3-1) * R = 2520 * 0.4 = 1008

Depreciation Reserve accumulated by the year n

$D^*_n = BV(0) * [1 - (1 - R)^n$

Depreciation Reserve accumulated by the year 3

$D^*_3 = BV(0) * [1 - (1 - 0.4)^3$

$= 7000 * [1 - (0.6)^3] = Rs. 5488$

Alternately,

Depreciation Reserve accumulated by the year n

D^*_n = DC(1) +DC(2) +DC(3)+ ....+DC(n-1) +DC(n)

D^*_3 = DC(1) +DC(2) +DC(3)

= 2800 + 1680 + 1008 = 5488

Book Value at the end of year n

$BV(n) = BV(0) * (1 - R)^n$

Book Value at the end of year 3

$BV(3) = BV(0) * (1 - 0.4)^3$

$= 7000 * (0.6)^3 =$ RS. 1512

EOY	DDB DC(n)	D*n	DDB BV(n)
0			BV (O)
1	(2/N) * BV(0)	DC(1)	BV (O) - DC (1)
2	(2/N) * BV(1)	D*1 + DC(2)	BV (1) - DC (2)
3	(2/N) * BV(2)	D*2 + DC(3)	BV (2) - DC (3)
4	(2/N) * BV(3)	D*3 + DC(4)	BV (3) - DC (4)
5	(2/N) * BV(4)	D*4 + DC(5)	BV (4) - DC (5)

When n =1:

DDB DC(n) = (2/N) * BV(0) = (0.40) * (7000) = Rs.2800

DDB BV(n) = BV (O) - DC (1) = 7000 - 2800 = Rs.4200

Similarly, calculate other values.

EOY	DDB DC(n)	D*n	DDB BV(n)
0			Rs.7000
1	Rs.2800	Rs.2800	Rs.4200
2	Rs.1680	Rs.4480	Rs.2520
3	Rs.1008	Rs.5488	Rs.1512
4	Rs. 605	Rs.6093	Rs. 907
5	Rs. 363	Rs.6456	Rs. 544

(b) When $BV_N > S$

Example:

Cost Basis (B) = Rs. 7000

Salvage Value (S) = Rs. 0

Useful life (N) = 5 years

Annual Depreciation Rate = **R = 2 * (1/N)**

= 2 * (1/5)

= 2 * (0.2)

= 0.4

= 40%

EOY	DDB DEPRECIATION RATE	BOOK VALUE WITH DDB	STRAIGHT LINE DEPRECIATION RATE	DEPRECIATION CHARGE ALLOCATED	DDB → SL DEPRECIATION
	DDB Dep.%	DDB BV(N)	SL Dep. %	(DDB → SL) %	DDB → SL Dep.
0		BV (O)			BV (O)
1	(2/N) * BV(0)	BV (O) - DC (1)	BV (O)/N	=DDB% If DDB% > SL% OR =SL% If SL% > DDB%	(DDB → SL) %* BV (O)
2	(2/N) * BV(1)	BV (1) - DC (2)	BV (1)/(N-1)		(DDB → SL) %* BV (1)
3	(2/N) * BV(2)	BV (2) - DC (3)	BV (2)/ (N-2)		(DDB → SL) %* BV (2)
4	(2/N) * BV(3)	BV (3) - DC (4)	BV (4)/ (N- 3)		(DDB → SL) %* BV (3)
5	(2/N) * BV(4)	BV (4) - DC (5)	BV (5)/ (N-4)		(DDB → SL) %* BV (4)

Switch from double declining balance to straight line depreciation							
EOY	DDB DC(n)	DDB BV(N)	SL DC (n)	COMPARE DDB Dep. with SL Dep.	DC (n) DDB → SL	D*n	BV (n) DDB → SL
0		7000					7000
1	**2800**	4200	7000/5 = 1400	**DDB DC(1)** > SL DC(1)	**2800**	2800	4200
2	**1680**	2520	4200/4 = 1050	**DDB DC(2)** > SL DC(2)	**1680**	4480	2520
3	**1008**	1512	2520/3 = 840	**DDB DC(3)** > SL DC(3)	**1008**	5488	1512
4	605	907	1512/2 = **756**	DDB DC(4) < **SL DC(4)**	**756**	6093	756
5	363	544	756/1 = **756**	DDB DC(5) < **SL DC(5)**	**756**	6456	0

Example:

(c) When $BV_N < S$

Cost Basis (B) = Rs. 7000

Salvage Value (S) = Rs.2000

Useful life (N) = 5 years

Annual Depreciation Rate

R = 2 * (1/N) = 2 * (1/5) = 2 * (0.2) = 0.4 = 40%

EOY	DDB DC(n)		DDB BV(N)
0			**Rs.7000**
1	**Rs.2800**	Rs.2800	**Rs.4200**
2	**Rs.1680**	Rs.4480	**Rs.2520**
3	**Rs.520**	Rs.5000	**Rs.2000**
4	**0**	Rs.5000	**Rs.2000**
5	**0**	Rs.5000	**Rs.2000**

Note : Year 3 depreciation charge is only Rs. 520 (not Rs.1008)

7.2.3 Sinking Fund (SF) Method

Sinking Fund Method assumes that a sinking fund is established in which funds will accumulate for replacement purposes. The total depreciation that has taken place up to any given time is assumed to be equal to the accumulated value of the sinking fund (including interest earned) at that time.

With this method, I, N, S, i on the sinking fund (I - S) are known, a uniform yearly deposit can be computed. the cost of depreciation for ay year is the sum of this deposit and accumulated interest for that year.

Example:

Cost Basis (B) = Rs. 7000
Salvage Value (S) = Rs.2000
Useful life (N) = 5 years
i = 10%

Annual Depreciation Charge allocated
d = **(B - S) * (A/F, i%, N)** = (7000-2000) (A/F, 10%, 5) = Rs. 819

Depreciation charge including interest for that year n
DC (n) = d [F/P, i%, (n-1)]
Dep. charge including interest for that year 3
DC (3) = d [F/P, 10%, (3-1)] = 819 * 1.10^2 = RS.919

Cumulative depreciation through year n
D^*_n = (B-S) * (A/F, i%, N) * (F/A, i%, N) = d * (F/A, i%, N)
Cumulative dep. through year 3
D^*_3 =d * (F/A, i%, N) = 819 * (F/A, i%, N) = 819 * 3.31 = Rs.2711

Alternately, Cumulative depreciation through year n
D^*_n = B - BV(n)
Cumulative dep. through year 3
D^*_3 = B - BV(n) = 7000 - 4289 = Rs.2711

Book value at the end of year 3
BV(n) = B - D^*_n
Book value at the end of year 3
BV(3) = B - D^*_n = 7000 - D^*_3 = 7000 - 2711 = Rs.4289

EOY	D	DC(n)	D^*_n	BV(n)
0				**7000**
1	**819**	819 * 1.1^0 = **819**	819 * 1.000 = **819**	**6181**
2	**819**	819 * 1.1^1 = **901**	819 * 2.100 = **1720**	**5280**
3	**819**	819 * 1.1^2 = **991**	819 * 3.310 = **2711**	**4289**
4	**819**	819 * 1.1^3 = **1090**	819 * 4.641 = **3801**	**3199**
5	**819**	819 * 1.1^4 = **1199**	819 * 6.105 = **5000**	**2000**

$F = P (1+i)^N = A^* (1/i) * [(1+i)^N-1]$

N	(F/P,i,N) = $(1+i)^N$	(F/A,i,N) = (1/i)*[F/P)-1]
1	1.1^1 = **1.1000**	**1.000**
2	1.1^2 = **1.2100**	**2.1000**
3	1.1^3 = **1.3310**	**3.3100**
4	1.1^4 = **1.4641**	**4.6410**
5	1.1^5 = **1.6105**	**6.1051**

7.2.4 Sum of the Year Digit (SYD) Method

Sum of the Year Digit (SYD) Method enables properties to be depreciated to zero value and easier to use than the declining balance (DB) method. Use of SYD method permits very rapid (accelerated) depreciation during the earlier period of life. In effect reduces the computed profits during early years of asset life and thus reduces income taxes in those early years.

To compute the depreciation deduction by the SYD method, the digits corresponding to the number of each permissible year of life are first listed in reverse order. The sum of these digits is then determined. The depreciation factor for any year is the number from the reverse-ordered listing for that year divided by the sum of the digits.

Example:

Cost Basis (B) = Rs. 17000
Salvage Value (S) = Rs.2000
Useful life (N) = 5 years

Sum of Year Digits for life N = 1+2+3+............N
SYD = N (N+1)/2 = 5+(5+1)/2 = 15

SYD Depreciation Factor for Year n = **n/SYD**
Depreciation Factor for Year 3 = 3/15

SYD Depreciation Charge for year n
DC(n) = SYD Depreciation factor for year n* (B-S)
SYD Depreciation Charge for year 3
DC(3) = SYD Depreciation factor for year 3* (B-S)
DC(3) = (3/15)*(17000-2000) = Rs.3000

Number of the year in reverse order (Digit) = **N - (n-1)**

EOY	**N - (n-1)**	**n/SYD**	**DC(n)**	**D^*_n**	**BV(n)**
0	-				17000
1	5	5/15	5000	5000	12000
2	4	4/15	4000	9000	8000
3	3	3/15	3000	12000	5000
4	2	2/15	2000	14000	3000
5	1	1/15	1000	15000	2000

DC(n) = [{2(N-n+1)}/{N(N+1)}]* (B - S)

SYD Depreciation Charge for year 3

DC(3) = [{2(5-3+1)}/{5(5+1)}]* (17000-2000)

= (3/15)*(17000-2000) = Rs.3000

Book Value at the end of year n

BV (n) = B - n * {[2 (B - S) } / N]
+ n * (n + 1) * [(B - S)/{ N * (N + 1)}]

Book Value at the end of year 3

BV(3) = 17000 - 3 * { [2 (17000 - 2000) } / 5] + 3 * (3 + 1) * [(17000 - 2000)/{ 5 * (5 + 1)}]

= 17000 - 18000 + 6000 = Rs. 5000

Cumulative Depreciation Reserve accumulated by the year n

D^*_n = B - BV (n)

Cumulative Depreciation Reserve accumulated by the year 3,

D^*_3 = B - BV (3) = 17000 - 5000 = Rs. 12000

SYD Depreciation Charge for year n

7.2.5 Modified Accelerated Cost Recovery System (MACRS) Method

MACRS uses switching from declining balance (DB) to Straight line (SL) method with half year convention. i.e. all the assets are placed in service at mid-year and they have zero salvage value. i.e. only half year depreciation is allowed for the 1st year, full year depreciation and the remaining half year depreciation in the year following the end of the recovery period. MACRS method includes 8 categories of assets:

3- year property includes special material handling devices and special tools for manufacturing.

5-year property includes automobiles, light and heavy trucks, computers, copiers, semiconductor manufacturing equipment, qualified technological equipment, and equipment used in research.

7-year property includes property that is not assigned to another class, such as office furniture, fixtures, single-purpose agricultural structures, and rail-road track.

10-year property includes assets used in petroleum refining, in the manufacture of castings, forgings, vessels, barges, and tugs.

15-year property includes service station buildings, telephone distribution equipment, and municipal water and sewage treatment plant.

20-year property includes farm buildings and municipal sewers

For real property:

27.5-year property includes residential property: apartment building and rental houses.

39-year property includes non residential building; warehouses, manufacturing facilities, refineries, mills, parking facilities, fences, and roads.

Depreciation methods:

Class life $\leq$ 20 years: 200% declining –balance switching to straight-line with half- year convention.

20 years $\leq$ Class life $< 25\ years\ and\ 25\ years \leq class\ life$:

150% declining –balance switching to straight-line with half-year convention.

Real property: Straight –line depreciation with half year convention over 27.5 & over 39 years.

DEPRECIATION PERCENTAGE FOR MARCS CLASSES						
Recovery Year	**3 year class (200%DB)**	**5 year class (200%DB)**	**7 year class (200%DB)**	**10 year class (200%DB)**	**15 year class (200%DB)**	**20 year class (200%DB)**
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.92	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.28
7			8.92	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.55	5.90	4.46
10				6.55	5.90	4.46
11				3.28	5.90	4.46
12					5.90	4.46
13					5.90	4.46
14					5.90	4.46
15					2.92	4.46
16						4.46
17						4.46
18						4.46
19						4.46
20						4.46
21						2.23

MACRS Depreciation Rates (%) Applied to the First Cost

Example:

It is classed as 5 year property.
Calculate MACRS depreciation and Book Value.

R = Annual Depreciation Rate = 2 * (1/N)
R = 2 * (1/5) = 2 * (0.2) = 0.4 = 40%
Calculate DDB Depreciation Rate

n	DDB Depreciation Rate
1	½ * 0.40 = **0.2000**
2	(1.00 - 0.20) * 0.40 = **0.3200**
3	(1 - 0.20 - 0.32) * 0.40 = **0.1920**
4	(1 - 0.20 - 0.32 - 0.192) * 0.40 = **0.1152**
5	(1 - 0.20 - 0.32 - 0.192 - 0.1152) * 0.40 = 0.0691
6	Not Applicable

Calculate SL Depreciation Rate

n	SL Depreciation Rate
1	½ * 0.20 = 0.1000
2	$1/4.5$ * (1.00 - 0.20) = 0.1778
3	$1/3.5$ * (1 - 0.20 - 0.32) = 0.1371
4	$1/2.5$ * (1 - 0.20 -0.32 - 0.192) = **0.1152**
5	$1/1.5$ * (1 - 0.20 -0.32 - 0.192 -0.1152) = **0.1152**
6	½ * (0.11521) = **0.0576**

Calculate MACRS Depreciation Rate

	Depreciation Rate		
n	DDB	SL	MACRS
1	**0.2000**	0.1000	0.2000
2	**0.3200**	0.1778	0.3200
3	**0.1920**	0.1371	0.1920
4	**0.1152**	**0.1152**	0.1152
5	0.0691	**0.1152**	0.1152
6	Not Applicable	**0.0576**	0.0576

Calculate DC(n), D*n & BV(n)

N	MACRS DEPRECIATION RATE	DC(n)	D*n	BV(n)
1	0.2000	2000	2000	8000
2	0.3200	3200	5200	4800
3	0.1920	1920	7120	2880
4	0.1152	1152	8272	1728
5	0.1152	1152	9424	576
6	0.0576	576	10000	0

Cost Basis of a car (B) = Rs.
10,000 Salvage Value (S) = 0

7.3 Introduction to Corporate Income Tax

A corporation includes associations, joint stock companies, issuance companies, and trust and partnerships that actually operate as association or corporations. Organizations of doctors, lawyers, engineers and other professionals are generally recognized as corporations. Such organizations have following characteristics

- associates organized to carry on business
- gains from the business that are divided
- continuity of life and centralized management
- limited liability and free transferability of interest

Organizations possessing a majority of these characteristics must file corporate tax returns. Income taxes are due from corporations and businesses whenever revenue exceeds allowable tax deductions. Revenues includes sales to customers of goods and services, dividends received on stocks, interest from loans and securities, rents, royalties and other gains from ownership of capital or property.

Deductions embraces a wide range of expenses incurred in the production of revenue: wages, salaries, rents, repairs, interest on loan taken, taxes, materials employee benefit, advertising etc. Also deductible, sometimes under special provisions, are losses from fire, theft, contribution, depreciation, depletion, research and development expenditures and outlays to satisfy legislated objectives such as pollution control.

The difference between revenue and deductions is taxable income. In general,

Taxable income = Gross income - Expenses - interest on debt - depreciations - other allowable decoctions.

7.4 After Tax Cash Flow Estimate

The transfer from estimating cash flow before taxes (CFBT) to cash flow after taxes (CFAT) involves a consideration of significant tax effects that may alter the final decision, as well as estimate the magnitude of the tax effect on cash flow over the life of the alternative. The after tax cash flow is the net proceeds from an income generating asset, after all costs (taxes, mortgages, interest, maintenance costs etc.) of owning and operating the property.

Some basic tax terms and relationships are explained here.

Gross income (GI) is the total income realized from all revenue-producing sources of the corporation, plus any income from other sources such as sale of assets, royalties, and license fees.

Income tax is the amount of taxes based on some form of income or profit levied by the government. A large percentage of tax revenue is based upon taxation of corporate and personal income. Taxes are actual cash flows.

Operating expenses (E*)* include all corporate costs incurred in the transaction of business. These expenses are tax deductible for corporations. For engineering economy alternatives, these are the AOC (annual operating cost) and M&O (maintenance and operating) costs.

Taxable income (TI) is the amount upon which income taxes are based. For corporations, depreciation *D* and operating expenses (E) are tax-deductible.

Taxable income(TI) = Gross Income(GI) – Operating Expenses (O) – Depreciation (D)

TI = GI – E - D

Tax rate (T) is a percentage, or decimal equivalent, of TI owed in taxes. The tax rate is graduated; that is, higher rates apply as TI increases.

Taxes = (Taxable Income) * (Applicable Tax Rate)

Taxes = (TI) * (T)

Example:

Develop cash flow after tax (CFAT). Use MACRS depreciation. Purchase Price of a car (I) = Rs. 10,000. Salvage Value (S) = 0.
It is classed as 5 year property. Annual Revenue = Rs 3500.
Tax Rate = 50%

R = Annual Depreciation Rate = 2 * (1/N)
= 2 * (1/5) = 2 * (0.2) = 0.4 = 40%

Calculate MACRS Dep. Rate				
EOY	**Cash Flow Before Tax**	**DDB Dep. Rate**	**St. Line Dep. Rate**	**MACRS Dep. Rate**
1	3,500	**0.2000**	0.1000	**0.2000**
2	3,500	**0.3200**	0.1778	**0.3200**
3	3,500	**0.1920**	0.1371	**0.1920**
4	3,500	**0.1152**	**0.1152**	**0.1152**
5	3,500	0.0691	**0.1152**	**0.1152**
6	3,500	Not Applicable	**0.0576**	**0.0576**

Calculate after tax cash flow					
EOY	**MACRS Dep. Rate**	**MACRS Dep. DC(n)**	**Taxable Income**	**Tax 50%**	**Cash Flow After Tax (CFAT)**
1	**0.2000**	2000	1500	750	2750
2	**0.3200**	3200	300	150	3350
3	**0.1920**	1920	1,580	790	2710
4	**0.1152**	1152	2,348	1174	2326
5	**0.1152**	1152	2,348	1174	2326
6	**0.0576**	576	2,948	1462	2038

7.5 General Procedure for Making After Tax Economic Analysis

The transfer from estimating cash flow before taxes (CFBT) to cash flow after taxes (CFAT) involves a consideration of significant tax effects that may alter the final decision, as well as estimate the magnitude of the tax effect on cash flow over the life of the alternative. Mutually exclusive alternative comparisons using after-tax PW, AW, and ROR methods are explained with major tax implications considered.

Example:

Evaluate after tax PW. Use MACRS depreciation. Purchase price of a car (I) = Rs. 10,000. Salvage Value (S) = 0. It is classed as 5 year property. Revenue/Yr = Rs 3500. Tax Rate = 50 & MARR = 10%.

R = Annual Depreciation Rate = 2 * (1/N)
= 2 * (1/5) = 2 * (0.2) = 0.4 = 40%

Calculate Cash Flow After Tax (CFAT) as in previous problem

EOY	CFBT	DDB Dep. Rate	SL Dep. Rate	MACRS Dep. Rate	MACRS Dep. DC(n)	Taxable Income	Tax 50%	CFAT
1	3,500	0.2000	0.1000	0.2000	2000	1500	750	2750
2	3,500	0.3200	0.1778	0.3200	3200	300	150	3350
3	3,500	0.1920	0.1371	0.1920	1920	1,580	790	2710
4	3,500	0.1152	0.1152	0.1152	1152	2,348	1174	2326
5	3,500	0.0691	0.1152	0.1152	1152	2,348	1174	2326
6	3,500	Not Applicable	0.0576	0.0576	576	2,948	1462	2038

EOY	PW Factor	Tax	PW of Tax	CFAT	NPW After Tax
0	1.1^{-0}		-	10,000	10,000
1	1.1^{-1}	750	682	2750	2500
2	1.1^{-2}	150	372	3350	2769
3	1.1^{-3}	790	594	2710	2036
4	1.1^{-4}	1174	802	2326	1589
5	1.1^{-5}	1174	729	2326	1444
6	1.1^{-6}	1462	825	2038	1150
		Total	4004	Total	1498

After tax Net Present Worth (NPW) = 1498 > 0.
Therefore, the project is feasible.

Example:

Evaluate before and after tax IRR using (FW formulation) for above problem.

Before Tax IRR using FW formulation

FW ($i^*\%_{\textbf{Before Tax}}$)= 0

FW ($i^*\%_{\textbf{Before Tax}}$) =

- 10,000 *(F/P, $i^*\%_{\textbf{Before Tax}}$, **6**) + 3500*(F/A, $i^*\%_{\textbf{Before Tax}}$, 6) = 0

After Tax IRR using FW formulation

FW ($i^*\%_{\textbf{After Tax}}$)= 0

FW ($i^*\%_{\textbf{After Tax}}$) =

- 10,000 * (F/P, $i^*\%_{\textbf{After Tax}}$, 6) + 2750 * (F/P, $i^*\%_{\textbf{After Tax}}$, 5)

+ 3350 * (F/P, i*% $i^*\%_{\textbf{After Tax}}$, 4) + 2710 * (F/P, $i^*\%_{\textbf{After Tax}}$, 3)

+ 2326 * (F/P, $i^*\%_{\textbf{After Tax}}$, 2) + 2326 * (F/P, $i^*\%_{\textbf{After Tax}}$, 1)

+ 2038 * (F/P, $i^*\%_{\textbf{After Tax}}$, 0) = 0

EOY	Before Tax Cash Flow	After Tax Cash Flow
0	-10,000	-10,000
1	3,500	2750
2	3,500	3350
3	3,500	2710
4	3,500	2326
5	3,500	2326
6	3,500	2038
IRR = i*	**26.43%**	**15.29%**

CHAPTER 8

INFLATION

8.1 CONCEPT OF INFLATION

Inflation is the term used to describe a decline in purchasing power evidenced in an economic environment of rising prices. Inflation exhibits a loss in the purchasing power of money over time. Inflation means that the cost of an item tends to increase over time, or put it another way, the same dollar amount buys less of an item over time. Inflation causes prices to rise and the decrease the purchasing power of a unit of money with passage of time. Deflation has the opposite effect. Deflation is the opposite of inflation in that prices usually decrease over time, hence, a specified dollar amount gains in purchasing power. Inflation and deflation are terms that describe changes in price levels in an economy. Inflation is far more common than deflation in the real world.

Prior to this chapter, we have assumed that prices for goods & services in the market place are unchanged over extended periods of time. Unfortunately, this is not generally a realistic assumption. General Price Inflation is defined as the phenomenon of a general increase in the prices paid for goods and services bringing about a reduction in the purchasing power of the monetary unit, is a business reality that can affect the economic comparison of alternatives.

8.2 MEASURING INFLATION

Inflation is difficult to measure because the prices of different goods and services do not increase or decrease by the same amount, nor do they change at the same time. Inflation rates are measured by Wholesale Price Index (WPI), Producer's Price Index (PPI), Consumer's Price Index (CPI).

CPI is the based on typical market basket of goods & services required by the average common consumer. The market basket normally consists of items food, housing apparel, transportation, medical care, entertainment, personnel care and other goods & serves. CPI is composite price index that measure price changes in these items.

CPI is a good measure of the general increase in prices of consumer products. However, it is not a good measure of industrial price increase. In performing engineering economic price indexes must be selected to estimate the price increase of raw materials, finished products, and operating costs.

To measure historical price-level changes for particular commodities, it is necessary to calculate a price index. A price index is the ration of historical price of some commodities or services at some point in time to the price at some point in time to the price at some earlier point. The earlier point is usually some selected base year. Thus, index or indexes can be relate

Price $\text{Index}_{2017} = \frac{Commodity\ price\ 2017}{Commodity\ price\ 1980} = \frac{Rs.\ 463/kg}{Rs.\ 100/kg} = 4.63$

It indicates that 2017 price is 4.63timmes greater than price of base year 1980.

Suppose an individual can invest Rs. 100 at the present time with the expectation of earning 15% annually for the next 5 years. At the end of 5 years, the accumulated amount will be FW = $(1.15)^5$ = Rs. 201.10. At present, his individual can purchase a commodity for Rs. 100, but suppose that cost of that commodity increases an annual rate of 10%. At the end of 5 years, the same commodity will cost FW = $(1.10)^5$ = Rs. 161.10. It may have false impression that if the invested now, he can purchase two commodities at the end of 5 years if he ignored the changes in prices. Actually, he can purchase only 1.25 commodities. If earning power is 10% and increase in commodity price is 15%, actually he can purchase only 0.80 commodities. Thus, when considering time value of money, one must include the impact of changes in prices (i.e. changes in purchasing power of money) as well as the effect of the earning power. When incorporating changes in price levels in the engineering economic studies the index selected should measure those changes that are pertinent to the individual or organization undertaking the study.

Calculating Yearly Inflation Rate

Inflation Rate of year_n to $\text{year}_{n-1} = \frac{\text{CPIn} - \text{CPIn}-1}{\text{CPIn}-1}$

Year	CPI	F
2000	100	-
2001	104	4.00%
2002	112.32	8.00%

Inflation Rate from 2000 to 2001

$f_{\textbf{2000 to 2001}} = \frac{\text{CPIn} - \text{CPIn}-1}{\text{CPIn}-1} = \frac{104-100}{100} = 4\%$

Inflation Rate from 2001 to 2002

$f_{\textbf{2001 to 2002}} = \frac{\text{CPIn} - \text{CPIn}-1}{\text{CPIn}-1} = \frac{112.32-104}{104} = 8\%$

Average Inflation rate: annual Inflation Rate varies from year to year. Since each individual rate is based on previous year's rate, all these rates have a compounding effect.

To find the price at the end of second year, we use the process of compounding
FW at the end of 2002 = [{Rs. 100 (1.04)}*(1.08)] = Rs.112.32
To find the average (compounding) inflation rate $f_{\textbf{2000 to 2002}}$,
we establish the following equivalence;
Rs. 100 $(1+f_{\textbf{2000 to 2002}})^2$ = Rs. 112.32

$f_{\textbf{2000 to 2002}}$ = 5.98%

Price increase in the last two years is equivalent to an average annual percentage of 5.98% per year. Note that the average is a geometric (not an arithmetic) average over a several-year period. Our computations are simplified by using a single average rate such as this, rather than a different rate for each year's cash flows. In terms of CPI, we define average annual inflation rate as

$$CPI_{n-1} = CPI_n (1+f)^2$$

$$f = \left[\frac{\mathrm{CPI(n-1)}}{\mathrm{CPIn}}\right]^{1/n} - 1$$

f = Average General annual Inflation Rate from Base period 2000 to 2002 = $\left[\frac{112.32)}{100}\right]^{1/n}$ - 1= 5.98%

Year	CPI	F
2000	100	-
2001	104	4.00%
2002	112.32	8.00%
2003	117.94	5.00%
2004	120.29	2.00%
2005	127.51	6.00%
2006	140.26	10.00%
2007	150.08	7.00%
2008	154.58	3.00%
2009	160.76	4.00%
2010	170.40	6.00%

Rs. 100 $(1+f)^{10}$ = Rs. 170.40, f = 5.47%

Average General annual Inflation Rate from Base period 2000 to 2010

$f_{\textbf{2000 to 2010}}$= $\left[\frac{170.40)}{100}\right]^{1/10}$ - 1= 5.47%.

8.3 Equivalence Calculations under inflation

Actual versus Constant Dollars

To introduce the effect of inflation into our economic analysis, we need to define several inflation- related terms:

- **Actual (current) dollars (An):** Out-of- pocket dollars paid at the time of purchasing goods and services. Actual dollars are estimates of future cash flows for year n that take into account any anticipated changes in amounts caused by inflationary or deflationary effects. Usually, these amounts are determined by applying an inflation rate to base-year dollar estimates.
- **Constant (real) dollars (A'n):** Dollars in some base year used to adjust for the effects of inflation. Constant dollars represent constant purchasing power that is independent of the passage of time. In situations where inflationary effects were assumed when cash flows were estimated, the estimates obtained can be converted to constant dollars (base-year dollar) by adjustment with some readily accepted general inflation rate. We assume that the base year is always time 0, unless we specify otherwise.

$$An = A'n\,(F/P,f,n) = A'n\,(1 + f)^{n}$$

$$A'n = An\,(P/F,f,n) = An\,(1 + f)^{-n}$$

Market & Inflation- Free Interest Rate

There are actually three different inflation-related rates that are important: the market interest rate (**i**), the real interest rate (**i'**), and the inflation rate (**f**). Only the first two are interest rates.

Inflation-Adjusted Interest Rate (i)

As its name implies, this is the interest rate that has been adjusted to take inflation into account. The *market interest rate,* which is the one we hear every day, is an inflation-adjusted rate. This rate is a combination of the real interest rate **(i')** and the inflation rate **(f)**, and, therefore, it changes as the inflation rate changes. Most firms use a market interest rate (also known as an inflation-adjusted MARR) in evaluating their investment project. Interest quoted by financial institutions that accounts both earning and purchasing power. This rate takes into account the combined effects of the earning value of the capital (earning power) and any anticipated inflation or deflation (purchasing power). Virtually all interest rates stated by financial institutions for loans & savings accounts are market interest rates.

$(1 + i) = (1 + f) + (1 + i')$

$i = (1 + f) + (1 + i') - 1$ $\quad\quad$ $\mathbf{i = i' + f + i'f}$

Real or Inflation-Free or Constant Dollar Interest Rate (i')

This is the rate at which interest is earned when the effects of changes in the value of currency (inflation) have been removed. Thus, the real interest rate presents an actual gain in purchasing power.

$(1 + i) = (1 + f) + (1 + i')$

$$i' = i - f - i'f$$

Inflation rate (f)

As described above, this is a measure of the rate of change in the value of the currency. A company's MARR adjusted for inflation is referred to as the inflation-adjusted MARR.

In calculating any cash flow equivalence, we need to identify the nature of project cash flows. The three common cases are as follows:

Case 1: Constant-Dollar Analysis. All cash flow elements are estimated in constant dollars.

Case 2: Actual-Dollar Analysis. All cash flow elements are estimated in actual dollars.

Case 3: Some of the cash flow elements are estimated in constant dollars, and others are estimated in actual dollars. In such case, we simply convert all cash flow elements into one type – either in constant or actual dollars. Then we proceed with either constant-dollar analysis as for case 1 or actual-dollar analysis as for case 2.

Constant-Dollar Analysis.

Suppose that all cash flow elements are already given in constant dollar, and that we want to compute the equivalent PW of the constant dollar (A'n) occurring in year n. In the absence of inflationary effect, we use inflation free interest rate (i') to account only for the earning power of money. Constant dollar analysis is common in the evaluation of many long-term public projects, because governments do not pay income taxes. Typically, income taxes are levied on basis of taxable incomes in actual dollars.

$$Pn = A'n\ (P/F,I',n) = A'n\ (1 + i')^{-n}$$

Example:

What would be PW if expected to earn a 12% inflation free rate of return. Net before tax cash flows in constant dollars are:

0	1	2	3	4	5
-250,000	100,000	110,000	120,000	130,000	120,000

inflation free interest rate (i') = 12%

PW(12%) = -25000 + 100,000(P/A,12%,5) + 10,000(P/G,12%,4) +20,000(P/F,12%,5)= $163,099 in constant dollars.

Actual-Dollar Analysis.

If all cash flow elements are estimated in actual dollars, to find the equivalent present worth of the actual dollar amount (An) in year n, there are two methods:

- Deflation Method
- Adjusted-discount method

Deflation Method

The deflation method requires two steps to convert actual dollars into equivalent present worth dollars.

Step 1: First we convert actual dollars into equivalent constant dollars by discounting the general inflation rate which removes the inflationary effect.

Step2: Using inflation-free interest rate (i'), find out equivalent present worth.

Adjusted-Discount Method

Adjusted-discount method performs deflation and discounting in one step. Mathematically, we can combine this two-step procedure into one with the formula

$$Pn = \frac{An}{\frac{(1+f)n}{(1+i')n}} = \frac{An}{[(1+f')n(1+i')n]} = \frac{An}{(1+i)n}$$

$$\frac{An}{[(1+f')n(1+i')n]} = \frac{An}{(1+i)n}$$

$(1 + i) = (1 + f)(1 + i')$

$i = i' + f + i'f$

If $f = 0$, $I = i'$

In practice, we often approximate the market interest rate (i) simply by adding the inflation rate (f) to the real interest rate (i') and ignoring the product (i'f). This practice is fine as long as either i' or f is relatively small.

Example:

Equivalence calculation when cash flows are stated in actual dollars:

A Project is expected to generate the following net cash flows in actual dollars.

0	1	2	3	4	5
-75,000	32,000	35,700	32,800	29,000	58,000

a. What are the equivalent year-0 dollars (constant dollars) if the general inflation rate (f) is 5% per year?

b. Compute the present worth of these cash flows in constant dollars at i' = 10%.

Step 1: Convert the actual dollars into constant dollars:

$A'n = An\,(P/F,f,n) = An\,(1 + f)^{-n}$

N	Net Cash Flow in Actual Dollars (An)	Deflation Factor $(1 + f)^{-n}$	Cash Flows in Constant Dollars (An')
0	-75,000	$(1+0.05)^{-0}$	-75,000
1	32,000	$(1+0.05)^{-1}$	30,476
2	35,700	$(1+0.05)^{-2}$	32,381
3	32,800	$(1+0.05)^{-3}$	28,334
4	29,000	$(1+0.05)^{-4}$	23 ,858
5	58,000	$(1+0.05)^{-5}$	45,445

Step 2: Calculate equivalent Present Worth

PW(10%) = -75000 +30,476*(P/F,10%,1) +32,381*(P/F,10%,2) +28,334*(P/F,10%,3) + 23,858*(P/F,10%,4) + 45,445*(P/F,10%,5)

N	Cash Flows in Constant Dollars (An')	Discounting Factor $1/(1 + i')^{n}$	Equivalent Present Worth
0	-75,000	$(1.10)^{-0}$	-75,000
1	30,476	$(1.10)^{-1}$	27,706
2	32,381	$(1.10)^{-2}$	26,761
3	28,334	$(1.10)^{-3}$	21,288
4	23 ,858	$(1.10)^{-4}$	16,296
5	45,445	$(1.10)^{-5}$	28,217
		∑PW(10%)	45,268

❖ Solve the above problem
using Adjusted-discount Method f = 5% & i' = 10%

Compute the market interest rate (i)

$i = i' + f + i*f$

$i = 0.10 + 0.05 + 0.10*0.05$

$i = 15.5\%$

PW(15.5%)

$= -75000*(1.155)^{-0} + 32{,}000*(1.155)^{-1} + 35{,}700*(1.155)^{-2}$
$+32{,}800*(1.155)^{-3} + 29{,}000*(1.155)^{-4}\ 58{,}000*(1.155)^{-5}$

= 45268