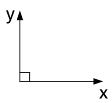
Chapter 6
Basic mechanics

BASIC PRINCIPLES OF STATICS

Statics is the branch of mechanics that deals with the equilibrium of stationary bodies under the action of forces. The other main branch – dynamics – deals with moving bodies, such as parts of machines.

Static equilibrium

A planar structural system is in a state of static equilibrium when the resultant of all forces and all moments is equal to zero, i.e.



$$\begin{array}{ccccccc} \Sigma F_x = 0 & & \Sigma F_x = 0 & & \Sigma F_y = 0 & & \Sigma M_a = 0 \\ \Sigma F_y = 0 & \text{or} & \Sigma M_a = 0 & \text{or} & \Sigma M_a = 0 & \text{or} & \Sigma M_b = 0 \\ \Sigma M_a = 0 & & \Sigma M_b = 0 & & \Sigma M_b = 0 & & \Sigma M_c = 0 \end{array}$$

where F refers to forces and M refers to moments of forces.

Static determinacy

If a body is in equilibrium under the action of coplanar forces, the statics equations above must apply. In general, three independent unknowns can be determined from the three equations. Note that if applied and reaction forces are parallel (i.e. in one direction only), then only two separate equations can be obtained and thus only two unknowns can be determined. Such systems of forces are said to be statically determinate.

Force

A force is defined as any cause that tends to alter the state of rest of a body or its state of uniform motion in a straight line. A force can be defined quantitatively as the product of the mass of the body that the force is acting on and the acceleration of the force.

$P = ma$

where
P = applied force
m = mass of the body (kg)
a = acceleration caused by the force (m/s^2)

The *Système Internationale* (SI) units for force are therefore kg m/s^2, which is designated a Newton (N). The following multiples are often used:

1 kN = 1 000 N, 1 MN = 1 000 000 N

All objects on earth tend to accelerate toward the centre of the earth due to gravitational attraction; hence the force of gravitation acting on a body with the mass (m) is the product of the mass and the acceleration due to gravity (g), which has a magnitude of 9.81 m/s^2:

$F = mg = v\rho g$

where:
F = force (N)
m = mass (kg)
g = acceleration due to gravity (9.81m/s^2)
v = volume (m^3)
ρ = density (kg/m^3)

Vector

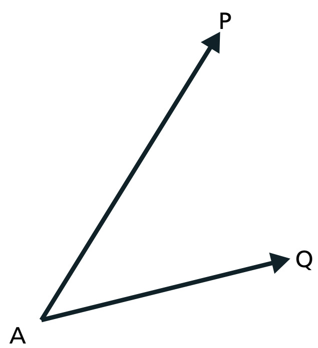
Most forces have magnitude and direction and can be shown as a vector. The point of application must also be specified. A vector is illustrated by a line, the length of which is proportional to the magnitude on a given scale, and an arrow that shows the direction of the force.

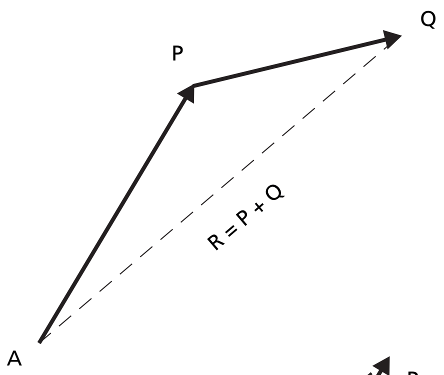
Vector addition

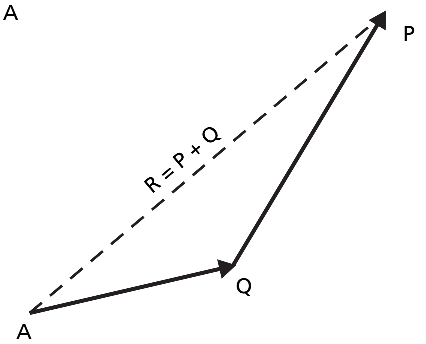
The sum of two or more vectors is called the resultant. The resultant of two concurrent vectors is obtained by constructing a vector diagram of the two vectors.

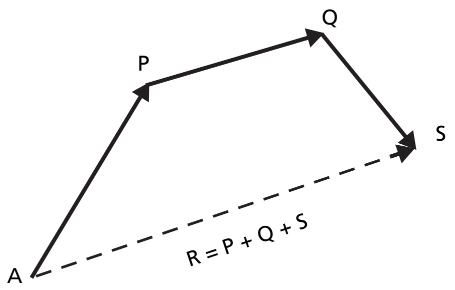
The vectors to be added are arranged in tip-to-tail fashion. Where three or more vectors are to be added, they can be arranged in the same manner, and this is called a polygon. A line drawn to close the triangle or polygon (from start to finishing point) forms the resultant vector.

The subtraction of a vector is defined as the addition of the corresponding negative vector.



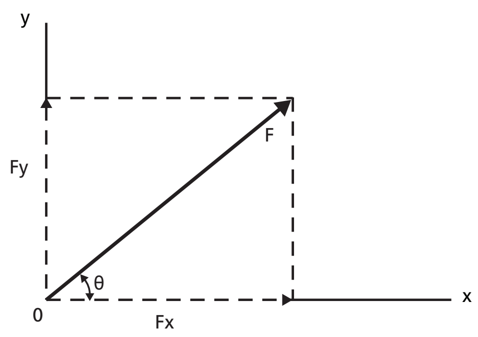






Resolution of a force

In analysis and calculation, it is often convenient to consider the effects of a force in directions other than that of the force itself, especially along the Cartesian (xx-yy) axes. The force effects along these axes are called vector components and are obtained by reversing the vector addition method.



F_y is the component of F in the y direction $F_y = F \sin\theta$
F_x is the component of F in the x direction $F_x = F \cos\theta$

Concurrent coplanar forces

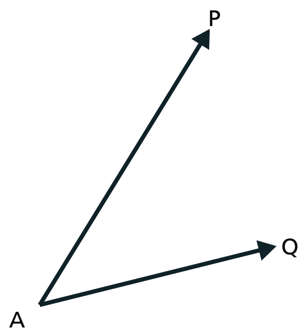
Forces whose lines of action meet at one point are said to be *concurrent. Coplanar* forces lie in the same plane, whereas *non-coplanar* forces have to be related to a three-dimensional space and require two items of directional data together with the magnitude. Two coplanar non-parallel forces will always be concurrent.

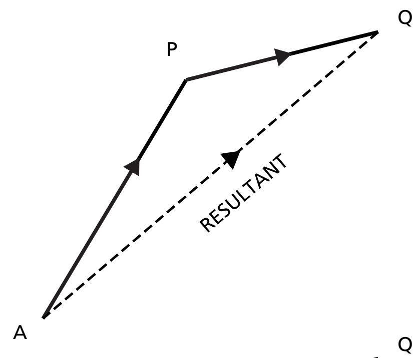
Equilibrium of a particle

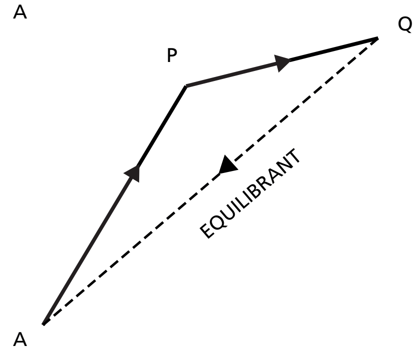
When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium,* i.e. it is not disturbed from its existing state of rest (or uniform movement).

The closed triangle or polygon is a graphical expression of the equilibrium of a particle.

The equilibrium of a particle to which a single force is applied may be maintained by the application of a second force that is equal in magnitude and direction, but opposite in sense, to the first force. This second force, which restores equilibrium, is called the *equilibrant*. When a particle is acted upon by two or more forces, the *equilibrant* has to be equal and opposite to the resultant of the system. Thus the *equilibrant* is the vector drawn closing the vector diagram and connecting the finishing point to the starting point.







Free-body diagram of a particle

A sketch showing the physical conditions of a problem is known as a *space diagram*. When solving a problem it is essential to consider all forces acting on the body and to exclude any force that is not directly applied to the body. The first step in the solution of a problem should therefore be to draw a *free-body diagram*.

A free-body diagram of a body is a diagrammatic representation or a sketch of a body in which the body is shown completely separated from all surrounding bodies, including supports, by an imaginary cut, and the action of each body removed on the body being considered is shown as a force on the body when drawing the diagram.

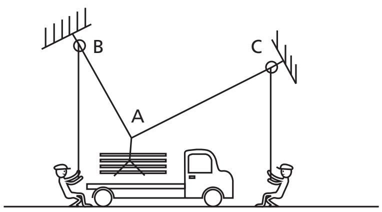
To draw a free-body diagram:

1. Choose the free body to be used, isolate it from any other body and sketch its outline.
2. Locate all external forces on the free body and clearly mark their magnitude and direction. This should include the weight of the free body, which is applied at the centre of gravity.
3. Locate and mark unknown external forces and reactions in the free-body diagram.
4. Include all dimensions that indicate the location and direction of forces.

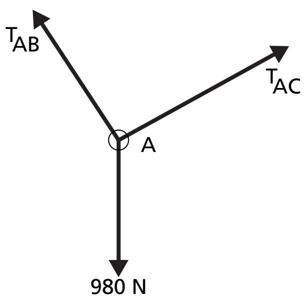
The free-body diagram of a rigid body can be reduced to that of a particle. The free-body of a particle is used to represent a point and all forces working on it.

Example 6.1

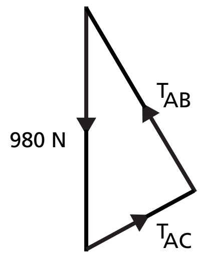
Determine the tension in each of the ropes AB and AC



Space diagram

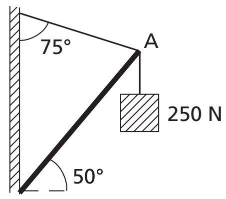


Free body diagram
for point A

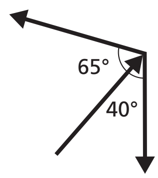


Example 6.2

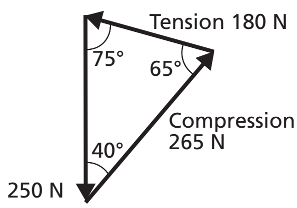
A rigid rod is hinged to a vertical support and held at 50° to the horizontal by means of a cable when a weight of 250 N is suspended as shown in the figure. Determine the tension in the cable and the compression in the rod, ignoring the weight of the rod.



Space diagram



Free-body diagram for point A



Force triangle

The forces may also be calculated using the law of sines:

$$\frac{\textit{Compression in rod}}{\sin 75°} = \frac{\textit{Tension in cable}}{\sin 40°} = \frac{250\text{ N}}{\sin 65°}$$

Point of concurrency

Three coplanar forces that are in equilibrium must all pass through the same point. This does not necessarily apply for more than three forces.

If two forces (which are not parallel) do not meet at their points of contact with a body, such as a structural member, their lines of action can be extended until they meet.

Collinear forces

Collinear forces are parallel and concurrent. The sum of the forces must be zero for the system to be in equilibrium.

Coplanar, non-concurrent, parallel forces

Three or more parallel forces are required. They will be in equilibrium if the sum of the forces equals zero and the sum of the moments around a point in the plane equals zero. Equilibrium is also indicated by two sums of moments equal to zero.

Reactions

Structural components are usually held in equilibrium by being secured to rigid fixing points; these are often other parts of the same structure. The fixing points or supports will react against the tendency of the applied forces (loads) to cause the member to move. The forces generated in the supports are called reactions.

In general, a structural member has to be held or supported at a minimum of two points (an exception to this is the cantilever). Anyone who has tried ‘balancing’ a long pole or a similar object will realize that, although only one support is theoretically necessary, two are needed to give satisfactory stability.

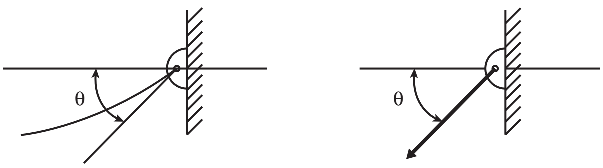
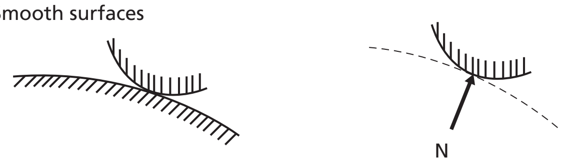
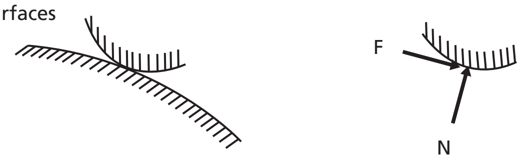
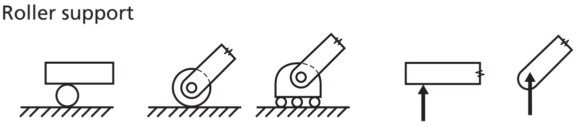
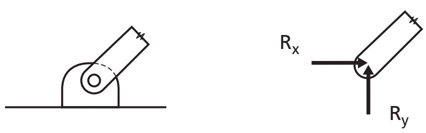
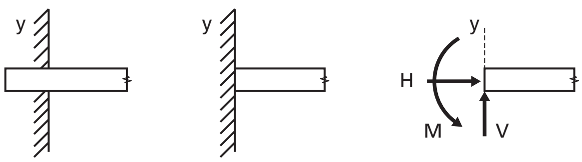
Resultant of gravitational forces

The whole weight of a body can be assumed to act at the centre of gravity of the body for the purpose of determining supporting reactions of a system of forces that are in equilibrium. Note that, for other purposes, the gravitational forces cannot always be treated in this way.

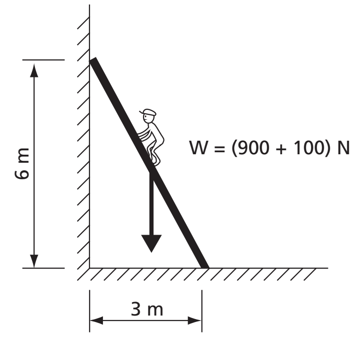
Example 6.3

A ladder rests against a smooth wall and a person weighing 900 N stands on it at the middle. The weight

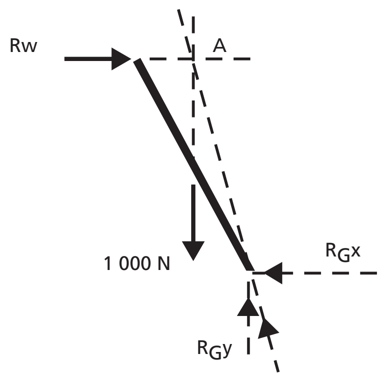
TABLE 6.1
Actions and reactions

Flexible cable or rope		Force exerted by the cable or rope is always tension away from the fixing, in the direction of the tangent to the cable curve.
Smooth surfaces		Reaction is normal to the surface, i.e., at right angles to the tangent.
Rough surfaces		Rough surface is capable of supporting a tangental force as well as a normal reaction. Resultant reaction is vectorial sum of these two.
Roller support		Reaction is normal to the supporting surface only.
Pin support		A freely hinged support is fixed in position, hence the two reaction forces, but is not restrained in direction - it is free to rotate.
Built-in support		The support is capable of providing a longitudinal reaction (H), a lateral or transverse reaction (V), and a moment (M). The body is fixed in position and fixed in direction.

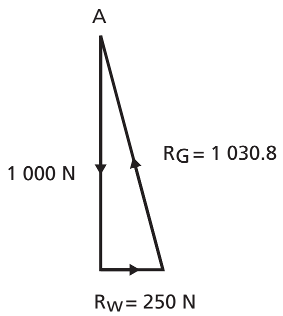
of the ladder is 100 N. Determine the support reactions at the wall (R_W) and at the ground (R_G).



Space diagram



Free-body diagram of ladder



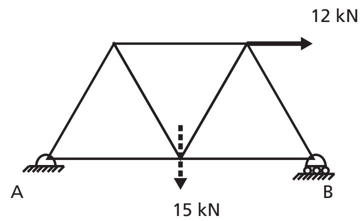
Force diagram

As the wall is smooth, the reaction R_W must be at right angles to the surface of the wall and is therefore horizontal. A vertical component would have indicated a friction force between the ladder and the wall. At the bottom, the ladder is resting on the ground, which is not smooth, and therefore the reaction R_G must have both a vertical and a horizontal component.

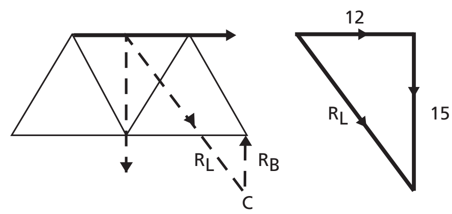
As the two weight forces in this example have the same line of action, they can be combined into a single force, reducing the problem from one with four forces to one with only three forces. The point of concurrency (A) can then be found, giving the direction of the ground reaction force. This in turn enables the force vector diagram to be drawn, and hence the wall and ground reactions determined.

Example 6.4

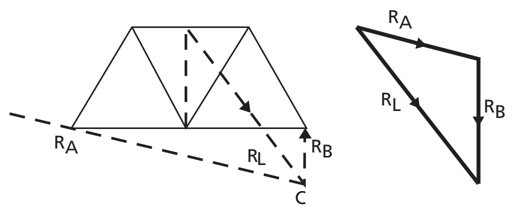
A pin-jointed framework (truss) carries two loads, as shown. The end A is pinned to a rigid support, while the end B has a roller support. Determine the supporting reactions graphically:



1. Combine the two applied forces into one and find the line of action.
2. Owing to the roller support reaction R_B will be vertical. Therefore the resultant line (R_L) must be extended to intersect the vertical reaction of support B. This point is the point of concurrency for the resultant load, the reaction at B and the reaction at A.



3. From this point of concurrency, draw a line through the support pin at A. This gives the line of action of the reaction at A.



4. Use these three force directions and the magnitude of R_L to draw the force diagram, from which R_A and R_B can be found.

Answer: R_A = 12.2 kN at 21° to horizontal.
R_B = 12.7 kN vertical.

The link polygon (see an engineering handbook) may also be used to determine the reactions to a beam or a truss, though it is usually quicker and easier to obtain the reactions by calculation, the method shown in Example 6.4, or a combination of calculation and drawing.

However, the following conditions must be satisfied.

1. All forces (apart from the two reactions) must be known completely, i.e. magnitude, line of action and direction.
2. The line of action of one of the reactions must be known.
3. At least one point on the line of action for the other reaction must be known (2 and 3 reduce the number of unknowns related to the equations of equilibrium to an acceptable level).

Moments of forces

The effect of a force on a rigid body depends on its point of application, as well as its magnitude and direction. It is common knowledge that a small force can have a large turning effect or leverage. In mechanics, the term 'moment' is used instead of 'turning effect'.

The moment of force with a magnitude (F) about a turning point (O) is defined as: $M = F \times d$, where d is the perpendicular distance from O to the line of action of force F. The distance d is often called lever arm. A moment has dimensions of force times length (Nm). The direction of a moment about a point or axis is defined by the direction of the rotation that the force tends to give to the body. A clockwise moment is usually considered as having a positive sign and an anticlockwise moment a negative sign.

The determination of the moment of a force in a coplanar system will be simplified if the force and its point of application are resolved into its horizontal and vertical components.

Example 6.5

As the ladder in Example 6.3 is at rest, the conditions of equilibrium for a rigid body can be used to calculate the reactions. By taking moments around the point where the ladder rests on the ground, the moment of the reaction R_G can be ignored as it has no lever arm (moment is zero). According to the third condition for equilibrium, the sum of moments must equal zero, therefore:

$(6 \times R_W) - (900 \text{ N} \times 1.5 \text{ m}) - (100 \text{ N} \times 1.5 \text{ m}) = 0$

$R_W = 250$ N

The vertical component of R_G must, according to the second condition, be equal but opposite to the sum of the weight of the ladder and the weight of the person on the ladder, because these two forces are the only vertical forces and the sum of the vertical forces must equal zero, i.e.

$R_{Gy} = 1\,000$ N

Using the first condition of equilibrium it can be seen that the horizontal component of R_G must be equal but opposite in direction to R_W, i.e.

$R_{GX} = 250$ N

Because R_G is the third side of a force triangle, where the other two sides are the horizontal and vertical components, the magnitude of R_G can be calculated as:

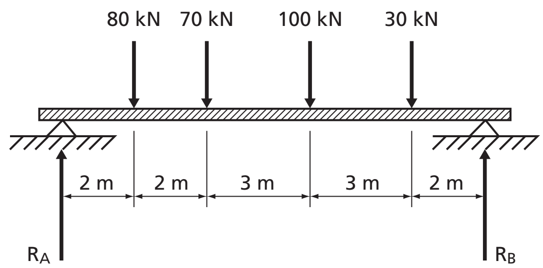
$(1\,000^2 + 250^2)^{1/2} = 1\,030.8$ N

Resultant of parallel forces

If two or more parallel forces are applied to a horizontal beam, then theoretically the beam can be held in equilibrium by the application of a single force (reaction) that is equal and opposite to the resultant R. The equilibrant of the downward forces must be equal and opposite to their resultant. This provides a method for calculating the resultant of a system of parallel forces. However, two reactions are required to ensure the necessary stability, and a more likely arrangement will have two or more supports.

The reactions R_A and R_B must both be vertical because there is no horizontal force component. Furthermore, the sum of the reaction forces R_A and R_B must be equal to the sum of the downward-acting forces.

Beam reactions



The magnitude of the reactions may be found by the application of the third condition for equilibrium, i.e. the algebraic sum of the moments of the forces about any point must be zero.

Take the moments around point A, then:

$(80 \times 2) + (70 \times 4) + (100 \times 7) + (30 \times 10) - (R_B \times 12) = 0;$

Giving $R_B = 120$ kN

R_A is now easily found with the application of the second condition for equilibrium.

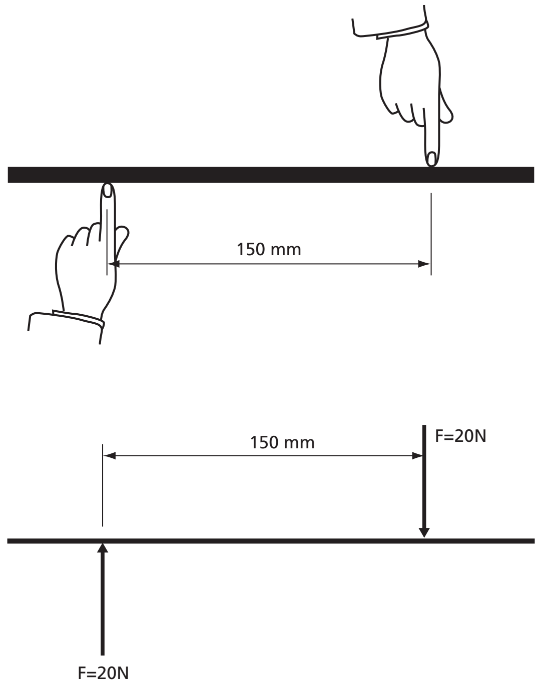
$R_A - 80 - 70 - 100 - 30 + R_B = 0$; with $R_B = 120$ kN gives:

$R_A = 160$ kN.

Couples

Two equal, parallel and opposite but non-collinear forces are said to be a couple.

A couple acting on a body produces rotation. Note that the couple cannot be balanced by a single force. To produce equilibrium, another couple of equal and opposite moment is required.



Loading systems

Before any of the various load effects (tension, compression, bending, etc.) can be considered, the applied loads must be rationalized into a number of ordered systems. Irregular loading is difficult to deal with exactly, but even the most irregular loads may be reduced and approximated to a number of regular systems. These can then be dealt with in mathematical terms using the principle of superposition to estimate the overall combined effect.

Concentrated loads are those that can be assumed to act at a single point, e.g. a weight hanging from a ceiling, or a person pushing against a box.

Concentrated loads are represented by a single arrow drawn in the direction, and through the point of action, of the force. The magnitude of the force is always indicated.

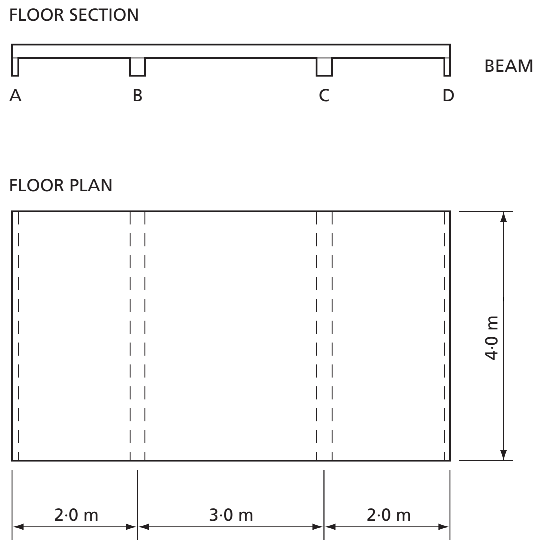
Uniformly distributed loads, written as UDL, are those that can be assumed to act uniformly over an area or along the length of a structural member, e.g. roof loads, wind loads, or the effect of the weight of water on a horizontal surface. For the purpose of calculation, a UDL is normally considered in a plane.

In calculating reactions, uniformly distributed loads can in most, but not all, cases be represented by a concentrated load equal to the total distributed load passing through the centre of gravity of the distributed load.

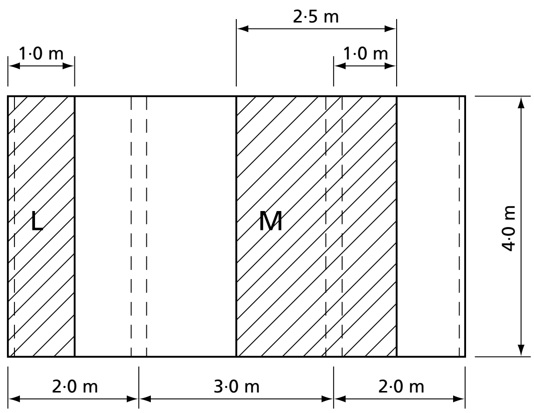
This technique must not be used for calculation of shear force, bending moment or deflection.

Example 6.6

Consider a suspended floor where the loads are supported by a set of irregularly placed beams. Let the load arising from the weight of the floor itself and the weight of any material placed on top of it (e.g. stored grain) be 10 kPa. Determine the UDL acting on beam A and beam C.



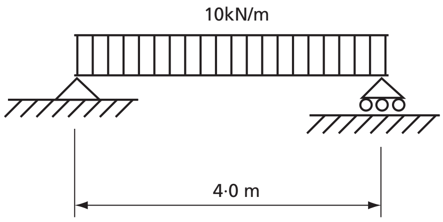
It can be seen from the figure below that beam A carries the floor loads contributed by half the area between the beams A and B, i.e. the shaded area L. Beam C carries the loads contributed by the shaded area M.



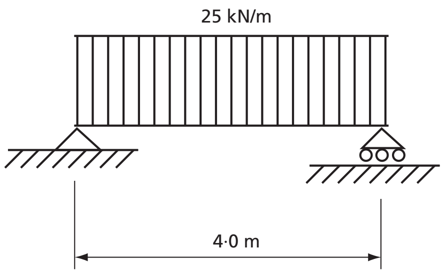
Therefore beam A carries a total load of:

1 m × 4 m × 10 kPa = 40 kN, or 40 kN / 4 = 10 kN / m.

In the same way, the loading of beam C can be calculated to 25 kN / m. The loading per metre run can then be used to calculate the required size of the beams.

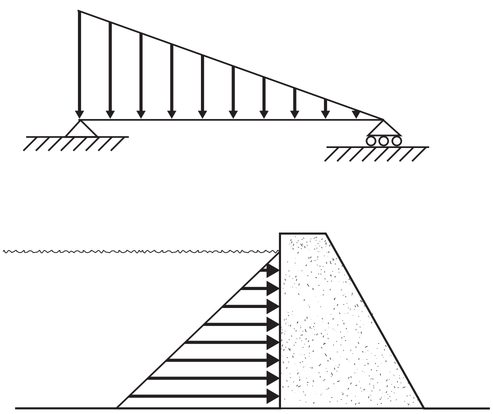


Loading of beam A



Loading of beam C

Distributed load with linear variation is another common load situation. The loading shape is triangular and is the result of such actions as the pressure of water on retaining walls and dams.



Distributed loads with linear variation

Shear force and bending moment of beams

A beam is a structural member subject to lateral loading in which the developed resistance to deformation is of a flexural character. The primary load effect that a beam is designed to resist is that of bending moments but, in addition, the effects of transverse or vertical shearing forces must be considered.

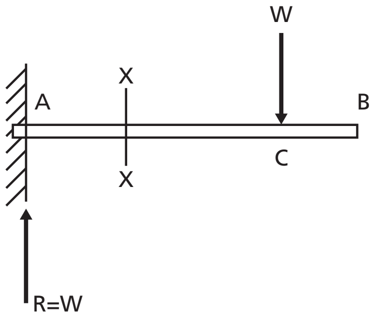
Shear force (V) is the algebraic sum of all the transverse forces acting to the left or to the right of the chosen section.

Bending moment (M) at any transverse cross-section of a straight beam is the algebraic sum of the moments, taken about an axis passing through the centroid of the cross-section, of all the forces applied to the beam on either side of the chosen cross-section.

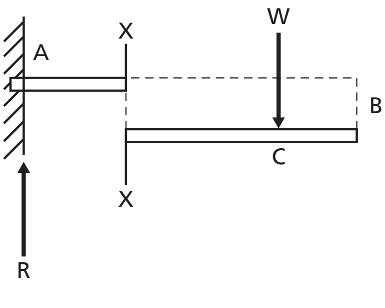
Consider the cantilever AB shown in (A). For equilibrium, the reaction force at A must be vertical and equal to the load W.

The cantilever must therefore transmit the effect of load W to the support at A by developing resistance (on vertical cross-section planes between the load and the support) to the load effect called shearing force. Failure to transmit the shearing force at any given section, e.g. section x-x, will cause the beam to fracture as in (B).

(A)



(B)



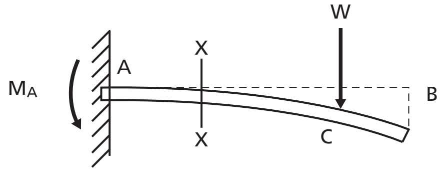
The bending effect of the load will cause the beam to deform as in (C). To prevent rotation of the beam at the support A, there must be a reaction moment at A, shown as M_A, which is equal to the product of load W and the distance from W to point A.

The shearing force and the bending moment transmitted across the section x-x may be considered as the force and moment respectively that are necessary to maintain equilibrium if a cut is made severing the beam at x-x. The free-body diagrams of the two portions of the beam are shown in (D).

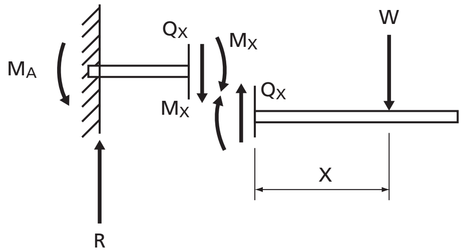
Then the shearing force between A and C = Q_x = W and the bending moment between A and C = M_x = $W \times AC$.

Note: Both the shearing force and the bending moment will be zero between C and B.

(C)



(D)



Definitions

Shear force (Q) is the algebraic sum of all the transverse forces acting to the left or to the right of the chosen section.

Bending moment (M) at any transverse cross section of a straight beam is the algebraic sum of the moments, taken about an axis passing through the centroid of the cross section, of all the forces applied to the beam on either side of the chosen cross section.

Table 6.2 shows the sign convention for shear force (Q) and bending moment (M) used in this book. Shearing forces, which tend to make the part of the beam to the left move up and the right part move down, are considered positive. The bending moment is considered positive if the resultant moment is clockwise on the left and anticlockwise on the right. These tend to make the beam concave upwards and are called *sagging* bending moments. If the moment is anticlockwise on the left and clockwise on the right, the beam will tend to become convex upwards – an effect called hogging.

TABLE 6.2
Shearing and bending forces

Load effect	Symbol	Sign convention: Positive (+)	Sign convention: Negative (–)	Units
Shearing force	Q	Up on the left	Down on the left	N kN
Bending moment	M	Sagging (top fibre in compression)	Hogging (top fibre in tension)	Nm kNm Nmm

Shear-force variation

Concentrated loads will change the value of the shear force only at points where they occur, i.e. the shear force remains constant in between. When the load is uniformly distributed, however, the shear force will vary at a uniform rate. Thus it will be seen that uniform loads cause gradual and uniform change of shear, while concentrated loads bring a sudden change in the value of the shear force.

Bending moment variation

Concentrated loads will cause a uniform change of the bending moment between the points of action of the loads. In the case of uniformly distributed loads, the rate of change of the bending moment will be parabolic. Maximum bending moment values will occur where the shear force is zero or where it changes sign.

Shear-force (SF) and bending-moment (BM) diagrams

Representative diagrams of the distribution of shearing forces and bending moments are often required at several stages in the design process. These diagrams are obtained by plotting graphs with the beams as the base and the values of the particular effect as ordinates. It is usual to construct these diagrams in sets of three, representing the distribution of loads, shearing forces and bending moments respectively. These graphical representations provide useful information regarding:

1. The most likely section where a beam may fail in shear or in bending.
2. Where reinforcement may be required in certain types of beam, e.g. concrete beams.
3. The shear-force diagram will provide useful information about the bending moment at any point.
4. The bending-moment diagram gives useful information on the deflected shape of the beam.

Some rules for drawing shear-force and bending-moment diagrams are:

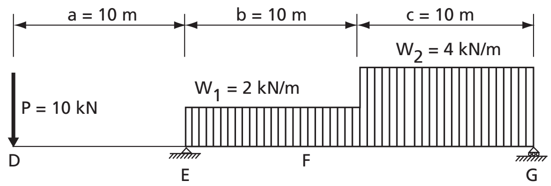
1. In the absence of distributed loads, the shear-force diagram consists of horizontal steps and the bending-moment diagram is a series of straight lines.
2. For a beam (or part of a beam) carrying a UDL only, the shear-force diagram is a sloping straight line and the bending diagram is a parabola.
3. At the point where the shear-force diagram passes through zero (i.e. where the SF changes sign), the BM has a maximum or minimum value.
4. Over a part of the span for which SF is zero, the bending moment has a constant value.
5. At a point where the bending-moment diagram passes through zero, the curvature changes from concave upwards to concave downwards or vice versa. This point is referred to as point of inflexion.
6. If a beam is subjected to two or more different systems of loading, the resulting shear and bending moment at a given section is the algebraic sum of the values at the section. This is referred to

as the principle of superposition and applies also to bending stresses, reactions and deflections.

The following example demonstrates the construction of diagrams representing shearing forces and bending moments.

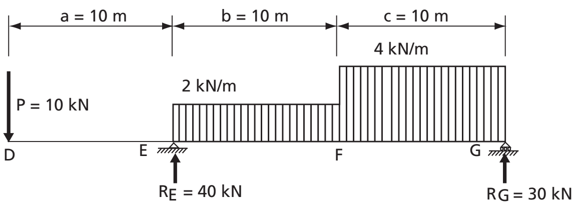
Example 6.7

The distribution of loads in a simply supported beam is as given in the diagram below. Determine the reactions at the supports and draw the shear-force and bending-moment diagram.



Solution:

(a) Draw the free-body diagram of the beam.



(b) Determine the reactions at the supports. First use the condition for equilibrium of moments about a point:

$\Sigma M_E = 0$

$M_E = (P \times a) + (w_1 \times b \times b / 2) + w_2 \times c(b + c / 2) - R_G (b + c) = 0$

$M_E = -(10 \times 10) + (2 \times 10 \times 5) + 4 \times 10 \times (15) - R_G (20) = 0$

$R_G = 30$ kN

$\Sigma F_y = 0$ hence

$\Sigma F_y = R_E + R_G - P - (w_1 \times b) - (w_2 \times c) = 0$

$\Sigma F_y = R_E + 30 - 10 - (2 \times 10) - (4 \times 10) = 0$

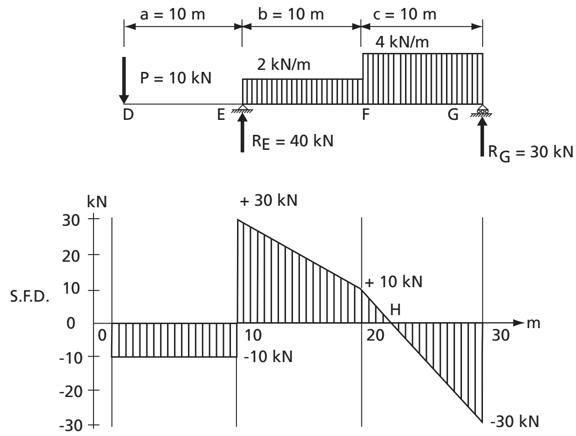
$R_E = 40$ kN

(c) Draw the shear-force diagram (SFD) directly below the loading diagram and choose a convenient scale to represent the shear force.

Calculate the values of the shear force to the left and to the right of all critical points. Critical points are:

- at concentrated loads;
- at reactions;
- at points where the magnitude of a distributed load changes.

1. Consider a section through the beam just to the left of D, and find the algebraic sum of all vertical forces to the left of this section. $\Sigma F_y = 0$, therefore, shear force to the left of D is zero.
2. Consider a section just to the right of D, algebraic sum of forces to the left of this section is 10 kN down to the left. Hence, shear force to the right of D is 10 kN (negative).
3. The same result as in point 2 above will be found for any such section between D and E. The shear-force diagram between D and E is thus a horizontal line at -10 kN.
4. Consider a section just to the right of E; the algebraic sum of forces to the left of this section is made up of P and R_E given that the shear force equals (-10 + 40) kN = + 30 kN, i.e. up to the left of section. Thus at E the shear-force diagram changes from -10 kN to + 30 kN.
5. As we approach the right-hand end of the beam we find the mathematics easier to consider on the right-hand side of any section. Section just to the left of F. Shear force = (4 kN / m × 10 m) - (30 kN) using the sign convention to determine positive or negative. Shear force here equals + 40 - 30 = + 10 kN.
6. Section just to the right of F. Shear force = + 40 - 30 = + 10 kN (i.e. no sudden change at F).
7. Section just to the left of G. Shear force = -30 kN
8. Variation of shear under a distributed load must be linear.



Note the following from the shear-force diagram:

- Maximum shear force occurs at E and G where the values are + 30 kN and - 30 kN respectively. These two transverse sections are the two most likely points for failure in shear.
- The maximum bending moment will occur where the shear force is zero or where the shear force changes sign. However, note that cantilevered beams will always have maximum bending at the fixed end.

The shear-force diagram in the example has two points where the shear force is zero. One is at E and the other is between H and G. The position of H can be calculated from the fact that at F the shear force is 10 kN and, under the action of UDL to the right of F, it reduces at the rate of 4 kN / m. It will read a value of zero after 2.5 m, i.e. the point H is 2.5 m to the right of F.

(d) Draw the bending-moment diagram directly under the shear-force diagram and choose a convenient scale to represent the bending moment. Calculate values of the bending moment at all critical points. Critical points for bending moment are:

- ends of the beam;
- where the shear force is zero or changes sign;
- other points that experience has shown to be critical.

Values of bending moment are calculated using the definition and sign convention, and considering each load (to one side of the point) separately. It is the effect that one load would have on the bent shape at the chosen point that determines the sign.

1. For the bending moment at D consider the left side of this point $M_D = 0$

2. For the bending moment at E consider the left side of this point $M_E = P \times a$ and the beam would assume a hogging shape:

   $M_E = -(10 \times 10) = -100$ kNm

3. For the bending moment at F consider the loads to the right of this point, a sagging beam results and:

   $M_F = -(4 \times 10 \times 10 / 2) + (30 \times 10) = 100$ kNm

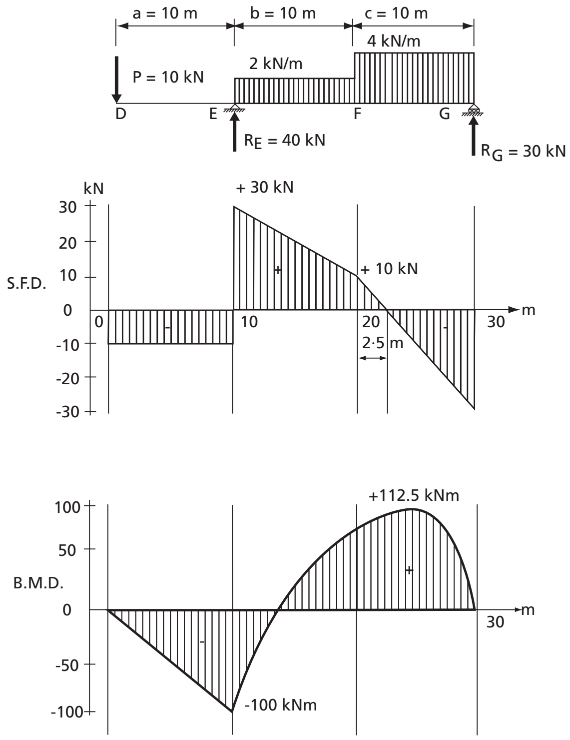
4. The bending moment at G is obviously zero

5. At point H we have the maximum bending moment: considering the forces to the right of this point gives

   $M_H = -(4 \times 7\ 512 \times 7\ 5) + (30 \times 7\ 5)$
   $= 112.5$ (sagging)

6. The variation of the bending moment under a UDL is parabolic

7. If the inclusion of other points would be helpful in drawing the curve, they should also be plotted.



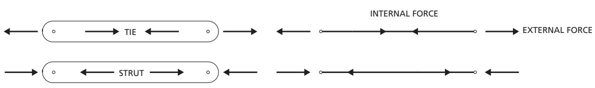
Note the following from the bending-moment diagram:

- The maximum negative bending-moment hogging (100 kNm) occurs at E and the maximum positive bending moment sagging (112.5 kNm) occurs at a point between F and G. When designing beams in materials such as concrete, the steel reinforcement would have to be placed according to these moments.
- The bending-moment diagram will also give an indication as to how the loaded beam will deflect. Positive bending moments (sagging) cause compression in the top fibres of the beam, hence they tend to bend the beam with the concave side downwards.
- At the supported ends of a simple beam and at the free end of a cantilevered beam, where there can be no resistance to bending, the bending moment is always zero.

Forces in pin-jointed frames

Designing a framework necessitates finding the forces in the members. For the calculation of primary stresses, each member is considered to be pin-jointed at each end so that it can transmit an axial force only in the direction of the line connecting the pin joints at each end. The force can be a pure tension (conventionally designated positive), in which case the member is called a tie, or a pure compression (conventionally designated negative), when the member is called a strut.

These are internal forces that must be in equilibrium with the external applied forces.

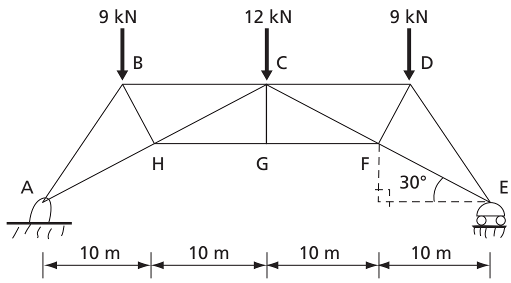


A number of different techniques can be used to determine the forces in the members.

Joint analysis: This is based on considering the equilibrium of each joint in turn and using the free-body diagram for each joint.

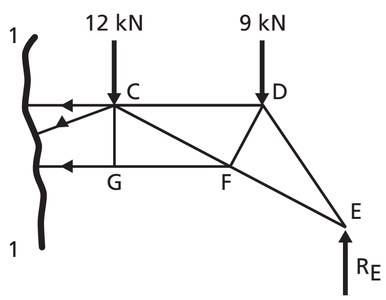
Method of sections: The free-body diagram considered is for a portion of the framework to one side or the other of a cut section. The forces in the members cut by the section are included in the free-body diagram. Application of the equations of equilibrium will solve the unknown forces in the cut section. This provides an analytical solution and is most useful when requiring the answers for one or two members only.

Example 6.8



(all 30°, 60°, 90° triangle)

Find the forces and their direction in the members BH and HG by using the method of sections.



F_{HG} is found by taking a moment about point C, considering the right hand section (RHS) of the cut 1-1 is in equilibrium. The forces F_{HC} and F_{BC} have no moment about point CBL because they intersect at and pass through the point.

$\Sigma M_c = 0\ (F_{HG} \times CG) + (9 \times CD) - (R_E \times 20) = 0$

$CG = FX = 10 \tan 30° = 5.774$

$CD = DE = FE / \cos 30°$

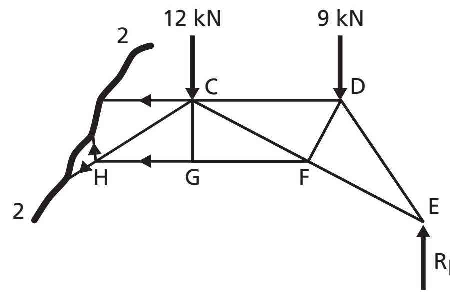
$FE = EX / \cos 30° = 11.547$ m

$CD = 11\,547 / \cos 30° = 13.333$ m

$RE = (9 + 12 + 12\ ) / 2 = 15$ kN

Hence $(F_{HG} \times 5.774) + (9 \times 13.333) - (15 \times 20) = 0$

$F_{HG} = 31.17$



Take section 2-2.

$HC = FE = 11.547\ (F_{BH} \times 11.547) + (9 \times 13.333)$
$- (15 \times 20) = 0\ F_{BH} = 15.59$ kN

It can therefore be seen that F_{GH} and F_{BH} must be clockwise to have equilibrium about point C. The members GH and HB are therefore in tension.

MECHANICS OF MATERIALS

Direct stress

When a force is transmitted through a body, the body tends to change its shape. Although these deformations are seldom visible to the naked eye, the many fibres or particles that make up the body transmit the force throughout the length and section of the body, and the fibres doing this work are said to be in a state of stress. Thus, a stress may be described as a mobilized internal reaction that resists any tendency towards deformation. As the effect of the force is distributed over the cross-section area of the body, stress is defined as force transmitted or resisted per unit area.

$$\text{Thus Stress} = \frac{\text{Force}}{\text{Area}}$$

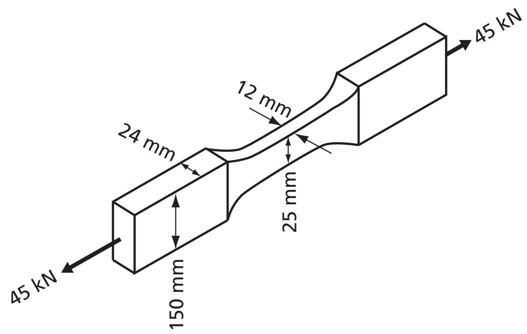
The SI unit for stress is Newtons per square metre (N / m^2). This is also called a Pascal (Pa). However, it is often more convenient to use the multiple N / mm^2.

Note that 1 N / mm² = 1 MN / m² = 1 MPa

Tensile and compressive stress, which result from forces acting perpendicular to the plane of cross-section in question, are known as normal stress and are usually symbolized with σ (the Greek letter sigma), sometimes given a suffix t for tension (σ_t) or c for compression (σ_c). Shear stress is produced by forces acting parallel or tangential to the plane of cross-section and is symbolized with τ (Greek letter tau).

Tensile stress

Example 6.9



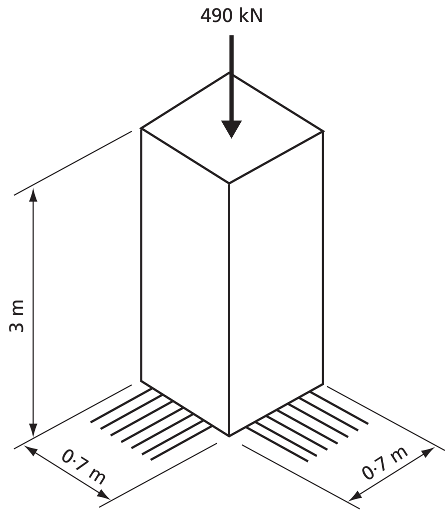
Consider a steel bar that is thinner at the middle of its length than elsewhere, and that is subject to an axial pull of 45 kN.

If the bar were to fail in tension, it would be as a result of breaking where the amount of material is at a minimum. The total force tending to cause the bar to fracture is 45 kN at all cross-sections but, whereas the effect of the force is distributed over a cross-sectional area of 1 200 mm² for part of the length of the bar, it is distributed over only 300 mm² at the middle position. Thus, the tensile stress is greatest in the middle and is:

$$\sigma_t = \frac{45\,\text{kN}}{300\,\text{mm}^2} = 150\,\text{MPa}$$

Compressive stress

Example 6.10
A brick pier is 0.7 metres square and 3 metres high and weighs 19 kN / m³. It is supporting an axial load from a column of 490 kN. The load is spread uniformly over the top of the pier, so the arrow shown in the diagram merely represents the resultant of the load. Calculate (a) the stress in the brickwork immediately under the column, and (b) the stress at the bottom of the pier.



Solution a
Cross-section area = 0.49 m²

$$\text{Stress} = \sigma_c = \frac{490\,\text{kN}}{0.49\,\text{m}^2} = 1\,000 \text{ kPa or } 1 \text{ MPa}$$

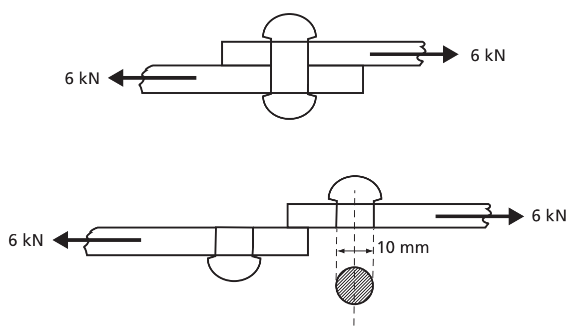
Solution b
Weight of pier = 0.7 m × 0.7 m × 3.0 m × 19 kN / m³
= 28 kN

Total load = 490 + 28 = 518 kN and

$$\text{Stress} = \sigma_c = \frac{518\,\text{kN}}{0.49\,\text{m}^2} = 1\,057 \text{ kPa}$$

Shear stress

Example 6.11
A rivet is used to connect two pieces of flat steel. If the loads are large enough, the rivet could fail in shear, i.e. not breaking but sliding of its fibres. Calculate the shear stress of the rivet when the steel bars are subject to an axial pull of 6 kN.



Note that although the rivets do, in fact, strengthen the connection by pressing the two steel bars together, this strength cannot be calculated easily owing to friction and is therefore neglected, i.e. the rivet is assumed to give all the strength to the connection.

Cross-section area of rivet = $1/4 \times \pi \times 10^2 = 78.5$ mm²

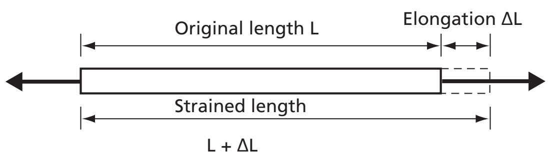
$$\text{Shear stress} = \tau = \frac{6\,\text{kN}}{78.5\,\text{mm}^2} = 76\ \text{MPa}$$

Strain

When loads of any type are applied to a body, the body will always undergo dimension changes; this is called *deformation*. Tensile and compressive stresses cause changes in length, torsional-shearing stresses cause twisting, and bearing stresses cause indentation in the bearing surface.

In farm structures, where a uniaxial state of stress is the usual stress considered, the major deformation is in the axial direction. Although there are always small deformations present in the other two dimensions, they are seldom significant.

$$\text{Direct strain} = \frac{\text{Change in length}}{\text{original length}} = \varepsilon = \frac{\Delta L}{L}$$



By definition *strain* is a ratio of change and thus it is a dimensionless quantity.

Elasticity

All solid materials deform when they are stressed and, as the stress increases, the deformation also increases. In many cases, when the load causing the deformation is removed, the material returns to its original size and shape and is said to be elastic. If the stress is steadily increased, a point is reached when, after the removal of the load, not all of the induced strain is recovered. This limiting value of stress is called the *elastic limit*.

Within the elastic range, strain is proportional to the stress causing it. This is called *the modulus of elasticity*. The greatest stress for which strain is still proportional is called the *limit of proportionality* (Hooke's law).

Thus, if a graph is drawn of stress against strain as the load is gradually applied, the first portion of the graph will be a straight line. The slope of this straight line is the constant of proportionality, modulus of elasticity (E), or Young's modulus and should be considered as a measure of the stiffness of a material.

$$\text{Modulus of elasticity} = E = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{A\Delta L}$$

The modulus of elasticity will have the same units as stress (Pa). This is because strain has no units.

A convenient way of demonstrating elastic behaviour is to plot a graph of the results of a simple tensile test carried out on a thin mild steel rod. The rod is hung vertically and a series of forces are applied at the lower end. Two gauge points are marked on the rod and the distance between them is measured after each force increment has been added. The test is continued until the rod breaks.

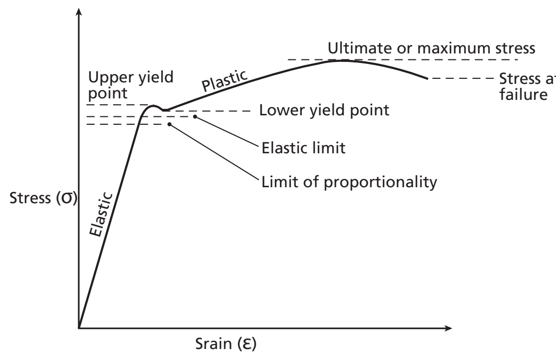


Figure 6.1 **Behaviour of a mild steel rod under tension**

Example 6.12

Two timber posts, measuring 150 millimetres square and 4 metres high, are subjected to an axial load of 108 kN each. One post is made of pine timber (E = 7 800 MPa) and the other is Australian blackwood (E = 15 300 MPa). How much will they shorten because of the load?

Cross-section area A = 22 500 mm²; length L = 4 000 mm

$$\text{Pine: } \Delta L = \frac{FL}{AE} = \frac{108\,000 \times 4\,000}{22\,500 \times 7\,800} = 2.5\ \text{mm}$$

$$\text{Australian blackwood: } \Delta L = \frac{FL}{AE} = \frac{108\,000 \times 4\,000}{22\,500 \times 15\,300} = 1.3\ \text{mm}$$

Factor of safety

The permissible stresses must, of course, be less than the stresses that would cause failure of the members of the structure – in other words there must be an ample safety margin. (In 2 000 BC, a building code declared the life of the builder to be forfeit should the house collapse and kill the owner).

Also, deformations must be limited because excessive deflection may give rise to problems such as cracking of ceilings, partitions and finishes, as well as adversely affecting the functional needs.

Structural design is not an exact science and, while calculated values of reactions, stresses, etc. may be mathematically correct for the theoretical structure (i.e. the model), they may be only approximate as far as the actual behaviour of the structure is concerned.

For these and other reasons, it is necessary to ensure that the design stress, working stress, allowable stress and permissible stress are less than the ultimate stress or the yield stress. This margin is called the *factor of safety.*

$$\text{Design stress} = \frac{\text{Ultimate (or yield) stress}}{\text{factor of safety}}$$

In the case of a material such as concrete, which does not have a well defined yield point, or brittle materials that behave in a linear manner up to failure, the factor of safety is related to the ultimate stress (maximum stress before breakage). Other materials, such as steel, have a yield point where a sudden increase in strain occurs, and at which point the stress is lower than the ultimate stress. In this case, the factor of safety is related to the yield stress in order to avoid unacceptable deformations.

The value of the factor of safety has to be chosen with a variety of conditions in mind, such as the:

- accuracy in the loading assumptions;
- permanency of the loads;
- probability of casualties or big economic losses in case of failure;
- purpose of the building;
- uniformity of the building material;
- workmanship expected from the builder;
- strength properties of the materials;
- level of quality control ensuring that the materials are in accordance with their specifications;
- type of stresses developed;
- cost of building materials.

Values of 3 to 5 are normally chosen when the factor of safety is related to ultimate stress, and values of 1.4 to 2.4 are chosen when related to yield-point stress.

In the case of building materials such as steel and timber, different factors of safety are sometimes considered for common loading systems and for exceptional loading systems, in order to save materials. Common loadings are those that occur frequently, whereas a smaller safety margin may be considered for exceptional loadings, which occur less frequently and seldom at full intensity, e.g. wind pressure, earthquakes, etc.

STRUCTURAL ELEMENTS AND LOADING

Applied loads

Applied loads fall into three main categories: dead loads, wind loads and other imposed loads.

Dead loads are loads resulting from the self-weight of all permanent construction, including roof, walls, floor, etc. The self-weight of some parts of a structure, e.g. roof cladding, can be calculated from the manufacturer's data sheets, but the self-weight of the structural elements cannot be accurately determined until the design is completed. Hence estimates of self-weight of some members must be made before commencing a design analysis and the values checked upon completion of the design.

Wind loads are imposed loads, but are usually treated as a separate category owing to their transitory nature and their complexity. Very often wind loading proves to be the most critical load imposed on agricultural buildings. Wind loads are naturally dependent on wind speed, but also on location, size, shape, height and construction of a building.

Specific information concerning various load types is presented in Chapter 8.

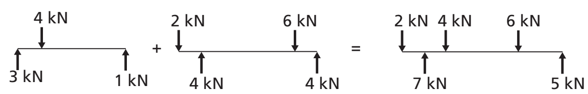
When designing a structure, it is necessary to consider which combination of dead and imposed loads could give rise to the most critical loading condition. Not all the imposed loads will necessarily reach their maximum values at the same time. In some cases (for example, light open sheds), wind loads may tend to cause the roof structure to lift, producing an effect opposite in direction to that of the dead load.

Imposed loads are loads related to the use of the structure and to the environmental conditions, e.g. weight of stored products, equipment, livestock, vehicles, furniture and people who use the building. Imposed loads include earthquake loads, wind loads and snow loads where applicable, and are sometimes referred to as superimposed loads because they are in addition to the dead loads.

Dynamic loading results from a change of loading, resulting directly from the movement of loads. For example, a grain bin may be affected by dynamic loading if filled suddenly from a suspended hopper; it is not sufficient to consider the load solely when the bin is either empty or full.

Principle of superposition

This principle states that the effect of a number of loads applied at the same time is the algebraic sum of the effects of the loads applied singly.

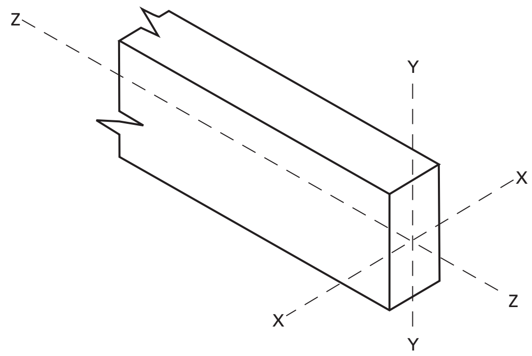


Using standard load cases and applying the principle of superposition, complex loading patterns can be solved. Standard case values of shear force, bending moment or deflection at particular positions along a member can be evaluated, after which the total value of such parameters for the actual loading system can be found by algebraic summation.

Effects of loading

After the loads have been transformed into definable load systems, the designer must consider how the loads will be transmitted through the structure. Loads are not transmitted as such, but as load effects.

It is usual practice to orientate the Cartesian z-z axis along the length of the member and the x-x and y-y axes along the horizontal and vertical cross-sectional axes respectively, when considering a structural member that occupies a certain space (see the figure below).



Primary load effects

A primary load effect is defined as being the direct result of a force or a moment, which has a specific orientation with respect to the three axes. Any single load or combination of loads can give rise to one or more of these primary load effects. In most cases, a member will be designed basically to sustain one load effect, usually the one producing the greatest effect.

In more complex situations, the forces and moments are resolved into their components along the axes, after which the load effects are first studied separately for one axis at a time, and subsequently their combined effects are considered when giving the member its size and shape.

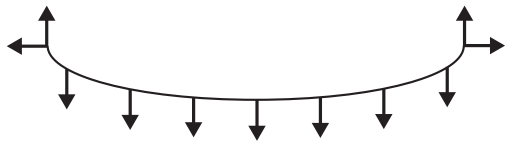
The choice of material for a member may be influenced to some extent by the type of loading. For instance, concrete has little or no strength in tension and is therefore unsuitable for use alone as a tie.

Tension, compression, shear, bending and torsion are all primary load effects. Secondary load effects, such as deflection, are derived from the primary load effects.

Structural elements

Cable

Cables, cords, strings, ropes and wires are flexible because of their small lateral dimensions in relation to their length, and therefore have very limited resistance to bending. Cables are the most efficient structural elements because they allow every fibre of the cross-section to resist the applied loads up to any allowable stress. However, their application is limited by the fact that they can be used only in tension.

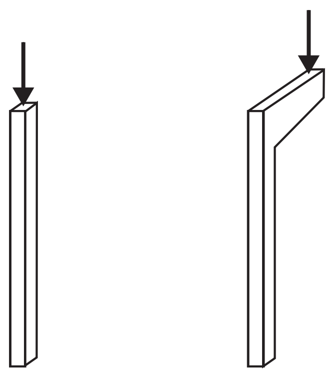


Rod

Rods, bars and poles are used to resist tensile or compressive loads. In a rod or a bar under axial tension, the full cross section can be considered and the full allowable stress for the material can be used in design calculations.

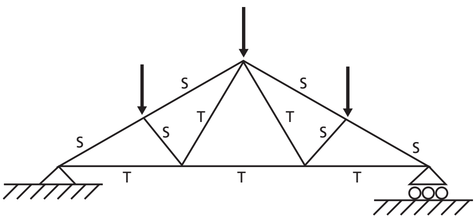
Column

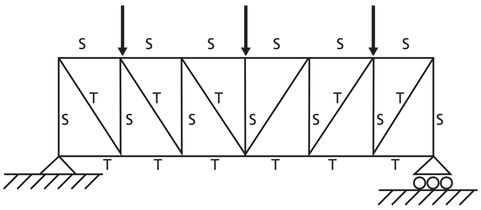
Rods or bars under compression are the basis for vertical structural elements such as columns, stanchions, piers and pillars. They are often used to transfer load effects from beams, slabs and roof trusses to the foundations. They may be loaded axially or they may have to be designed to resist bending when the load is eccentric.



Ties and struts

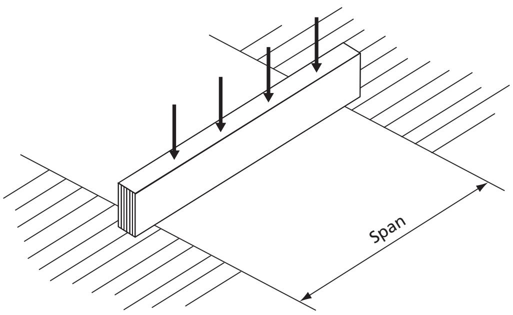
When bars are connected with pin joints and the resulting structure loaded at the joints, a structural framework called a pin-jointed truss or lattice frame is obtained. The members are subjected only to axial loads and members in tension are called *ties*, while members in compression are called *struts*.

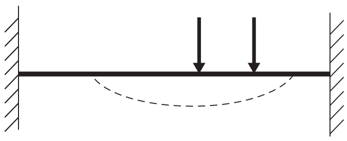




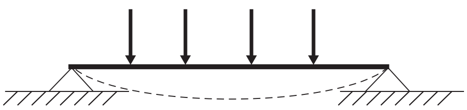
Beam

A beam is a member used to resist a load acting across its longitudinal axis by transferring the effect over a distance between supports – referred to as the *span.*

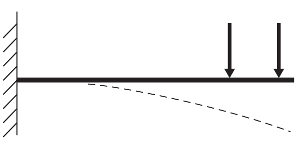




Built - in ends



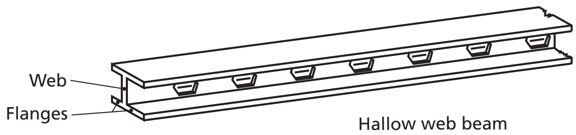
Deflection shape
Simply supported



Cantilever

The load on a beam causes longitudinal tension and compression stresses, and shear stresses. Their magnitudes will vary along, and within, the beam.

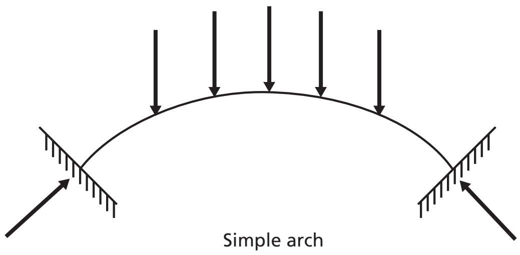
The span that a beam can usefully cover is limited by the self-weight of the beam, i.e. it will eventually reach a length when it is capable of supporting only itself. To a degree, this problem is overcome with the hollow web beam and the lattice girder or frame. The safe span for long, lightly loaded beams can be increased somewhat by removing material from the web, even though the shear capacity will be reduced.



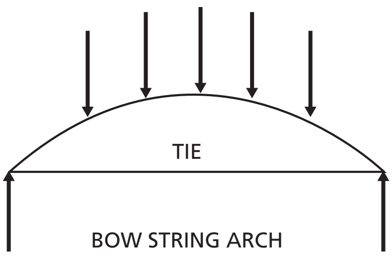
Hallow web beam

Arch

The arch can be shaped such that, for a particular loading, all sections of the arch are under simple compression with no bending. Arches exert vertical and horizontal thrusts on their supports, which can prove troublesome in the design of supporting walls. This problem of horizontal thrust can be eliminated by connecting a tension member between the support points.



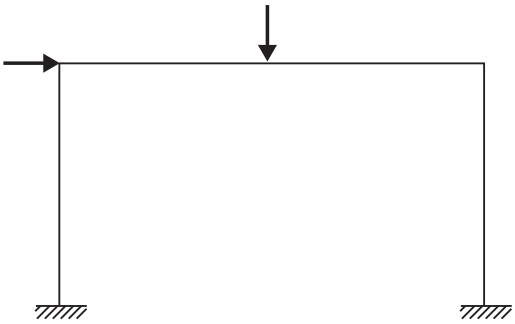
Simple arch



BOW STRING ARCH

Frames

Plane frames are also made up of beams and columns, the only difference being that they are rigidly connected at the joints. Internal forces at any cross-section of the plane frame member are: bending moment, shear force and axial force.



PROPERTIES OF STRUCTURAL SECTIONS

When designing beams in bending, columns in buckling, etc., it is necessary to refer to a number of basic geometrical properties of the cross-sections of structural members.

Area

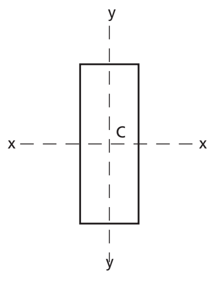
Cross-section areas (A) are generally calculated in square millimetres, because the dimensions of most structural members are given in millimetres, and values for design stresses found in tables are usually given in Newtons per millimetre square (N / mm²).

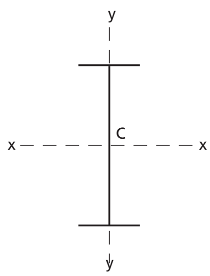
Centre of gravity or centroid

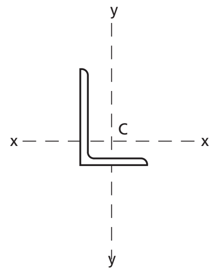
This is a point about which the area of the section is evenly distributed. Note that the centroid is sometimes outside the actual cross-section of the structural element.

Reference axes

It is usual to consider the reference axes of structural sections as those passing through the centroid. In general, the x-x axis is drawn perpendicular to the greatest lateral dimension of the section, and the y-y axis is drawn perpendicular to the x-x axis, intersecting it at the centroid.







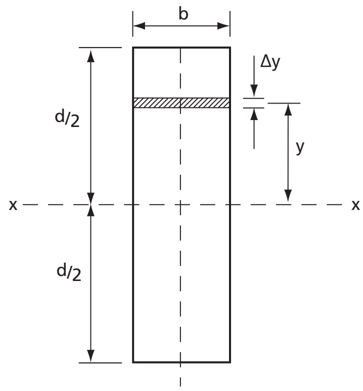
Moment of inertia

The area moment of inertia (I), or to use the more correct term, *second moment of area*, is a property that measures the distribution of area around a particular axis of a cross-section, and is an important factor in its resistance to bending. Other factors, such as the strength of the material from which a beam is made, are also important for resistance to bending, and are allowed for in other ways. The moment of inertia measures only how the geometric properties or shape of a section affect its value as a beam or slender column. The best shape for a section is one that has the greater part of its area as distant as possible from its centroidal, neutral axis.

For design purposes, it is necessary to use the moment of inertia of a section about the relevant axis or axes.

Calculation of moment of inertia

Consider a rectangle that consists of an infinite number of strips. The moment of inertia about the x-x axis of such a strip is the area of the strip multiplied by the square of the perpendicular distance from its centroid to the x-x axis, i.e. $b \times \Delta y \times y^2$



The sum of all such products is the moment of inertia about the x-x axis for the whole cross-section.

By applying calculus and integrating as follows, the exact value for the moment of inertia can be obtained.

$$I_{xx} = \int_{-d/2}^{+d/2} by^2\,dy = \frac{bd^3}{12}$$

For a circular cross-section:

$$I_{xx} = \frac{\pi D^4}{64}$$

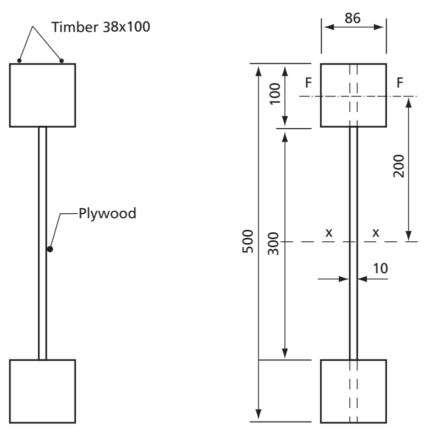
Moments of inertia for other cross-sections are given later and in Table 4.3. For structural rolled-steel sections, the moment of inertia can be found tabulated in handbooks. Some examples are given in Appendix V.3.

Principle of parallel axes

According to the principle of parallel axes, if the moment of inertia of any area (e.g. top flange of the beam shown below) about any axis is parallel to its centroidal axis, then the product of the area of the shape and the square of the perpendicular distance between the axes must be added to the moment of inertia about the centroidal axis of that shape.

Example 6.13

Determine the moment of inertia about the x-x axis and the y-y axis for the I-beam shown in the figure. The beam has a web of 10 mm plywood and the flanges are made of 38 mm by 100 mm timber, which are nailed and glued to the plywood web.



Solution:

The entire cross-section of both the beam and the cross-section of the web have their centroids on the x-x axis, which is therefore their centroidal axis. Similarly, the F-F axis is the centroidal axis for the top flange.

I_{xx} of the web using $\dfrac{bd^3}{12} = \dfrac{10 \times 300^3}{12} = 22.5 \times 10^6 \text{ mm}^4$

The moment of inertia of one flange about its own centroidal axis (F-F):

I_{FF} of one flange $= \dfrac{86 \times 100^3}{12} = 7.2 \times 10^6 \text{ mm}^4$

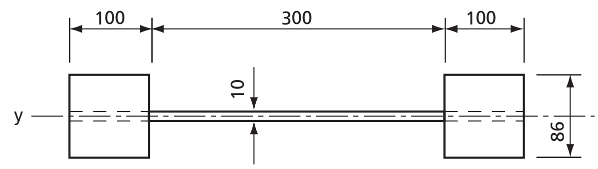
and from the principle of parallel axes, the I_{xx} of one flange equals:

$(7.2 \times 10^6) + (86 \times 100 \times 200^2) = 351.2 \times 10^6 \text{ mm}^4$

Thus the total I_{xx} of the web plus two flanges equals:

$I_{xx} = (22.5 \times 10^6) + (351.2 \times 10^6) + (351.2 \times 10^6)$
$= 725 \times 10^6 \text{ mm}^4$

The I_{yy} of the above beam section is most easily found by adding the I_{yy} of the three rectangles of which it consists, because the y-y axis is their common neutral axis, and moments of inertia may be added or subtracted if they are related to the same axis.



$$I_{yy} = 2 \times \frac{100 \times 86^3}{12} + \frac{300 \times 10^3}{12}$$
$$= 2 \times 5.3 \times 10^6 + 0.025 \times 10^6$$
$$= 10.6 \times 10^6 \text{ mm}^4$$

Section modulus

In problems involving bending stresses in beams, a property called *section modulus* (Z) is useful. It is the ratio of the moment of inertia (I) about the neutral axis of the section to the distance (C) from the neutral axis to the edge of the section.

Unsymmetrical cross-sections

Sections for which a centroidal reference axis is not an axis of symmetry will have two section moduli for that axis.

$$Z_{xx1} = \frac{I_{xx}}{y_1} \;;\; Z_{xx2} = \frac{I_{xx}}{y_2}$$

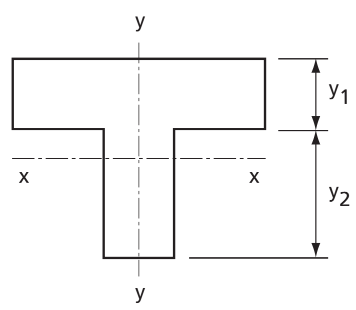
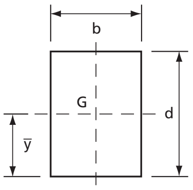
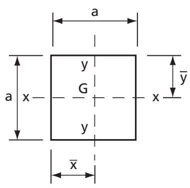
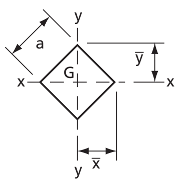
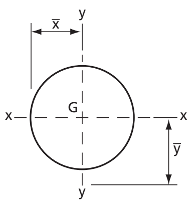
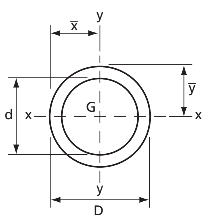


TABLE 6.3
Properties of structural sections

	Section	Area (mm^2) or (m^2)	Moment of inertia (mm^4) or (m^4)		Section modulus (mm^3) or (m^3)		Radius of gyration (mm) or (m)		Distance from extreme fibre to centroid (mm) or (m)
		A	I_{xx}	I_{yy}	Z_{xx}	Z_{yy}	r_{xx}	r_{yy}	$\bar{y}$ $\bar{x}$
	Rectangle	bd	$\frac{bd^3}{12}$	$\frac{db^3}{12}$	$\frac{bd^2}{6}$	$\frac{db}{6}$	$\frac{d}{12}$ $\frac{d}{\sqrt{12}}$	$\frac{b}{12}$ $\frac{b}{\sqrt{12}}$	$\bar{y} = \frac{d}{2}$ $\bar{y} = \frac{d}{2}$
	Square	a^2	$\frac{a^4}{12}$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}$	$\frac{a}{\sqrt{12}}$	$\bar{y} = \bar{x} = \frac{a}{2}$
	Square with diagonal axes	a^2	$\frac{a^4}{12}$	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}}$	$\frac{a^3}{6\sqrt{2}}$	$\frac{a}{\sqrt{12}}$	$\frac{a}{\sqrt{12}}$	$\bar{y} = \bar{x} = \frac{a}{2}$
	Circle	$\frac{\pi D^2}{4}$	$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$	$\frac{\pi D^3}{32}$	$\frac{\pi D^3}{32}$	$\frac{D}{4}$	$\frac{D}{4}$	$\bar{y} = \frac{D}{2}$ $\bar{x} = \frac{D}{2}$
	Annulus	$\frac{\pi (D^2 - d^2)}{4}$	$\frac{\pi (D^4 - d^4)}{64}$	$\frac{\pi (D^4 - d^4)}{32}$	$\frac{\pi (D^3 - d^3)}{32}$	$\frac{\pi (D^3 - d^3)}{32}$	$4\sqrt{(D^2 + d^2)}$	$4\sqrt{(D^2 + d^2)}$	$\bar{y} = \frac{D}{2}$ $\bar{x} = \frac{D}{2}$

Radius of gyration

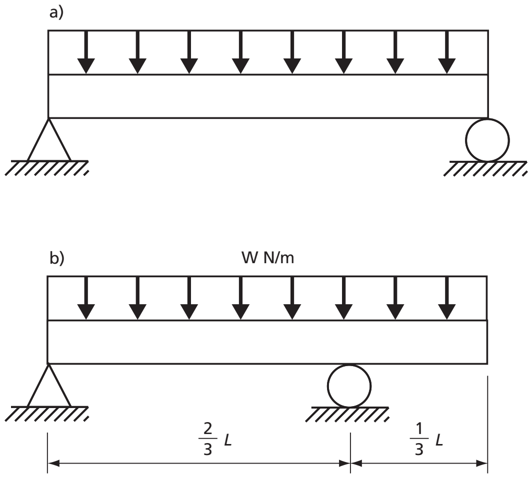
Radius of gyration (r) is the property of a cross-section that measures the distribution of the area of the cross-section in relation to the axis. In structural design, it is used in relation to the length of compression members, such as columns and struts, to estimate their slenderness ratio and hence their tendency to buckle. Slender compression members tend to buckle about the axis for which the radius of gyration is a minimum value. From the equations, it will be seen that the least radius of gyration is related to the axis about which the least moment of inertia occurs.

Therefore, $r_{xx} = \sqrt{\frac{I_{xx}}{A}}$ and $r_{yy} = \sqrt{\frac{I_{yy}}{A}}$

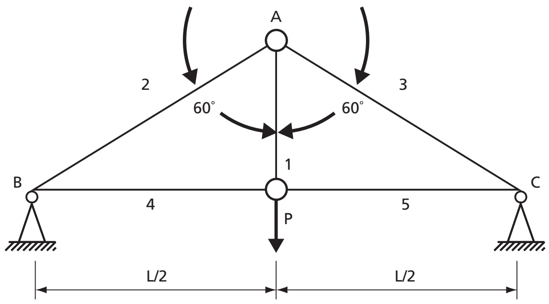
(general relationship $I = Ar^2$)

REVIEW QUESTIONS

1. Sketch the shear and bending moment diagrams for the beams below, indicating values of shear force and bending moment at the key points.



2. Find the reactions on beam BC.



3. Two concentrated loads of 100 kN and 200 kN advance along a girder with a 20-metre span, the distance between the loads being 8 metres. Find the position of the section that has to support the greatest bending moment, and calculate the value of the bending moment.

4. A load of 100 kN, followed by another load of 50 kN, at a distance of 10 metres, advances across a girder with a 100-metre span. Obtain an expression for the maximum bending moment at a section of the girder at a distance of *z* metres from an abutment.

FURTHER READING

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