University of Anbar
College of Engineering

Soil Mechanics

Prepared
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RAMADI – IRAQ

Soil Mechanics

University of Anbar

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Catalogue Description

- *Origin of Soil and Grain Size*
- *Weight-Volume Relationships, Plasticity and Structure of Soil*
- *Engineering Classification of Soil*
- *Permeability*
- *Seepage*
- *In Situ Stresses (Effective Stress Concept)*
- *Stresses in a Soil Mass*
- *Compressibility of Soil*
- *Shear Strength of Soil*
- *Soil Compaction*

Textbook and Reference Books

Textbook- Fundamentals of Geotechnical Engineering, Braja M. Das, 3rd ed., 2008

1. *Principles of geotechnical engineering, Braja M. Das, 7th edition*
2. *Soil mechanics, R.F. Craig, 8th ed.*
3. *Solving problems in soil mechanics, B.H.C. Sutton, 2nd ed.*

Evaluation

- *Homework & Reports: 10%*
- *Mid-Term Examination: 20%*
- *Unannounced quizzes: 10%*
- *Final Examination: 60%*

Class Notebook

- *You are required to keep and assemble a three ring (or other suitable binding) notebook with the following divisions in it:*
- *Homework*
- *Quizzes*
- *Tests*
- *Class Notes (Optional)*
- *You will turn this notebook in at the final exam. It will be inspected and returned to you.*

Appearance of Work

- *All homework and tests must be on engineering paper.*
- *Homework and tests must conform to format given in syllabus. Failure to do so will result in reduced credit.*
- *Each time you use an equation, write down what it is: don't just put a bunch of numbers on the page and expect anyone to know what you did. This too will result in reduced credit.*

Honour System

- You are encouraged to work homework with someone but your turned in work must be your own work.
- You are studying now so that you may enter and practice the engineering profession later. The engineering profession is highly regarded by the public because those who practice it do so with ethical and social consciousness. The same is expected of students in this course. Any direct copying of homework, tests or exams will be considered a violation of the honour code and a course grade of "F" will be given.

Types of Civil Engineering

- Structural Engineering
- Engineering Mechanics
- Transportation Engineering
- Environmental Engineering
- Coastal Engineering
- Geotechnical Engineering

Definition of Geotechnical Engineering

"The branch of Civil Engineering that deals with the properties of soils and rocks and their capability of supporting structures placed on or under them."

Characteristics of Geotechnical Engineering

- Works in a complex environment
- Requires a higher degree of judgment than other branches of engineering
- More than one "acceptable" solution to any problem
- The integrity of the structure above is dependent upon the quality of the foundation below

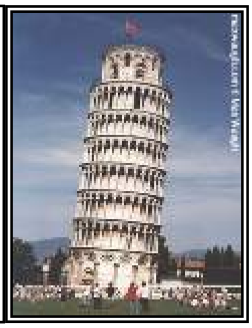
Development of Geotechnical Engineering

- The slowest branch of civil engineering to develop a theoretical basis that could be used in practical design
- Design of foundations traditionally was conservative and the result of trial and error
- Larger structures and catastrophic failures led to the investigation of the causes of failure and the establishment of theory which in turn would lead to design methods that resulted in workable foundations

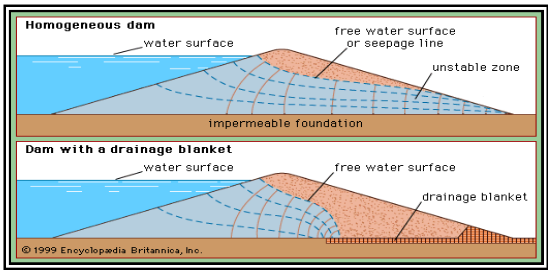
Problems in Geotechnical Engineering

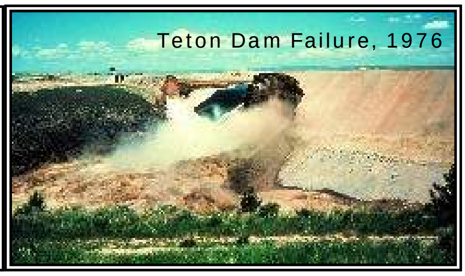
- Shear Failure-Loads have exceeded shear strength capacity of soil!

- Settlement



- Seepage Problems

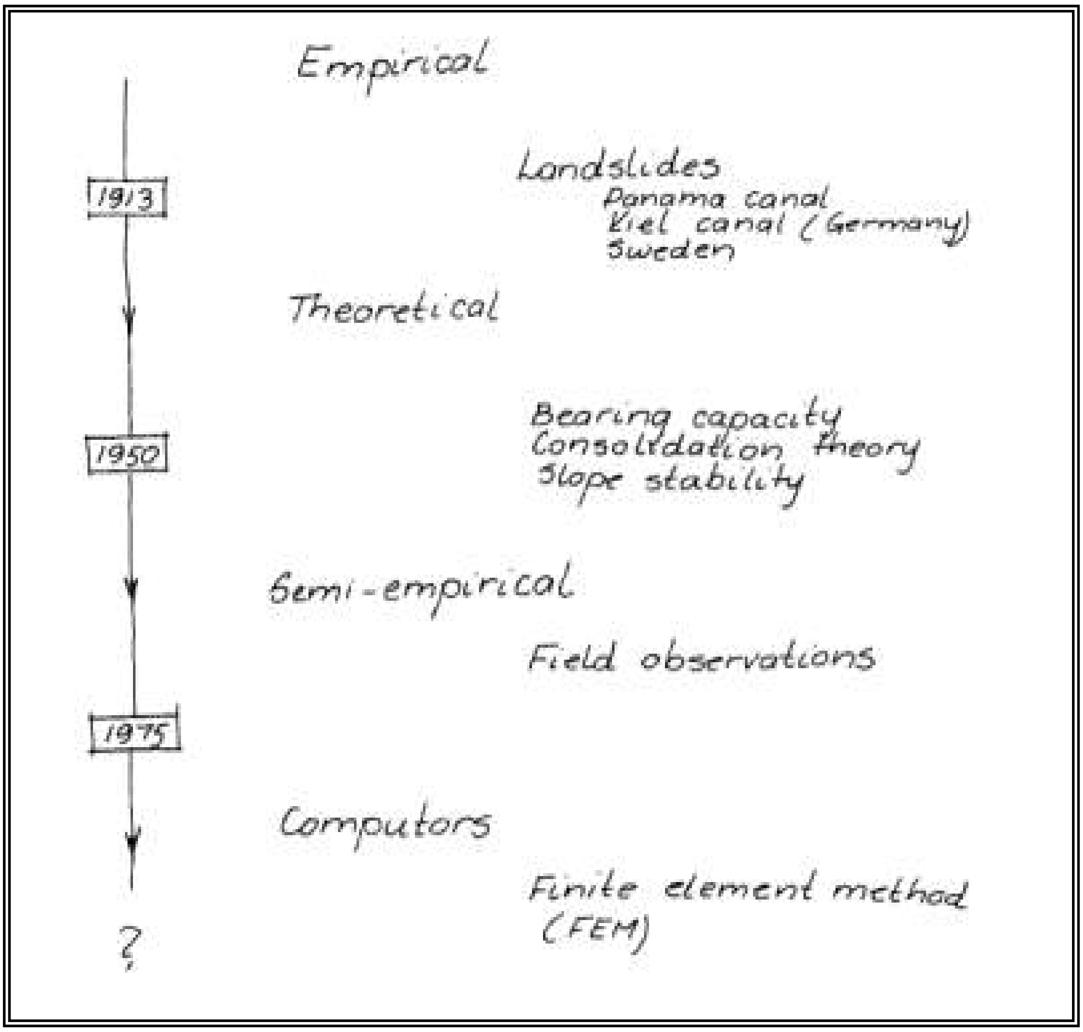




Historical Background

Karl Terzaghi

- The "father of geotechnical engineering"
- Developed both the theory and practice of the analysis of soils and the design of foundations
- Consolidation theory
- Bearing Capacity of Shallow Foundations
- Design of retaining walls and cellular cofferdams
- Wrote some of the first textbooks on soil mechanics and foundations design
- *Soil Mechanics in Engineering Practice* (1948)
- *Theoretical Soil Mechanics* (1943)


Empirical
1913
Landslides
Panama canal
Kiel canal (Germany)
Sweden
Theoretical
1950
Bearing capacity
Consolidation theory
Slope stability
Semi-empirical
Field observations
1975
Computors
Finite element method
(FEM)
?

1. ORIGIN OF SOIL AND GRAIN SIZE

- **Introduction**
- **Soils and Rocks**
- **Types of Rocks**
- **Soil – Rock Cycle**
- **Basic Soil Types**
- ***Soil-Particle Size or Grain Sizes***
- ***Structure of Clay Minerals***
- ***Types of Clay Minerals***
- ***How is water absorbed on the surface of a clay particle?***
- ***Gradation of Particle Size***

2. WEIGHT-VOLUME RELATIONSHIPS, PLASTICIY, AND STRUCTURE OF SOIL

- ***Weight-Volume Relationships***
- ***Important variables-(Water or Moisture Content-Unit Weight or Mass-Void ratio-Specific Gravity……..etc.***
- ***Relative Density***
- ***Particle Size and Shape***
- ***Grain Size Tests***
- ***Sieve Tests (Coarse-Grained Soils)***
- ***Hydrometer Tests (Fine-Grained Soils)***
- ***Plasticity and the Atterberg Tests***

3. ENGINEERING CLASSIFICATION OF SOIL

- ***Introduction***
- ***Textural classification***
- ***Unified Soil Classification System (USCS)***

4. PERMEABILITY AND SEEPAGE

PERMEABILITY

- ***Overview of Underground Water Flow***
- ***Permeability***
- ***Theory***
- ***Laboratory and Field Tests***
- ***Empirical Correlations***
- ***Equivalent Permeability in Stratified Soil***

SEEPAGE

- ***Laplace's Equation of Continuity***
- ***Continuity Equation for Solution of Simple Flow Problems***
- ***Flow Nets***
- ***Seepage Calculation***
- ***Seepage pressure and Uplift Pressure***
- ***Seepage through an Earth Dam***

5. IN SITU STRESSES

- ***Effective Stress Concept***
- ***Effective Stress in Saturated Soil with no Seepage***
- ***Effective Stress in Saturated Soil with Seepage***
- ***Seepage Force***
- ***Filter Requirements and Selection of Filter Material***
- ***Capillary Rise in Soil***
- ***Effective Stress in Capillary Zone***

6. STRESSES IN SOIL MASS

- ***Normal and Shear Stresses on a Plane***
- ***Stress distribution in soils***
- ***Stress Caused by a Point Load***
- ***Vertical Stress Caused by a Line Load***
- ***Vertical Stress Caused by a Strip Load***
- ***Vertical Stress Due to Embankment Loading***
- ***Vertical Stress below the Center of a uniformly Loaded Circular Area***
- ***Vertical Stress at any Point below a uniformly Loaded Circular Area***
- ***Vertical Stress Caused by a Rectangularly Loaded Area***
- ***Influence Chart for Vertical Pressure (Newmark Chart)***
- ***Approximate methods***

7. COMPRESSIBILITY OF SOIL

- ***Introduction***
- ***Immediate Settlement***
- ***Consolidation Settlement (Primary Consolidation)***
- ***Secondary Compression (Secondary consolidation) Settlement***
- ***Time Rate of Consolidation***
- ***Calculation of Consolidation Settlement under a Foundation***

8. SHEAR STRENGTH OF SOIL

- ***Introduction***
- ***Mohr-Coulomb Failure Criterion***
- ***Inclination of the plane of failure due to shear***
- ***Laboratory Tests for Determination of Shear Strength Parameters***
- ***Stress Path***

9. SOIL COMPACTION

- ***General Principles***
- ***Soil Compaction in the Lab:***
- ***Factors affecting Compaction***
- ***Structure of Compacted Clay Soil***
- ***Field Compaction***
- ***Specification for Field Compaction***
- ***Determination of Field Unit Weight of Compaction***

Origin of Soil and Grain Size

Soils and Rocks

Definition of "Soil" and "Rock"

● Soil

Naturally occurring mineral particles which are readily separated into relatively small pieces, and in which the mass may contain air, water, or organic materials (derived from decay of vegetation).

● Rock

Naturally occurring material composed of mineral particles so firmly bonded together that relatively great effort is required to separate the particles (i.e., blasting or heavy crushing forces).

Types of Rocks

- Igneous rocks
- Sedimentary rocks
- Metamorphic rocks

Igneous Rocks

- Definition-Rocks formed by the solidification of molten material, either by intrusion at depth in the earth's crust or by extrusion at the earth's surface.
- Examples
    - Acidic (high silica content) → sandy or gravelly soils
    - Granite (contains quartz and feldspar w/mica)
    - Basic (low silica content) → clay soils
    - Basalt (contains feldspar and augite with green olivine)
    - Intermediate → fine textured soils
    - Diorite (similar to granite except little or no quartz)

Sedimentary Rocks

- Definition- Rocks formed by deposition, usually under water, of products derived by the disaggregation of pre-existing rocks.
- Types
    - Shales ⟵ clay and silt particles
    - Sandstones
    - Limestone (Karst topography)
    - Dolstone (marl, chalk)

Metamorphic Rocks

- Definition-Rocks that may be either igneous or sedimentary rocks that have been altered physically and sometimes chemically by the application of intense heat and pressure at some time in their geological history
- Types
    - Coarse crystalline (gneiss)
    - Medium crystalline (schist, marble, soapstone)
    - Fine to microscopic (slate, anthracite coal)

Methods of Classifying Rocks

- Visual Classification
- Weathering Classification
- Discontinuity Classification
- Colour and Grain Size
- Hardness Classification
- Geological Classification
- Classification by Field Measurements and Strength Tests
- Strength
- Rock Quality Designation and Velocity Index Rock

Rock Quality Designation (RQD)

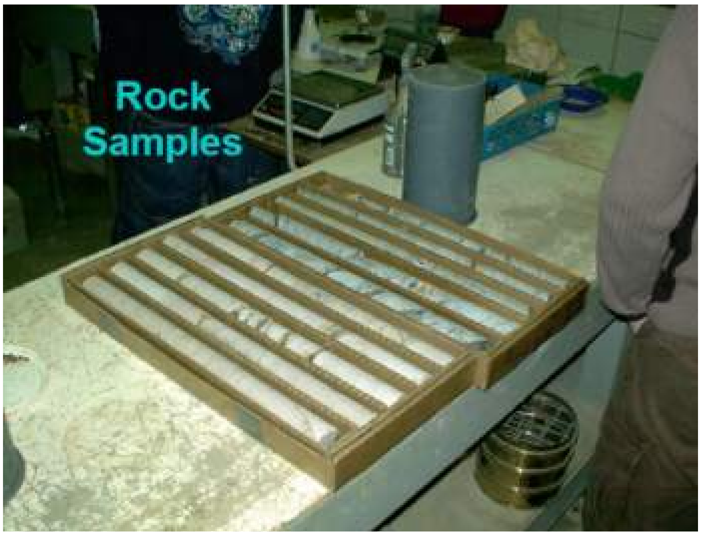
- Based on a modified core recovery procedure

L_i = length of a given recovered piece $\geq$ **4″**

$$RQD = \frac{\sum L_i}{L_t}$$

L_t = total length of core sample

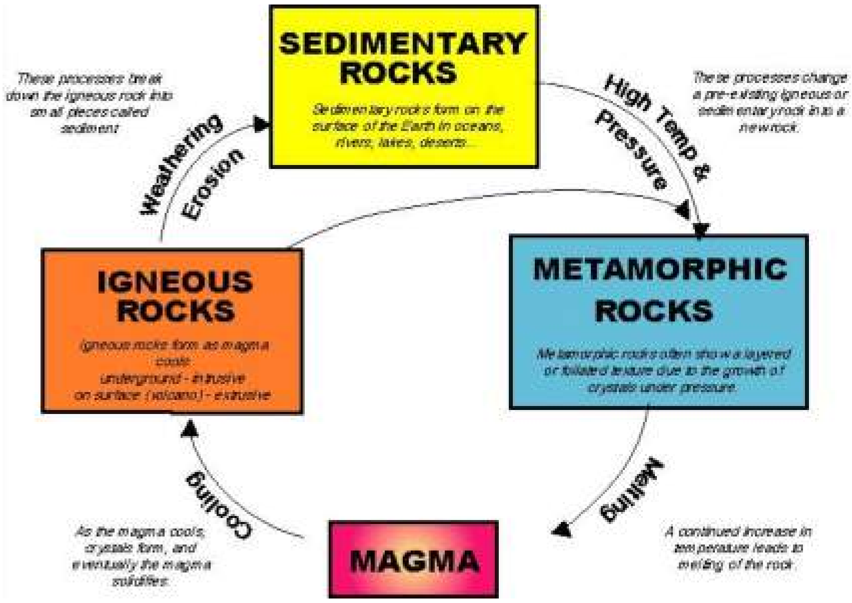
- Velocity index
    - Square of the ratio of the field compressional wave velocity to the laboratory compressional wave velocity
    - Typically used to determine rock quality using geophysical surveys



Rock Quality Designation (RQD)

RQD%	VELOCITY INDEX	ROCK MASS QUALITY
90 - 100	0.80 - 1.00	Excellent
75 - 90	0.60 - 0.80	Good
50 - 75	0.40 - 0.60	Fair
25 - 50	0.20 - 0.40	Poor
0 - 25	0 - 0.20	Very Poor

Soil – Rock Cycle



Weathering

Physical or Mechanical weathering causes disintegration of the rocks into smaller particle sizes, the processes that cause physical weathering are-

- Freezing and thawing
- Temperature changes
- Erosion (Abrasion)
- Activity of plants and animals including man

- ***Chemical weathering*** causes decomposition in rocks by –

  - Oxidation – union of oxygen with minerals in rocks forming another minerals
  - Hydration – water will enter the crystalline structure of minerals forming another group of minerals
  - Hydrolysis – the release Hydrogen from water will union with minerals forming another minerals
  - Carbonation – when Co_2 is available with the existence of water the minerals changed to Carbonates

Basic Soil Types

● Sedimentary Soils • Residual • Organic 	● Transported Soils • Alluvial • Aeolian • Glacial • Marine • Colluvial • Pyroclastic

Sedimentary Soils

● Residual Soils: Material formed by disintegration of underlying parent rock or partially indurated material.

- ➢ Sands
  - ● Residual sands and fragments of gravel size formed by solution and leaching of cementing material, leaving the more resistant particles; commonly quartz.
  - ● Generally, favourable foundation conditions.
- ➢ Clays
  - ●Residual clays formed by decomposition of silicate rocks, disintegration of shales, and solution of carbonates in limestone.
  - ●Variable properties requiring detailed investigation. Deposits present favorable foundation conditions except in humid and tropical climates.

● Organic Soils: Accumulation of highly organic material formed in place by the growth and subsequent decay of plant life.

- ➢ Peat. A somewhat fibrous aggregate of decayed and decaying vegetation matter having a dark colour and odour of decay.

- Muck. Peat deposits which have advanced in stage of decomposition to such extent that the botanical character is no longer evident.

● Very compressible, entirely unsuitable for supporting building foundations.

Transported Soils

● Alluvial Soils: Material transported and deposited by running water.

- **Flood plain deposits**. Deposits laid down by a stream within that portion of its valley subject to inundation by floodwaters.
  - Point Bar: Alternating deposits of arcuate ridges and swales (lows formed on the inside or convex bank of mitigating river bends.)
  - Channel Fill: Deposits laid down in abandoned meander loops isolated when rivers shorten their courses.
  - Back swamp: The prolonged accumulation of floodwater sediments in flood basins bordering a river.
  - Generally favourable foundation conditions, with important exceptions; frequently require deep foundations.

- **Alluvial Terrace deposits.**
  - Relatively narrow, flat-surfaced, river-flanking remnants of flood plain deposits formed by entrenchment of rivers and associated processes.
  - Usually drained, oxidised. Generally favourable foundation conditions.

- **Estuarine deposits.**
  - Mixed deposits of marine and alluvial origin laid down in widened channels at mouths of rivers and influenced by tide of body of water into which they are deposited.
  - Generally, fine-grained and compressible. Many local variations in soil conditions.

- **Alluvial-Lacustrine deposits.**
  - Material deposited within lakes (other than those associated with glaciation by waves, currents, and organo-chemical processes.
  - Clays are frequently varved, i.e., layered by the annual deposition of material
  - Usually very uniform in horizontal direction. Fine-grained soils generally compressible.

- **Piedmont deposits**
  - Alluvial deposits at foot of hills or mountains. Extensive plains or alluvial fans.
  - Generally favourable foundation conditions.

- **Deltaic deposits.**
  - Deposits formed at the mouths of rivers that result in extension of the shoreline.
  - Generally fine-grained and compressible. Many local variations in soil condition.

- Aeolian Soils: Material transported and deposited by wind.
  - **Loess**
    - A calcareous, unstratified deposit of silts or sandy or clayey silt traversed by a network of tubes formed by root fibres now decayed.
    - Relatively uniform deposits characterised by ability to stand in vertical cuts. Collapsible structure. Deep weathering or saturation can modify characteristics.

  - **Dune sands**
    - Mounds, ridges, and hills of uniform fine sand characteristically exhibiting rounded grains.
    - Very uniform grain size; may exist in relatively loose condition.

- Glacial soils: Material transported and deposited by glaciers, or by melt water from the glacier.

- **Glacial till**
  - An accumulation of debris, deposited beneath, at the side (lateral moraines,) or at the lower limit of a glacier (terminal moraine.) Material lowered to ground surface in an irregular sheet by a melting glacier is known as a ground moraine
  - Consists of material of all sizes in various proportions from boulder and gravel to clay. Deposits are unstratified. Generally present favourable foundation conditions; however, rapid changes in conditions are common.

- **Glacio-Fluvial deposits**
  - Coarse and fine-grained material deposited by streams of melt water from glaciers. Material deposited on ground surface beyond terminal of glacier is known as an outwash plain. Gravel ridges known as kames and eskers.
  - Many local variations. Generally, these present favourable foundation conditions.

- **Glacio-Lacustrine deposits**
  - Material deposited within lakes by melt water from glaciers. Consisting of clay in central portions of lake and alternate layers of silty clay or silt and clay (varved clay in peripheral zones.
  - Very uniform in a horizontal direction.

- Marine Soils: Material transported and deposited by ocean waves and currents in shore and offshore areas.
  - **Shore deposits**
    - Deposits of sands and/or gravels formed by the transporting, destructive, and sorting action of waves on the shoreline.
    - Relatively uniform and of moderate to high density.
  - **Marine clays**
    - Organic and inorganic deposits of fine-grained material.
    - Generally very uniform in composition. Compressible and usually very sensitive to remolding.

- Colluvial Soils: Material transported and deposited by gravity.
  - **Talus**
    - Deposits created by gradual accumulation of unsorted rock fragments and debris at base of cliffs.
    - Previous movement indicates possible future difficulties. Generally unstable foundation conditions.
  - **Hillwash**
    - Fine colluvium consisting of clayey sand, sand silt, or clay.

- Pyroclastic Soils: Material ejected from volcanoes and transported by gravity, wind and air.
  - **Ejecta**
    - Loose deposits of volcanic ash, lapilli, bombs, etc.
  - **Pumice**
    - Frequently associated with lava flows and mudflows, or may be mixed with nonvolcanic sediments.
    - Typically shardlike particles of silt size with larger volcanic debris. Weathering and redeposition produce highly plastic, compressible clay. Unusual and difficult foundation conditions.

Special Soils (problematic soil)

- Expansive Soils
- Collapsing Soils
- Permafrost and Frost Penetration
- Man-made and Hydraulic Fills
- Limestone and Related Soils
- Karst Topography
- Calcareous Soils
- Quick Clays
- Dispersive Clays
- Submarine Soils

Expansive Soils

- Expansive soils are distinguished by their potential for great volume increase upon access to moisture.
- Soils exhibiting such behaviour are mostly Montmorillonite clays and clay shales.
- Expansive soils can be identified by either their plasticity limit or a swell test

Collapsing Soils

- Collapsing soils are distinguished by their potential to undergo large decrease in volume upon increase in moisture content even without increase in external loads.

- Examples:
  - Loess
  - Weakly cemented sands and silts where cementing agent is soluble (e.g., soluble gypsum, halite, etc.)
  - Certain granite residual soils.
  - Deposits of collapsible soils are usually associated with regions of moisture deficiency.

Permafrost and Frost Penetration

- Volume Increase from underground ice formation leads to heave of structure
  - In non-frost susceptible soil: Typically 4% (porosity 40%, water volume increase in turning to ice = 10%, total heave = 40% x 10% = 4%).
  - In susceptible soil heave is much greater as water flows to colder zones (forming ice lenses). The associated loss of support upon thaw can be more detrimental to structure than the heave itself.
- Silts are the most susceptible to frost heave. Soils of types SM, ML, GM, SC, GC, and CL are classified as having frost heave potential.

Man-made and Hydraulic Fills

- Found in coastal facilities, levees, dikes and tailings dams.
- High void ratio.
- Subject to large amount of settlement.
- Uniform gradation but variable grain size within same fill.
- High liquefaction potential
- Lateral spreading.
- Easily eroded.

Limestone and Related Soils

● Karst Topography

- Limestone is very soluble
- Uneven underground erosion leads to erratic depth and quality of "bedrock"
- Erosion also leads to underground caverns and water flows
- Expansion of underground voids can lead to sinkholes

● Calcareous Soils

- Calcareous soils are those which are composed of primarily sand size particles of calcium carbonate, which may be indurated to varying degrees.
- They can originate from biological processes such as sedimentation of skeletal debris and coral reef formation.
- Because of their association with coral reefs, these soils appear mostly between the latitudes of 30°N and 30°S.
- These soils are some of the most challenging types of soils for the design and installation of foundations.

Quick Clays

- Quick clays are characterised by their great sensitivity or strength reduction upon disturbance.
- All quick clays are of marine origin.
- Because of their brittle nature, collapse occurs at relatively small strains. Slopes in quick clays can fail without large movements.
- Generally found in northern regions (Canada, Scandinavia, Alaska)

Dispersive Clays

- Easily eroded by low water velocities
- When placed into embankments, tunnels and gullies easily form (piping)
- Can be dealt with chemical treatment of the soil, use of geotextiles or blockage using different types of walls

Submarine Soils

- Found in continental shelf deposits at water depths up to several hundred feet

- Distribution and physical properties of sand, silt and clay may change with time and local geologic conditions
- Soil deposits have typical properties
- Some areas (Gulf of Mexico) have weak, underconsolidated deposits

Soil-Particle Size or Grain Sizes

We are often interested in the particle or grain sizes present in a particular soil as well as the distribution of those sizes.

Its range

Boulders or cobbles ⟷ Ultra fine – grained colloidal materials

D > 75 mm 10^8 max. log scale D < 0.001 mm

Table 2.3 Particle-Size Classifications

	Cohesionless soils		Cohesive soils	
	Grain size (mm)			
Name of organization	**Gravel**	**Sand**	**Silt**	**Clay**
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

Note: Sieve openings of 4.75 mm are found on a U.S. No. 4 sieve; 2-mm openings on a U.S. No. 10 sieve; 0.075-mm openings on a U.S. No. 200 sieve. See Table 2.5.

Soil Cohesion

Cohesionless Soils	*Cohesive Soils*
♦ Generally are granular or coarse grained ♦ Particles do not naturally adhere to each other ♦ Have higher permeability	♦ Generally are fine grained ♦ Particles have natural adhesion to each other ♦ due to presence of clay minerals ♦ Have low permeability

Coarse-grained, Granular or Cohesionless Soils

- Excellent foundation material for supporting structures and roads.
- The best embankment material.
- The best backfill material for retaining walls.
- Might settle under vibratory loads or blasts.
- Dewatering can be difficult due to high permeability.
- If free draining not frost susceptible

Fine-Grained or Cohesive Soils

- Very often, possess low shear strength.
- Plastic and compressible.
- Loses part of shear strength upon wetting.
- Loses part of shear strength upon disturbance.
- Shrinks upon drying and expands upon wetting.
- Very poor material for backfill.
- Poor material for embankments.
- Practically impervious.
- Clay slopes are prone to landslides.

Silts

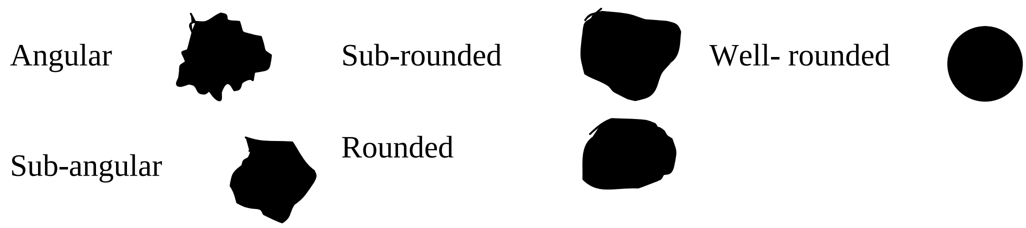
- Characteristics
  - Relatively low shear strength
  - High Capillarity and frost susceptibility
  - Relatively low permeability
  - Difficult to compact

- Compared to Clays
  - Better load sustaining qualities
  - Less compressible
  - More permeable
  - Exhibit less volume change

Aspects of Cohesionless Soils

Angularity

- Angular – Sharp Edges
- Subangular – Edges distinct but well rounded
- Subrounded
- Rounded
- Well Rounded



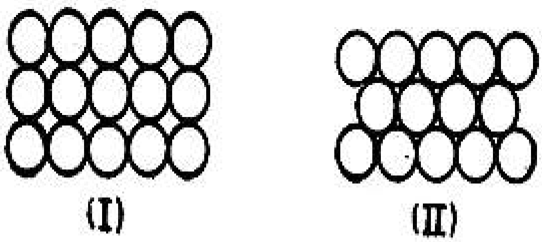
Angular particled soils generally exhibit better engineering properties; also can frequently pass larger particles through a given sieve size

Density

- Both unit weight and strength of soil can vary with particle arrangement
- Denser soils have both higher load carrying capacity and lower settlement

$e_{max} = 0.91 \qquad n_{max} = 48\%$

$e_{min} = 0.35 \qquad n_{min} = 26\%$



Relative Density

$$D_r = \frac{e_{max} - e_o}{e_{max} - e_{min}} x100$$

- e_{max} = void ratio of the soil in its loosest condition
- e_{min} = void ratio of the soil in its densest condition
- e_0 = void ratio in the natural or condition of interest of the soil
- Convenient measure for the strength of a cohesionless soil

Example

- Given
  - Sand Backfill
  - Unit Weight = 109 pcf
  - Water Content = 8.6%
  - Specific Gravity of Solids = 2.6
  - e_{max} = 0.642 (loosest state)

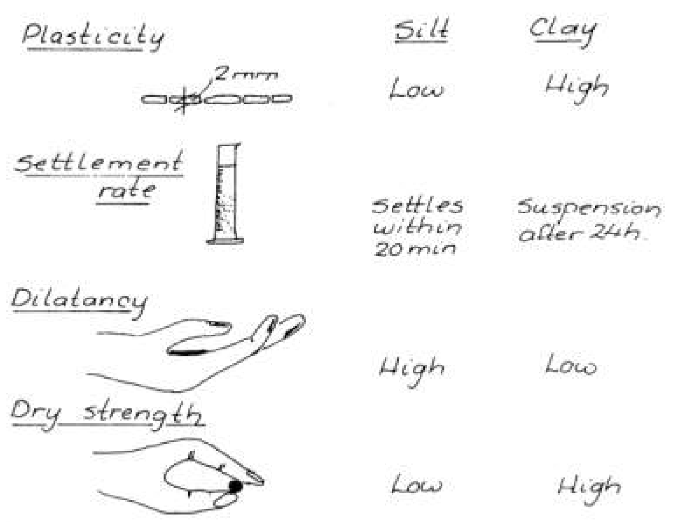
e_{min} = 0.462 (densest state)

- Find
  - Void Ratio
  - Relative Density

- Solution
  - Assume V_t = 1 ft^3; thus, W_t = 109 lbs.
  - Weight balance: 109 = $W_s + W_w$
  - Water content $\omega = W_w/W_s = 0.086$
  - Solving two previous equations:
  - W_s = 100.4 lbs; W_w = 8.6 lbs.
  - $V_s = W_s/\gamma_s = 100.4/((2.6)(62.4)) = 0.618\ ft^3$
  - $V_w = W_w/\gamma_w = 8.6/62.4 = 0.138\ ft^3$
  - $V_a = V_t - V_w - V_s = 1 - 0.138 - 0.618 = 0.243\ ft^3$

  - $e = V_v/V_s = (V_a + V_w)/V_s = (0.243 + 0.138)/0.618 = 0.616$
  - $D_r = \dfrac{e_{\max} - e_o}{e_{\max} - e_{\min}} x100 = \dfrac{0642 - 0618}{0642 - 0.462} x100 = 14.2\%$

Properties of Fine Soils



Aspects of Cohesive and Fine Grained Soils

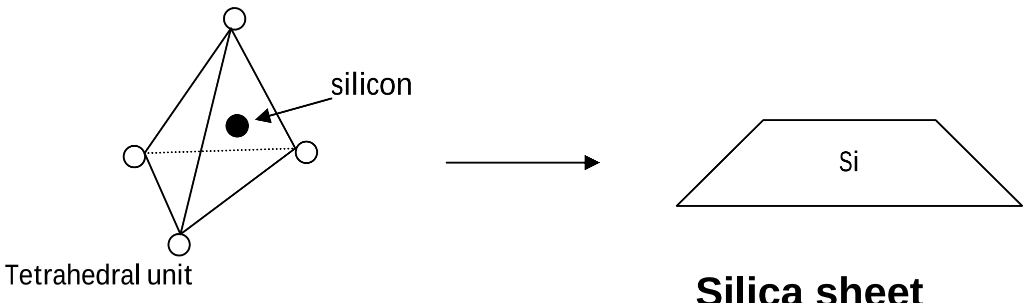
- Structure of Clay Minerals
- Types of Clay Minerals
- Clay Minerals and Water
- Particle Orientation of Clay Soils
- Thixotropy

Structure of Clay Minerals

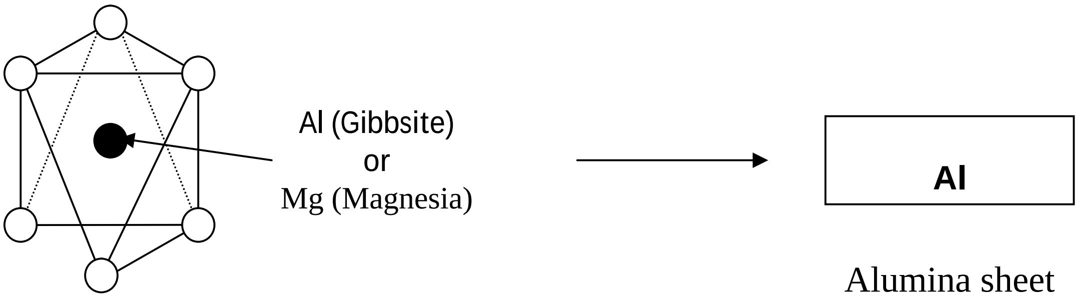
Clay minerals are very tiny crystalline substances evolved primarily from chemical weathering of certain rock forming minerals, they are complex alumino – silicates plus other metallic ions.

All clay minerals are very small with colloidal – sized (D < 1μm). Because of their small size and flat shape, they have very large specific surfaces. There is usually a negative electric charge on the crystal surfaces and electro – chemical forces on these surfaces are therefore predominant in determining their engineering properties. In order to understand why these materials behave as they do, it will be necessary to examine their crystal structure in some detail.

- Atoms of clay minerals form sheets
  - Silica tetrahedral sheets



- Alumina octahedral sheets



Alumina sheet

Octahedral unit

- Sheets can layer in different ways, forming different types of clay minerals
- Clay minerals tend to form flat, platelike, and niddle shapes

- Electro – Chemical Forces
  - Primary valency bonds
  - Van der Waals forces or molecular bonds
  - Polar forces
  - Hydrogen bonds

- Isomorphic substitutions and absorbed ions

  It is the replacement of the silicon and aluminum ions in the crystal by other elements, with no change in the crystalline structure

Types of Clay Minerals

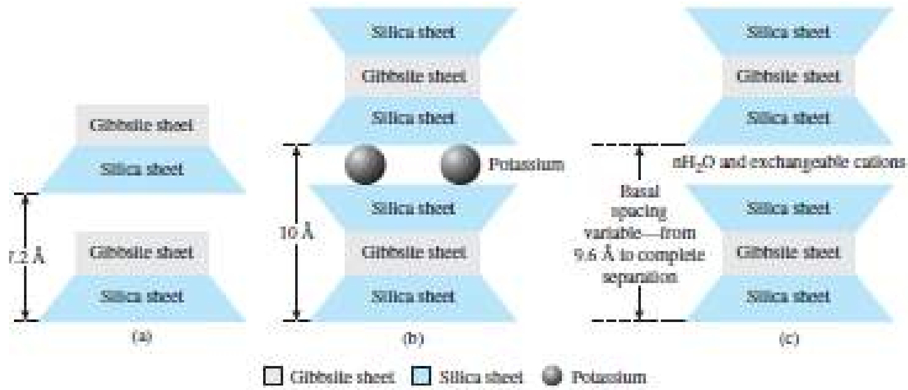


Figure 2.12 Diagram of the structures of (a) kaolinite; (b) illite; (c) montmorillonite

Kaolinite group	*Illite group*	*Montmorillonite group*
Kaolinite • One sheet alumina, one silica • Most prevalent clay mineral Halloysite • One sheet alumina, one silica, sheet of water in between • Properties affected by presence or removal of water sheet • Reverts to kaolinite when water is removed	Illite • One silica, one alumina, one silica sheet, bonded with potassium • More plastic than kaolinite • Most prevalent in marine deposits	Montmorillonite • Same as Illite except no potassium; iron or magnesium • replace the alumina • Very prone to expansion with changes in water content

Specific surface

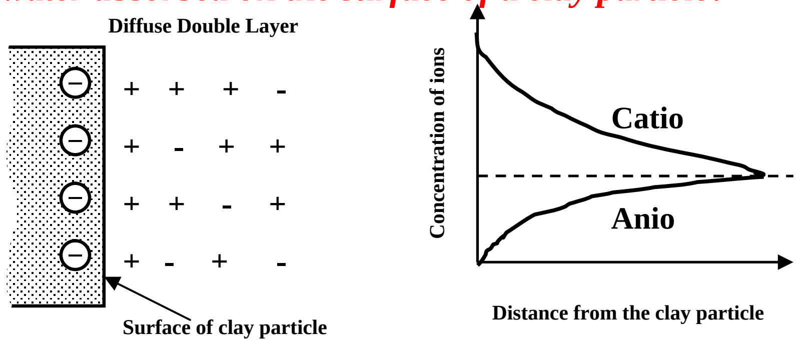
It defines as the ratio of the surface area (As) of a material to either its volume (V) for regular shape or mass (m) for irregular shape of soil particles.

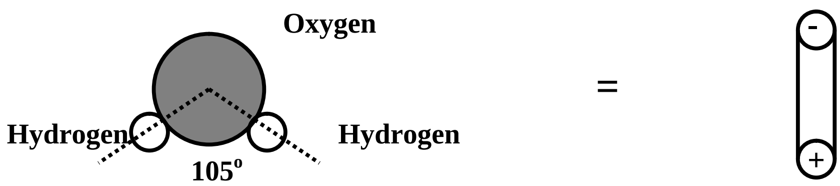
$$S.S = \frac{As}{V} \left(\frac{1}{length}\right) \quad ; \quad S.S = \frac{As}{m} \left(\frac{length^2}{mass}\right)$$

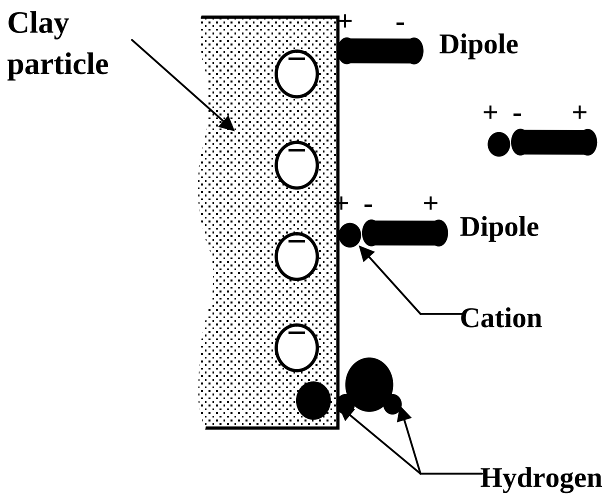
To demonstrate this, S.S for cubes with different dimensions were computed as follows:-

Cube	S.S
1x1x1 cm^3	$\frac{6(1cm^2)}{1cm^3} = 6/cm = 0.6/mm$
1x1x1 mm^3	$\frac{6(1mm^2)}{1mm^3} = 6/mm$
1x1x1 µm^3	$\frac{6(1\mu m^2)}{1\mu m^3} = 6/\mu m = 6000/mm$

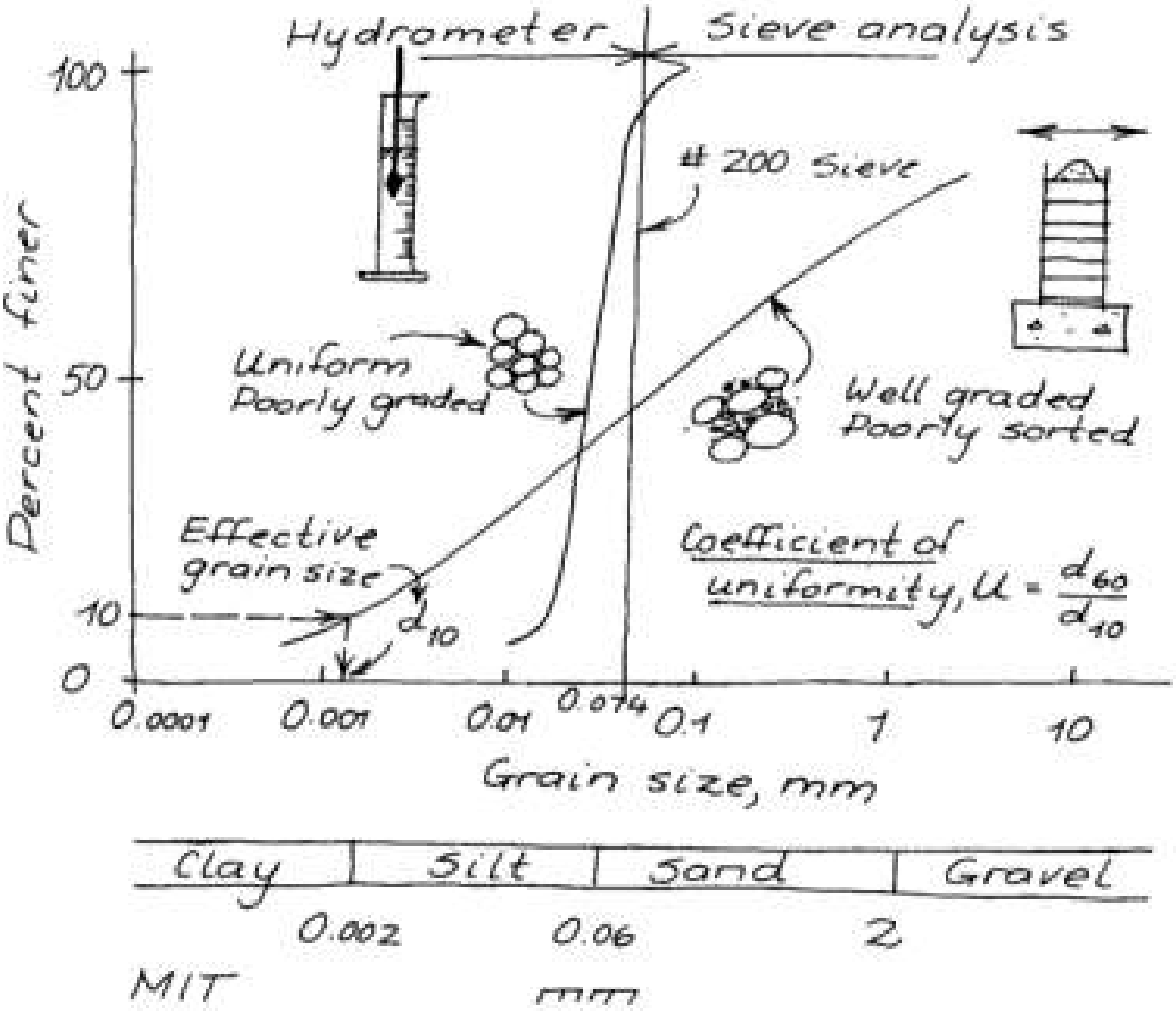
How is water absorbed on the surface of a clay particle?





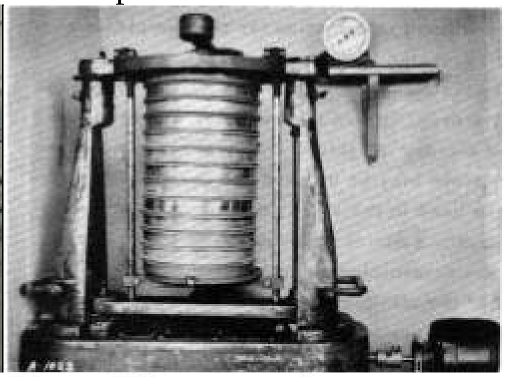


Gradation of Particle Size



Sieve Analysis

- Primarily applied to granular (cohesionless) soils
- Passes soil sample through a series of sieves of varying mesh fineness
- Different portions of soil with different grain size pass through each mesh
- Distribution of grain sizes constructed and plotted



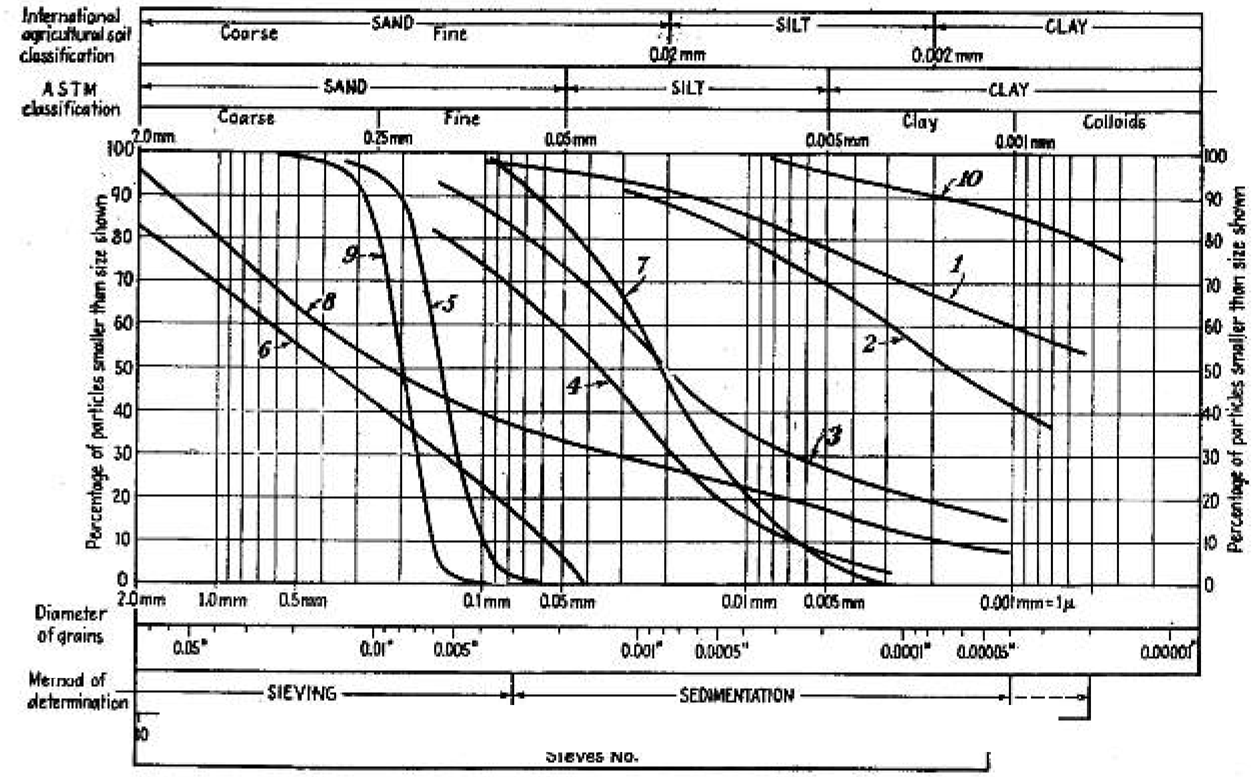


FIG. 3-1. Some grain-size-accumulation curves. 1 and 2—clay soils of the Nile Delta; 3 and 4—silts from the Nile Delta; 5—Port Said beach sand; 6—sand artificially graded for maximum density; 7—Vicksburg loess; 8—New Mexico adobe brick; 9—Daytona Beach sand; 10—Wyoming bentonite. Soils 1 to 9 were tested by G. P. Tschebotarioff. Data on soil 10 are taken from Ref. 6.

Dx – designates particle size for which x percent of sample has passed

- D_{10} – effective size – particle size at which 10% of the sample has passed. It is useful to determine permeability

Uniformity Coefficient Cu

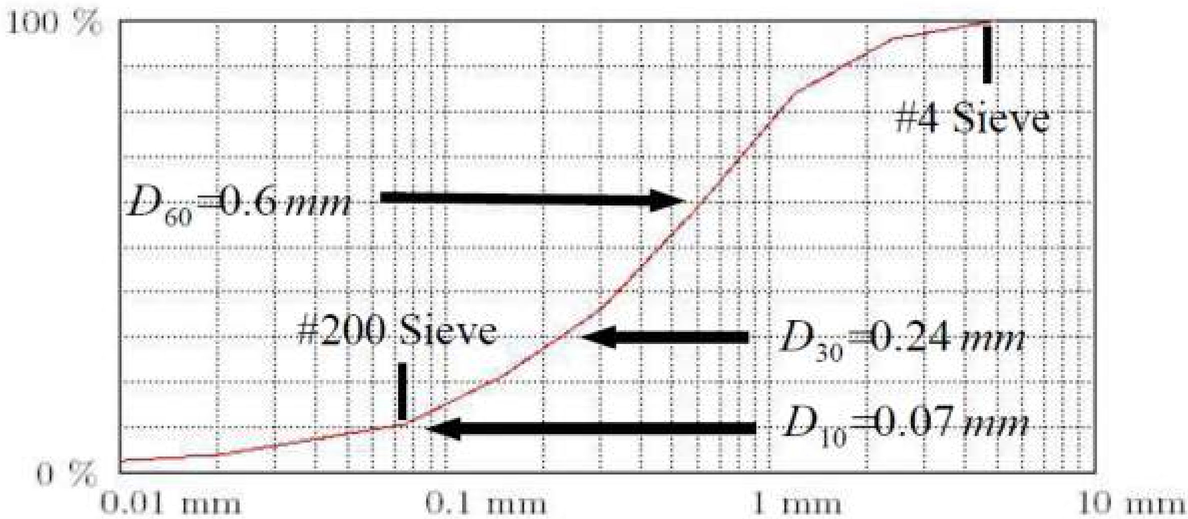
$$C_u = \frac{D_{60}}{D_{10}}$$

- Well graded – even distribution of different particle sizes—Cu > 10
- Poorly graded – most particles in a narrow size range— Cu < 5
- Gap Graded – some particle size ranges are missing

Coefficient of Curvature Cc

$$C_u = \frac{D^2{}_{30}}{D_{10}D_{60}}$$

Sieve Analysis Example



$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.6}{0.07} = 8.5$$ (Below well graded)

$$C_u = \frac{D^2{}_{30}}{D_{10}D_{60}} = \frac{0.24^2}{0.07x0.6} = 1.37$$

Passing #4 and #200 Sieve

- Portion Passing #200 (0.074 mm) Sieve
  - Measure of whether soil is cohesive or Cohesionless (50%)
  - In this case, portion is approximately 10% of sample, so soil is definitely cohesionless
- Portion Remaining on #4 Sieve
  - Measure of whether a soil is a gravel or a sand (50%)
  - Usually taken as a percentage of soil not passing #200 sieve
  - For this sample, percentage is negligible, so soil is sand

Hydrometer Analysis

Hydrometer analysis is based on the principle of sedimentation of soil grains in water. When a soil specimen is dispersed in water, the particles settle at different velocities, depending on their shape, size, and weight, and the viscosity of the water, (detailed discus will be hold on lab.)

Weight-Volume Relationships, Plasticity, and Structure of Soil

Topics in Soil Composition

- Weight-Volume Relationships
- Important variables-(Water or Moisture Content-Unit Weight or Mass-Void ratio-Specific Gravity……..etc.
- Relative Density
- Particle Size and Shape
- Grain Size Tests
- Sieve Tests (Coarse-Grained Soils)
- Hydrometer Tests (Fine-Grained Soils)
- Plasticity and the Atterberg Tests

Basic Concepts

- Soil is a collection of particles that do not form a totally solid substance

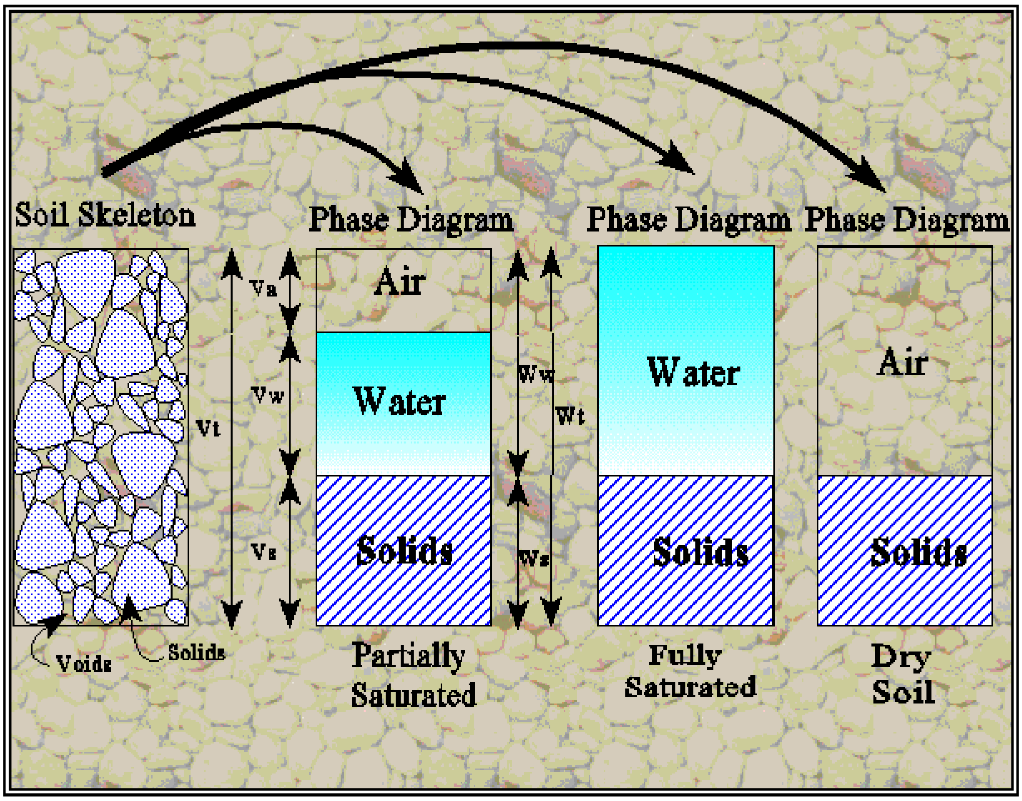
- Soil is a combination of:
    - ♦ Soil material in particles
    - ♦ Air
    - ♦ Water

- The relationship between this combination defines much of what any particular soil can do to support foundations

Phase Diagram

Assumptions and Definitions:

- Weight of air = 0
- Dry Soil: Water weight and volume = 0
- "Volume of voids" include all non-soil volume, both air and water



Saturated Soil

- Saturated Soil: Air volume = 0
- Only water and solids appear in completely saturated soil

Basic Formulas

$V_{total} = V_{air} + V_{water} + V\ soil$

$W_{total} = W_{water} + W_{soil}$ or $M_{total} = M_{water} + M_{soil}$

$W_x = \gamma_x \times V_x$ or $M_x = \rho_x \times V_x$

Specific Gravity and Density

- Unit Weight of Water (γ_w)
    - 62.4 lb/ft^3
    - 9.81 $kN/m^3 \approx 10\ kN/m^3$
- Density of Water
    - 1.95 $slugs/ft^3$
    - 1 g/cm^3 = 1 Mg/m^3 = 1 Metric Ton/m^3

Typical Specific Gravities for Soil Solids

- Quartz Sand: 2.64 – 2.66
- Silt: 2.67 – 2.73
- Clay: 2.70 – 2.9
- Chalk: 2.60 – 2.75
- Loess: 2.65 – 2.73
- Peat: 1.30 – 1.9
- Except for organic soils, range is fairly narrow

Weight and Volume Relationships

$W_x = G_x \times \gamma_w \times V_x$
$M_x = G_x \times \gamma_w \times V_x$

In most cases, calculations in soil mechanics are done on a weight basis. Exceptions include wave propagation problems (earthquakes, pile dynamics,……. etc.)

Important Variables

1. ***Void ratio, e***

$$e = \frac{Vv}{Vs}$$ Expressed as decimal Sands (0.4 – 1.0) Clays (0.3 – 1.5)

2. ***Porosity , n***

$$n = \frac{Vv}{Vt} x100\%$$ Expressed as percentage (0-100%)

Prove that $n = \dfrac{e}{1+e}$ or $e = \dfrac{n}{1-n}$

3. Degree of saturation, S

$S = \dfrac{Vw}{Vv} x100\%$ S = 0 % Dry Soil, S = 100 % Saturated soil

4. Air Content, Ac

$$Ac = \frac{Va}{V} x100\%$$

So we can show that $\boldsymbol{Ac = n(1-S)}$

5. Water Content, ω

$w = \dfrac{Ww}{Ws} x100\%$ ω can be equal to zero in dry soil and may be reached 500% in some marine and organic soils.

6. Unit weight, γ

Total unit weight, $\gamma_t = \dfrac{Wt}{Vt} = \dfrac{Ws + Ww}{Vt}$

Solid unit weight, $\gamma_s = \dfrac{Ws}{Vs}$ γ_s range (25.4 kN/m^3 - 28.5 kN/m^3)

Water unit weight, $\gamma_w = \dfrac{Ww}{Vw}$

There are three other useful densities in soils engineering; they are

- Dry Unit weight, $\gamma_d = \dfrac{Ws}{Vt}$
- Saturated Unit Weight, $\gamma_{sat} = \dfrac{Ws+Ww}{Vt} = \dfrac{W_t}{V_t}$ (Va = 0, S = 100 %)
- Submerged Unit Weight, $\boldsymbol{\gamma^{/} = \gamma_{sat} - \gamma_w}$

If we replaced the weight in these relationships by mass we could find basic definitions for density (ρ) instead of unit weight (γ).

7. Specific gravity

$$G = \frac{g}{g_w} \quad \text{apparent}$$

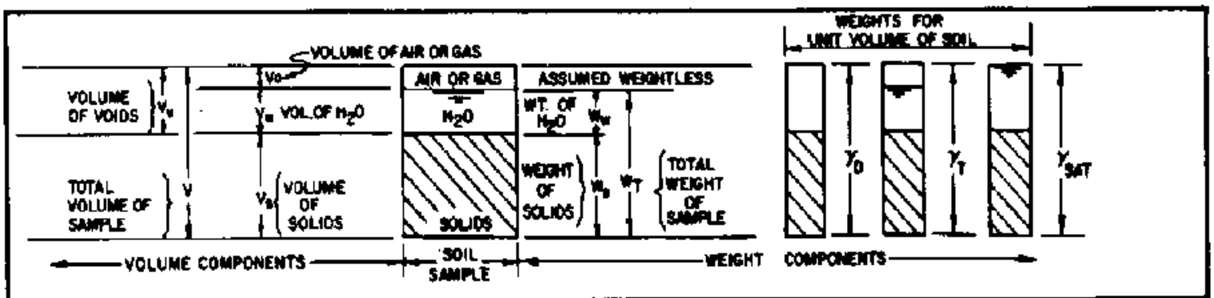
$$G_s = \frac{g_s}{g_w} \quad \text{Solid}$$

Typical Specific Gravities for Soil Solids

- Quartz Sand: 2.64 – 2.66
- Silt: 2.67 – 2.73
- Clay: 2.70 – 2.9
- Chalk: 2.60 – 2.75
- Loess: 2.65 – 2.73
- Peat: 1.30 – 1.9
- Except for organic soils, range is fairly narrow

Computing Soil Composition

TABLE 6
Volume and Weight Relationships



		PROPERTY	SATURATED SAMPLE (W_s, W_w, G, ARE KNOWN)	UNSATURATED SAMPLE (W_s, W_w, G, V, ARE KNOWN)	SUPPLEMENTARY FORMULAS RELATING MEASURED AND COMPUTED FACTORS			
VOLUME COMPONENTS	V_s	VOLUME OF SOLIDS	$\frac{W_s}{G\gamma_w}$		$V-(V_a+V_w)$	$V(1-n)$	$\frac{V}{(1+e)}$	$\frac{V_v}{e}$
	V_w	VOLUME OF WATER	$\frac{W_w}{\gamma_w}$		V_v-V_a	SV_v	$\frac{SVe}{(1+e)}$	$SV_s e$
	V_a	VOLUME OF AIR OR GAS	ZERO	$V-(V_s+V_w)$	V_v-V_w	$(1-S)V_v$	$\frac{(1-S)Ve}{(1+e)}$	$(1-S)V_s e$
	V_v	VOLUME OF VOIDS	$\frac{W_w}{\gamma_w}$	$V-\frac{W_s}{G\gamma_w}$	$V-V_s$	$\frac{V_s n}{1-n}$	$\frac{Ve}{(1+e)}$	$V_s e$
	V	TOTAL VOLUME OF SAMPLE	V_s+V_w	MEASURED	$V_s+V_a+V_w$	$\frac{V_s}{1-n}$	$V_s(1+e)$	$\frac{V_v(1+e)}{e}$
	n	POROSITY	$\frac{V_v}{V}$		$1-\frac{V_s}{V}$	$1-\frac{W_s}{GV\gamma_w}$	$\frac{e}{1+e}$	
	e	VOID RATIO	$\frac{V_v}{V_s}$		$\frac{V}{V_s}-1$	$\frac{GV\gamma_w}{W_s}-1$	$\frac{W_w G}{W_s S}$	$\frac{n}{1-n}$; $\frac{wG}{S}$

TABLE 6 (continued)
Volume and Weight Relationships

		PROPERTY	SATURATED SAMPLE (W_s, W_w, G, ARE KNOWN)	UNSATURATED SAMPLE (W_s, W_w, G, V, ARE KNOWN)	SUPPLEMENTARY FORMULAS RELATING MEASURED AND COMPUTED FACTORS			
WEIGHTS FOR SPECIFIC SAMPLE	W_s	WEIGHT OF SOLIDS	MEASURED		$\frac{W_T}{(1+w)}$	$GV\gamma_w(1-n)$	$\frac{W_w G}{eS}$	
	W_w	WEIGHT OF WATER	MEASURED		wW_s	$S\gamma_w V_v$	$\frac{eW_s S}{G}$	
	W_t	TOTAL WEIGHT OF SAMPLE	$W_s + W_w$		$W_s(1+w)$			
WEIGHTS FOR SAMPLE OF UNIT VOLUME	γ_D	DRY UNIT WEIGHT	$\frac{W_s}{V_s + V_w}$	$\frac{W_s}{V}$	$\frac{W_t}{V(1+w)}$	$\frac{G\gamma_w}{(1+e)}$	$\frac{G\gamma_w}{1+wG/S}$	
	γ_T	WET UNIT WEIGHT	$\frac{W_s+W_w}{V_s+V_w}$	$\frac{W_s+W_w}{V}$	$\frac{W_T}{V}$	$\frac{(G+Se)\gamma_w}{(1+e)}$	$\frac{(1+w)\gamma_w}{w/S+1/G}$	
	γ_{SAT}	SATURATED UNIT WEIGHT	$\frac{W_s+W_w}{V_s+V_w}$	$\frac{W_s+V_v\gamma_w}{V}$	$\frac{W_s}{V}+\left(\frac{e}{1+e}\right)\gamma_w$	$\frac{(G+e)\gamma_w}{(1+e)}$	$\frac{(1+w)\gamma_w}{w+1/G}$	
	γ_{SUB}	SUBMERGED (BUOYANT) UNIT WEIGHT	$\gamma_{SAT}-\gamma_w$		$\frac{W_s}{V}-\left(\frac{1}{1+e}\right)\gamma_w$	$\left(\frac{G+e}{1+e}-1\right)\gamma_w$	$\left(\frac{1-1/G}{w+1/G}\right)\gamma_w$	
COMBINED RELATIONS	w	MOISTURE CONTENT	$\frac{W_w}{W_s}$		$\frac{W_t}{W_s}-1$	$\frac{Se}{G}$	$S\left[\frac{\gamma_w}{\gamma_D}-\frac{1}{G}\right]$	
	S	DEGREE OF SATURATION	1.00	$\frac{V_w}{V_v}$	$\frac{W_w}{V_v\gamma_w}$	$\frac{wG}{e}$	$\frac{w}{\left[\frac{\gamma_w}{\gamma_D}-\frac{1}{G}\right]}$	
	G	SPECIFIC GRAVITY	$\frac{W_s}{V_s\gamma_w}$		$\frac{Se}{w}$			

Example 1

- Given:
  - Total Volume = 1 cu. ft.
  - Total Weight = 140 lb.
  - Dry Weight = 125 lb.
- Find
  - Water Content
  - Wet Unit Weight
  - Dry Unit Weight

- By Definition:
  - Dry Unit Weight = Dry Weight = 125 lb/ft^3
  - Wet Unit Weight = Total Weight = 140 lb/ft^3
- Solve for Weight of Water
  - $W_T = W_s + W_w$
  - $140 = 125+W_w$
  - $W_w = 15\ lb/ft^3$
- Solve for Water Content
  - $w = W_w/W_s = W_w/125 = 15/125 = 0.12 = 12\%$

Example 2

- Given:
  - Total Mass = 18.18 kg
  - Total Volume = 0.009 m^3
  - Dry Mass = 16.13 kg
  - Specific Gravity of Solids = 2.7
- Find
  - Wet Density
  - Dry Unit Weight
  - Void Ratio
  - Water Content

- Compute Mass of Water
  - $Mt = M_s+M_w$
  - $18.18 = 16.13+M_w$
  - $M_w = 2.05$ kg

- Compute Water Content
    - $w = M_w/M_s$
    - $w = 2.05/16.13 = .127 = 12.7\%$

- Compute Volumes
    - Volume of Water
        - $V_w = M_w / \rho_w$
        - $V_w = 2.05/1000 = 0.00205\ m^3$
    - Volume of Solids
        - $V_s = M_s/\rho_s = M_s/(G_s\rho_w)$
        - $V_s = 16.13/((1000)(2.7)) = 0.00597\ m^3$
    - Volume of Air
        - $V_a = V_t - V_w - V_s$
        - $V_a = 0.009-0.00205-0.00597 = .00098\ m^3$

Example 3

- Given
    - Saturated Soil
    - Void Ratio = 0.45
    - Specific Gravity of Solids = 2.65
- Find
    - Wet Unit Weight
    - Water Content
- Assumptions
    - $V_a = 0$
    - $V_t = 1$
    - Vs + Vw = 1
    - γ_w water = 62.4 lb/ft^3
- Solve for Volumes
    - for saturated soil $V_v = V_w$
    - $e = V_w/V_s = 0.45$
    - $V_w = 0.31\ ft^3$
    - $V_s = 0.69\ ft^3$
- Compute Wet Unit Weight
    - Weight of Soils = $\gamma_w V_s G_s = (62.4)(0.69)(2.7) = 114$ lb
    - Weight of Water = $\gamma_w V_w = (62.4)(0.31) = 19.4$ lb

- Total Weight = 114 + 19.4 = 133.4 lb
- Since volume is unity, total weight is also net unit
- weight = 133.4 pcf

● Compute Water Content
- $\omega = W_w/W_s = 19.4/114 = 0.17 = 17\%$

Example 4

● Given
- Well Graded Sand
- Specific Gravity of Solids = 2.65
- Void Ratio = 0.57
- Porosity = 36.5%

● Find
- Degree of Saturation
- Wet and Dry Unit Weight of Soil

●Solution
- Set sample volume = 1 m^3
- Total Volume = $1= V_w + V_a + V_s$
- Void ratio $e = 0.57 = V_v/V_s$
- $V_t = 1 = 2.754$ (Vw + Va) .... (1)
- Porosity = $n = V_v/V_t = (V_a+V_w)/V_t = 0.365 = V_a + V_w$ .... (2)
- Solving (1) and (2) for V_a and V_w,
- $V_a = 0.00305\ m^3$
- $V_w = 0.362\ m^3$
- then $V_s = 0635\ m^3$

● Degree of Saturation
- $S=V_w/V_v = V_w/(V_w+V_a) = 0.362/(0.362+.0031) = 0.99 = 99\%$
- Soil is for practical purposes saturated

● Dry Unit Weight
- $W_s = \gamma_w G_s V_s = (9.81)(2.65)(.635) = 16.51\ kN/m^3$
- Weight of Water
- $W_w = \gamma_w V_w = (9.81)(.362) = 3.55\ kN/m^3$

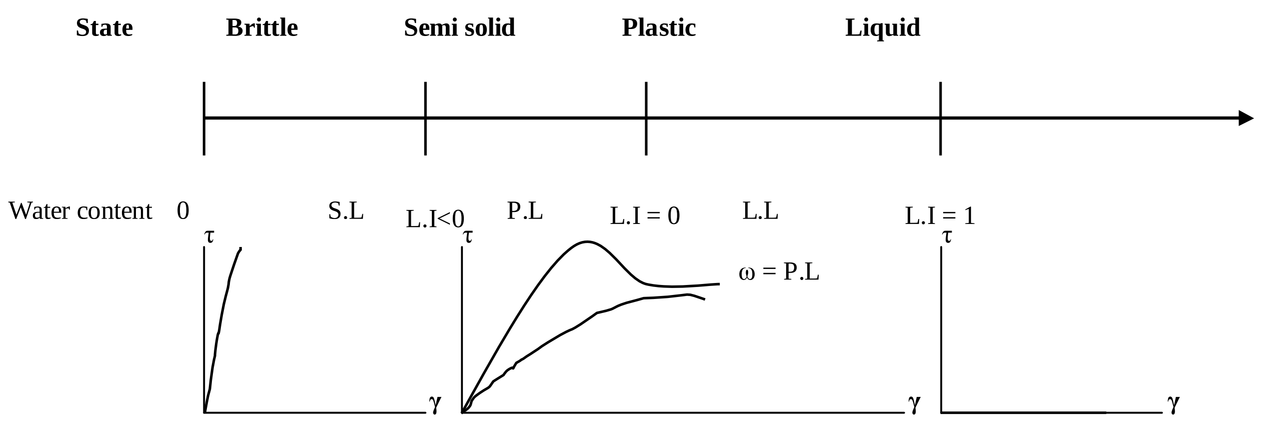
●Wet Unit Weight
- $W_t = W_w + W_v = 20.06\ kN/m^3$

Atterberg limits and Consistency indices

They are water contents at certain limiting or critical stages in soil behavior (especially, fine- grained soils). They, along with the natural water content (ω_n) are the most important items in the description of fine- grained soils and they are correlate with the engineering properties & behavior of fine- grained soils.

They are-

1- Liquid Limit (L.L or ω_L).
2- Plastic Limit (P.L or ω_P).
3- Shrinkage limit (S.L or ω_S).



Stress - strain response

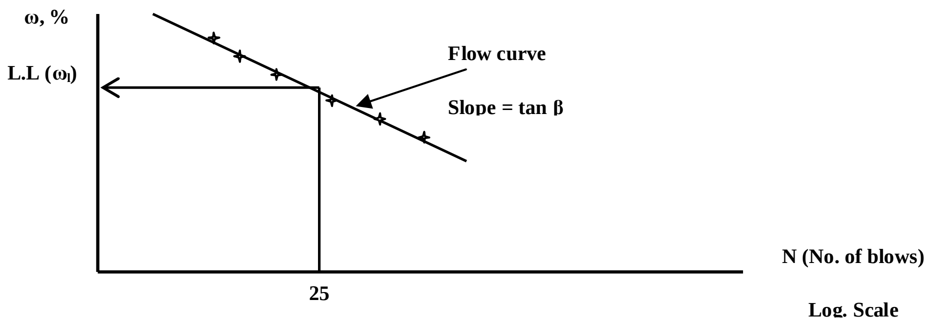
Liquid Limit

<u>Definition</u>

Atterberg defined the liquid limit as a water content at which the soil becomes a viscous liquid.

Casagrande- defined the liquid limit as a water content at which a standard groove cut in the remolded soil sample by a grooving tool will close over a distance of 13 mm (1/2”) at 25 blows of the L.L cup falling 10 mm on a hard rubber base. (See the figure below)

In practice, it is difficult to mix the soil so that the groove closure occurs at exactly 25 blows, so Casagrande did the following:



Sometimes one – point liquid limit test can be used because, for soils of similar geologic origin, the slopes of the flow curves are similar.

$$L.L(w_L) = w_n\left(\frac{n}{25}\right)^{\tan b}$$

Where tan β = slope of flow curve = 0.121

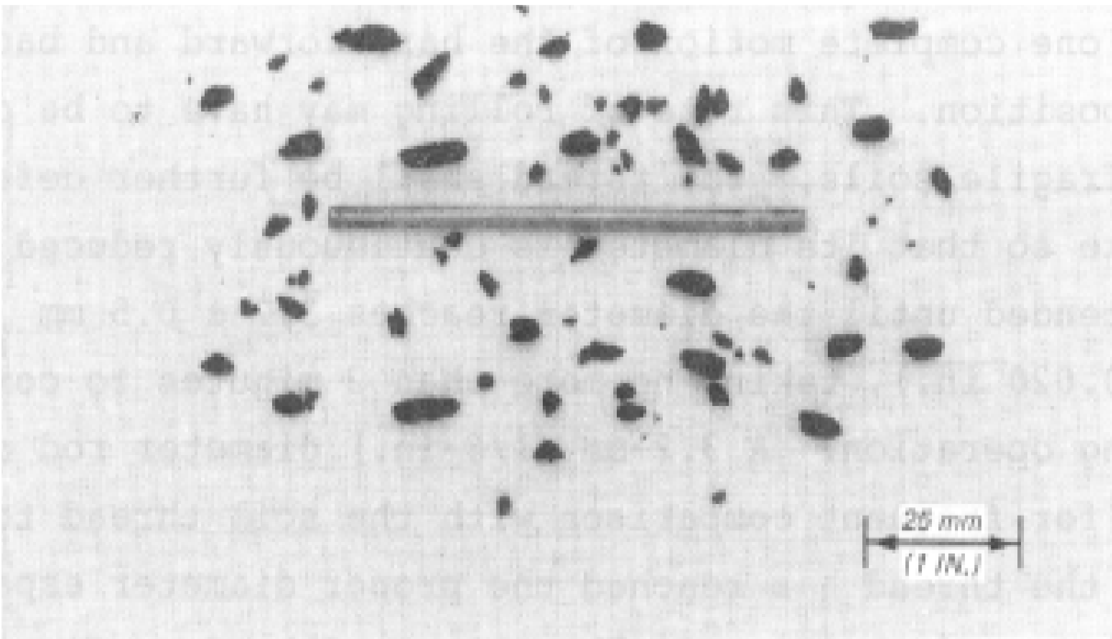
not equal for all soils

n = 20 – 30 for best results

Plastic Limit

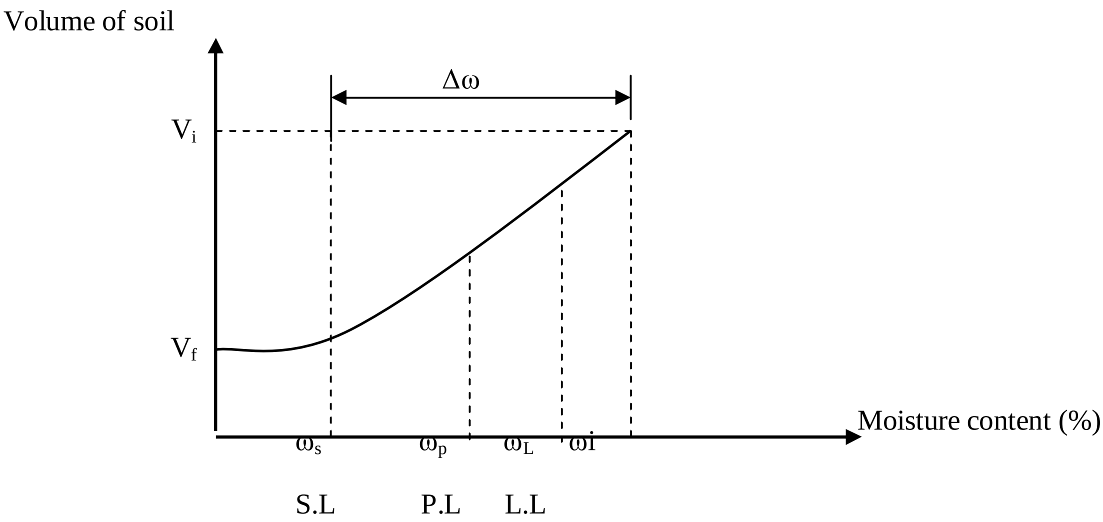
Atterberg defined the plastic limit as water content at which soil becomes in plastic state .

Casagrande defined the plastic limit as water at which a thread of soil just crumbles when it is carefully rolled out to a diameter of 3 mm(1/8”). It should break up into segments about 3 – 10 mm (1/8 – 3/8 inch) long. If the thread crumbles at diameter smaller than 3 mm, the soil is too wet. If the thread crumbles at diameter grater than 3 mm, the soil past the P.L

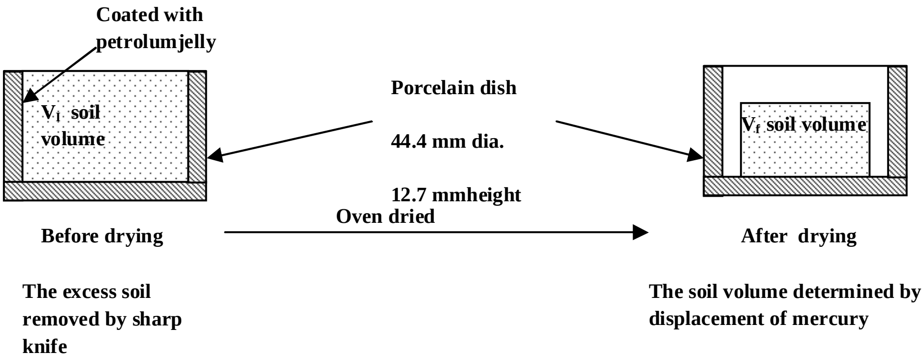


Shrinkage Limit

It defines as a water content at which no further volume change occurs with continuous loss of moisture.



The following figure illustrate the concept of the tests



Referring to the figure that illustrate the test

S.L = $\omega_i - \Delta\omega$ where ω_i = initial water content

$\Delta\omega$ = change in water content

However

$$w_i(\%) = \frac{m_1 - m_2}{m_2} x 100\%$$

also

$$\Delta w(\%) = \frac{(V_i - V_f)}{m_2} x r_w x 100\%$$

$$\therefore S.L = w_i - \Delta w$$

We can also estimate the magnitude of S.L using the plasticity chart, as we will described in lab.

Other index properties for the soil

- Plasticity index, $P.I = L.L - P.L$

- Flow index, $F.I = \frac{w_2 - w_1}{\log N_2 - \log N_1} = \frac{\Delta w}{\log \frac{N_2}{N_1} = 1 \, for \, \mathbf{K} \, one \, \mathbf{K} \, cycle} = \Delta w$

slope of flow curve, it shows how close the clayey soil

from the plastic state

- Toughness index, $T.I = \frac{P.I}{F.I}$ express the soil consistency in the plastic State.
- Consistency index, $C.I = \frac{L.L - w_n}{L.L - P.L} = \frac{L.L - w_n}{P.I}$
- Liquidity index, $L.I = \frac{w_n - P.L}{P.I}$

L.I < 0 --- the soil is in Brittle state
L.I (0 –1) – the soil is in plastic state
L.I >1 --- the soil is in viscous liquid state

Factors affecting the Atterberg Limits

1. Shape and size of grains.

   As the grains size get smaller the plasticity increases while grains with flaky shape had more plasticity characteristics than other shapes.

2. Content of clay minerals.

   As the content of clay minerals increase the plasticity characteristics increase.

3. Type of clay minerals.

   As we will describe later the characteristics of each type of clay mineral group the type will effect the plasticity characteristics and for instance

   Montmorillonite
   Illite
   Kaolinite

   Plasticity increase

4. Type of ions.

   The type of absorbed ions will effect the plasticity characteristics such as Na , Mg will give high plasticity while Ca will give low plasticity.

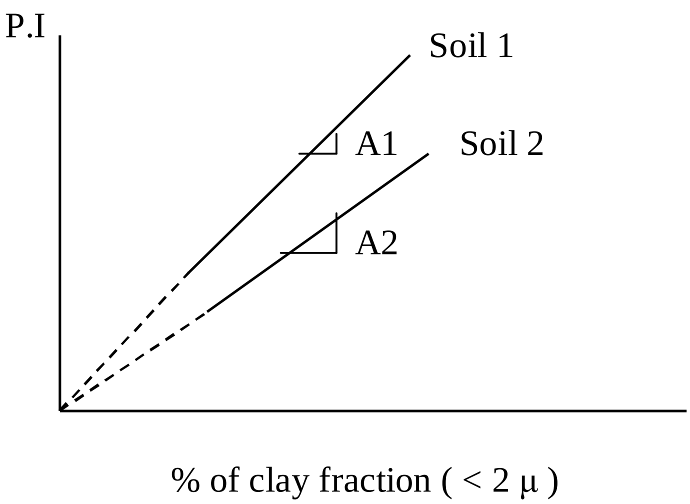
5. Content of organic matter.

   As the organic matter content increase the plasticity characteristics Increase.

Activity

Skempton (1953) observed the following relationship. He defined a quantity called "***Activity***" which the slope of the line correlating P.I & % finer than 2 μm.

$$A = \frac{P.I}{\% of clay - size fraction, by weight}$$



This term used for identifying the swelling potential of clay soils and for certain classification properties.

A	Soil classification
< 0.75	Non Active
0.75 – 1.25	Normally Active
1.25 – 2.0	Active

A	Type of clay minerals
0.4 – 0.5	Kaolinite
0.5 – 1.0	Illite
1.0 – 7.0	Montmorillonite

Example

The following data were obtained from the liquid & plastic limits tests for a soil with ω_n = 15 %

Liquid limit test		Plastic limit test
No. of blows	Moisture content;ω %	
15	42	P.L = 18.7 %
20	40.8	
28	39.1	

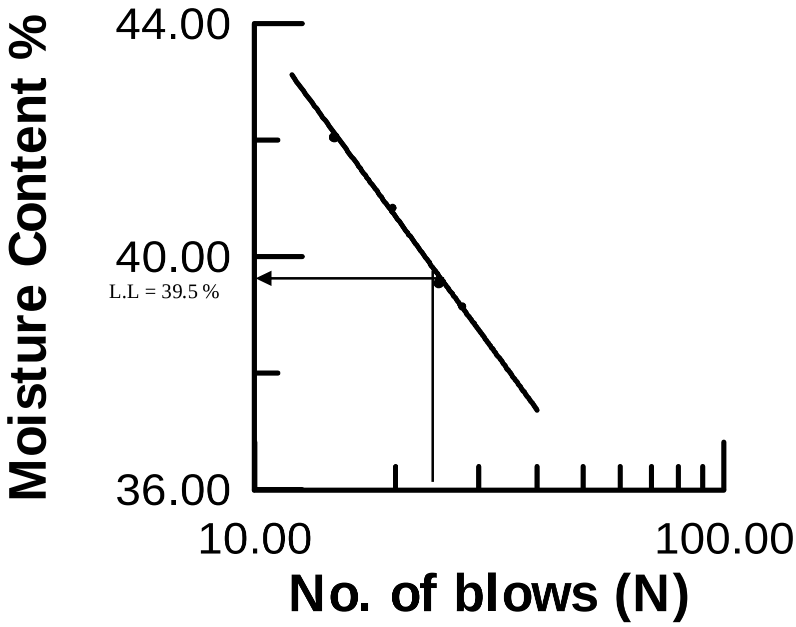
Required

a- Draw the flow curve & find the liquid limit.

b- Find the plasticity index of the soil

c- Find L.I, C.I, F.I, T.I

Solution



$$P.I = L.L - P.L = 39.5 - 18.7 = 20.8$$

$$L.I = \frac{w_n - P.L}{P.I} = \frac{15 - 18.7}{20.8} = -0.178 < 1$$

$$C.I = \frac{L.L - w_n}{P.I} = \frac{39.5 - 15}{20.8} = 1.178$$

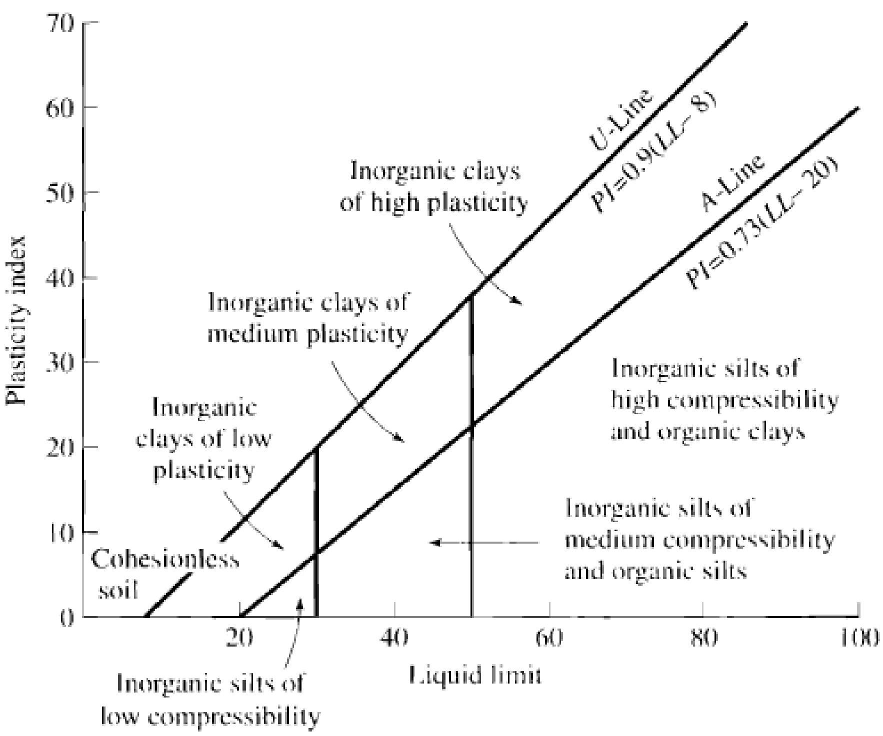
$$F.I = \frac{42 - 40.8}{\log 15 - \log 20} = -9.6$$

$$T.I = \frac{P.I}{F.I} = \frac{20.8}{9.6} = 2.167$$

The soil is heavily preconsolidated, since ω_n is smaller than P.L & lower than L.L.

Plasticity Chart

Casagrande (1932)



Soil Structure and Fabric

In geotechnical engineering, the structure of a soil affects or governs the engineering behavior of particular soil and is taken to mean both –

1. ***Geometric arrangement*** of the particles or mineral grains with respect to each other (soil fabric).
2. ***Interparticle forces*** *which may act between the particles or minerals grains. They probably have two main causes : Orientation of the adsorbed water and Cementation*

Factors that affect the soil structure are-

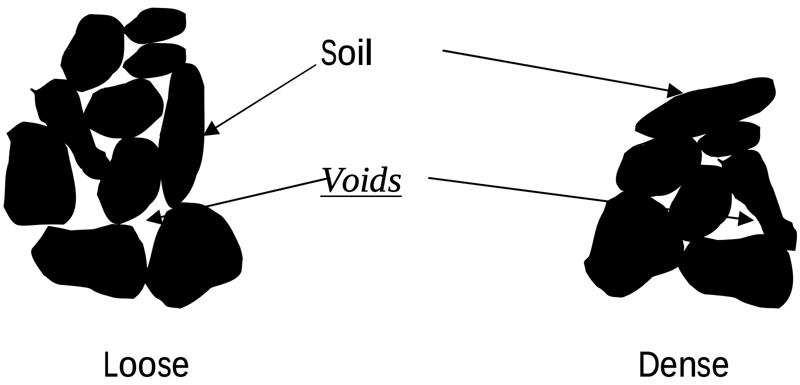
- The shape, size, and mineralogical composition of soil particles,
- The nature and composition of soil water.

Structures in Cohesionless Soil

The structures generally encountered in cohesionless soils can be divided into two major categories:-

1. *Single – grained structure*
2. *Honeycombed structure*

Single – grained structure

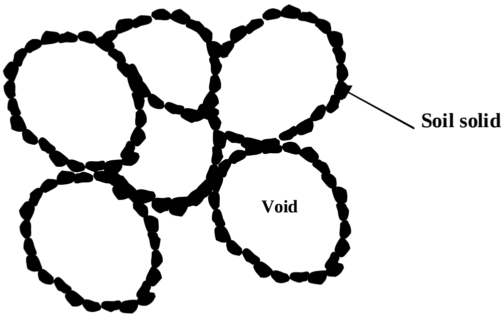


A useful way to characterize the density of a natural granular soil is with relative density D_r as described before.

Honeycombed structure

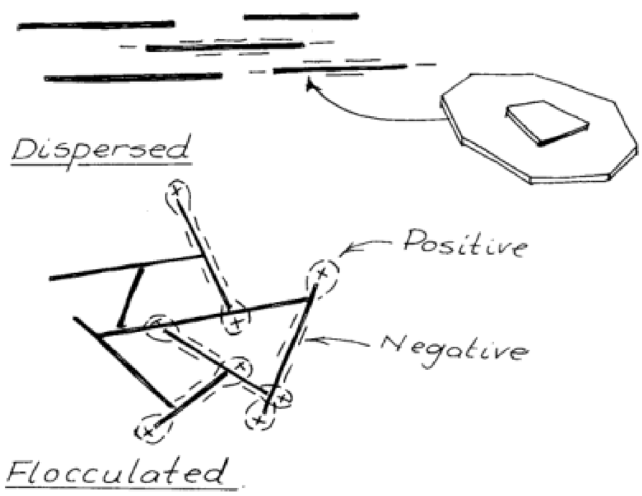
In this structure, relatively fine sand and silt form small arches with chains of particles as shown in the figure below. Soils exhibiting honeycombed structure have large void ratios and they can carry ordinary static load.

However, under heavy load or when subjected to shock loading, the structure breaks down, resulting in large settlement.



Structures in Cohesive Soils

1. *Dispersed structure*
2. *Flocculated structure*



Thixotropy

Thixotropy is the ability of certain substances to liquefy when agitated and to return to a gel form when at rest. The term thixotropy is derived from the Greek words thixis, meaning "the act of handling," and trope, meaning "change." Thixotropic substances are colloidal gels when solid and sols when liquefied. Examples of thixotropic substances include catsup, some hand creams, certain paints and printer's inks, and suspensions of clay in water. The reversibility and essentially isothermal nature of the of the gel-sol-gel transformation distinguish thixotropic materials from those that liquefy upon heating--for example gelatin.

Thixotropic systems are quite diverse. Therefore, it is unlikely that a single descriptive theory can include them all. However, in general, the phenomenon is found only in colloidal suspensions.

Various mechanisms can cause thixotropic behavior. For a gel system, agitation disrupts the three-dimensional structure that binds the system into a gel. Agitation might also introduce order into the system. In a system containing long polymeric molecules, these molecules can be disordered in the gel. When the gel is agitated, the molecules can align in the direction of flow, reducing the resistance to flow.

Some substances possess a property which is nearly the opposite of thixotropy. This property is called dilatancy. A dilatant substance is one that develops increasing resistance to flow as the rate of shear increases. A household example of a dilatant material is a thick dispersion of cornstarch in water. This appears to be a free-flowing liquid when poured, but when it is stirred, it becomes very firm. Another familiar example of dilatancy is the phenomenon of wet sand appearing to dry and become firm when it is walked on.

Soil Classification

Introduction

A soil classification system-

- is the arrangement of different soils with similar properties into groups & subgroups based on their application or to their probable engineering behavior.
- provides a common language to briefly express the general characteristics of soils, which are infinitely varied, without detailed descriptions.
- Most of the soils classification systems that have been developed for engineering purposes are based on simple index properties such as ***particle size distribution & plasticity.***
- Although there are several classification systems now in use, none is totally definitive of any soil for all possible applications, because of the wide diversity of soil properties.

The role of classification system in geotechnical engineering practice is-

Classification & index properties

e, n, γ, S, GDS, L.L, P.I, ……etc

↓

Classification system

↓

Engineering properties

Permeability, compressibility, Shear strength,….etc.

↓

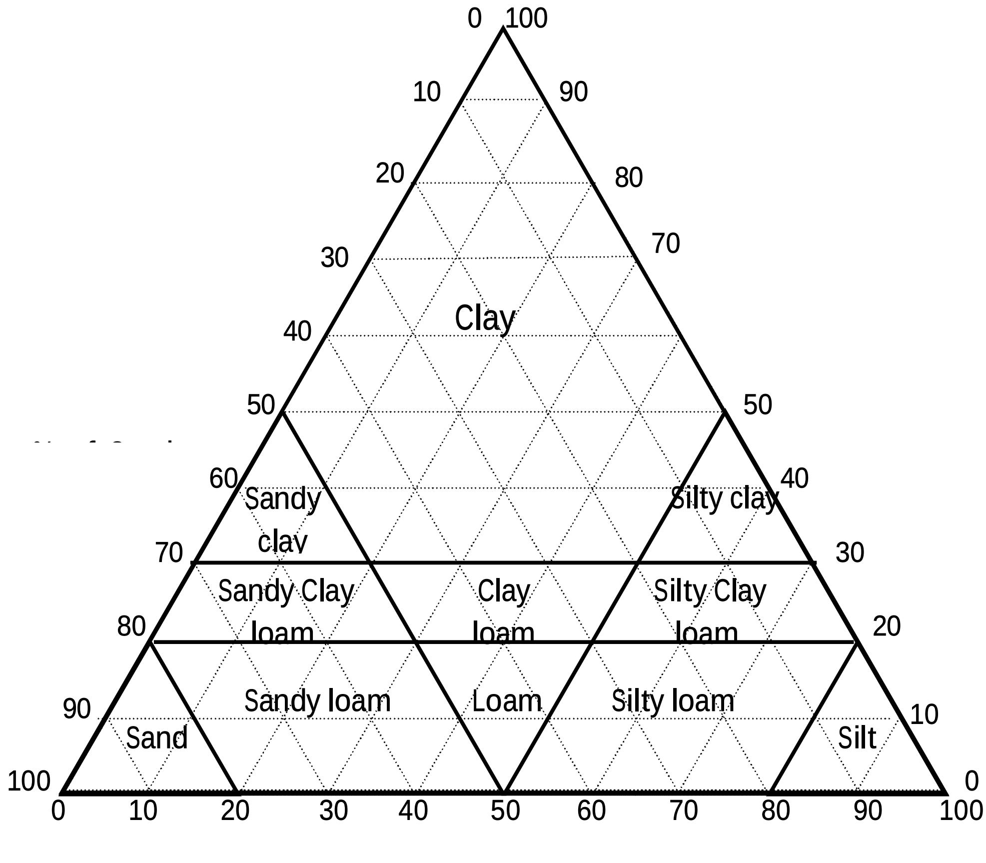
Engineering purposes

Highways, airfield, dams, foundations,…etc.

A- <u>Textural classification</u>

In general classification systems divided soils into the following categories on the basis of particle size. ***Gravel; Sand; Silt; and Clay*** , but the nature of soils are mixtures of particles from several size groups, so if we know the principle components of the soils, we can named the soils such as Sandy Clay, Silty Clay ; and so forth. One of these systems is the system developed by AASHTO (**A**merican **A**ssociation of **S**tate **H**ighway and **T**ransportation **O**fficial).the following chart is used to classify the soil, It is based on the particle size limits

Sand – size 2.0 – 0.05 mm in diameter
Silt – size 0.05 – 0.002 mm in diameter
Clay – size smaller than 0.002 mm in diameter



The chart is based only on the fraction of soil that passes through the no. 10 sieve. Otherwise a correction will be necessary if a certain percentage of the soil particles are larger than 2 mm in diameter, as shown below-

The modified textural composition are-

$$\text{Modified \% Sand} = \frac{\%\boldsymbol{sand}}{100 - \%\boldsymbol{gravel}}\, \boldsymbol{x}100\%$$

$$\text{Modified \% Silt} = \frac{\%\boldsymbol{silt}}{100 - \%\boldsymbol{gravel}}\, \boldsymbol{x}100\%$$

$$\text{Modified \% Clay} = \frac{\%\boldsymbol{clay}}{100 - \%\boldsymbol{gravel}}\, \boldsymbol{x}100\%$$

Then the soil is classified by proceeding in manner indicated by the arrows & the soil named according to the zone that fall in it as shown in the following example.

Example

Given

	Particle – size distribution (%)			
Soil	***Gravel***	***Sand***	***Silt***	***Clay***
A	0	18	24	58
B	18	~~51~~ 62.2	~~22~~ 26.83	~~9~~ 10.96

Required-

Classify the soils using textural classification of AASHTO

Solution-

Soil B percentages need to be corrected while percentages of **soil A** need no correction and we can use the % directly

Soil B

Modified % Sand $= \dfrac{51}{100-18} \boldsymbol{x} 100 = 62.2\%$

Modified % Silt $= \dfrac{22}{100-18} = 26.83\%$

Modified % Clay $= \dfrac{9}{100-18} = 10.96\%$

Using AASHTO chart we classified the soil A as clay and soil B As gravelly Sandy loam

B- <u>Other classification systems</u>

Although the textural classification of soil is relatively simple, it is based entirely on particle – size distribution. The amount & type of clay minerals present in fine – grained soils dictates to a great extent their physical properties. Hence, it is necessary to consider plasticity, which results from the presence of clay minerals, in order to interpret soil characteristics.

At the present time two classification systems are commonly used by soil engineers which take into consideration the particle – size distribution & Atterberg limits. They are –

1- AASHTO System
2- Unified Soil Classification System (USCS)

At present we will consider (USCS) only

			SAND					
AASHTO	***BOULDERS***	***GRAVEL***	***COARSE***	***MEDIUM***	***FINE***	***SILT***	***CLAY***	***COLLOIDAL***
	75	**4.75**	**2**	**0.425**	**0.075**	**0.005**	**0.001**	

			GRAVEL		SAND			
USCS	***BOULDERS***	***COBBLLES***	***COARSE***	***FINE***	***COARSE***	***MEDIUM***	***FINE***	***FINES (SILT & CLAY)***
	300	**75**	**19**	**4.75**	**2**	**0.425**	**0.075**	

Unified Soil Classification System (USCS)

The original form of this system was proposed by Casagrande in 1942 during World War 2, it was revised in 1952. At present it widely used among engineers.

This system classifies soils under two broad categories

1- **Coarse – grained soils** that are gravelly and sandy in nature with less than 50% passing through the no.200 sieve. The group symbols start with prefixes of either **G** or **S**. besides cobble and boulder without symbol.(see the following table)

2- Fine – grained soils with 50% or more passing through the no. 200 sieve. The group symbols start with prefixes **M; C; O & Pt.** (see the following table).

Soil component	Symbol	Grain size range & description	Significant properties
Boulder	None	Rounded to angular, bulky, hard, rock particle, average diameter more than 300 mm	Boulders and cobbles are very stable components, used for fills, ballast, and to stabilize slopes (riprap). Because of size and weight, their occurrences in natural deposits tends to improve the stability of foundations. Angularity of particles increases stability.
Cobble	None	Rounded to angular, bulky, hard, rock particle, average diameter smaller than 300 mm but larger than 75 mm.	
Gravel	**G**	Rounded to angular, bulky, hard, rock particle, passing 75 mm sieve and retained on sieve no. 4 (4.75 mm). **Coarse** 75 – 19 mm **Fine** 19 – 4.75 mm	Gravel and sand have essentially same engineering properties differing mainly in degree. The 4.75-mm sieve is arbitrary division and does not correspond to significant change in properties. They are easy to compact, little affected by moisture, not subject to frost action. Gravels are generally more previously stable, resistant to erosion and piping than are sands. The well- graded sands and gravels are generally less pervious and more stable than those which are poorly graded (uniform gradation). Irregularity of particles increases the stability slightly. Finer, uniform sand approaches the characteristics of silt: i.e., decrease in permeability and reduction in stability with increase in moisture.
Sand	**S**	Rounded to angular, bulky, hard, rock particle, passing sieve no. 4 and retained on sieve no. 200 sieve (0.075 mm). **Coarse** 4.75 – 2 mm **Medium** 2 – 0.425 mm **Fine** 0.425 – 0.075 mm	

Soil component	Symbol	Grain size range & description	Significant properties
Silt	**M**	Particles smaller than 0.075 mm, identified by behavior: that is, slightly or non – plastic regardless of moisture and exhibits little or no strength when air dried.	Silt is inherently unstable, particularly when moisture is increased, with tendency to become quick when saturated. It is relatively impervious, difficult to compact, highly susceptible to frost heave, easily erodible and subject to piping and boiling. Bulky grains reduce compressibility, flaky grains, i.e., mica, diatoms, increase compressibility, produce an “elastic” silt. Produce
Clay	**C**	Particles smaller than 0.075 mm, identified by behavior: that is, it can be made to exhibit plastic properties within certain range of moisture and exhibits considerable strength when air-dried.	The distinguishing characteristics of clay is cohesion or cohesive strength, which increase with decrease in moisture. The permeability of clay is very low, it is difficult to compact when wet and impossible to drain by boundary means, when compacted is resistant to erosion and piping, is not suspectible to frost heave, is subject to expansion and shrinkage with changes in moisture. The properties are influenced not only by the size and shape (flat, plate- like particles) but also by their mineral compositions: i.e., the type of clay – mineral, and chemical environment or base exchange capacity. In general, he Montmorillonite clay mineral has greatest, Illite and Kaolinite the least, adverse effect on the properties.
Organic matter	**O**	Organic matter in various sizes and stages of decomposition.	Organic matter present even in moderate amounts increase compressibility and reduces the stability of the fine – grained components. It may decay causing voids or by chemical alteration change the properties of a soil, hence organic soils are not desirable for engineering uses.

Other symbols used for the classification are –

W – well graded

P – poorly graded

L – low plasticity (L.L < 50%)

H – high plasticity (L.L > 50%)

So the group symbols may be one of the following for-

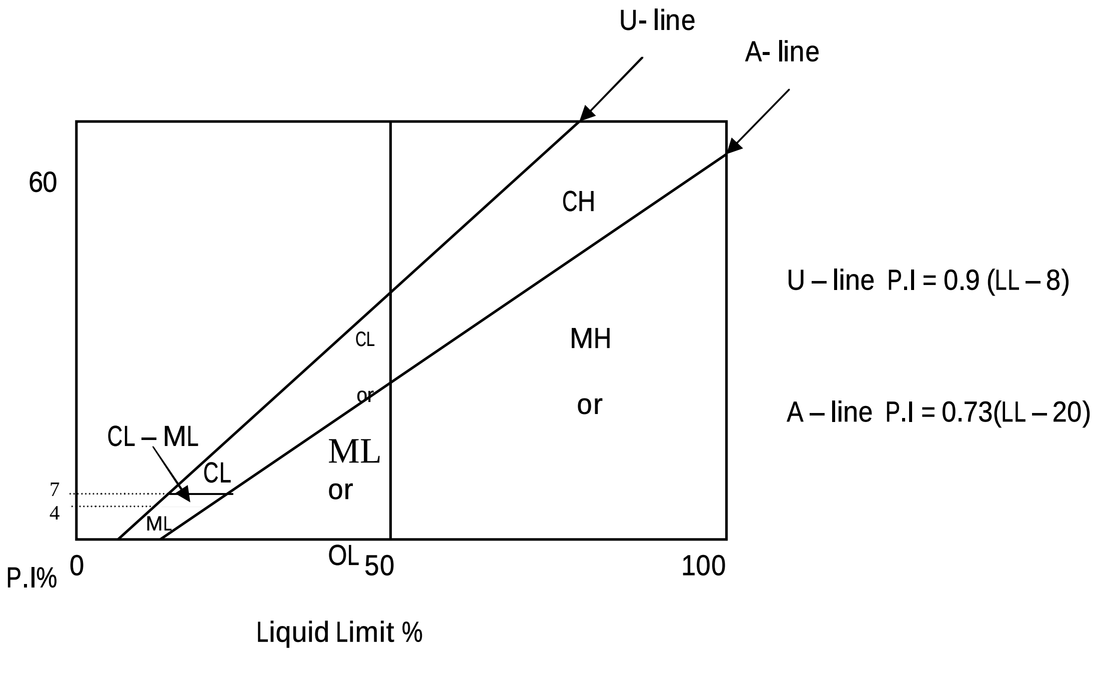
- ***Coarse – grained soils***

GW , SW	**GW – GM , SW – SM**	**GM , SM**
GP , SP	**GW – GC , SW – SC**	**GC , SC**
	GP – GM , SP – SM	
	GP – GC , SP – SC	

- ***Fine – grained soils***

CL , ML , OL	**CH , MH , OH**	**CL – ML &Pt**

The plasticity chart used in USCS is shown below which is developed by Casagrande (1948) and modified to some extent here.



The following is a step – by – step procedure for classification of soils

<u>Step 1-</u> determine the percent of soil passing no. 200 sieve (F).
If F < 50% , the soil will classify as Coarse – grained soil gravelly or sandy soil , then go to ***step 2.***
If F > = 50% , the soil will classify as Fine – grained soil silty or clayey soil , then go to ***step 3***

<u>Step 2 –</u> Determine the percent of soil passing no. 4 & retained on no 200 sieve (F_1).
If $F_1 < \frac{100-F}{2}$ the soil will take the symbol **G** (gravel or gravelly soil).
If $F_1 \geq \frac{100-F}{2}$ the soil will take the symbol **S** (sand or sandy soil)

To state the degree of gradation whether to be well (W) or poor (P) the following criteria shall be meet together and the soil will be well – graded otherwise the soil will be poorly – graded.

C_u greater than 4 for gravel & greater than 6 for sand
C_c between 1 and 3

Then if **F** < 5% examine GSD & find C_u & C_c and the soil will take one of the following symbol **GW, SW, GP, SP** according to the above criteria.
If **F** is between 5% - 12% besides the GSD characteristics (C_u & C_c) we shall use the plasticity characteristics such as (L.L & P.I) with the plasticity chart to define the dual symbol such as **GW – GM , SW – SM , GP – GM , SP – SM , GW – GC , SW – SC , GP – GC , SP – SC .**

If **F** > 12% we use the plasticity characteristics (L.L & P.I) with the plasticity chart to state the soil symbol such as **GM, GC, SM, SC, GM – GC** or **SM – SC .**

<u>Step 3 –</u> For fine –grained we use the plasticity characteristics (L.L & P.I) with the plasticity chart to state the soil symbol such as **OL** or **ML**, **CL – ML , CL** when L.L <50% but if L.L >50% the symbol will be **OH** or **MH , CH.** To state whether the soil is inorganic (**M** or **C**) or organic (**O**) we shall examine the color and changes in L.L & P.I after drying for the soil such test will not describe here.

After we classify the soil and give it a symbol , knowing its significant properties we can state the engineering use of it.

<u>Example</u>

Following are the results of a sieve analysis and L.L & P.L tests for two soils

Sieve size	Soil 1 % passing	Soil 2 % passing
No.4 (4.75 mm)	99	97
No. 10 (2 mm)	92	90
No. 40 (0.475 mm)	86	40
No. 100	78	8
No. 200 (0.075 mm)	60	5
L.L	20	-
P.L	15	-
P.I	5	NP (Not Plastic)

<u>Required</u>

Classify the soil according to USCS

Solution

1- Plot the GSD curve for the two soils.

2- For soil 1 % passing no. 200 sieve is greater than 50% so it is fine grained soil and by using plasticity chart the soil plots in the zone **(CL – ML).**

3- For soil 2 % passing no. 200 sieve is less than 50% so it is coarse – grained soil.

F_1 =92% (% passing no. 4 & retained on No.200 sieve) > $\frac{100-5}{2}$ = 47.5% so the symbol is S (Sand)

Referring to the GSD curve we find $D_{10} = 0.18$ mm

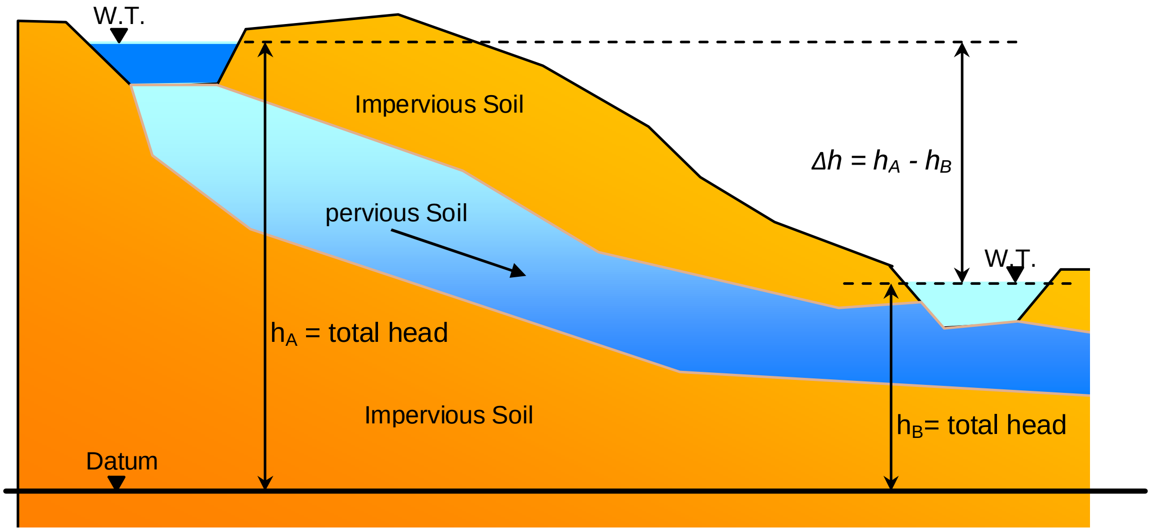
$D_{30} = 0.34$ mm

$D_{60} = 0.71$ mm

$$C_u = \frac{D_{60}}{D_{10}} = 3.9 < 6 \;;\; C_c = \frac{D_{30}^{\;2}}{D_{10}.D_{60}} \cong 0.91 \approx 1$$

as C_u & C_c does not meet the requirements of well- graded the soil is poorly graded , the symbol will be SP, but since % passing no. 200 sieve = 5% the soil will take a dual symbol, since the soil is NP so the symbol is SM

so the symbol will be **SP – SM** .



Permeability and Seepage

Topics

1. Permeability

- Overview of Underground Water Flow
- Permeability
- Theory
- Laboratory and Field Tests
- Empirical Correlations
- Equivalent Permeability in Stratified Soil

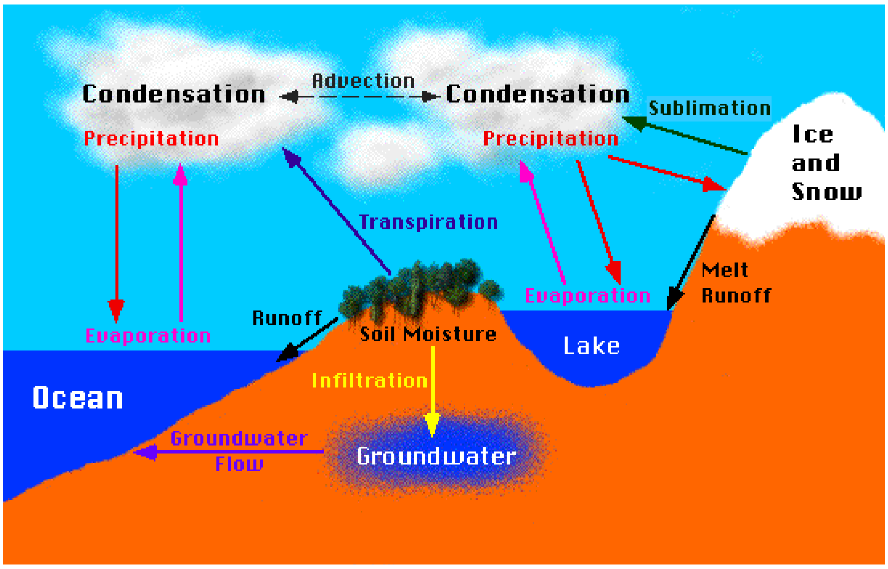
2. Seepage

- Laplace's Equation of Continuity
- Continuity Equation for Solution of Simple Flow Problems
- Flow Nets
- Seepage Calculation
- Seepage pressure and Uplift Pressure
- Seepage through an Earth Dam

Permeability

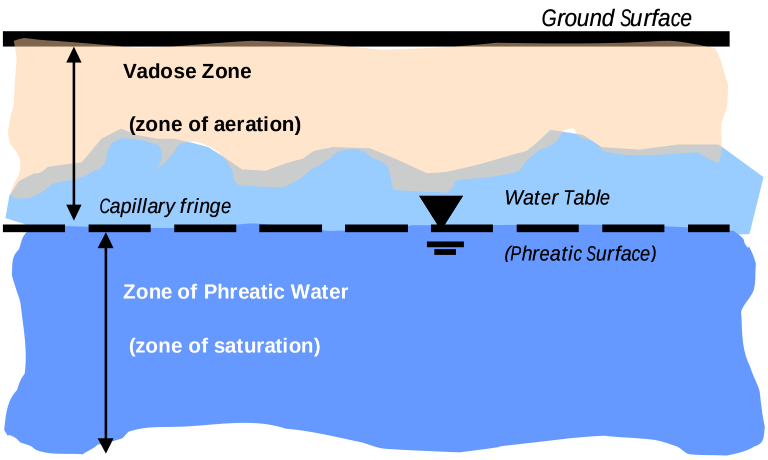
● Overview of Underground Water Flow

Hydrologic Cycle



Aspects of Hydrology

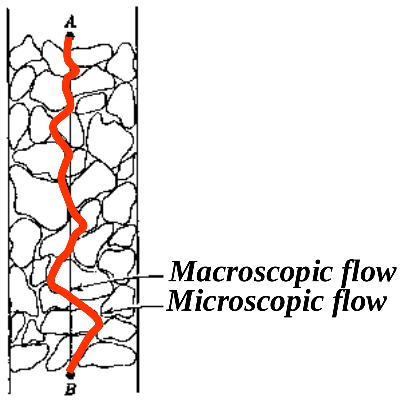
- ♦ A relatively small amount of the earth's water (<1%) is contained in the groundwater, but the effects of this water are out of proportion to their amount
- ♦ The permeability of soil affects the distribution of water both between the surface and the ground mass and within the ground mass itself



● ***Permeability***

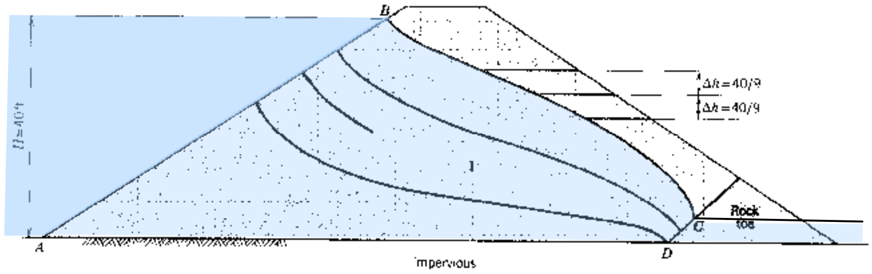
Definition-

- The property of soils that allows water to pass through them at some rate.
- This property is a product of the granular nature of the soil, although it can be affected by other factors (such as water bonding in clays)
- Different soils have different perm abilities, understanding of which is critical to the use of the soil as a foundation or structural element
- Soil and rock are porous materials
- Fluid flow takes place through interconnected void spaces between particles and *not through the particles themselves*
- No soil or rock material is strictly "impermeable"

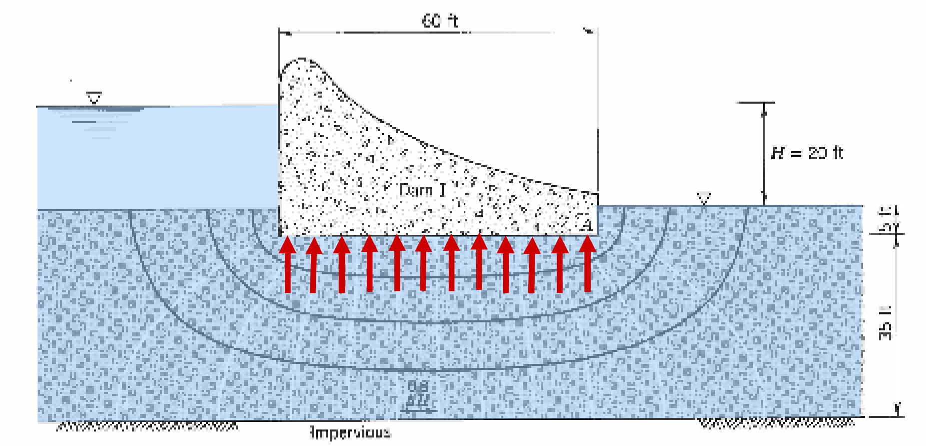


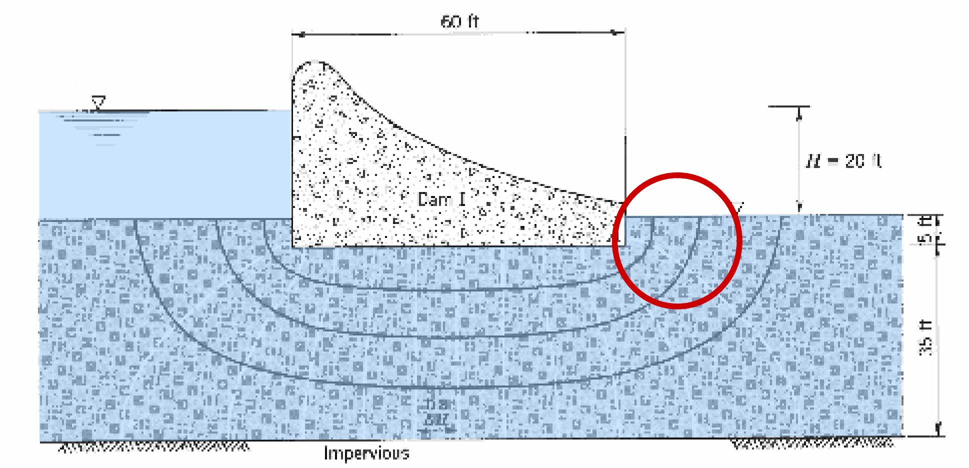
The study of flow of water through porous media is necessary for-

- **Estimation Seepage Loss**
- **Estimation Pore Water Pressures**
- **Evaluation Quicksand Conditions**
- **Dewatering System Design**
- **Drainage System Design**

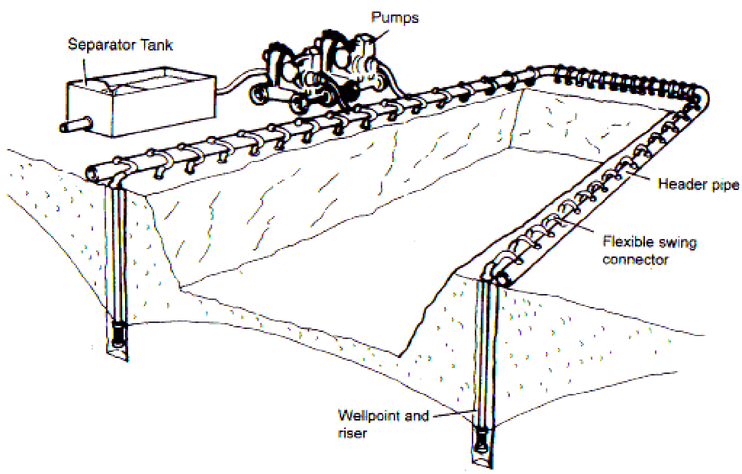


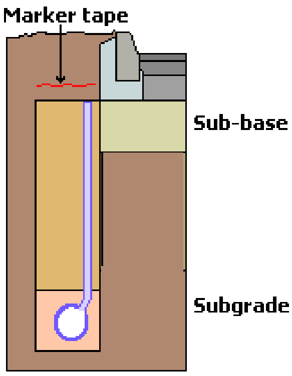
Seepage through the body of the dam



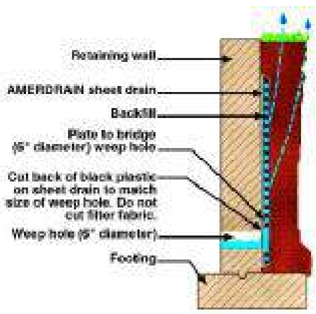


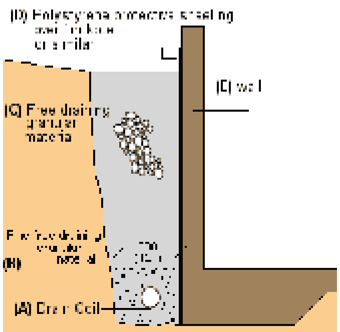
Typical Wellpoint System





Pavement Drainage





Drainage behind Retaining Walls

● *Theory*

Bernoulli's Law

According to Bernoulli's equation, the total head (h_t) at a point in water under motion is

$$h_t = \frac{p}{\gamma_w} + \frac{v^2}{2g} + Z$$

Where

Pressure head (Kinetic vomponent) = $\frac{p}{\gamma_w}$ =h_p

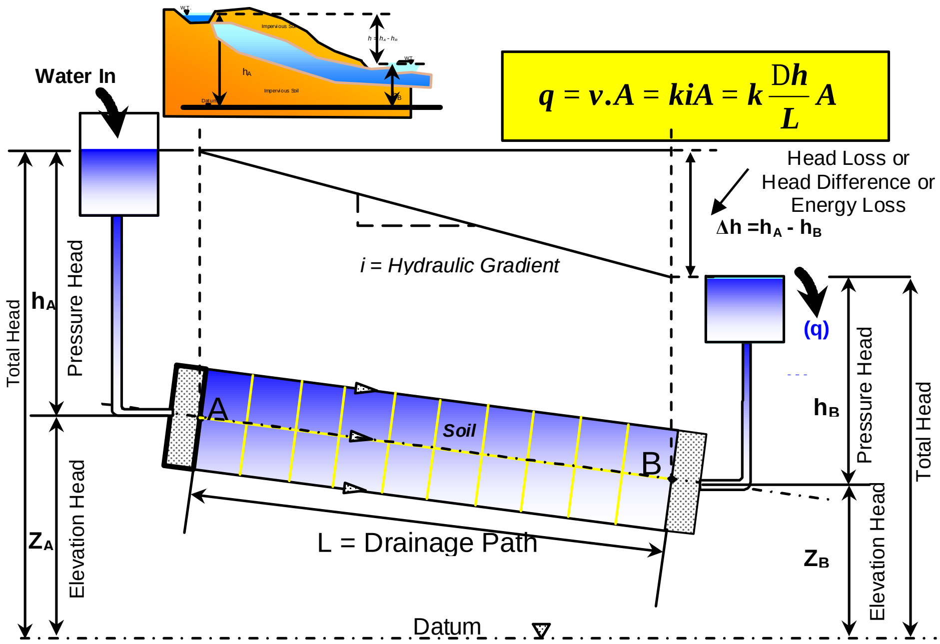
Velocity head (pressure component) = $\frac{v^2}{2g}$ =h_v

Elevation head (Gravitational (potential) component) = Z=h_e

● In reality, an energy balance of the soil as it flows through the ground

● Kinetic Component can usually be ignored then $h_t = \frac{p}{\gamma_w} + Z = h_p + h_e$

Head Differential



The loss of head between A & B, can be given by

$$\Delta h = h_A - h_B = \left(\frac{P_A}{g_w} + Z_A\right) - \left(\frac{P_B}{g_w} + Z_B\right)$$

Δ h can be expressed in nondimensional form as

Hydraulic gradient

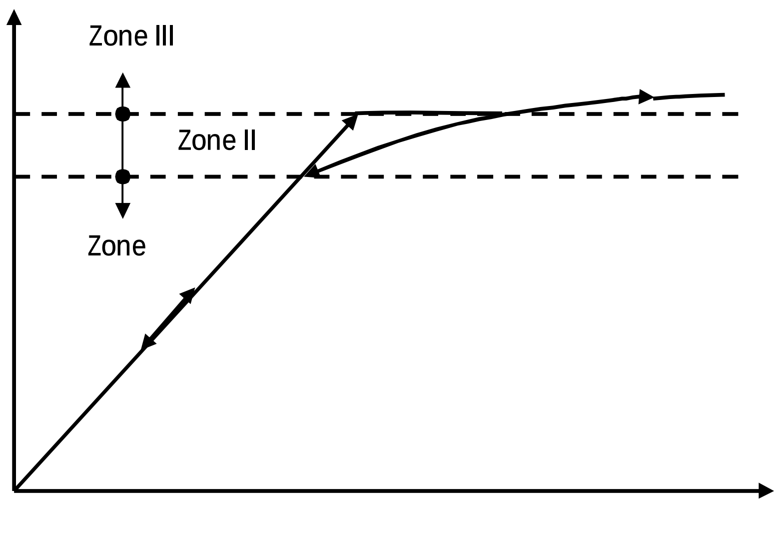
$$i = \frac{\Delta h}{L}$$

Where

i = hydraulic gradient

L = distance between A&B (the length of flow over which loss of head occurred)

In general, the variation of velocity (v) with the hydraulic gradient (i) will be as shown in the figure below



Nature of variation of velocity with hydraulic gradient

This figure has been divided into three zones:

laminar flow (Zone I)

transition zone (Zone II)

turbulent flow zone (Zone III)

In most soils, the flow of water through the void spaces can be considered laminar and thus $v \propto i$

Darcy's Law

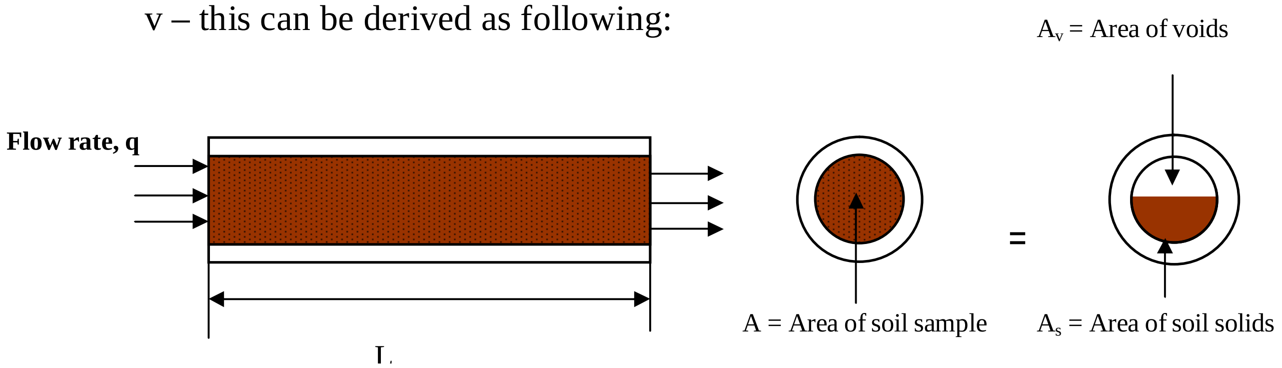
In 1856, Darcy published a simple equation for discharge velocity of water through saturated soils, which may expressed as

$$v = ki$$

Where v = discharge velocity = quantity of water flowing in unit time through a unit gross – sectional area of soil at right angles to the direction of flow

k = coefficient of permeability

(v) is based on the gross – sectional area of the soil, however the actual velocity of water (seepage velocity, vs)through the void spaces is higher than v – this can be derived as following:



If the flow rate is q then

$$q = vA = A_v . v_s$$

$$A = A_v + A_s$$

$$\therefore q = v(A_v + A_s) = A_v . v_s$$

so

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \left(\frac{v(V_v + V_s)}{V_v}\right) \div Vs$$

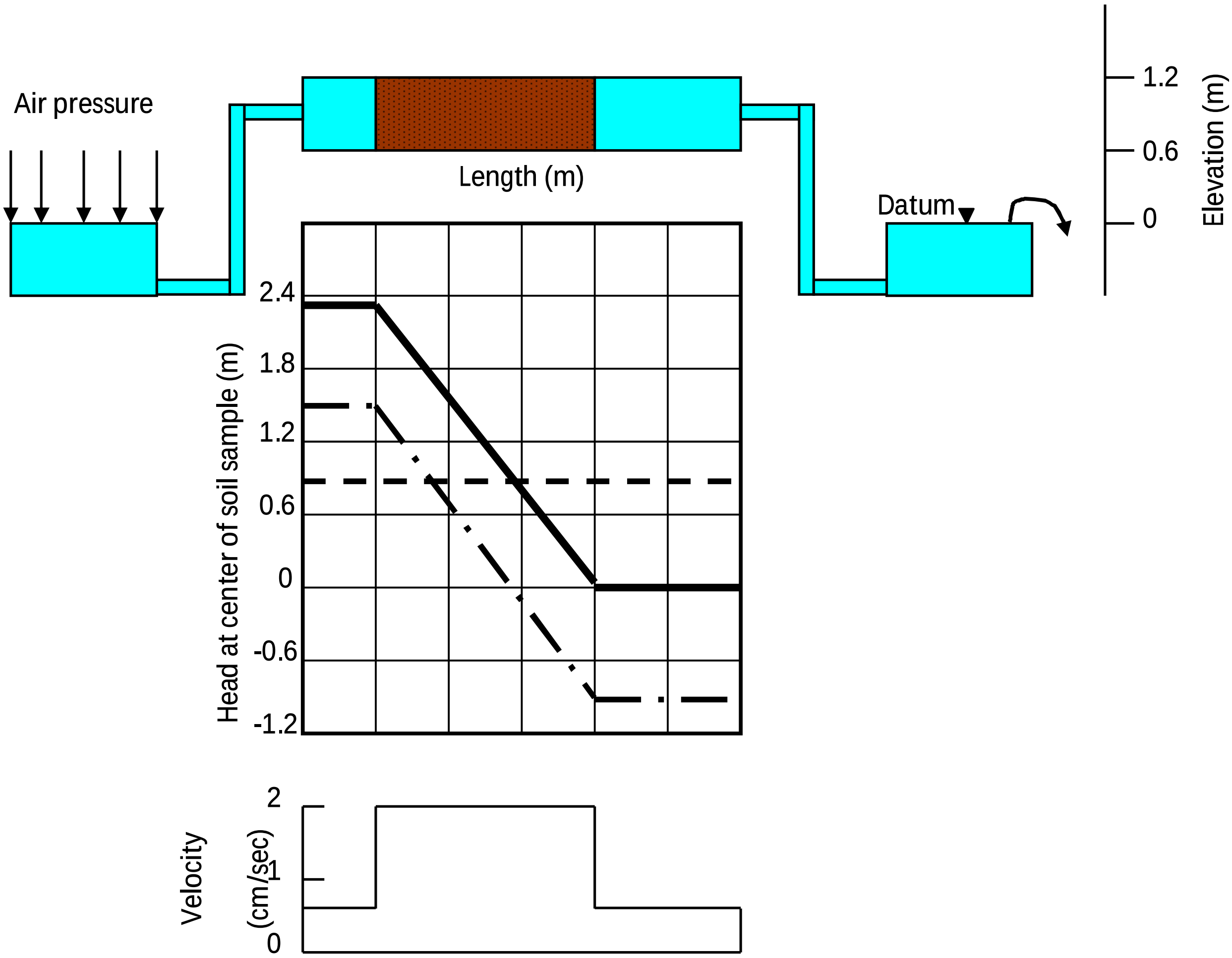
$$v_s = v\left[\frac{1 + \frac{V_v}{V_s}}{\frac{V_v}{V_s}}\right] = v\left(\frac{1+e}{e}\right) = \frac{v}{n}$$

$$v_s = \frac{v}{n}$$

What causes flow of water through soil?

§ *Answer*: *A difference in TOTAL HEAD*

Horizontal flow

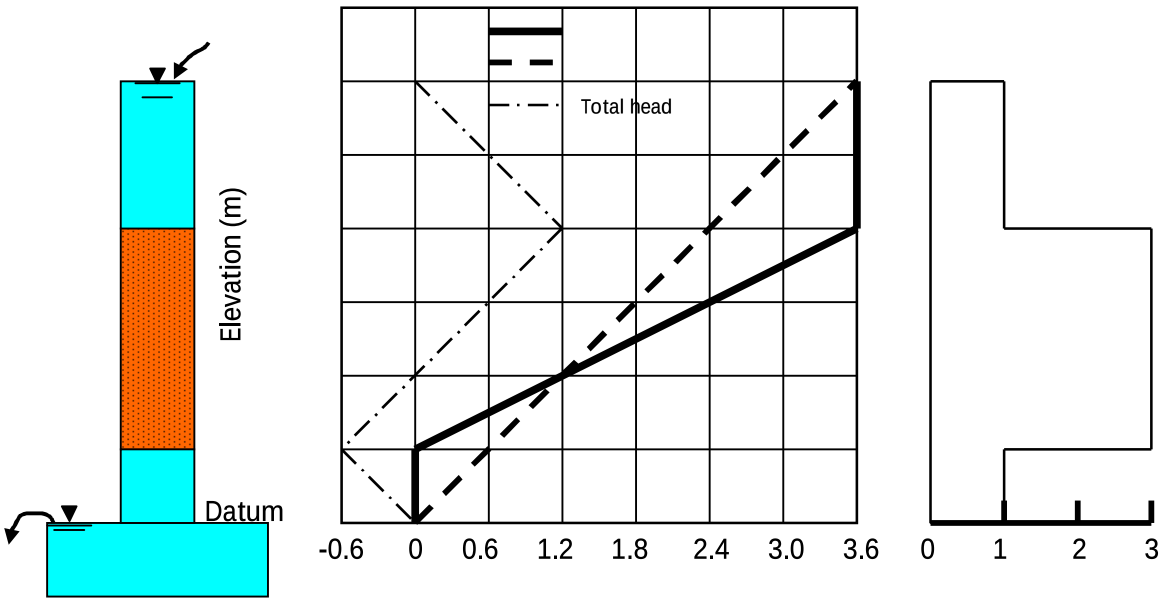


In this case the air pressure will produce the required head for horizontal flow. Thus

$$\textit{Total head loss} = \frac{23.4}{9.81} = 2.385\ m\ .$$

$$v = k.i = 0.5\frac{2.385}{1.8} = 0.663 \Rightarrow v_s = \frac{v}{n} = \frac{0.663}{0.33} = 2\ cm/\sec$$

Downward Flow

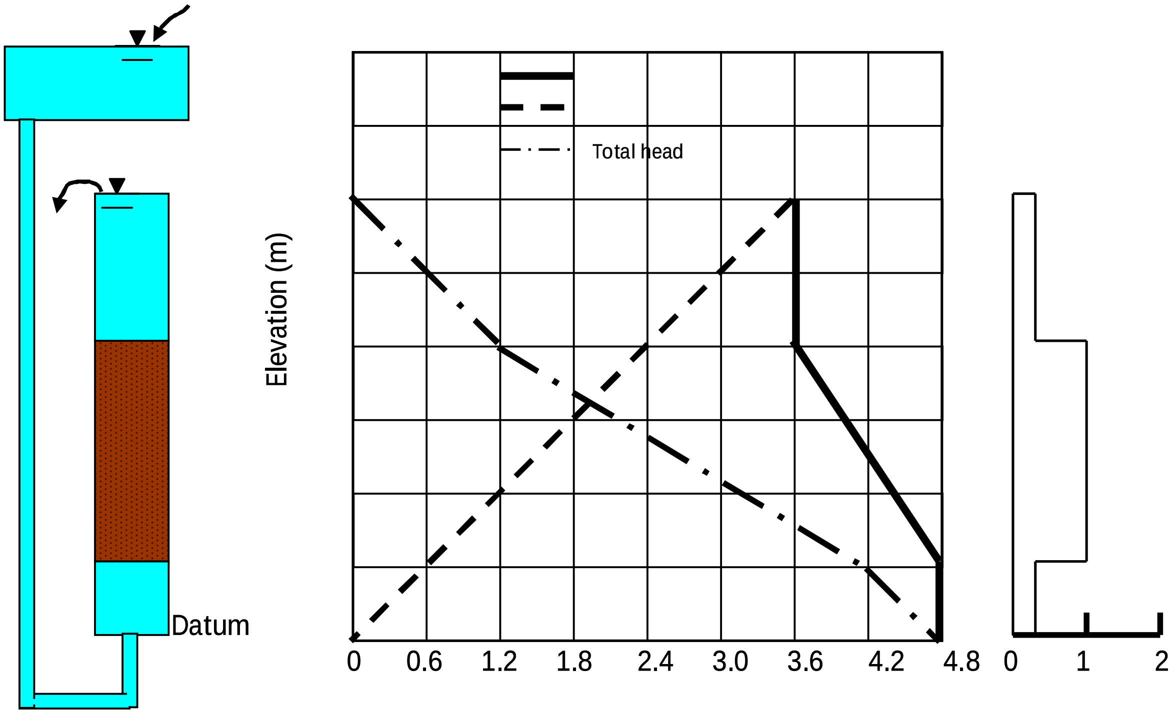


$$v = k.i = 0.5.\frac{3.6}{1.8} = 1 \quad \text{cm/sec at the entrance and the exit parts of the tube .}$$

$$\therefore\ v_s = \frac{v}{n} = \frac{1}{0.33} = 3 \quad \text{cm/sec through the soil sample}$$

Upward flow

The same tube was tested under upward flow as shown in the figure below



$$v = k.i = 0.5.\frac{1.2}{1.8} = 0.33 \Rightarrow v_s = \frac{v}{n} = \frac{0.33}{0.33} = 1 \quad \text{cm/sec}$$

Hydraulic Conductivity or Coefficient of permeability (k)

- It is defined as the rate of flow per unit area of soil under unit hydraulic gradient, it has the dimensions of velocity (L/T) such (cm/sec or ft/sec).
- It depends on several factors as follows:

1. Shape and size of the soil particles.
2. Distribution of soil particles and pore spaces.
3. Void ratio. Permeability increases with increase of void ratio.

4. Degree of saturation. Permeability increases with increase of degree of saturation.
5. Composition of soil particles.
6. Soil structure
7. Fluid properties. When the properties of fluid (water) affecting the flow are included, we can express ***k*** by the relation

$$\boldsymbol{k}(\boldsymbol{cm}/\boldsymbol{s}) = \frac{\boldsymbol{K}\rho \boldsymbol{g}}{\mu} = \frac{\boldsymbol{K}\gamma_w}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Where K = intrinsic or absolute permeability, cm^2
ρ = mass density of the fluid, g/cm^3
g = acceleration due to gravity, cm/sec^2
μ = absolute viscosity of the fluid, poise [that is, g/(cm.s)]

(***k***) varies widely for different soils, as shown in the table below

Typical values of permeability coefficient (k)	
Soil type	k (mm/sec)
Coarse gravel	10 to 10^{3}
Fine gravel, coarse and medium sand	10^{-2} to 10
Fine sand, loose silt	10^{-4} to 10^{-2}
Dense silt, clayey silt	10^{-5} to 10^{-4}
Silty clay, clay	10^{-8} to 10^{-5}

The coefficient of permeability of soils is generally expressed at a temperature of 20^{o}C. at any other temperature T, the coefficient of permeability can be obtained from eq.(12) as

$$\frac{\boldsymbol{k}_{20}}{\boldsymbol{k}_{\boldsymbol{T}}} = \frac{(r_{20})(m_{\boldsymbol{T}})}{(r_{\boldsymbol{T}})(m_{20})}$$

Where

k_T , k_{20} = coefficient of permeability at T^{o}C and 20^{o}C, respectively
ρ_T , ρ_{20} = mass density of the fluid at T^{o}C and 20^{o}C, respectively
μ_T , μ_{20} = coefficient of viscosity at T^{o}C and 20^{o}C, respectively
Since the value of ρ_{20} / ρ_T is approximately 1, we can write

$$\boldsymbol{k}_{20} = \boldsymbol{k}_{\boldsymbol{T}} \frac{m_{\boldsymbol{T}}}{m_{20}}$$

Where $\quad \dfrac{m_{\boldsymbol{T}}}{m_{20}} = \boldsymbol{f}(\boldsymbol{T}) \approx 1.682 - 0.0433\boldsymbol{T} + 0.00046\boldsymbol{T}^2$

● ***<u>Laboratory and Field Tests</u>***

The four most common laboratory methods for determining the permeability coefficient of soils are the following:

1. Constant – head test.
2. Falling – head test.
3. Indirect determination from consolidation test
4. Indirect determination by horizontal capillary test.

Laboratory Tests

Constant – head test

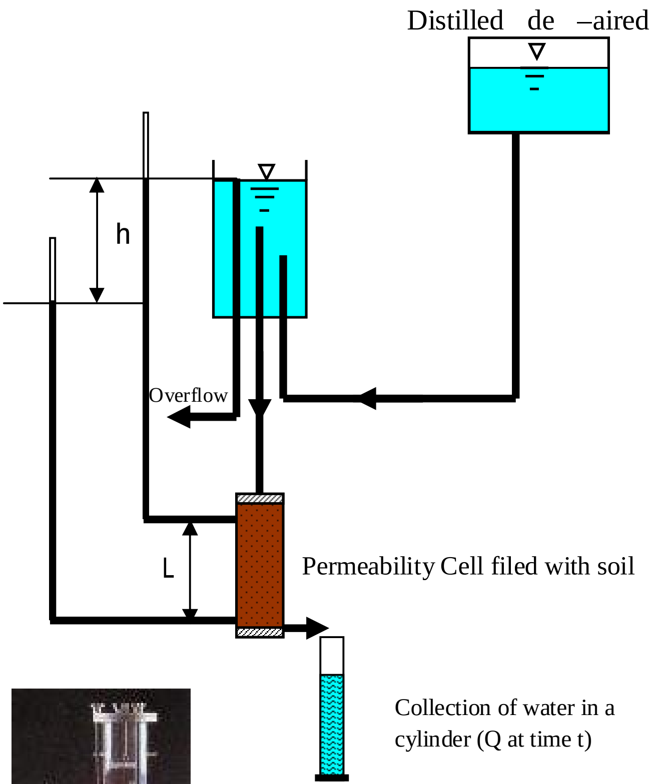
● Direct measure of permeability using Darcy's Law

$$Q = qt = kiAt \Rightarrow k = \frac{QL}{hAt}$$

● Suitable for cohesionless soils with permeabilities > 10 x10^{-4} cm/sec

● The simplest of all methods for determining the coefficient of permeability

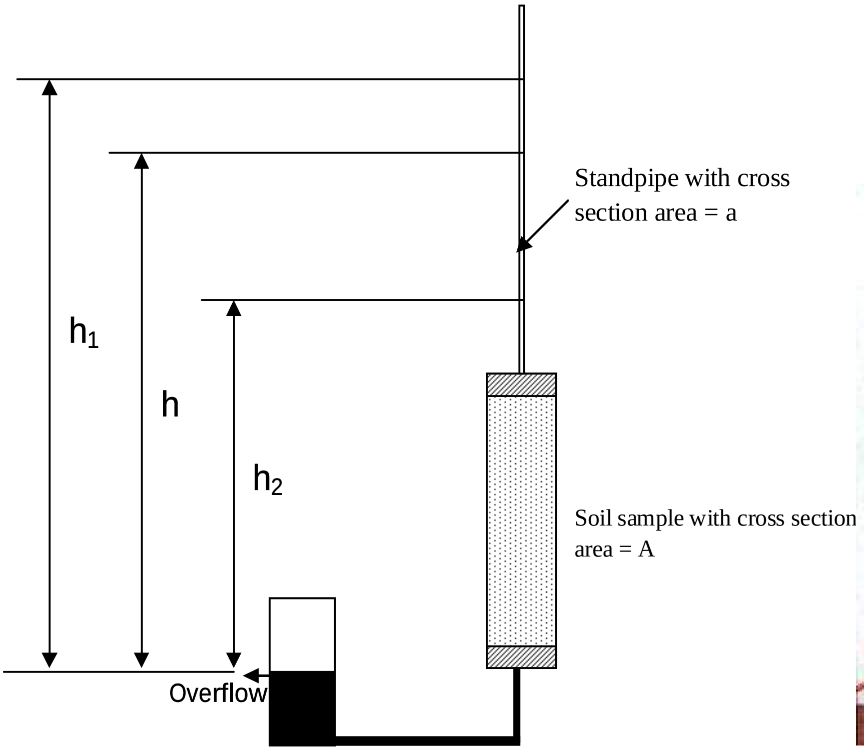
● This test is performed by measuring the quantity of water, Q, flowing through the soil specimen, the length of the soil specimen, L, the head of water, h, and the elapsed time, t. The head of water is kept constant throughout the test.

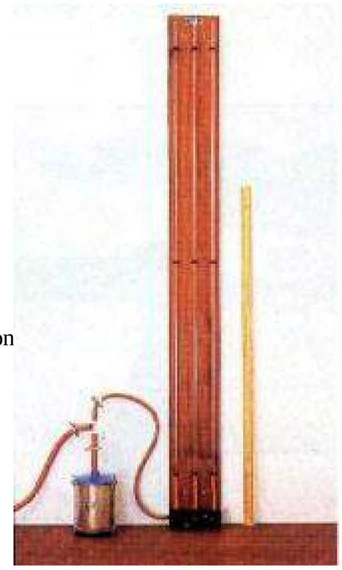


Permeability Cell with

Falling – head test

- Indirect measurement of permeability using time of flow
- Suitable for cohesive soils with permeabilities < 10 x 10-4 cm/sec




Falling head apparatus (ELE)

The rate of flow through the soil is

$$q = kiA = k\frac{h}{L}A = -a\frac{dh}{dt}$$

where h = head difference at any time t

A = area of specimen

a = area of standpipe

L = length of specimen

From eq.(15),

$$\int_0^t dt = \int_{h_1}^{h_2} \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

Or

$$k = 2.303\frac{aL}{Ak}\log\frac{h_1}{h_2}$$

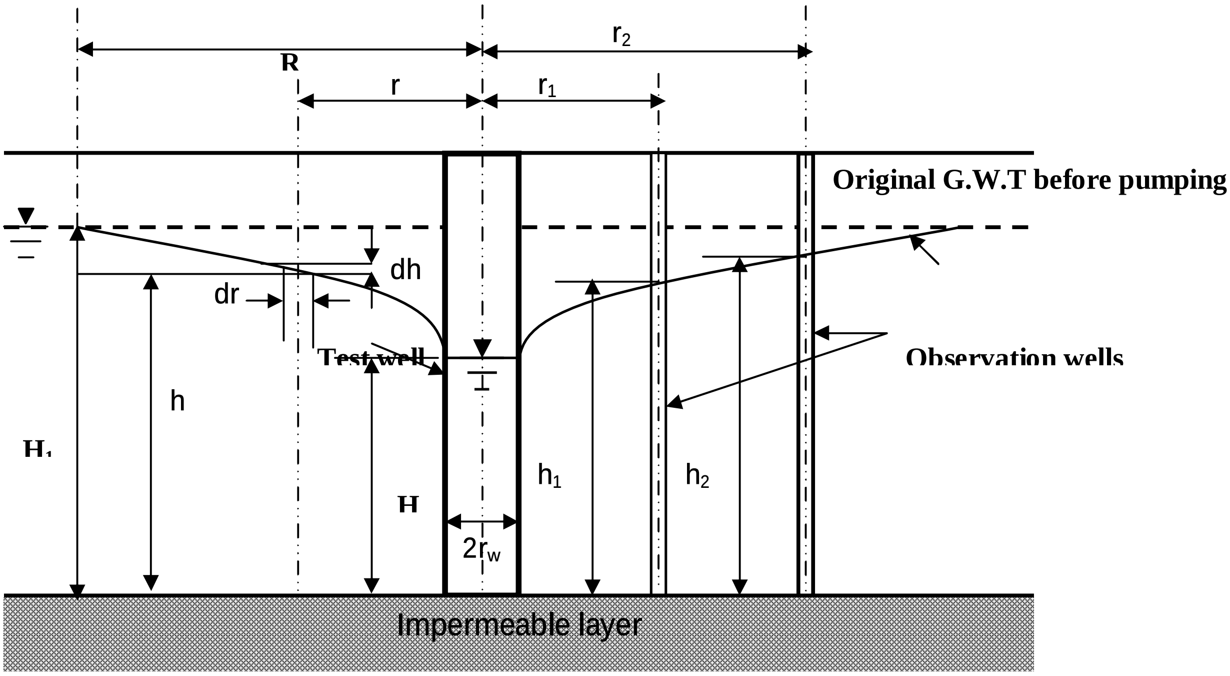
Field tests

There are many useful methods to determine the permeability coefficient in field such as

1. pumping from wells
2. Bore hole test
3. Open – end test
4. Packer test
5. Variable – head tests by means of piezometer observation well

Pumping from wells

- ***Gravity wells (unconfined aquifer)***



$$q = kiA$$

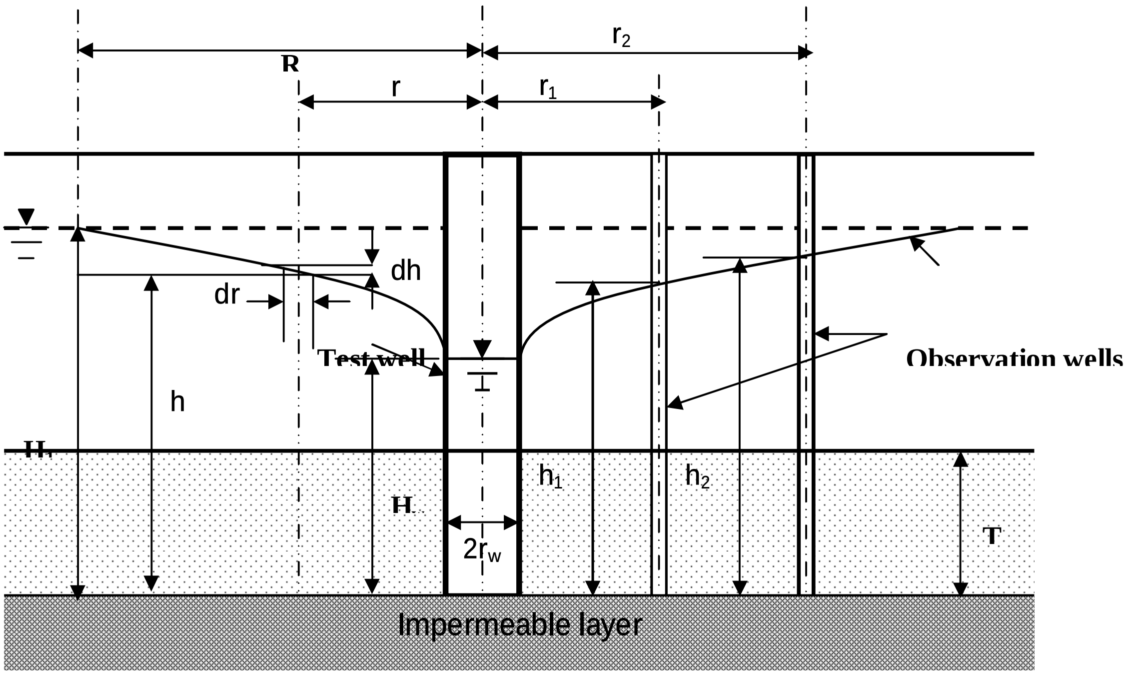
$$q = k\frac{dh}{dr}2phr$$

$$\int_{r_1}^{r_2}\frac{dr}{r} = \frac{2pk}{q}\int_{h_1}^{h_2} h\,dh$$

So

$$k = \frac{2.303q\left[\log\left(r_2/r_1\right)\right]}{p\left(h_2^2 - h_1^2\right)}$$

- ***Artesian wells (confined aquifer)***



$$q = kiA = k\frac{dh}{dr}2prT$$

$$\int_{r_1}^{r_2}\frac{dr}{r} = \int_{h}^{h_2}\frac{2pkT}{q}dh$$

$$k = \frac{q \log(r_2 / r_1)}{2.727T(h_2 - h_1)}$$

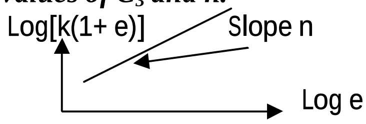
If we substitute $h_1 = H_w$ at $r_1 = r_w$ and $h_2 = H_1$ ar $r_2 = R$ in, we get

$$k = \frac{q \log(R / r_w)}{2.727T(H_1 - H_w)}$$

● *Empirical Correlations*

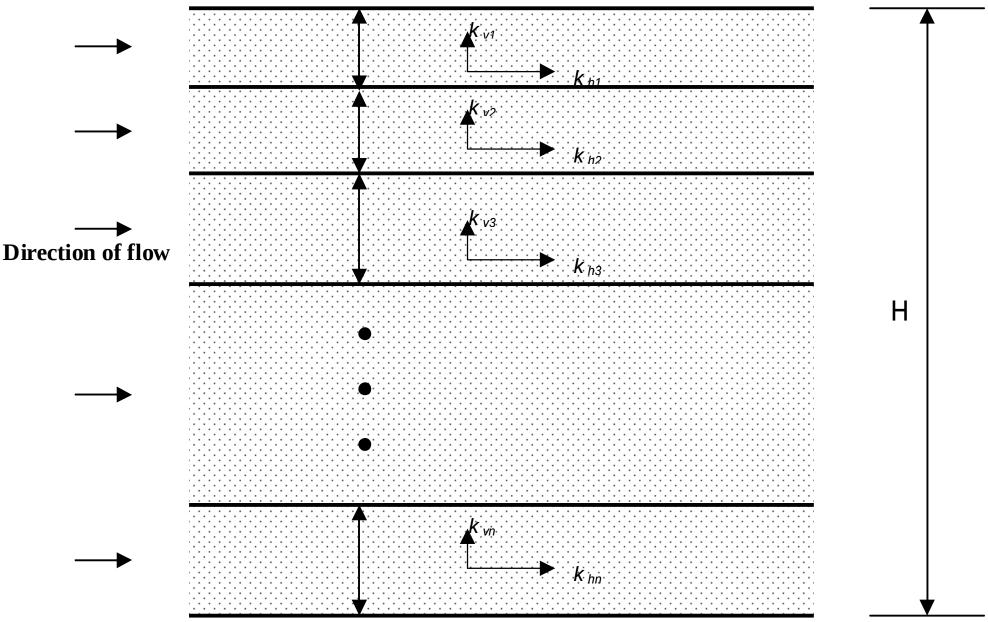
Several empirical equations for estimation of the permeability coefficient have been proposed in the past. Some of these will be briefly discussed in this section.

Hazen (1930)	$k(cm/\sec) = cD_{10}^2$ ***c = constant that varies from 1.0 to 1.5*** ***D_{10} = effective size, in millimeters***	***For fairly uniform sand (that is, small C_u). This eq. Is based on observations made on clean filter sands. A small quantity of silts and clays, when present in a sandy soil, may change the permeability coefficient substantially.***
Casagrande	$k = 1.4e^2 k_{0.85}$ k = permeability coefficient at void ratio e ***$k_{0.85}$ = the corresponding value at void ratio of 0.85***	***For fine – to medium – clean sand***
Application of Kozeny – Carman equation	$k = C_1 \frac{e^3}{1+e}$ $C_1 = C_2 D_{10}^{2.32} C_u^{0.6} \frac{e^3}{1+e}$ ***k = permeability coefficient at a void ratio of e*** ***C_1 = constant*** ***C_2 = a constant*** ***C_u = uniformity coefficient*** ***D_{10} = effective size***	***For sandy soils***
Shahabi et. al. (1984)	$k = 1.2C_u^{0.735} D_{10}^{0.89} \frac{e^3}{1+e}$	***For medium – and fine – sand.***

Samarasingh et. al. (1984)	$k = C_3 \frac{e^n}{1+e}$ ***C_3 & n are constants to be determined experimentally. This equation can be rewritten as*** ***Log[k(1+e)] = log C_3 + n log e***	***For normally consolidated clays. For any given clay, if the variation of k with the void ratio is known, a log – log graph can be plotted with k*1+e) against e to determine the values of C_3 and n.*** 
Mesri & Olson (1971)	$\log k = A' \log + B'$ ***A' & B' are constants***	***For clays***
Taylor (1948)	$\log k = \log k_o - \frac{e_o - e}{C_k}$ ***Where k_o = in situ permeability coefficient at void ratio e_o*** ***k = permeability coefficient at void ratio e*** ***C_k = permeability change index***	***For clays.***

Equivalent Permeability in Stratified Soil

Horizontal direction.



$$q = v.1.H = v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + \ldots\ldots + v_n.1.H_n$$

Where v = average discharge velocity

v_1 , v_2 , v_3 , …..v_n = discharge velocities of flow in layers denoted by the subscripts.

From Darcy's law

$$v = k_{H(eq)}.i_{eq}$$

$$v_1 = k_{h1}.i_1$$

$$v_1 = k_{h2}.i_2$$

$$v_1 = k_{h3}.i_3$$

$$\vdots$$

$$v_1 = k_{hn}.i_n$$

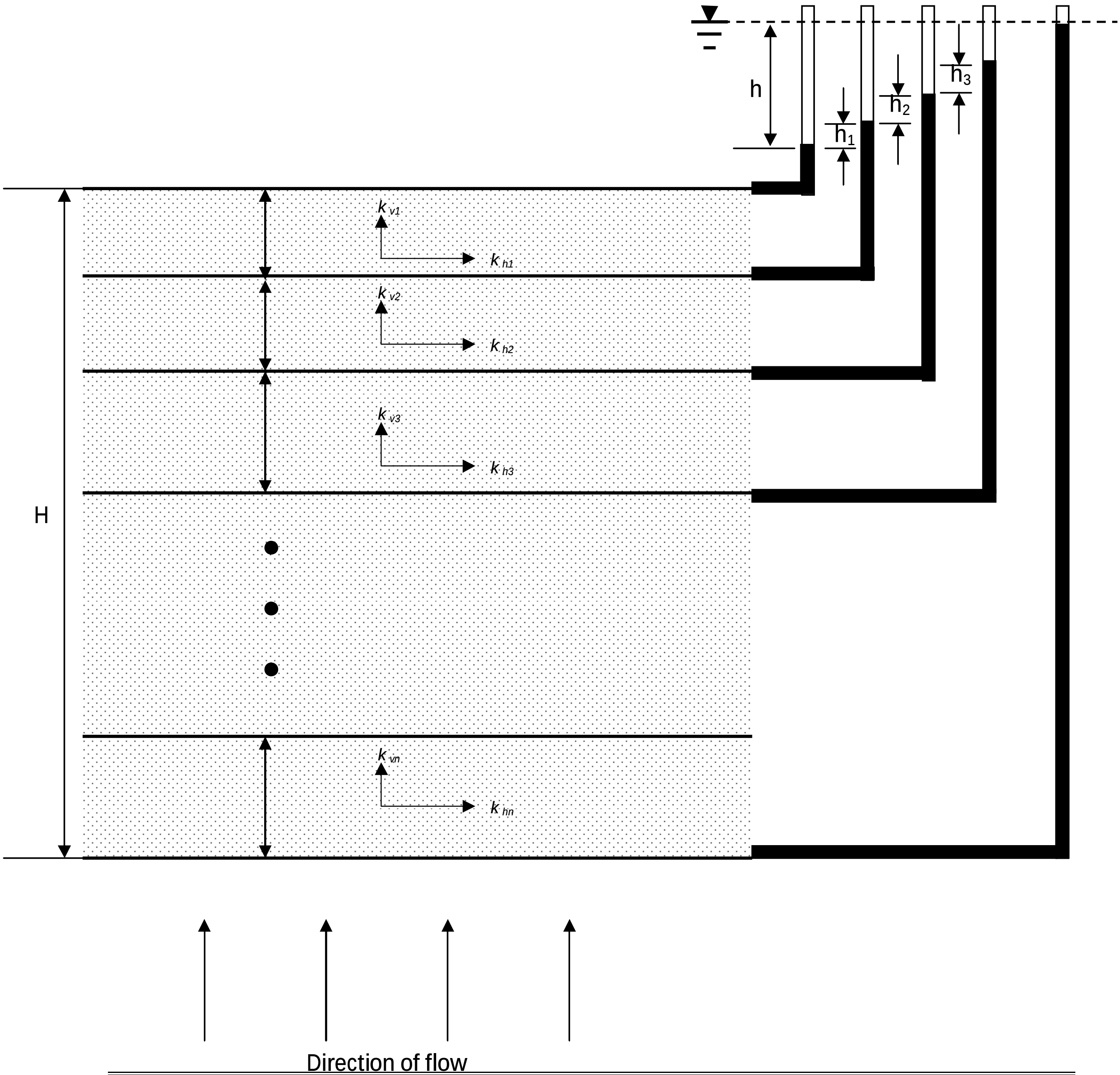
Since $i_{eq} = i_1 = i_2 = i_3 = \cdots = i_n$ then

$$k_{H(eq)} = \frac{1}{H}\left(k_{h1}H_1 + k_{h2}H_2 + k_{h3}H_3 + \cdots + k_{hn}H_n\right)$$

Or

$$\boxed{k_{H(eq)} = \frac{\sum_{i=1}^{n} k_{hi} H_i}{H}}$$

Vertical direction



$$v = v_1 = v_2 = v_3 = \ldots = v_n$$

and

$$h = h_1 + h_2 + h_3 + \ldots + h_n$$

using Darcy's law $v = ki$, we can write

$$k_{v(eq)} \cdot \frac{h}{H} = k_{v1}.i_1 = k_{v2}.i_2 = k_{v3}.i_3 = \ldots = k_{vn}.i_n$$

again

$$h = H_1.i_1 + H_2.i_2 + H_3.i_3 + \ldots + H_n.i_n$$

the solutions of these equations gives

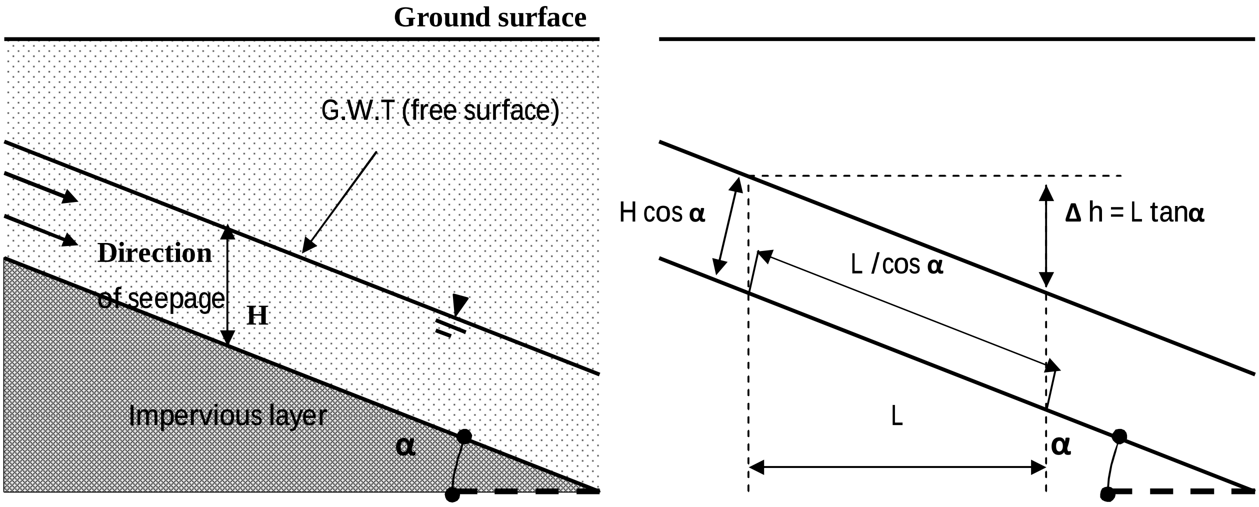
$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v1}}\right) + \left(\frac{H_2}{k_{v2}}\right) + \left(\frac{H_3}{k_{v3}}\right) + \ldots + \left(\frac{H_n}{k_{vn}}\right)}$$

or

$$\boxed{k_{v(eq)} = \frac{H}{\sum_{i=1}^{n} \frac{H_i}{k_{vi}}}}$$

Examples

1.An impervious layer as shown in the figure underlies a permeable soil layer. With k = 4.8x10^{-3} cm/sec for the permeable layer, calculate the rate of seepage through it in cm^3/sec/cm length width. Given H = 3 m and α = 5^o .



Solution

From the above figure

$$\boldsymbol{i} = \frac{\boldsymbol{headloss}}{\boldsymbol{length}} = \frac{\boldsymbol{L}\tan a}{\left(\dfrac{\boldsymbol{L}}{\cos a}\right)} = \sin a$$

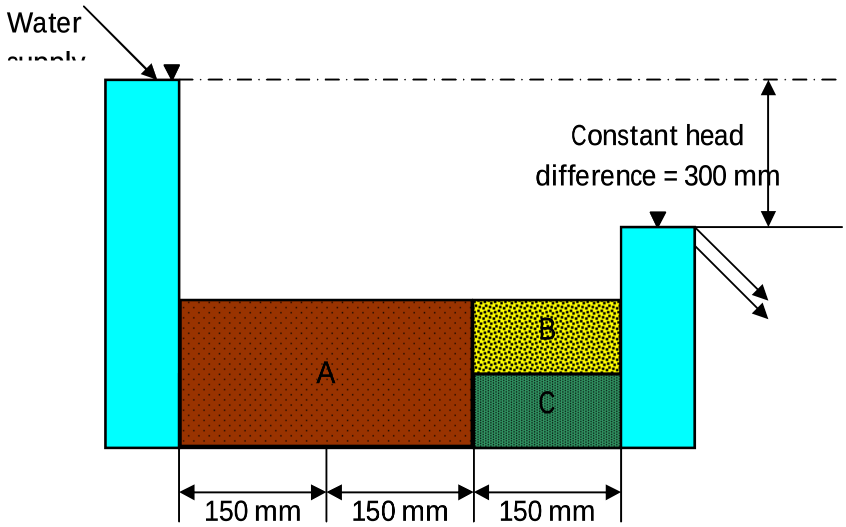
$$\boldsymbol{q} = \boldsymbol{kiA} = (\boldsymbol{k})(\sin a)(\boldsymbol{H}\cos a.1) = \left(4.8\boldsymbol{x}10^{-4}\right)(\sin 5).(3\cos 5.) = 12.5\boldsymbol{x}10^{-4}$$

$$\boldsymbol{q} = 12.5 \ \text{cm}^3\text{/sec/cm length}$$

2. The following figure shows the layers of soil in a tube 100mmx100mm in cross – section. Water is supplied to maintain a constant head difference of 300 mm across the sample. The permeability coefficient of the soils in the direction of flow through them are as follows:

Soil	***k (cm/sec)***
A	***$1x10^{-2}$***
B	***$3x10^{-3}$***
C	***$5x10^{-4}$***

Find the rate of supply.



Solution

For the soil layers B & C (the flow is parallel to the stratification)

$$k_{H(eq)} = \frac{1}{H}\left(k_{h1}H_1 + k_{h2}H_2\right) = \frac{1}{10}(3x10^{-3}(5) + 5x10^{-4}(5)) = 1.75x10^{-3} \quad \text{cm/sec}$$

For the layer A with equivalent layer of B&C

$$\therefore k_{eq} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{45}{\frac{30}{1x10^{-2}} + \frac{15}{1.75x10^{-3}}} = 3.8x10^{-3}$$

$$k_{eq} = 0.003888 cm/\sec$$

$$q = k_{eq} iA = 0.003888\frac{300}{450}(10)^2 = 0.259\ cm^3/\sec$$

3. The permeability coefficient of a sand at a void ratio of 0.55 is 0.1 ft/min. estimate its permeability coefficient at avoid ratio of 0.7. Use Casagrande empirical relationship

<u>Solution</u>

From Casagrande relation k=1.4e²k0.85 $\Rightarrow k \propto e^2$ ***.So***

$$\frac{k_1}{k_2} = \frac{e_1^2}{e_2^2} \Rightarrow \frac{0.1}{k_2} = \frac{(0.55)^2}{(0.7)^2} \Rightarrow k_2 = \frac{(0.1)(0.7)^2}{(0.55)^2} = 0.16 \text{ ft/min at e = 0.7}$$

4. for normally consolidated clay soil, the following are given:

Void ratio	***k (cm/sec)***
1.1	***0.302x10⁻⁷***
0.9	***0.12x10⁻⁷***

Estimate the permeability coefficient of clay at void ratio of 1.2 . Use Samarasingh et. al. relation.

<u>Solution</u>

Samarasingh et.al. eq. $k = C_3 \frac{e^n}{1+e} \quad \therefore \frac{k_1}{k_2} = \frac{\left(\frac{e_1^n}{1+e_1}\right)}{\left(\frac{e_2^n}{1+e}\right)}$

$$\frac{03.02x10^{-7}}{0.12x10^{-7}} = \frac{\frac{(1.1)^n}{1+1.1}}{\frac{(0.9)^n}{1+0.9}} \Rightarrow 2.517 = \left(\frac{1.9}{2.1}\right)\left(\frac{1.1}{0.9}\right)^n$$

$$\therefore 2.782 = (1.222)^{n}$$

$$n = \frac{\log(2.782)}{\log(1.222)} = \frac{0.444}{0.087} = 5.1$$

So

$$k = C_3\left(\frac{e^{5.1}}{1+e}\right)$$

To find C_3

$$0.302x10^{-7} = C_3\left[\frac{(1.1)^{5.1}}{1=1.1}\right] = \left(\frac{1.626}{2.1}\right)C_3$$

$$C_3 = \frac{(0.302x10^{-7})(2.1)}{1.626} = 0.39x10^{-7}\, cm/\sec$$

Hence

$$k = (0.39x10^{-7})\left(\frac{e^{5.1}}{1+e}\right)$$

At a void ratio of 1.2

$$k = (0.39x10^{-7})\left(\frac{1.2^{5.1}}{1+1.2}\right) = 0.449x10^{-7}\text{ cm /sec.}$$

5. ***A pumping test from Gravity well in a permeable layer underlain by an impervious stratum was made. When steady state was reached, the following observations were made q = 100 gpm; h_1 = 20 ft; h_2 = 15 ft; r_1 = 150 ft; and r_2 = 50 ft. Determine the permeability coefficient of the permeable layer.***

Solution

Since $k = \dfrac{2.303 q \log_{10}\left(\dfrac{r_1}{r_2}\right)}{p\left(h_1^2 - h_2^2\right)}$

Given: q = 100gpm = 13.37 ft^3 / min, so

$$k = \frac{2.303x13.37 \log_{10}\left(\dfrac{150}{50}\right)}{p\left(20^2 - 15^2\right)} = 0.0267\ ft/\min \approx 0.027\ ft/\min$$

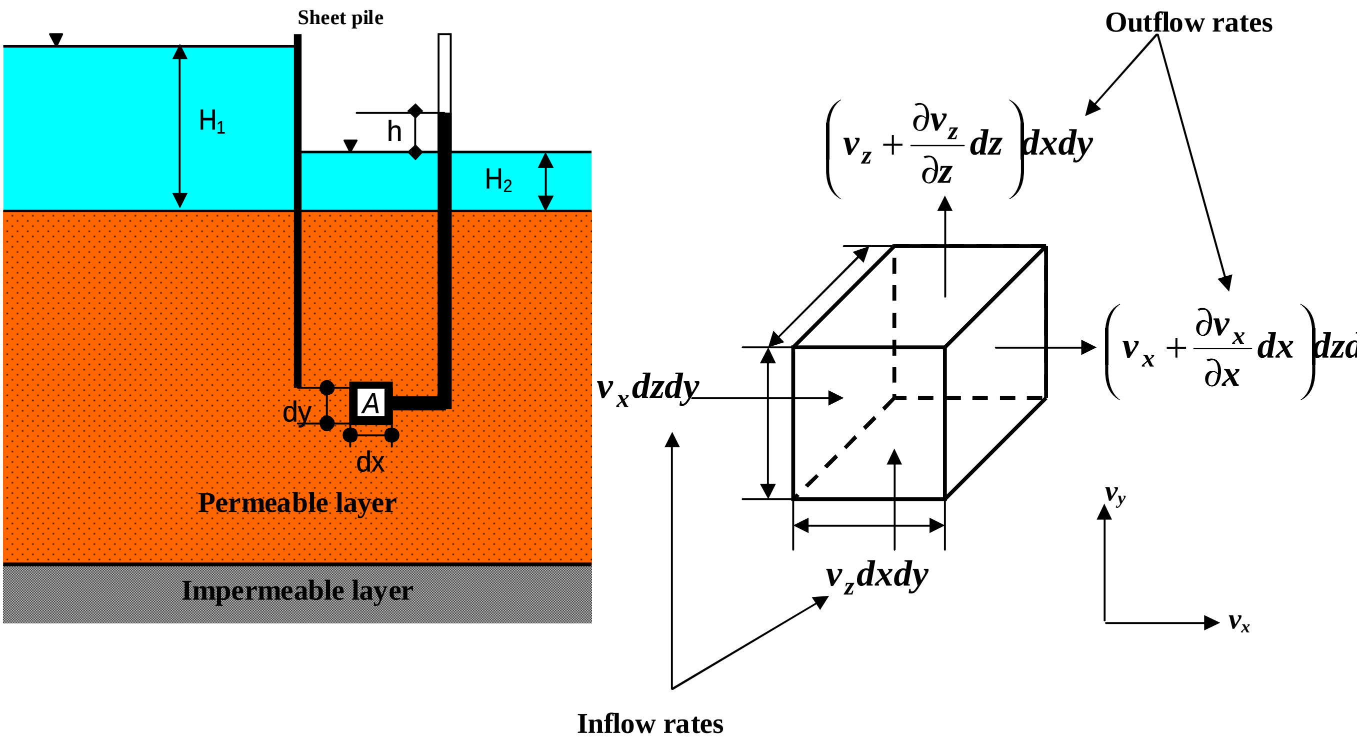
Seepage

- Laplace's Equation of Continuity
  - ***Introduction***

    In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In such cases, calculation of ground water flow is generally made by use of graphs referred to as ***flow nets***. The concept of the flow net is based on ***Laplace's equation of continuity***, which describes the steady flow condition for a given point in the soil mass.

  - ***Derivation***

    To derive the Laplace differential equation of continuity, let us take a single row of sheet piles that have been driven into a permeable soil layer, as shown in the figure below.



Flow at element A

Assumptions:

1. The row of sheet piles is impervious
2. The steady state flow of water from the upstream to the downstream side through the permeable layer is a two – dimensional flow.
3. The water is incompressible
4. No volume change occurs in the soil mass. Thus, the total rate of inflow should be equal to the total rate of outflow

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dz.dy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx.dy\right] - \left[v_x.dz.dy + v_z.dx.dy\right] = 0$$

Or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \text{.........(1)}$$

Using Darcy's law, the discharge velocities can be expressed as

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \text{ and } v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \text{ ....(2)}$$

Where k_x, k_z are the permeability coefficients in the horizontal and vertical directions respectively.

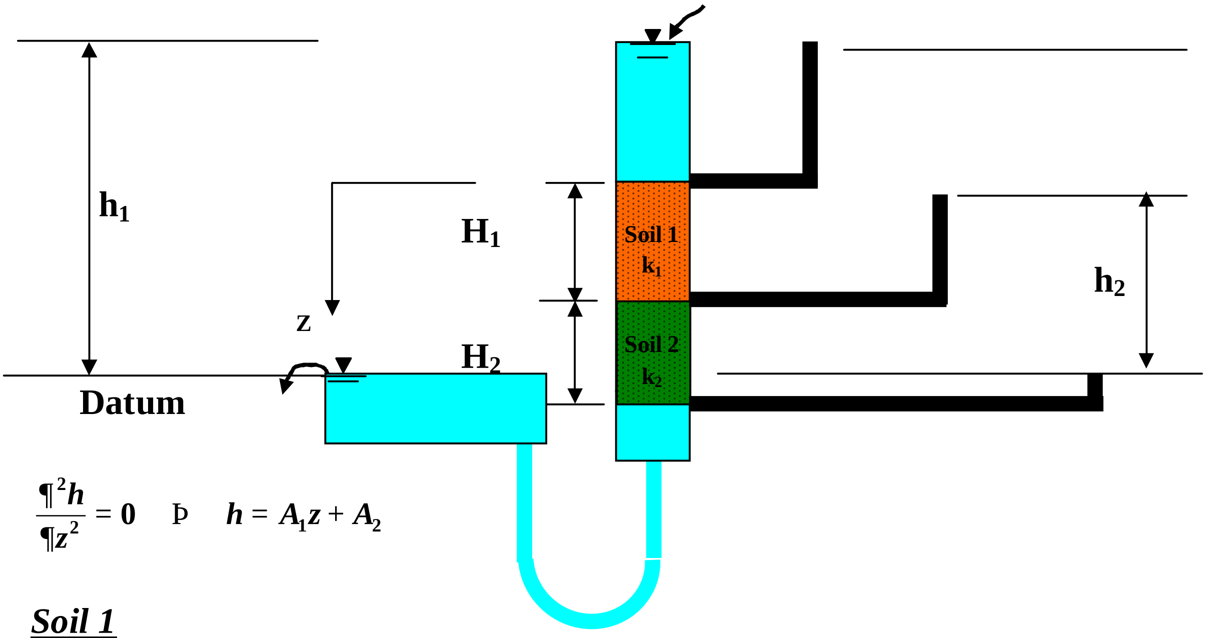
From Eqs. 1 and 2, we can write that

$$\boxed{k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0}$$

If the soil is isotropic with respect to the permeability coefficients – that is, $k_x = k_z$ - the preceding continuity equation for two dimensional flow simplifies to

$$\boxed{\frac{\partial^2 \boldsymbol{h}}{\partial \boldsymbol{x}^2}+\frac{\partial^2 \boldsymbol{h}}{\partial \boldsymbol{z}^2}=0}$$

- Continuity Equation for Solution of Simple Flow Problems



$$\frac{\partial^2 h}{\partial z^2}=0 \quad \Rightarrow \quad h = A_1 z + A_2$$

<u>Soil 1</u>

@ $z = 0$	$h = h_1$
@ $z = H_1$	$h = h_2$

$$h_1 = A_2$$

$$h_2 = A_1 H_1 + h_1 \quad \Rightarrow \quad A_1 = -\frac{(h_1 - h_2)}{H_1}$$

$$\therefore \quad h = -\frac{(h_1 - h_2)}{H_1} z + h_1 \qquad for \quad 0 \le z \le H_1$$

<u>Soil 2</u>

@ $z = H_1$	$h = h_2$
@ $z = H_1+H_2$	$h = 0$

$$h_2 = A_1 H_1 + A_2 \quad \Rightarrow \quad A_2 = h_2 - A_1 H_1$$

$$0 = A_1(H_1 + H_2) + A_2 \quad \Rightarrow \quad A_1 = -\frac{h_2}{H_2} \quad \textit{and} \quad A_2 = h_2\left(1 + \frac{H_1}{H_2}\right)$$

$$\therefore \quad h = -\frac{h_2}{H_2} z + h_2\left(1 + \frac{H_1}{H_2}\right) \quad \textit{for} \quad H_1 \le z \le H_1 + H_2$$

At any given time

$$q_1 = q_2$$

$$k_1 \frac{h_1 - h_2}{H_1} A = k_2 \frac{h_2 - 0}{H_2} A$$

$$h_2 = \frac{h_1 k_1}{H_1\left(\dfrac{k_1}{H_1} + \dfrac{k_2}{H_2}\right)}$$

$$\therefore \quad h = h_1\left(1 - \frac{k_2 z}{k_1 H_2 + k_2 H_1}\right) \qquad \textit{for} \quad 0 \le z \le H_1$$

$$h = h_1\left[\left(\frac{k_1}{k_1 H_2 + k_2 H_1}\right)(H_1 + H_2 - z)\right] \qquad \textit{for} \quad H_1 \le z \le H_1 + H_2$$

● Flow Nets

The following methods are available for the determination of flow nets:

1. Graphical solution by sketching
2. Mathematical or analytical methods
3. Numerical analysis
4. Models
5. Analogy methods

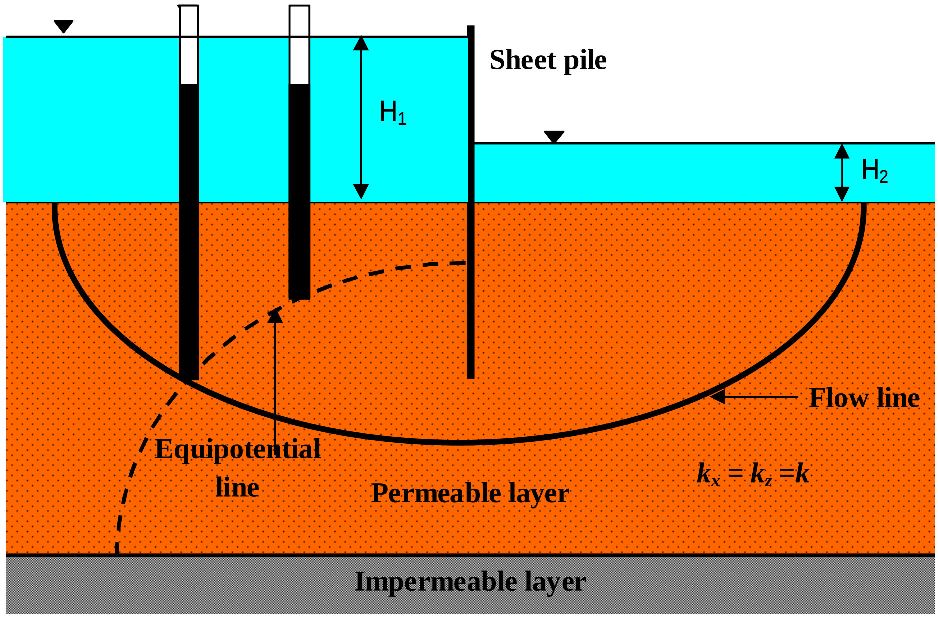
All the methods are based on Laplace's continuity equation.

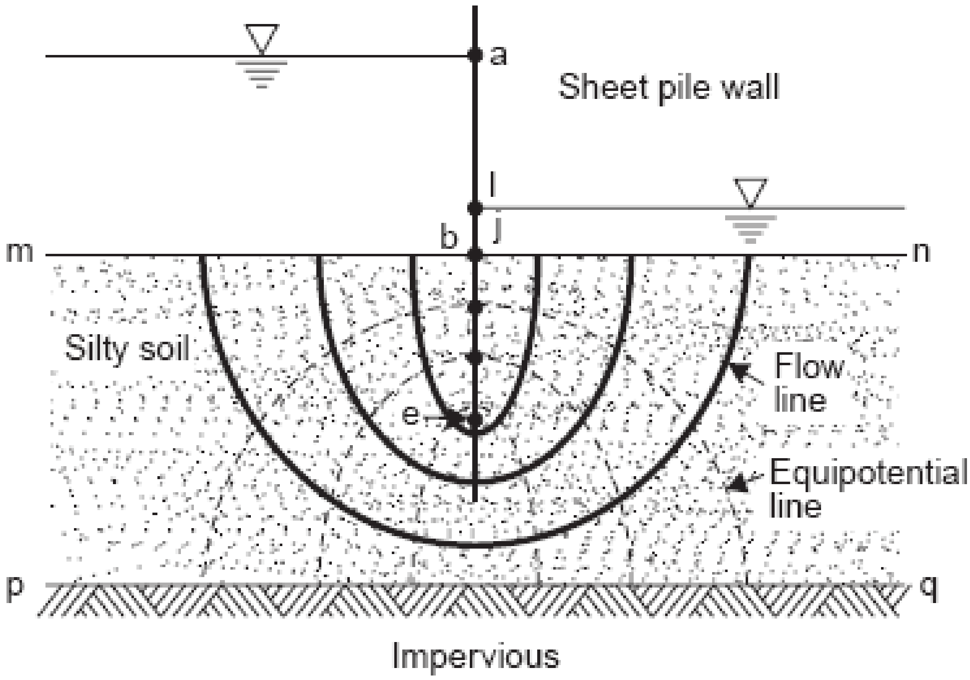
Flow net in isotropic medium

$\dfrac{\partial^2 h}{\partial x^2} + \dfrac{\partial^2 h}{\partial z^2} = 0$ represents two orthogonal families of curves – that is, the *flow lines* and the *equipotential lines.*

Flow line is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium.

Equipotential line is a line along which the potential head at all points is the same.



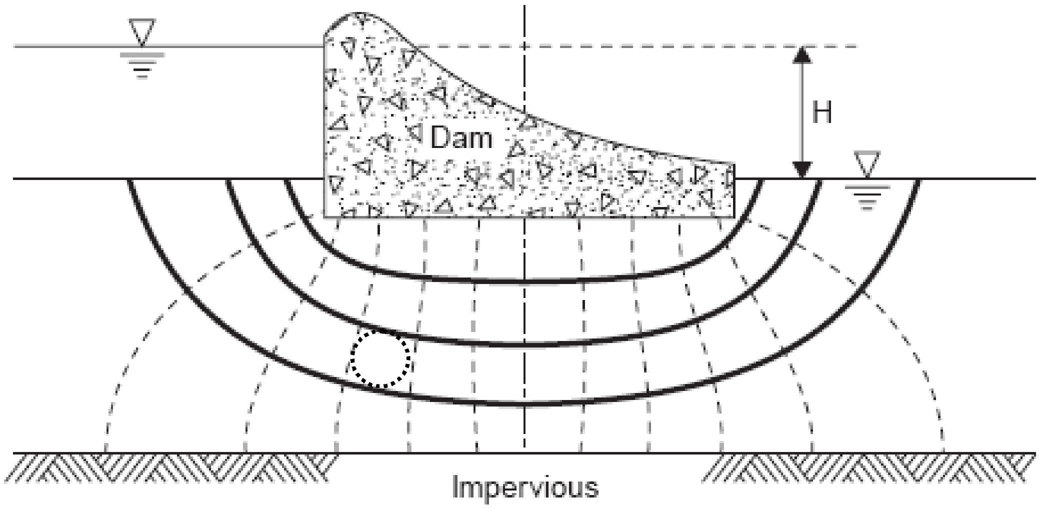


A combination of number of flow lines and equipotential lines is called a ***flow net***.

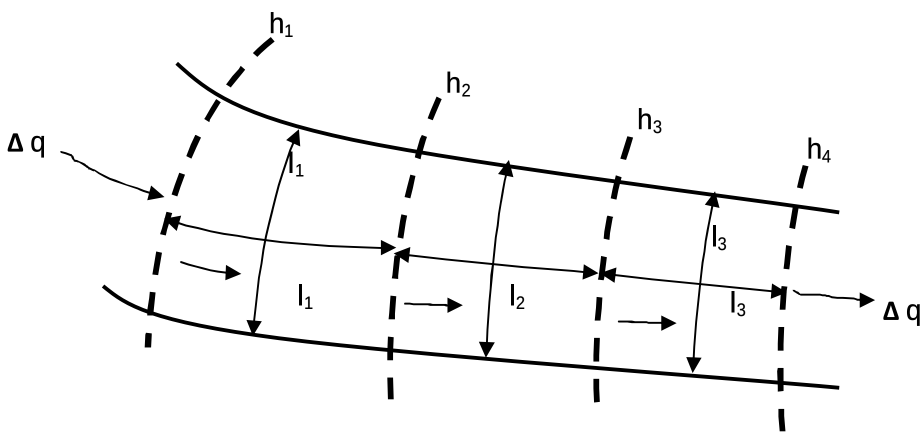
To construct a flow net, the flow and equipotential lines are drawn (see the above figure which is an example of a completed flow net) in such a way that

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.

The following figure shows another example of a flow net in an isotropic permeable layer.



● Seepage Calculation



Let $h_1, h_2, h_3, h_4, \ldots\ldots, h_n$ be the Piezometric levels

The rate of seepage through the flow channel per unit width $\Delta q_1 = \Delta q_2 = \Delta q_3 = \ldots = \Delta q$

From Darcy's law, the rate of flow is equal to $\boldsymbol{k.i.A}$. Thus

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right) l_1 = k\left(\frac{h_2 - h_3}{l_2}\right) l_2 = k\left(\frac{h_3 - h_4}{l_3}\right) l_3 = \ldots \text{ So}$$

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \ldots = \frac{H}{N_d}$$ potential drop between any adjacent equipotential lines

And

$$\Delta q = k\,\frac{H}{N_d}$$

Where

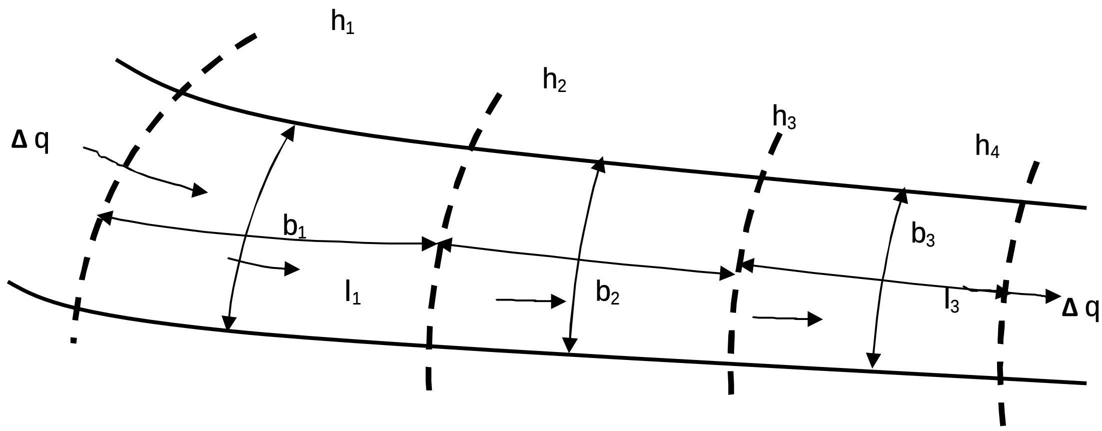
H = the difference of head between the upstream and downstream sides

N_d = number of potential drops

If the number of flow channels in a flow net is equal to N_f , the total rate of flow through all the channels per unit width can be given by

$$\boxed{q = k.H.\frac{N_f}{N_d} = k.H.\oint}$$ Where $\oint$ = shape factor of the flow net = $\dfrac{N_d}{N_f}$

Although convenient, it is not always to draw square elements for a flow net. It is also possible to draw a rectangular mesh for a flow channel as shown in the figure below, provided that the width – to – length ratio for all the rectangular elements in the flow net are the same.



In that case, the flow rate through the channel can write as follows

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)b_1 = k\left(\frac{h_2 - h_3}{l_2}\right)b_2 = k\left(\frac{h_3 - h_4}{l_3}\right)b_3 = \mathrm{L}$$

If $\frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \mathrm{L} = n$. So

$$\Delta q = k.H.\left(\frac{n}{N_d}\right)$$

$$\boxed{\therefore q = k.H.\left(\frac{N_f}{N_d}\right).n = k.H.\oint.n}$$

for square elements n =1

In general the flow nets may contain square and rectangular elements, in that case we can solve the problem by treating each part separately then we get the sum of the parts.

Flow nets in anisotropic meduim

In nature, most soils exhibit some degree of anisotropy. So to account for soil anisotropy with respect to permeability, some modification of the flow net construction is necessary.

The differential equation of continuity for two – dimensional flow in anisotropic soil, where $k_x \neq k_z$, is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

in that case the equation represents two families of curves that do not meet at 90^o . However, we can rewrite the preceding equation as

$$\frac{\partial^2 h}{(k_z / k_x)\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

Substituting $x' = \sqrt{k_z / k_x}.x$ then

$$\boxed{\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

To construct the flow net, use the following procedures:

1. Adopt a vertical scale (that is, z – axis) for drawing the cross – section.
2. Adopt a horizontal scale (that is, x – axis) such that horizontal scale = $\sqrt{k_z / k_x}$.(vertical scale).
3. With scales adopted in steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.
4. Draw the flow net for the permeable layer on the section obtained from step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

Depending on the problem geometry, we can also adopt transformation in the z – axis direction in the same manner describe above by adopting horizontal scale and then vertical scale will equal horizontal scale multiplying by $\sqrt{k_x/k_z}$ i.e. that the continuity equation will be written as follow:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z'^2}=0 \quad \text{where } z'=\sqrt{k_x/k_z}.z$$

The rate of seepage per unit width can be calculated by the following equation

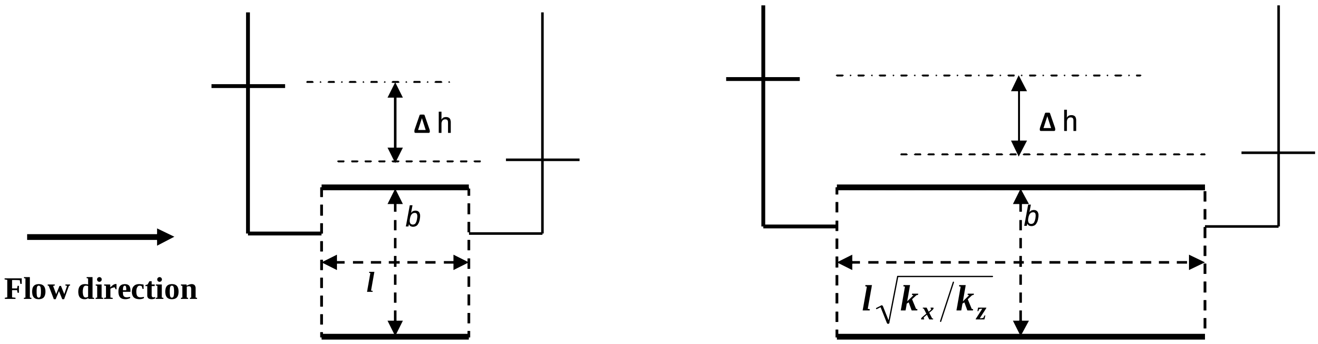
$$q = k_e.H.\oint = \sqrt{k_x.k_z}.H.\frac{N_f}{N_d}$$

Where

k_e = effective permeability to transform the anisotropic soil to isotropic soil

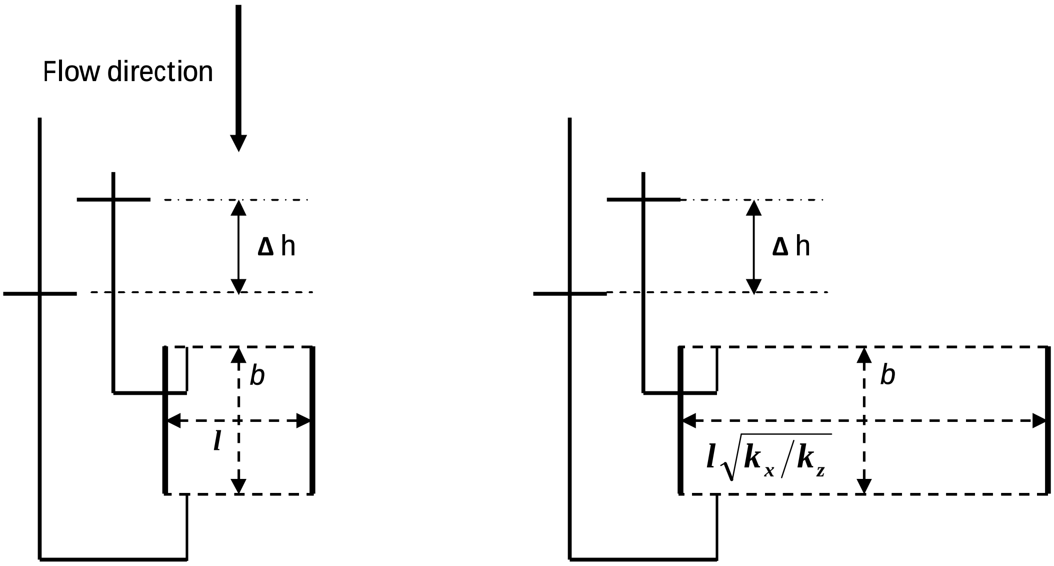
To prove that $k_e = \sqrt{k_x.k_z}$ whatever is the direction of flow let us consider two elements one from a flow net drawn in natural scale the other one drawn in transformed scale as shown below.

Transformed Scale **Natural Scale**



$$\boxed{k_e.\frac{\Delta h}{l}b(1) = k_x\frac{\Delta h}{l\sqrt{k_x/k_z}} \Rightarrow k_e = \sqrt{k_x.k_z}}$$

Transformed Scale **Natural Scale**



$$\boxed{k_e \frac{\Delta h}{b} l(1) = k_z \frac{\Delta h}{b} . l\sqrt{k_x/k_z} \Rightarrow k_e = \sqrt{k_x . k_z}}$$

In the anisotropic soil, the permeability coefficient having a maximum value in the direction of stratification and a minimum value in the direction normal to that of stratification: these directions are devoted by x & z i.e.

$k_x = k_{\max}$ and $k_z = k_{\min}$

From Darcy' s law

$$v_x = k_x . i_x = k_x . \left(-\frac{\partial h}{\partial x}\right)$$

$$v_z = k_z . i_z = k_z . \left(-\frac{\partial h}{\partial z}\right)$$

Also, in any direction S, inclined at angle α to the x – direction

$$v_s = k_s.i_s = k_s.\left(-\frac{\partial h}{\partial s}\right)$$

Now

$$\frac{\partial h}{\partial S} = \frac{\partial h}{\partial x}.\frac{\partial x}{\partial S} + \frac{\partial h}{\partial z}.\frac{\partial z}{\partial S}$$

$$\frac{\partial x}{\partial S} = \cos a$$

$$\frac{\partial z}{\partial S} = \sin a$$

$$\frac{v_s}{k_s} = \frac{v_x}{k_x}\cos a + \frac{v_z}{k_z}\sin a$$

Also

$$v_x = v_s \cos a$$

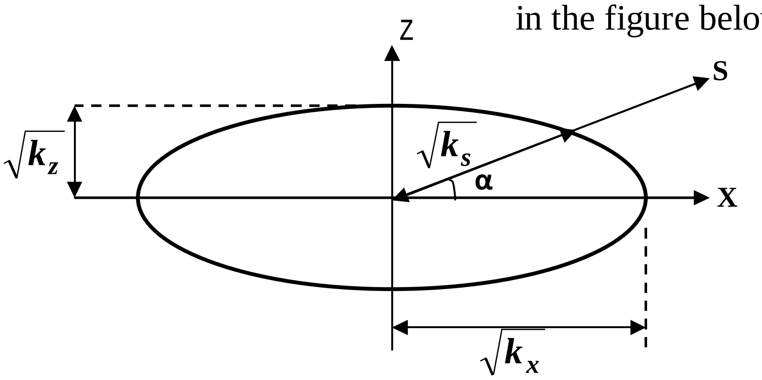
$$v_z = v_s \sin a$$

$$\therefore \frac{1}{k_s} = \frac{\cos^2 a}{k_x} + \frac{\sin^2 a}{k_z}$$

Or

$$\frac{S^2}{k_s} = \frac{x^2}{k_x} + \frac{z^2}{k_z}$$

is in the form of the ellipse as shown in the figure below

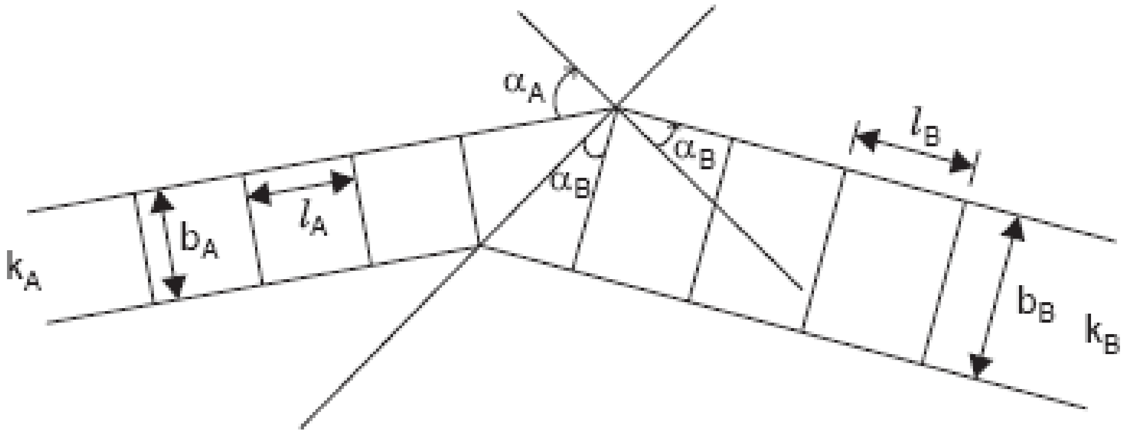


Permeability Ellipse

Transfer Condition

In case of flow perpendicular to soil strata, the loss of head and rate of flow are influenced primarily by the less pervious soil whereas in the case of flow parallel to the strata, the rate of flow is essential controlled by comparatively more pervious soil.

The following shows a flow channel (part of two – dimensional flow net) going from soil A to soil B with $\boldsymbol{k_A \neq k_B}$ (two layers). Based on the principle of continuity, i.e., the same rate of flow exists in the flow channel in soil A as in soil B, we can derive the relationship between the angles of incident of the flow paths with the boundary for the two flow channels. Not only does the direction of flow change at a boundary between soils with different permeabilities, but also the geometry of the figures in the flow net changes. As can be seen in the figure below, the figures in soil B are not squares as is the case in soil A, but rather rectangles.



$$\Delta q_A = \Delta q_B$$

$$\Delta q_A = k_A \frac{\Delta h}{l_A} b_A$$

$$\Delta q_B = k_B \frac{\Delta h}{l_B} b_B$$

$$k_A \frac{\Delta h}{l_A} b_A = k_B \frac{\Delta h}{l_B} b_B$$

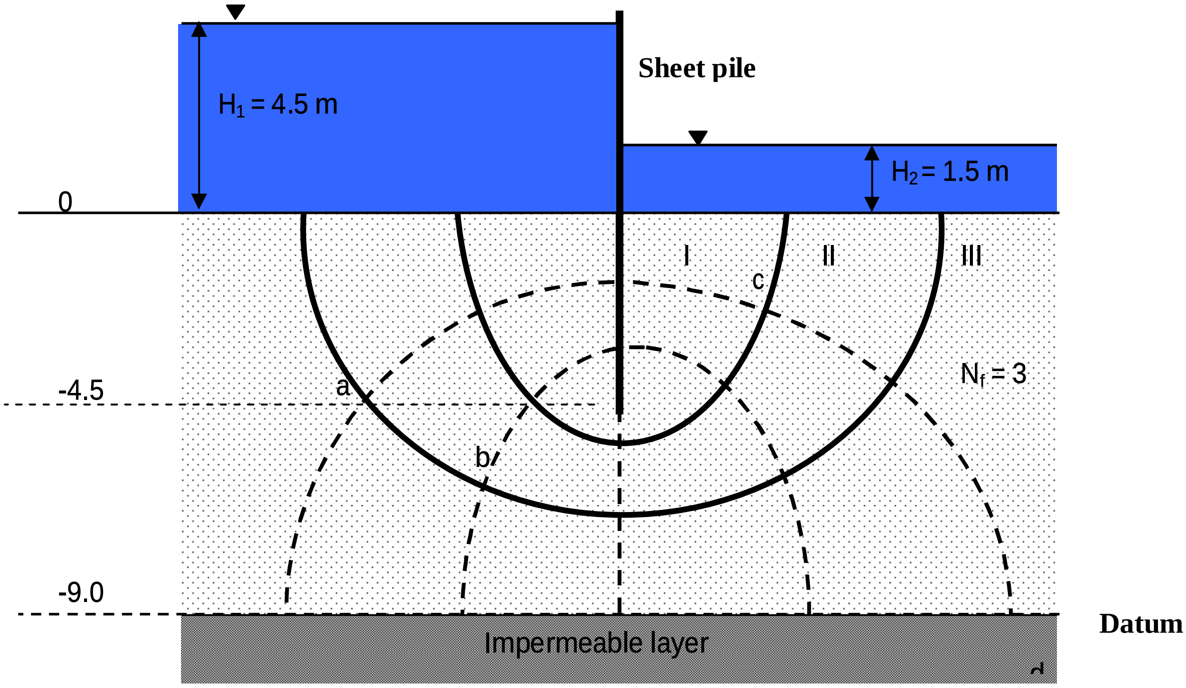
$$\frac{l_A}{b_A} = \tan a_A \; \mathrm{L} \; \textit{and} \; \mathrm{L} \; \frac{l_B}{b_B} = \tan a_B$$

$$\frac{k_A}{\tan a_A} = \frac{k_B}{\tan a_B} \Rightarrow \frac{k_A}{k_B} = \frac{\tan a_A}{\tan a_B}$$

Example

A flow net for flow around single row of sheet piles in a permeable soil layer is shown in the figure. Given $k_x = k_z = k = 5x10^{-3}$ cm/sec. Determine:

1. How high (above the ground surface) the water will rise if piezometers are placed at points a, b, c, and d.
2. The total rate of seepage through the permeable layer per unit width.
3. The rate of seepage through the flow channel II per unit width (perpendicular to the section shown)



Point	Potential drop, m	Rise above the ground surface, m
A	1x 0.5 = 0.5	4.5 – 0.5 = 4.0
B	2 x 0.5 = 1.0	4.5 – 1.0 = 3.5
C	5 x 0.5 = 2.5	4.5 – 2.5 = 2.0
D	5 x 0.5 = 2.5	4.5 – 2.5 = 2.0

Solution

a. H = 4.5 – 1.5 = 3.0 m

So, head loss / drop = $\frac{3}{6}$ = 0.5 m drop

b. $q = k.H.\oint = k.H.\frac{N_f}{N_d} = 0.05x10^{-3}(3.0)\frac{3}{6} = 7.5x10^{-5}$ m^3 / sec / m length

c. $\Delta q = k\frac{H}{N_d} = 0.05x10^{-3}.\frac{3}{6} = 2.5x10^{-5}$ m^3 /sec /m length

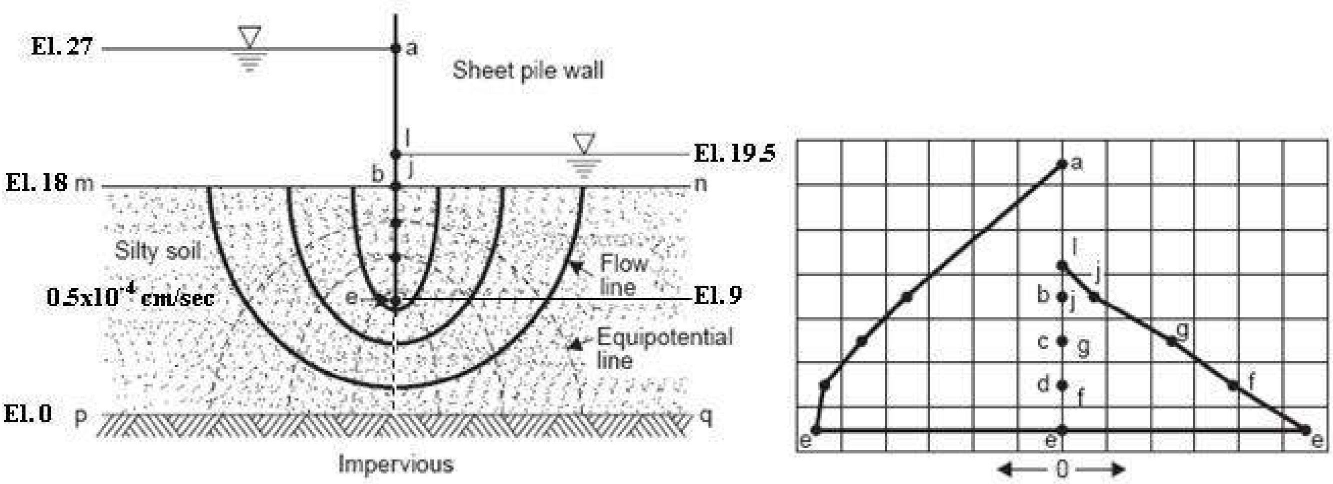
● Seepage pressure and Uplift Pressure

1. Seepage Pressure on Sheet Piles

Example

Given. Flow net in the following figure

Find. Pore pressure at points a to i; quantity of seepage; exit gradient.



(a) Flow net (b) Water pressure on the wall

Total head loss H = 27 – 19.5 =7.5 m

Head loss /drop = **7.5/8** = **0.9375** m

Let $\gamma_w = 10$ kN/m^2

Point	h_e , m	h_t , m	h_p , m	Water pressure kN/m^2
a	27	27.0	0	0
b	18	27.0	9.0	90
c	14.7	27 – 1x0.9375 = 26.0625	11.325	113.25
d	11.7	27 – 2x0.9375 = 25.125	13.425	134.25
e	9.0	27 – 4x0.9375 = 23.25	14.25	142.5
f	11.7	27 – 6x0.9375 = 21.375	9.675	96.75
g	14.7	27 – 7x0.9375 = 20.4375	5.7375	57.375
h	18.0	27 – 8x0.9375 = 19.5	1.50	15.0
i	19.5	19.50	0	0

Seepage under wall

$$q = kH\theta = 5x10^{-9}(7.5)\frac{4}{8} = 18.75x10^{-9} \text{ m}^3 \text{/sec / m. length}$$

Exit gradient

$$i = \frac{Dh}{l} = \frac{1.25}{3.45} = 0.362$$

The water pressure plot, such shown in the above figure, is useful in the structural design of the wall and in study of water pressure differential tending to cause leakage through the wall.

2. Uplift Pressure under Hydraulic structures

Example

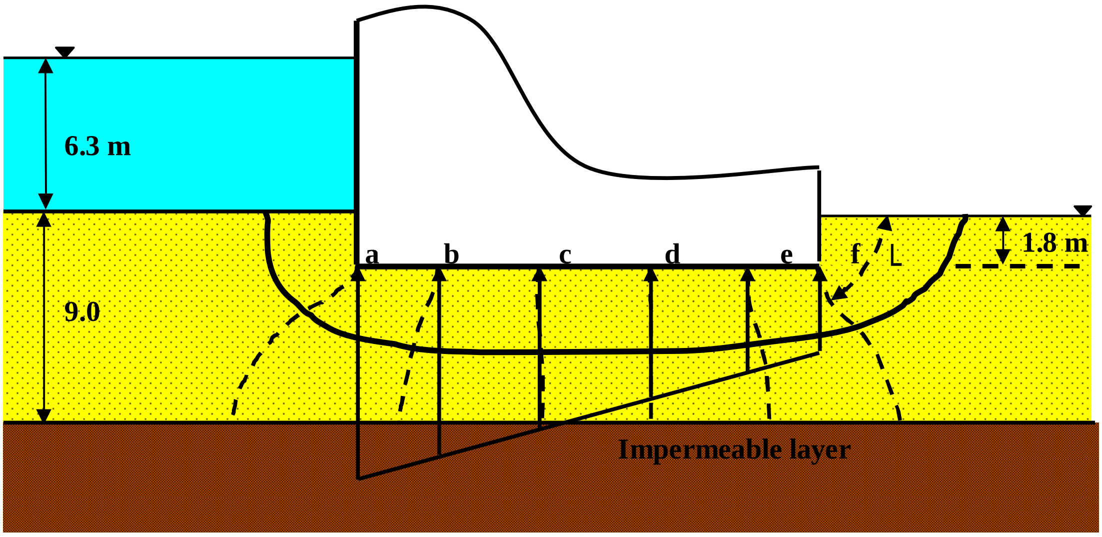
The following figure shows a weir, the base of which is 1.8 m below the ground surface. The necessary flow net also been drawn (assuming $k_x = k_z = k$).

H = 6.3 m.

So, the loss of head for each potential drop is $H/7 = 6.3/7 = 0.9$ m.

Let the datum be at the base level of the weir, so the elevation head of points (a to g) will be zero and since $h_t = h_p + h_e$ then $h_t = h_p$

The total head at the ground level in the upstream side = 6.3 + 1.8 = 8.1 m



Let γ_w = 10 kN/m^2

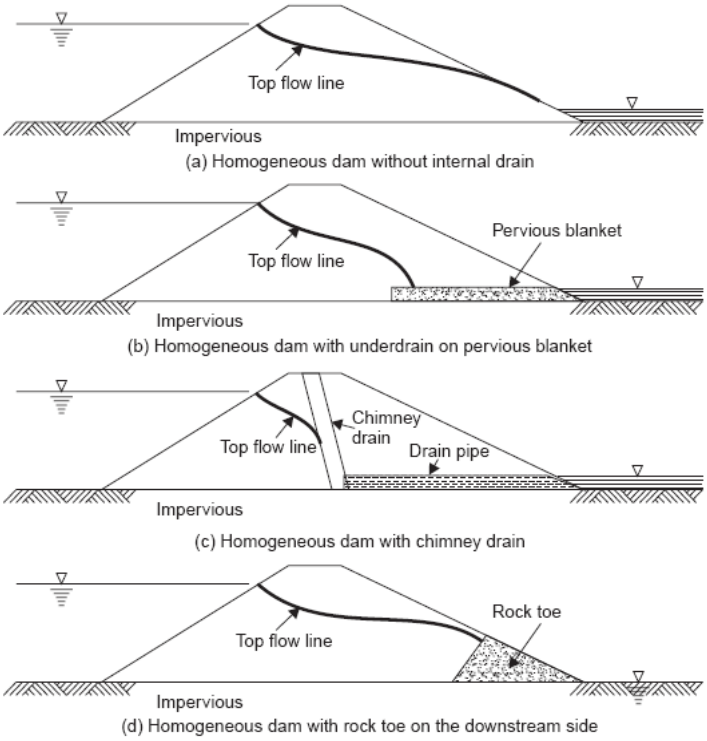
Point	Total head, h_t	Pressure head, h_p	Uplift pressure, kN/m^2 $U = h_p \times \gamma_w$
A	$8.1 - 1x0.9 = 7.2$	7.2	72
B	$8.1 - 2x0.9 = 6.3$	6.3	63
C	$8.1 - 3x0.9 = 5.4$	5.4	54
D	$8.1 - 4x0.9 = 4.5$	4.5	45
E	$8.1 - 5x0.9 = 3.6$	3.6	36
F	$8.1 - 6x0.9 = 2.7$	2.7	27

i_{exit} = 0.9 / L

High value of exit gradient will affect the stability of the structure and a factor of safety will be applied. This will discussed later

● Seepage through an Earth Dam

The flow through an earth dam differs from the other cases in that the top flow line is not know in advance of sketching the flow net. Thus, it is a case of ***unconfined flow***. The top flow line as well as the flow net will be dependent upon the nature of internal drainage for the earth dam. Typical cases are shown in Fig. 6.8; the top flow line only is shown.



Assuming that the top flow line is determined, a typical flow net for an earth dam with a rock toe, resting on an impervious foundation is shown in Fig. 6.9:

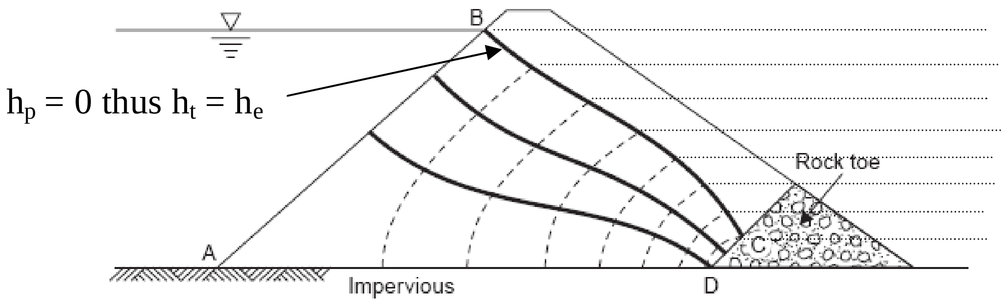


Fig. 6.9 Flow net for an earth dam with rock toe (for steady state seepage)

AB is known to be an equipotential and **AD** a flow line. **BC** is the top flow line; at all points of this line the pressure head is zero. Thus **BC** is also the ‘phreatic line’; or, on this line, the total head is equal to the elevation head. Line **CD** is neither an equipotential nor a flow line, but the total head equals the elevation head at all points of **CD**.

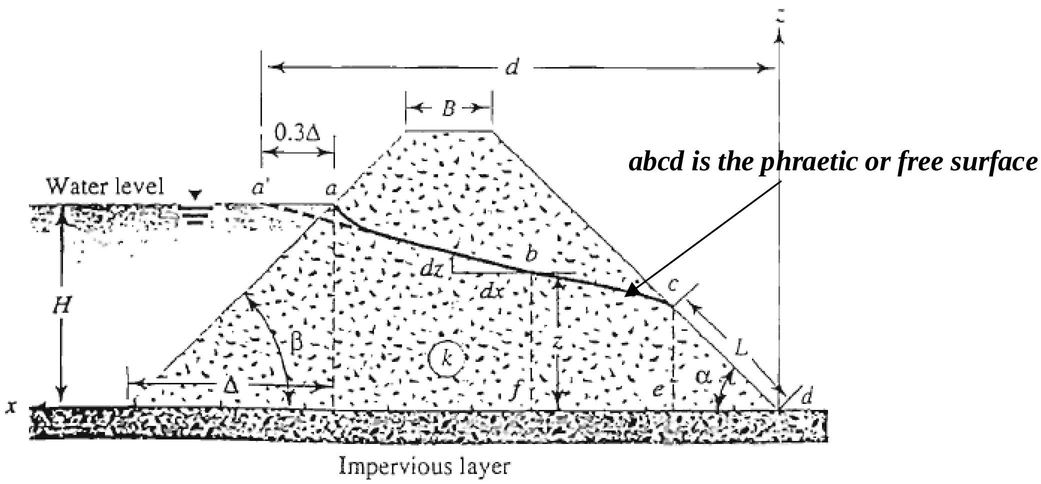


Figure 7.14 Flow through an earth dam constructed over an impervious base

Schaffernak’s solution

using Dupuit's assumption $$i \cong \frac{dz}{dx} = \sin \alpha$$

Considering Δcde

$$\boldsymbol{q = kiA}$$

$$\boldsymbol{i = \frac{dz}{dx}}$$

$$\boldsymbol{A = (\overline{ce})(1) = L\sin\alpha}$$

so

$$\boldsymbol{q = k(\tan\alpha)(L\sin\alpha) = kL\tan\alpha\,\sin\alpha \quad \ldots\ldots (1)}$$

Again,

$$q = kiA = k\left(\frac{dz}{dx}\right)(zx1) = kz\frac{dz}{dx} \ldots\ldots\ldots(2)$$

For continuous flow

$$q_1 = q_2$$

$$kz\frac{dz}{dx} = kL\tan a \sin a$$

$$\int_{z=L\sin a}^{z=H} kzdz = \int_{z=L\cos a}^{x=d} (kL\tan a \sin a)dx$$

....

....

$$L = \frac{d}{\cos a} - \sqrt{\frac{d^2}{\cos^2 a} - \frac{H^2}{\sin^2 a}} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Steps to find rate of seepage q (per m length of the dam)

1. obtain α
2. calculate Δ(see the Fig.) and then 0.3Δ
3. calculate d
4. with known values of α and d , calculate L from Eq. 3
5. with known values of L, calculate q from Eq.1

L. Casagrande'sSolution

Casagrande show that when α is more than 30^o the deviation from Dupuit's Assumption is more noticeable, he suggested that

$$i = \frac{dz}{ds} = \sin a \qquad \text{where } ds = \sqrt{dx^2 + dz^2}$$

$$q = kiA = k\sin a)L\sin a) = kL\sin^2 a$$

again

$$q = kiA = k\left(\frac{dz}{ds}\right)(1xz)$$

Combining these questions e get,

$$\int_{z=L\sin a}^{z=H} kzdz = \int_{L}^{s} L\sin^2 a \qquad \text{where s = length of the curve a}^{/}\text{bc}$$

$$L = s - \sqrt{s^2 - \frac{H^2}{\sin^2 a}}$$

With an error about 4-5%, e can write

$$s = \sqrt{d^2 + H^2}$$

Then

$$L = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \cot^2 a}$$

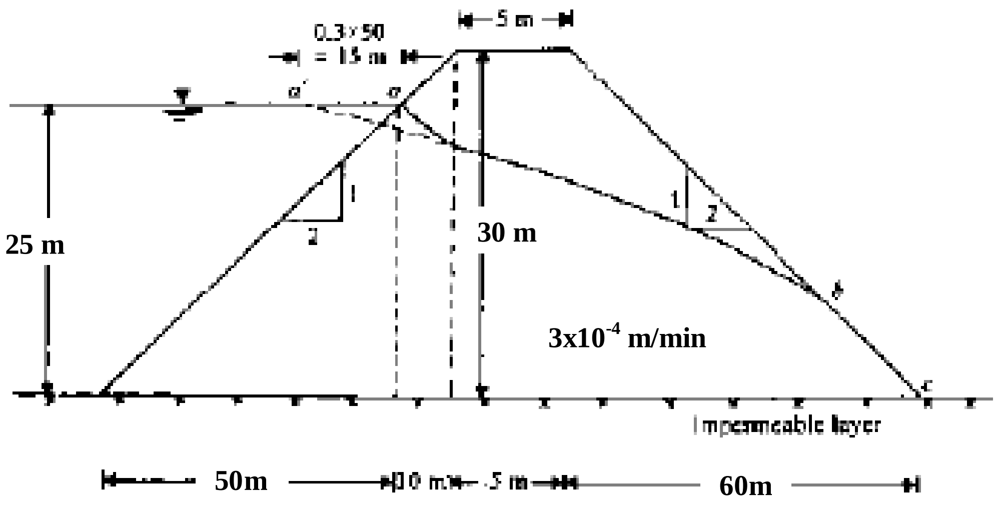
Once L is known, the rate of seepage can be calculated

$$q = kL \sin^2 a$$

Example

The cross-section of an earth dam is shown in Figure. Calculate the rate of seepage through the dam [q in $m^3/min \cdot m$] by

1. Schaffernak's solution
2. L. Casagrande's method;



Schaffernak's solution

$$L = \frac{d}{\cos a} - \sqrt{\frac{d^2}{\cos^2 a} - \frac{H^2}{\sin^2 a}}.$$

d = 125 – 0.7x50= 90 m; α = 26.57°

$$L = \frac{90}{\cos a} - \sqrt{\frac{90^2}{\cos^2 26.57} - \frac{25^2}{\sin^2 26.57}} = 16.95$$

$$q = kL \tan a \sin a$$

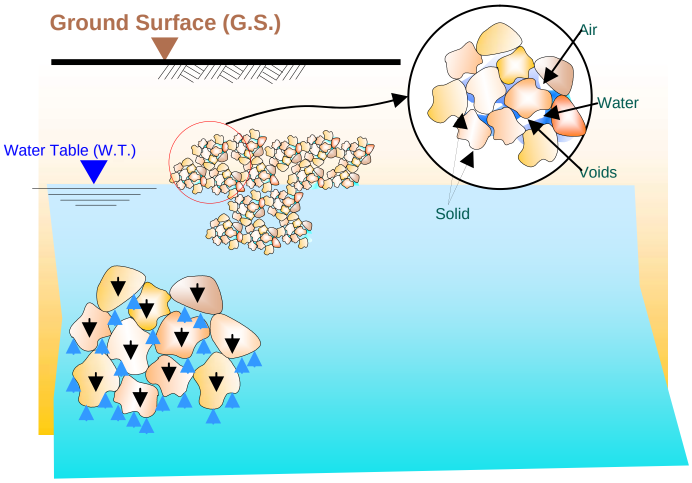
$$q = 3x10^{-4}(16.95)(\tan 26.57)(sn 26.57) = 11.37x10^{-4}$$

L. Casagrande's method;

$$L = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \cot^2 a}$$

$$L = \sqrt{90^2 + 25^2} - \sqrt{90^2 - 25^2 \cot^2 26.57} = 19m$$

$$q = kL\sin^2 a = 3x10^{-4}(19)(\sin 26.57) = 11.4x10^{-4}\,\mathrm{K}m^3/(m.\min)$$



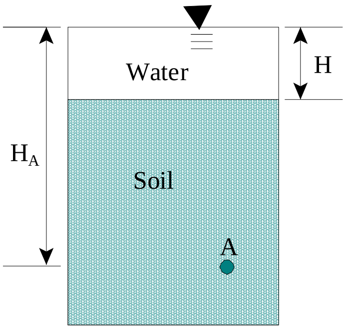
Effective Stress Concept

Topics

- Effective Stress Concept
- Effective Stress in Saturated Soil with no Seepage
- Effective Stress in Saturated Soil with Seepage
- Seepage Force
- Filter Requirements and Selection of Filter Material
- Capillary Rise in Soil
- Effective Stress in Capillary Zone

● ***Effective Stress Concept***

- Soil is a multi phase system
- To perform any kind of analysis - we must understand stress distribution
- The concept of effective stress:
    - The soil is "loaded" (footing for example)
    - The resulting stress is transmitted to the soil mass
    - The soil mass supports those stresses at the point to point contacts of the individual soil grains



The total stress at A is calculated from:

The weight of the soil above A

The weight of the water above A

$\sigma = H\gamma_w + (H_A - H)\,\gamma_{sat}$

σ = Total Stress at A

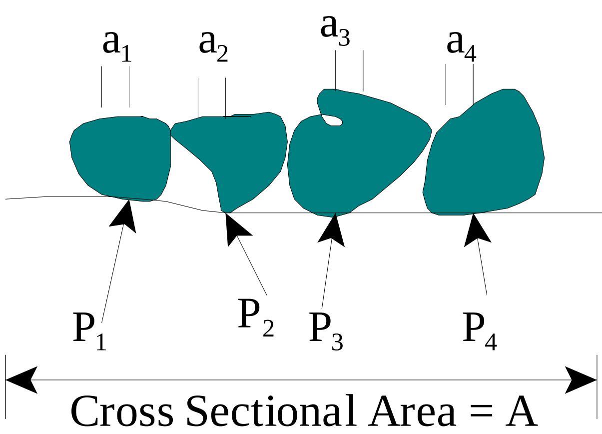
γ_w = Unit Weight of Water

γ_{sat} = Saturated Unit Weight

H_A = Height of A to Top of water

H = Height of water

- **σ** is the stress applied to the soil by its own weight
- As you go deeper in the soil mass, the stress increases
- Like in a swimming pool, as you go deeper, the stress of the weight of the water increases
- The soil carries the stress in 2 ways:
    - A portion is carried by the water (acts equally in all directions)
    - A portion is carried by the soil solids at their point of contact.



a_n = Area of points of contact

A = Cross Sectional area of soil mass

P_n = Forces acting at points of contact

- The sum of the vert. components of the forces at their points of contact per unit of X-sectional area is the effective stress.
- The sum of vertical components of forces over the area is the effective stress F'

$\sigma' = (P_{1v}+P_{2v}+P_{3v}\ .....+P_{nv}) / A$

If $a_s = a_1 + a_2 + a_3 + ...a_n$

Then the space occupied by water = $A - a_s$

Assume $u = H_A \gamma_w$ $\qquad$ H_A = Height of water

$\sigma = \sigma' + u(A - a_s) / A$

Since a_s is very small, assume = 0

$\sigma = \sigma' + u$

Recall the following equation:

$\sigma = H\gamma_w + (H_A - H)\, \gamma_{sat}$

Now, $\sigma' = \sigma - u$

Substituting: $\sigma' = [H\gamma_w + (H_A - H)\, \gamma_{sat}] - H_A\, \gamma_w$

Rearranging: $\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)$ = Height of Soil Column ***x*** γ' Where γ' = $\gamma_{sat} - \gamma_w$ = SUBMERGED OR EFFECTIVE UNIT WEIGHT

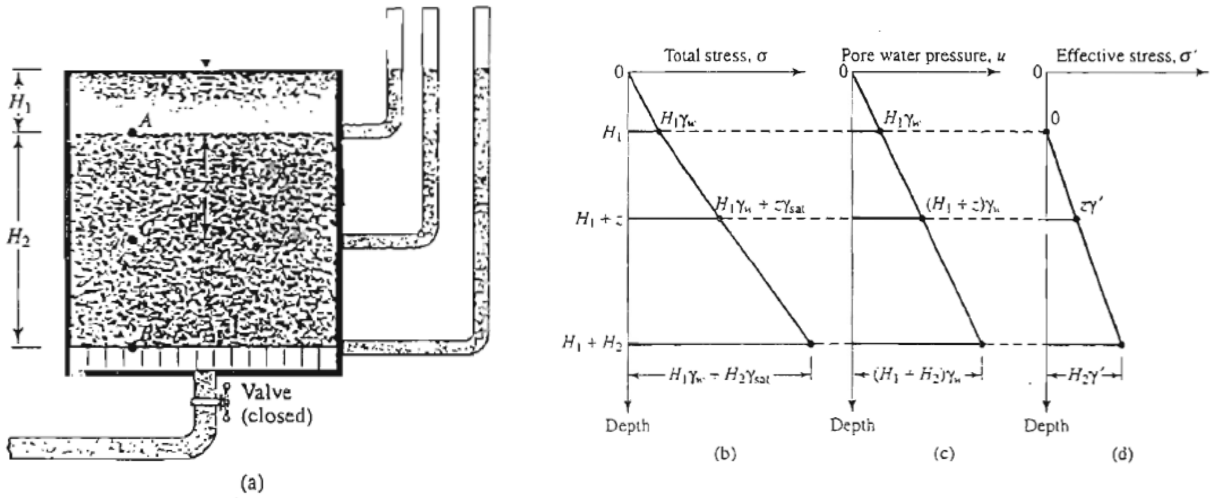
Effective Stress is independent of height of water

In the equation: $\sigma = \sigma' + u$

σ' is the soil skeleton stress

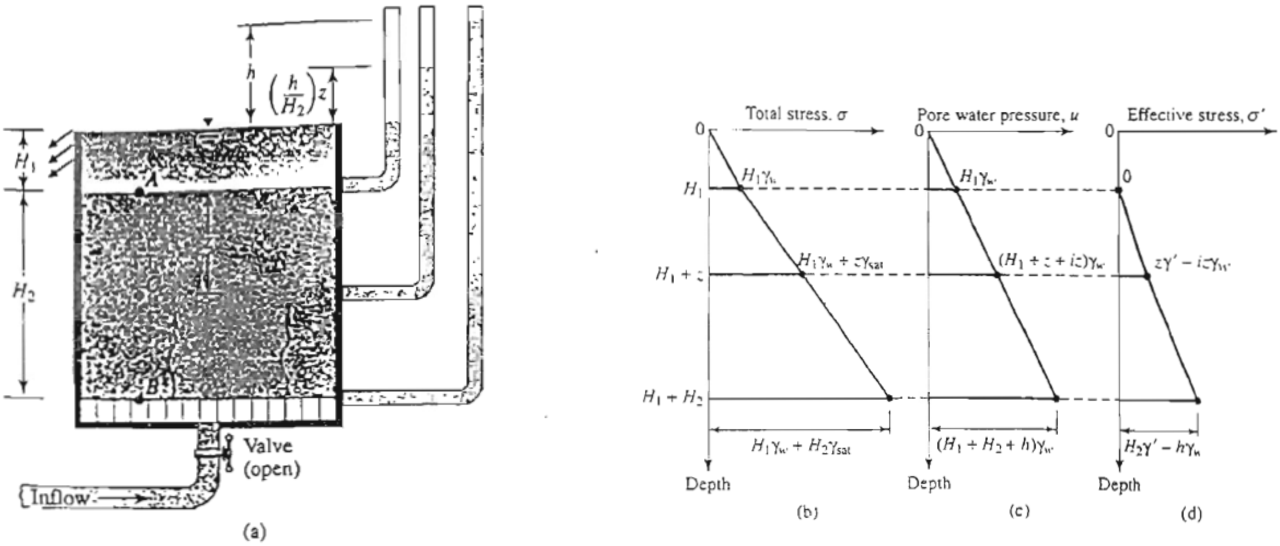
u is the stress in the water, or pore water pressure

● Effective Stress in Saturated Soil with no Seepage



●*Effective Stress in Saturated Soil with Seepage*

Upward flow

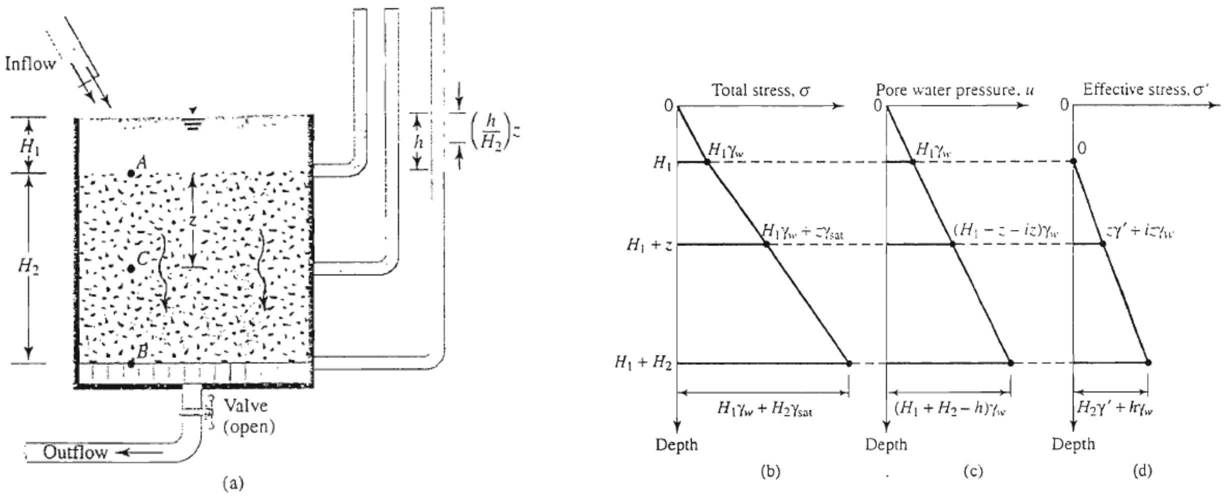


And limiting conditions may occur when $\sigma'_c = z\gamma' - \boldsymbol{i}\boldsymbol{z}\gamma_w = \mathbf{0}$ which lead to

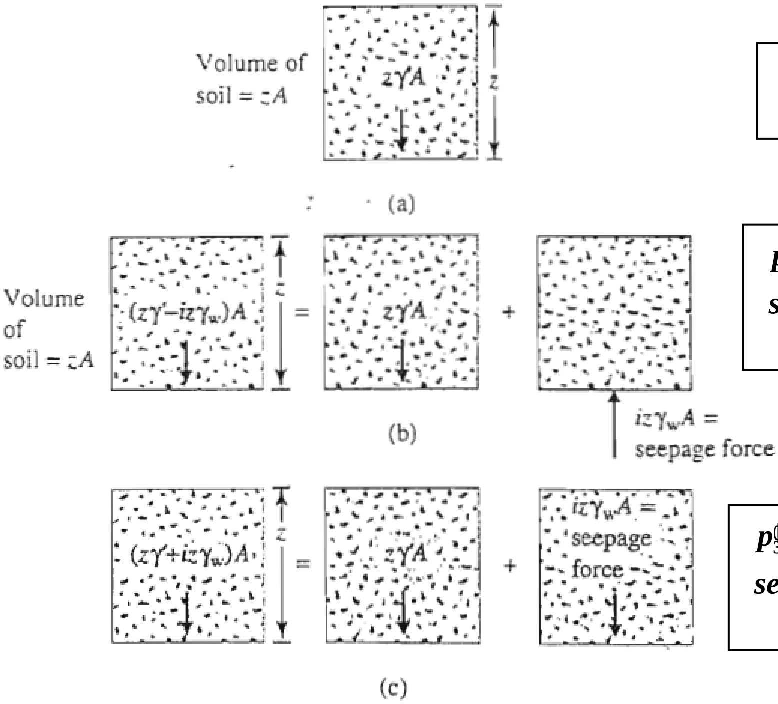
i_{cr} = *critical hydraulic gradient*

$\boldsymbol{i}_{cr} = \dfrac{\gamma'}{\gamma_w}$ *for most soils 0.9-1.1 ith average value of 1*

Downward flow



● Seepage Force



$$p'_1 = z\gamma' A$$

$$p'_2 = (z\gamma' - iz\gamma_w)A = p'_1 + \text{seepage force}$$
$$\text{seepage force} = -iz\gamma_w A$$

$$p'_3 = (z\gamma' + iz\gamma_w)A = p'_3 + \text{seepage force}$$
$$\text{seepage force} = iz\gamma_w A$$

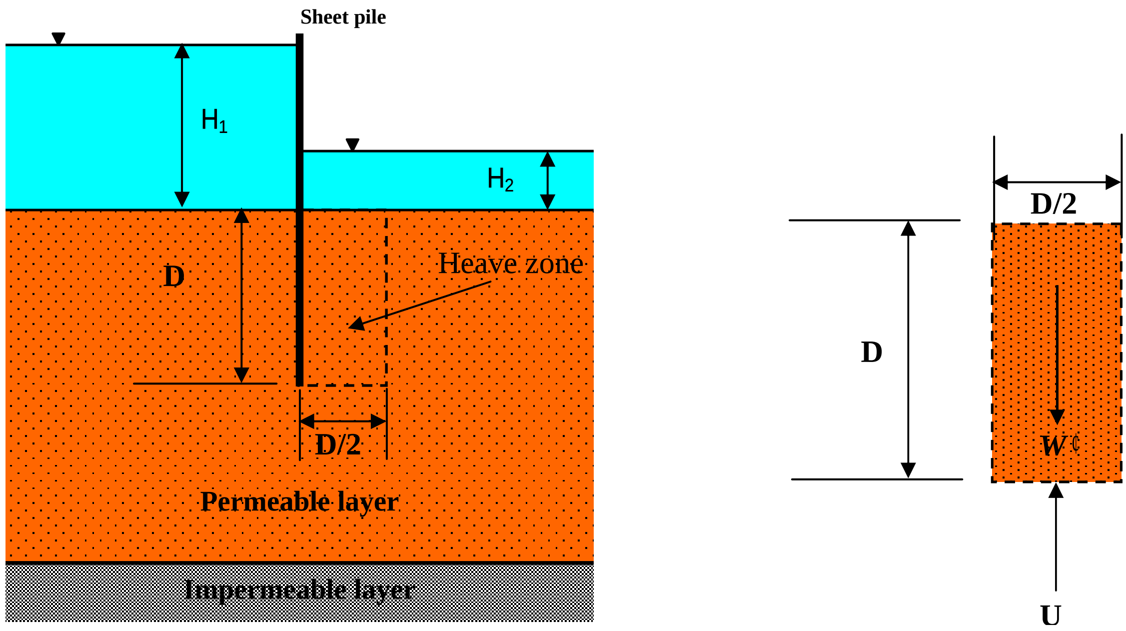
The volume of the soil contributing to the effective stress force equals zA, so the seepage force per unit volume of the soil is

$$\frac{iz\gamma_w A}{zA} = i\gamma_w \quad \text{in the direction of seepage (see the fig.)}$$

Therefore, in isotropic soil and in any direction, the force acts in the same direction as the direction of flow. Thus, the flow nets can be used to find the hydraulic gradient at any point to find seepage force at that point. This concept is useful to estimate F.S against heave

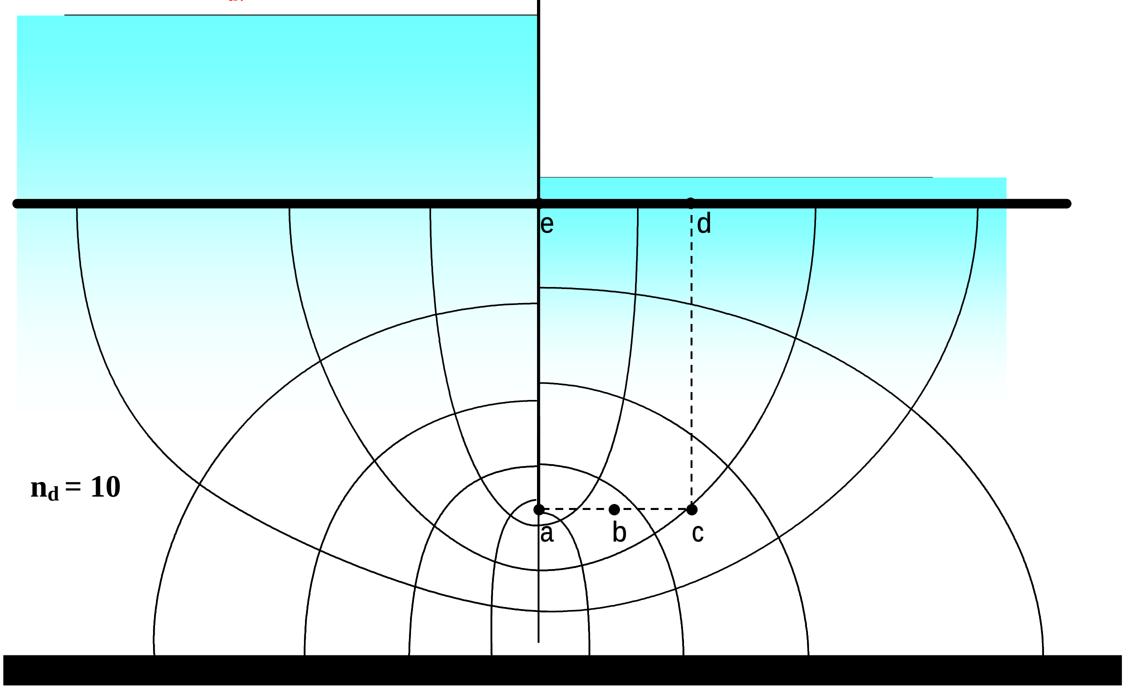
Factor of Safety against heave at the downstream of hydraulic structures

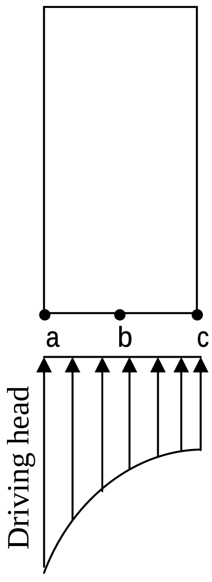
Terzaghi (1922)



$$F.S = \frac{\textit{Submerged weight}}{\textit{Uplift force caused by seepage}} = \frac{W'}{U} = \frac{D(D/2)(\gamma_{sat} - \gamma_w)}{\textit{soil volume } x\ (i_{av}\gamma_w)} = \frac{\frac{1}{2}D^2\gamma'}{\frac{1}{2}D^2 i_{av}\gamma_w} = \frac{\gamma'}{i_{av}\gamma_w}$$

Estimation of i_{av}



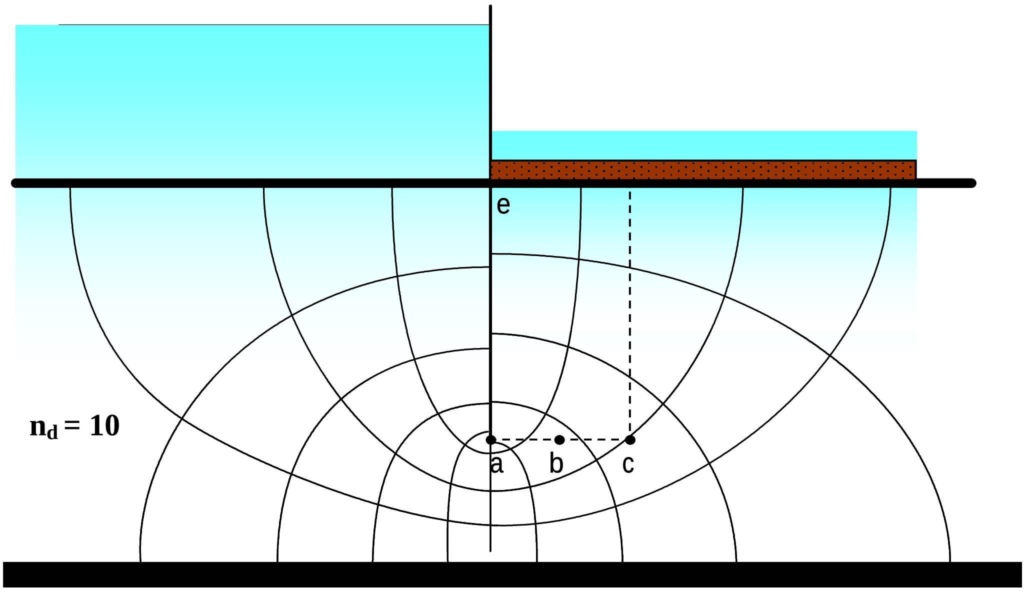


point	driving head
a	$\frac{4}{10}H$
b	$\frac{6.7}{10}H$
c	$\frac{2.5}{10}H$
$h_{av} = \frac{(h_a + h_c)/2 + h_b}{2}$ $i_{av} = \frac{h_{av}}{D}$	

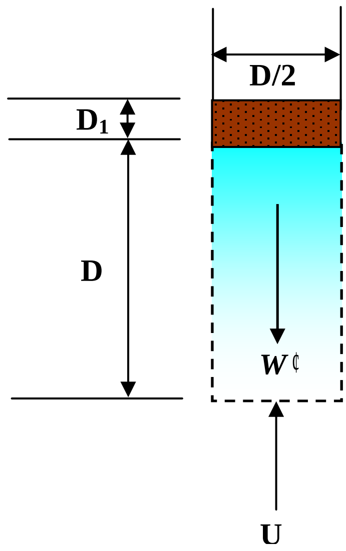
• *Filter Requirements and Selection of Filter Material*

In practice, for the safe of the hydraulic structure, a minimum value of 4 to 5 for F.S against heaving is used, because of the uncertainty in the analysis. One way to increase the F.S is using filter.

Filter:- is a granular material with opening small enough to prevent the movement of the soil particles upon which is placed and, at the same time, is previous enough to offer little resistance to seepage through it.



$$F.S = \frac{W' + W'_F}{U} = \frac{\frac{1}{2}D^2\gamma' + \frac{1}{2}DD_1\gamma'_F}{\frac{1}{2}D^2 i_{av}\gamma_w} = \frac{\gamma' + \left(\frac{D_1}{D}\right)\gamma'_F}{i_{av}\gamma_w}$$



Selection of Filter Material

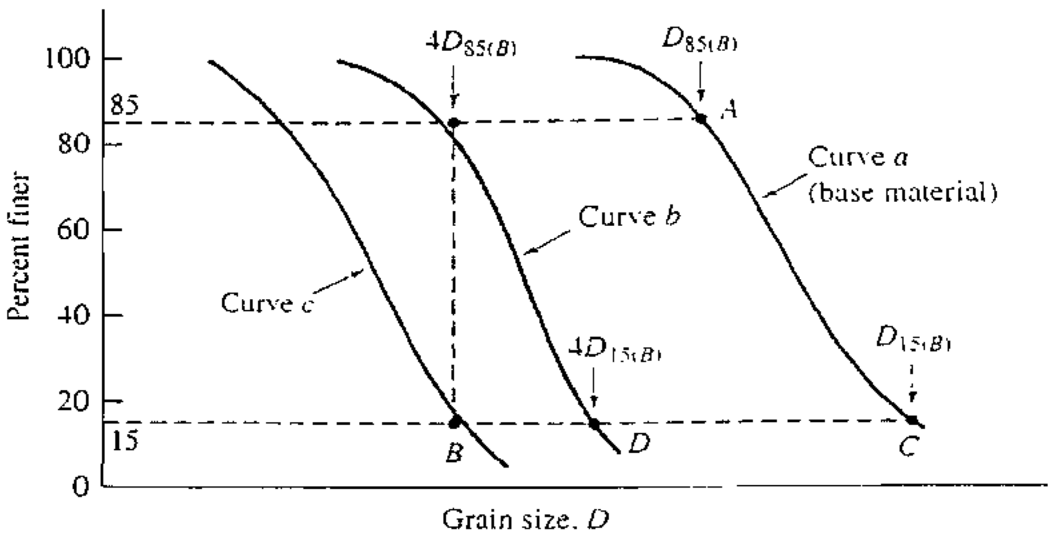
1. $\dfrac{D_{15(F)}}{D_{85(B)}} < 4$

2. $\dfrac{D_{15(F)}}{D_{15(B)}} > 4$

Here. $D_{15(F)}$, $D_{15(B)}$ = diameters through which 15% of the filter and base material, respectively, will pass

$D_{85(B)}$ = diameter through which 85% of the base material will pass

Figure 8.13 Definition of base material and filter material



Capillary Rise in Soil

$$\left(\frac{\pi}{4}d^2\right)h_c\gamma_w = \pi dT\cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{d\gamma_w}$$

where T = surface tension (force/length)
α = angle of contact
d = diameter of capillary tube
γ_w = unit weight of water

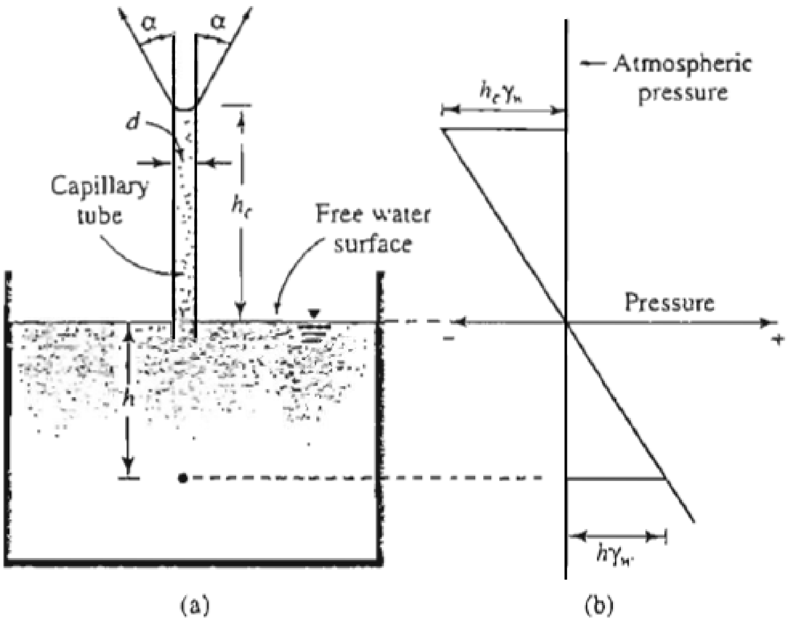


Figure 8.15 (a) Rise of water in the capillary tube; (b) pressure within the height of rise in the capillary tube (atmospheric pressure taken as datum)

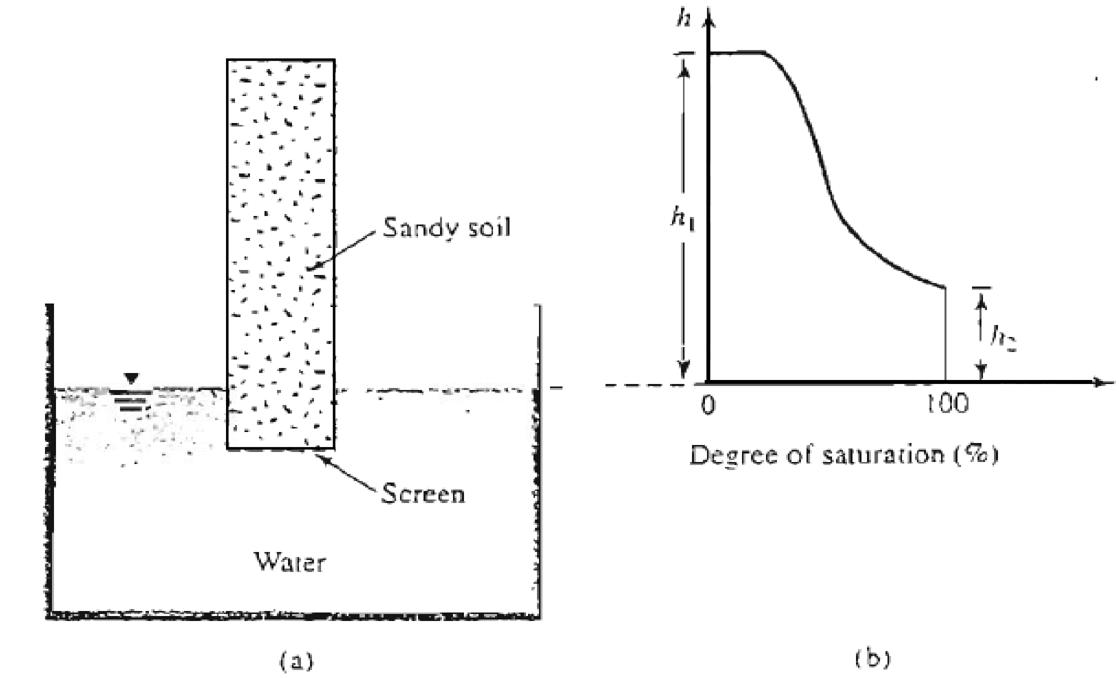
For pure water and clean glass α = 0

$$\therefore\ h_c = \frac{4T}{d\gamma_w}$$

For water T = 72 m.N/m

$$h_c \propto \frac{1}{d}$$ the smaller the capillarity tube, the larger capillary rise

For soils, the capillary tubes formed because of the continuity of voids have variable cross sections. The results of the nonuniformity on capillary can be demonstrated as shown in the fig.



(a) a soil column in contact with water

Variation of ***S*** in the soil column

Capillary effect in sandy soil

Hazen (1930) give a formula to estimate the height of capillary

$$h_c(mm) = \frac{C}{eD_{10}}$$

where D_{10} = effective size (mm)

e = void ratio

C = a constant that varies from 10 to 50 mm^2

Approximate Range of Capillary Rise in Soils

Soil type	Range of capillary rise (m)	Range of capillary rise (ft)
Coarse sand	0.1–0.2	0.3–0.6
Fine sand	0.3–1.2	1–4
Silt	0.75–7.5	2.5–25
Clay	7.5–23	25–75

● ***Effective Stress in Capillary Zone***

The general relationship of effective stress is

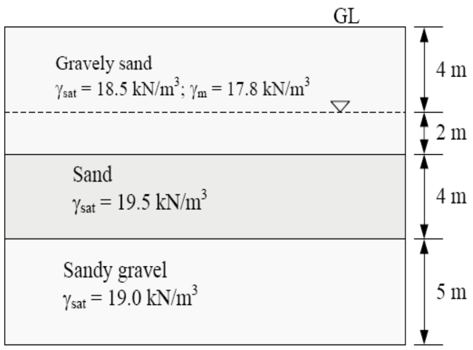
$$\sigma' = \sigma - u$$

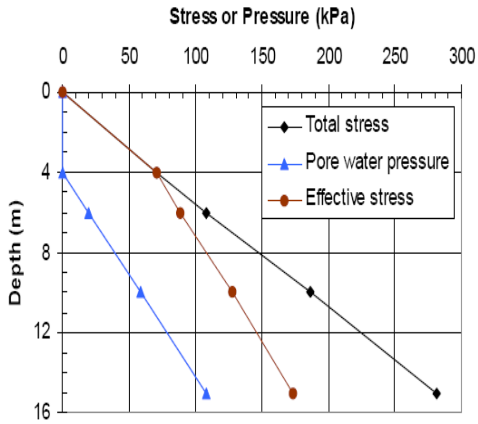
For soil fully saturated by capillary $\quad u = -h_c \gamma_w$

For soil partially saturated by capillary $\quad u = -\left(\frac{S}{100}\right) h_c \gamma_w$

● ***Examples***

EXAMPLE1. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.





Solution:
Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,

$\sigma_v = 0$; $\sigma_v' = 0$; and u=0

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$

$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

$u = (2)(9.81) = 19.6$ kPa

$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

$u = (6)(9.81) = 58.9$ kPa

$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

At 15 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

$u = (11)(9.81) = 107.9$ kPa

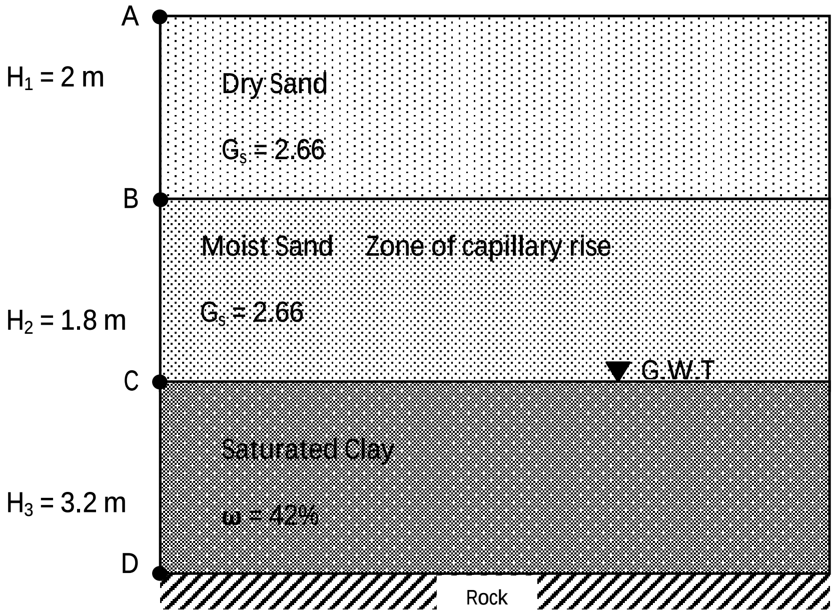
$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

The values of σ_v, u and σ'_v computed above are summarized in Table 1.

Table 6.1 Values of σ_v, u and σ'_v in Ex. 1

depth (m)	σ_v (kPa)	u (kPa)	σ_v' (kPa)
0	0	0	0
4	71.2	0	71.2
6	108.2	19.6	88.6
10	186.2	58.9	127.3
15	281.2	107.9	173.3

EXAMPLE2. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.



Dry sand $$\gamma_d = \frac{G_s}{1+e}\gamma_w = \frac{2.66}{1+0.55}9.81 = 16.84 \text{ kN/m}^3$$

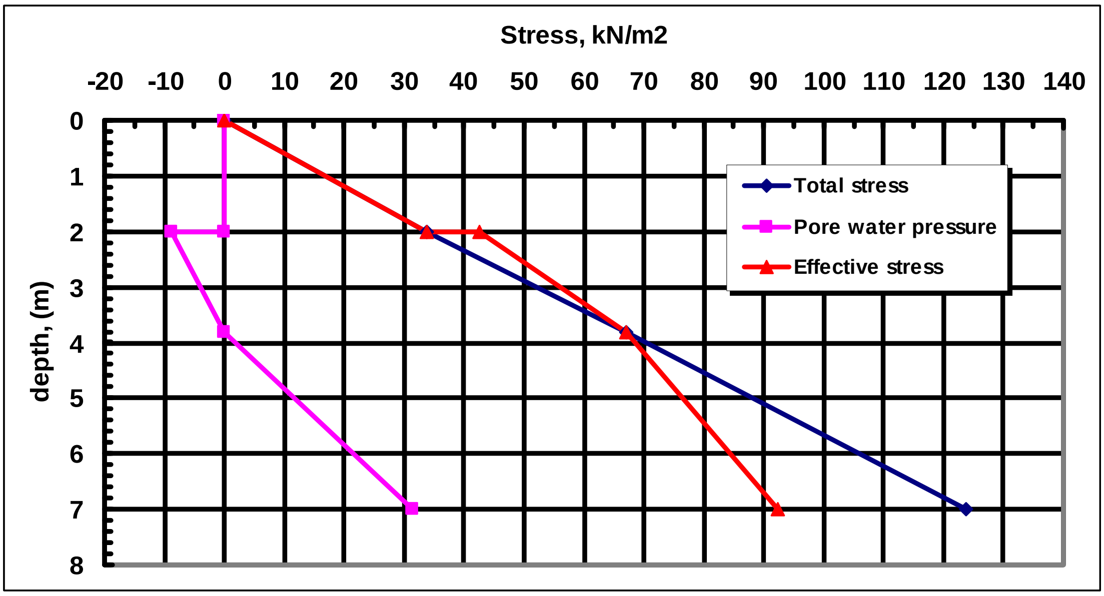
Moist sand $$\gamma_t = \frac{G_s + Se}{1+e}\gamma_w = \frac{2.66+0.5*0.55}{1+0.55}9.81 = 18.58 \text{ kN/m}^3$$

Saturated Clay $$e = \frac{G_s w}{S} = \frac{2.71*0.42}{1} = 1.138$$

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w = \frac{2.66+1.138}{1+1.138}9.81 = 17.66 \text{ kN/m}^3$$

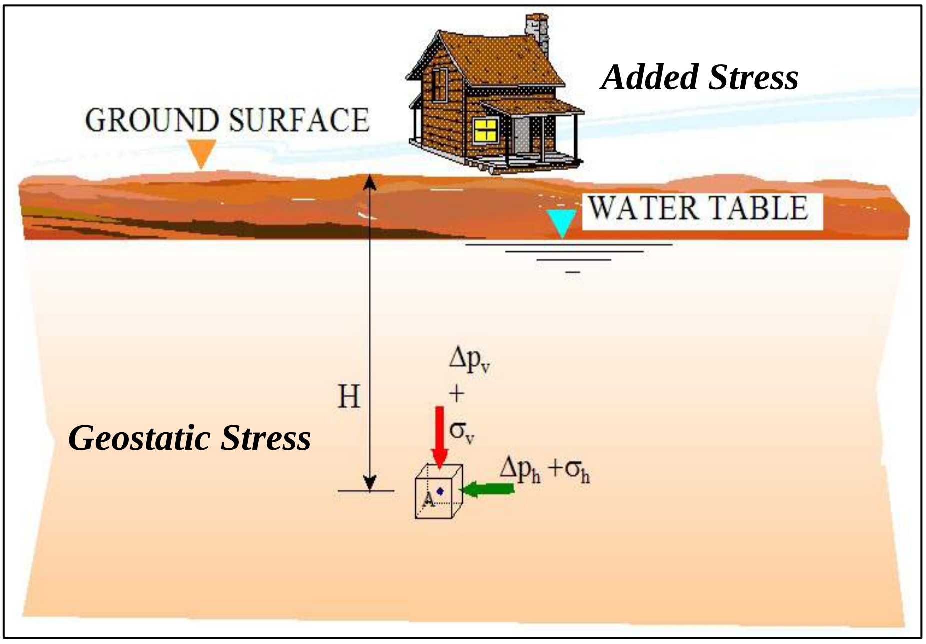
Point	σ_v kN/m^2	u kN/m^2	σ'_v kN/m^2
A	0	0	0
B	2*16.84=33.68	0	33.68
		- S γ_w H_2 = - 0.5*9.81*1.8 = - 8.83	33.68-(-8.83) = 42.51
C	2*16.84+1.8*18.58 = 67.117	0	67.117
D	2*16.84+1.8*18.58+3.2*17.66 =123.68	3.2*9.81=31.39	123.68-31.39 = 92.24

The plot is shown below in Fig.



Variation of σ_v, u and σ'_v with depth

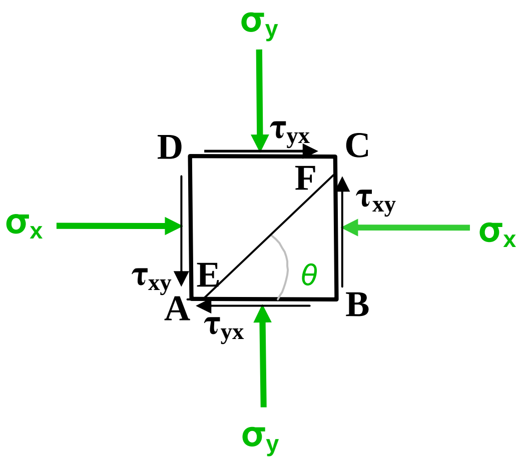
Stresses in a Soil Mass

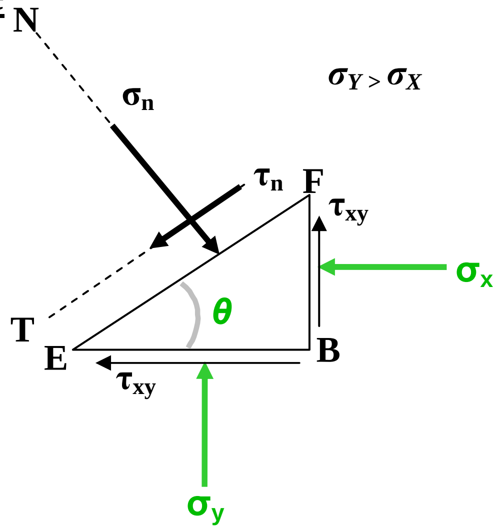


Topics

- Normal and Shear Stresses on a Plane
- Stress distribution in soils
- Stress Caused by a Point Load
- Vertical Stress Caused by a Line Load
- Vertical Stress Caused by a Strip Load
- Vertical Stress Due to Embankment Loading
- Vertical Stress below the Center of a uniformly Loaded Circular Area
- Vertical Stress at any Point below a uniformly Loaded Circular Area
- Vertical Stress Caused by a Rectangularly Loaded Area
- Influence Chart for Vertical Pressure (Newmark Chart)
- Approximate methods

● Normal and Shear Stresses on a Plane





From geometry for the free body diagram EBF

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Summing forces in N and T direction, we have

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Again

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If $\tau_n = 0$ then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

This eq. gives 2 values of θ that are 90^{o} apart, this means that there are 2 planes that are right angles to each other on which shear stress = 0, such

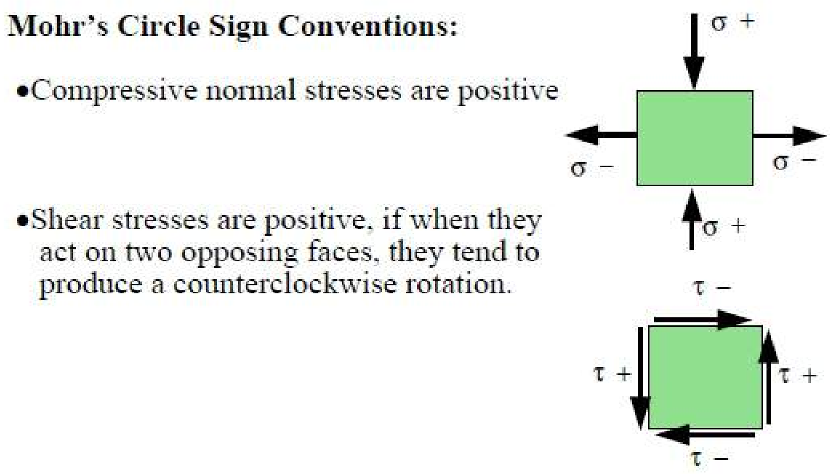
planes are called *principle planes* and the normal stress that act on the principle planes are to as *principle stresses*.
To find the principle stress substitute eq.3 into eq.1, we get

$$\sigma_n=\sigma_1=\frac{\sigma_y+\sigma_x}{2}+\sqrt{\left[\frac{\sigma_y-\sigma_x}{2}\right]^2+\tau^2_{xy}} \qquad \textit{major principle stress}$$

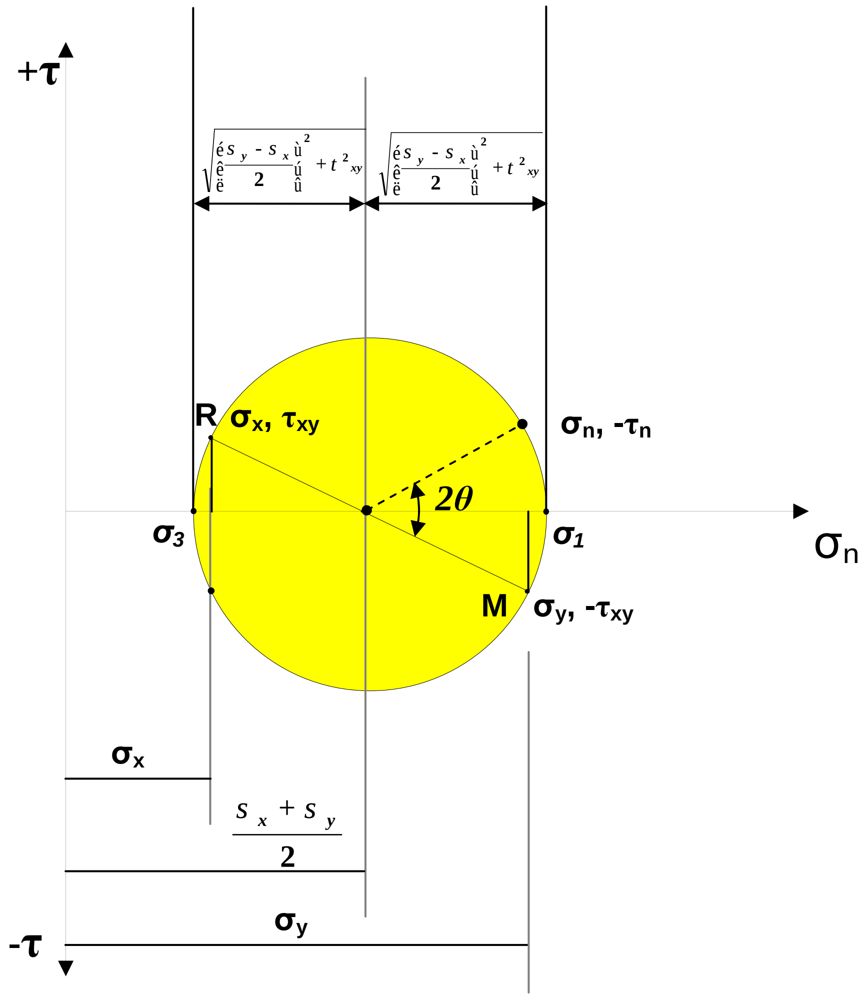
$$\sigma_n=\sigma_3=\frac{\sigma_y+\sigma_x}{2}-\sqrt{\left[\frac{\sigma_y-\sigma_x}{2}\right]^2+\tau^2_{xy}} \qquad \textit{minor principle stress}$$

These stresses on any plane can be found using *Mohr's circle*

- ***<u>Mohr's circle</u>***



Refer to the element shown in Fig.   above



♦ ***Pole Method***

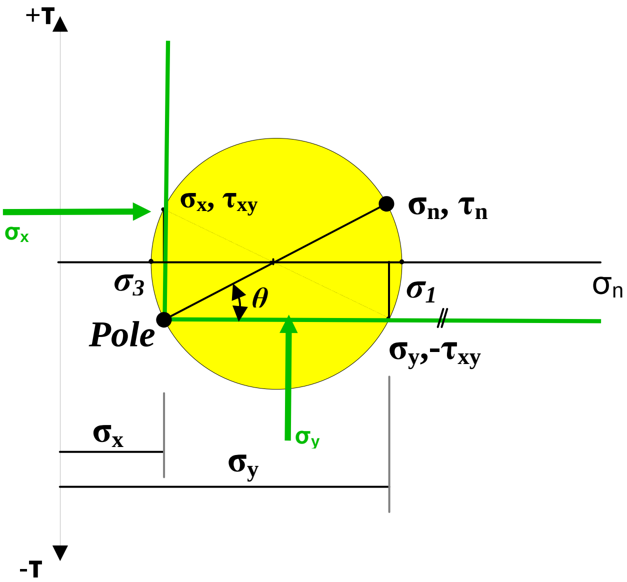
a) Draw the circle.

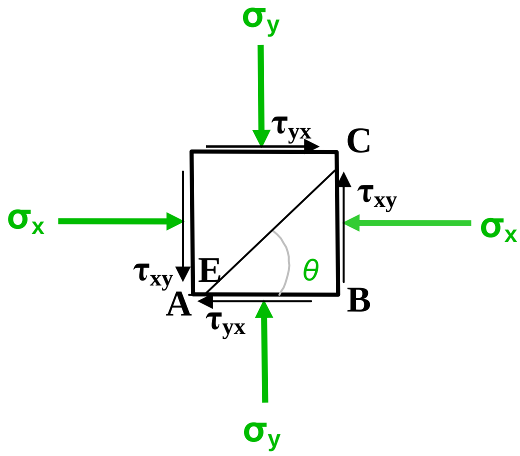
b) To locate the pole P:

1) Through the point representing the stresses on the first reference plane (x–plane), draw the orientation of the first reference plane (x–plane is vertical).
2) The point where this line intersects the Mohr's Circle is the pole P.

c) To find the stresses on a plane of any orientation:

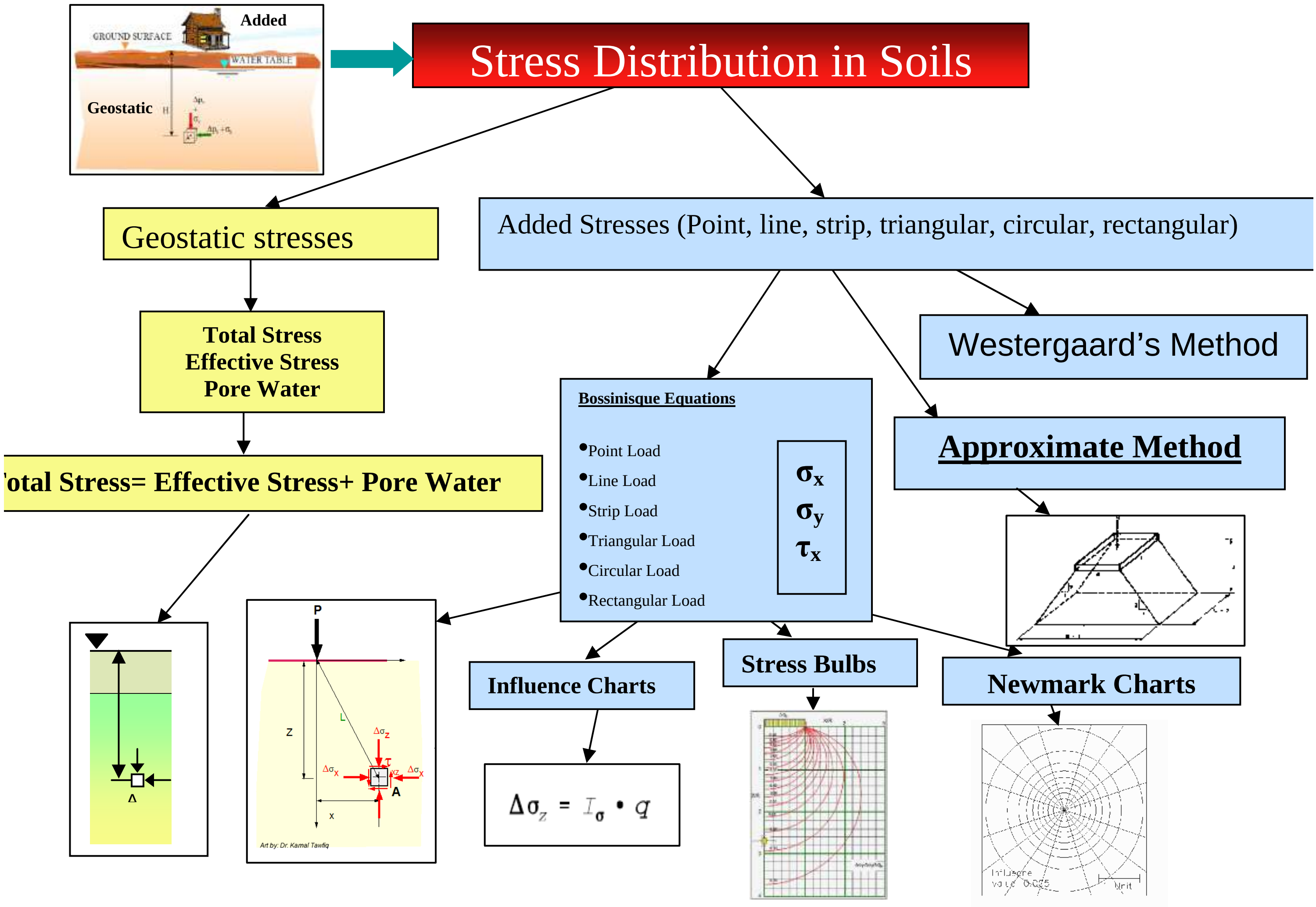
1) Draw a line through the pole P parallel to the plane;
2) The point where this line intersects the Mohr's circle gives the stresses (σ_n, τ_n) on the plane of interest.


+τ
σx, τxy
σn, τn
σx
σ3
θ
σ1
σn
Pole
σy,-τxy
σy
σx
σy
-τ


σy
τyx
C
τxy
σx
σx
τxy
E
θ
A
B
τyx
σy

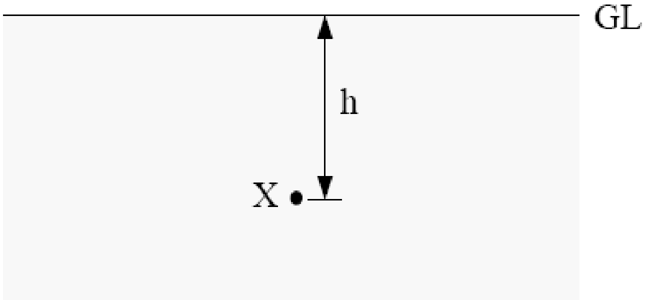
Stress Distribution in Soils

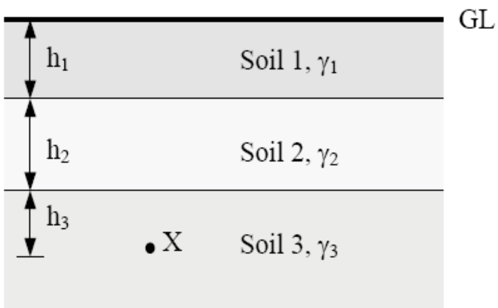
- *Foundations and structures placed on the surface of the earth will produce stresses in the soil, usually net stress*
- These net stresses will depends on the load magnitude and nature, depth below the foundation and other factors
- These stresses are necessary to estimate the settlement of the foundation



- **Geostatic stresses**

The vertical geostatic stress at point X will be computed as following





$\sigma_V = \gamma h$ homogenous soils $\sigma_V = \sum_1^n \gamma_i h_i$ stratified soils

$\sigma_v = \int_0^h \gamma \, dz$ density varies continuously with depth

The horizontal geostatic stress can be computed as following

$\sigma_h = K\sigma_v$ where K is the coefficient of lateral stress or lateral stress ratio

$$K = \frac{\sigma_h}{\sigma_v} \qquad 1 < K \le 1$$

- Geostatic stress are principle stresses (σ_1, σ_2 and σ_3 major, intermediate and minor principle stresses) and hence the horizontal and vertical planes through any point are principle planes.

$$K < 1 \qquad s_v = s_1 \qquad s_h = s_3$$

$$s_2 = s_3 = s_h$$

$$K = 1 \qquad s_v = s_h = s_1 = s_2 = s_3$$

Isotropic

$$K > 1 \qquad s_h = s_1 \qquad s_v = s_3$$

$$s_2 = s_1 = s_h$$

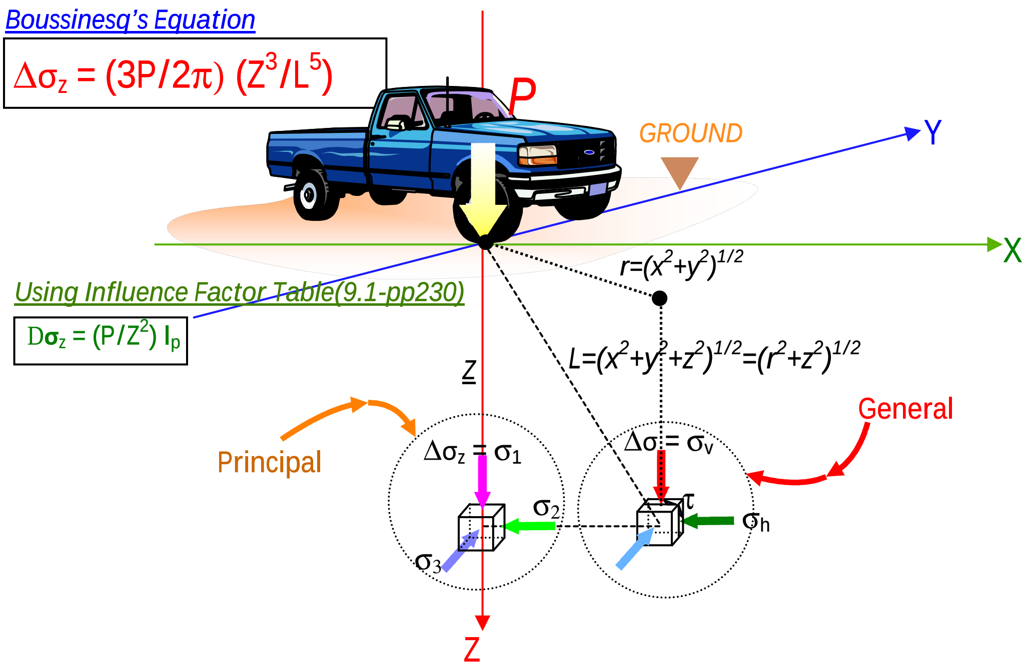
The largest shear stress will found on plane lying at 45^o to the horizontal

$$K < 1 \qquad t_{max} = \frac{s_v}{2}(1 - K)$$

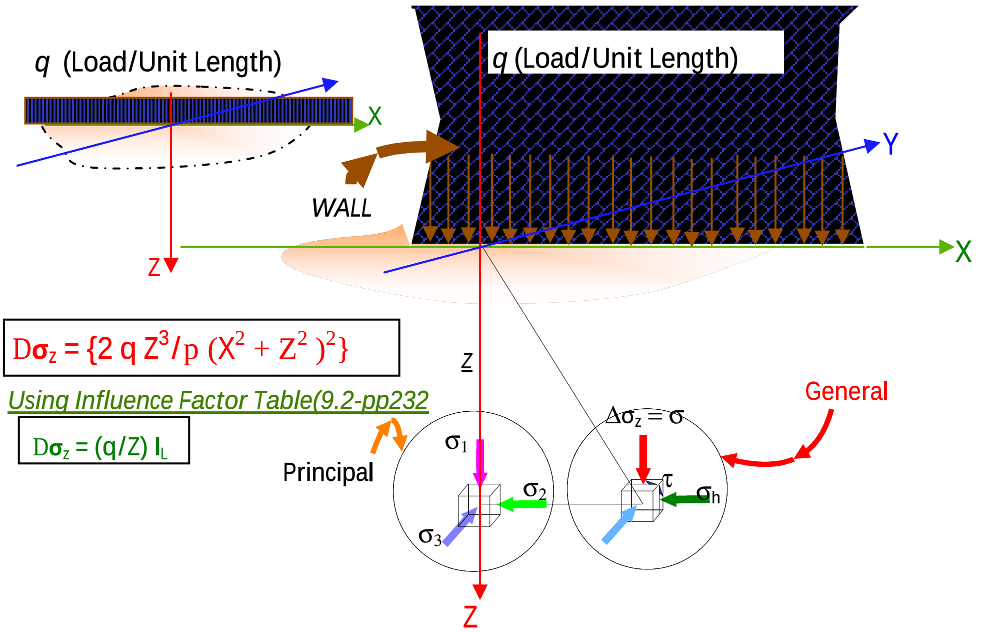
$$K = 1 \qquad t_{max} = 0$$

$$K > 1 \qquad t_{max} = \frac{s_v}{2}(K - 1)$$

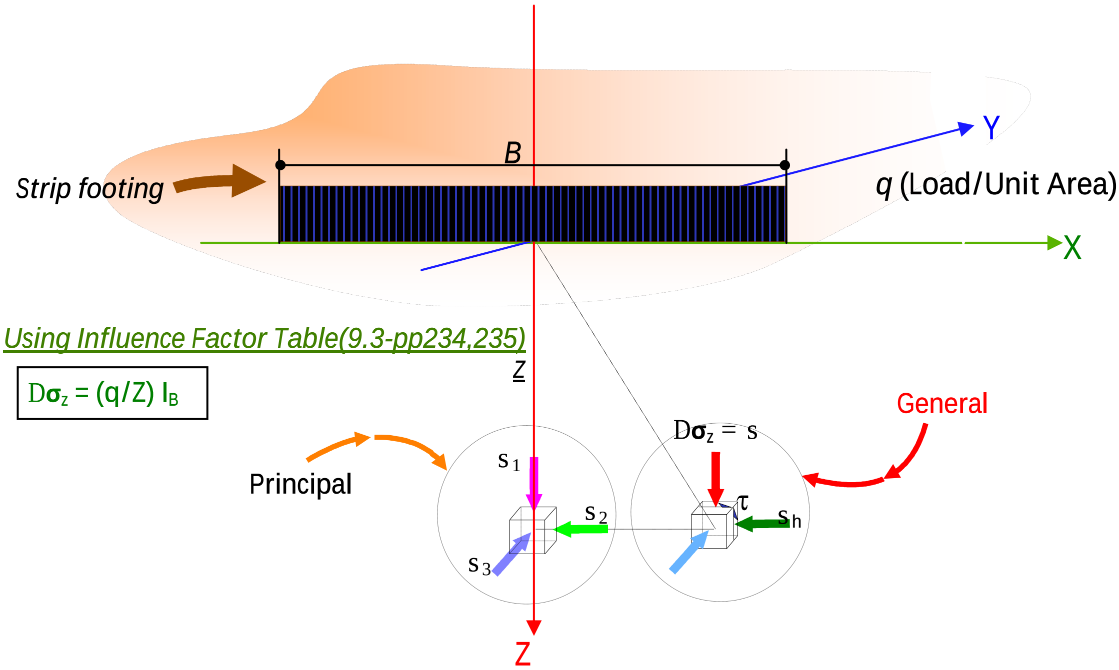
● Stress Caused by a Point Load



● Vertical Stress Caused by a Line Load



• Vertical Stress Caused by a Strip Load



Using Influence Factor Table(9.3-pp234,235)

$\boxed{D\sigma_z = (q/Z)\, I_B}$

• Vertical Stress Due to Embankment Loading

$\boxed{D\sigma_z = q/p[\{(B_1+B_2)/B_2\}(\alpha_1+\alpha_2)-(B_1/B_2)(\alpha_2)]}$

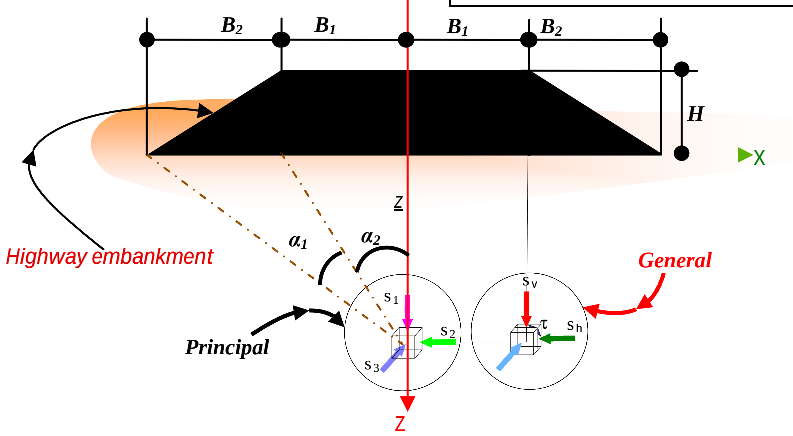
Using Influence Factor fig.(9.11-pp238)

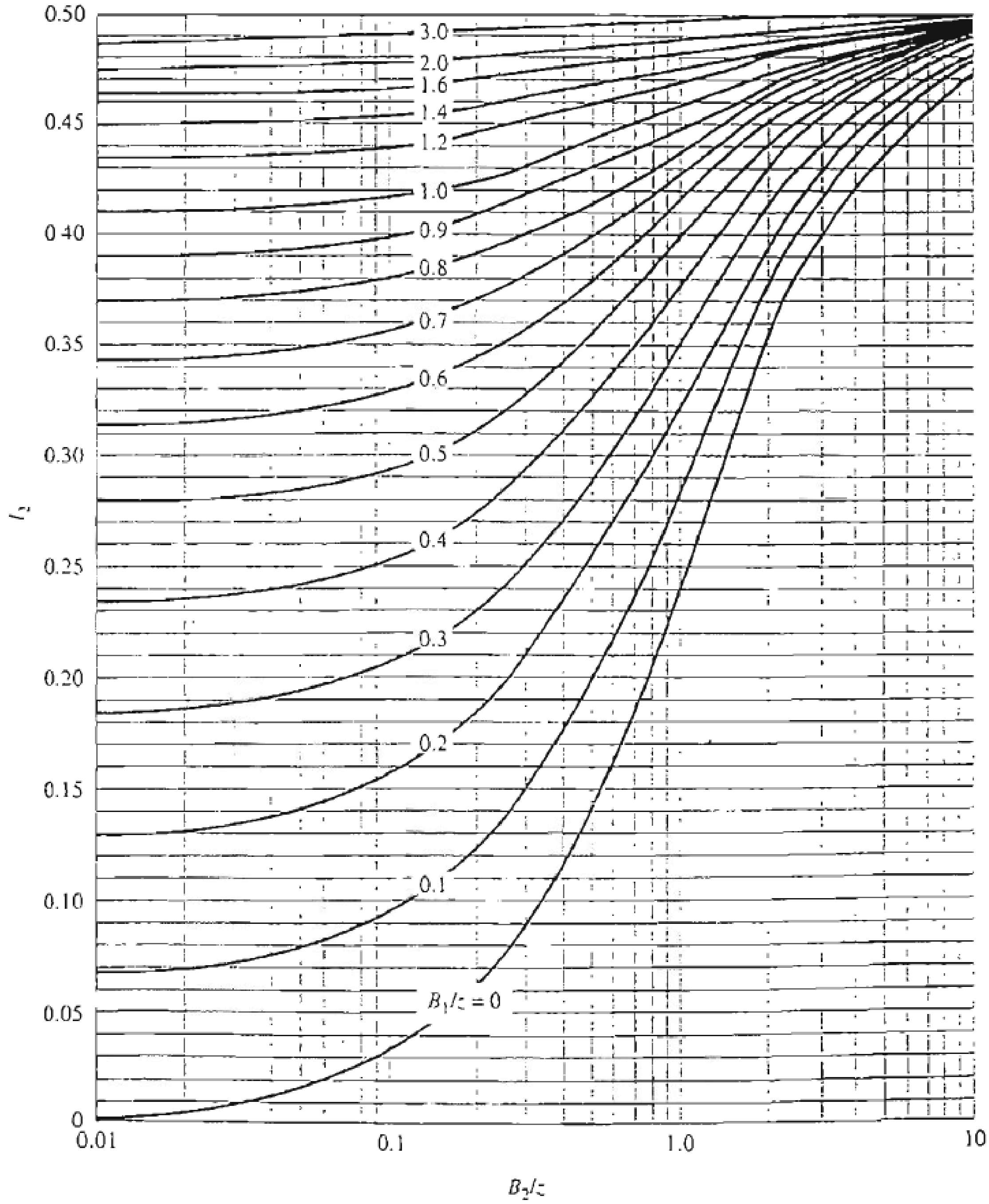
$\boxed{D\sigma_z = qI_2}$

$q=\gamma H$

$\alpha_1(\text{radians})=\tan^{-1}\{(B_1+B_2)/z\}-\tan^{-1}(B_1/z)$

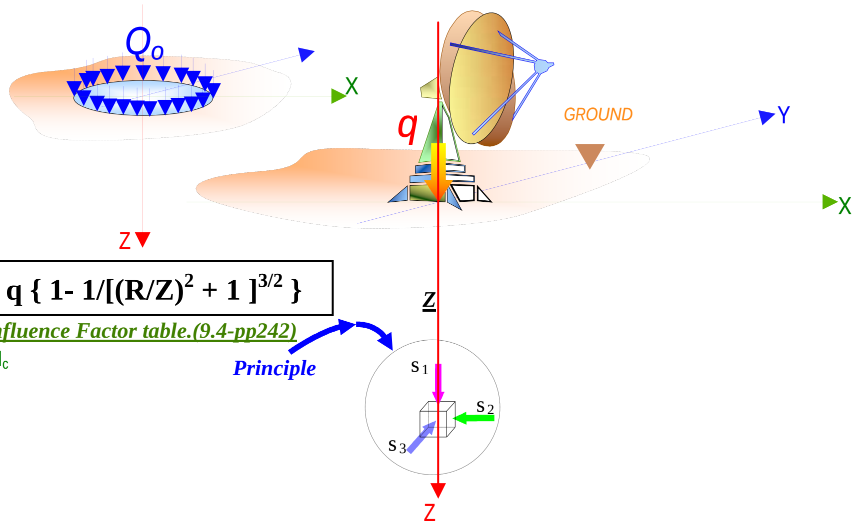
$\alpha_2=\tan^{-1}(B_1/z)$





Osterberg's Chart

• Vertical Stress below the Center of a uniformly Loaded Circular Area

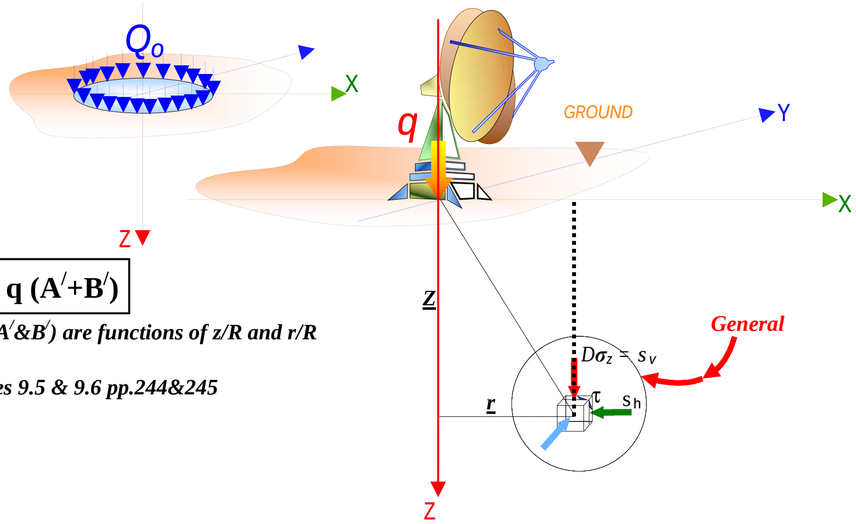


$$\boxed{\Delta\sigma_z = q\,\{\,1 - 1/[(R/Z)^2 + 1\,]^{3/2}\,\}}$$

Using Influence Factor table.(9.4-pp242)

$D\sigma_z = qI_c$

• Vertical Stress at any Point below a uniformly Loaded Circular Area

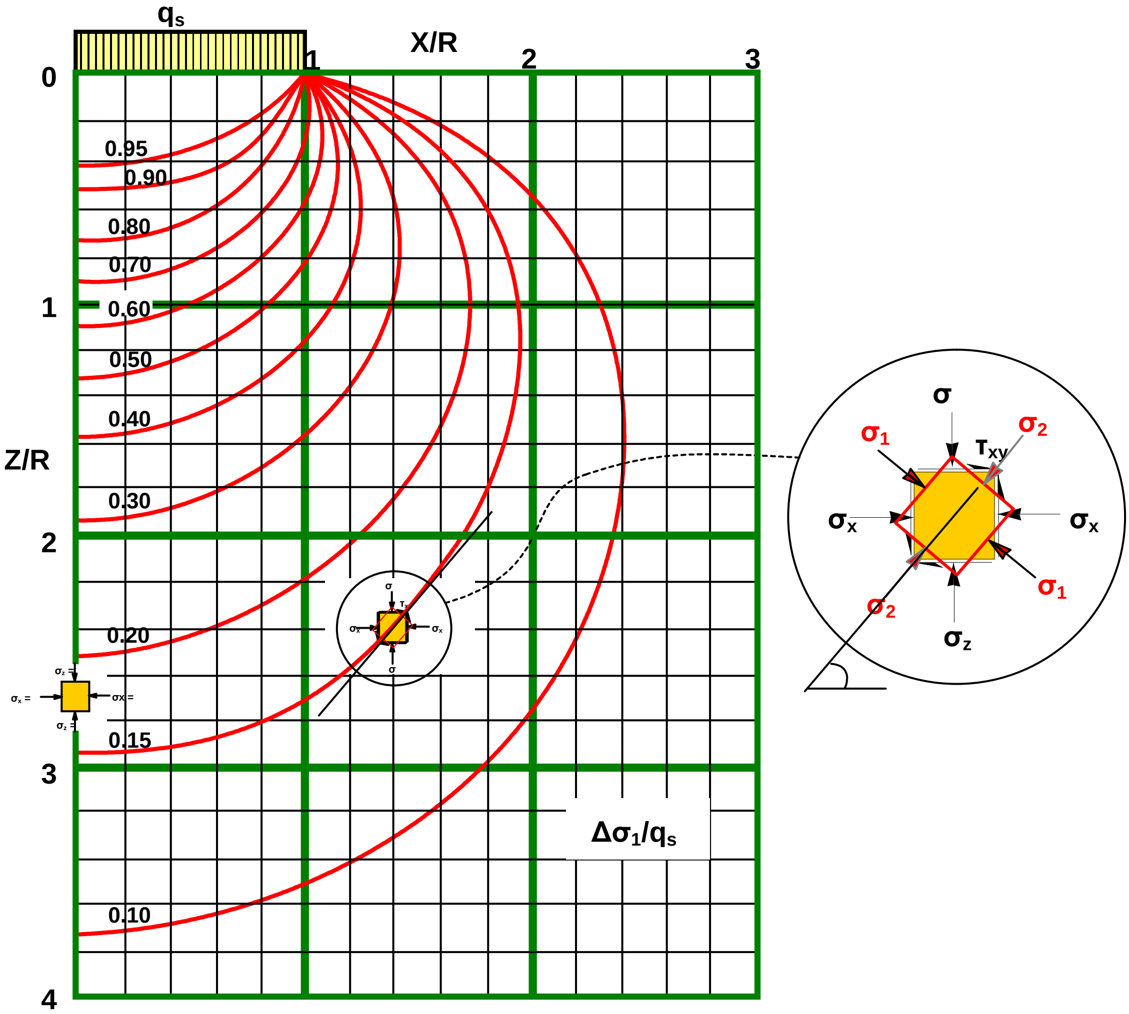


$$\boxed{\Delta\sigma_z = q\,(A' + B')}$$

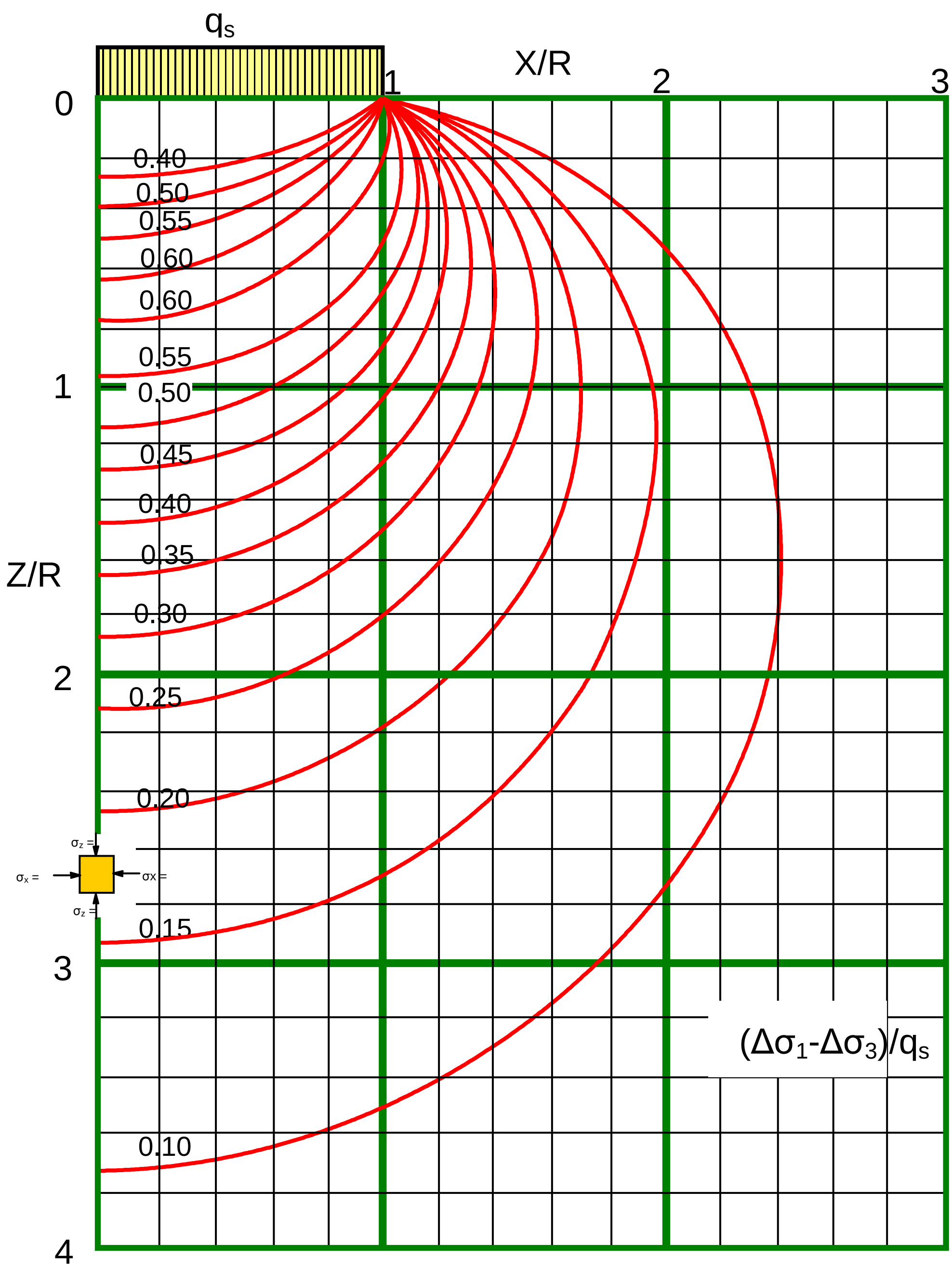
Where (A′&B′) are functions of z/R and r/R

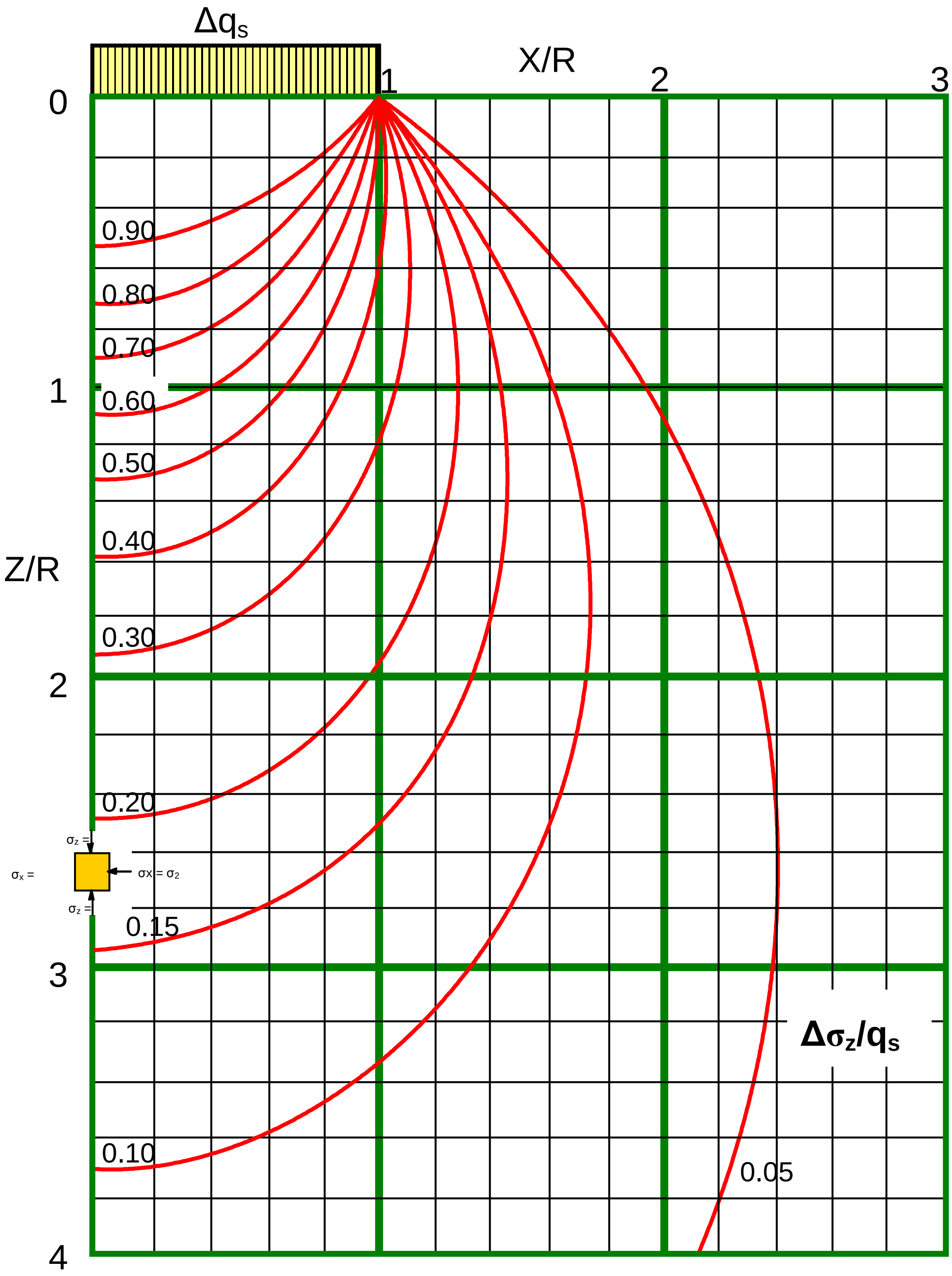
See tables 9.5 & 9.6 pp.244&245

Or we can use the stress bulb charts



Circular Load: (Major Principal Stress)/(Surface Stress)


qs
X/R
0
1
2
3
4
Z/R
0.40
0.50
0.55
0.60
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
σz =
σx =
σx =
σz =
(Δσ1-Δσ3)/qs

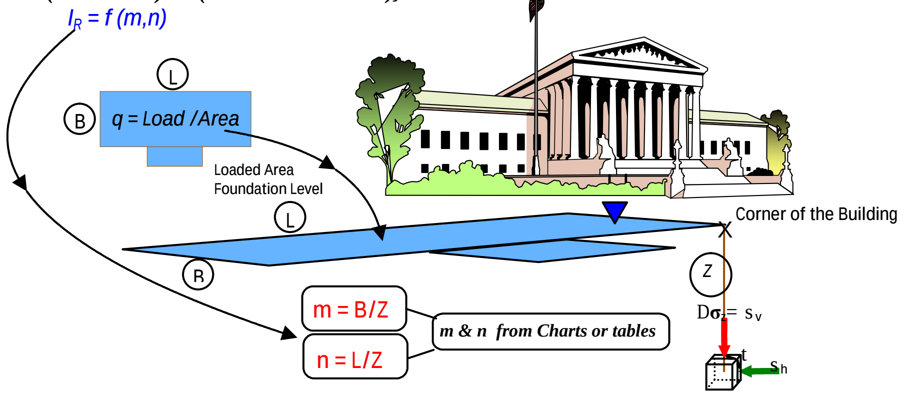


Circular Load: (Vertical Stress)/(Surface Stress)

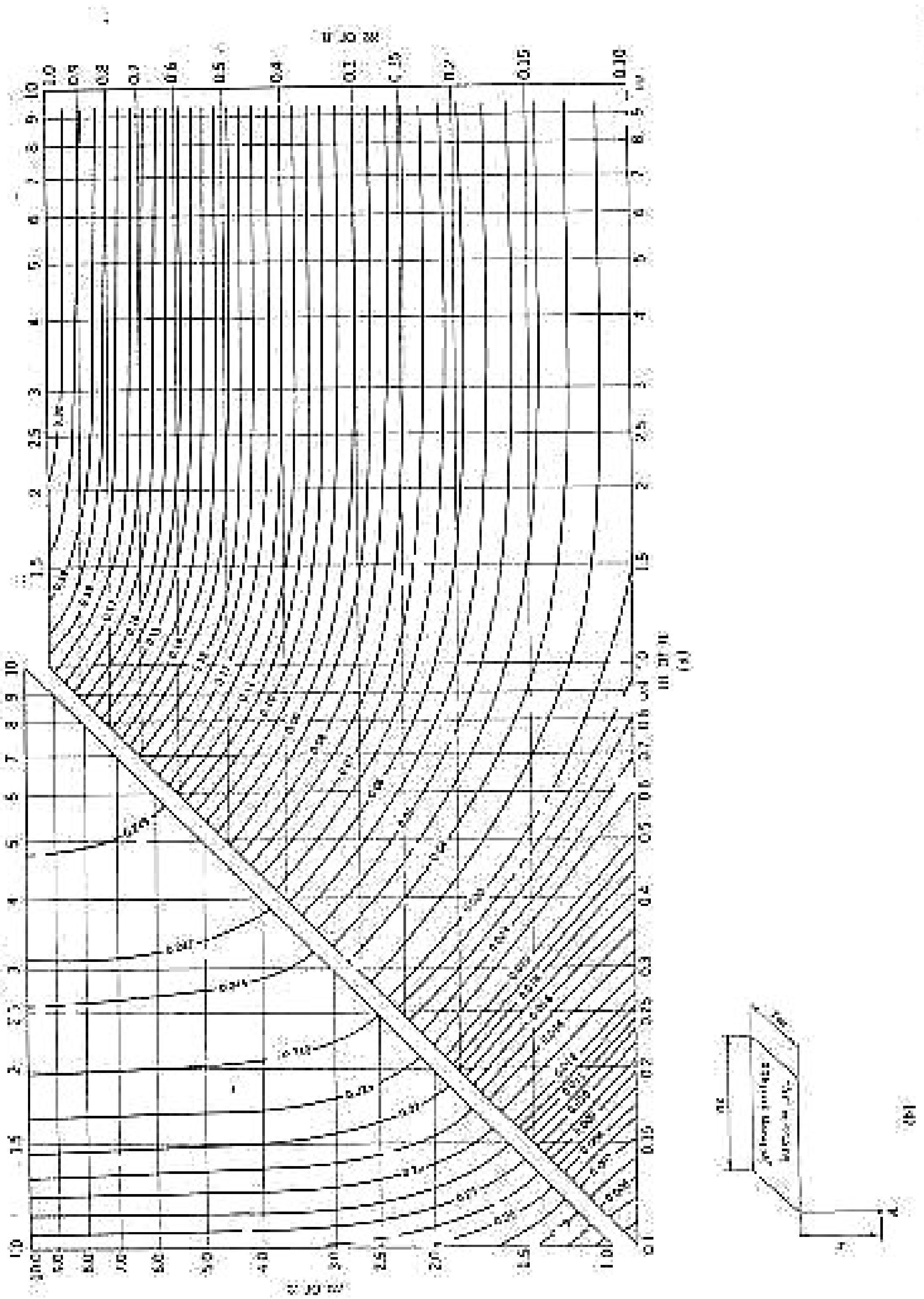
• Vertical Stress Caused by a Rectangularly Loaded Area

$$\Delta P = q\, I_R$$

$$I_R = 1/4\pi \{[(2.\, m.n\, (m^2 + n^2 + 1)^{1/2}) / (m^2 + n^2 + m^2.\, n^2 + 1)]\, [(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] + \tan^{-1} (2.m\, .n\, (m^2 + n^2 + 1)^{1/2} / (m^2 + n^2 - m^2 .\, n^2 + 1)\}$$



See tables 9.7 pp. 246,247 or one can use the charts below



- ***Calculation of Stress below an interior point of the loaded area***

$$\Delta\sigma_z = q[I(OXAY) + I(OYBZ) + I(OZCT) + I(OTDX)$$

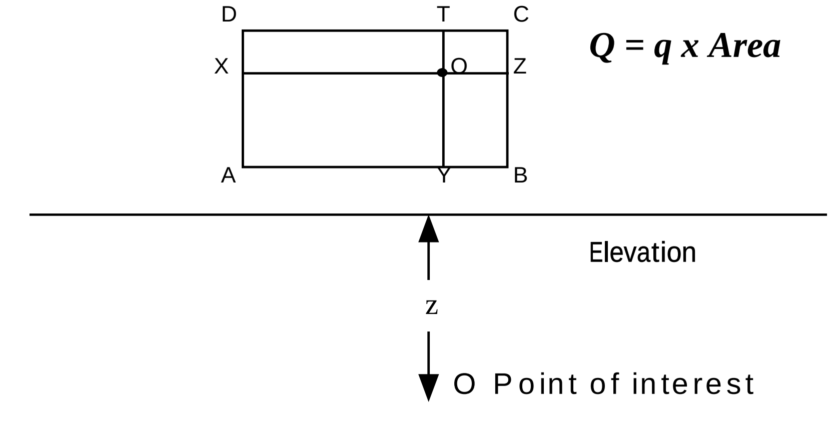
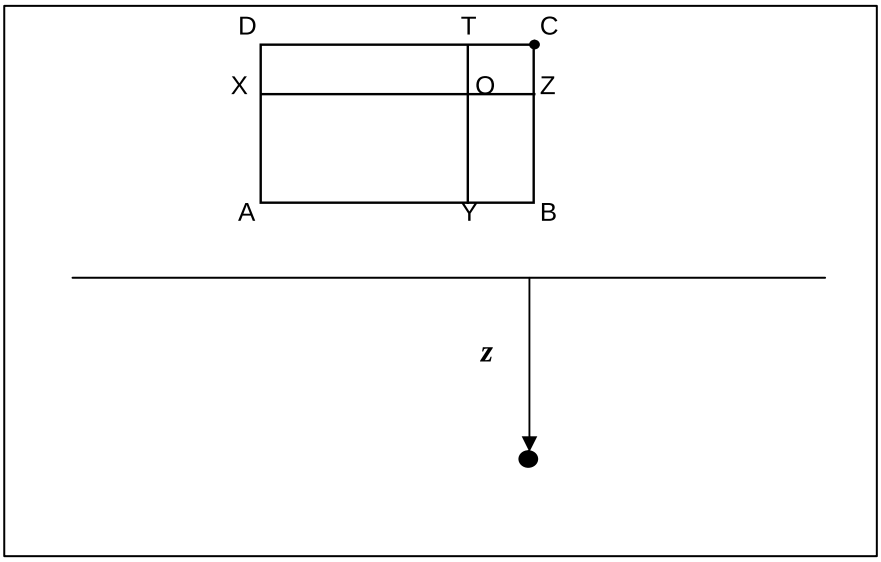


Fig. 10 stress increase at a point below a loaded rectangular region

- ***Calculation of Stress below a point outside of the loaded area***

$$\Delta\sigma_z = q[I(ABCD) + I(TYBZ) + I(XZCD) - I(OZCT)$$



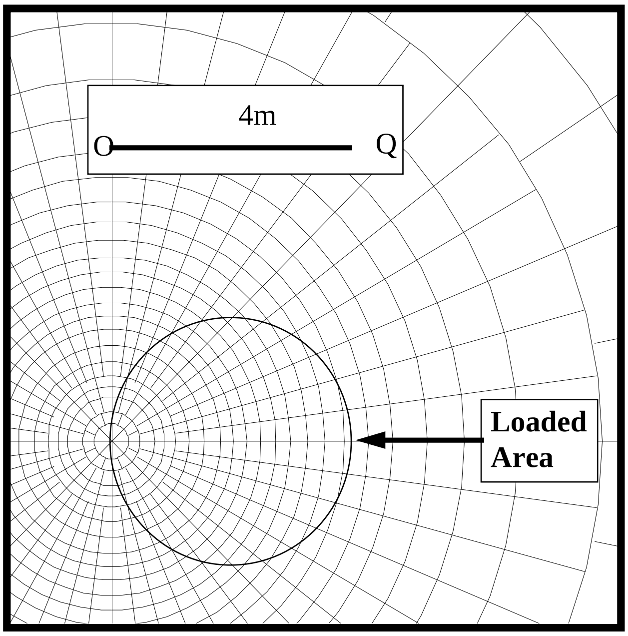
● Influence Chart for Vertical Pressure (Newmark Chart)

*Stresses due to foundation loads of **arbitrary shape** applied at the ground surface*

Newmark's chart provides a graphical method for calculating the stress increase due to a uniformly loaded region, of arbitrary shape resting on a deep homogeneous isotropic elastic region.

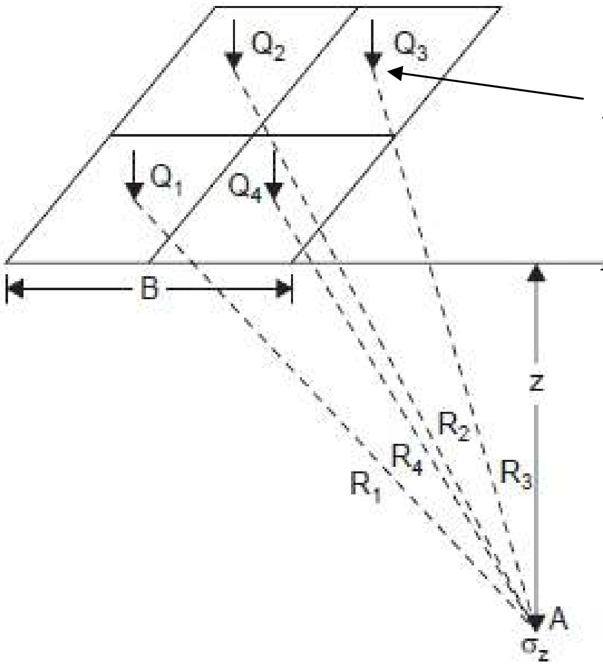
Newmark's chart is given in the data sheets and is reproduced in part in Fig 15. The procedure for its use is outlined below

1. The scale for this procedure is determined by the depth z at which the stress is to be evaluated, thus z is equal to the distance OQ shown on the chart.
2. Draw the loaded area to scale so that the point of interest (more correctly its vertical projection on the surface) is at the origin of the chart, the orientation of the drawing does not matter
3. Count the number of squares (N) within the loaded area, if more than half the square is in count the square otherwise neglect it.
4. The vertical stress increase $\Delta\sigma_z = N \times$ [scale factor(0.001)] $\times$ [surface stress (p)]



● Approximate Methods

♦ ***Equivalent Point Load Method***



Each Q is the resultant of the uniform load on the unit area ac at the center of it and treated as point load

Fig. 10.20 Equivalent point load method

In dividing the loaded area into smaller units, we have to remember to do it such that $z/B \geq 3$; that is to say, in relation to the specified depth, the size of any unit area should not be greater than one-third of the depth.

$$\Delta s_z = \sum \frac{Q_i}{z^2} I_{pi}$$

- ***2:1 Method***

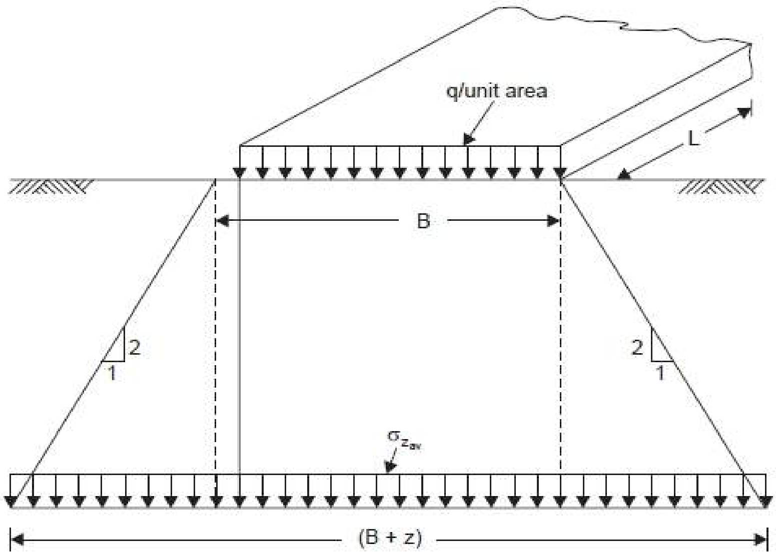
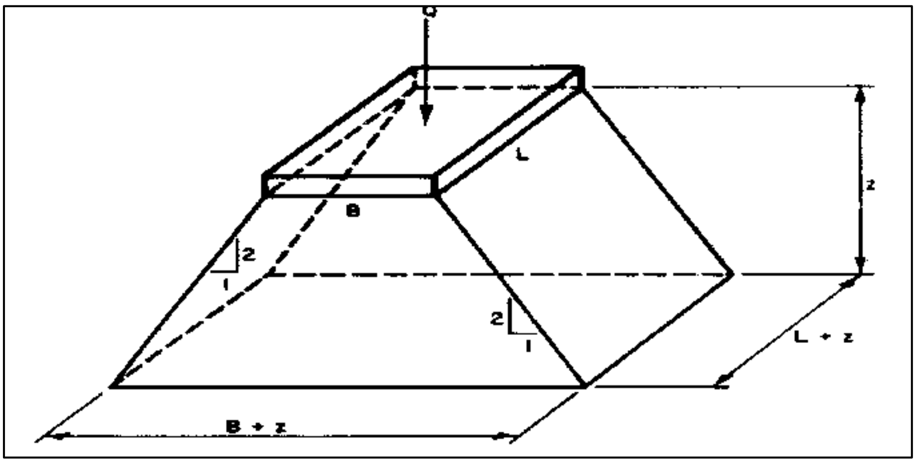


Fig. 10.21 Two is to one method



$$\Delta\sigma_z = \frac{Q}{(B+Z)(L+Z)}$$ Rectangular area

$$\Delta\sigma_z = \frac{Q}{(B+Z)^2}$$ Square area

$$\Delta\sigma_z = \frac{Q}{\frac{\pi}{4}(D+Z)^2}$$ Circular area

Examples (1-3)

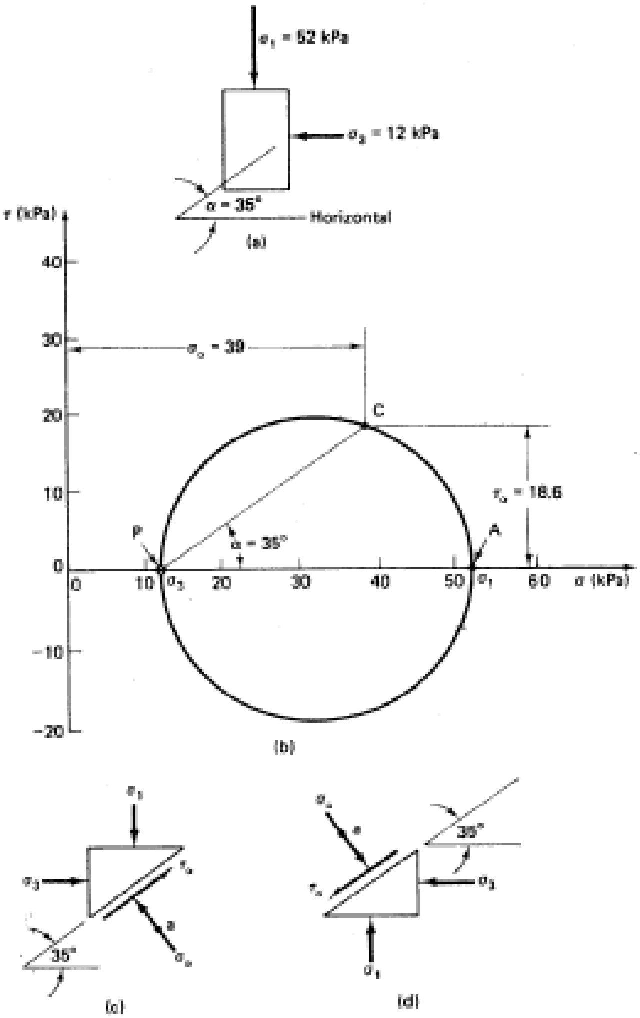
Given:

Stresses on an element as shown in Fig.

Plot the Mohr circle to some convenient scale

$$\text{center of circle} = \frac{\sigma_1 + \sigma_3}{2} = \frac{52 + 12}{2} = 32 \text{ kPa}$$

$$\text{radius of circle} = \frac{\sigma_1 - \sigma_3}{2} = \frac{52 - 12}{2} = 20 \text{ kPa}$$

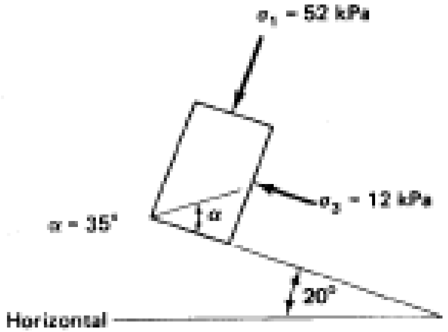


Given:

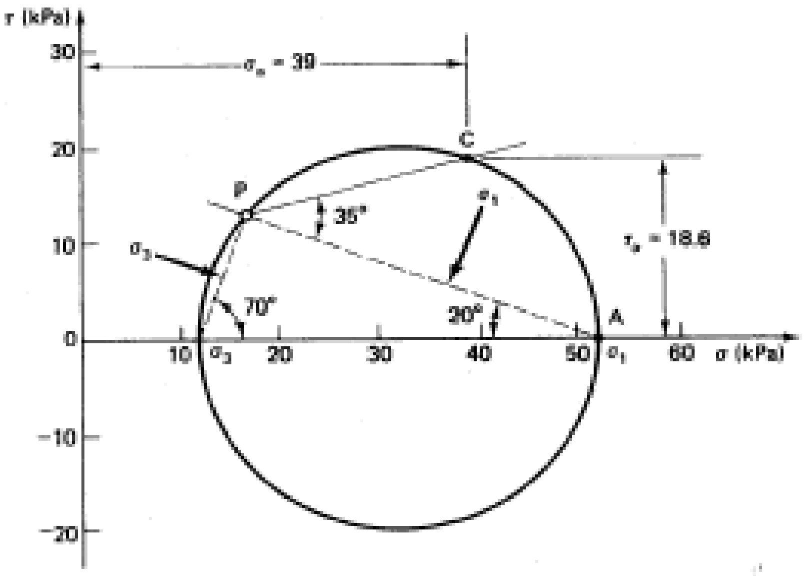
The same element and stresses as in Fig. Ex. 1 , except that the element is rotated 20° from the horizontal, as shown in Fig.

Required:

As in Example 10.1, find the normal stress σ_α and the shear stress τ_α on the plane inclined at $\alpha = 35°$ from the base of the element.



(a)



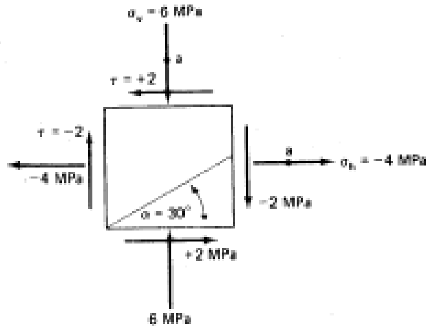
(b)

Given:

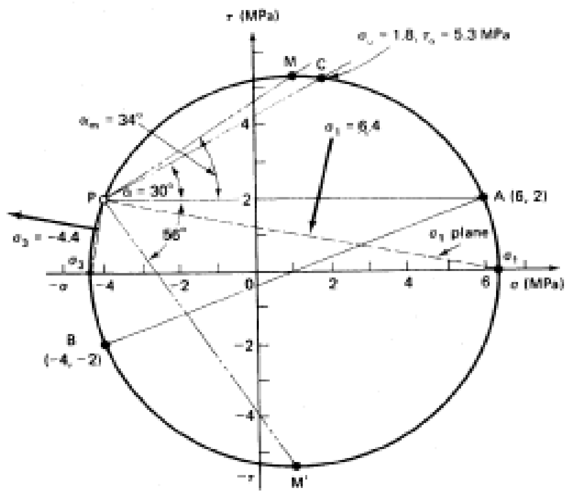
The stress shown on the element in Fig.

Required:

a. Evaluate σ_a and τ_a when $\alpha = 30°$.
b. Evaluate σ_1 and σ_3 when $\alpha = 30°$.



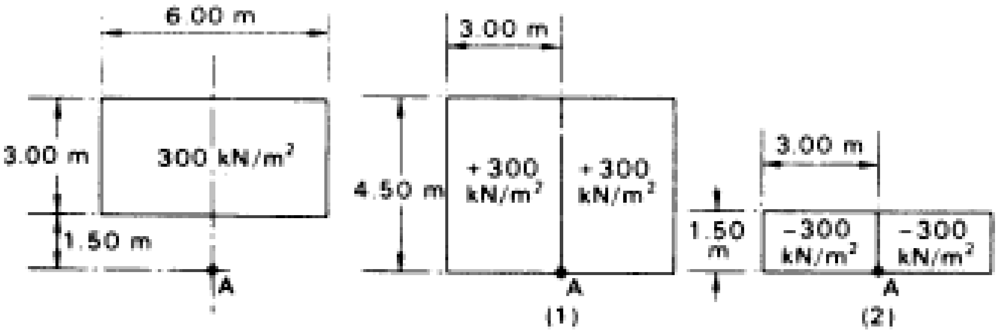
(a)



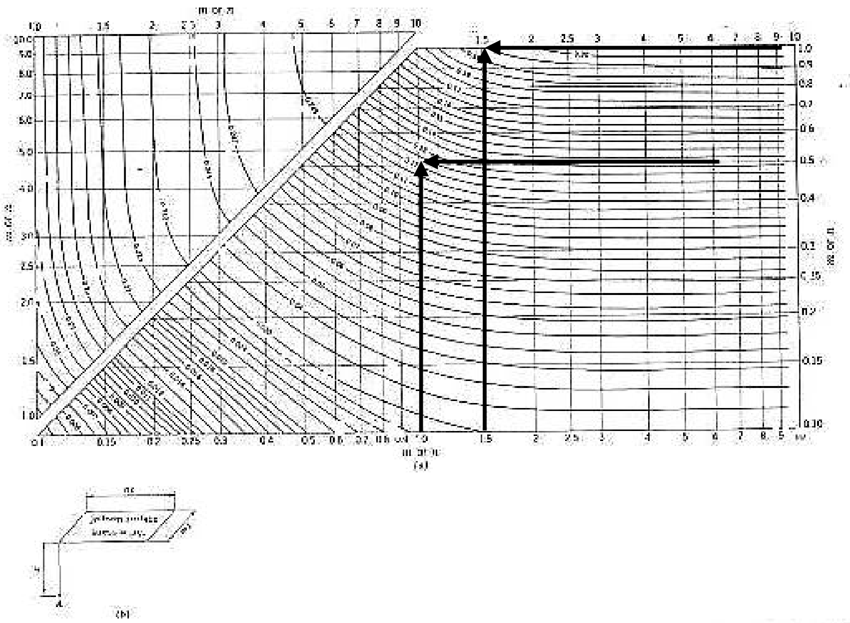
(b)

Example 4

A rectangular foundation 6 x 3m carries a uniform pressure of 300 kN/m^2 near the surface of a soil mass. Determine the vertical stress at a depth of 3m below a point (A) on the centre line 1.5m outside a long edge of the foundation using influence factors

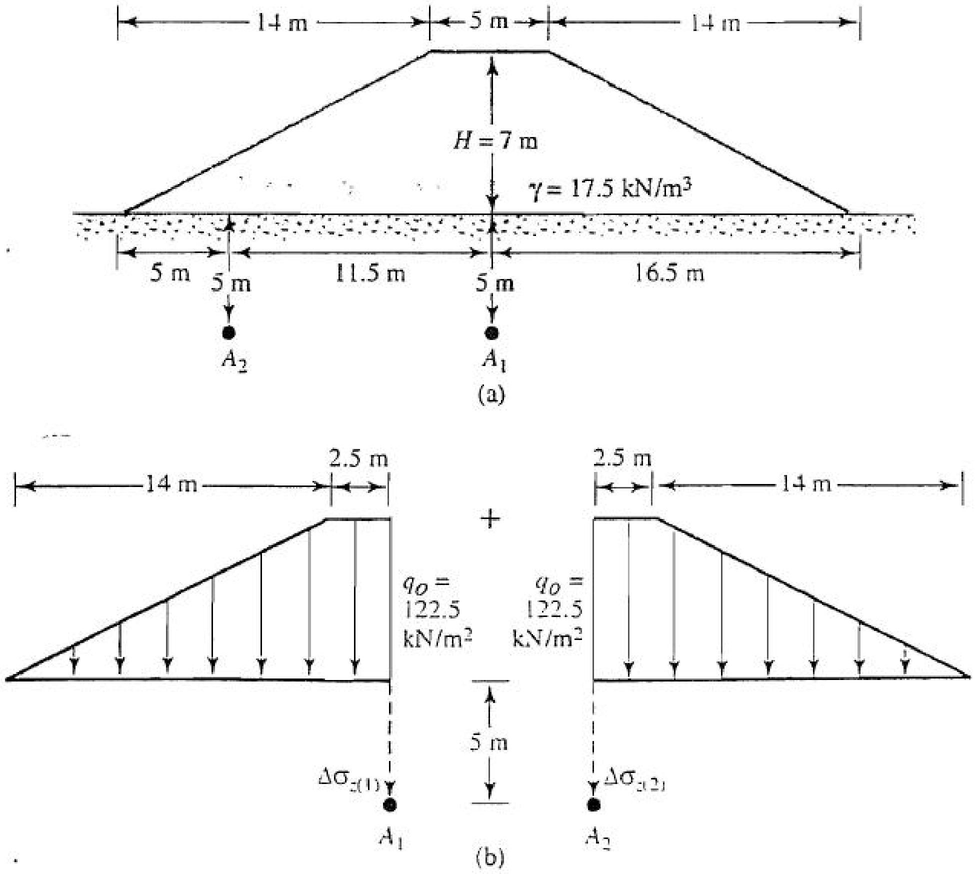


(1)	(2)
m = 1	***m=1***
n =1.5	***n=0.5***
I = 0.193	***I=0.120***



Example 5

Determine the stress increase under the embankment at points A_1 and A_2



Solution

$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

Stress Increase at A_1

The left side of Figure a indicates that $B_1 = 2.5$ m and $B_2 = 14$ m. So

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

According to Figure b in this case, $I_2 = 0.445$. Because the two sides in Figure b are symmetrical, the value of I_2 for the right side will also be 0.445. So

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{2(\text{Left})} + I_{2(\text{Right})}]$$

$$= 122.5[0.445 + 0.445] = \mathbf{109.03\ kN/m^2}$$

Stress Increase at A_2

Refer to Figure c. For the left side, $B_2 = 5$ m and $B_1 = 0$. So

$$\frac{B_2}{z} = \frac{5}{5} = 1; \ \frac{B_1}{z} = \frac{0}{5} = 0$$

According to **osterberg chart** for these values of B_2/z and B_1/z, $I_2 = 0.25$. So

$$\Delta\sigma_{z(1)} = 43.75(0.25) = 10.94 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I_2 = 0.495$. So

$$\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$$

For the right side

$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \frac{B_1}{z} = \frac{0}{5} = 0$$

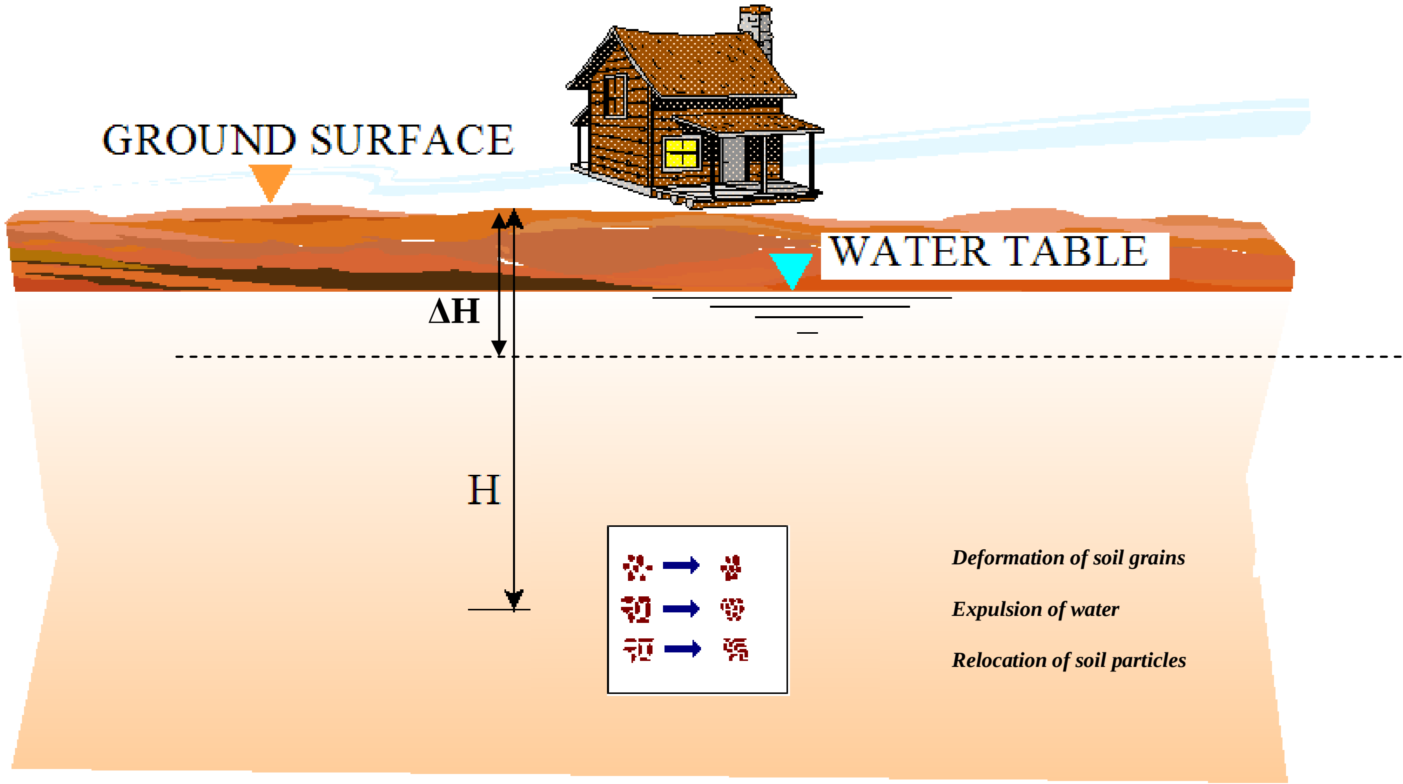
and $I_2 = 0.335$. So

$$\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.94 + 60.64 - 26.38 = \mathbf{45.2 \text{ kN/m}^2}$$

Compressibilty of Soils



Topics

- **Introduction**
- **Immediate Settlement**
- **Consolidation Settlement (Primary Consolidation)**
- **Secondary Compression (Secondary consolidation) Settlement**
- **Time Rate of Consolidation**
- **Methods for Accelerating Consolidation Settlement**

● Introduction

- ♦ Soil deformation may occur by change in:
  a) Stress
  b) Water content
  c) Soil mass
  d) Temperature
- ♦ The compression is caused by
  a) Deformation of soil particles
  b) Relocation of soil particles
  c) Expulsion of water or air from the voids
- ♦ Types of settlement:
  a) Immediate (Elastic) Settlement $\boldsymbol{\delta_e}$
  b) Consolidation Settlement (primary consolidation) $\boldsymbol{\delta c}$
  c) Secondary Compression (Consolidation) Settlement $\boldsymbol{\delta_s}$

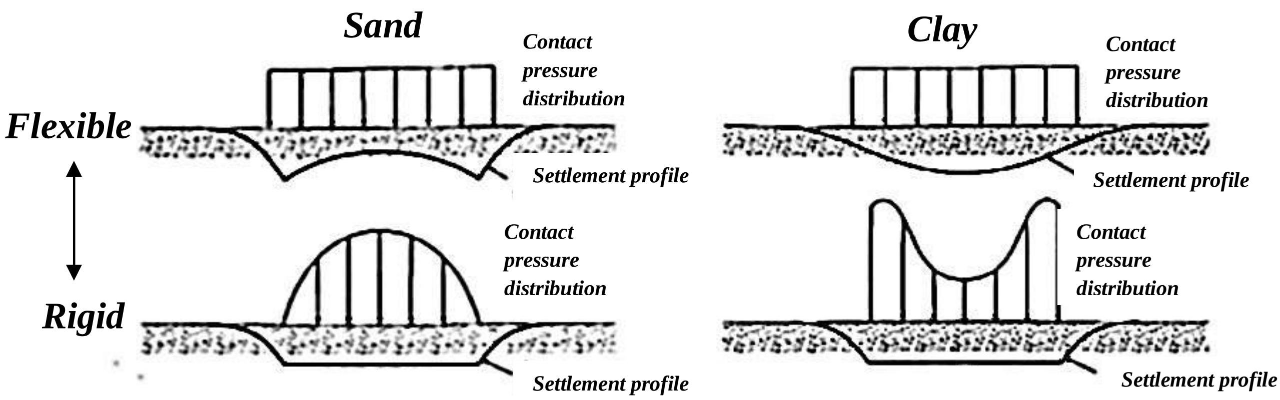
  Thus, the total settlement will be $\delta_T = \delta_e + \delta_c + \delta_s$

● Immediate (Elastic) Settlement

♦ Due elastic deformation of soil grains without any change in moisture content

♦ It is usually small and occurs directly after the application of a load.

♦ The magnitude of the contact settlement will depend on the flexibility of the foundation and the type of material on which it is resting, these distributions are true if $\boldsymbol{E}$ is constant with depth.



♦ all the previous relationship discussed in previous chapter were based on the following assumptions:-

a) The load is applied at the ground surface
b) The loaded area is flexible.

c) The soil medium is homogenous, elastic, isotropic, and extends to a great depth.

- ***Relations for Immediate Settlement Calculation***

$$\delta_e = \Delta\sigma B \frac{1-\mu^2{}_s}{E_s} I_p$$

Schleicher (1926)

$$I_p = \frac{1}{\pi}\left[m_1 \ln\left(\frac{1+\sqrt{m_1^2+1}}{m_1}\right) + \ln\left(m_1 + \sqrt{m_1^2+1}\right)\right]$$

Table 10.1 Influence Factors for Foundations [Eq. (10.2)]

		I_p		
		Flexible		
Shape	m_1	Center	Corner	Rigid
Circle	–	1.00	0.64	0.79
Rectangle	1	1.12	0.56	0.88
	1.5	1.36	0.68	1.07
	2	1.53	0.77	1.21
	3	1.78	0.89	1.42
	5	2.10	1.05	1.70
	10	2.54	1.27	2.10
	20	2.99	1.49	2.46
	50	3.57	1.8	3.0
	100	4.01	2.0	3.43

Table 10.2 Representative Values of the Modulus of Elasticity of Soil

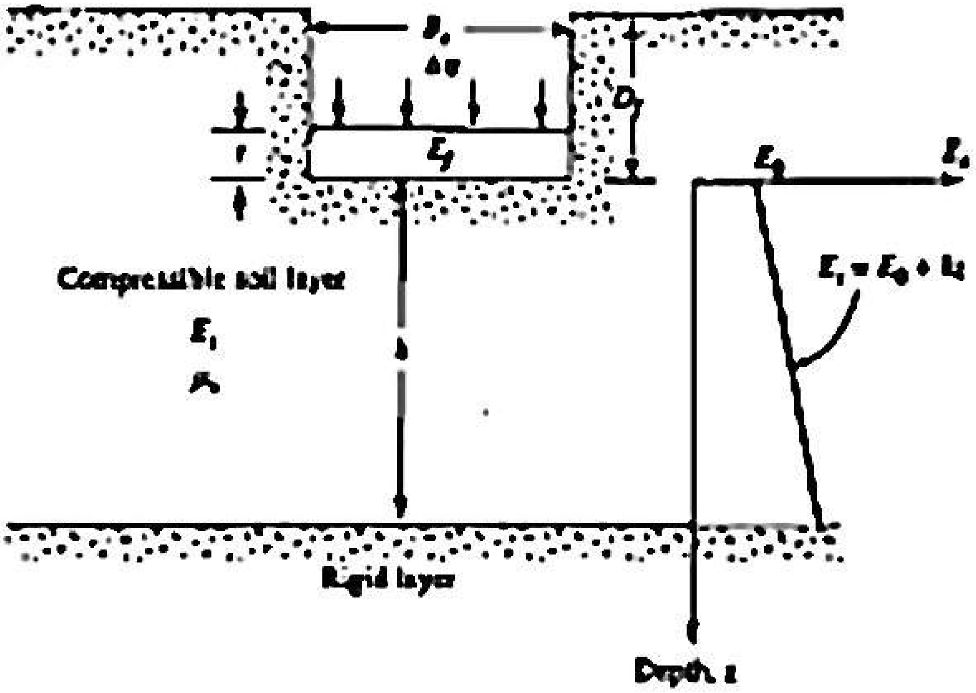
	E_s	
Soil type	kN/m²	lb/in.²
Soft clay	1,800–3,500	250–500
Hard clay	6,000–14,000	850–2,000
Loose sand	10,000–28,000	1,500–4,000
Dense sand	35,000–70,000	5,000–10,000

Table 10.3 Representative Values of Poisson's Ratio

Type of soil	Poisson's ratio, μ_s
Loose sand	0.2–0.4
Medium sand	0.25–0.4
Dense sand	0.3–0.45
Silty sand	0.2–0.4
Soft clay	0.15–0.25
Medium clay	0.2–0.5

- ***Improved Relationship for Immediate Settlement***

  Mayne and Polous (1999) present an improved relationship for calculating δ_e taking into account –
  - rigidity of the foundation
  - depth of embedment of the foundation
  - the increase in modulus of elasticity ***E*** with depth
  - the location of rigid layers at limited depth

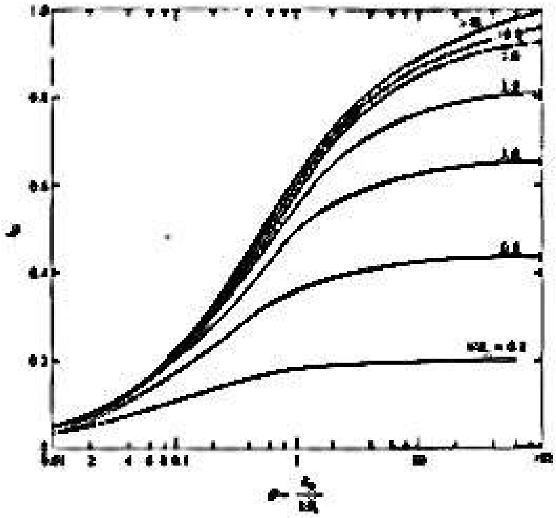


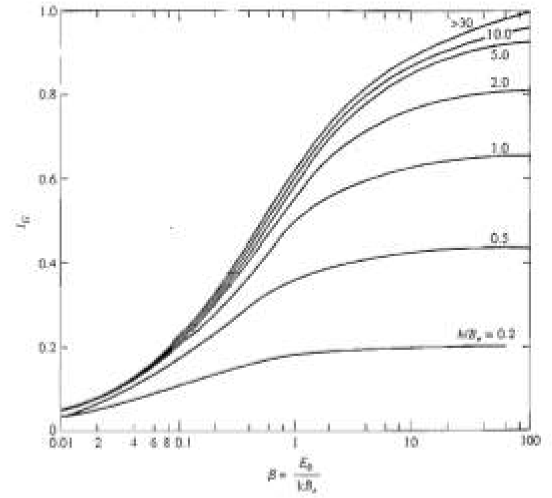
Improved relationship for immediate settlement

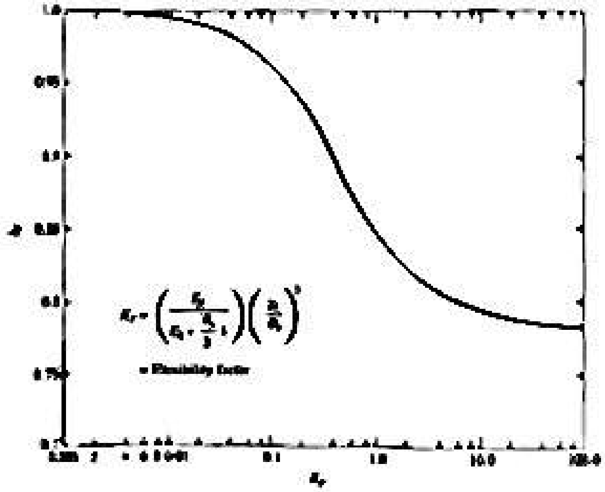
$$\delta_e = \Delta\sigma B_e \frac{1-\mu^2{}_s}{E_o} I_G I_F I_E$$

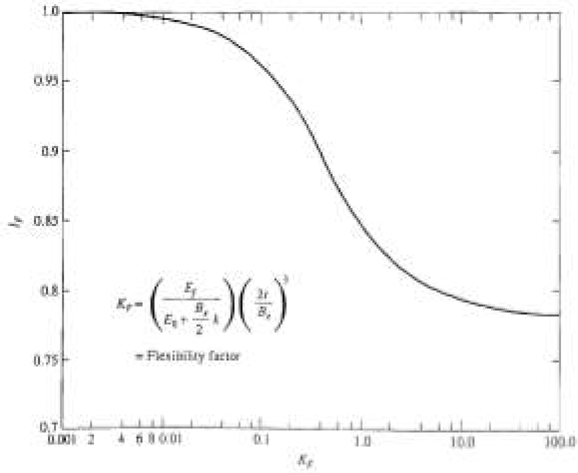
$B_e = \sqrt{\dfrac{4BL}{\pi}}$ for rectangular footing

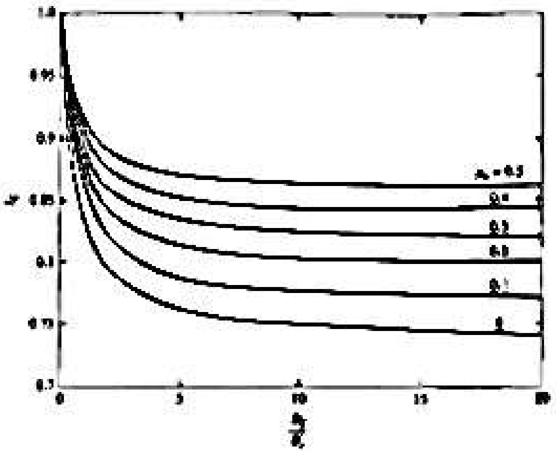
$B_e = Diameter$ for circular footing

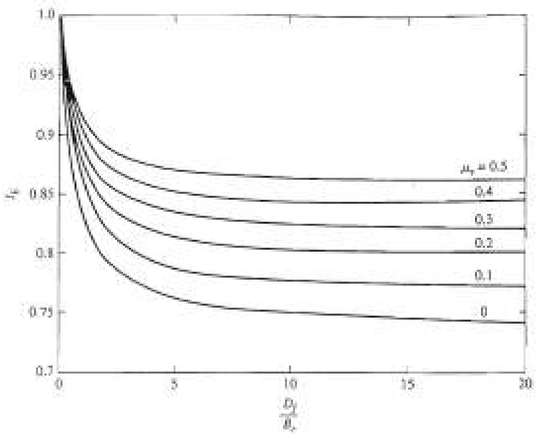



>30
10.0
5.0
2.0
1.0
0.5
h/B_e = 0.2
I_G
β = E_0 / kB_e




I_F
K_F = (E_f / (E_0 + (B_e/2) k)) (2t / B_e)^3
= Flexibility factor
K_F

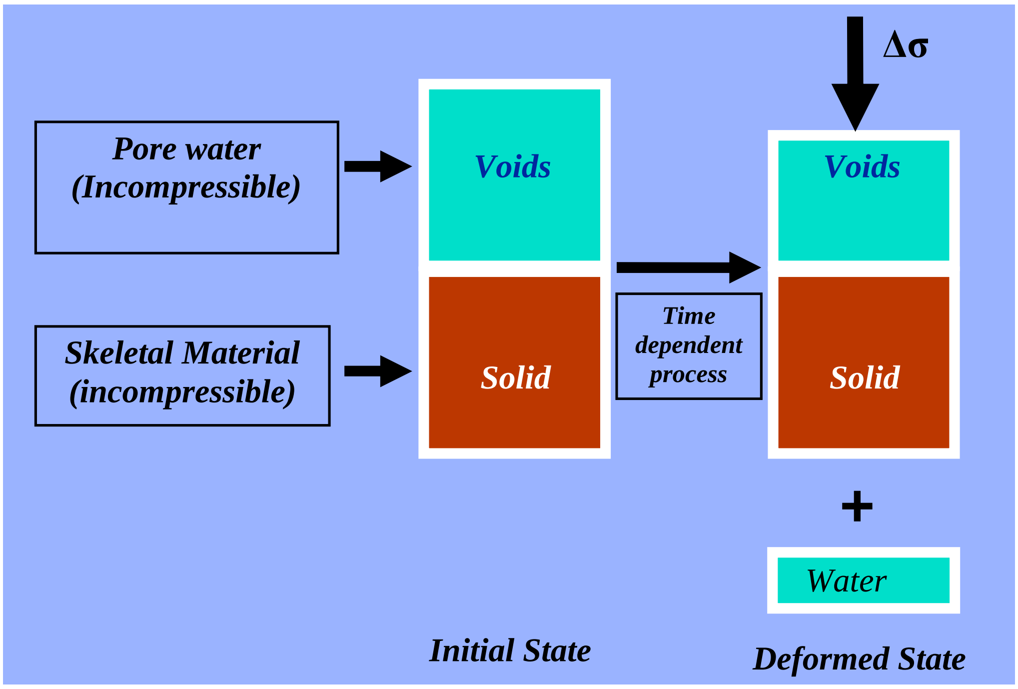



μ_s = 0.5
0.4
0.3
0.2
0.1
0
I_E
D_f / B_e

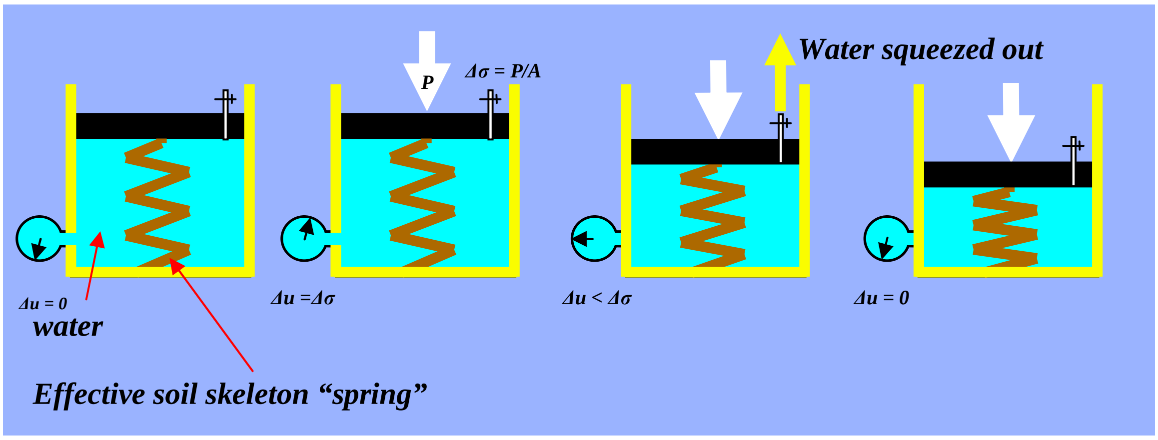
● Consolidation Settlement (Primary Consolidation)

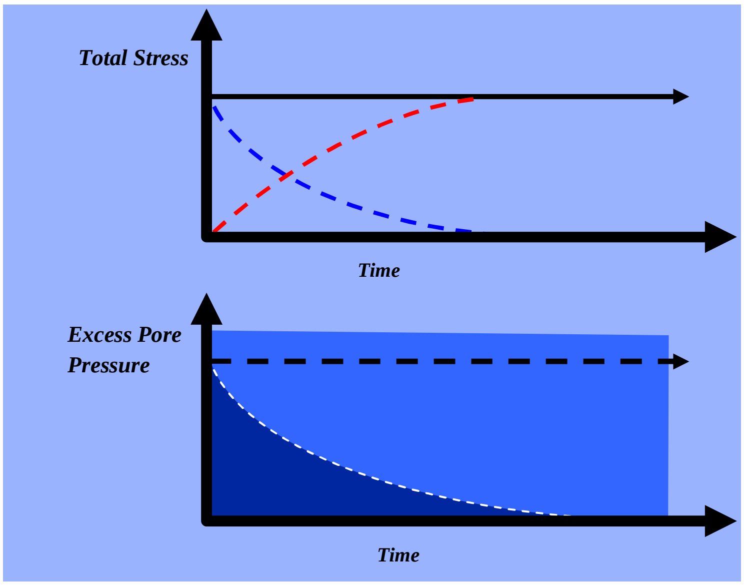
- ***Fundamentals of Consolidation***

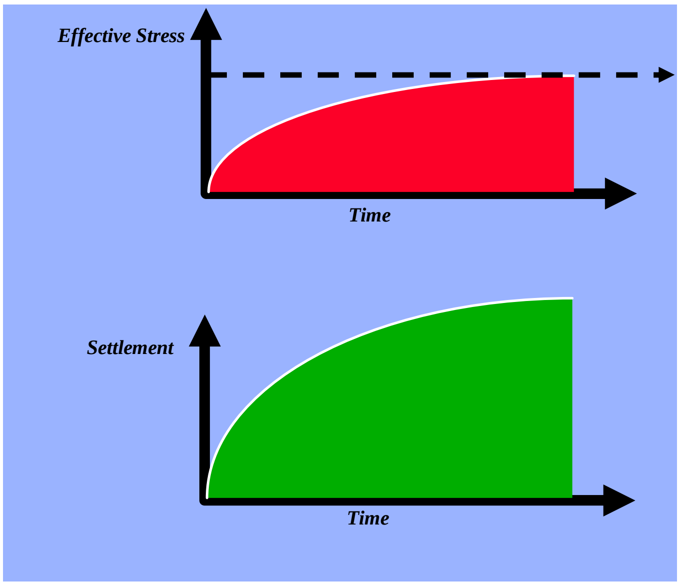
  Deformation of saturated soil occurs by reduction of pore space & the squeezing out of pore water. The water can only escape through the pores which for fine-grained soils are very small, while for coarse-grained soils are large enough for the process to occur immediately after the application of load.



- ***Spring model***



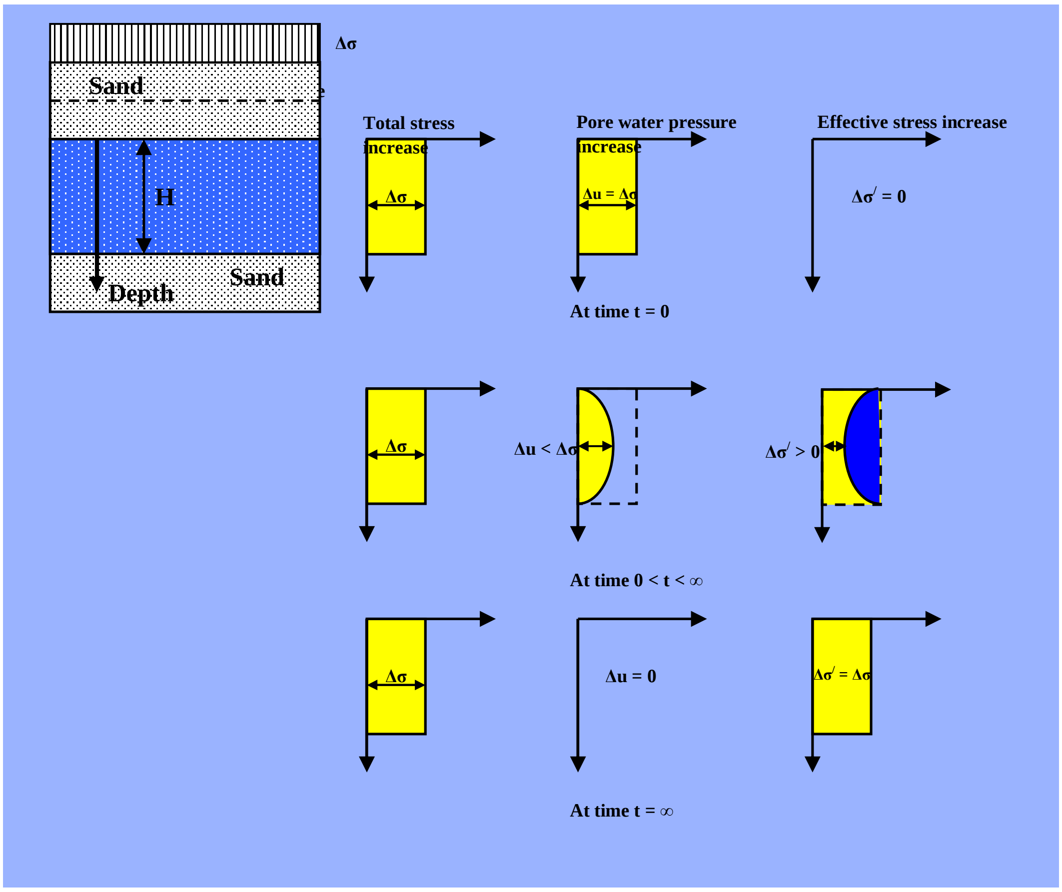




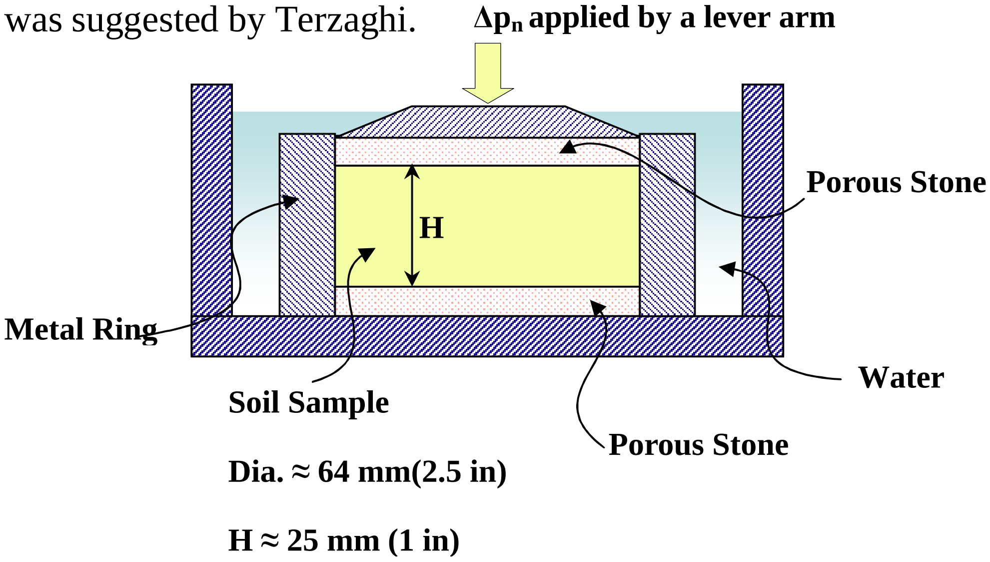
Conclusions

- Especially in low permeability soils (silts and clays) settlement is delayed by the need to squeeze the water out of the soil
- Consolidation is the process of gradual transfer of an applied load from the pore water to the soil structure as pore water is squeezed out of the voids.
- The amount of water that escapes depends on the size of the load and compressibility of the soil.
- The rate at which it escapes depends on the coefficient of permeability, thickness, and compressibility of the soil.

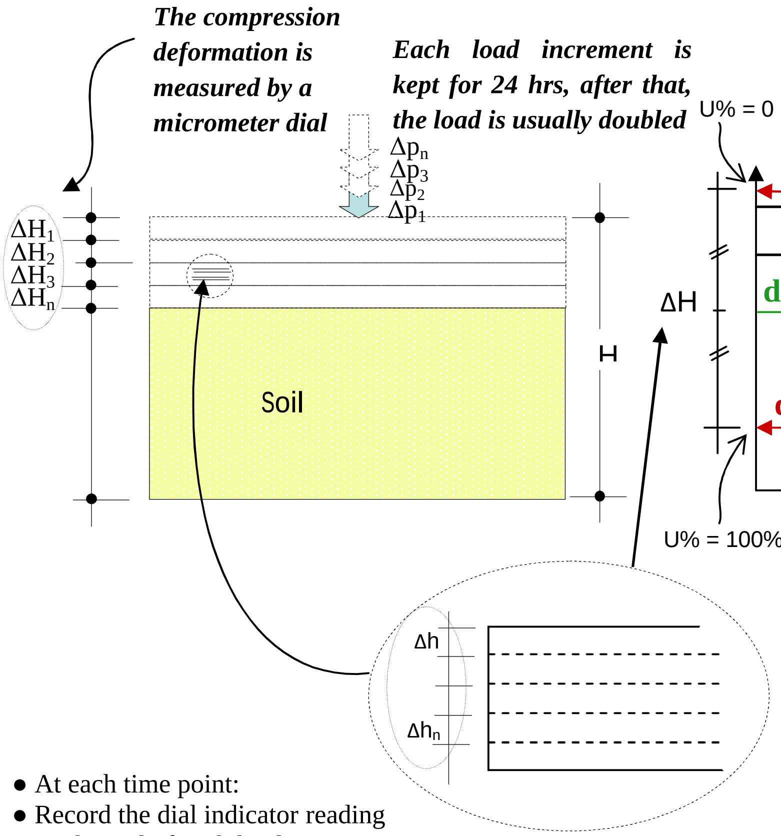
Consider a clay layer with thickness H subjected to an instantaneous increase of total stress Δσ.

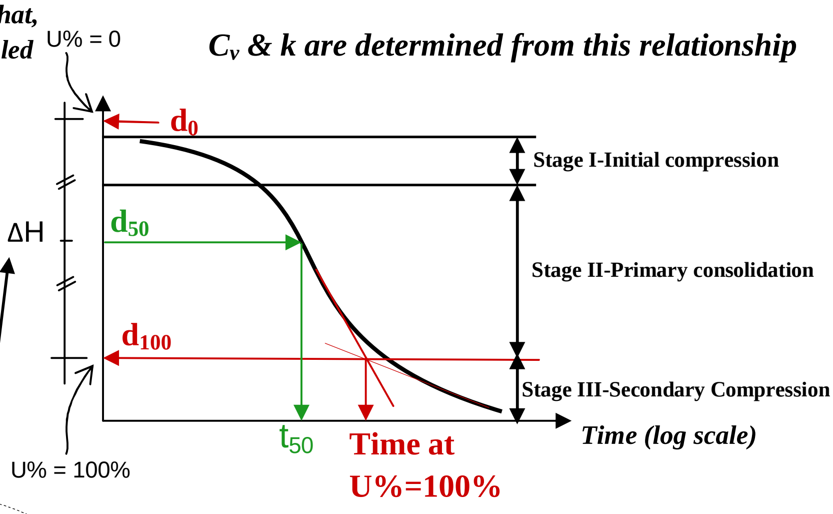


- ***One-Dimensional Laboratory Consolidation Test***
  It was suggested by Terzaghi.



<u>Consolidometer (Oedometer)</u>





Settlement (deformation) vs. time

Time, min	*Deformation, mm*
o	
¼	
½	
1	
2	
5	
10	
20	
30	
1 hr	
2 hr	
……..	
24 hr	

- At each time point:
- Record the dial indicator reading
- At the end of each load increment:
- Record the dial reading at completion of primary consolidation for each load (usually 24 hours)
- At the unloading of the specimen:
- Remove load in decrements, recording dial indicator readings
- Conduct a water content test on specimen after unloading is complete

- ***Void Ratio-Pressure (e-log***
  1. calculate the height of solids, H_s

$$H_s = \frac{W_s}{AG_s\gamma_w}$$ Prove it.

  2. calculate initial height of voids, H_v

$$H = H - H_s$$

  3. calculate initial void ratio, e_o

$$e_o = \frac{V_v}{V_v} = \frac{H_v A}{H_s A} = \frac{H_v}{H_s}$$

4. for the 1st incremental loading, σ_1 ($\frac{\Delta P_1}{A}$) which causes a deformation ΔH_1

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

5. calculate new void ratio after consolidation caused by σ_1

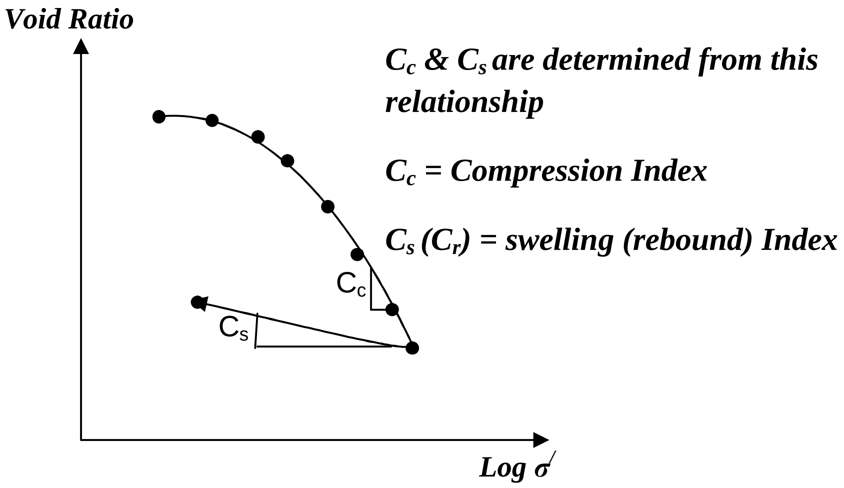
$$e_1 = e_o - \Delta e_1$$

6. for the next loading, σ_2 ($\frac{\Delta P_1 + \Delta P_2}{A}$), which causes additional deformation ΔH_2, the void ratio at the end of consolidation is

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

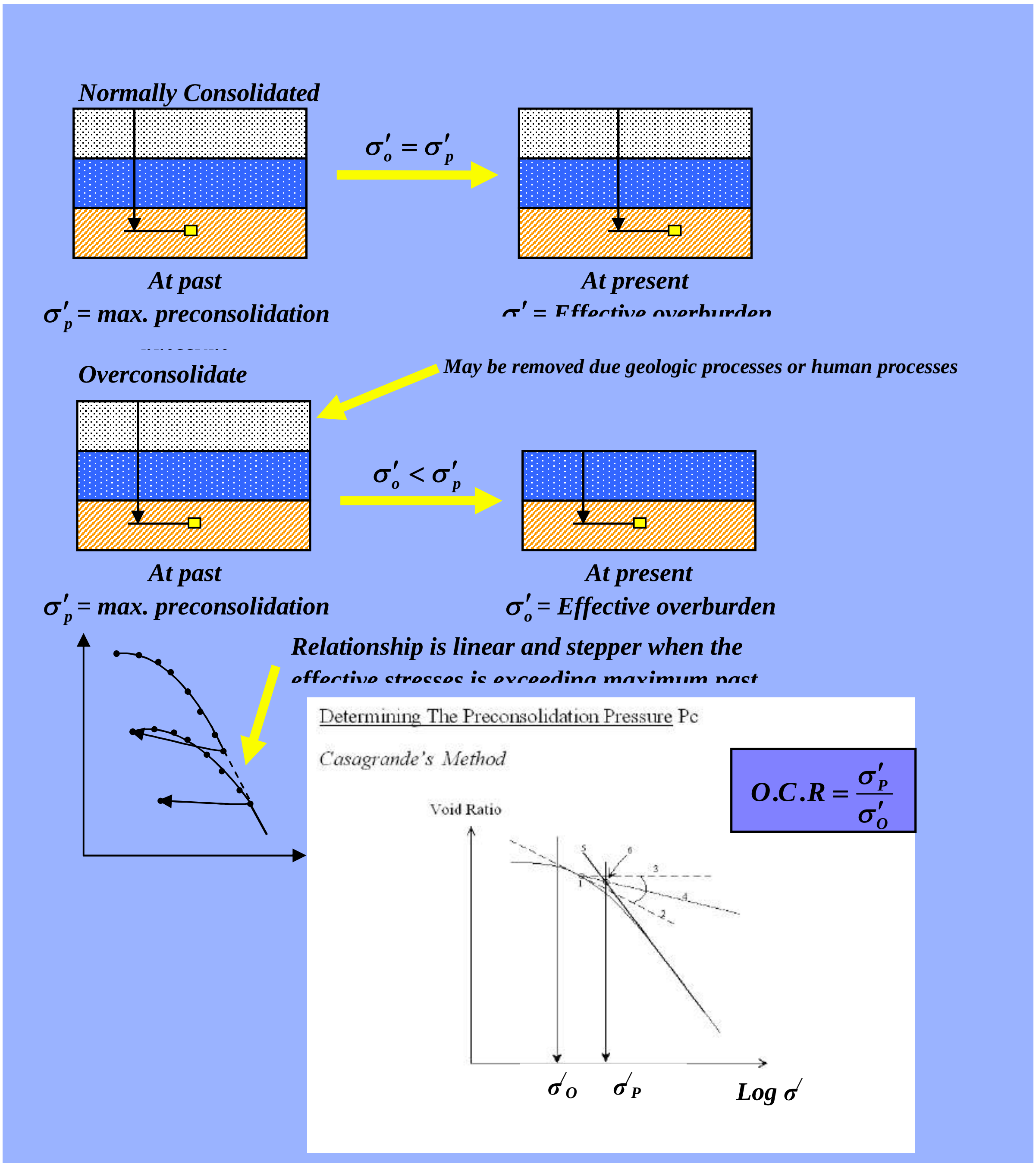
Note:- at the end of consolidation $\sigma = \sigma'$

Plot the corresponding e with σ' on semi-logarithmic paper. The typical shape of e-log σ' will be as shown in the figure.



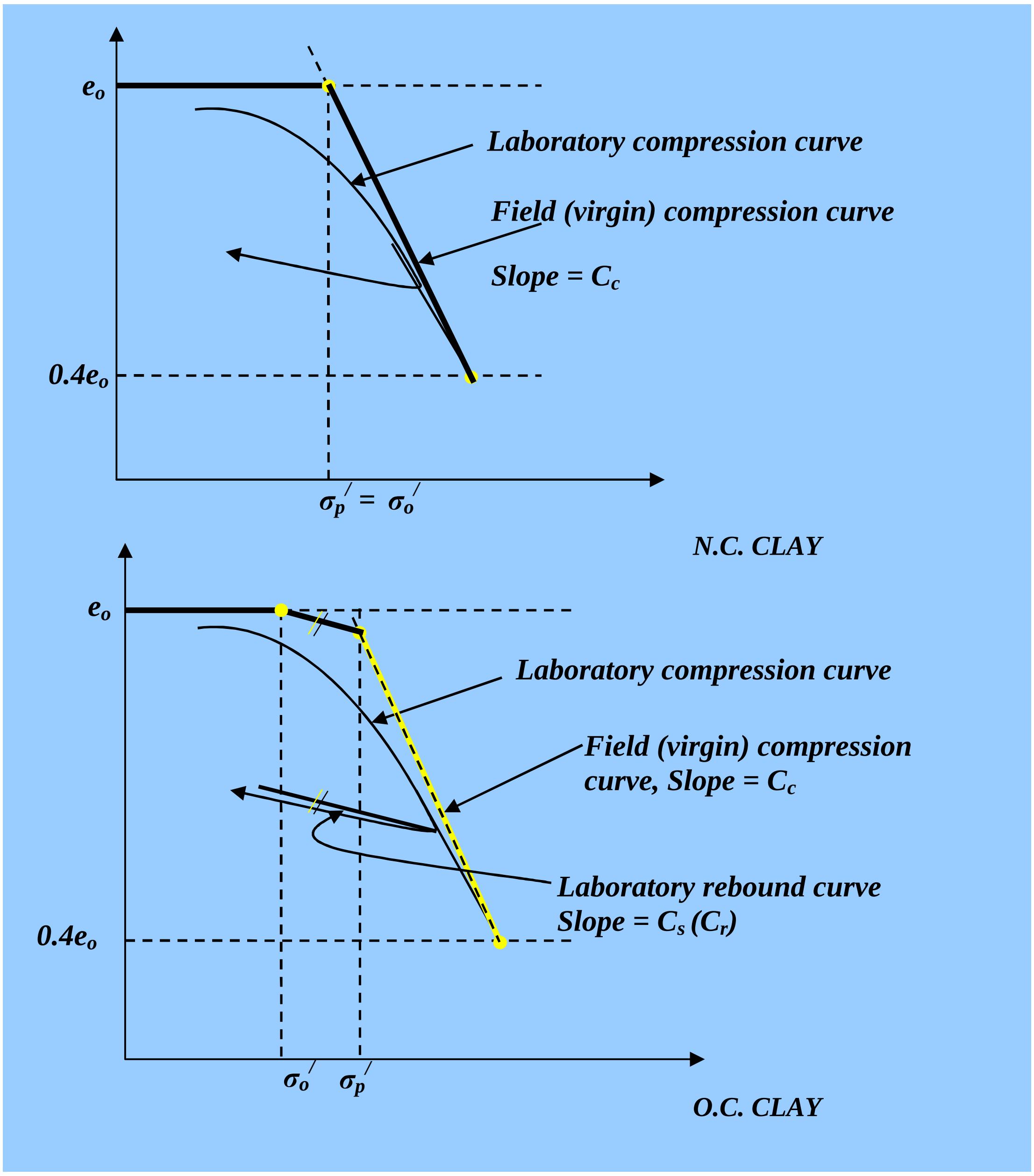
Change in void ratio vs. vertical effective stress

- ***Normally Consolidated and Overconsolidated Clays***

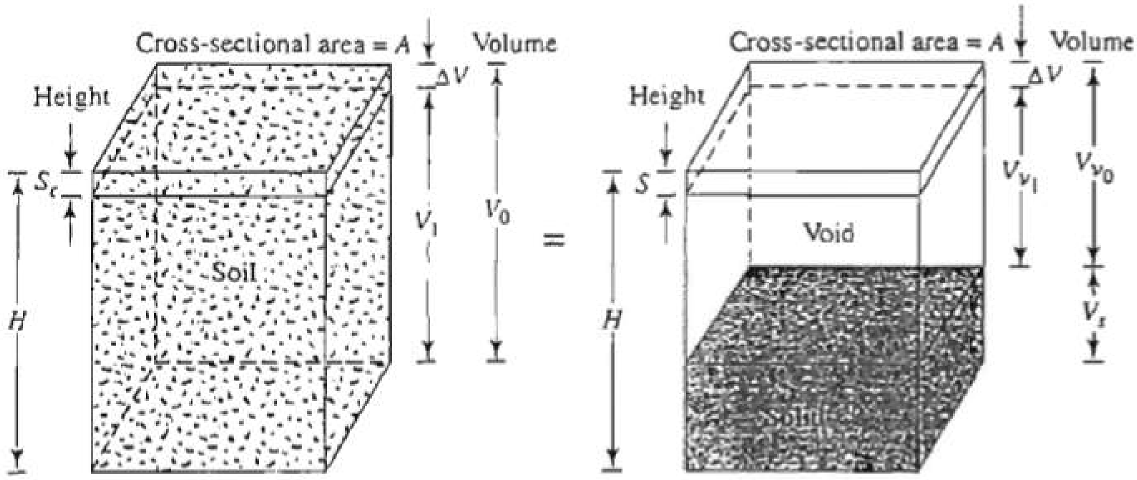


- ***Effect of Disturbance on Void Ratio-Pressure Relationship***

  - Usually e-log σ' founded by performing consolidation test on undisturbed sample or remolded sample, does not reflects the field (virgin) compression curve.
  - This difference is attribute to disturbance due to-
    - Handling and transferring samples into consolidation cells.
    - Sampling and stress relief



- ***Calculation of Settlement from One-Dimensional Primary Consolidation***



Settlement caused by one-dimensional consolidation

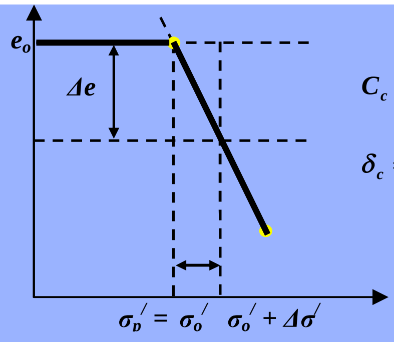
$$\Delta V = V_o - V_1 = HA - (H - \delta_c)A = \delta_c A$$

$$\Delta V = V_{vo} - V_{v1} = \Delta V_v = \Delta e V_s$$

$$V_s = \frac{V_o}{1+e_o} = \frac{AH}{1+e_o}$$

$$\Delta V = \delta_c A = \Delta e V_s = \Delta e \frac{AH}{1+e_o}$$

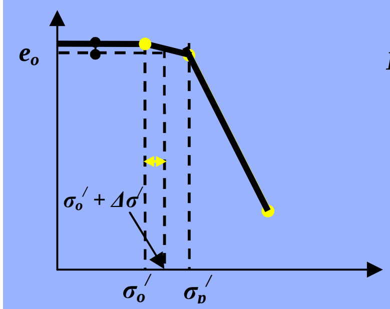
$$\delta_c = H \frac{\Delta e}{1+e_o}$$



N.C. CLAY

$$C_c = \frac{\Delta e}{\Delta \log \sigma'} \Rightarrow \Delta e = C_c[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o] = C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o})$$

$$\delta_c = H \frac{\Delta e}{1 + e_o} = \frac{C_c}{1 + e_o} H \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o})$$

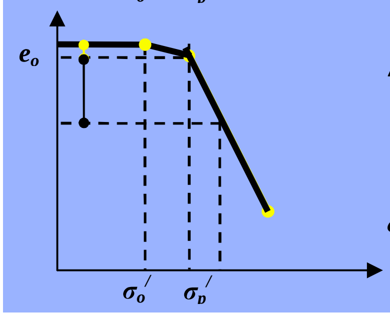


O.C. CLAY

If $\sigma_o' + \Delta\sigma' \leq \sigma'_p$

$$C_s = \frac{\Delta e}{\Delta \log \sigma'} \Rightarrow \Delta e = C_s[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o] = C_s \log(\frac{\sigma'_o}{}$$

$$\delta_c = H \frac{\Delta e}{1 + e_o} = \frac{C_s}{1 + e_o} H \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o})$$



If $\sigma_o' + \Delta\sigma' \leq \sigma'_p$

$$\Delta e = \Delta e_1 + \Delta e_2 = C_s[\log \sigma'_p - \log \sigma'_o] + C_c[\log(\sigma'_o + \Delta\sigma') - \mathrm{l}$$

$$= C_s \log(\frac{\sigma'_p}{\sigma'_o}) + C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_p})$$

$$\delta_c = H \frac{\Delta e}{1 + e_o} = \frac{H}{1 + e_o}\left[C_s \log(\frac{\sigma'_p}{\sigma'_o}) + C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_p})\right]$$

Compression Index (C_c) and Swell Index (C_s)

Several correlations were suggested for C_c (See table 10.4 PP. 282) besides other eq. see PP. 282. and in most cases $C_s \cong \frac{1}{5} to \frac{1}{10} C_c$

$C_c = 0.009(LL - 10)$ undisturbed clays LL = liquid limit

$C_c = 0.007(LL - 7)$ remolded clays LL = liquid limit

Table 10.4 Correlations for Compression Index, C_c*

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01w_N$		Chicago clays
$C_c = 1.15(e_O - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_O - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_O - 0.5)$		Soils with low plasticity
$C_c = 0.208e_O + 0.0083$		Chicago clays
$C_c = 0.156e_O + 0.0107$		All clays

*After Rendon-Herrero (1980)

Note: e_O = *in situ* void ratio; w_N = *in situ* water content.

● Secondary Compression (Consolidation) Settlement

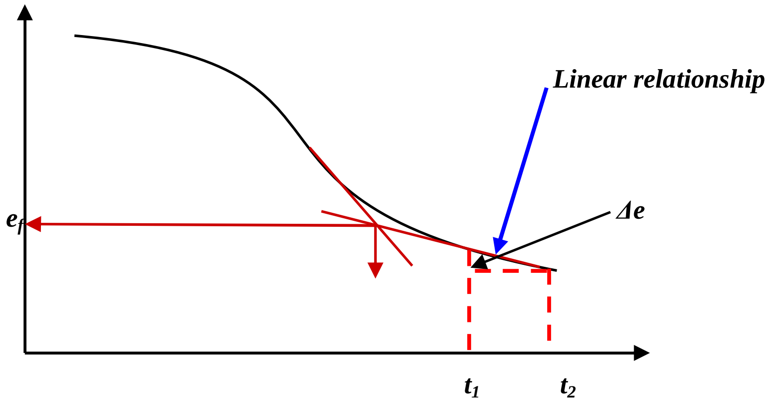
●Secondary compression settlement is a form of soil creep that is largely controlled by the rate at which the skeleton of compressible soils, particularly clays, silts, and peats, can yield and compress.

●Secondary compression is often conveniently identified to follow primary consolidation when excess pore fluid pressure can no longer be measured; however, both processes may occur simultaneously.

●Also referred to as the "secular effect"

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log \frac{t_2}{t_1}}$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$



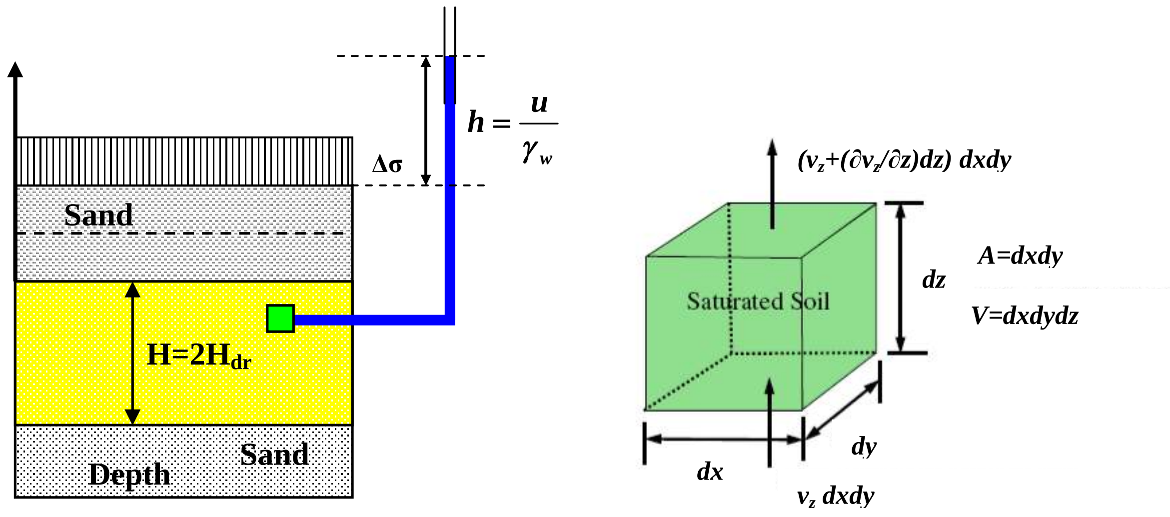
$$\delta_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

Type of soil	C'_α
O.C clays	0.001 or less
N.C clays	0.005 to 0.03
Organic soil	0.04 or more

• Time Rate of Consolidation

- Terzaghi (1925) proposed the first theory to consider the rate of one-dimensional consolidation for saturated clay soils.
- Assumptions:-
    1. The clay-water system is homogenous.
    2. Saturation is complete.
    3. Compressibility of water is negligible
    4. The flow of water is in one direction only (direction of compression)
    5. Darcy's law is valid

To begin, consider a very small element of soil being subjected to one-dimensional consolidation in the z-direction.



Volume of pore fluid which flows out = Volume decrease of the soil
and thus
Rate at which pore fluid flows out = Rate of volume decrease of soil

$$[(v_z + \frac{\partial v_z}{\partial z}dz) \; - v_z]dxdy = \frac{\partial V}{\partial t}$$

$$\frac{\partial v_z}{\partial z}dxdydz = \frac{\partial V}{\partial t}$$

It will also be assumed that Darcy’s law holds and thus that

$$v_z \quad = ki = \quad -k\frac{\partial h}{\partial z} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial z}$$

$$h = \frac{u}{\gamma_w}$$

u = excess pore water pressure caused by the increase of stress

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

During consolidation

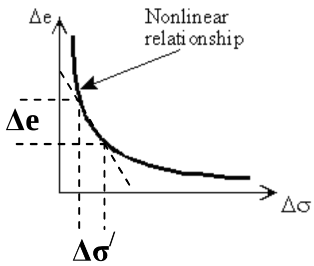
$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$

but $\frac{\partial V_s}{\partial t} = 0$

$$\frac{\partial V}{\partial t} = V_s\frac{\partial e}{\partial t}$$

but

$$V_s = \frac{V}{1+e_o} = \frac{dxdydz}{1+e_o}$$



$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1+e_o}\frac{\partial e}{\partial t}$$

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_o}\frac{\partial e}{\partial t}$$

But $\partial e = a_v \partial(\Delta\sigma') = -a_v \partial u$

a_v = coefficient of compressibility

$$\therefore \quad -\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

where $m_v = \frac{a_v}{1+e_o}$

m_v = coefficient of volume compressibility

or $\boxed{\dfrac{\partial u}{\partial t} = c_v \dfrac{\partial^2 u}{\partial z^2}}$ where $c_v = \dfrac{k}{\gamma_w m_v} = \dfrac{k}{\gamma_w \left(\dfrac{a_v}{1+e_o}\right)}$

c_v = coefficient of consolidation has units L^2/T

k = coefficient of permeability

This equation is the basic differential equation of *Terzaghi's consolidation theory* and can be solved with the following boundary conditions

z = 0 u = 0 at a permeable boundary

z = 2Hdr u = 0 at a permeable boundary

t = 0 $u = u_o = \Delta\sigma$ initial excess pore water pressure

The solutions yields

$$\boxed{u = \sum_{m=0}^{\infty}\left[\frac{2u_o}{M}\sin\left(\frac{MZ}{H_{dr}}\right)\right]e^{-M^2T_v}}$$

Where

$$M = \frac{\pi}{2}(2m+1)$$

$\boxed{Z = \dfrac{z}{H_{dr}}}$, ***a depth factor dimensionless number***

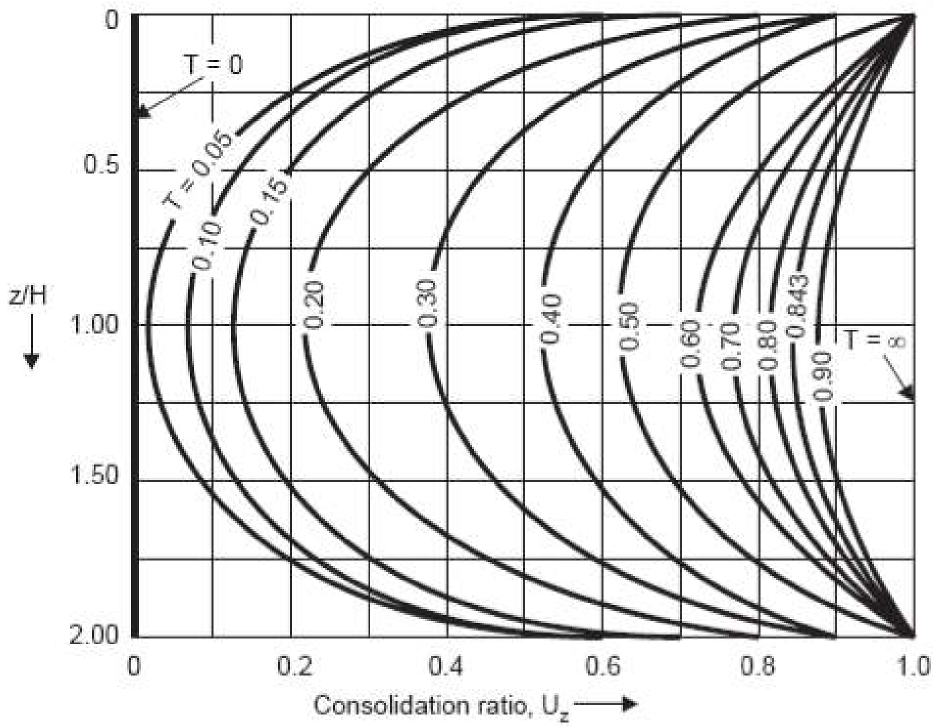
$\boxed{T_v = \dfrac{c_v t}{H_{dr}^{\ 2}}}$, ***a time factor is a nondimensoinal number***

Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at distance ***z*** at any time ***t*** is

$$\boxed{U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}}$$

u_z = *excess pore pressure at time* ***t***

The variation of excess pore pressure within the layer is shown in Figure below



Graphical solution for consolidation equation

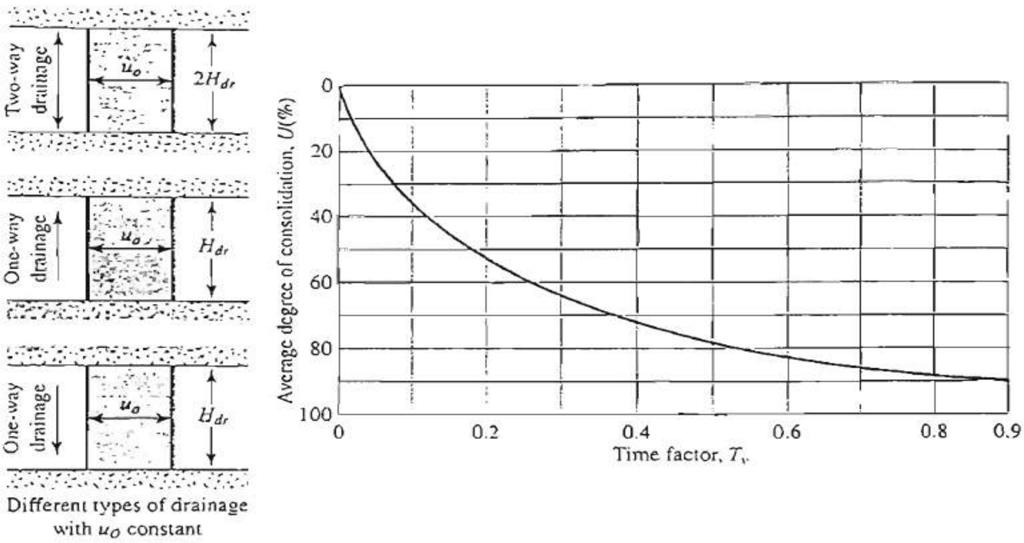
The average degree of consolidation for the entire depth of the clay layer at any time t can be written as

$$\boxed{U = \frac{\delta_{c(t)}}{\delta_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z dz}{u_o} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} e^{-M^2 T_v}}$$

U = average degree of consolidation
$\delta_{c(t)}$ = settlement of the layer at time t

δ_c = ultimate (final) primary consolidation settlement

The U – T_v relationship is represented in the figure below for the case where u_o is uniform for the entire depth of the consolidating layer.



Variation of average degree of consolidation with time factor, T_v

The values of T_v and their corresponding average U for the case presented above may also be approximately by the following relationship:

For U = 0 to 60%

$$\boxed{T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2}$$

For U > 60%

$$\boxed{T_v = 1.781 - 0.933\log(100 - U\%)}$$

Table 10.5-PP 293 gives the variation T_v – U according to the above equations

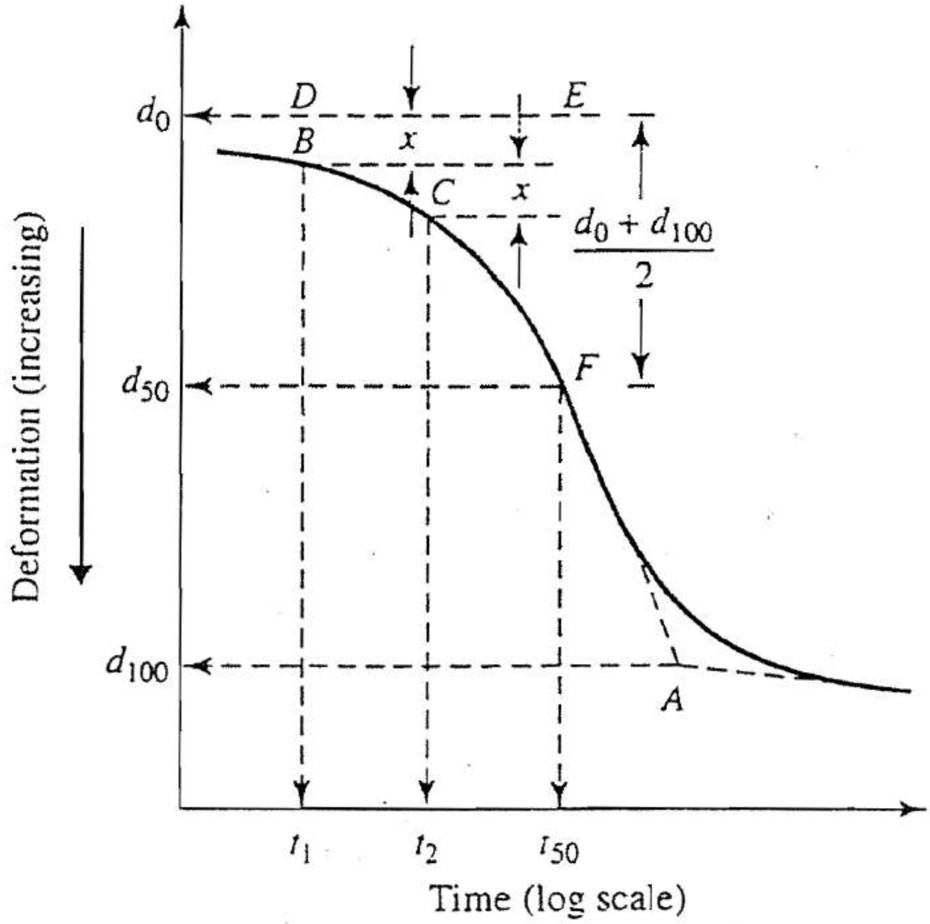
- ***Coefficient of Consolidation***

  - ***Logarithm of Time method***

    It is particularly useful when there is <u>significant secondary compression (creep)</u>. The d_o point is located by selected two points on the curve for which the times (t) are in the ratio 1:4, e.g. 1 min and 4 min; or 2 min and 8 min.; the vertical intervals DB and BC will be equal. The d_{100} point can be located in the final part of the curve flattens sufficiently (i.e. no secondary compression). When there is significant secondary compression, d_{100} may be located at the intercept of straight line drawn through the middle and final portions of the curve. Now d_{50} and log t_{50} can be located.

The coefficient of consolidation is therefore:

$$\boxed{T_{50} = \frac{c_v t_{50}}{H^2} \quad \Rightarrow \quad c_v = \frac{0.197 H_{dr}^2}{\;}}$$

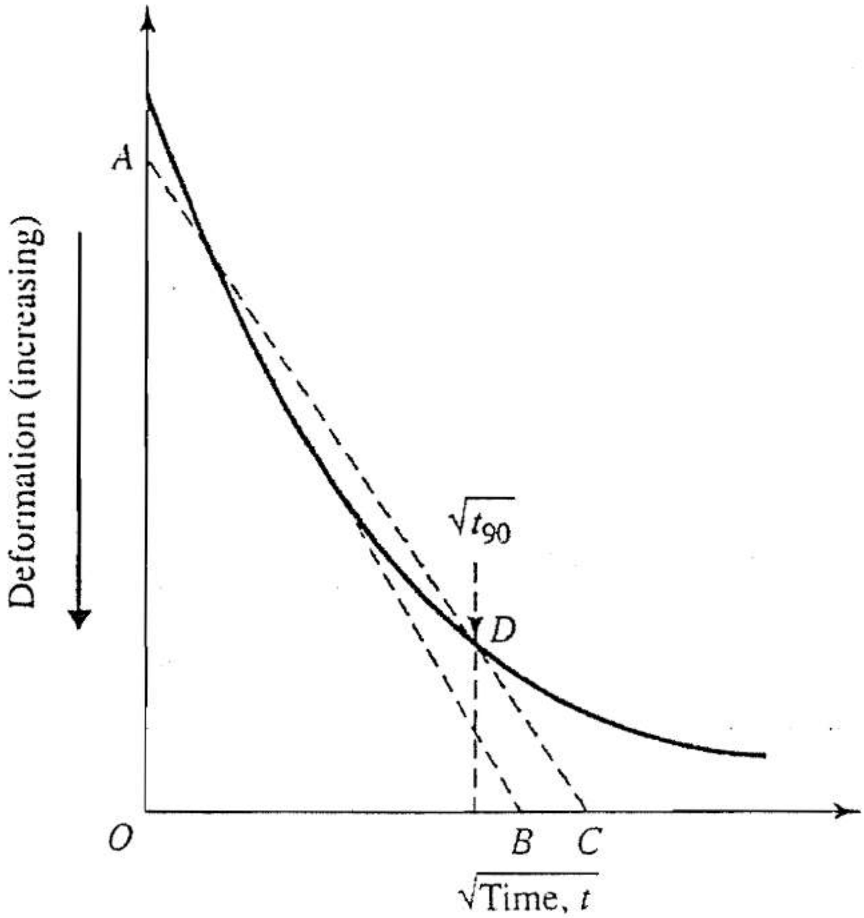


- ***Square Root of Time method***

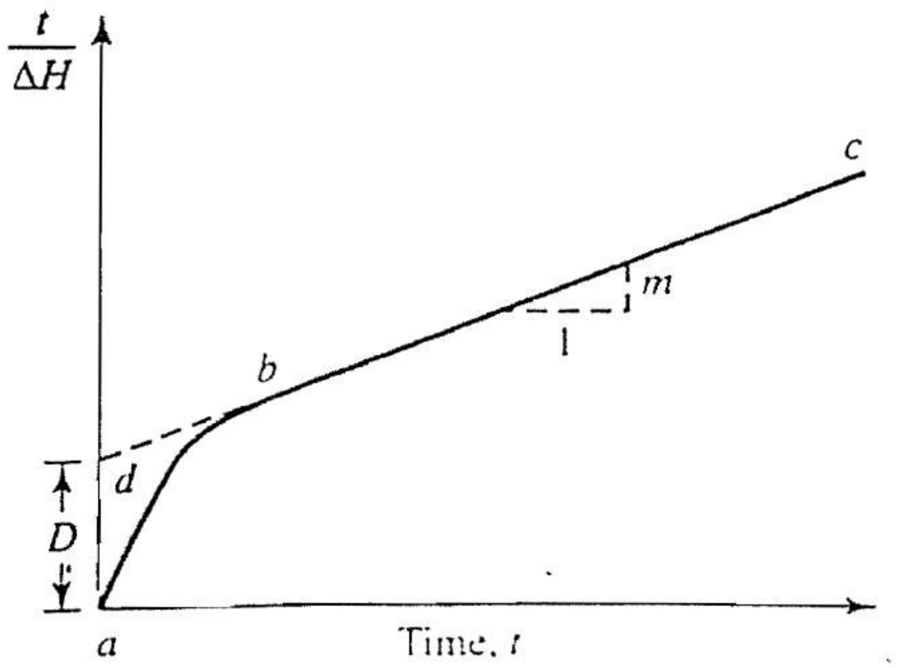
  After the laboratory results curve has been plotted, line AB is drawn, followed by line AC in such a way that $\overline{OC} = 1.15\overline{OB}$: AC crosses the laboratory curve at point D and locates $\sqrt{t_{90}}$

  The coefficient of consolidation is therefore:

$$\boxed{T_{90} = \frac{c_v t_{90}}{H_{dr}^2} \quad \Rightarrow \quad c_v = \frac{0.848 H_{dr}^2}{t_{90}}}$$



- ***Hperbola method***
  it gives good results for U =60% - 90%.
  The results are plotted on $\frac{t}{\Delta H}$ - t, then identify the straight portion *bc* of the curve and project back to point *d*, and determine the intercept *D*, then determine the slope *m* of *bc*
  The coefficient of consolidation is therefore: $c_v = 0.3\left(\frac{mH_{dr}^2}{D}\right)$

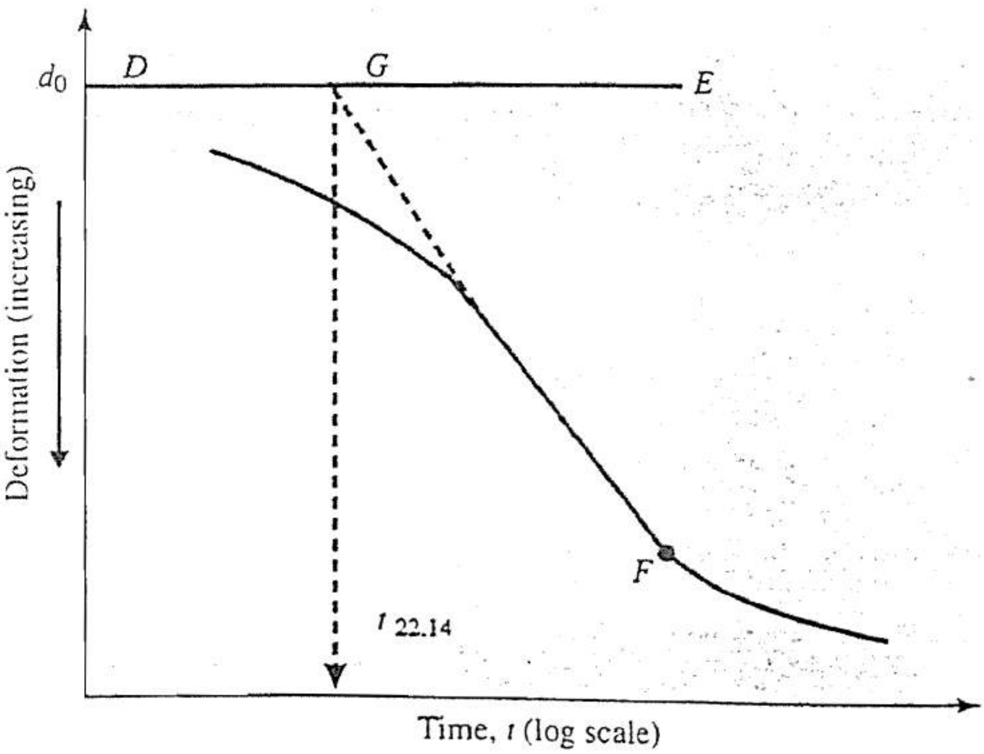


- ***Earl Stage log-t method***
  It gives the *highest value* while the conventional log-t method gives the lowest value, this is due to the contribution of the lower part of the consolidation curve in the conventional log-t method that means the secondary compression plays a role in the value of c_v, while in this method c_v obtained from the early stage log-t method, which gives more realistic values of the fieldwork.

  Follow the same steps in log-t method to locate d_o, draw a horizontal line *DE* through d_o, then draw tangent through the point of inflection *F*, the tangent intersects line *DE* at point

G, determine the corresponding time t corresponding to G, which is the time at U = 22.14%, The coefficient of consolidation is therefore:

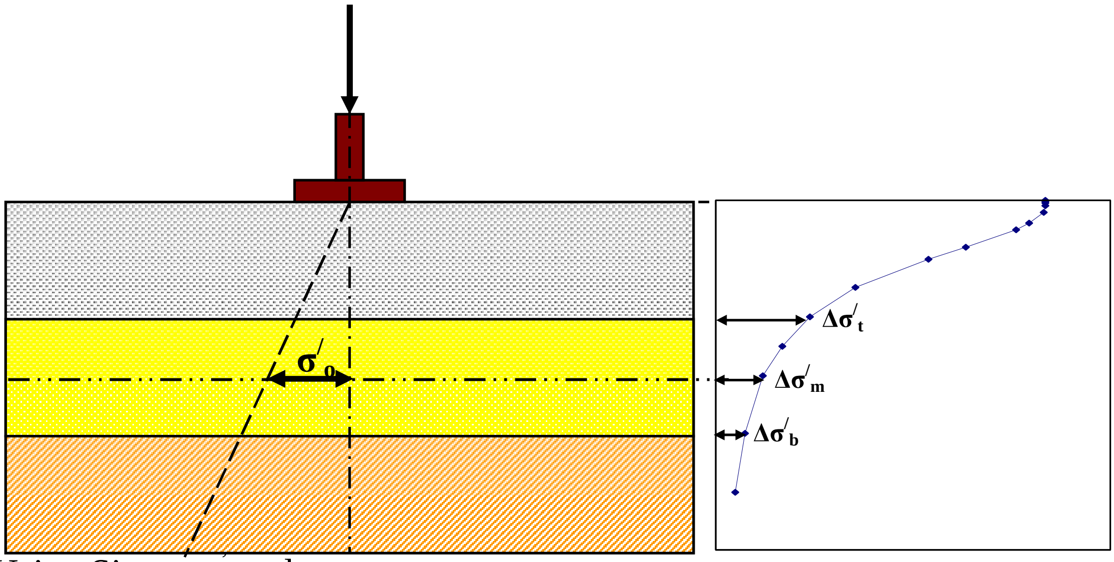
$$\boxed{c_v = \frac{0.0385 H_{dr}^2}{t_{22.14}}}$$



● Calculation of Consolidation Settlement under a Foundation

For limited area foundations (circular, square and rectangular), the increase of effective stress ($\Delta\sigma'$) decrease with depth as shown in figure below which can be estimated as described before in previous chapter.

Estimate σ'_o and $\Delta\sigma'_{av}$ at the middle of the clay layer, then use the previous equations in to determine final consolidation settlement.



Using Simpson᾽s rule

$$\boxed{\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}}$$

Alternative approach

Simply divide the clay layer to a number of sub layers, and then estimate δ_c for each sub layer taking into account effective overburden pressure and an increase in effective stress at the middle of each sub layer, then get the summation of settlements of the sub layers to get the final consolidation of the clay layer.

Rate of consolidation

It is important to determine δ_c – ***time*** relationship, which can be helpful in estimating the differential settlement between adjacent footings if the drainage condition at one footing differs from the other.

$$\boxed{U = \frac{\delta_{c(t)}}{\delta_c}}$$

Examples

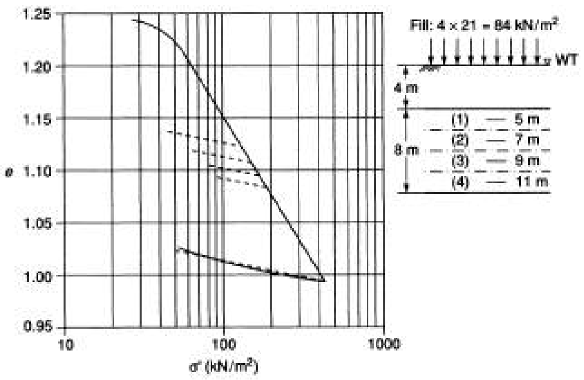
The following results were obtained from an oedometer test on a specimen of saturated clay:

Pressure (kN/m^2)	*27*	*54*	*107*	*214*	*429*	*214*	*107*	*54*
Void ratio	*1.243*	*1.217*	*1.144*	*1.068*	*0.994*	*1.001*	*1.012*	*1.024*

A layer of this clay 8m thick lies below a 4m depth of sand, the water table being at the surface. The saturated unit weight for both soils is $19kN/m^3$. A 4m depth of fill of unit weight 21 kN/m^3 is placed on the sand over an extensive area. Determine the final settlement due to consolidation of the clay. If the fill were to be removed some time after the completion of consolidation, what heave would eventually take place due to swelling of the clay?

$$\delta_c = \frac{e_o - e_1}{1 + e_o} H$$

Appropriate values of e are obtained from e-logσ/ drawn from the result. The clay will be divided into four sub-layers, hence H =2000 mm.



Settlement

Layer	σ'_0 (kN/m^2)	σ'_1 (kN/m^2)	e_0	e_1	$e_0 - e_1$	s_c (mm)
1	46.0*	130.0†	1.236	1.123	0.113	101
2	64.4	148.4	1.200	1.108	0.092	84
3	82.8	166.8	1.172	1.095	0.077	71
4	101.2	185.2	1.150	1.083	0.067	62
						318

Notes
* 5 × 9.2.
† 46.0 + 84.

Heave

Layer	σ'_0 (kN/m^2)	σ'_1 (kN/m^2)	e_0	e_1	$e_0 - e_1$	s_c (mm)
1	130.0	46.0	1.123	1.136	−0.013	−12
2	148.4	64.4	1.108	1.119	−0.011	−10
3	166.8	82.8	1.095	1.104	−0.009	−9
4	185.2	101.2	1.083	1.091	−0.008	−7
						−38

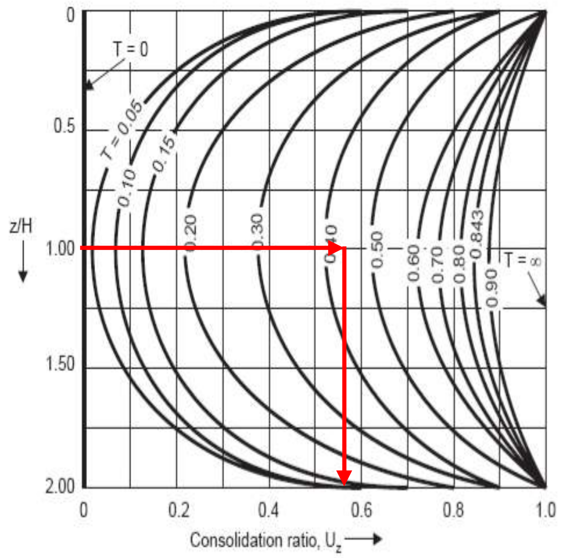
Assuming the fill in pervious example is dumped very rapidly, what would be the value of excess pore water pressure at the centre of the clay layer after a period of

$$d = \frac{8}{2} = 4\,\text{m}$$

$$T_v = \frac{c_v t}{d^2} = \frac{2.4 \times 3}{4^2} = 0.450$$

$$u_i = \Delta\sigma = 84\,\text{kN/m}^2$$

$$\boxed{U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{84} = 0.58 \qquad u_z = 35.2 \cdots kN/m^2}$$



Graphical solution for consolidation equation

In an oedometer test a specimen of saturated clay 19mm thick reaches 50% consolidation in 20 min. How long would it take a layer of this clay 5m thick to reach the same degree of consolidation under the same stress and drainage conditions? How long would it take the layer to reach 30% consolidation?

$$U = f(T_v) = f\left(\frac{c_v t}{d^2}\right)$$

Hence if c_v is constant,

$$\frac{t_1}{t_2} = \frac{d_1^2}{d_2^2}$$

where '1' refers to the oedometer specimen and '2' the clay layer.

For double Drainage (Two-way Drainage)

$$d_1 = 9.5\,\text{mm} \quad \text{and} \quad d_2 = 2500\,\text{mm}$$

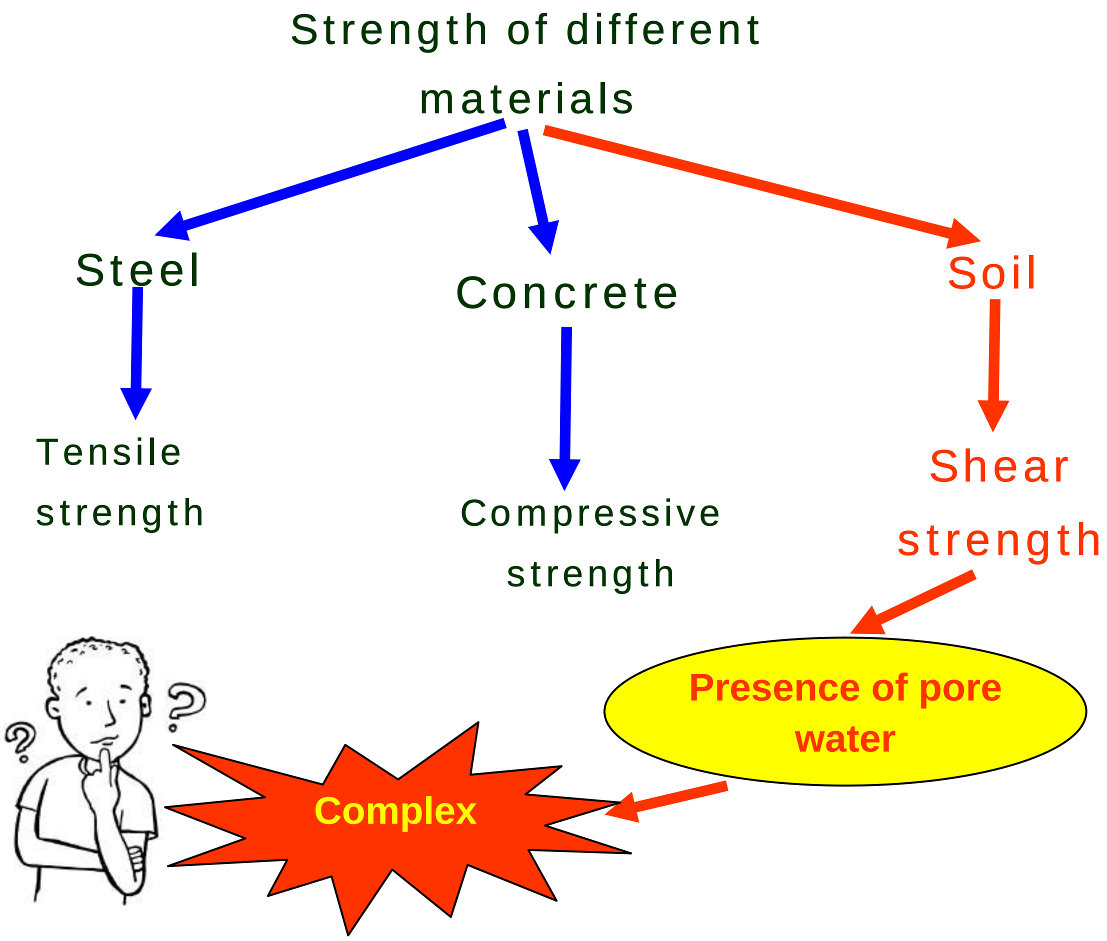
$$\therefore \text{ for } U = 0.50,\ t_2 = t_1 \times \frac{d_2^2}{d_1^2}$$

$$= \frac{20}{60 \times 24 \times 365} \times \frac{2500^2}{9.5^2} = 2.63\,\text{years}$$

$$\text{for } U < 0.60,\ T_v = \frac{\pi}{4} U^2$$

$$\therefore t_{0.30} = t_{0.50} \times \frac{0.30^2}{0.50^2}$$

$$= 2.63 \times 0.36 = 0.95\,\text{years}$$



Shear Strength of Soils

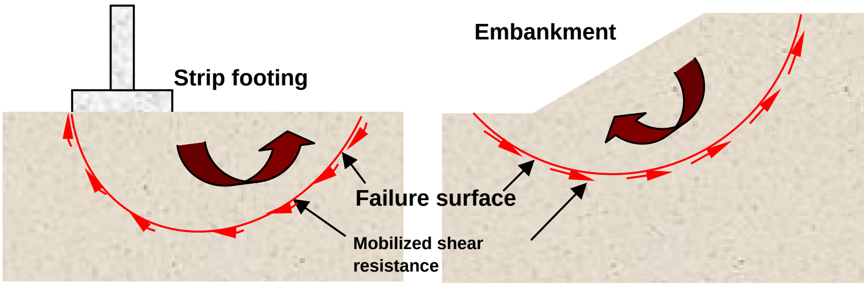
Topics

- **Introduction**
- **Mohr-Coulomb Failure Criterion**
- **Inclination of the plane of failure due to shear**
- **Laboratory Tests for Determination of Shear Strength Parameters**
- **Stress Path**
- **Other Methods for Determining Undrained Shear Strength**
- **Sensitivity and Thixotropy of Clay**
- **Empirical Relationships between Undrained Cohesion (C_u) and Effective Overburden Pressure (σ'_o)**
- **Shear Strength of Unsaturated Cohesive Soils**

• Introduction

Soils are essentially frictional materials. They are comprised of individual particles that can slide and roll relative to one another. In the discipline of soil mechanics, it is generally assumed that the particles are not cemented.

Thus, the ***Shear strength of a soil*** *mass* is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.



One consequence of the frictional nature is that the strength depends on the effective stresses in the soil. As the effective stresses increase with depth, so in general will the strength.

The strength will also depend on whether the soil deformation occurs under fully drained conditions, constant volume (undrained) conditions, or with some intermediate state of drainage. In each case, different excess pore pressures will occur resulting in different effective stresses, and hence different strengths. In assessing the stability of soil constructions analyses are usually performed to check the short term (undrained) and long term (fully drained) conditions.

Shear strength components

The shear strength components are-

- Friction resistance-

  It occurs between the particles of the soil due to the external load consists of-

  - Friction due to sliding
  - Friction due to rolling
  - Friction due to interlocking

- Cohesion

True Cohesion	Apparent Cohesion
• Cementation Due to the presence of cementing agents such as calcium carbonate or iron oxide • Electrostatic and electromagnetic attractions • Primary valence bonding (adhesion) Occurs primarily during overconsolidation	• Negative pore water pressure • Negative excess pore water pressures due to dilation (expansion) • Apparent mechanical forces • Can not be relied on for soil strength

● Mohr-Coulomb Failure Criterion

Mohr (1900) presented a theory for rupture in materials that held "*a material fails through a critical combination of normal stress (σ) and shear resistance (τ_f), and not through either maximum normal or shear stress alone.*

The functional relationship on a failure plane can be expressed in the form

$$\tau_f = f(\sigma)$$

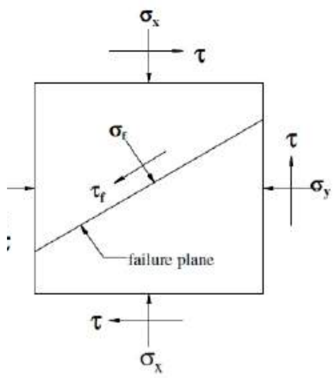
In soils the relationship is approximated as a linear relationship as following

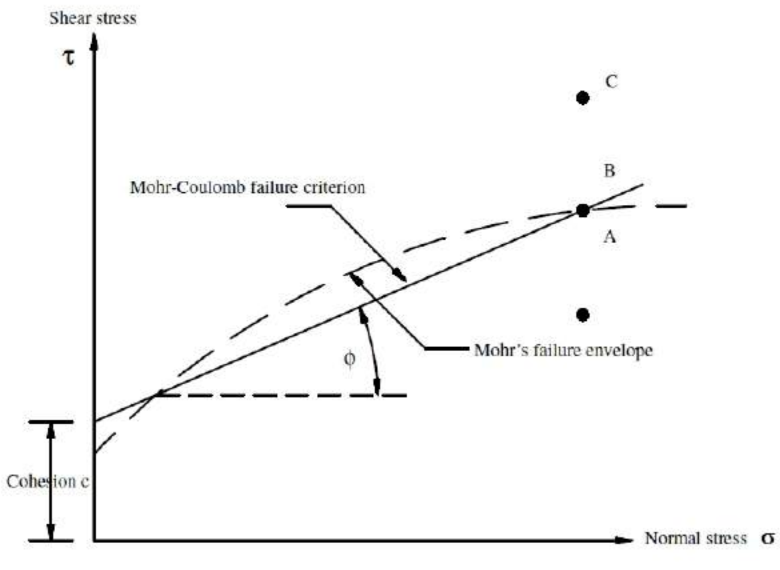
$$\tau_f = c + \sigma \tan\phi$$

This equation is known as the ***Mohr-Coulomb Failure Criterion*.**

where c = cohesion, and
ϕ = angle of internal friction

What does the failure envelope mean?





Comparison between the Mohr's failure envelope and the Mohr-Coulomb failure criterion.

● Inclination of the plane of failure due to shear

As stated by the Mohr-Coloumb failure criteria, failure by shear will take place hen the shear stresses on a plane reaches the value given by the equation

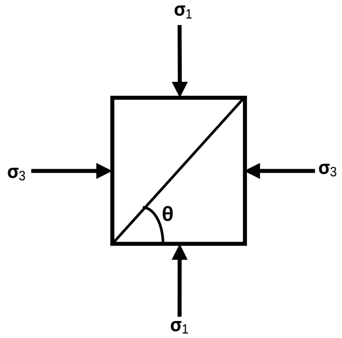
$$\tau_f = c + \sigma \tan\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

To determine the inclination of failure plane with major principle plane

$$\sigma_n = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta$$

$$\ldots\ldots\ldots\ldots 2$$

$$\tau_f = \frac{\sigma_1-\sigma_3}{2}\sin 2\theta$$



Sub. Eq.2 in Eq.1

$$\frac{\sigma_1-\sigma_3}{2}\sin 2\theta = c + \left[\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta\right]\tan\phi$$

or

$$\sigma_1 = \sigma_3 + \frac{\sigma_3\tan\phi + c}{\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

For a given value of σ_3 and c, the failure condition will be determined by the minimum value of the major principle stress σ_1 , for a minimum value of σ_1 , the term $[\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi]$ in eq.3 has to be maximum. Thus,

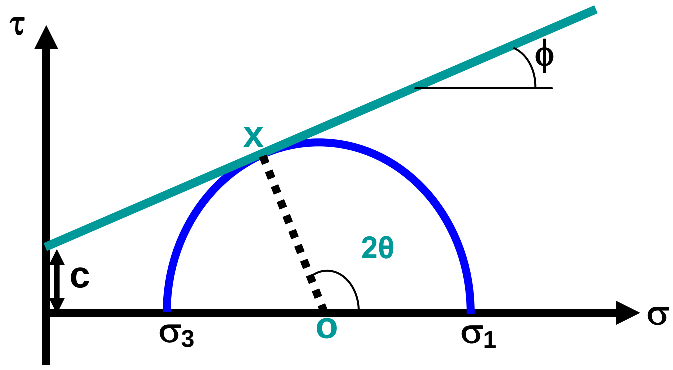
$$\frac{d}{d\theta}\left(\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots 4$$

or

$$\cos^2\theta - \sin^2\theta + 2\sin\theta\cos\theta\tan\phi = 0 \ldots\ldots\ldots\ldots 5$$

Eq.5 gives the relation

$$\theta = 45 + \frac{\phi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$



Sub. Eq.6 in Eq.3 we get

$$\sigma_1 = \sigma_3 \tan^2(45+\frac{\phi}{2}) + 2c\tan(45+\frac{\phi}{2}) \text{.........................}7$$

$$let \quad N_\phi = \tan^2(45+\frac{\phi}{2}) = \frac{1+\sin\phi}{1-\sin\phi} \quad \textit{prove it from geometry}$$

$$\therefore \quad \sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \text{..}7$$

Shear Failure Law in Saturated Soil

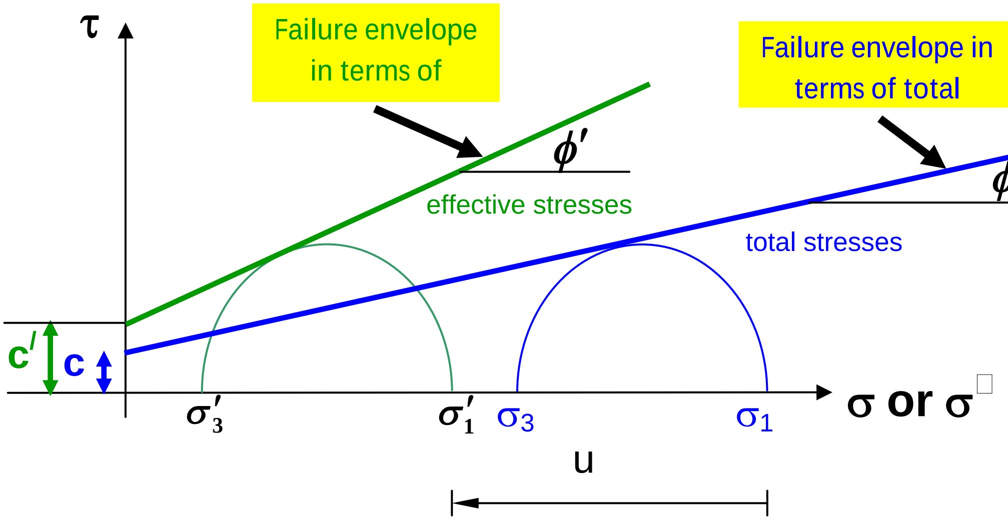
In saturated soil $\sigma = \sigma' + u$ and as stated before shear strength of the soil is a function of effective stress, the shear strength will be in terms of effective stress and eq.1 will be

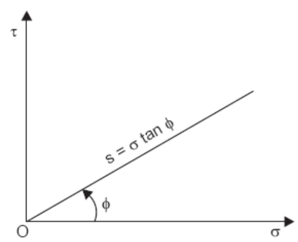
$$\tau_f = c' + (\sigma - u)\tan\phi'$$

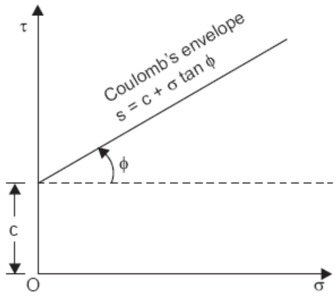
$$\tau_f = c' + \sigma' \tan\phi'$$

c and ϕ or c' and ϕ' are measures of shear strength, Higher the values, higher the shear strength.

Failure envelopes in terms of total & effective stresses



Type of soil	Cohesion	
Sand and Inorganic silt	Zero	
Normally consolidated clays	Very small ≈ Zero	

Over consolidated clays	> Zero	

● <u>Laboratory Tests for Determination of Shear Strength Parameters</u>(c, ϕ or c', ϕ')

Shear tests

Laboratory tests on specimens taken from representative undisturbed samples

Field tests

Most common laboratory tests to determine the shear strength parameters are,

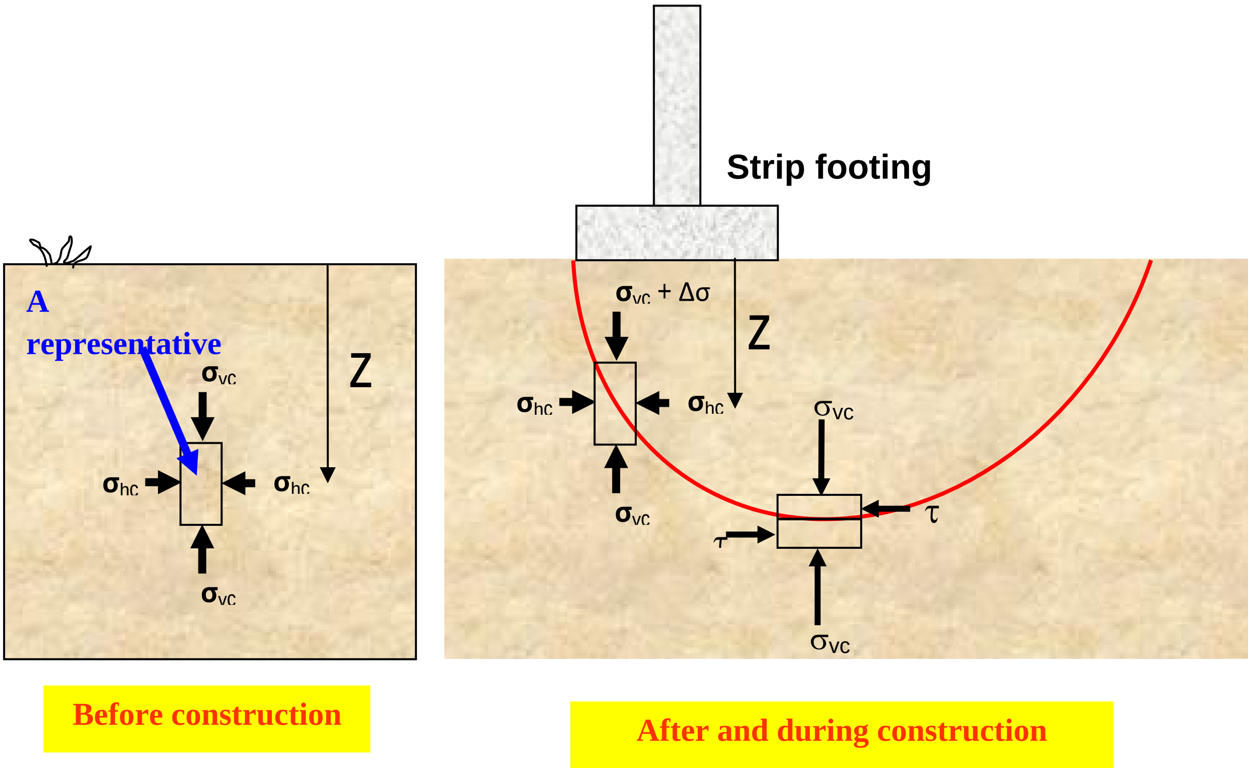
- **Direct shear test**
- **Triaxial shear test**

Other laboratory tests include,

- **Direct simple shear test,**
- **torsional ring shear test,**
- **plane strain triaxial test,**
- **laboratory vane shear test,**
- **laboratory fall cone test**

- **Vane shear test**
- **Torvane**
- **Pocket penetrometer**
- **Fall cone**
- **Pressuremeter**
- **Static cone penetrometer**

Simulating field conditions in the laboratory



Step 1 Representative soil sample taken from the site

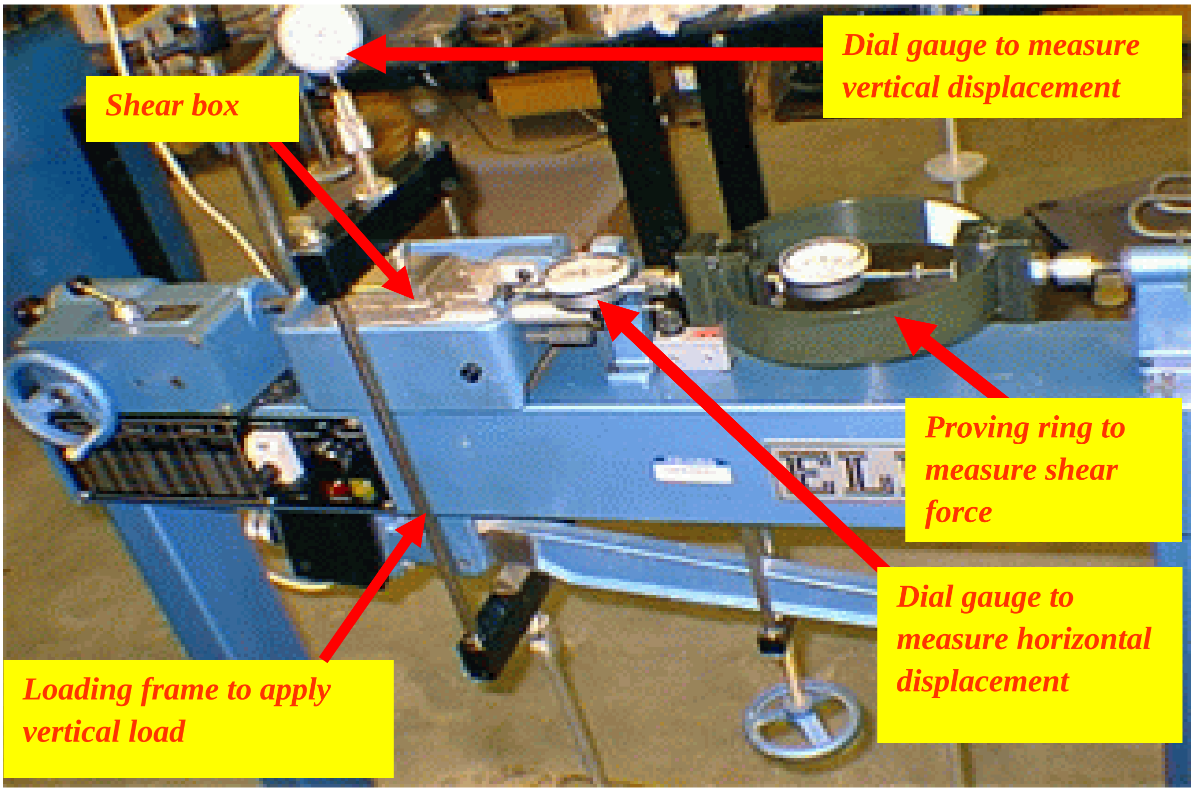
Step 2 Set the specimen in the apparatus and apply the initial stress condition

Step 3 Apply the corresponding field stress conditions

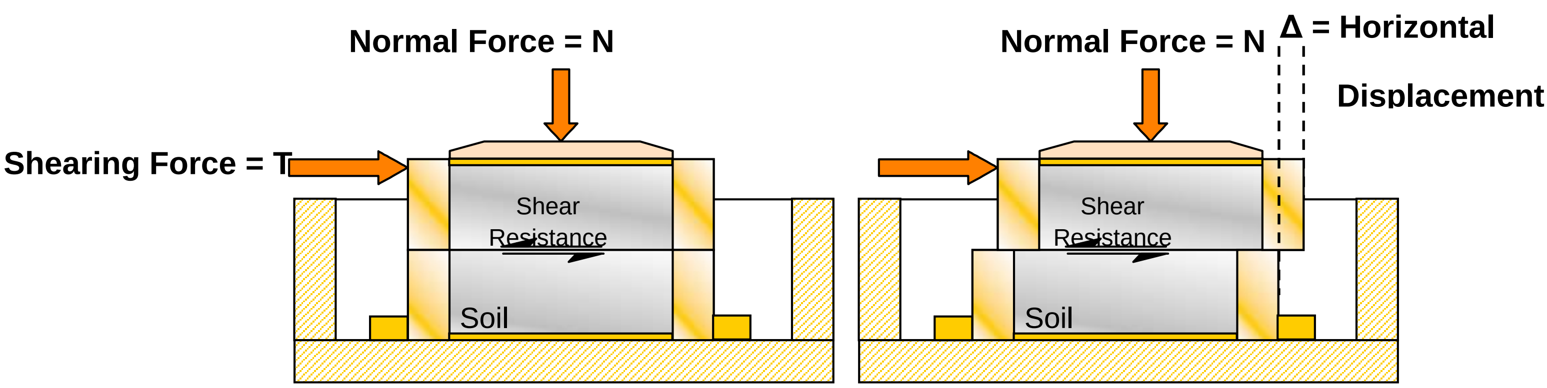
Direct shear test

Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays

Schematic diagram of the direct shear apparatus



Step 1: Apply a vertical load to the specimen and wait for consolidation
Step 2: Lower box is subjected to a horizontal displacement at a constant rate.



Step 3: Repeat this test three times. Each time increase "N"
Analysis of test results

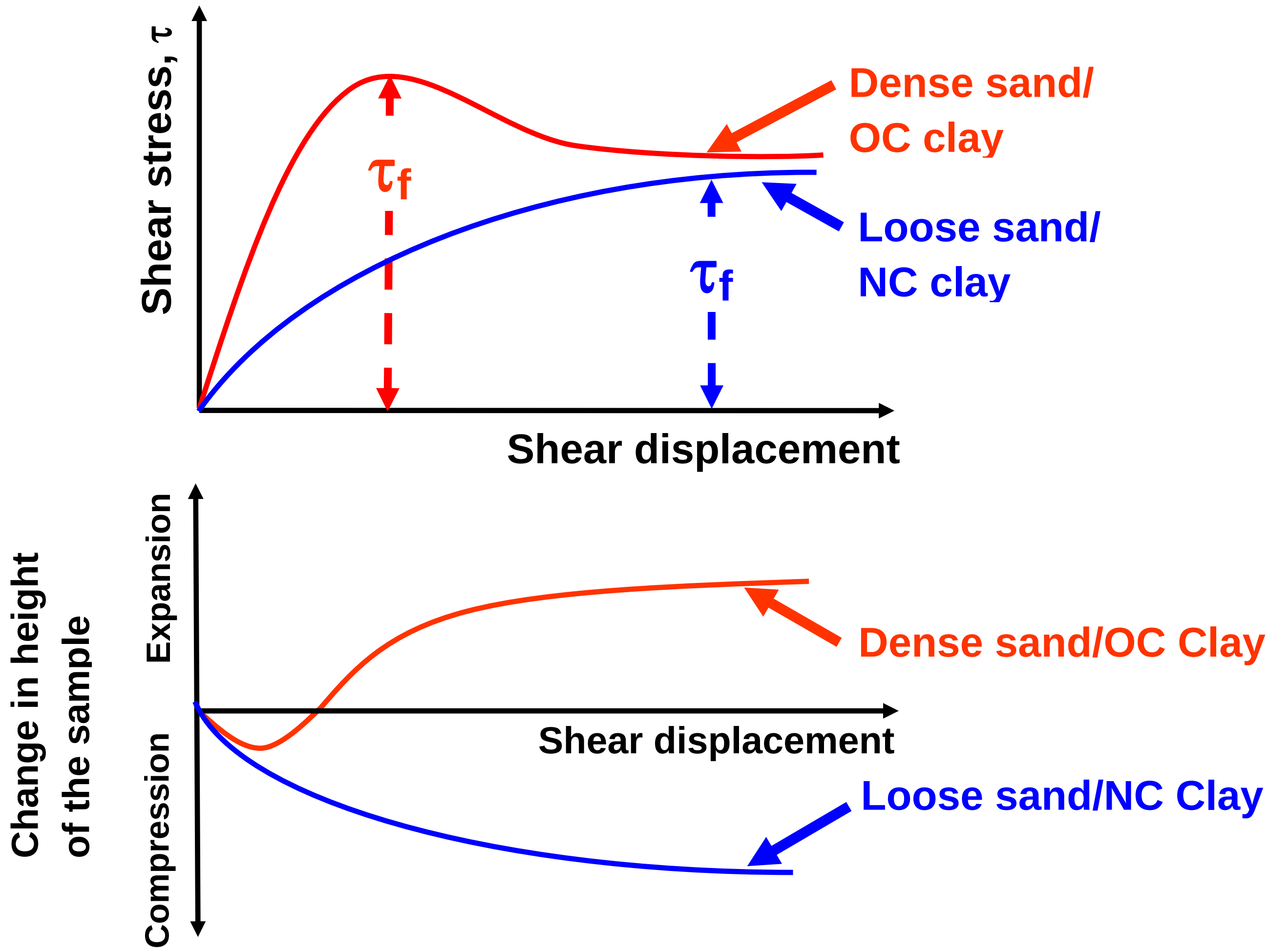
$$\sigma = \frac{Normal\ \ force}{Cross\ \sec tional\ \ area} = \frac{N}{A}$$

$$\tau = \frac{Shearing\ \ force}{Cross\ \sec tional\ area} = \frac{T}{A}$$

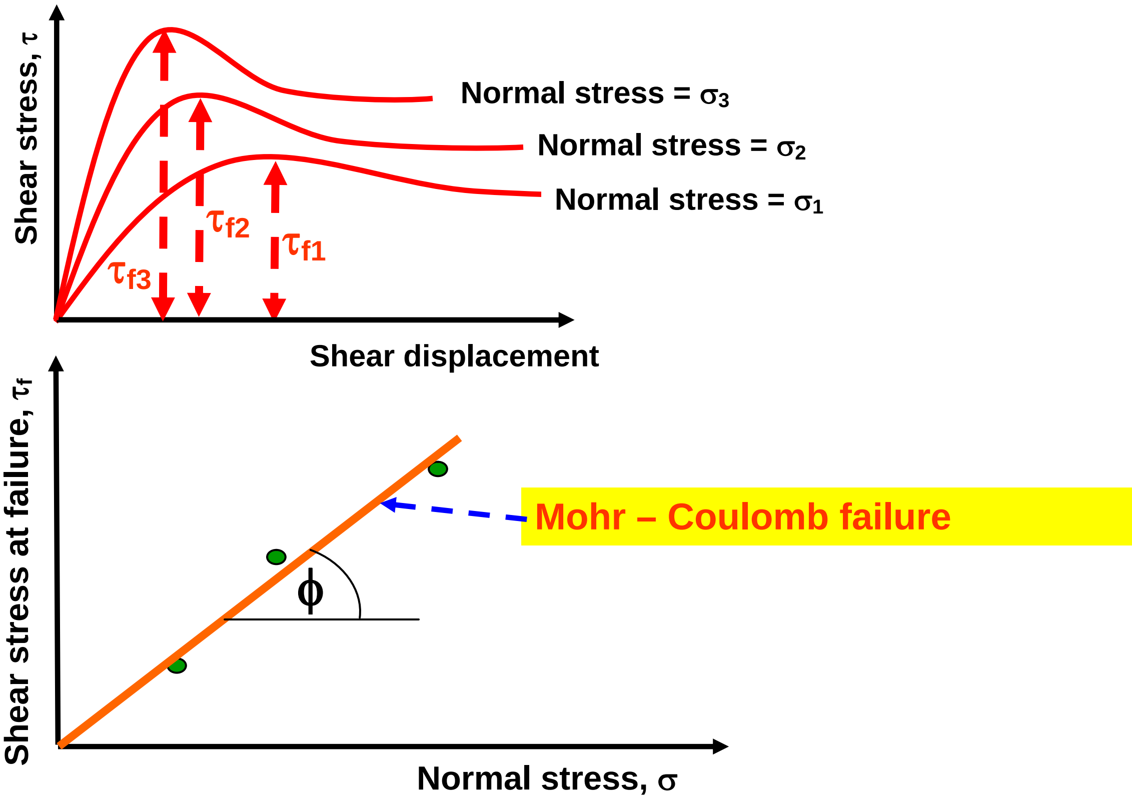
Note: Cross-sectional area of the sample changes with the horizontal displacement $A = (L - \Delta L)^2$

Direct shear tests on sands

Stress-strain relationship



How to determine strength parameters c and φ

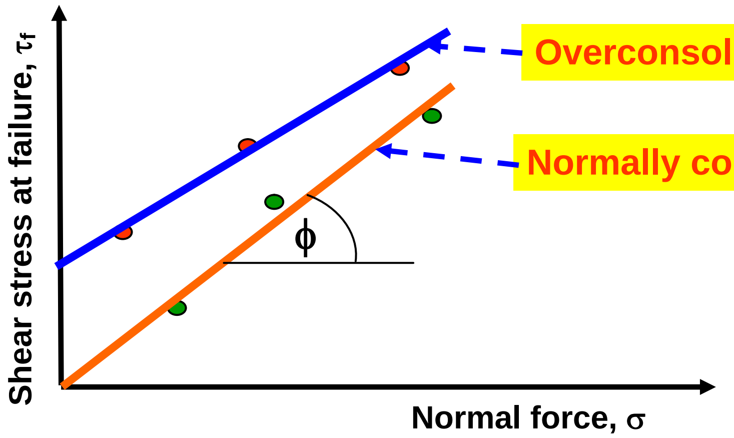


Some important facts on strength parameters c and f of sand

- **Sand is cohesionless hence c = 0**
- **Direct shear tests are drained and pore water pressures are dissipated, hence u = 0, Therefore, $\phi' = \phi$ and $c' = c = 0$**

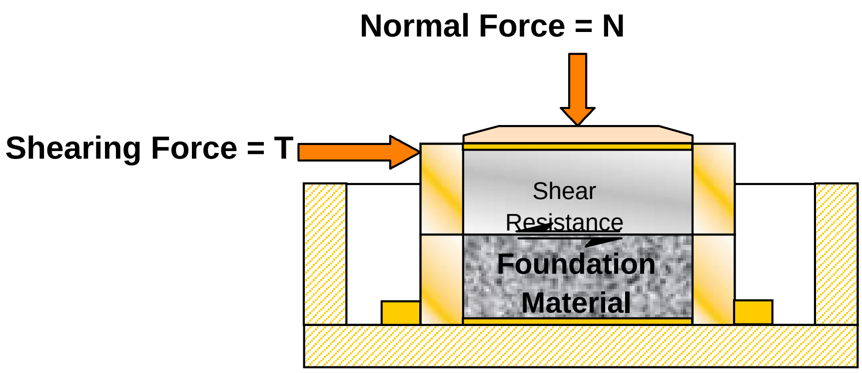
Direct shear tests on clays

In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)



Interface tests on direct shear apparatus

- **In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood)**



$$\boxed{\tau_f = c_a + \sigma' \tan\delta}$$

where c_a = adhesion

δ = angle of wall friction $\leq \phi$

Advantages of direct shear apparatus	Disadvantages of direct shear apparatus
• Due to the smaller thickness of the sample, rapid drainage can be achieved • Can be used to determine interface strength parameters • Clay samples can be oriented along the plane of weakness or an identified failure plane	• Failure occurs along a predetermined failure plane • Area of the sliding surface changes as the test progresses • Non-uniform distribution of shear stress along the failure surface

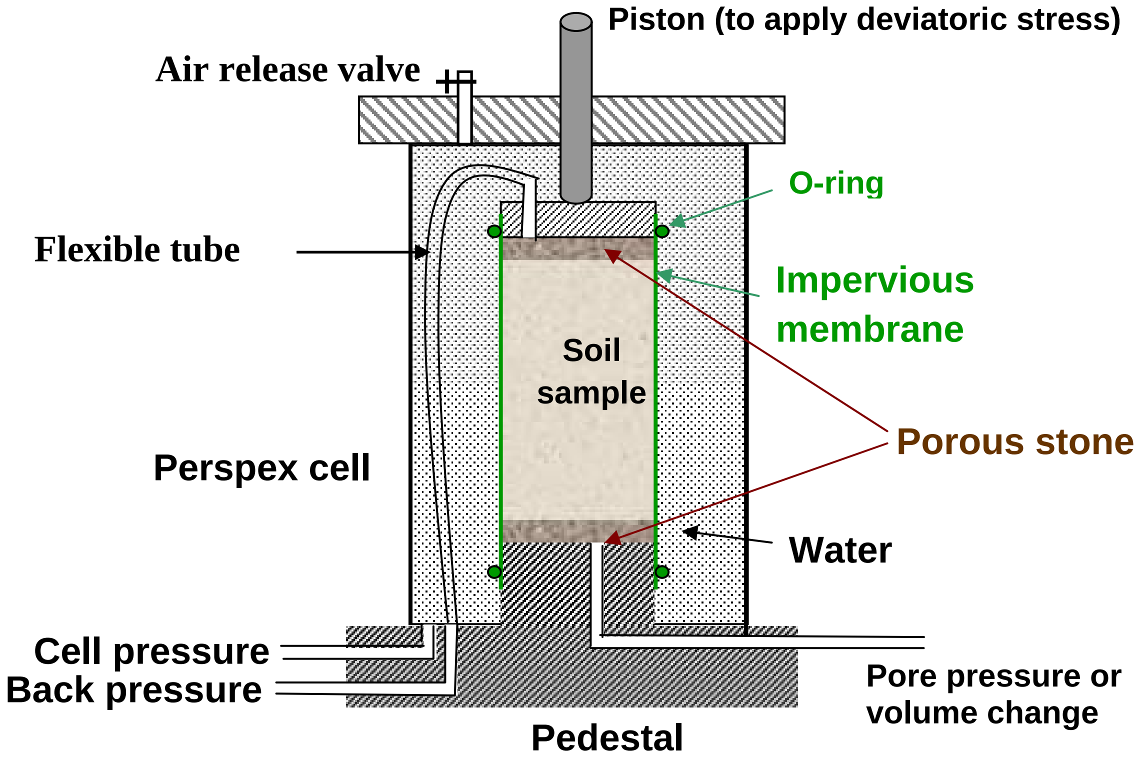
Triaxial Shear Test

Proving ring to measure the deviator load

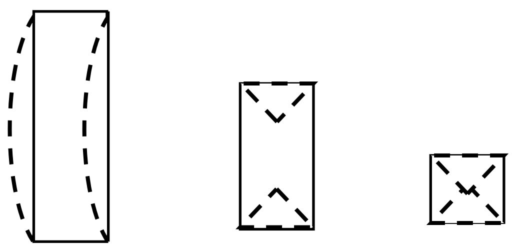
Dial gauge to measure vertical displacement

Failure plane

Soil sample at failure

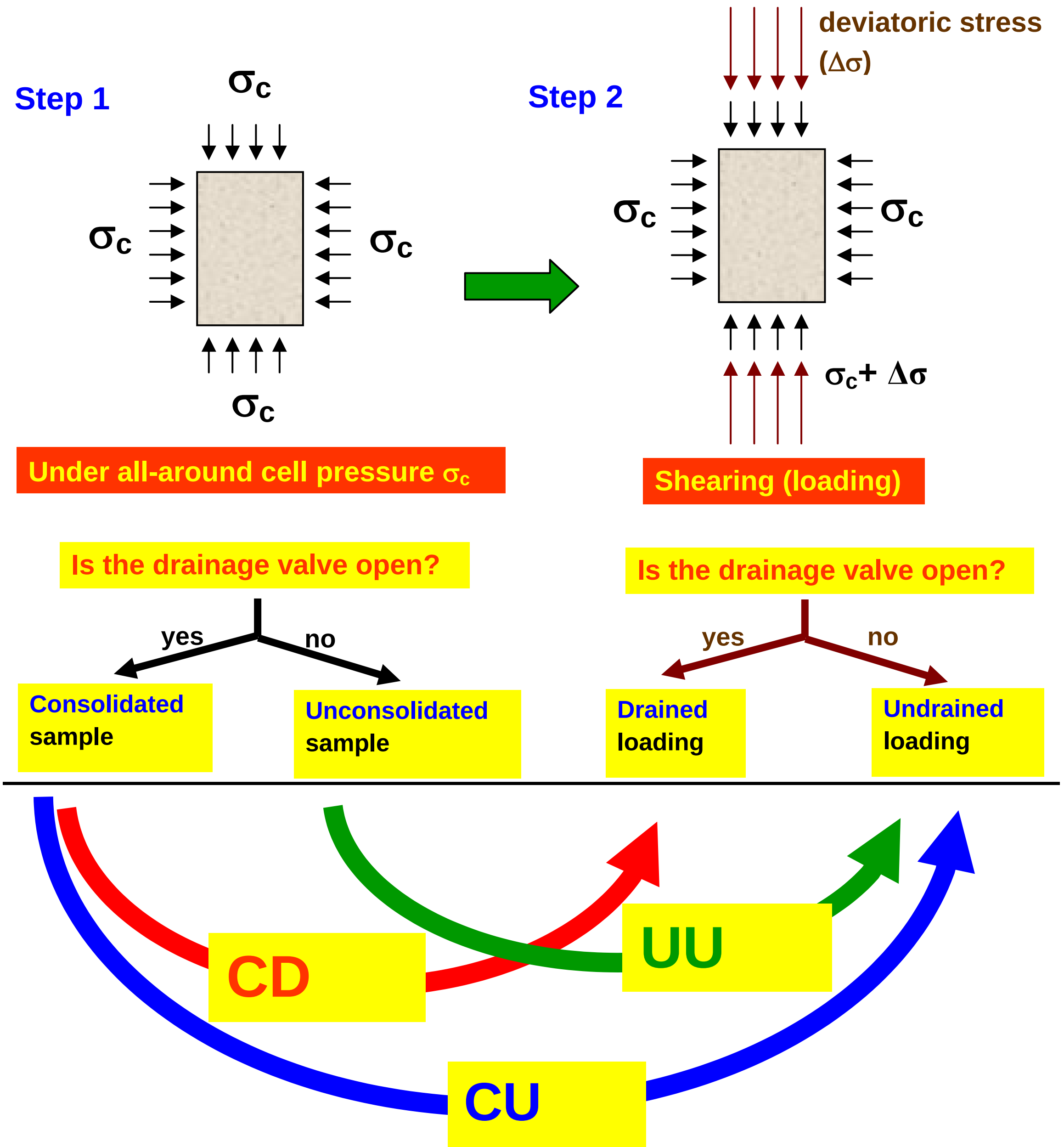


- In this test, a soil sample about 38 mm ($1.5^{//}$) in diameter and 76 mm ($3^{//}$) is generally used (L = 2D – 3D)



- Sample is encased by thin rubber membrane and placed inside a plastic cylindrical chamber that is usually filled with water or glycerine.
- Confining pressure is applied by compression of fluid in the chamber (air sometimes used as a compression medium)
- To cause shear failure in the sample, axial stress is applied through a vertical loading ram (called deviator stress). This can be done in one of two ways
    - Stress-controlled – load is applied in increments and the deformation is measured
    - Strain-controlled – load is applied at a constant rate of deformation

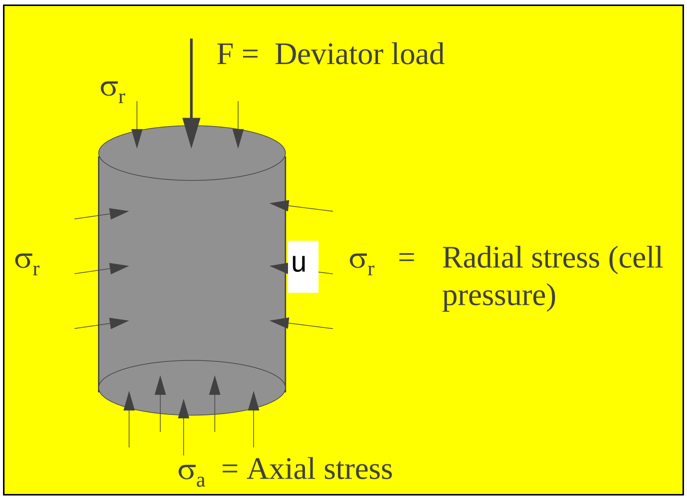
Types of Triaxial Tests



σ_C = confining pressure or all around pressure or cell pressure = σ_3

$\Delta\sigma$ = deviatoric stress = $\sigma_1 - \sigma_3$

Stresses



From vertical equilibrium we have $\sigma_a = \sigma_r + \frac{F}{A}$

The term F/A is known as the deviator stress, and is usually given the symbol $\Delta\sigma$.

Hence we can write $\Delta\sigma = \sigma_a - \sigma_r = \sigma_1 - \sigma_3$ (The axial and radial stresses are principal stresses)

If $\Delta\sigma = 0$ increasing cell pressure will result in:

- Volumetric compression if the soil is free to drain. The effective stresses will increase and so will the strength
- Increasing pore water pressure if soil volume is constant (that is, undrained). As the effective stresses cannot change it follows that $\Delta u = \Delta\sigma_3$

Strains

From the measurements of change in height, dh, and change in volume dV we can determine

Axial strain $\boldsymbol{\varepsilon_a = -dh/h_0}$

Volume strain $\boldsymbol{\varepsilon_v = -dV/V_0}$

Where h_0 is the initial height, and V_0 the initial volume. The conventional small strain assumption is generally used.

It is assumed that the sample deforms as a right circular cylinder. The cross-sectional area, A, can then be determined from

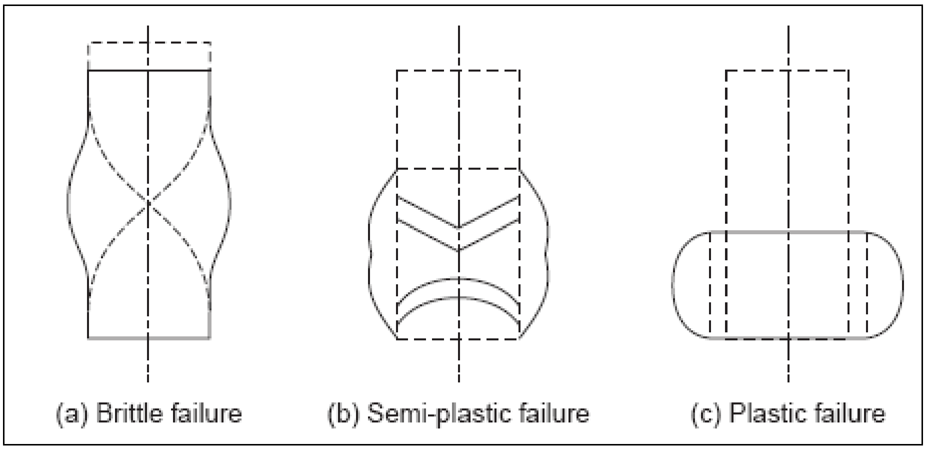
$$A(h_o + \Delta h) = V = V_0 + \Delta V$$

$$A = A_o \left(\frac{1 + \frac{dV}{V_0}}{1 + \frac{dh}{h_0}} \right) = A_o \left(\frac{1 - \varepsilon_v}{1 - \varepsilon_a} \right)$$

For an undrained test $\Delta V = 0$, then $A = A_o \left(\frac{1}{1 - \varepsilon_a} \right)$

It is important to make allowance for the changing area when calculating the deviator stress, $\Delta\sigma = \sigma_1 - \sigma_3 = F/A$

A triaxial compression test specimen may exhibit a particular pattern or shape as failure is reached, depending upon the nature of the soil and its condition, as illustrated in Fig. below



Failure patterns in triaxial compression tests

- brittle failure with well-defined shear plane,
- semi-plastic failure showing shear cones and some lateral bulging,
- Plastic failure with well-expressed lateral bulging.

In the case of plastic failure, the strain goes on increasing slowly at a reduced rate with increasing stress, with no specific stage to pin-point failure. In such a case, failure is assumed to have taken place when the strain reaches an arbitrary value such as 20%.

Merits of Triaxial Compression Test

The following are the significant points of merit of triaxial compression test:

(1) Failure occurs along the weakest plane unlike along the predetermined plane in the case of direct shear test.

(2) The stress distribution on the failure plane is much more uniform than it is in the direct shear test: the failure is not also progressive, but the shear strength is mobilised all at once. Of course, the effect of end restraint for the sample is considered to be a disadvantage; however, this may not have pronounced effect on the results since the conditions are more uniform to the desired degree near the middle of the height of the sample where failure usually occurs.

(3) Complete control of the drainage conditions is possible with the triaxial compression test; this would enable one to simulate the field conditions better.

(4) The possibility to vary the cell pressure or confining pressure also affords another means to simulate the field conditions for the sample, so that the results are more meaningfully interpreted.

(5) Precise measurements of pore water pressure and volume changes during the test are possible.

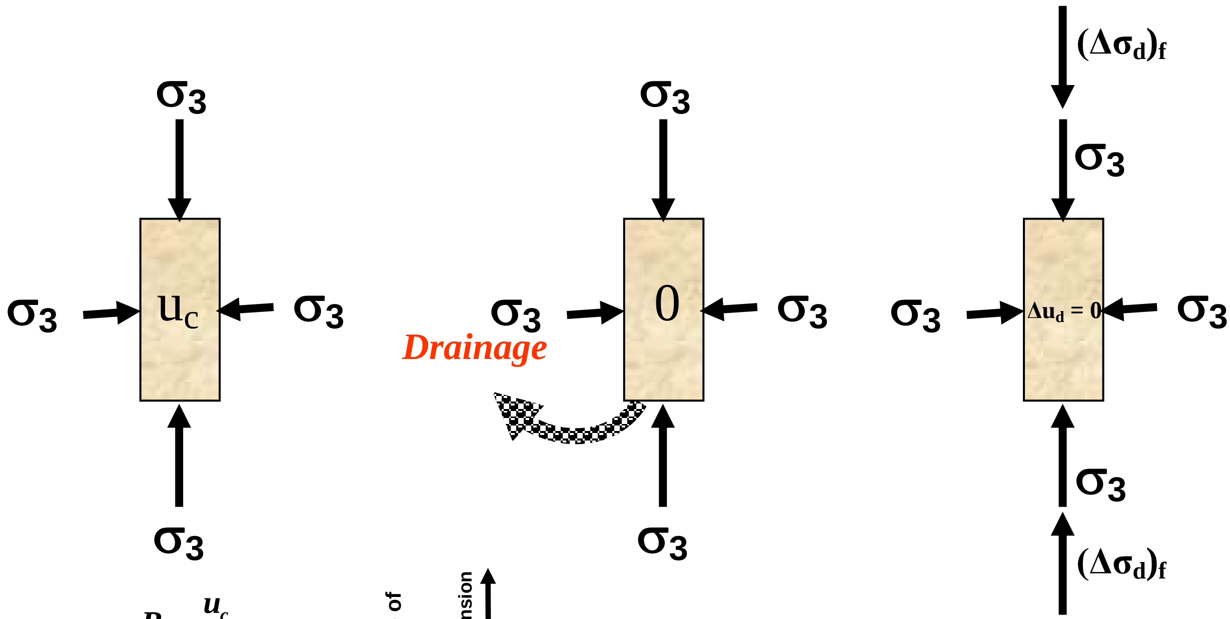
(6) The state of stress within the specimen is known on all planes and not only on a predetermined failure plane as it is with direct shear tests.

(7) The state of stress on any plane is capable of being determined not only at failure but also at any earlier stage.

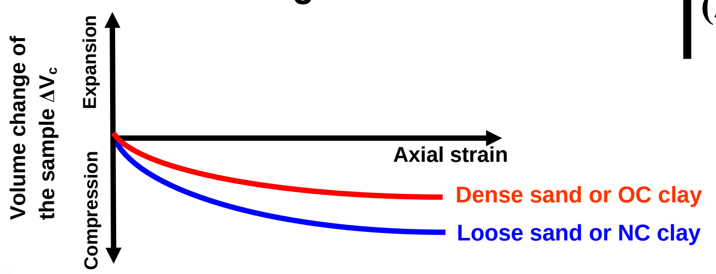
(8) Special tests such as extension tests are also possible to be conducted with the triaxial testing apparatus.

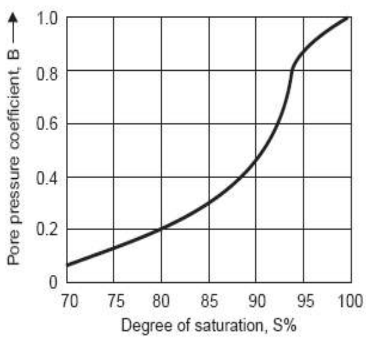
(9) It provides an ingenious and a symmetrical three-dimensional stress system better suited to simulate field conditions.

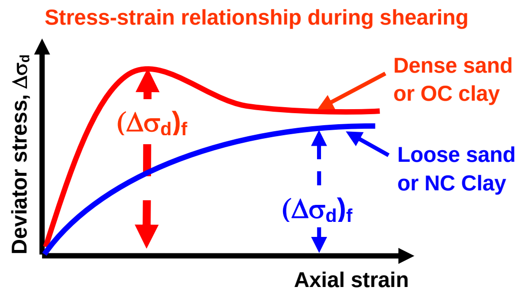
Consolidated - Drained test (CD Test)

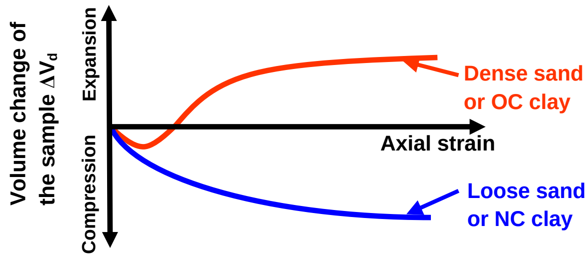


$$B = \frac{u_c}{\sigma_3}$$





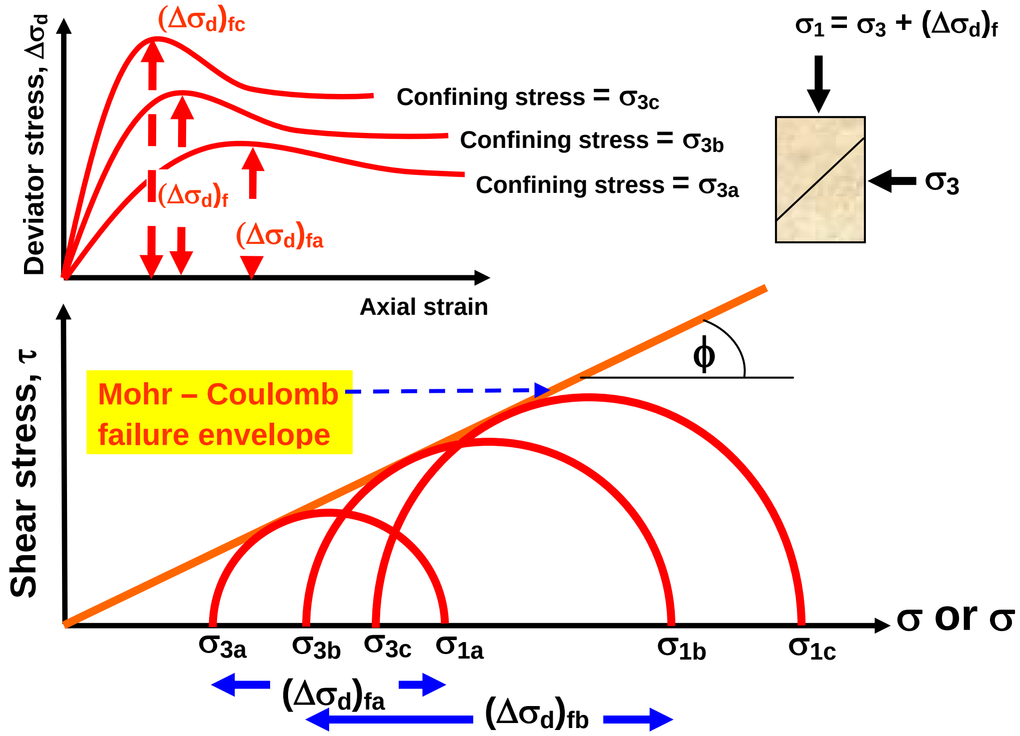




Since u = 0 in CD tests, σ = σ′ Therefore, c = c′ and $\phi = \phi'$ and c_d and ϕ_d are used to denote them

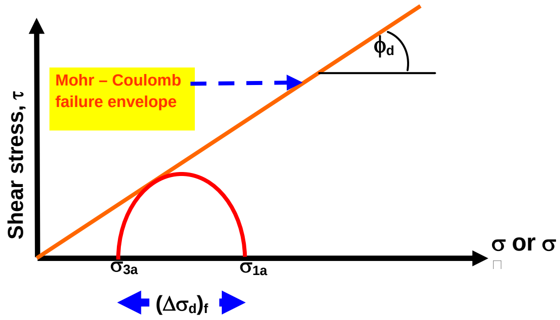
$$\boxed{\begin{aligned}&\sigma_3 = \sigma_3' \\ &\sigma_1 = \sigma_1' = \sigma_3 + (\Delta\sigma_d)_f\end{aligned}}$$

CD tests How to determine strength parameters c and ϕ



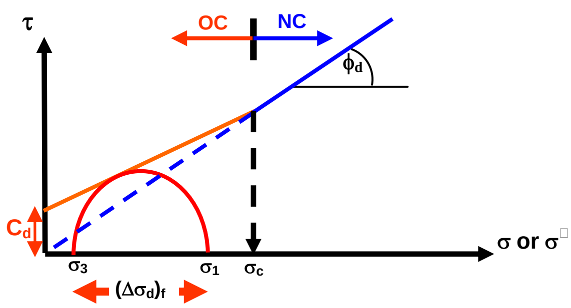
CD tests Failure envelopes

For sand and NC Clay, $c_d = 0$



CD tests Failure envelopes

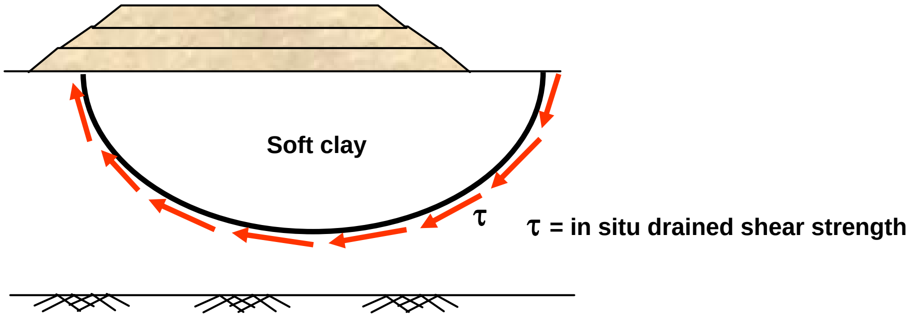
For OC Clay, $c_d \neq 0$



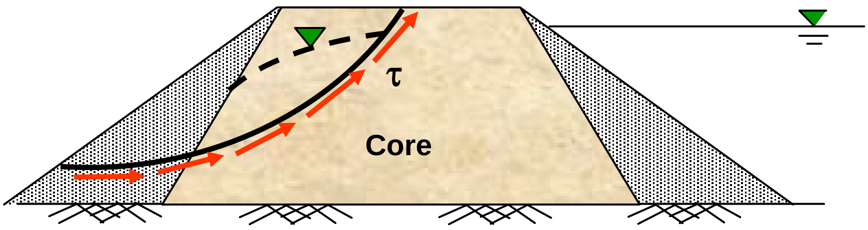
Therefore, one CD test would be sufficient to determine ϕ_d of sand or NC clay

Some practical applications of CD analysis for clays

1. Embankment constructed very slowly, in layers over a soft clay deposit

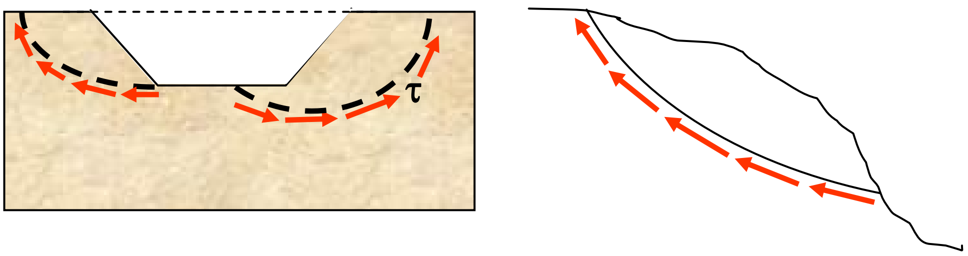


2. Earth dam with steady state seepage



τ = drained shear strength of clay core

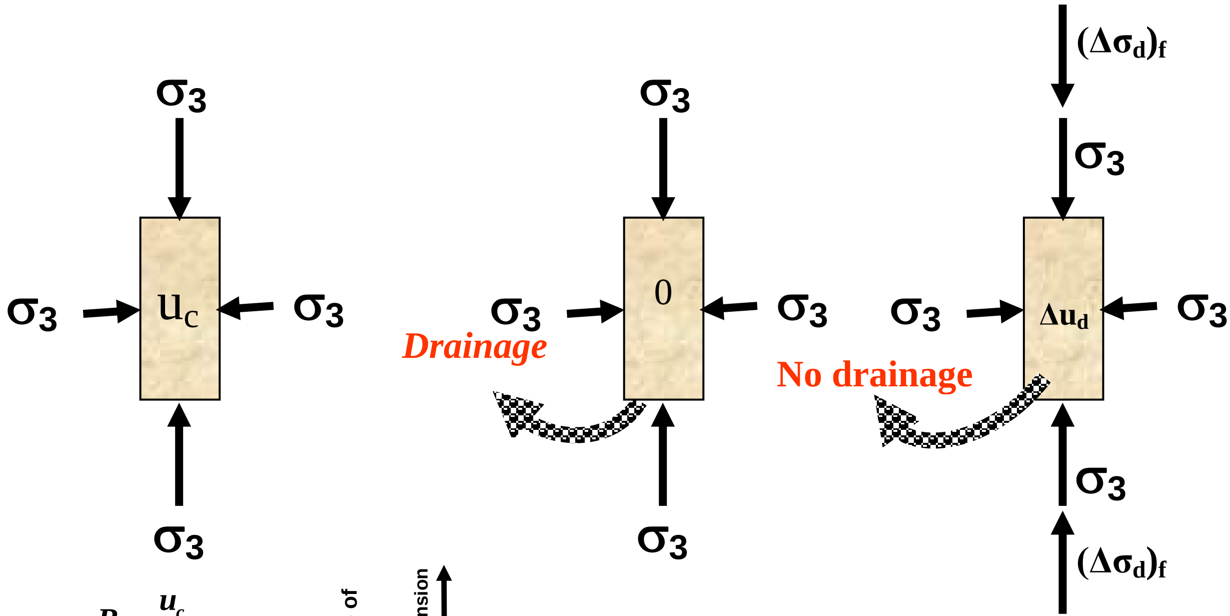
3. Excavation or natural slope in clay



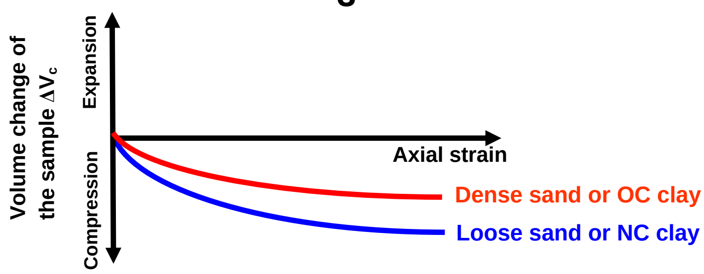
τ = In situ drained shear strength

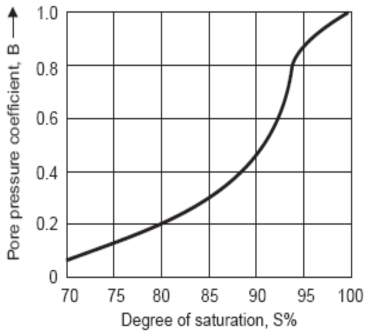
Note: CD test simulates the long term condition in the field. Thus, c_d and ϕ_d should be used to evaluate the long term behavior of soils

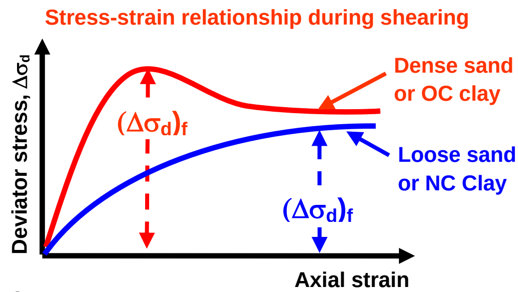
Consolidated- Undrained test (CU Test)

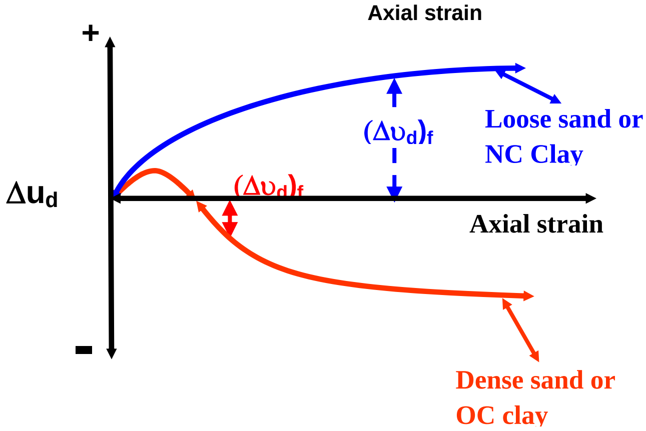


$$B = \frac{u_c}{\sigma_3}$$







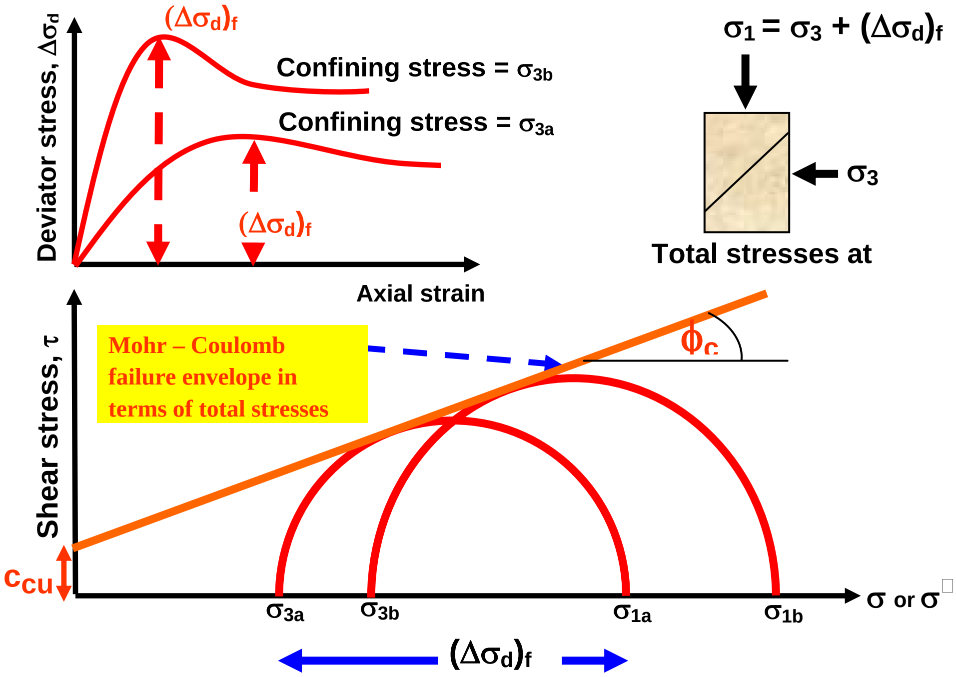


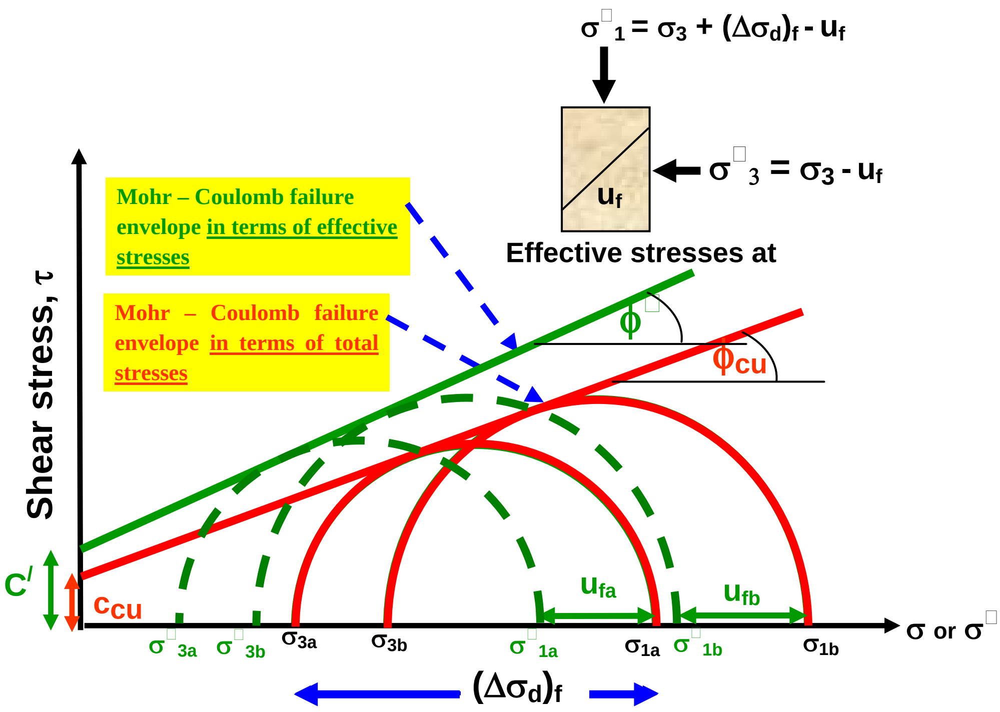
$$A = \frac{\Delta u_d}{\Delta \sigma_d}$$

$$A_f = \frac{(\Delta u_d)_f}{(\Delta \sigma_d)_f}$$

Soil	A_f
N.C	0.5-1
O.C	0-0.5

CU tests How to determine strength parameters c

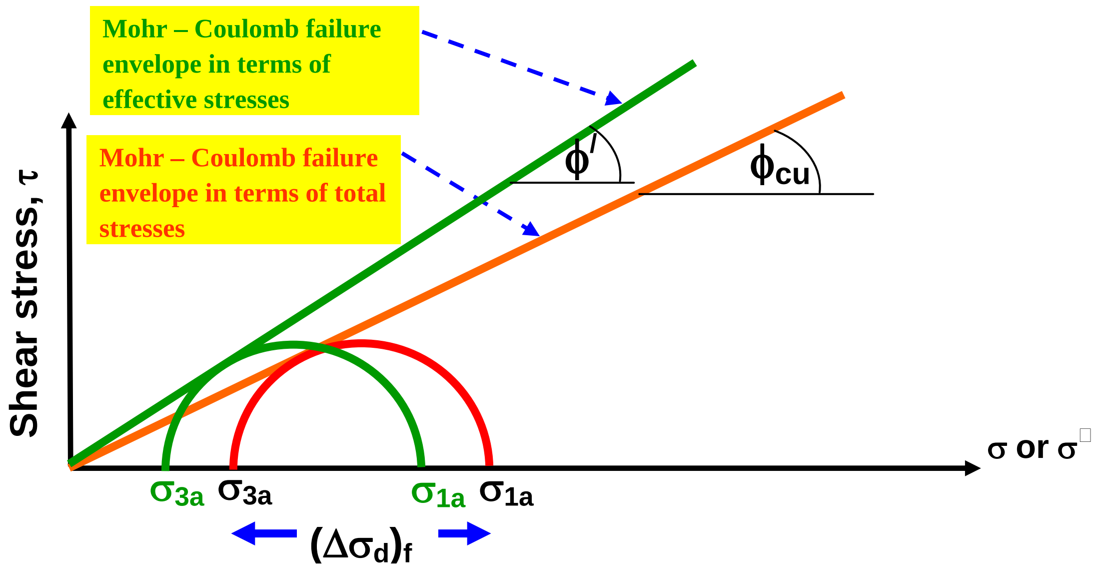




- **Shear strength parameters <u>in terms of total stresses</u> are C_{cu} and Φ_{cu}**
- **Shear strength parameters in terms of effective stresses are $C^{/}$ and $\Phi^{/}$**
- **$C^{/} = C_{drained}$ and $\Phi^{/} = \Phi_{drained}$**

CU tests Failure envelopes

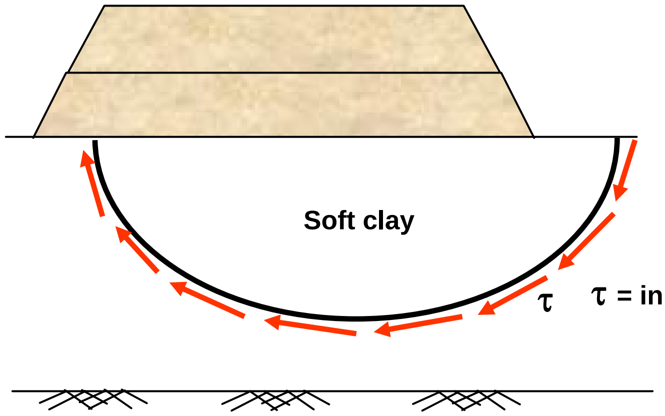
For sand and NC Clay, c_{cu} and $c^{/} = 0$



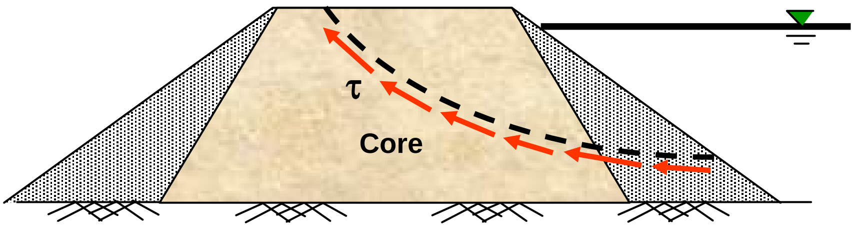
Therefore, one CU test would be sufficient to determine ϕ_{cu} and $\phi^{/} = \phi_d$ of sand or NC clay

Some practical applications of CU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit

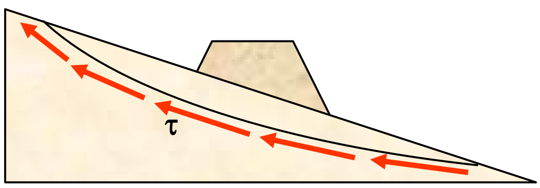


2. Rapid drawdown behind an earth dam



τ = Undrained shear strength of clay core

3. Rapid construction of an embankment on a natural slope



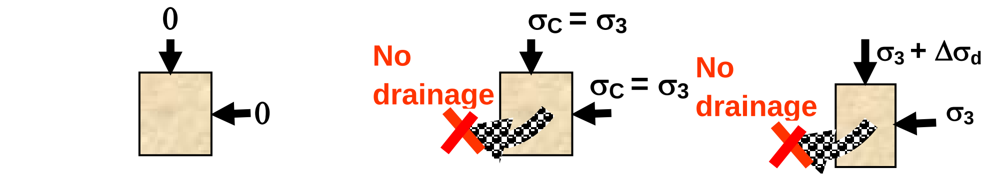
τ = In situ undrained shear strength

Note: Total stress parameters from CU test (C_{cu} and Φ_{cu}) can be used for stability problems where, Soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring

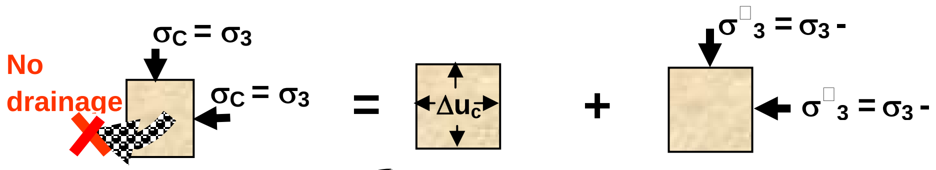
Unconsolidated- Undrained test (UU Test)

Data analysis

Step 1 Immediately after sampling	Step 2 Initial specimen condition	Step 3 Specimen condition during shearing



Step 2: After application of hydrostatic cell pressure



$$\Delta u_c = B \, \Delta\sigma_3$$

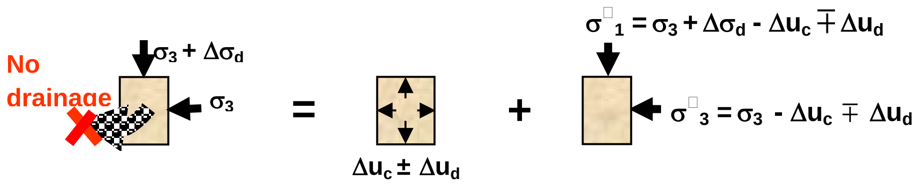
Increase of pwp due to increase of cell

Skempton's pore water pressure

Increase of cell

Note: If soil is fully saturated, then B = 1 (hence, $\Delta u_c = \Delta\sigma_3$)

Step 3: During application of axial load



$$\Delta u_d = A \, B \, \Delta\sigma_d$$

Increase of pwp due to increase of deviator stress

Increase of deviator stress

Skempton's pore water pressure parameter, A

Combining steps 2 and 3,

$\Delta u_c = B\ \Delta\sigma_3$ $\qquad$ $\Delta u_d = AB\Delta\sigma_d$

Total pore water pressure increment at any stage, Δu

$$\Delta u = \Delta u_c + \Delta u_d$$

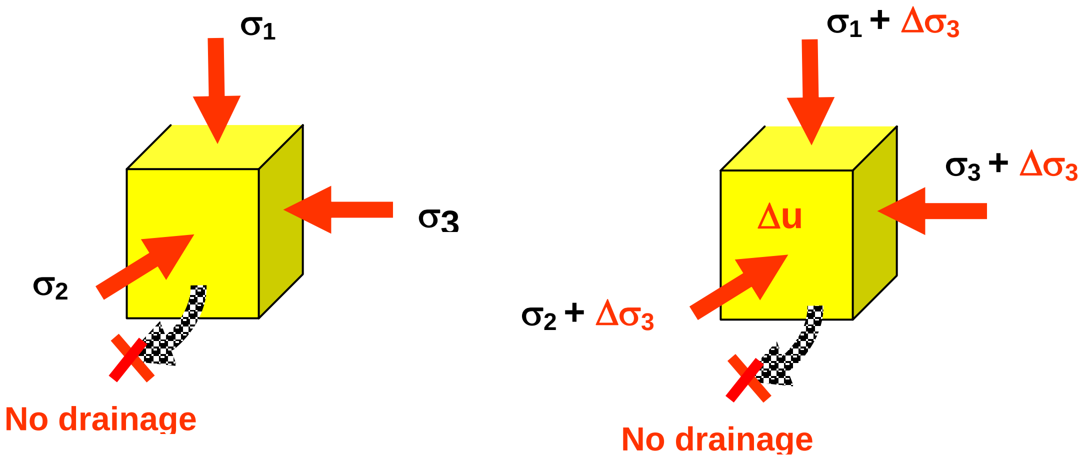
$$\Delta u = B\ [\Delta\sigma_3 + A\Delta\sigma_d]$$

$$\Delta u = B\ [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Skempton's pore water pressure equation

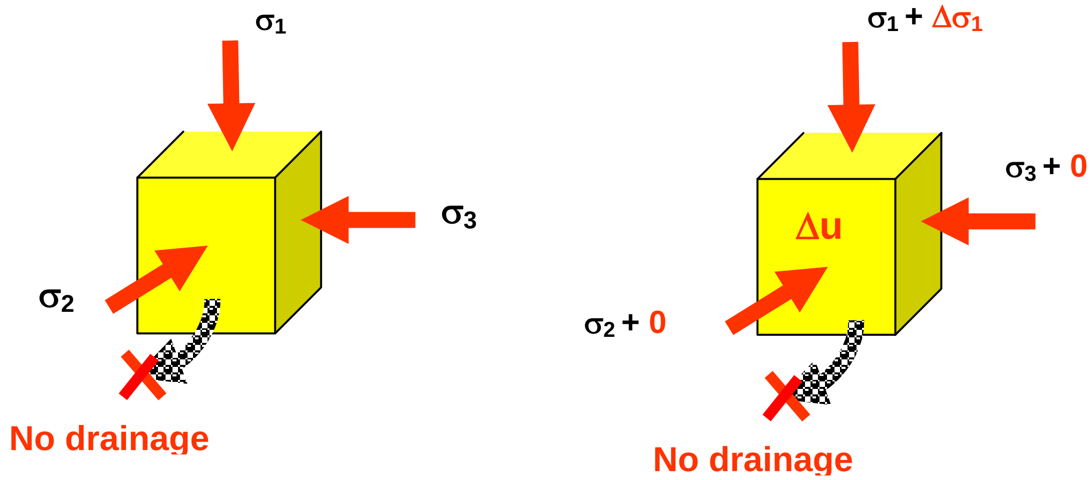
Derivation of Skempton's pore water pressure equation

Step 1 :Increment of isotropic stress



Increase in effective stress in each direction = $\Delta\sigma_3 - \Delta u_c$

Step 2 :Increment of major principal stress

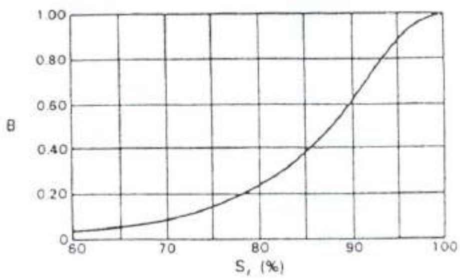


Increase in effective stress in σ_1 direction = $\Delta\sigma_1$

Increase in effective stress in σ_2 and σ_3 directions = 0

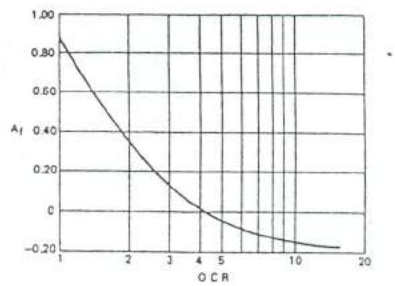
Average Increase in effective stress = $(\Delta\sigma_1 - \Delta u_d - \Delta u_d - \Delta u_d)/3$

Typical values for parameter B

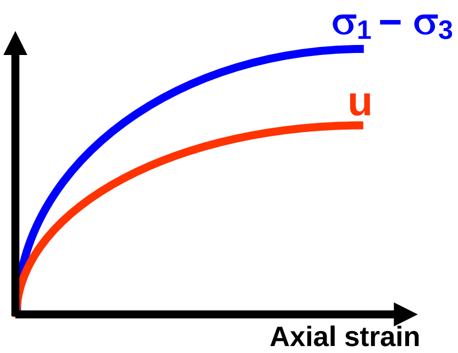


Typical relationship between B and degree of saturation.

Typical values for parameter A

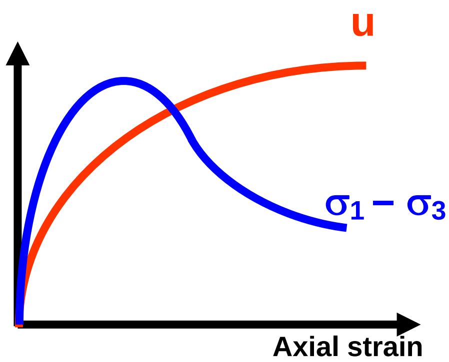


Typical relationship between A at failure and overconsolidation



NC Clay (low sensitivity)

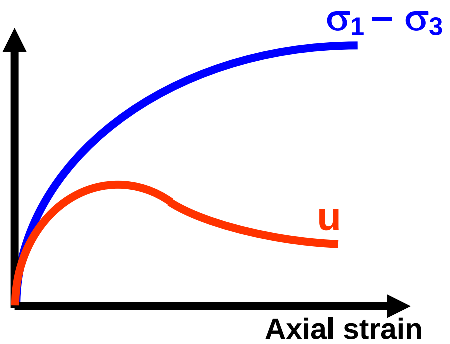
(A = 0.5 – 1.0)



NC Clay (High sensitivity)

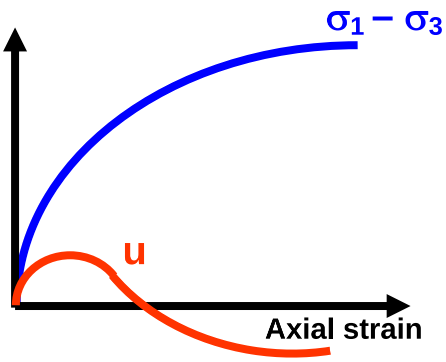
(A > 1.0)

Collapse of soil structure may occur in high sensitivity clays due to very high pore water pressure generation



OC Clay (Lightly overconsolidated)

(A = 0.0 – 0.5)



OC Clay (Heavily overconsolidated)

(A = -0.5 - 0.0)

During the increase of major principal stress pore water pressure can become negative in heavily overconsolidated clays due to dilation of specimen

● Stress Path

A ‘‘Stress–Path’’ is a curve or a straight line which is the locus of a series of stress points depicting the changes in stress in a test specimen or in a soil element in-situ, during loading or unloading, engineered as in a triaxial test in the former case or caused by forces of nature.

An elementary way to monitor stress changes is by showing the Mohr’s stress circles at different stages of loading/unloading. But this may be cumbersome as well as confusing when a number of circles are to be shown in the same diagram.

Stress-path approach enables the engineer to predict and monitor the shear strength mobilized at any stage of loading/unloading in order to ensure the stability of foundation soil.

Lambe and Whitman (1969) have suggested the locus of points representing the maximum shear stress acting on the soil at different stages be treated as a ‘stress path’, which can be drawn and studied in place of the corresponding Mohr’s circles. This is shown in Fig. below.

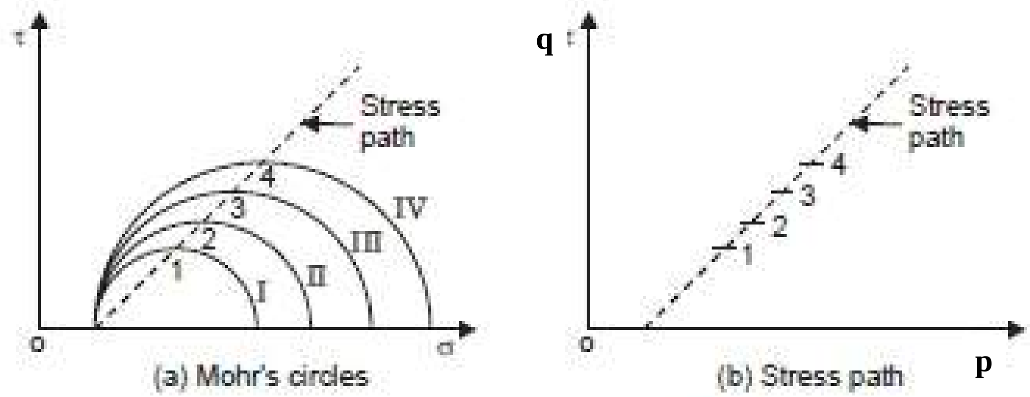


Fig. 8.23 Stress path (Lambe and Whitman, 1969) for the case of σ_1 increasing and σ_3 constant

The co-ordinates of the points on the stress path

$$q = \frac{\sigma_1 - \sigma_3}{2} \qquad p = \frac{\sigma_1 + \sigma_3}{2}$$

If σ_1 and σ_3 are the vertical and horizontal principal stresses, these become

$$q = \frac{\sigma_v - \sigma_h}{2} \qquad p = \frac{\sigma_v + \sigma_h}{2}$$

Either the effective stresses or the total stresses may be used for this purpose. The basic types of stress path and the co-ordinates are:

(a) Effective Stress Path (ESP) $\left[\left(\frac{\bar{\sigma}_1+\bar{\sigma}_3}{2}\right),\left(\frac{\bar{\sigma}_1-\bar{\sigma}_3}{2}\right)\right]$ **p′ , q′**

(b) Total Stress Path (TSP) $\left[\left(\frac{\sigma_1+\sigma_3}{2}\right),\left(\frac{\sigma_1-\sigma_3}{2}\right)\right]$ **p = p′+u , q = q′**

(c) Stress path of total stress less static pore water pressure (TSSP)

$$\left[\left(\frac{\sigma_1+\sigma_3}{2}-u_0\right),\left(\frac{\sigma_1-\sigma_3}{2}\right)\right]$$

u_0 : Static pore water pressure

u_0 = zero in the conventional triaxial test, and (b) and (c) coincide in this case. But if back pressure is used in the test, u_0 = the back pressure.

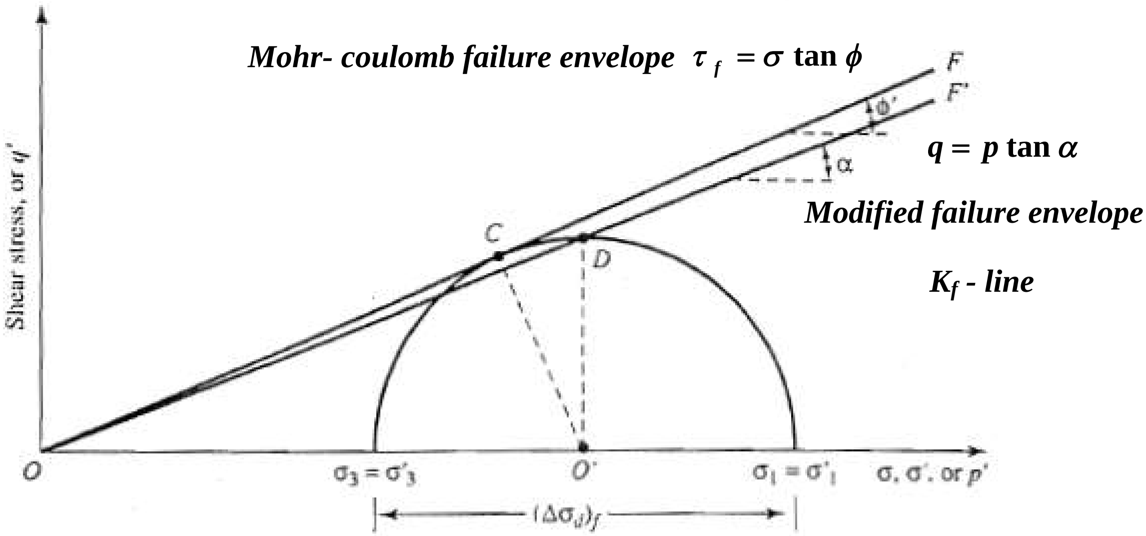
For an in-situ element, the static pore water pressure depends upon the level of the ground water table.

Slope of stress path line $= \frac{\Delta q}{\Delta p} = \frac{q_f - q_o}{p_f - p_o}$

Where q_f and p_f are coordinates at failure and q_o and p_o are coordinates at initial condition.

Modified Failure Envelope

For N.C soil



$$\frac{DO'}{OO'} = \tan \alpha$$

$$\tan \alpha = \frac{\frac{\sigma_1' - \sigma_3'}{2}}{\frac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

Again,

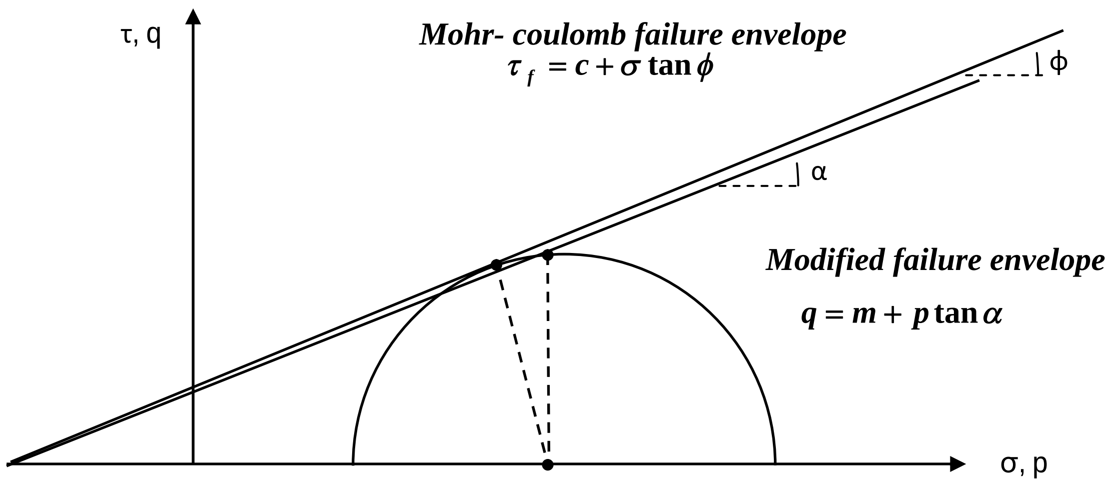
$$\frac{CO'}{OO'} = \sin \phi'$$

or

$$\sin \phi' = \frac{\frac{\sigma_1' - \sigma_3'}{2}}{\frac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

$$\sin \phi' = \tan \alpha$$

For O.C. clay



Where m = c cosφ

Typical stress paths for triaxial compression and extension tests (loading as well as unloading cases) are shown in Fig. below

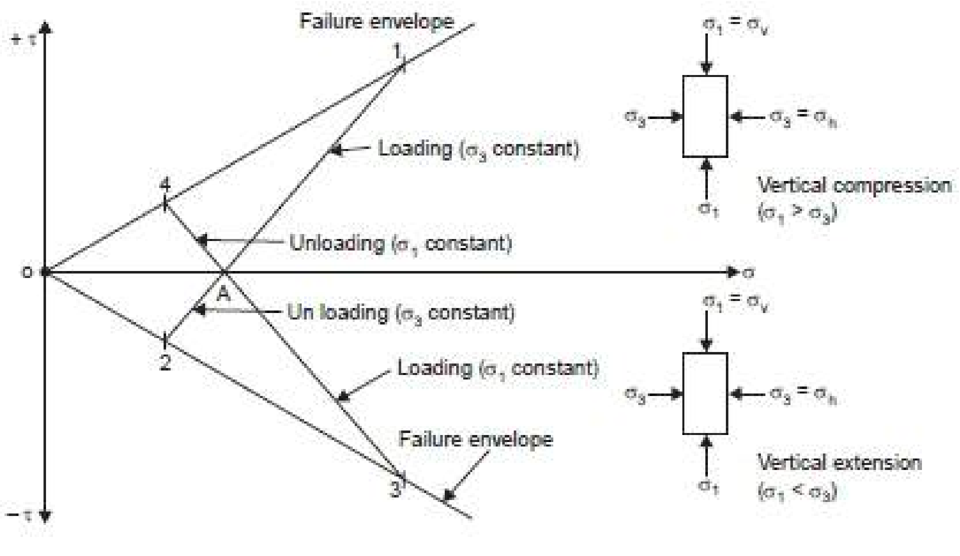


Fig. 8.24 Typical stress paths for triaxial compression and extension tests (loading/unloading)

A-1 is the effective stress path for conventional triaxial compression test during loading. (Δσv = positive and Δσh = 0, i.e., σh is constant). A typical field case is a footing subjected to vertical loading.

A-2 is the unloading case of the triaxial extension text (Δσh = 0 and Δσv = negative). Foundation excavation is a typical field example.

A-3 is the loading case of the triaxial extension test (Δσv = 0 and Δσh = positive). Passive earth resistance is represented by this stress path.

A-4 is the unloading case of the triaxial compression test (Δσu = 0 and Δσh = negative). Active earth pressure on retaining walls is the typical field example for this stress path

For a drained test

Figure below shows the typical stress paths. Point **A** corresponds to the stress condition with only the confining pressure acting (σ1 = σ3 and τ = 0). Point **F** represents failure. Stress paths for effective stresses, total stresses, and total stresses less static pore water pressure are shown separately in the same figure.

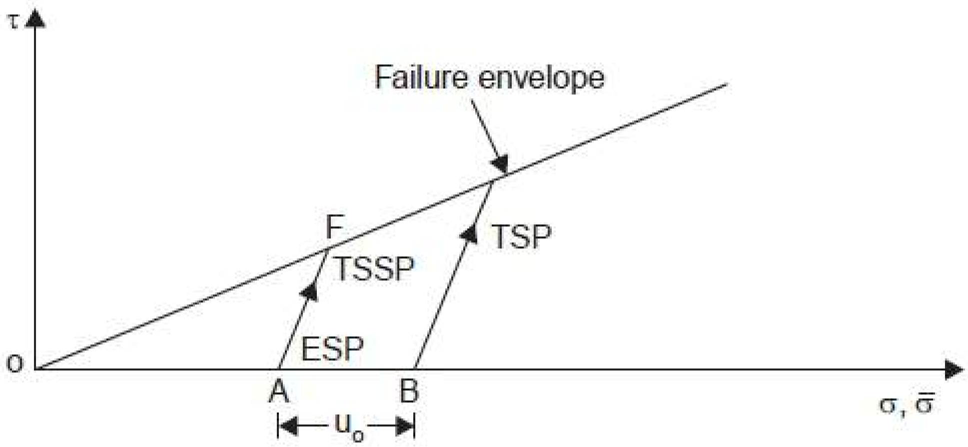


Fig. 8.25 Stress paths for drained test

For a consolidated Undrained test on a normally consolidated clay.
Figure below shows the typical stress paths.

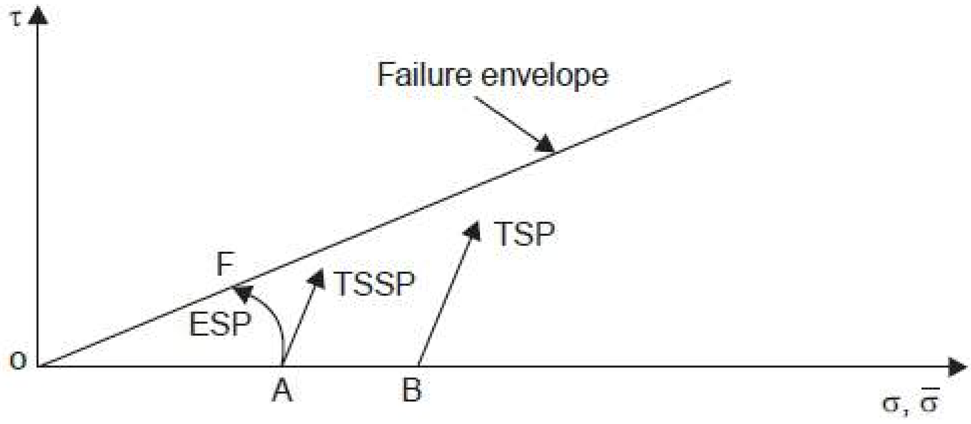


Fig. 8.26 Stress paths for consolidated undrained test on a normally consolidated clay

For a consolidated Undrained test on over consolidated clay
Figure below shows the typical stress paths.

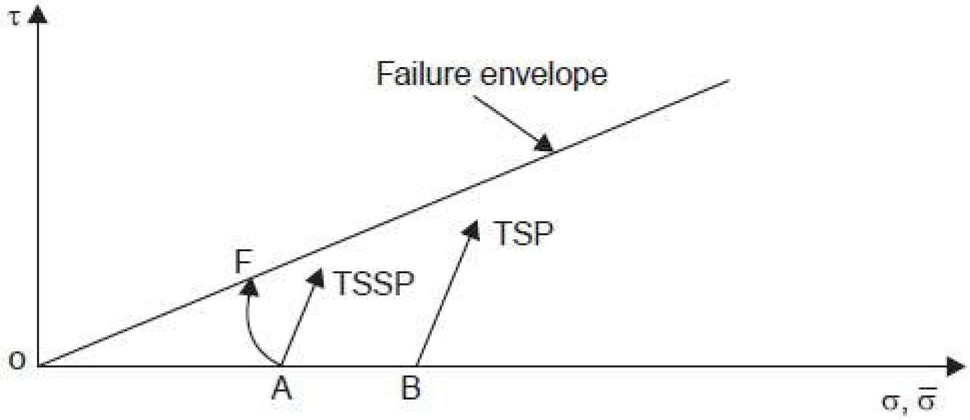


Fig. 8.27 Stress paths for consolidated undrained test on an overconsolidated clay

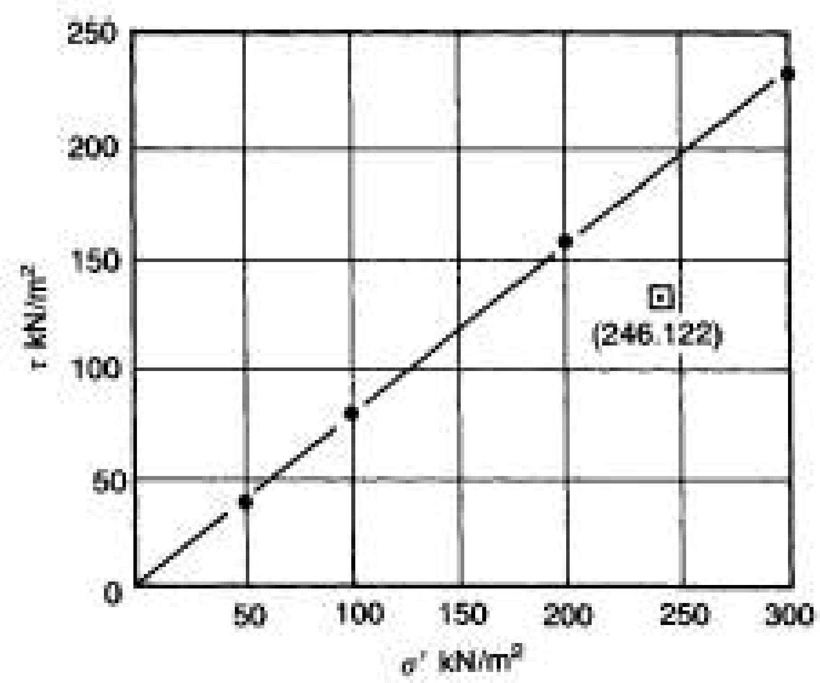
[Note : TSSP to the right of ESP indicates of positive excess pore pressure; TSSP to the left of ESP indicates negative excess pore pressure. Both coincide for zero excess pore pressure].

Example 1

The following results were obtained from direct shear tests on specimens of a sand compacted to the in-situ density. Determine the value of the shear strength parameter $\varphi^{/}$.

Normal stress (kN/m^2)	50	100	200	300
Shear stress at failure (kN/m^2)	**36**	**80**	**154**	**235**

Would failure occur on a plane within a mass of this sand at a point where the shear stress is 122 kN/m^2 and the effective normal stress 246 kN/m^2?



The values of shear stress at failure are plotted against the corresponding values of normal stress, as shown in Figure above. The failure envelope is the line having the best fit to the plotted points; in this case a straight line through the origin. If the stress scales are the same, the value of $\varphi^{/}$ can be measured directly and is 38^o.

The stress state τ=122kN/m^2, $\varphi^{/}$ = 246 kN/m^2 plots below the failure envelope, and therefore would not produce failure.

Example 2

The results shown in Table below were obtained at failure in a series of triaxial tests on specimens of a saturated clay initially 38mm in diameter by 76mm long. Determine the values of the shear strength parameters with respect to (a) total stress and (b) effective stress.

Solution

The initial values of length, area and volume for each specimen are:

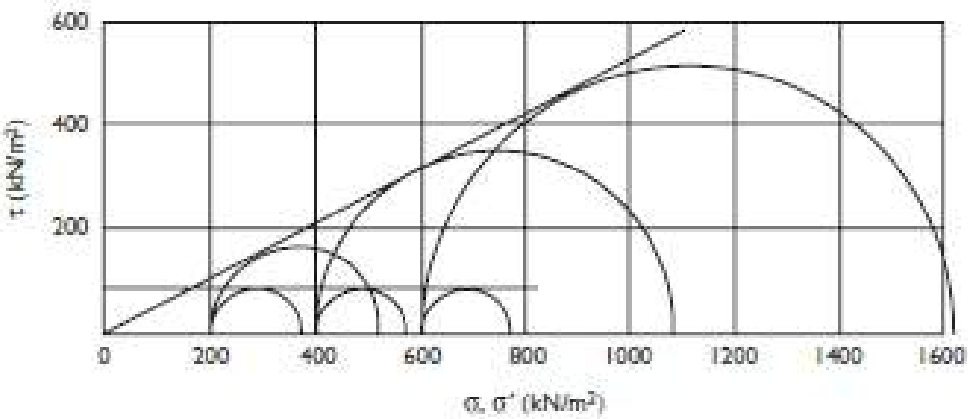
l_o = 76mm; A_0 = 1135mm^2; V_0 = 86 x 103 mm^3

Table 4.2

	Type of test	All-round pressure (kN/m^2)	Axial load (N)	Axial deformation (mm)	Volume change (ml)
(a)	Undrained	200	222	9.83	–
		400	215	10.06	–
		600	226	10.28	–
(b)	Drained	200	403	10.81	6.6
		400	848	12.26	8.2
		600	1265	14.17	9.5

Table 4.3

	σ_3 (kN/m^2)	$\Delta l/l_0$	$\Delta V/V_0$	Area (mm^2)	$\sigma_1 - \sigma_3$ (kN/m^2)	σ_1 (kN/m^2)
(a)	200	0.129	–	1304	170	370
	400	0.132	–	1309	164	564
	600	0.135	–	1312	172	772
(b)	200	0.142	0.077	1222	330	530
	400	0.161	0.095	1225	691	1091
	600	0.186	0.110	1240	1020	1620



Cu = 85 kN/m^2; φ_u = 0 **c$^/$ = 0; $\varphi^/$ = 27^0**

Example 3

The results shown in Table below were obtained for peak failure in a series of consolidated–undrained triaxial tests, with pore water pressure measurement, on specimens of saturated clay. Determine the values of the effective stress parameters.

All-round pressure (kN/m^2)	Principal stress difference (kN/m^2)	Pore water pressure (kN/m^2)
150	192	80
300	341	154

σ_3	σ_1	σ_3'	σ_1'
150	342	70	262
300	641	146	487

$$\sigma_1' = \sigma_3' \tan^2(45+\frac{\phi'}{2}) + 2c'\tan(45+\frac{\phi'}{2})$$

$$262 = 70\tan^2(45+\frac{\phi'}{2}) + 2c'\tan(45+\frac{\phi'}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$487 = 146\tan^2(45+\frac{\phi'}{2}) + 2c'\tan(45+\frac{\phi'}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solve eqs. 1 and 2 simultaneously we get,

$\varphi' = 29.67^o \approx 30^o$ $\qquad$ $c' \approx 16$ kN/m^2

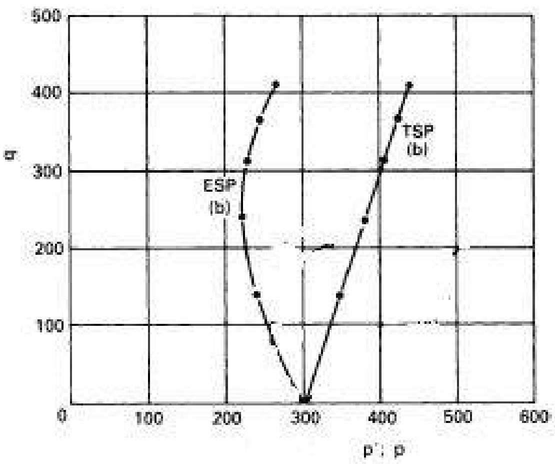
Example 4

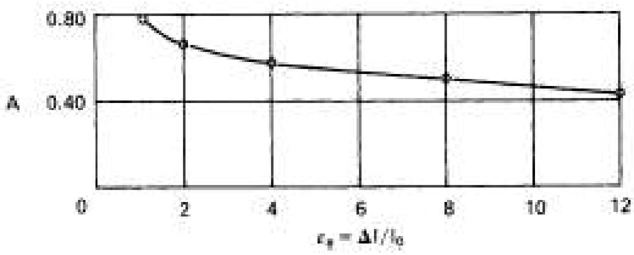
The following results refer to a consolidated-undrained triaxial test on a saturated clay specimen under an all-round pressure of 300 kN/m^2:

$\Delta l/l_o$	0	0.01	0.02	0.04	0.08	0.12
$\sigma_1 - \sigma_3$ (kN/m^2)	0	138	240	312	368	410
u (kN/m^2)	0	108	158	178	182	172

Draw the total and effective stress paths and plot the variation of the pore pressure coefficient A during the test.

$\Delta l/l_0$	0	0.01	0.02	0.04	0.08	0.12
q	0	138	240	312	368	410
p	300	346	380	404	423	437
p'	300	238	222	226	241	265
A	–	0.78	0.66	0.57	0.50	0.42





From the shape of the effective stress path and the value of A at failure it can be concluded that the clay is overconsolidated.




Soil Compaction

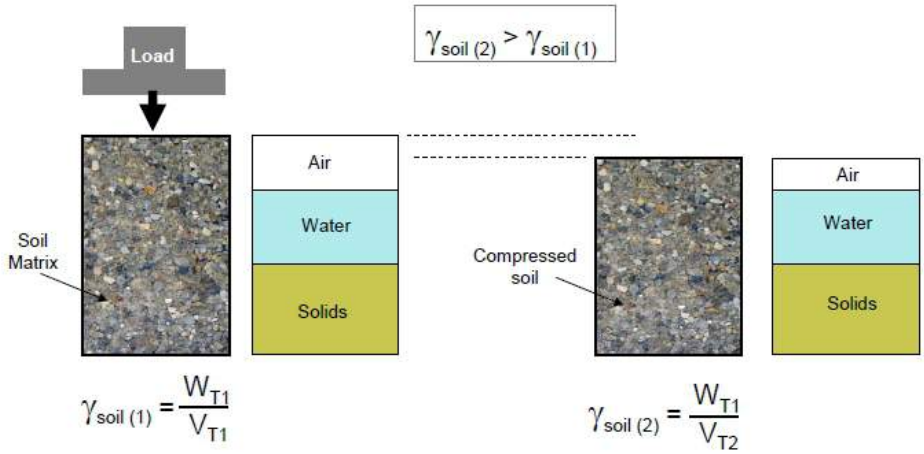
Topics

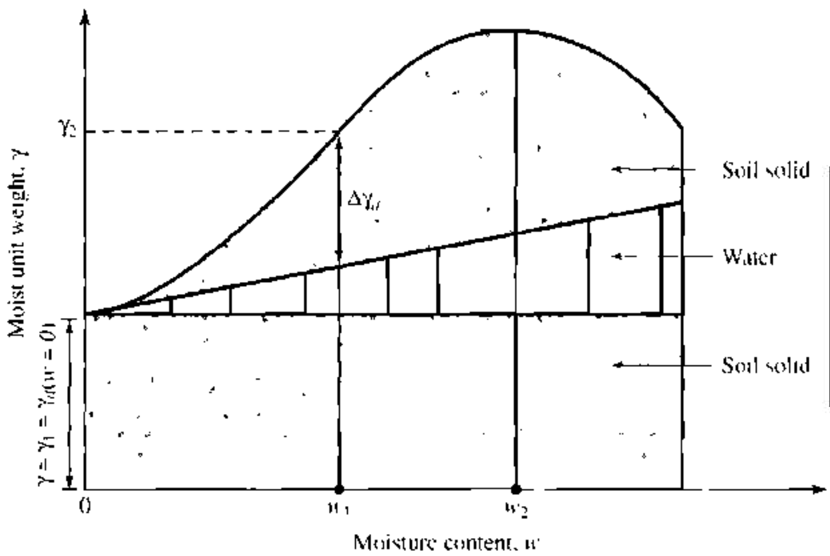
- General Principles
- Soil Compaction in the Lab:
- Factors affecting Compaction
- Structure of Compacted Clay Soil
- Field Compaction
- Specification for Field Compaction
- Determination of Field Unit Weight of Compaction

♦ *General Principles*

Definition:

Soil compaction is defined as the method of mechanically increasing the density of soil by reducing volume of air.



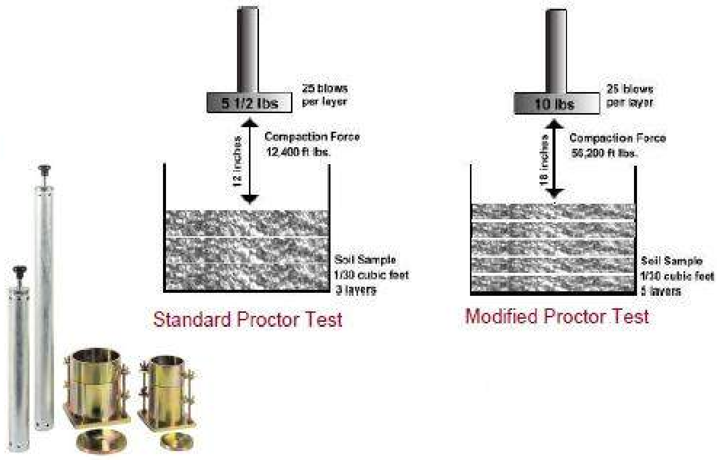


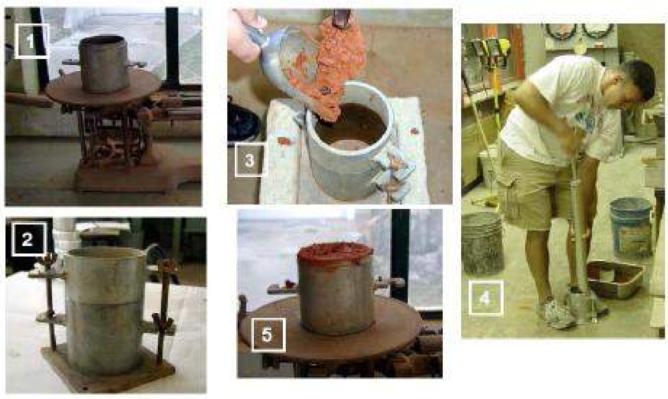
Water is added to lubricate the contact surfaces of soil particles and improve the compressibility of the soil matrix

Figure 5.1 Principles of compaction

♦ ***Soil Compaction in the Lab:***

1- Standard Proctor Test
2- Modified Proctor Test





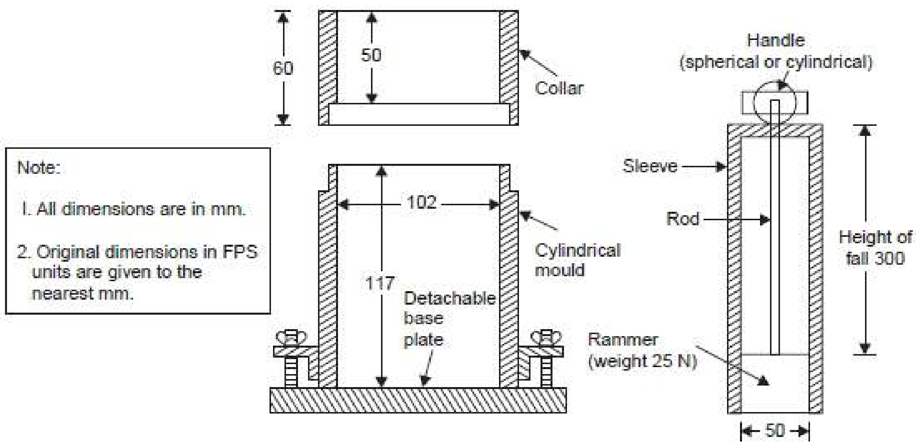
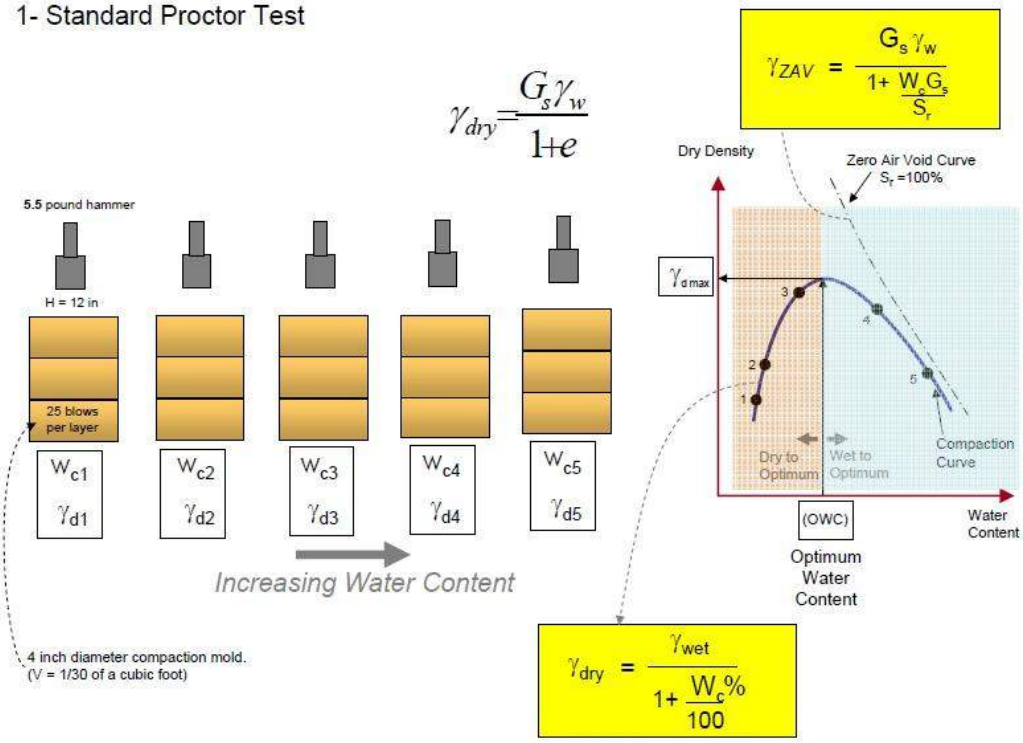


Fig. 12.4 Apparatus for standard proctor test

1- Standard Proctor Test

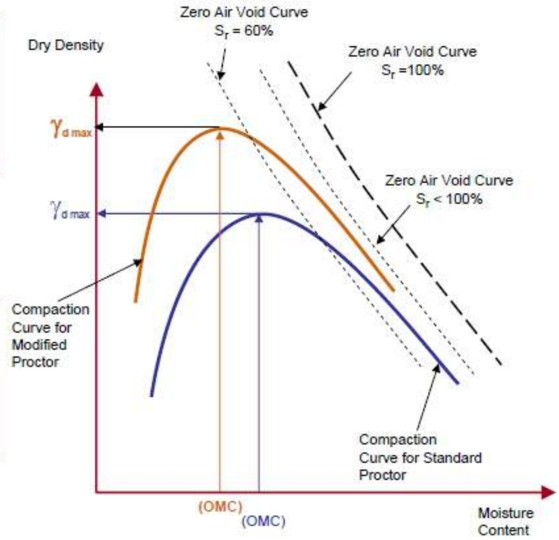


1- Standard Proctor Test
ASTM D-698 or AASHTO T-99

Energy = 12,375 foot-pounds per cubic foot

2- Modified Proctor Test
ASTM D-1557 or AASHTO T-180

Energy = 56,520 foot-pounds per cubic foot



$$\text{Energy} = \frac{\text{Number of blows per layer} \times \text{Number of layers} \times \text{Weight of hammer} \times \text{Height of drop hammer}}{\text{Volume of mold}}$$

♦ ***Factors affecting Compaction***

1- Soil Type
2- Water Content (w_c)
3- Compaction Effort Required (Energy)

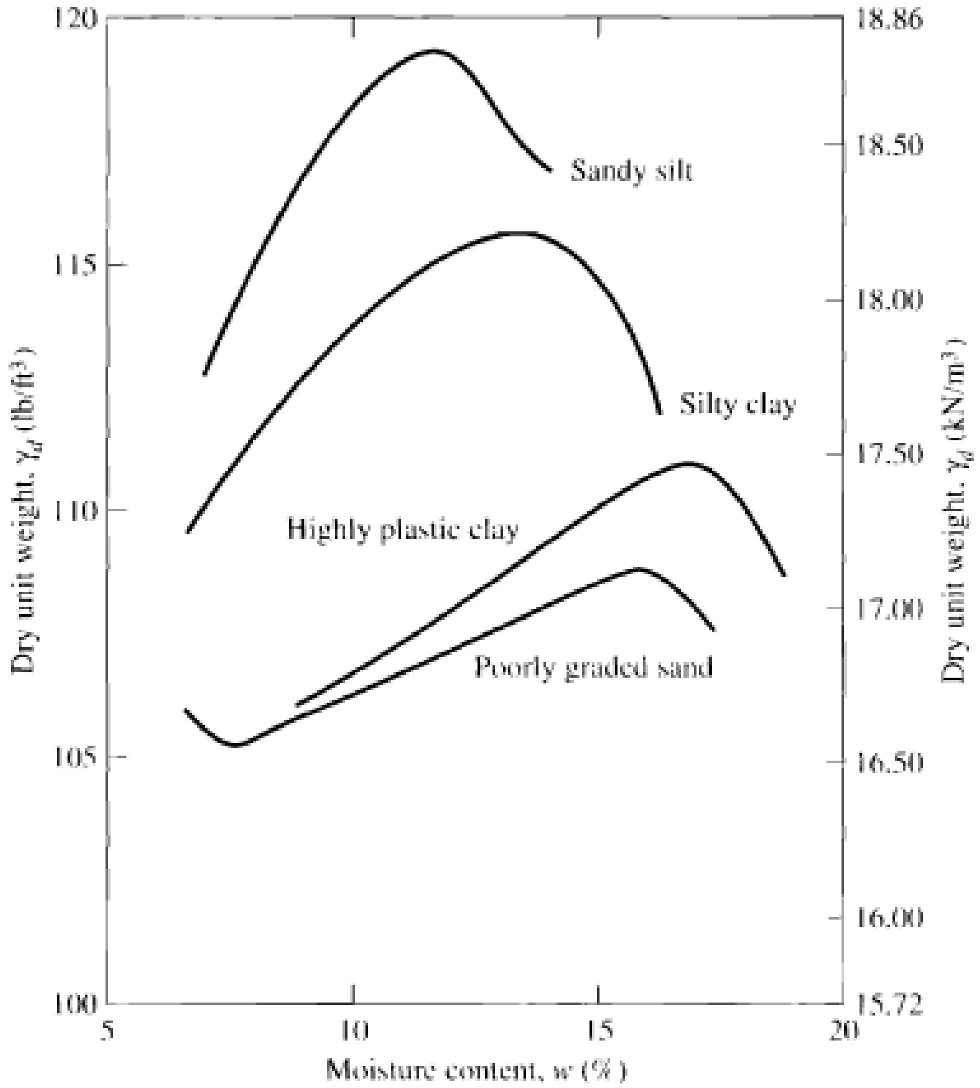


Figure 5.4 Typical compaction curves for four soils (ASTM D-698)

Curve type	Soil properties
A (one peak)	Soil with liquid limit 30-70
B (one & half Peak)	Soil with liquid limit < 30
C (double peak)	Soil with liquid limit >70
D (odd shape)	

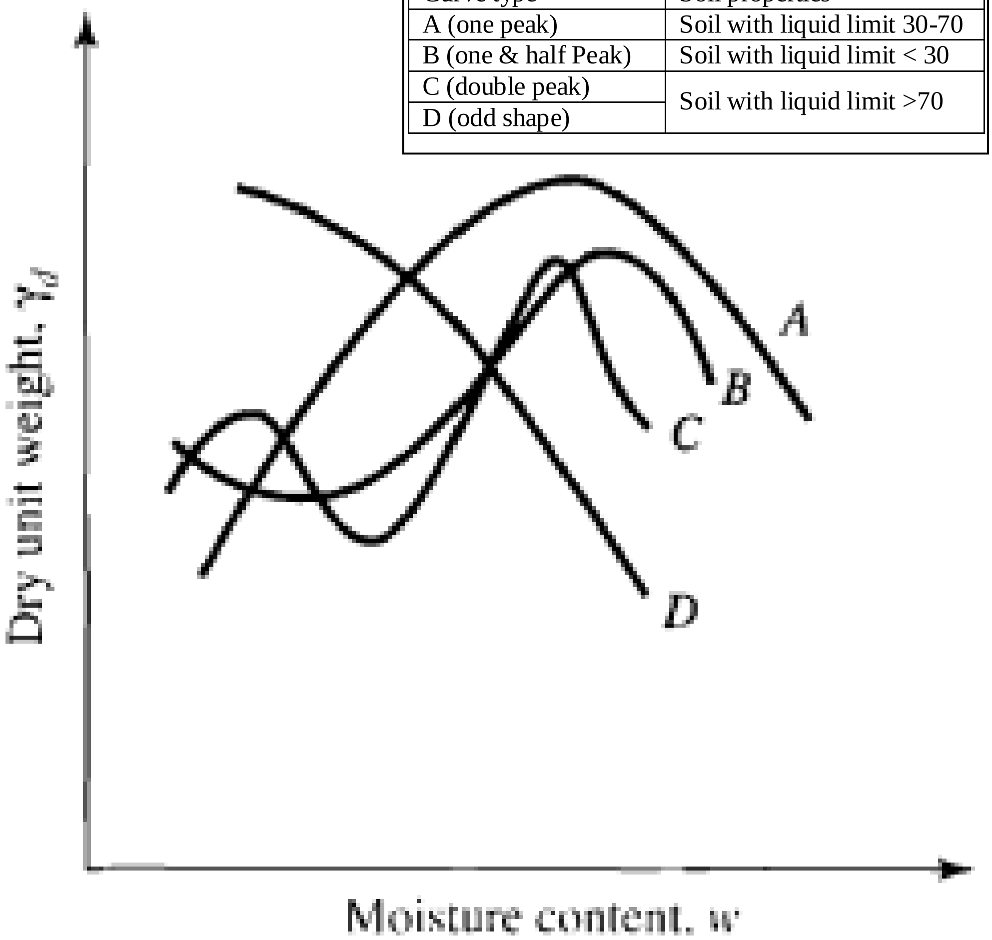


Figure 5.5 Types of compaction curve

Effect of compaction effort

$$\text{Energy} = \frac{\text{Number of blows per layer x Number of layers x Weight of hammer x Height of drop hammer}}{\text{Volume of mold}}$$

in SI units,

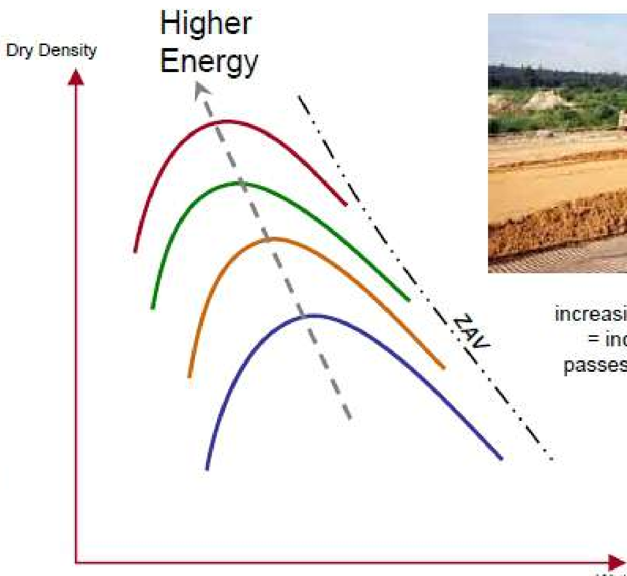
$$E = \frac{(25)(3)\left(\frac{2.5 \times 9.81}{1000}\ \text{kN}\right)(0.305\ \text{m})}{944 \times 10^{-6}\ \text{m}^3} = 594\ \text{kN-m/m}^3 \approx 600\ \text{kN-m/m}^3$$

In English units,

$$E = \frac{(25)(3)(5.5)(1)}{\left(\frac{1}{30}\right)} = 12{,}375\ \text{ft-lb/ft}^3 \approx 12{,}400\ \text{ft-lb/ft}^3$$

Increasing compaction energy → <u>Lower OWC</u> and <u>higher dry density</u>

In the lab
increasing compaction energy
= increasing number of blows



In the field
increasing compaction energy
= increasing number of
passes or reducing lift depth

♦ ***Structure of Compacted Clay Soil***

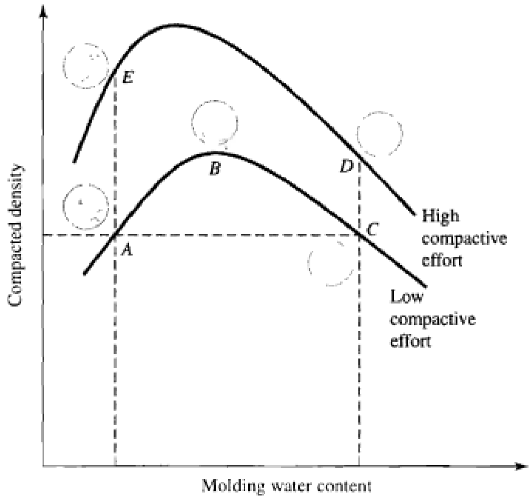


Figure 5.8 Effect of compaction on structure of clay soils (redrawn after Lambe, 1958)

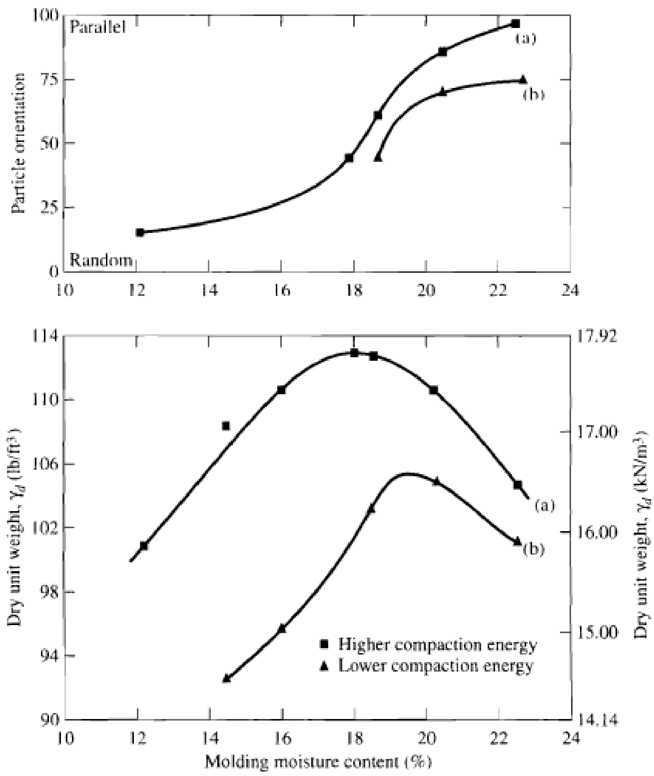
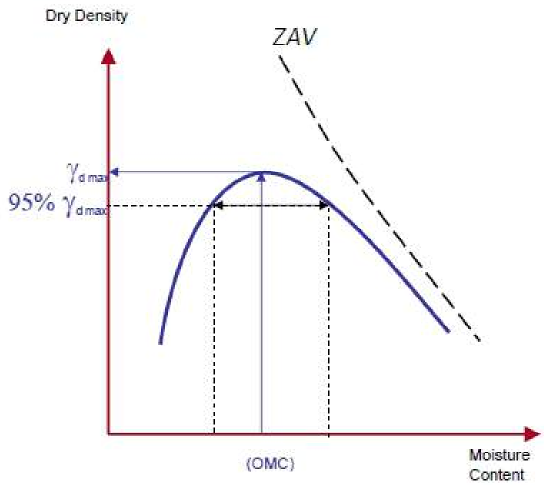


Figure 5.9 Orientation against moisture content for Boston blue clay (after Lambe, 1958)

♦ ***Field Compaction***

Because of the differences between lab and field compaction methods, the maximum dry density in the field may reach 90% to 95%.



1- Rammers

2- Vibratory Plates

3- Smooth Rollers

4- Rubber-Tire

5- Sheep foot Roller

6- Dynamic Compaction

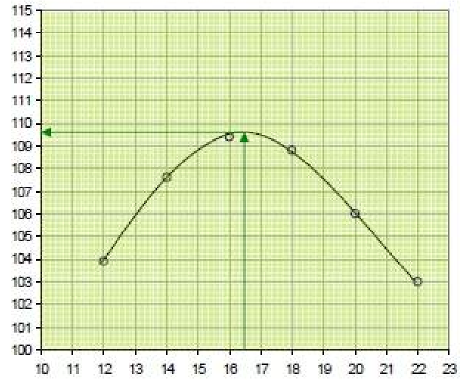
Example:

The laboratory test for a standard proctor is shown below. Determine the optimum water content and maximum dry density. If the G_s of the soil is **2.70**, draw the ZAV curve.

Volume of Proctor Mold (ft^3)	Weight of wet soil in the mold (lb)	Water Content (%)
1/30	3.88	12
1/30	4.09	14
1/30	4.23	16
1/30	4.28	18
1/30	4.24	20
1/30	4.19	22

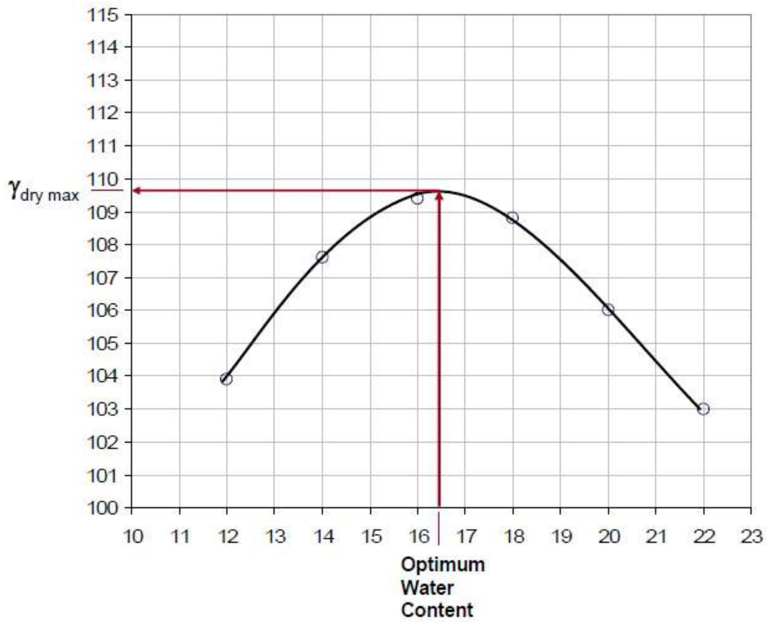
Solution:

Volume of Mold (ft^3)	Weight of wet soil in the mold (lb)	Wet Unit Weight (lb/ft^3)	Water Content (%)	Dry Unit Weight (lb/ft^3)
1/30	3.88	116.4	12	103.9
1/30	4.09	122.7	14	107.6
1/30	4.23	126.9	16	109.4
1/30	4.28	128.4	18	108.8
1/30	4.24	127.2	20	106.0
1/30	4.19	125.7	22	103.0



$$\gamma_{dry} = \frac{\gamma_{wet}}{1 + \frac{W_c\%}{100}}$$

$$\gamma_{ZAV} = \frac{G_s \gamma_w}{1 + \frac{W_c G_s}{S_r}}$$



♦ ***Specification for Field Compaction***

Compaction performance parameters are given on a construction project in one of two ways:

1- Method Specification

detailed instructions specify machine type, lift depths, number of passes, machine speed and moisture content. A "recipe" is given as part of the job specifications to accomplish the compaction needed.

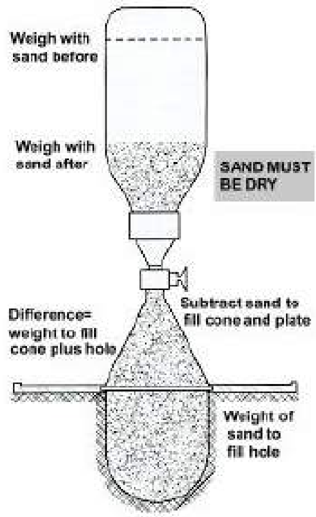
2- End-result Specification

Only final compaction requirements are specified (95% modified or standard Proctor). This method, gives the contractor much more flexibility in determining the best, most economical method of meeting the required specs.

RELATIVE DESIRABILITY OF SOILS AS COMPACTED FILL

Group	Symbol	* if gravelly ** erosion critical *** volume change critical - not appropriate for this type of use Soil Type	Relative Desirability for Various Uses (1=best; 14=least desirability)									
			Rolled Earth Fill Dams			Canal Sections		Foundations		Roadways		
										Fills		
			Homogenous Embankment	Core	Shell	Erosion Resistance	Compacted Earth Lining	Seepage Important	Seepage Not Important	Frost Heave Not Possible	Frost Heave Possible	Surfacing
GRAVELS	GW	Well-graded gravels, gravel/ sand mixes, little or no fines	-	-	1	1	-	-	1	1	1	3
GRAVELS	GP sand	Poorly-graded gravels, gravel/ mixtures, little or no fines	-	-	2	2	-	-	3	3	3	-
GRAVELS	GM	Silty gravels, poorly-graded gravel/sand/silt mixtures	2	4	-	4	4	1	4	4	9	5
GRAVELS	GC	Clay-like gravels, poorly graded gravel/sand/clay mixtures	1	1	-	3	1	2	6	5	5	1
SANDS	SW	Well-graded sands, gravelly sands, little or no fines	-	-	3*	6	-	-	2	2	2	4
SANDS	SP	Poorly-graded sands, gravelly sands, little or no fines	-	-	4*	7*	-	-	5	6	4	-
SANDS	SM	Silty sands, poorly-graded sand/ silt mixtures	4	5	-	8*	5**	3	7	6	10	6
SANDS	SC	Clay-like sands, poorly-graded sand/clay mixtures	3	2	-	5	2	4	8	7	6	2
CLAYS & SILTS – LEAN	ML	Inorganic silts and very fine sands, rock flour, silty or clay-like fine sands with slight plasticity	6	6	-	-	6**	6	9	10	11	-
CLAYS & SILTS – LEAN	CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	5	3	-	9	3	5	10	9	7	7
CLAYS & SILTS – LEAN	OL	Organic silts and organic silt-clays of low plasticity	8	8	-	-	7**	7	11	11	12	-
CLAYS & SILTS – FAT	MN	Organic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	9	9	-	-	-	8	12	12	13	-
CLAYS & SILTS – FAT	CH	Inorganic clays of high plasticity, fat clays	7	7	-	10	8**	9	13	13	8	-
CLAYS & SILTS – FAT	OH	Organic clays of medium high plasticity	10	10	-	-	-	10	14	14	14	-

♦ ***Determination of Field Unit Weight of Compaction***

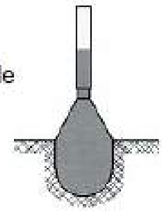


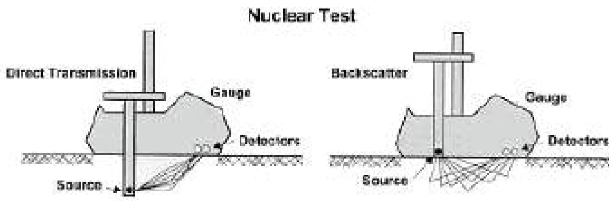
1- Sand Cone (ASTM D1556-90)

A small hole (6" x 6" deep) is dug in the compacted material to be tested. The soil is removed and weighed, then dried and weighed again to determine its moisture content. A soil's moisture is figured as a percentage. The specific volume of the hole is determined by filling it with calibrated dry sand from a jar and cone device. The dry weight of the soil removed is divided by the volume of sand needed to fill the hole. This gives us the density of the compacted soil in lbs per cubic foot. This density is compared to the maximum Proctor density obtained earlier, which gives us the relative density of the soil that was just compacted.

2- Balloon Dens meter

The same as the sand cone, except a rubber balloon is used to determine the volume of the hole





3- Nuclear Density (ASTM D2292-91)

Nuclear Density meters are a quick and fairly accurate way of determining density and moisture content. The meter uses a radioactive isotope source (Cesium 137) at the soil surface (backscatter) or from a probe placed into the soil (direct transmission). The isotope source gives off photons (usually Gamma rays) which radiate back to the mater's detectors on the bottom of the unit. Dense soil absorbs more radiation than loose soil and the readings reflect overall density. Water content (ASTM D3017) can also be read, all within a few minutes.

Nuclear Density

Sand Cone