The Physics of Flight
100 Years Since the Wright Brothers

by

Donald A. Gurnett

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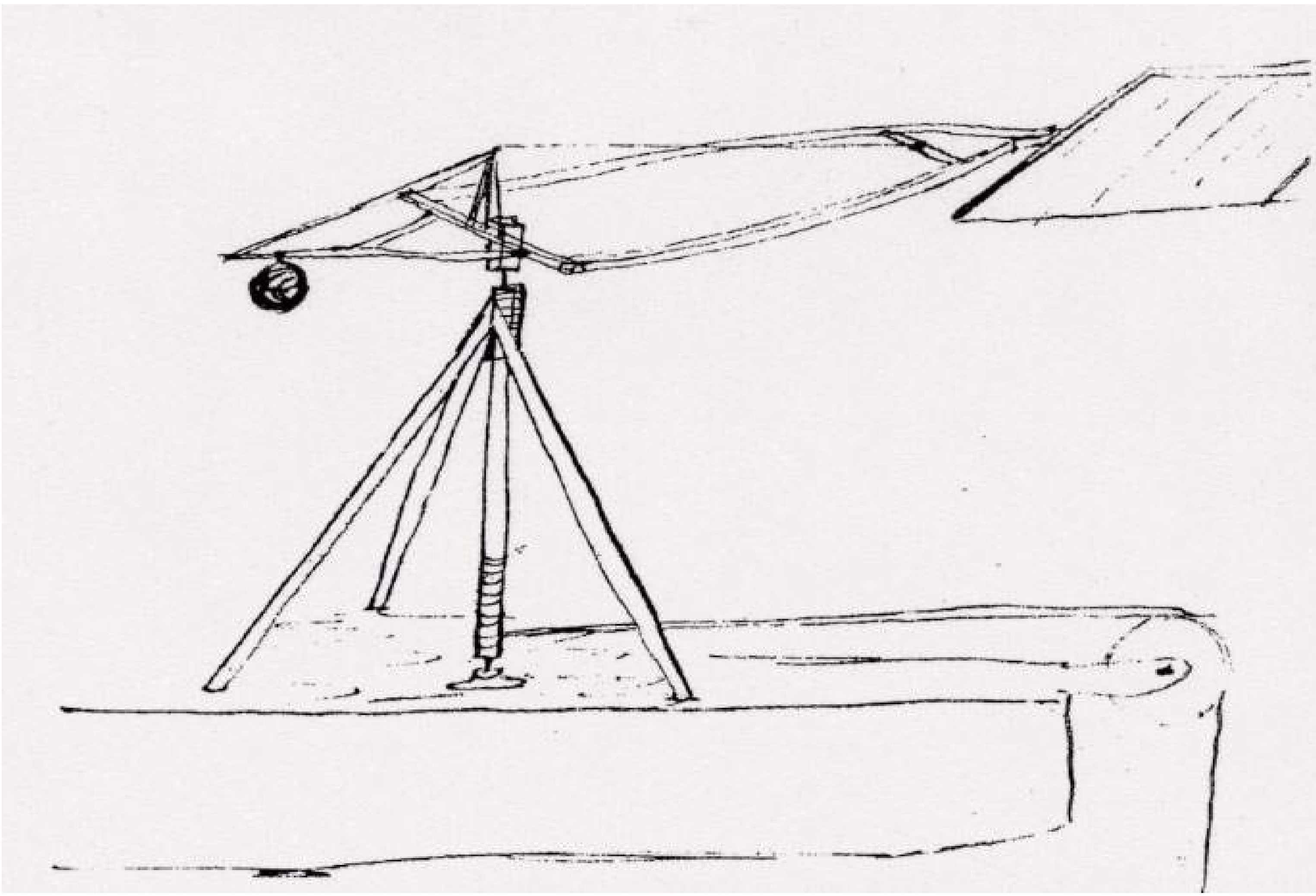
University of Iowa, Iowa City, Iowa, December 12, 2003.

Sir George Cayley, 1773-1857

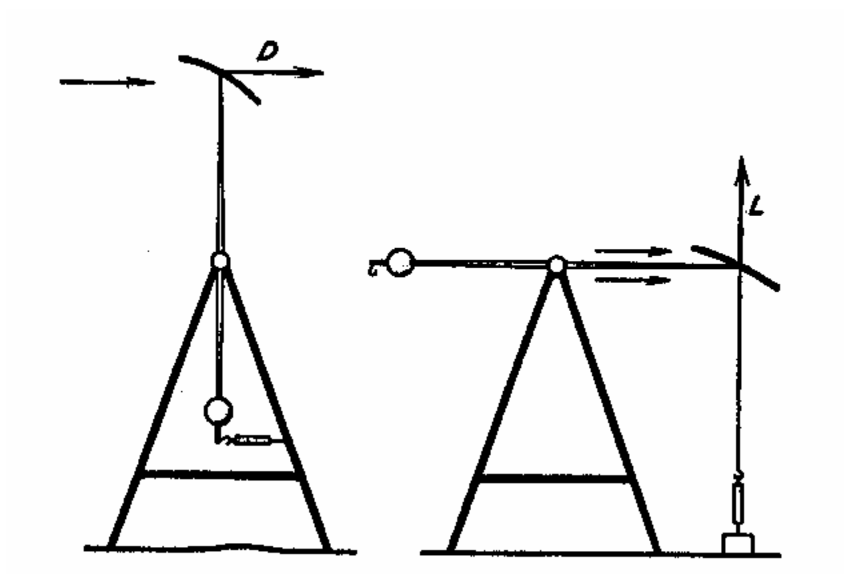
- Showed lift is proportional to velocity squared and sin α, 1804
- Wrote a three-part paper on “Aerial Navigation,” 1809-1810
- Designed the first successful glider



A replica of Caley's glider, flown by Derek Piggott, 1973

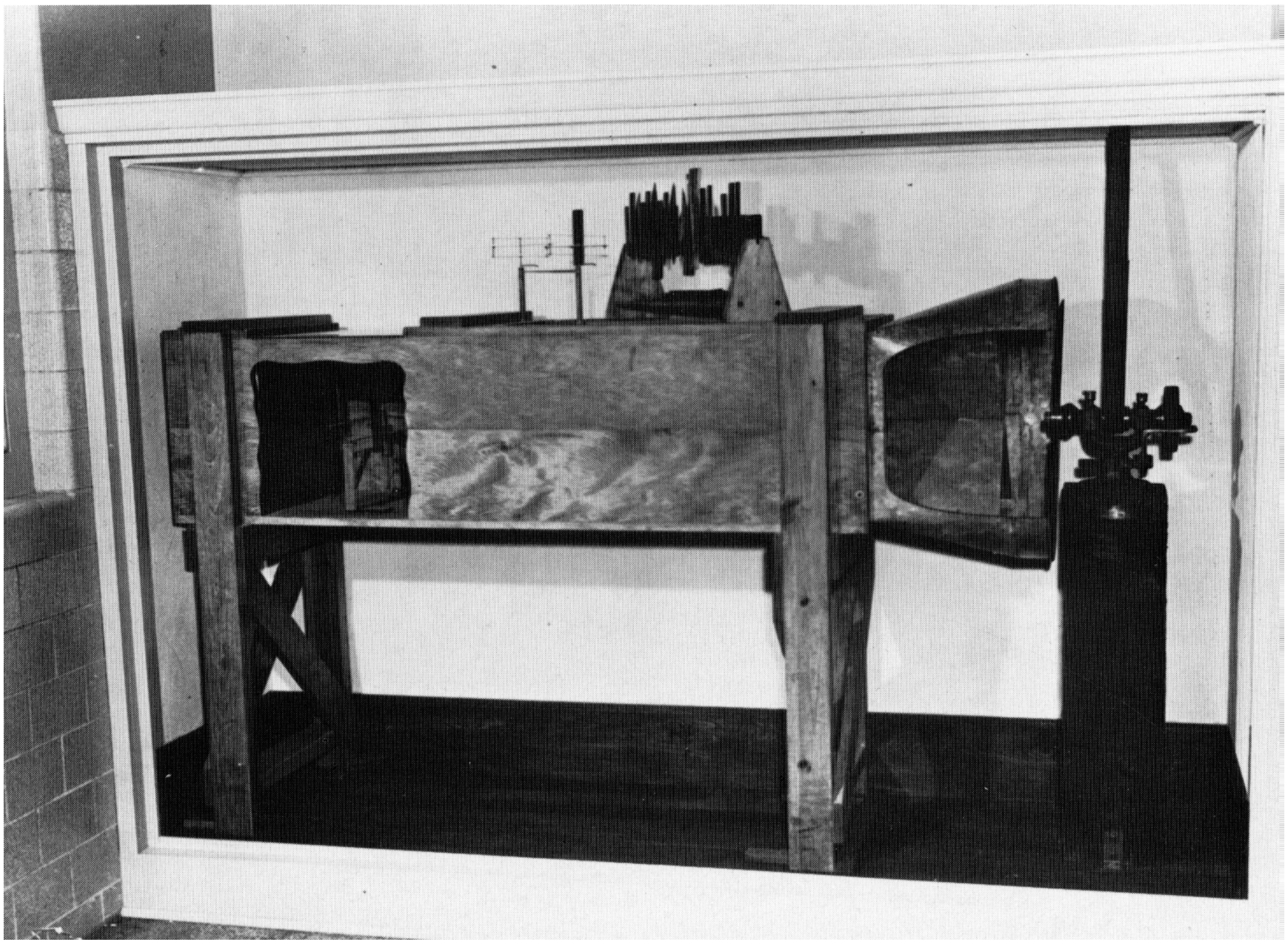
Otto Lilienthal (1848-1896)

- Measured the lift and drag of a wing
- Made over 2000 flights in a glider, some as far as 350m
- Wrote a book “The flight of birds as the basis for the art of flying,” 1886.
- First person killed in an aircraft accident, 1896.

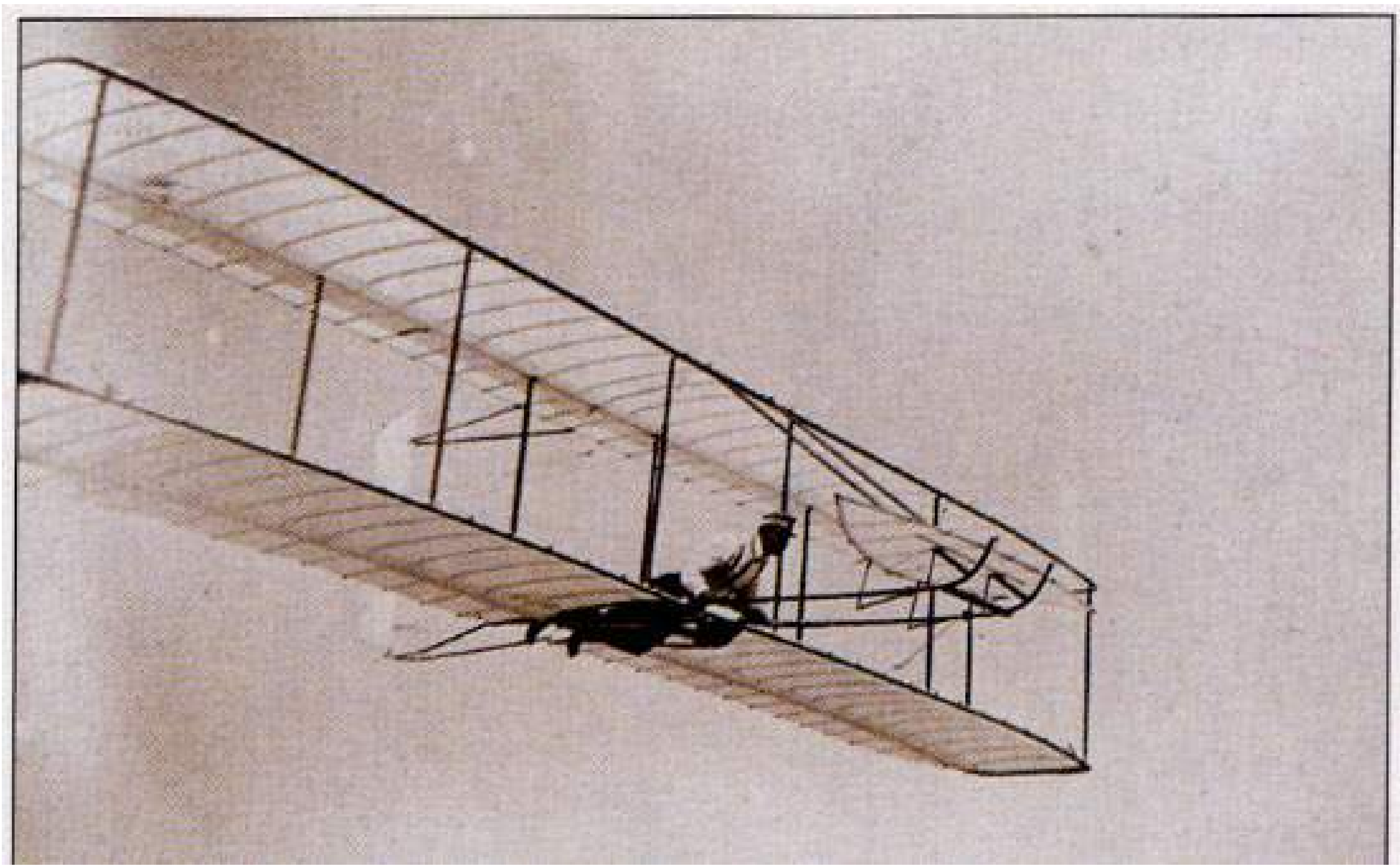


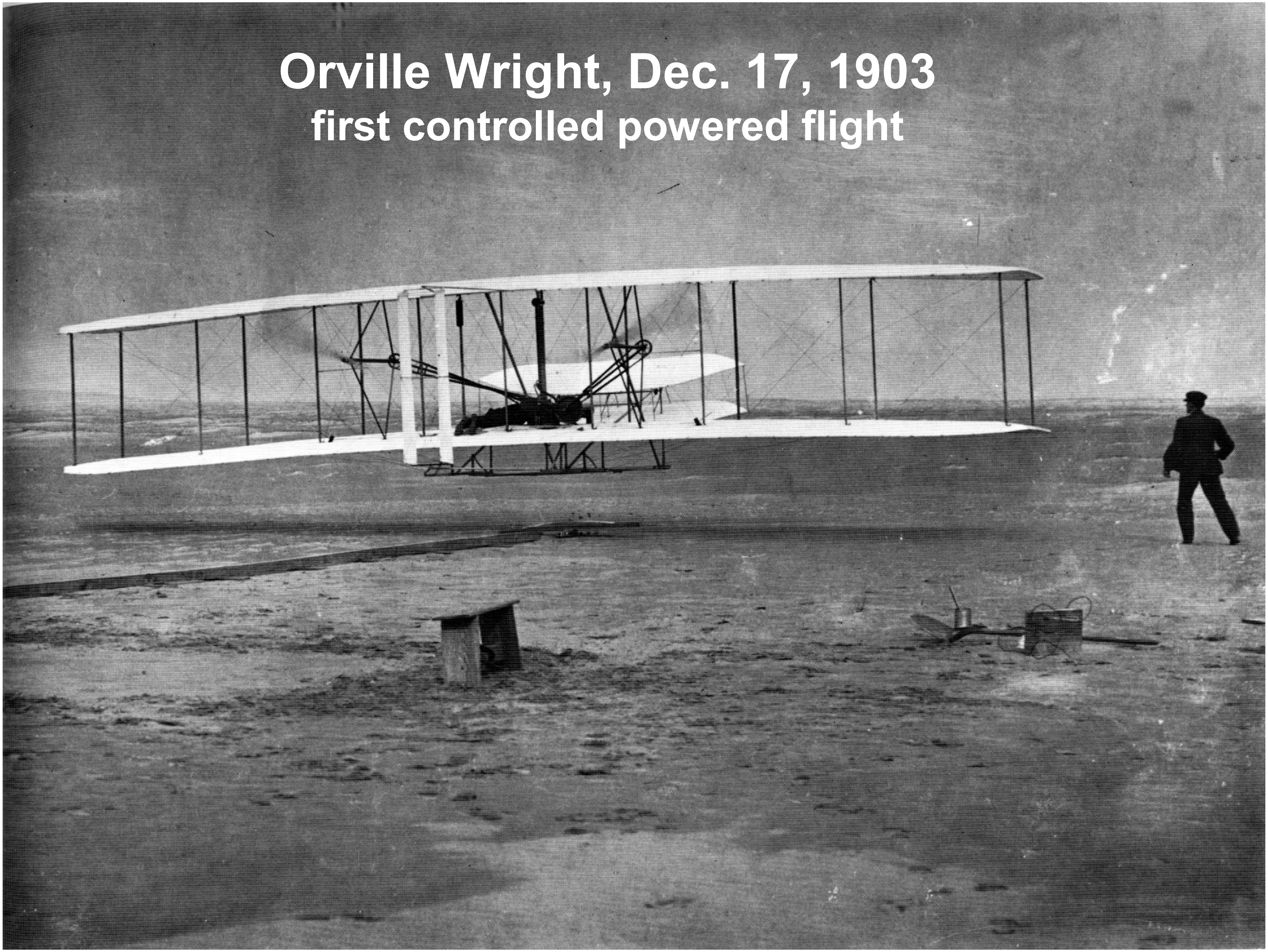
Orville and Wilbur Wright (1871-1948; 1867-1912)

- Knew of Cayley and Lilienthal's work
- Made one of the first wind tunnels, 1901
- Invented wing warping as a method of roll control,1902



Wright Brothers’ 1902 Glider
first full 3-axis controlled flight




Orville Wright, Dec. 17, 1903
first controlled powered flight

Status of Aerodynamic Theory in the Early 1900s

- All of the basic equations of fluid mechanics were known.

- However, no acceptable theory existed for the lift force on a wing.

- Worst than that, the existing theories predicted that the lift was exactly zero.

Bernoulli's Theorem and the Lift on a Wing

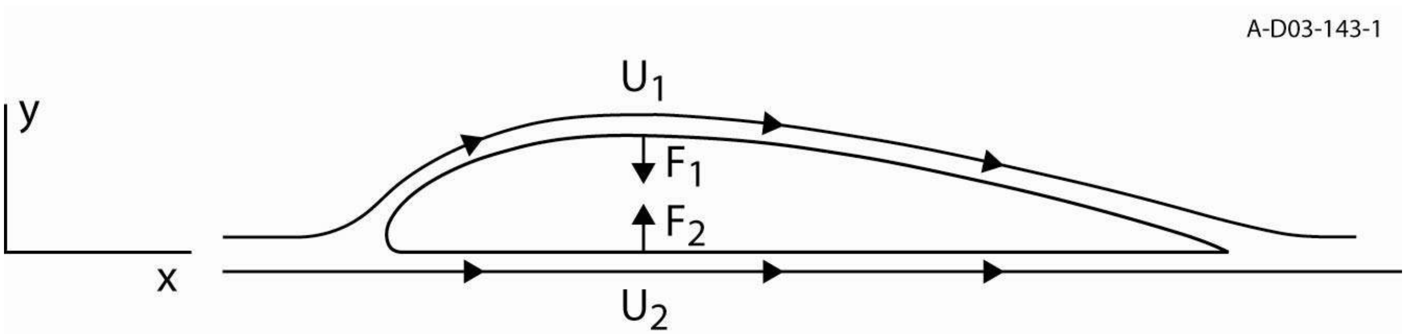
$$\frac{1}{2}\rho_m U^2 + p = \text{constant}$$

ρ_m = mass density, U = velocity

$p = F/A$ is the pressure

F = force, A = Area

The usual argument:



Since $\ell_1 > \ell_2$ it follows that $U_1 > U_2$ so $p_1 < p_2$. Therefore, $F_2 > F_1$. **(WRONG).** One cannot assume that the stagnation point (where the streamlines separate) is exactly at the leading edge. One must solve for the entire flow pattern over the wing, which varies considerably with the angle of attack.

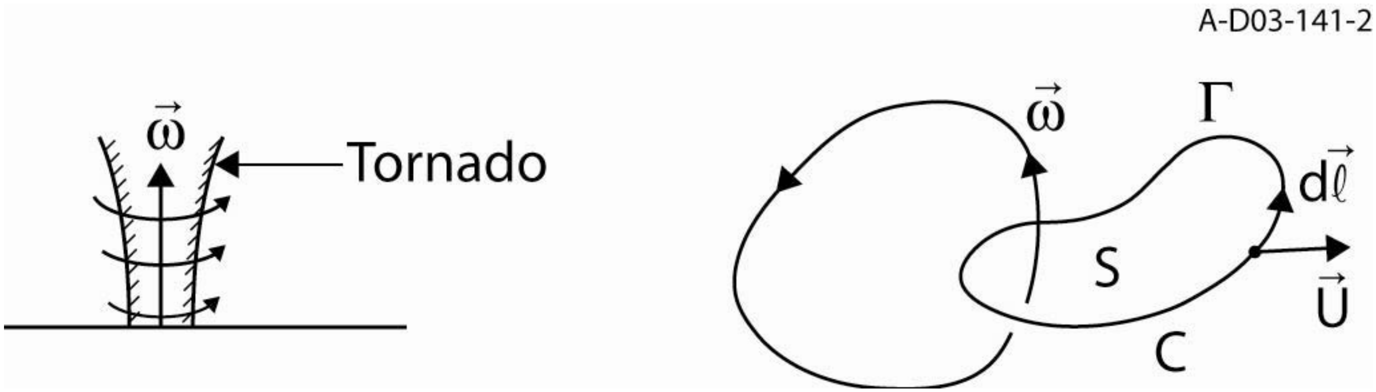
Some Key Concepts

Vorticity

$$\vec{\omega} = \vec{\nabla} \times \vec{U}$$

Circulation

$$\Gamma = \int_c \vec{U} \cdot d\vec{\ell}$$



Theorem: $\vec{\nabla} \cdot \vec{\omega} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{U}) = 0$

Vortex lines are continuous

Theorem: $\Gamma = \int_c \vec{U} \cdot d\vec{\ell} = \int_s \vec{\nabla} \times \vec{U} \cdot d\vec{A}$

$$\Gamma = \int_s \vec{\omega} \cdot d\vec{A}$$

Basic Equations and Assumptions

- Continuity equation

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla} \cdot (\rho_m \vec{U}) = 0$$

- Navier Stokes equation

$$\rho_m \left[\frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \vec{\nabla})\vec{U} \right] = -\vec{\nabla} p + \rho_m \nu \left[\nabla^2 \vec{U} + \frac{1}{3} \vec{\nabla} (\vec{\nabla} \cdot \vec{U}) \right]$$

- The assumption of incompressibility

  If ρ_m = constant, then

$$\vec{\nabla} \cdot \vec{U} = 0$$

- The vorticity equation

$$\frac{\partial \vec{\omega}}{\partial t} = \underbrace{\vec{\nabla} \times (\vec{U} \times \vec{\omega})}_{\text{Convection}} + \underbrace{\nu \nabla^2 \vec{\omega}}_{\text{Diffusion}}$$

- Reynolds Number

$$R_N = \frac{|\,\text{convection}\,|}{|\,\text{diffusion}\,|} \sim \frac{UL}{\nu}$$

Reynolds Number

Typical values (at sea level)

	U	L	R_N
Commercial Jet	600 mph	20 ft	1.1×10^8
Light plane	100 mph	5 ft	4.7×10^6
Glider	60 mph	3 ft	1.6×10^6
Model airplane	40 mph	8 in	2.5×10^5
Seagull	20 mph	4 in	6.2×10^4
Butterfly	5 mph	1 in	3.9×10^3
Housefly	5 mph	0.3 in	1.2×10^2

Basic Equations and Assumptions (con't)

- The inviscid assumption, $R_N \gg 1$

  If $\nu = 0$, then

  $$\rho_m \left[\frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \vec{\nabla})\vec{U} \right] = -\vec{\nabla} p \qquad \text{(Euler's equation)}$$

- Kelvin's theorem

  If ρ_m = constant and ν = 0, then

  $$\Gamma = \int_s \vec{\omega} \cdot d\vec{A} = \text{constant}$$

- Velocity potentials

  If Γ = 0 in the upstream flow, then $\vec{\omega} = \vec{\nabla} \times \vec{U} = 0$ at all points in the flow. It follows then that

  $$\vec{U} = \vec{\nabla}\Phi$$

- Laplace's equation

  From the continuity equation, $\vec{\nabla} \cdot \vec{U} = 0$, one then has

  $$\nabla^2 \Phi = 0, \quad \text{or} \quad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

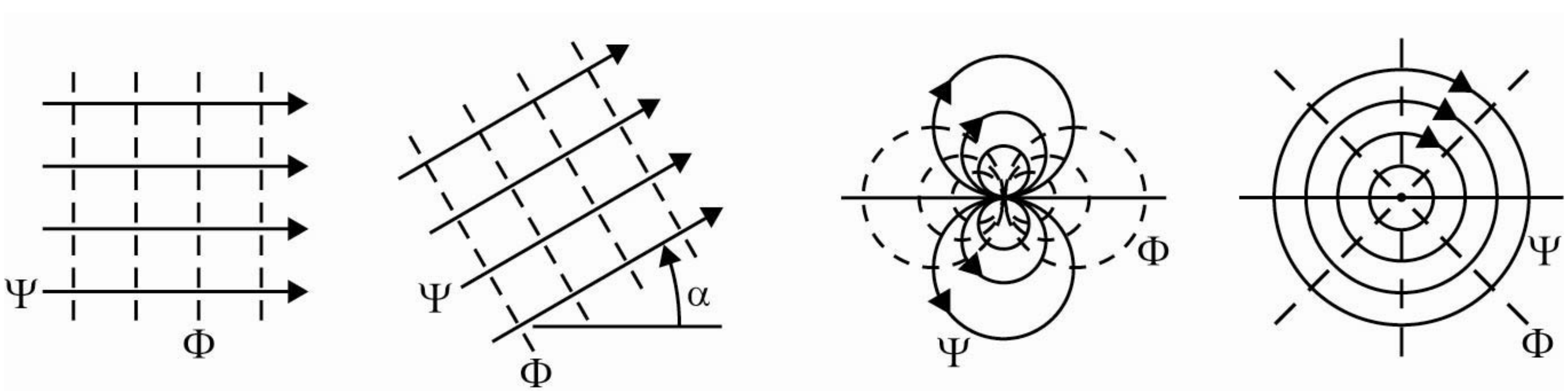
Complex Potentials

If z = x + iy, any analytic complex function $F(z) = \Phi(x,y) + i\Psi(x,y)$ provides a solution to Laplace's equation

$$\nabla^2\Phi = 0 \quad \text{and} \quad \nabla^2\Psi = 0$$

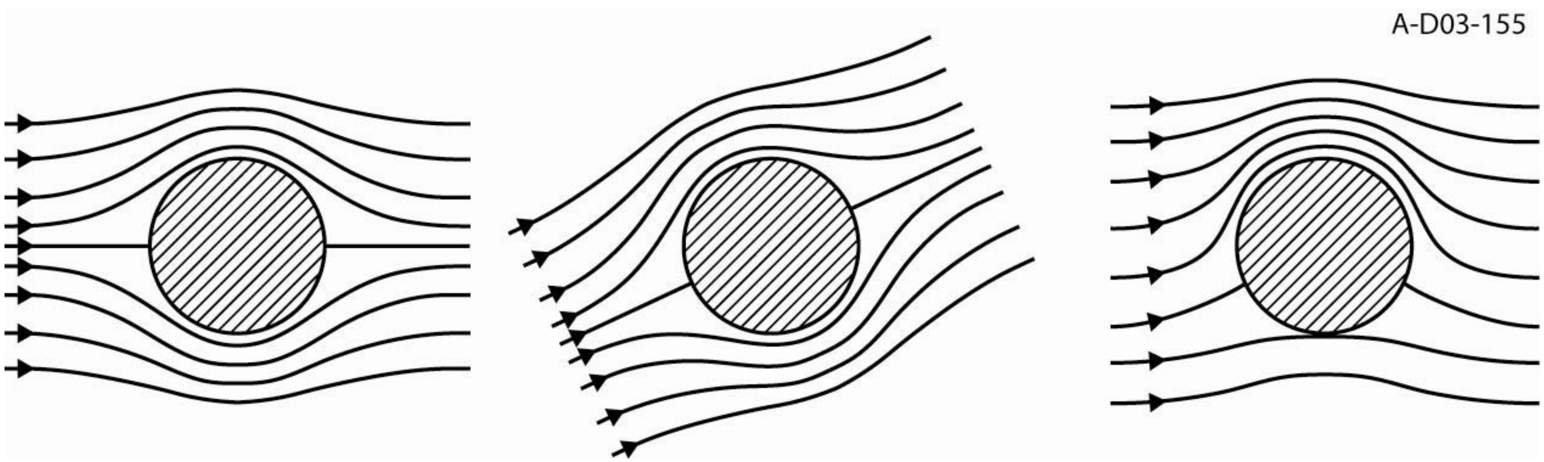
Examples:

Uniform flow	At an angle α	Dipole	Vortex
$U_0 z$	$U_0 z\, e^{-i\alpha}$	$\frac{\mu_0}{z}$	$i\frac{\Gamma_0}{2\pi}\ell n\, z$

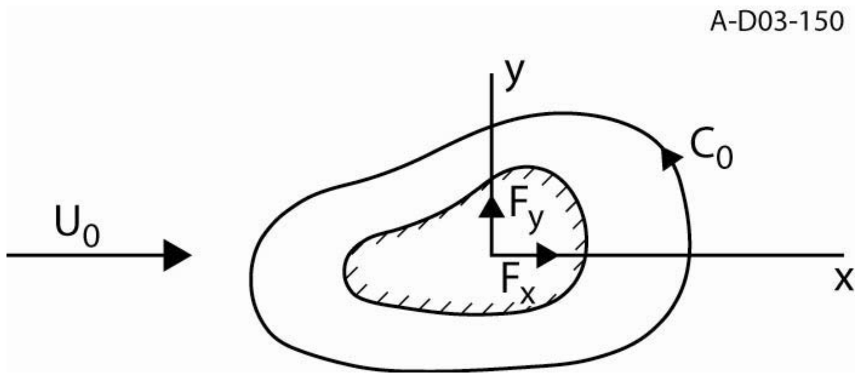


Flow Around a Cylinder

Horizontal flow	At an angle α	With circulation
$U_0\left(z+\frac{a^2}{z}\right)$	$U_0\left(z+\frac{a^2}{z}\right)e^{-i\alpha}$	$U_0\left(z+\frac{a^2}{z}\right)+\frac{i\Gamma_0}{2\pi}\ell n\left(\frac{z}{a}\right)$



The Blasius Force Equation



$$F_x - iF_y = i\frac{\rho_m}{2}\int_c W^2\,dz,$$

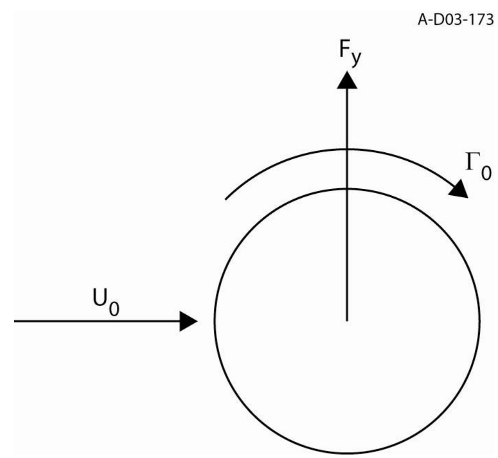
where $W(z) = dF/dz = U_x - iU_y$ is the complex velocity.

$$W(z) = U_0 + i\frac{\Gamma_0}{2\pi}\frac{1}{z} + \sum_n b_n\left(\frac{1}{z}\right)^n$$

$$\int_c W^2\,dz = 2\pi i \sum \mathrm{Res}(W^2) = -i2U_0\Gamma_0$$

$$F_x = 0$$

$$F_y = \rho_m U_0 \Gamma_0$$

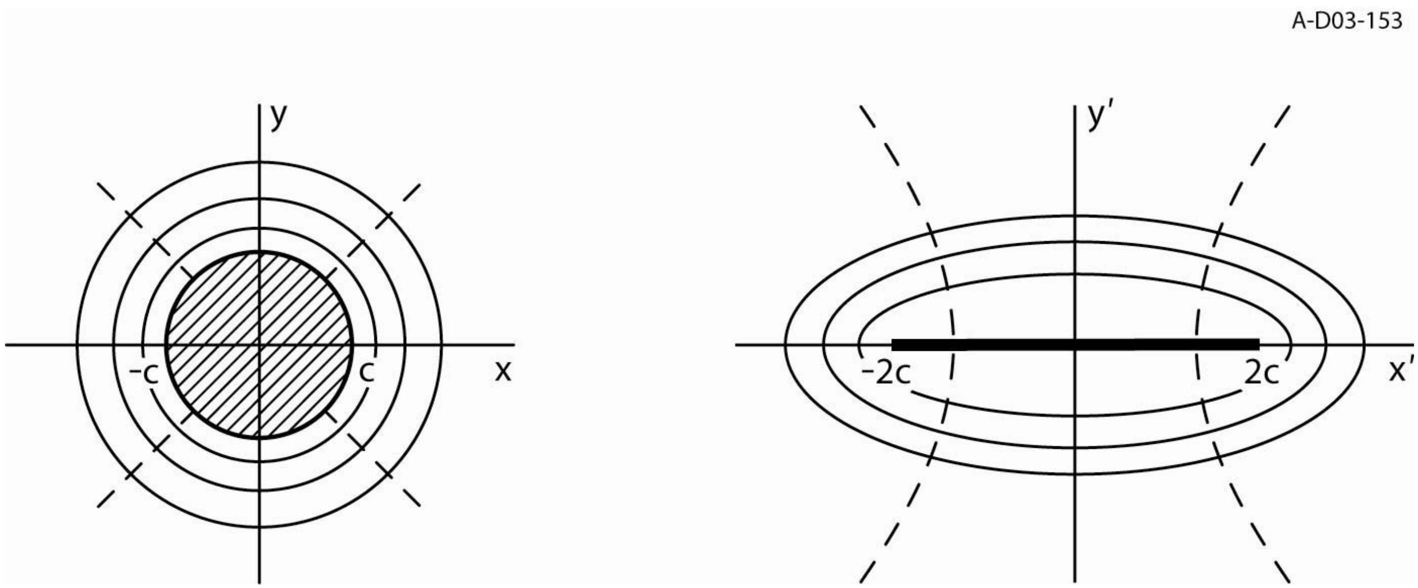


The Joukowski Transformation, 1910

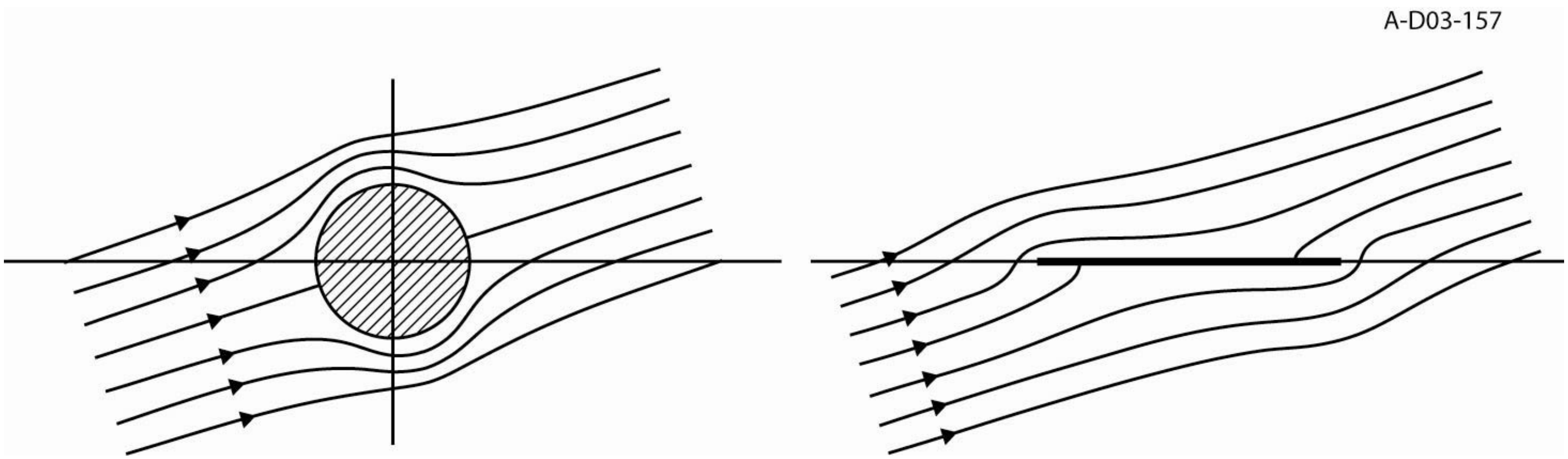
The Joukowski transformation

$$z' = z + \frac{c^2}{z}$$,where z' = x' + iy',

transforms a cylinder into a flat plate



Flow Around a Flat Plate Airfoil



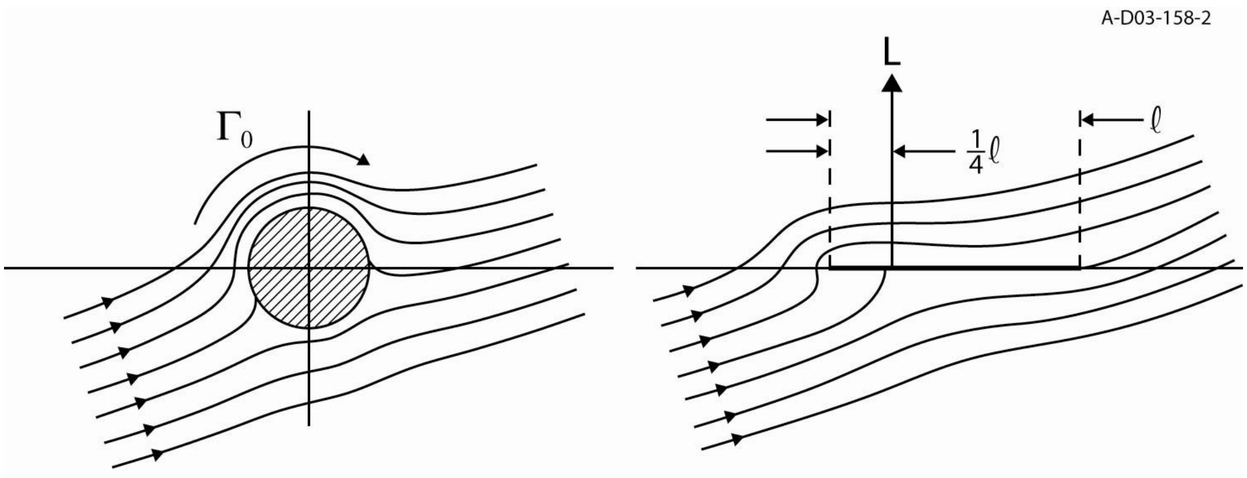
Since $\Gamma_0 = 0$, by the Blasius theorem $F_y = 0$.

The dilemma: Since the upstream vorticity is zero the circulation must be zero, so there can be no lift.

But a flat plate airfoil produces lift, so there must be circulation.

The Kutta-Joukowski Condition, 1910

The flow must be smooth and continuous at the trailing edge.
Requires a circulation, $\Gamma_0 = 4\pi U_0 a \sin\alpha$.

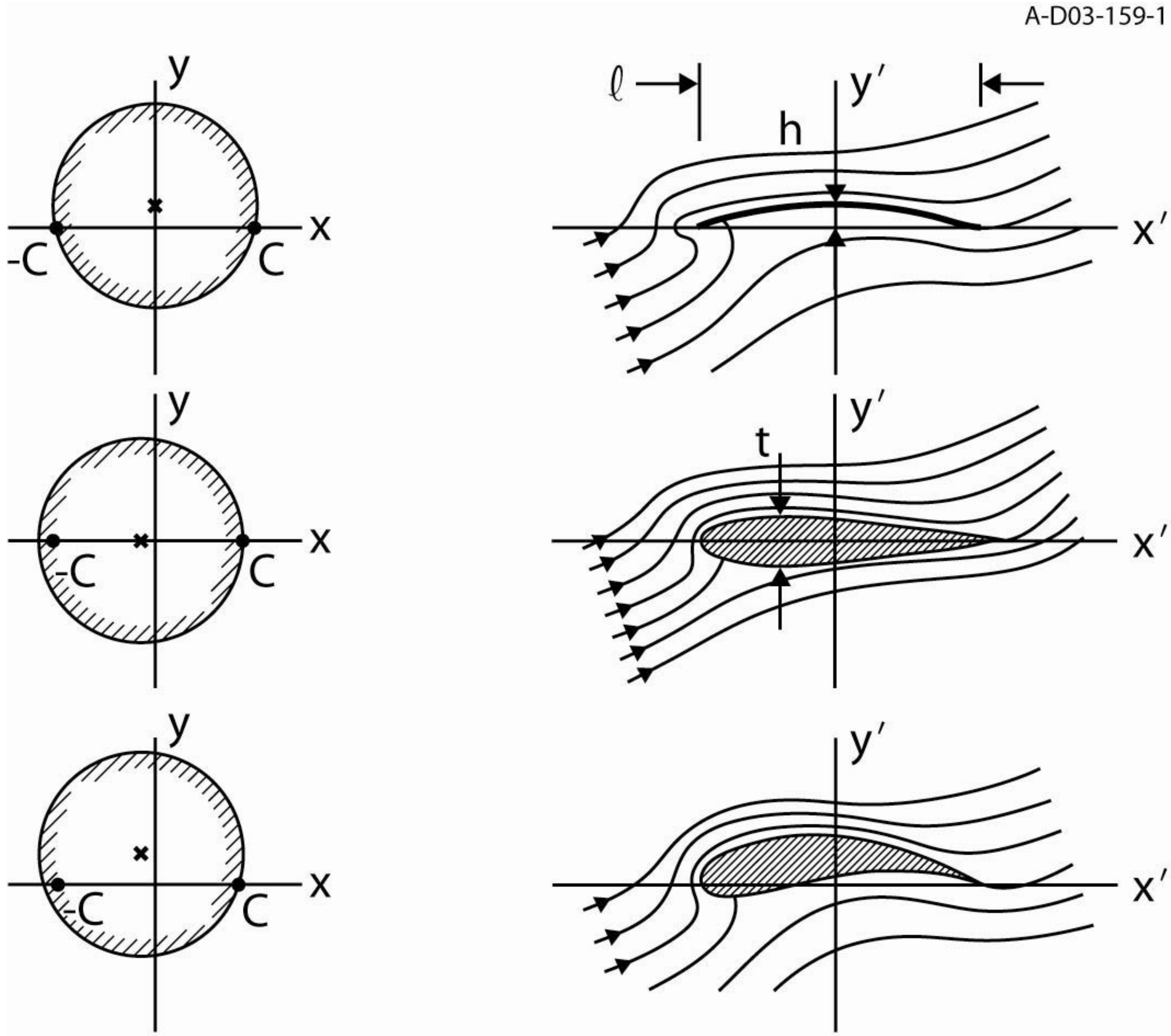


$$F_y = \rho_m U_0 \Gamma_0$$

$$L = \left(\frac{1}{2}\rho_m U^2\right) A\, 2\pi \sin\alpha$$

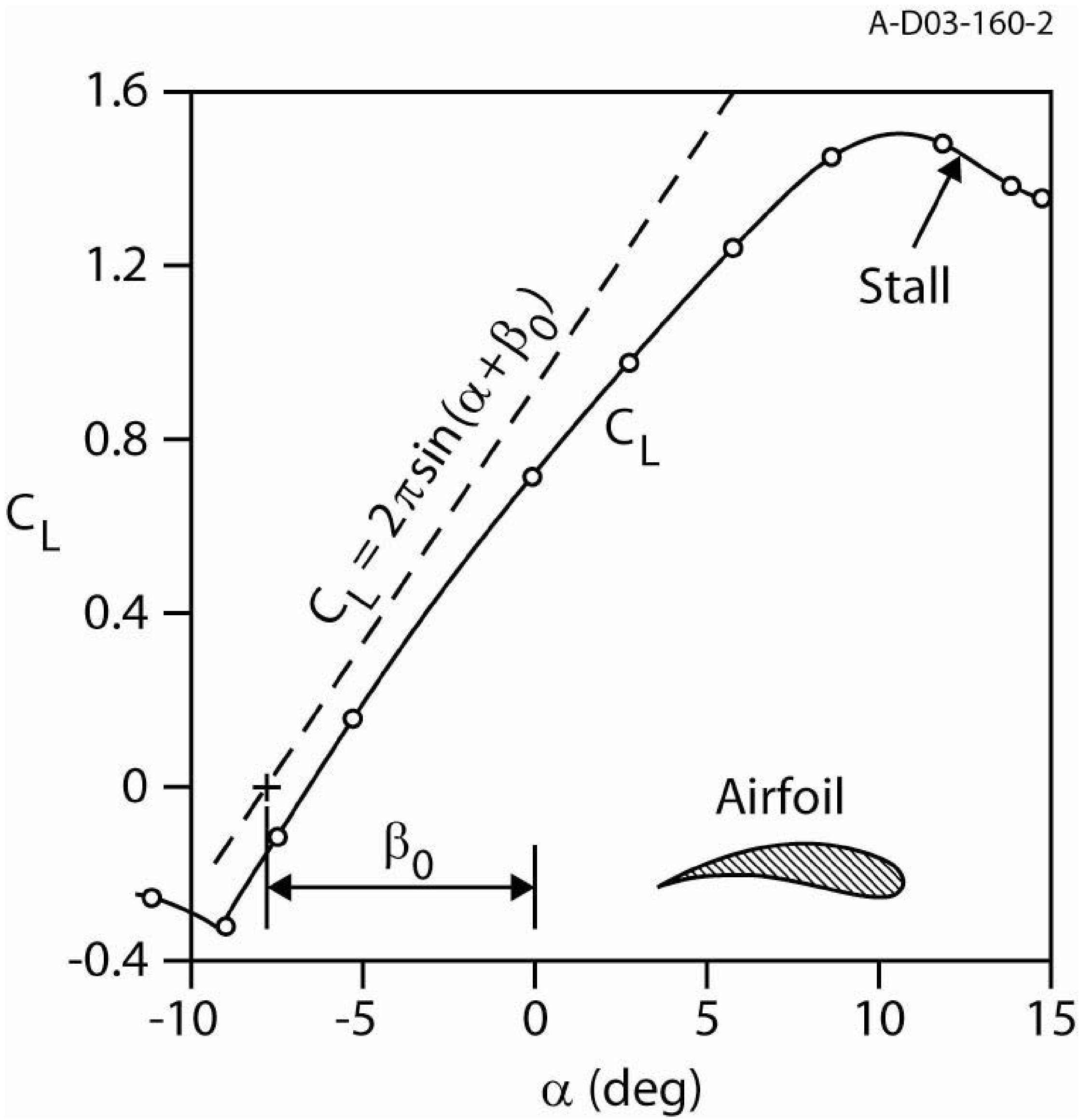
$$C_L = \frac{L}{\left(\frac{1}{2}\rho_m U_0^2\right) A} = 2\pi \sin\alpha$$

The Joukowski Family of Airfoils



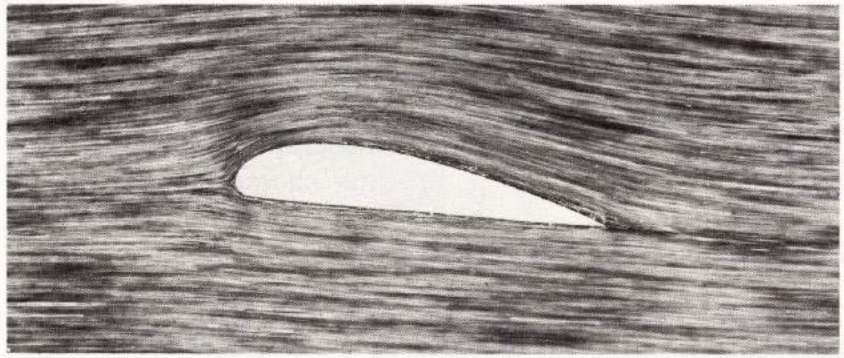
$$C_L = 2\pi\left(1 + 0.77\frac{t}{\ell}\right)\sin(\alpha + \beta_0),\ \beta_0 = 2h/\ell$$

Comparison with Experimental Data



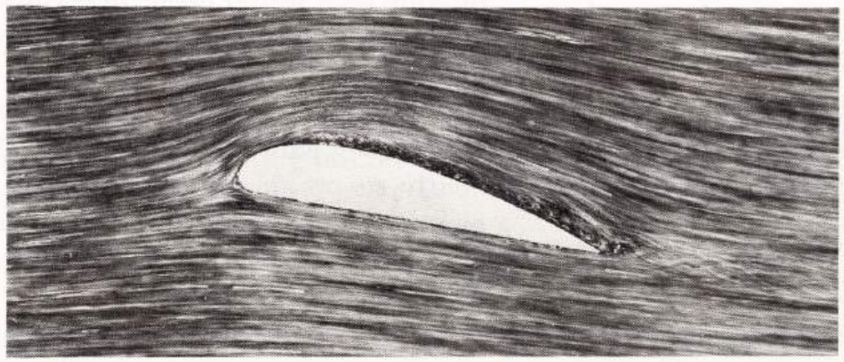
Wind Tunnel Observations

$\alpha = 5^\circ$



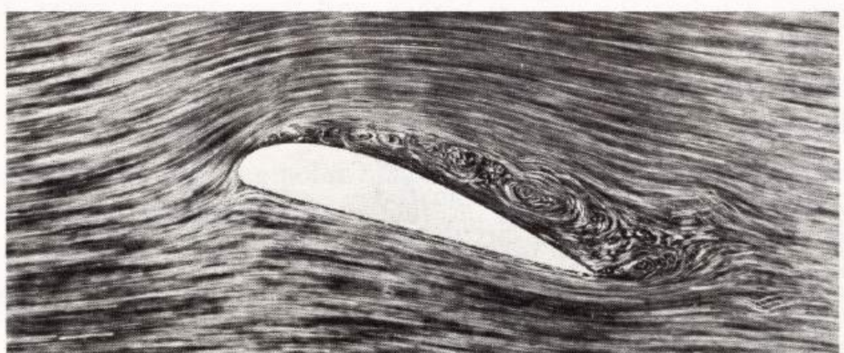
Kutta condition satisfied

$\alpha = 10^\circ$



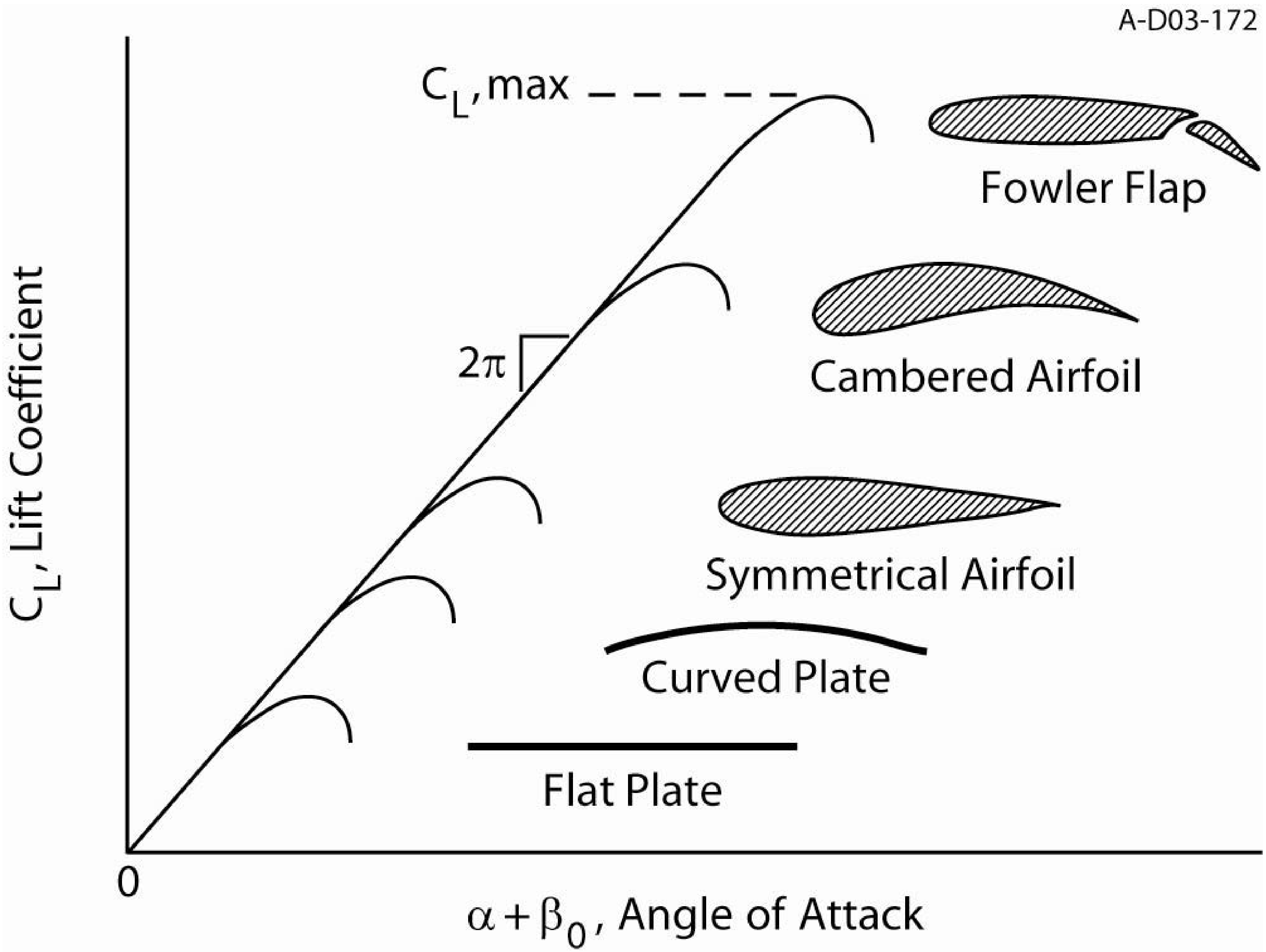
Slight flow separation

$\alpha = 15^\circ$

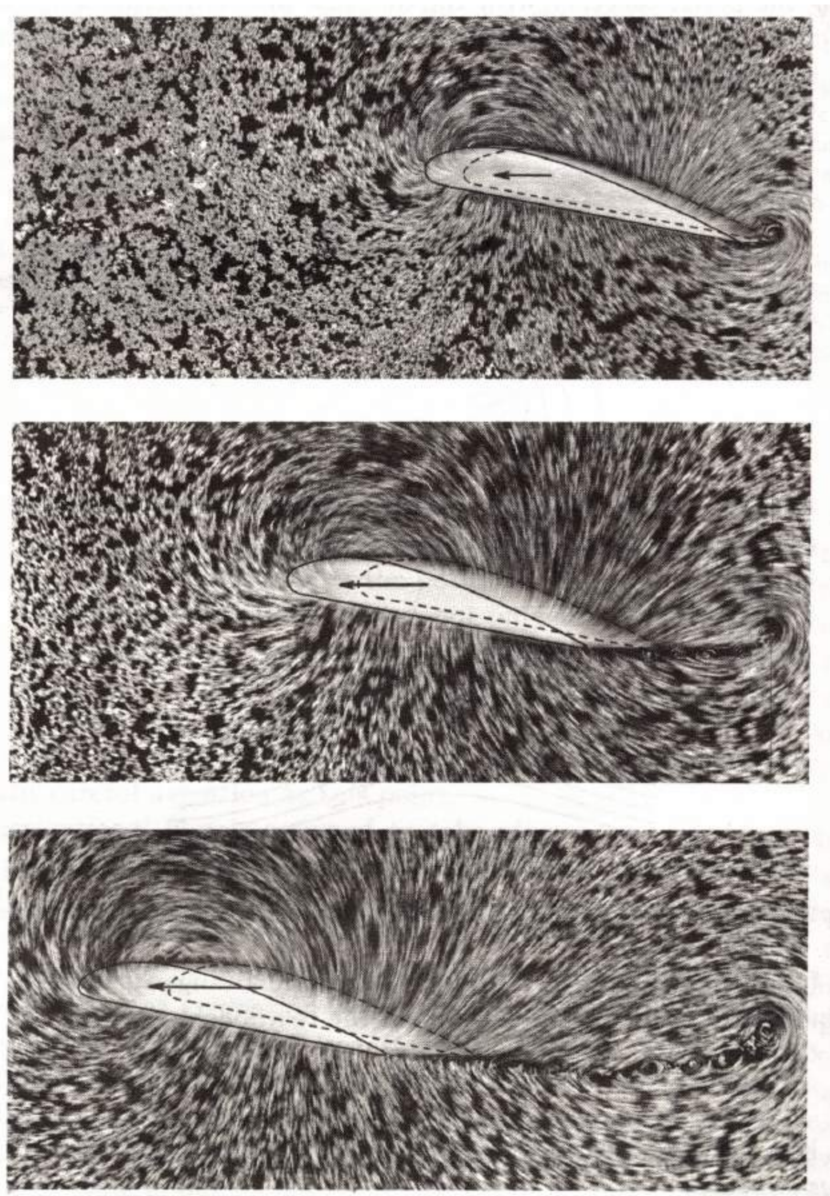


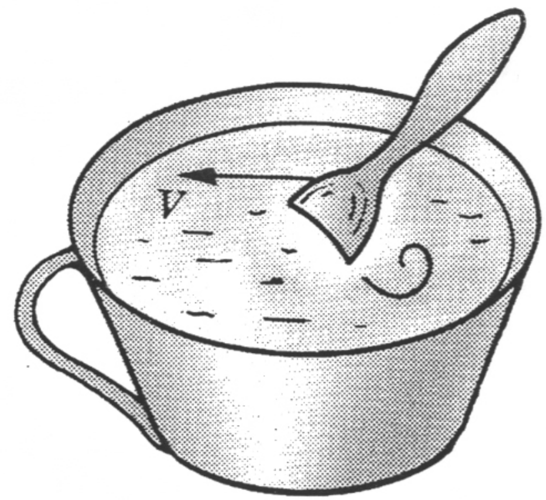
Complete flow separation (stall)

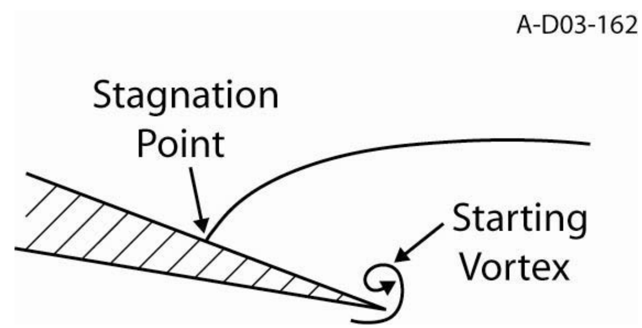
The Maximum Lift Coefficient



Origin of the Circulation





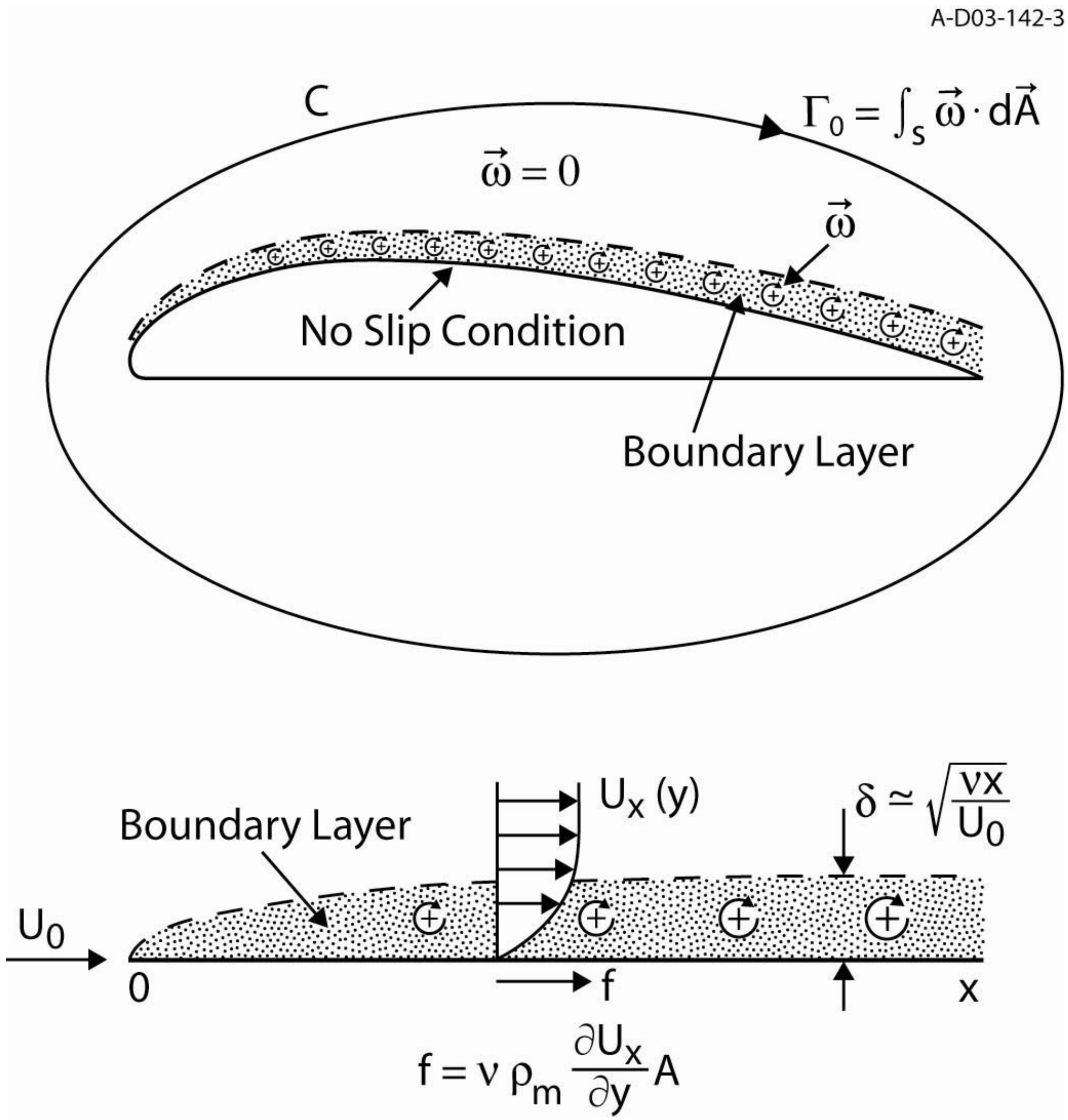


$$\frac{\partial \vec{\omega}}{\partial t} = \vec{\nabla} \times (\vec{U} \times \vec{\omega}) + \nu \nabla^2 \vec{\omega}$$

$$\frac{1}{L} \qquad \frac{1}{L^2}$$

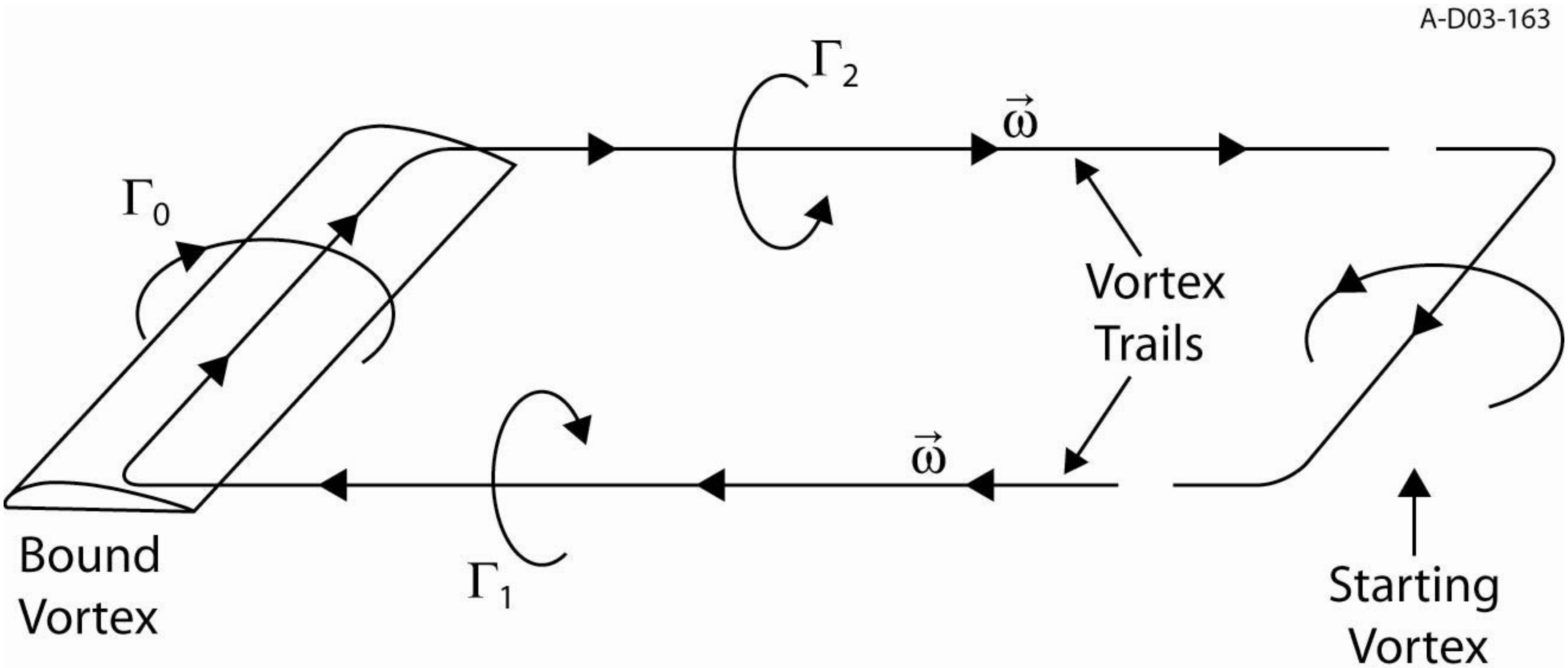
Where is the Vorticity?

(In the boundary layer)



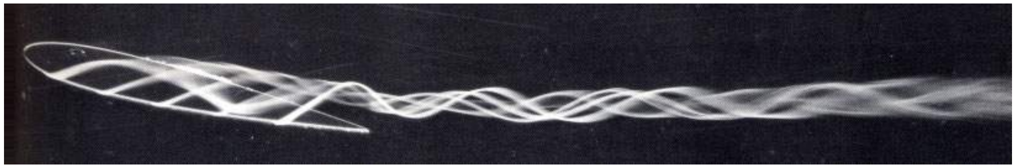
Finite Wing Span Effects

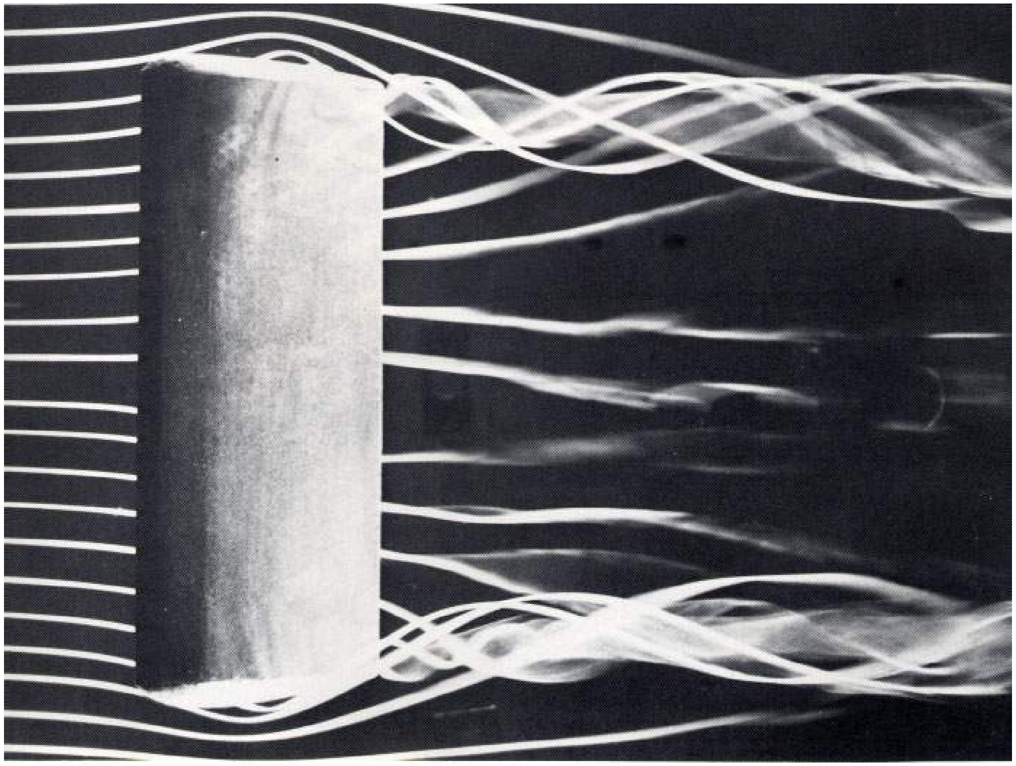
(continuity of $\vec{\omega}$)

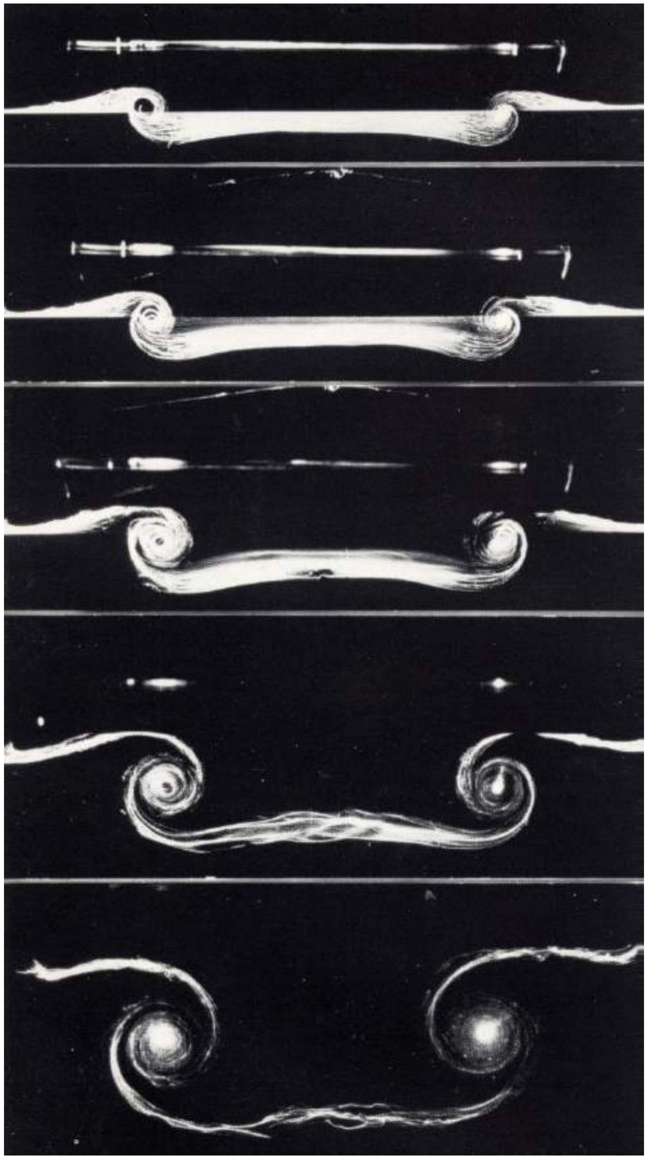


Note: $\Gamma_1 = \Gamma_2 = \Gamma_0$, vortex trails cannot be avoided.

Vortex Trails

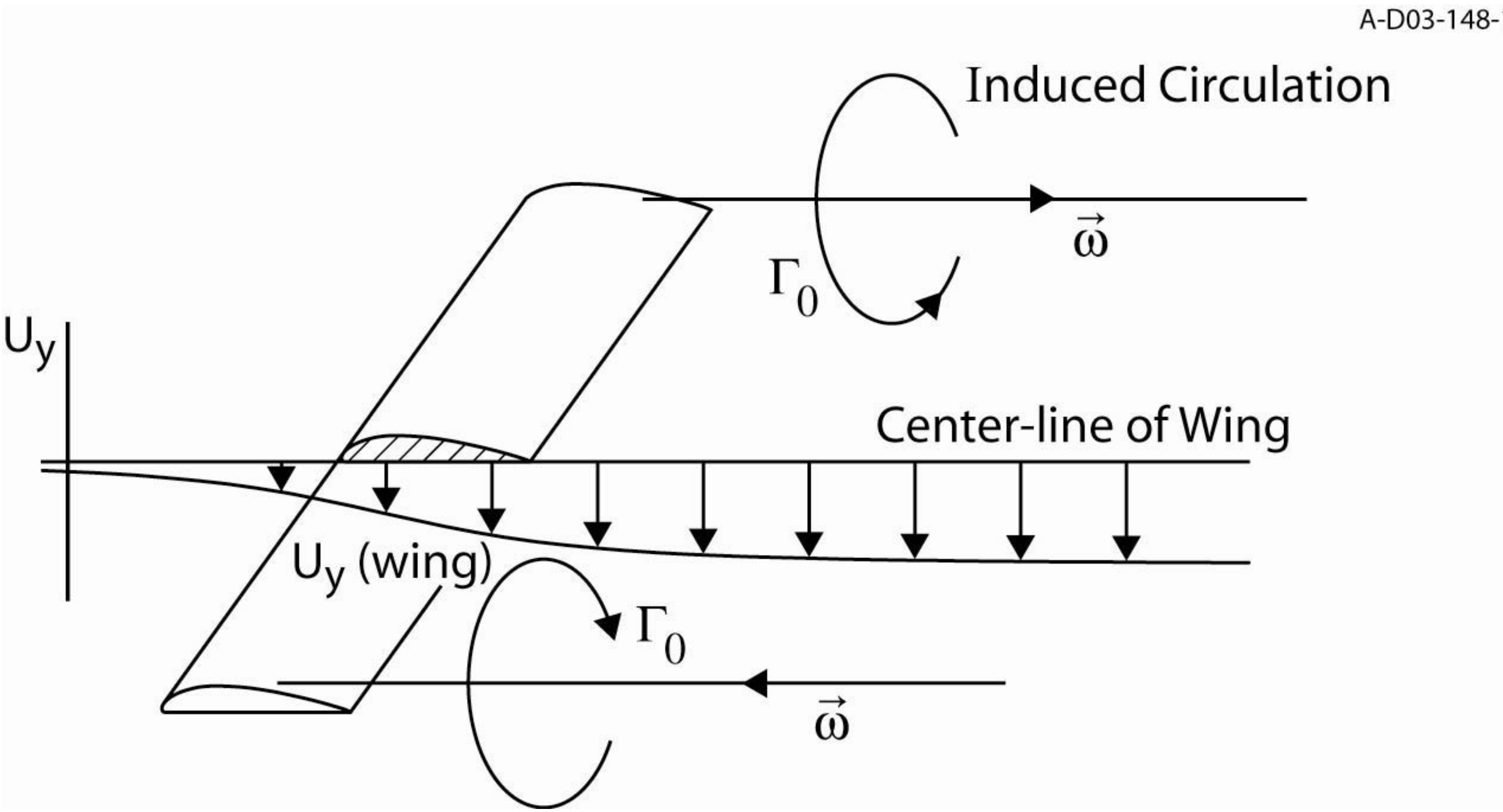






Vortex Trails

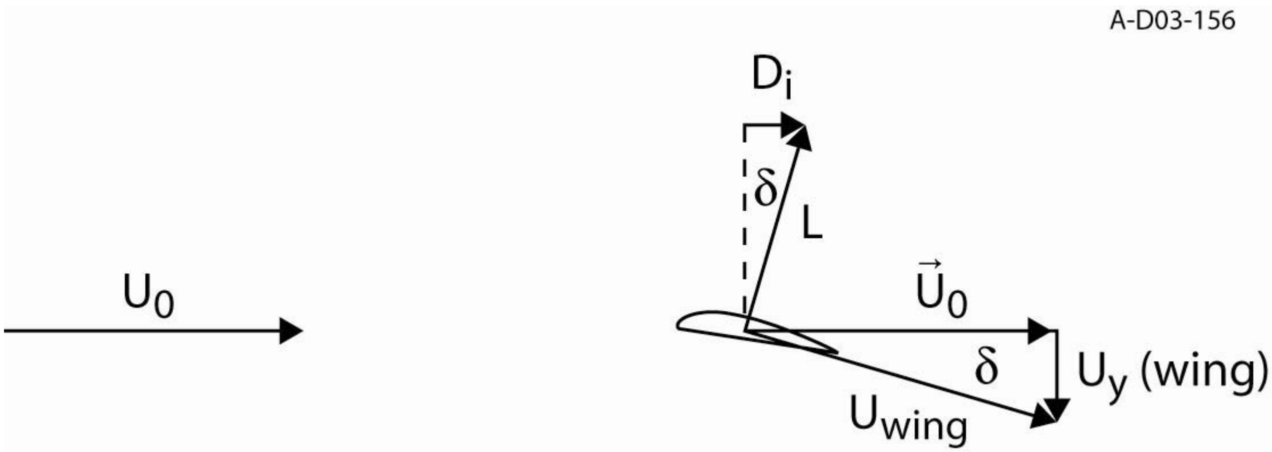
The Induced Downflow at the Wing



Note: The downflow velocity at the wing is exactly (1/2) of the downstream value (for a straight wing)

Induced Drag, D_i

Upstream **At the wing**

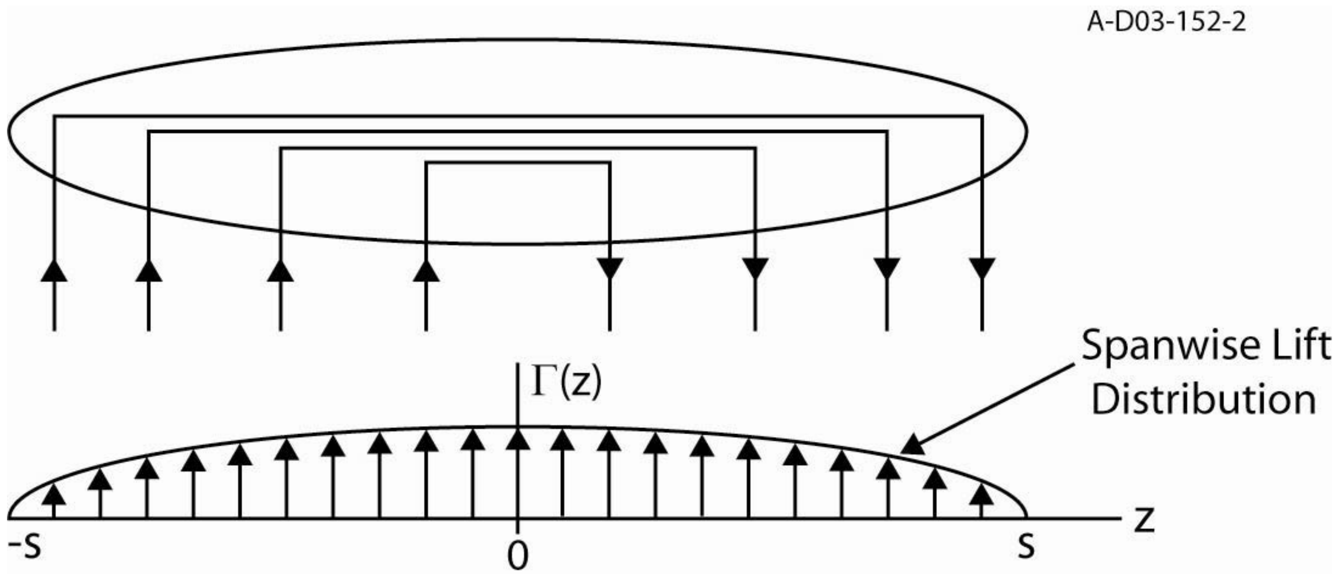


D_i = component of L in the U_0 direction

$\delta \approx U_y / U_0 =$ angle of the downflow at the wing

The Elliptical Wing Theorem

Prandtl, 1918-1919



$$L = \rho_m U_0 \int_{-s}^{s} \Gamma(z)\,dz$$

$$D_i = -\frac{\rho_m}{4} \int_{-s}^{s} \int_{-s}^{s} \frac{d\Gamma}{dz} \frac{\Gamma(z')}{z - z'}\,dz\,dz'$$

Problem: Minimize D_i, while holding L constant

Answer: $\Gamma(z) = \Gamma_c \left[1 - (z/s)^2\right]^{1/2}$, i.e., an ellipse

The Total Drag Force

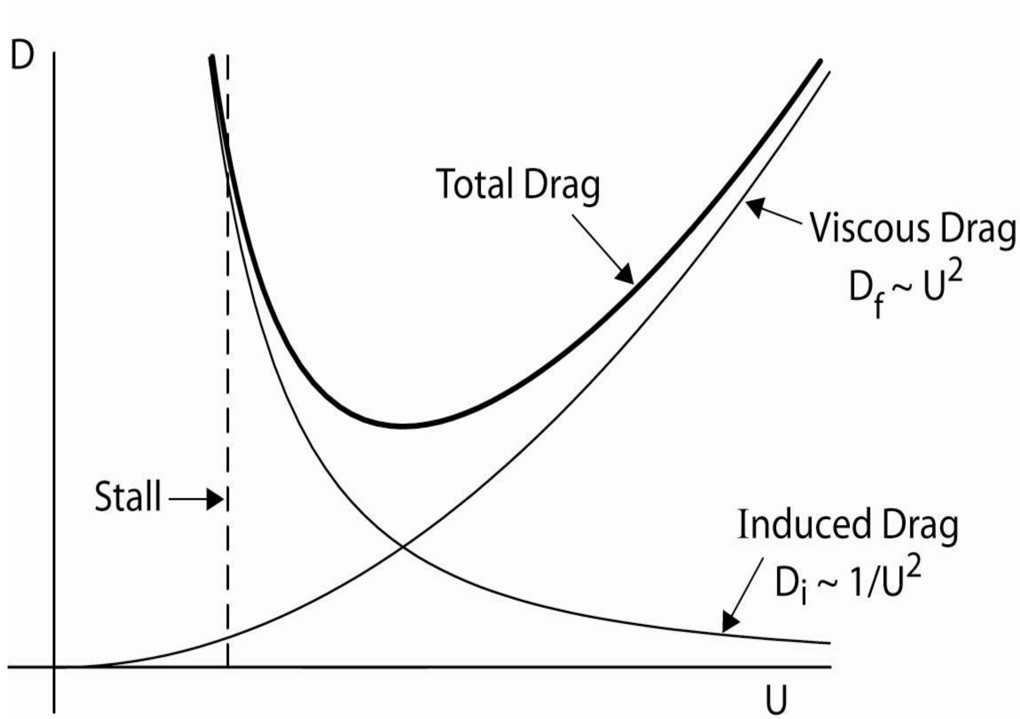
Induced Drag

$$D_i = \frac{1}{4\pi}\left(\frac{A}{s^2}\right)\frac{L^2}{\left(\frac{1}{2}\rho_m U^2 A\right)}$$

Viscous Drag

$$D_f = \frac{1}{2}\rho_m U^2 A\, C_{Df}$$

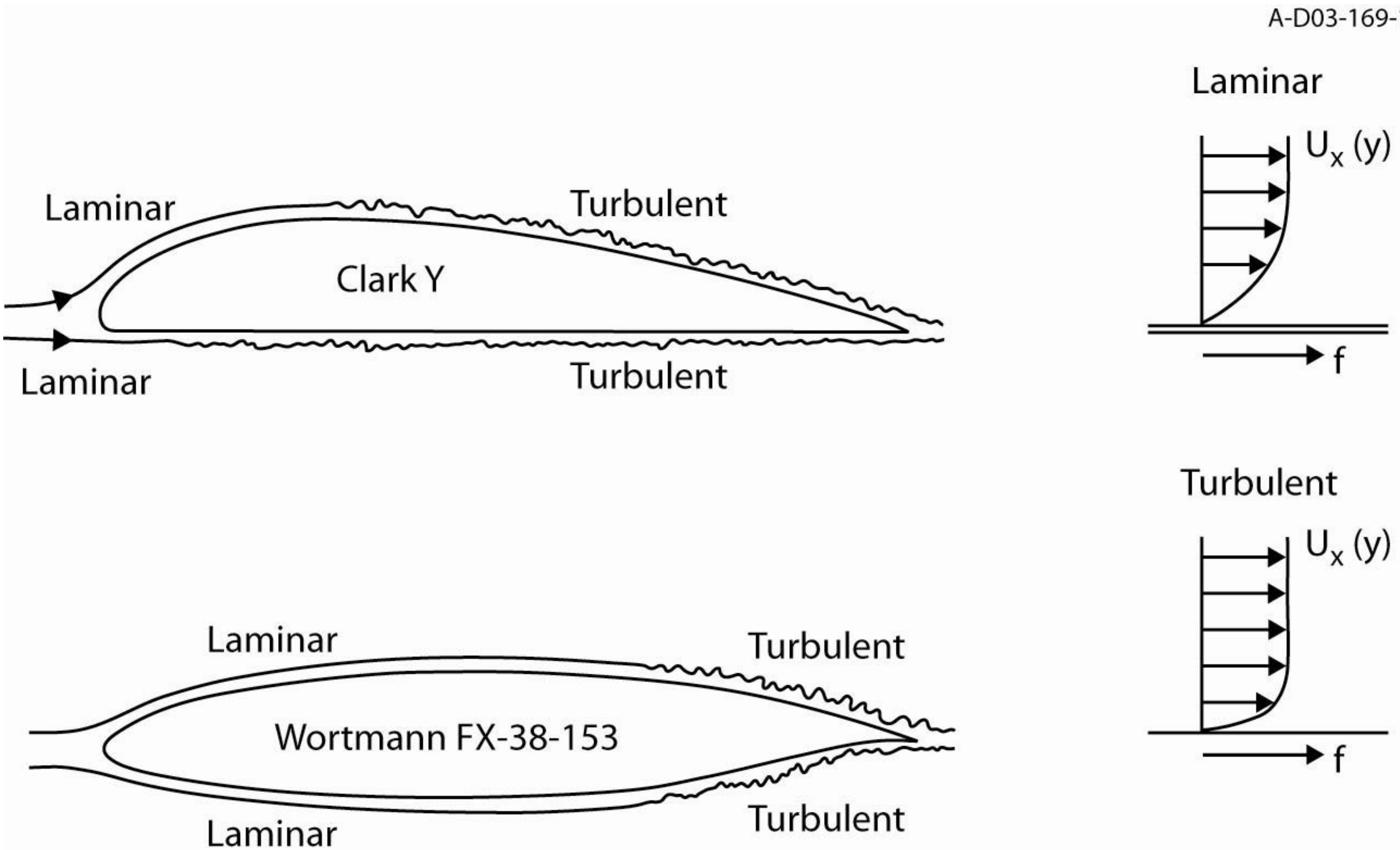
Note, A/s^2 = 1/Aspect ratio



Some Elliptical Wings

Winglets

Viscous Drag Reduction
Laminar Flow Airfoils, NACA 1930s

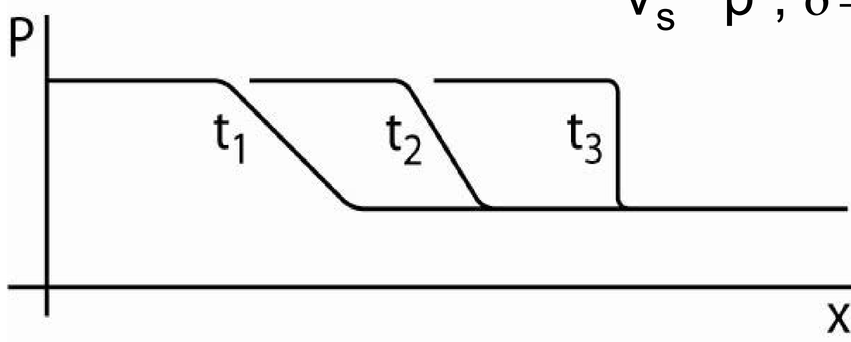


First Use of a Laminar Flow Airfoil, P-51, 1940

Compressibility Effects

Shock Wave

$v_s \sim p^{\delta}, \ \delta = 1/5$

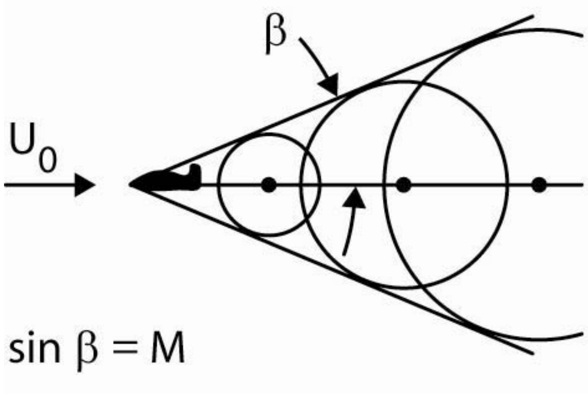


Mach Number

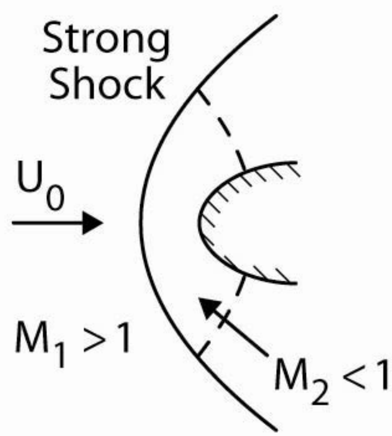
(Ernst Mach, 1889)

$$M = U_0/V_S$$

Mach Cone



Detached Shock



Attached Shock

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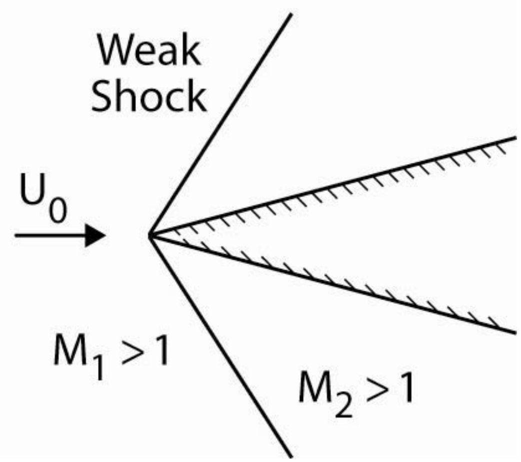
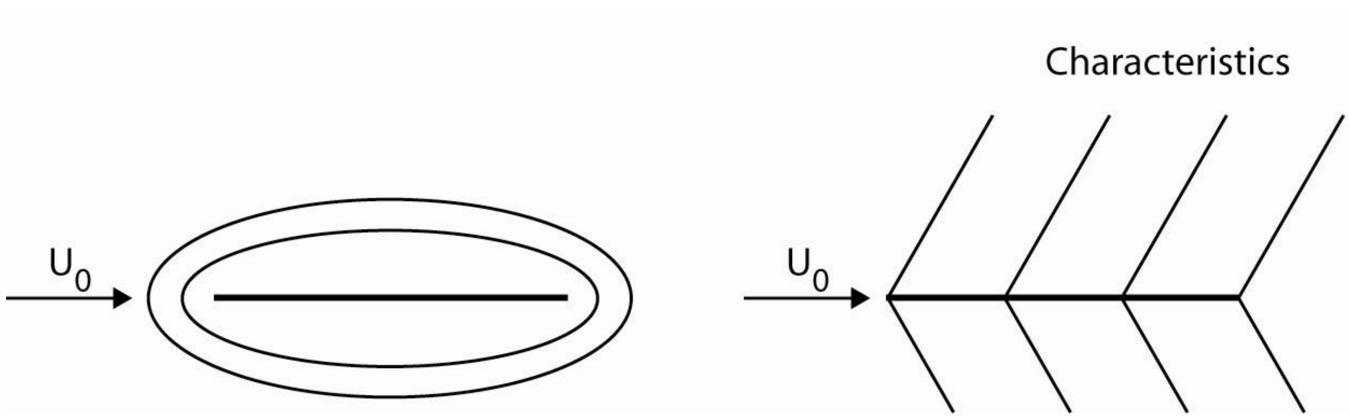


Thin Wing Theory
(Theodor von Karman, 1940s)

Laplace's equation modified for compressibility effects

$$\left(1-M^2\right)\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0$$

M < 1	M > 1
Elliptical differential equation	Hyperbolic differential equation



Solution: transform to M = 0

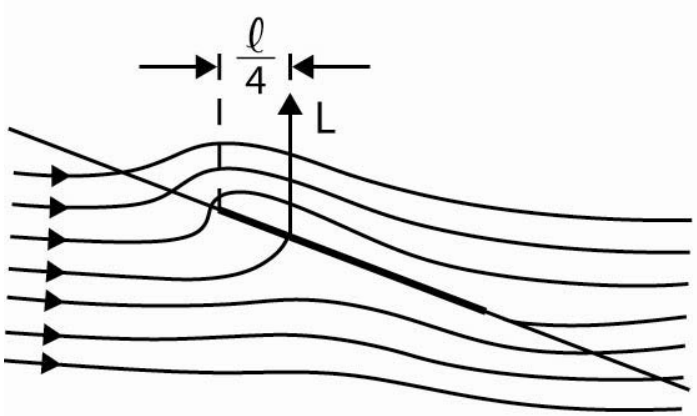
$$x' = \frac{x}{\sqrt{1-M^2}}$$

Solution: wave equation

$$\Phi = f\left(y - \frac{1}{\sqrt{M^2-1}}x\right)$$

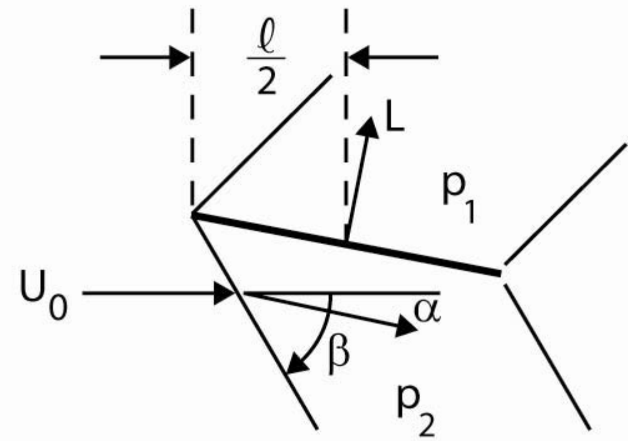
The Flat Plate Airfoil

Subsonic ($M < 1$)



$$C_L = \frac{2\pi}{\sqrt{1-M^2}} \sin\alpha$$

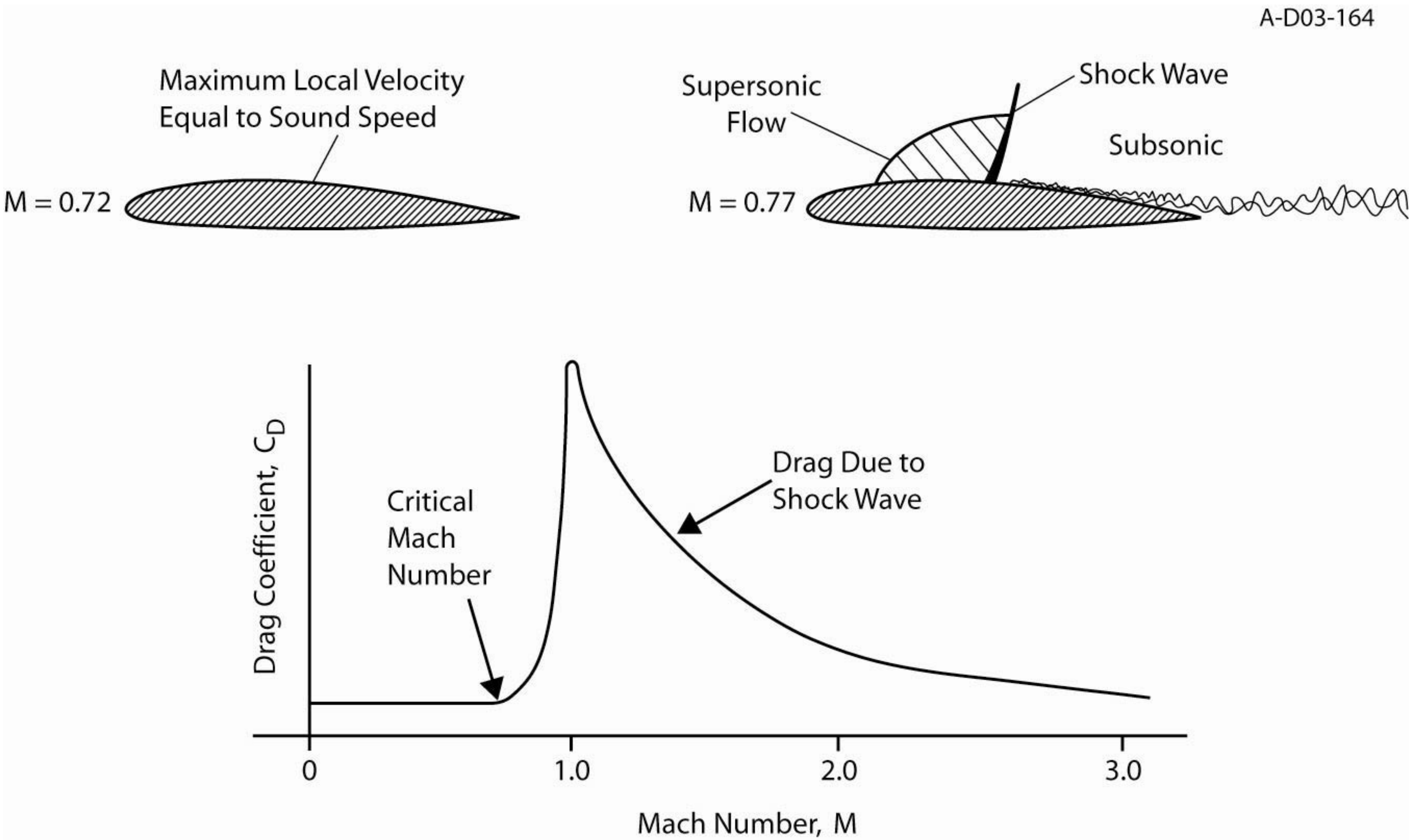
Supersonic ($M > 1$)



$$C_L = \frac{4\alpha}{\sqrt{M^2 - 1}}$$

Note the change in the center of pressure, from $\ell/4$ to $\ell/2$.

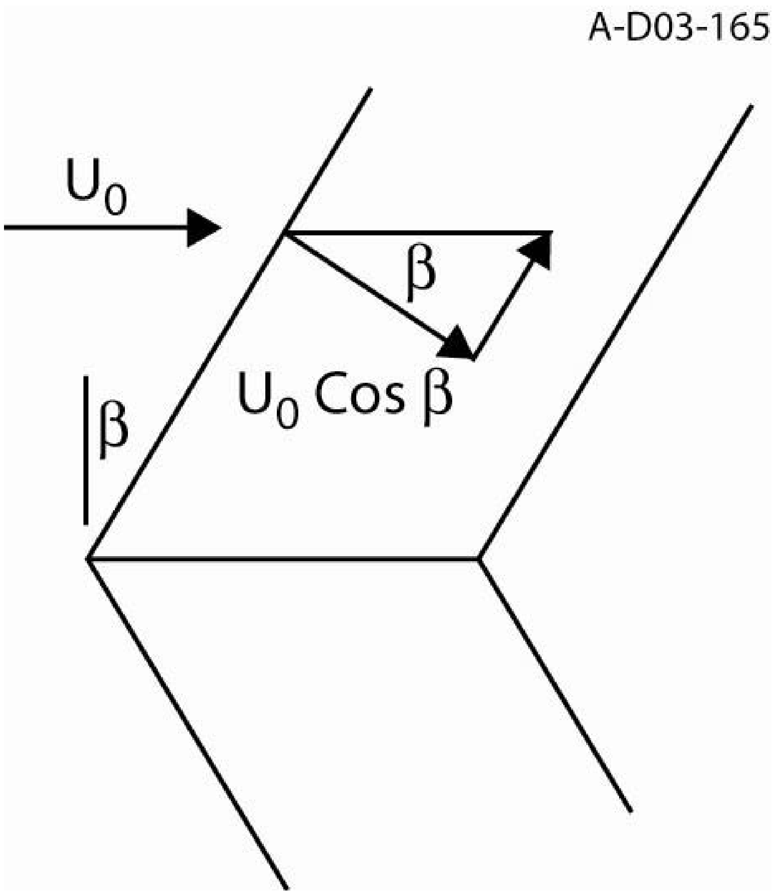
The Critical Mach Number



A Shock at the Critical Mach Number

Sweepback
(Busemann, 1935)

Sweepback increases the critical Mach number

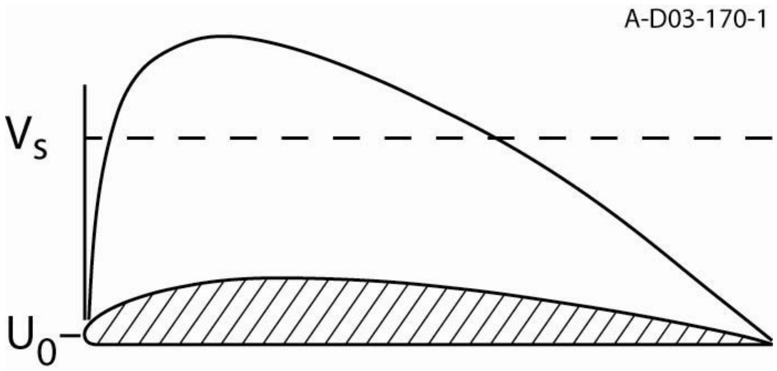


First use of sweepback Me-262, 1941

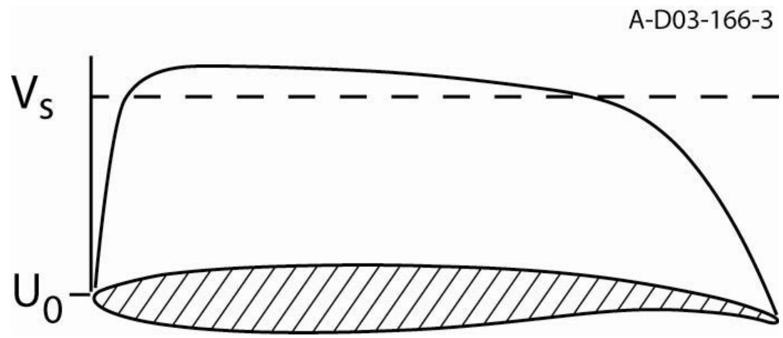
Supercritical Airfoil

(Whitcomb, 1971)

Conventional Airfoil



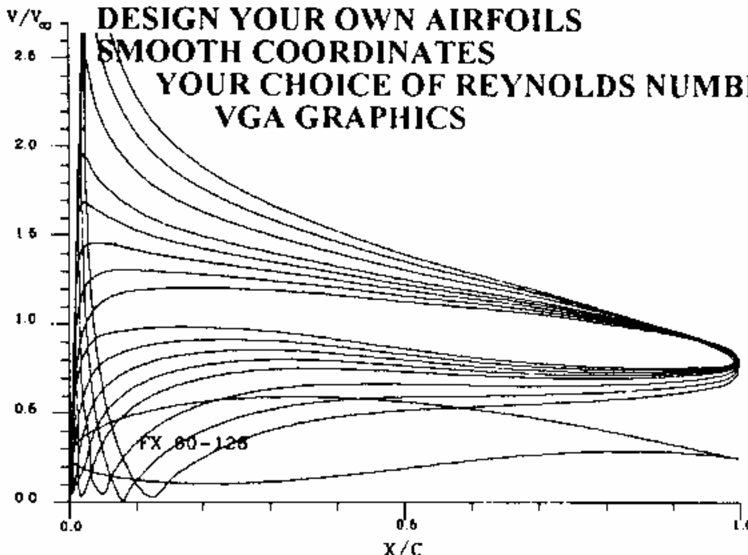
Supercritical Airfoil



Boeing 777 (M_c = 0.85)

A Wind Tunnel in Your Computer

DESIGN YOUR OWN AIRFOILS
SMOOTH COORDINATES
YOUR CHOICE OF REYNOLDS NUMBERS
VGA GRAPHICS



PREDICT THE PERFORMANCE OF AIRFOILS
SCALE TO FULL SIZE
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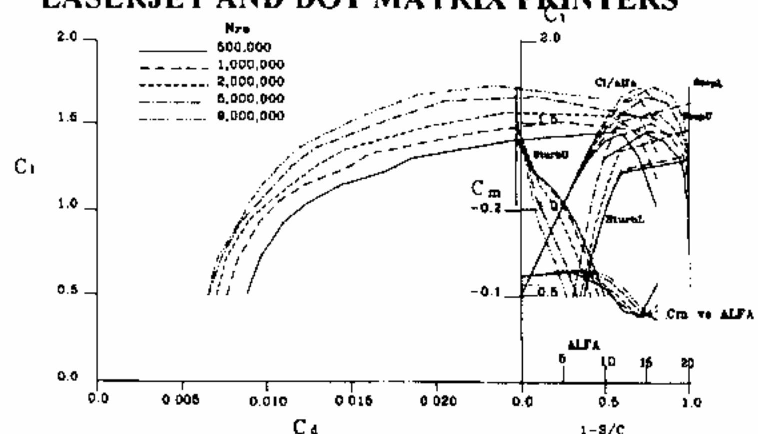

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Course Advertisement

- Consider taking Mechanics of Continua, 29:211
- 12:15 to 1:30 p.m., T-Th, 618 Van Allen Hall
- Topics covered include the fundamental equations of fluid mechanics, incompressible and compressible flows in 2 and 3 dimensions, wave propagation, shock waves, instabilities, turbulence, and boundary layer physics
- Prerequisites: working knowledge of vector calculus, i.e., curl, divergence, gradient and associated identities