Introduction to Structural Analysis

-Matrix Methods

By

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INTRODUCTION

Mechanics, is the branch of physics concerned with the behaviour of physical bodies when subjected to forces or displacements, and the subsequent effect of the bodies on their environment.

- Statics - bodies at rest or moving with uniform velocity
- Dynamics - bodies accelerating
- Strength of materials - deformation of bodies under forces.
- Structural Mechanics - focus on behavior of structures under loads.

Structural Analysis is a process by which the structural engineer determines the response of a structure to be specified loads or actions.

Response :

- Magnitude of force development (collapse)
- Magnitude of deformation (serviceability)

Structural Engineering Projects can be divided into 4 stages.

1. Planning phase
   - Material
   - Structural form
   - Loads
2. **Analysis**
3. Design
4. Construction

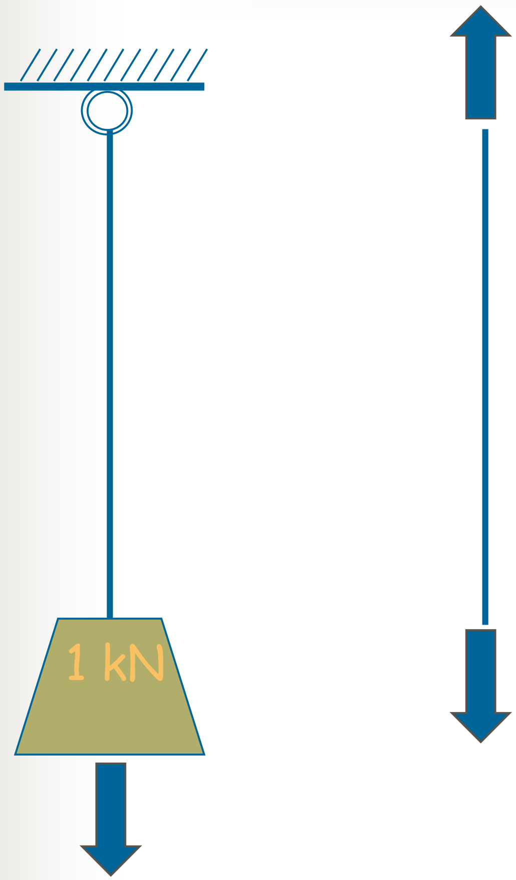
The design of a structure involves many considerations, among which are 4 major objectives that must be satisfied.

1) Safety (the structure must carry loads safely)
2) Economy (the stucture should be economical in material and overall costs)
3) Utility (the structure must meet the performance requirement)
4) Beauty (the stucture should have a good performance)

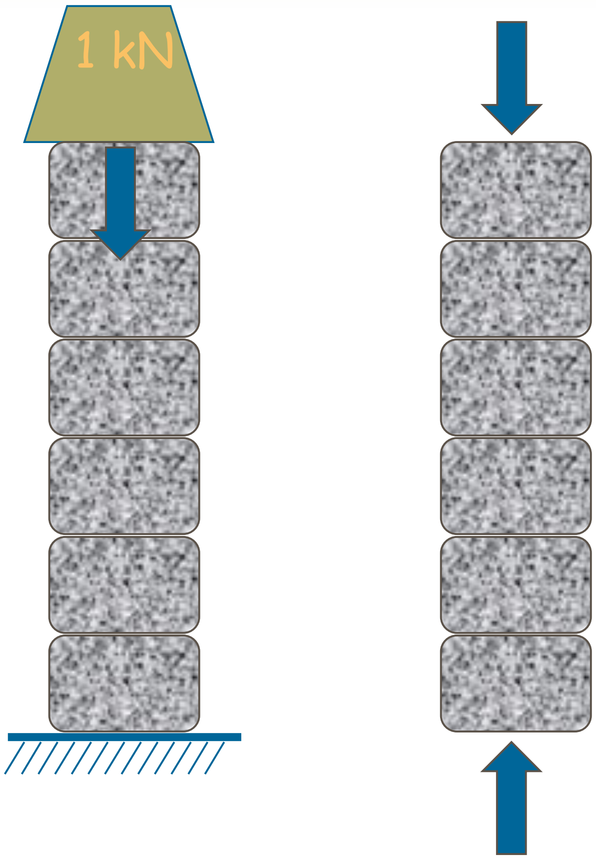
Therefore, the purpose of structural analysis is to determine the reactions, internal forces and deformations at any point of a given structure caused by applied loads and forces.

Types of Structural forms

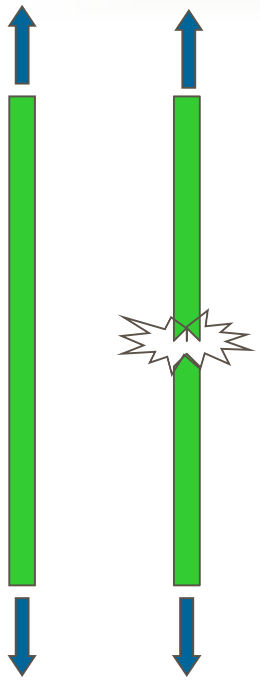
- Tension and Compression structures
- Flexural beam and frame structures (load carrying is achieved by bending)
- Surface structures (load carrying is by membrane action)



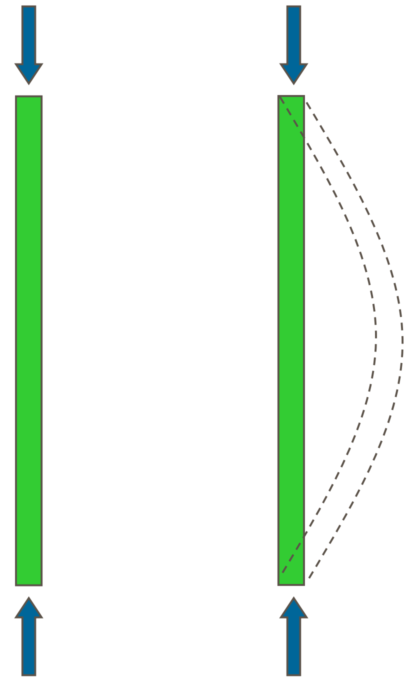
Tension



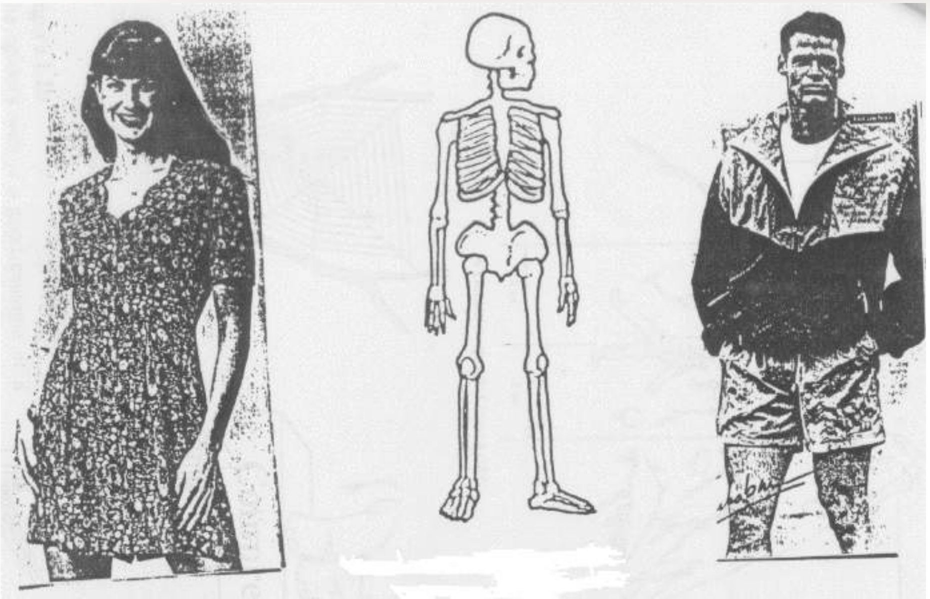
Compression



Tensile Failure



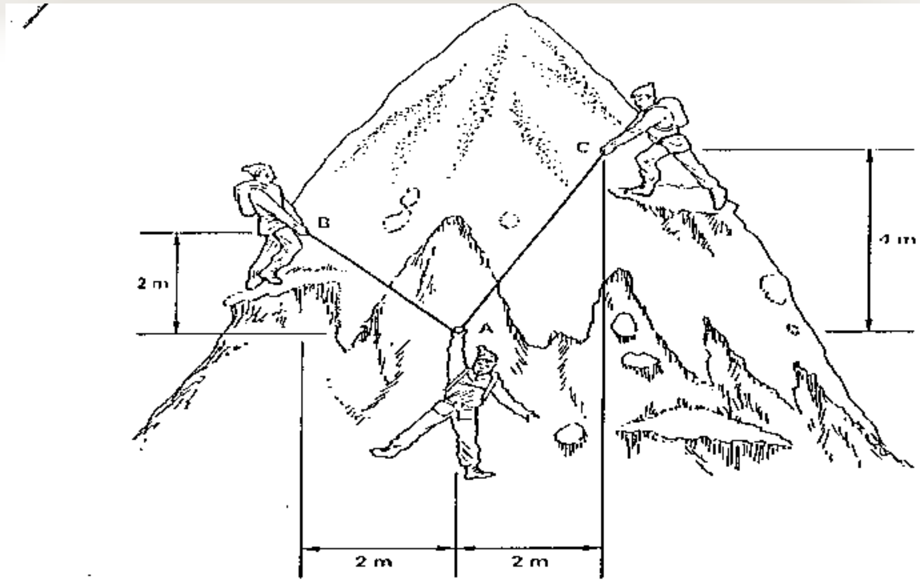
Compressive Failure



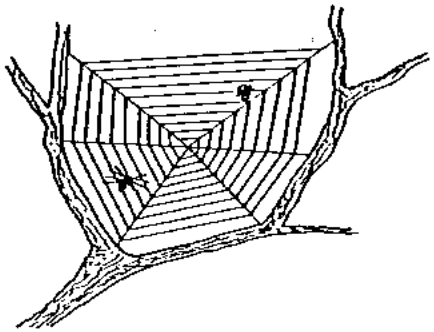
Various components carry different types of loads

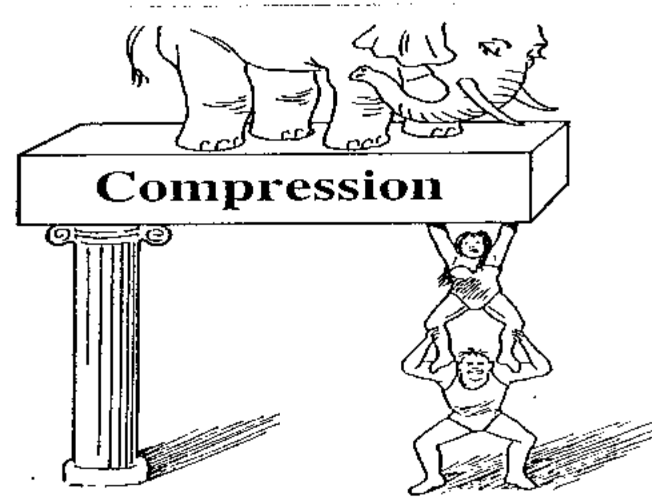
The human skeleton is a structure which maintains the shape of the body, keeps the various organs and muscles in the right place and transmits loads down to the ground

The spider’s web is a good example of a tension structure. The weight of the spider and its prey is supported by tensile strength of the web



A climber suspended from a cable





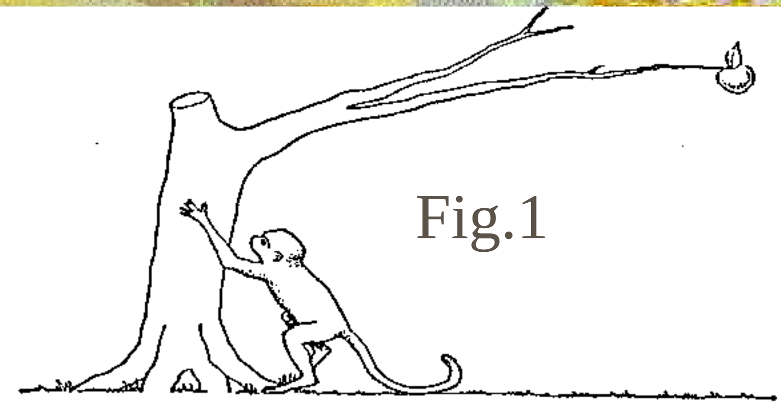


Fig.1

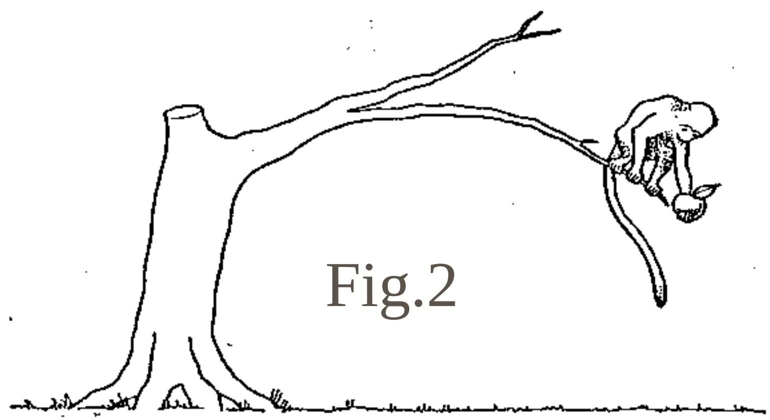
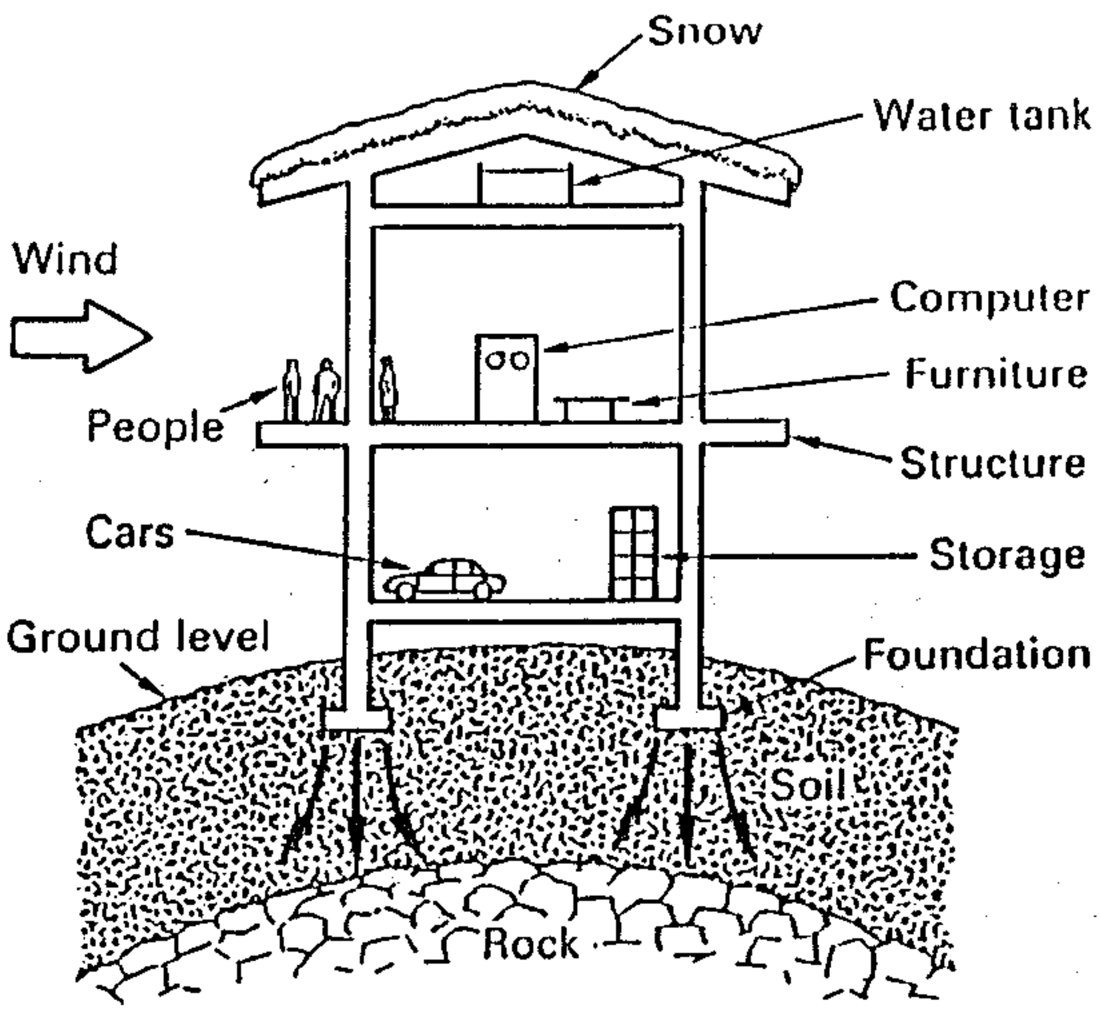


Fig.2

Figures 1and 2

- All materials and structures deflect, to greatly varying extents, when they are loaded. The science of elasticity is about the interactions between forces and deflections. The material of the bough is stretched near its upper surface and compressed or contracted near its lower surface by the weight of the monkey

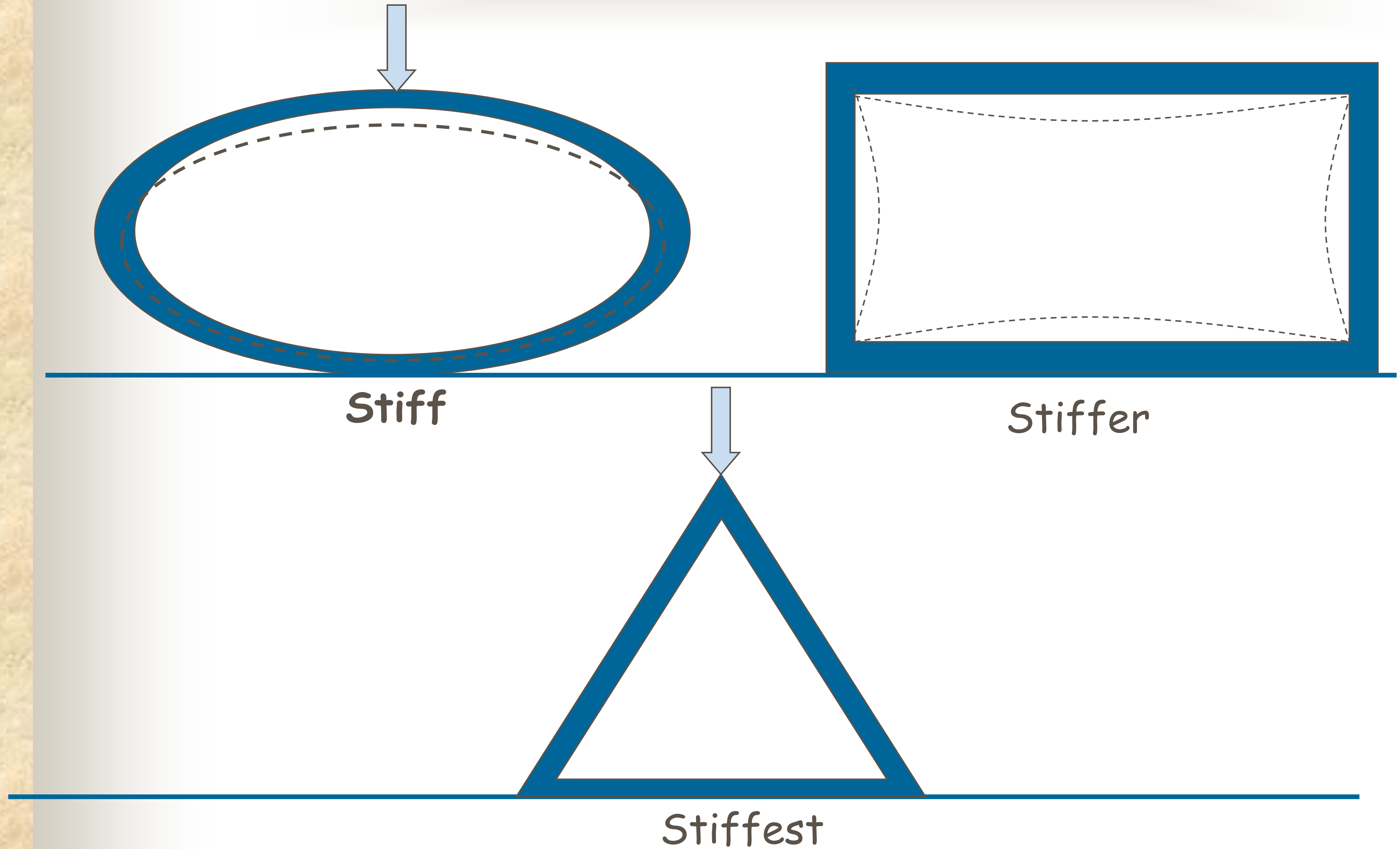


- A building structure safely transmits loads down to Earth

Important Structural Properties

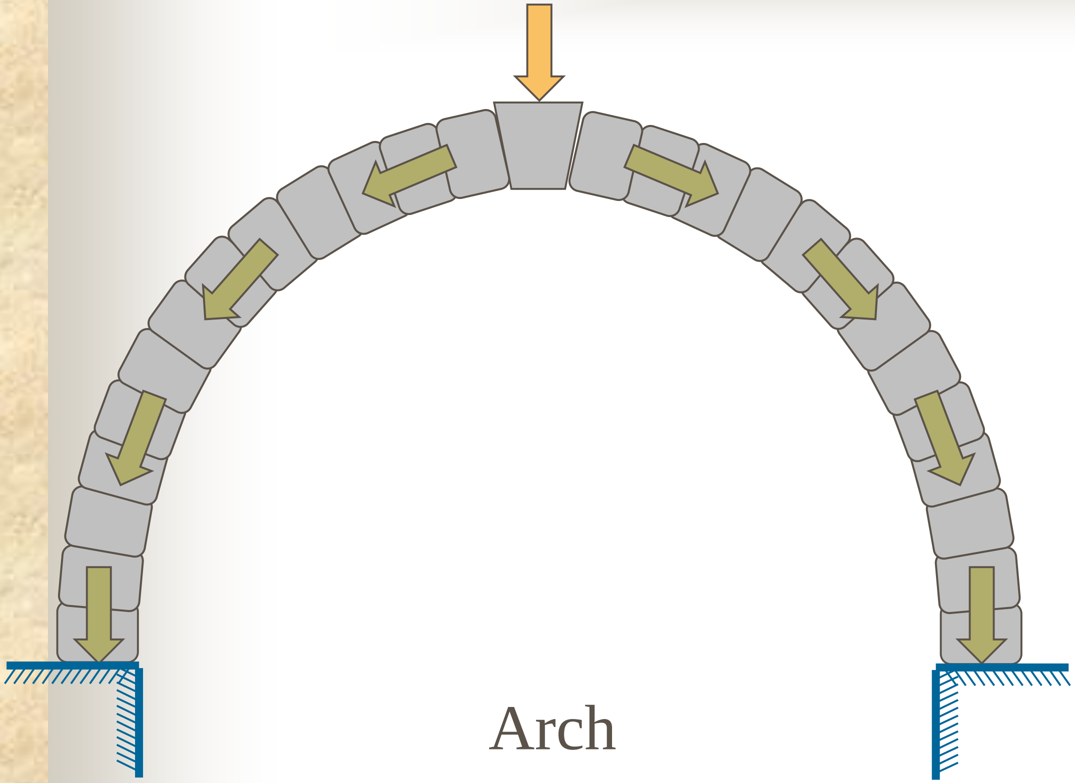
Strength : Ability to withstand a given stress without failure. Depends on type of material and type of force (tension or compression).

Stiffness : Property related to deformation. Stiffer structural elements deform less under the same applied load. Stiffness depends on type of material (E), structural shape, and structural configuration.Two main types; Axial stiffness and Bending stiffness.

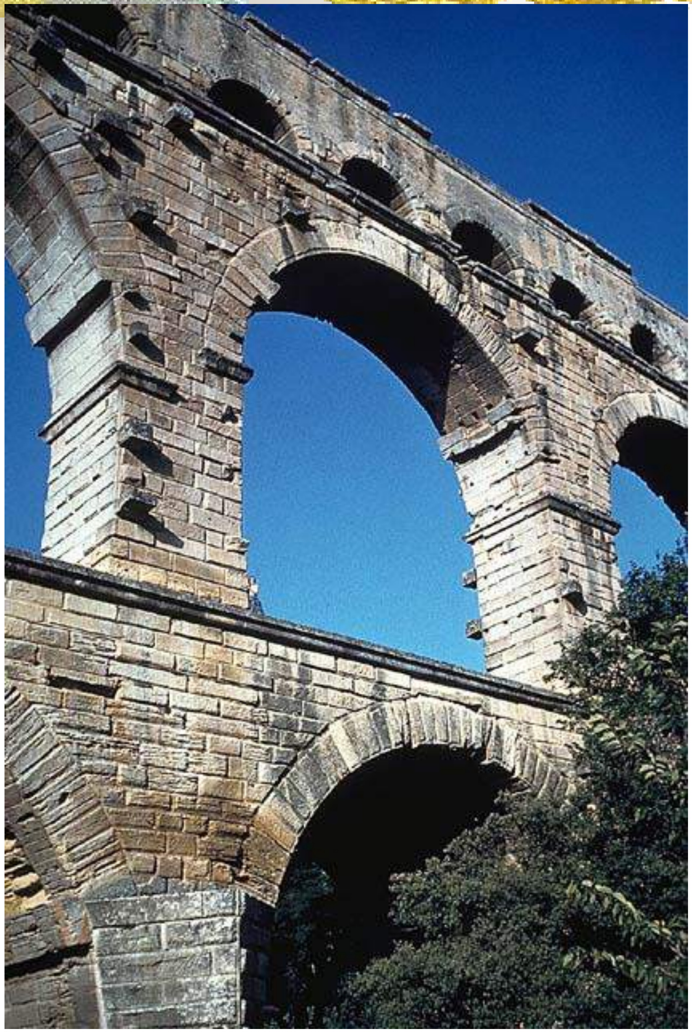

Stiff
Stiffer
Stiffest

Cables

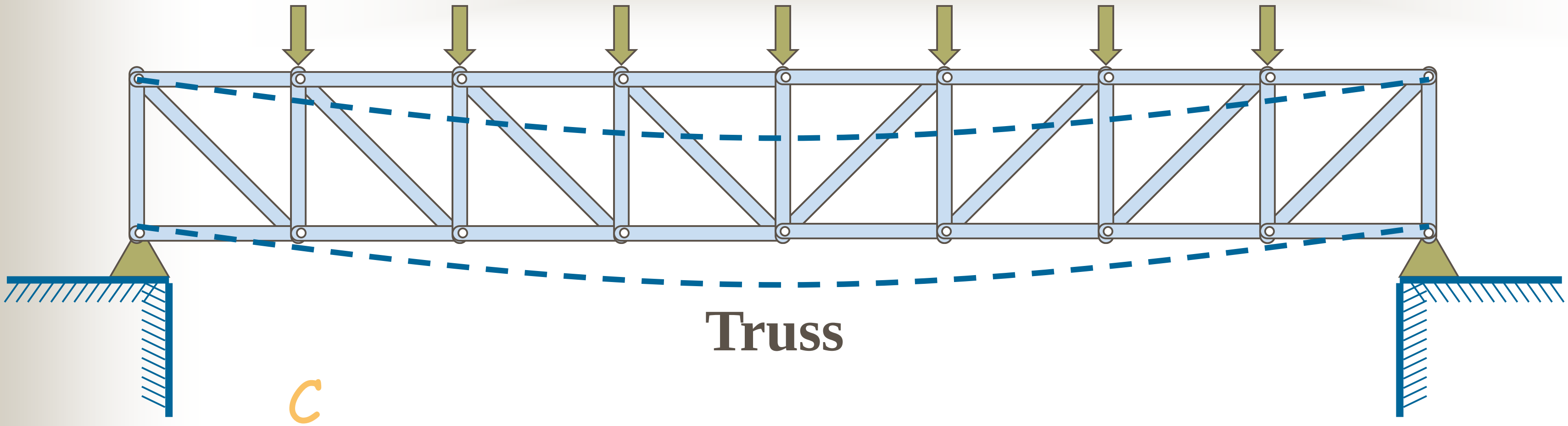
- simple
- uses
  - suspension bridges
  - roof structures
  - transmission lines
  - guy wires, etc.
- have same tension all along
- can't stand compression

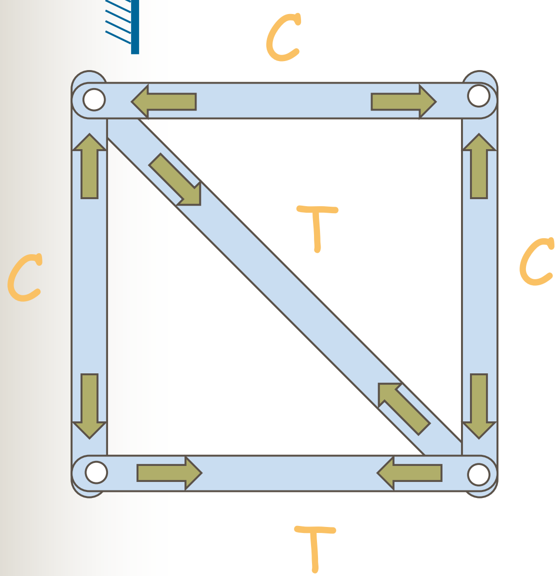


Arch

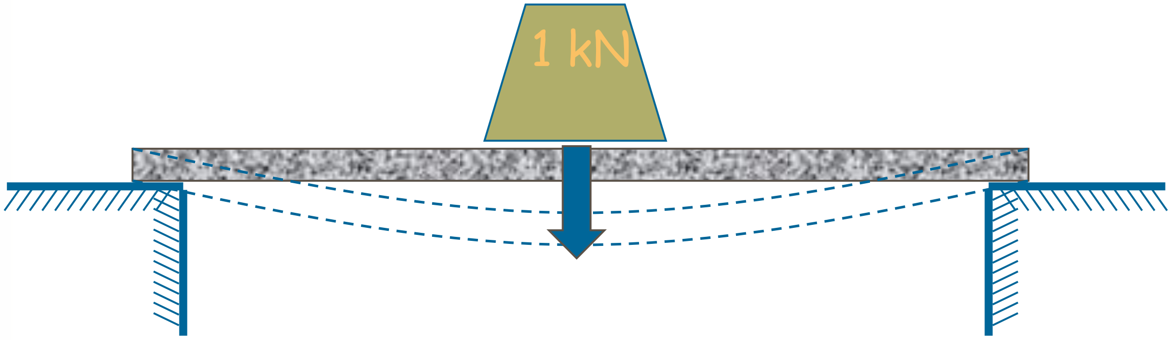


Arches carry the dominant permanent load case (usually full dead load) in pure axial compression.

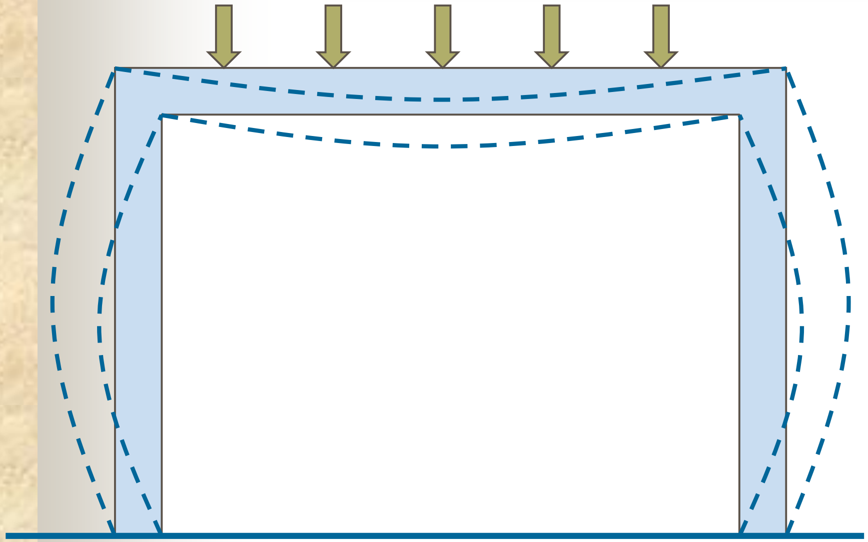




Forces in Truss Members



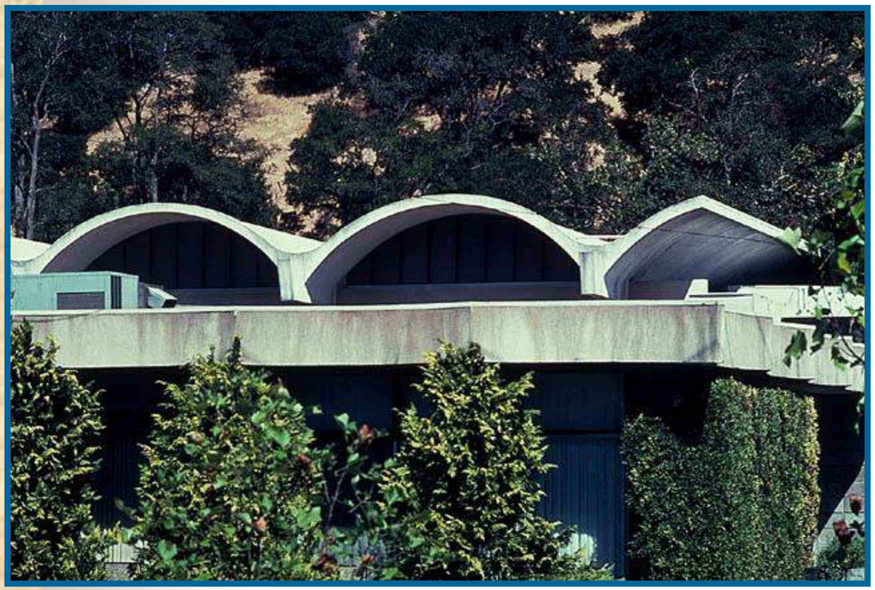
Bending



Frame

Flat Plate

Folded Plate

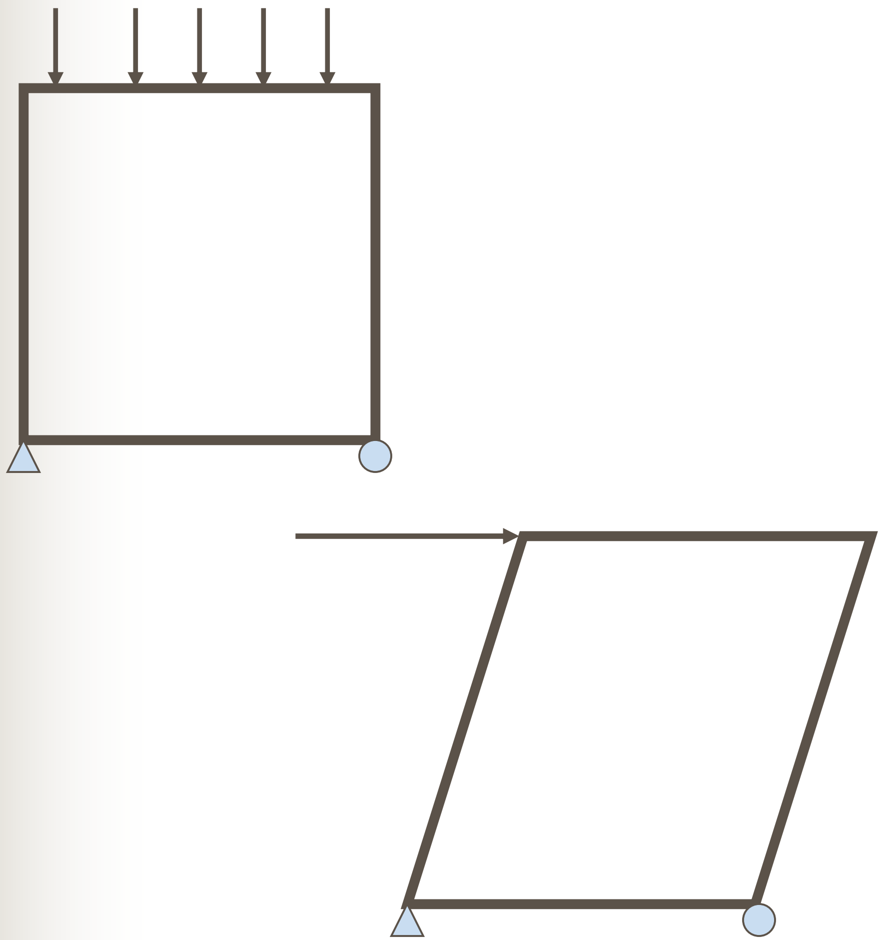


Shells

Structural Shapes

- Rectangle / Square
- Triangle
  - Interested in stability
- Truss
- Geodesic Dome

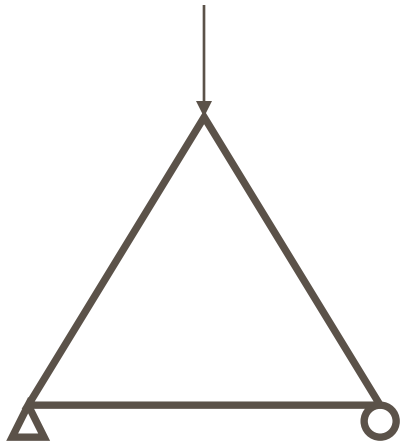
Rectangle

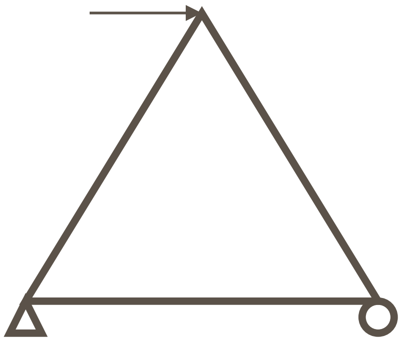


- Advantages
  - Proficient in resisting vertical load.
- Disadvantages
  - No lateral support

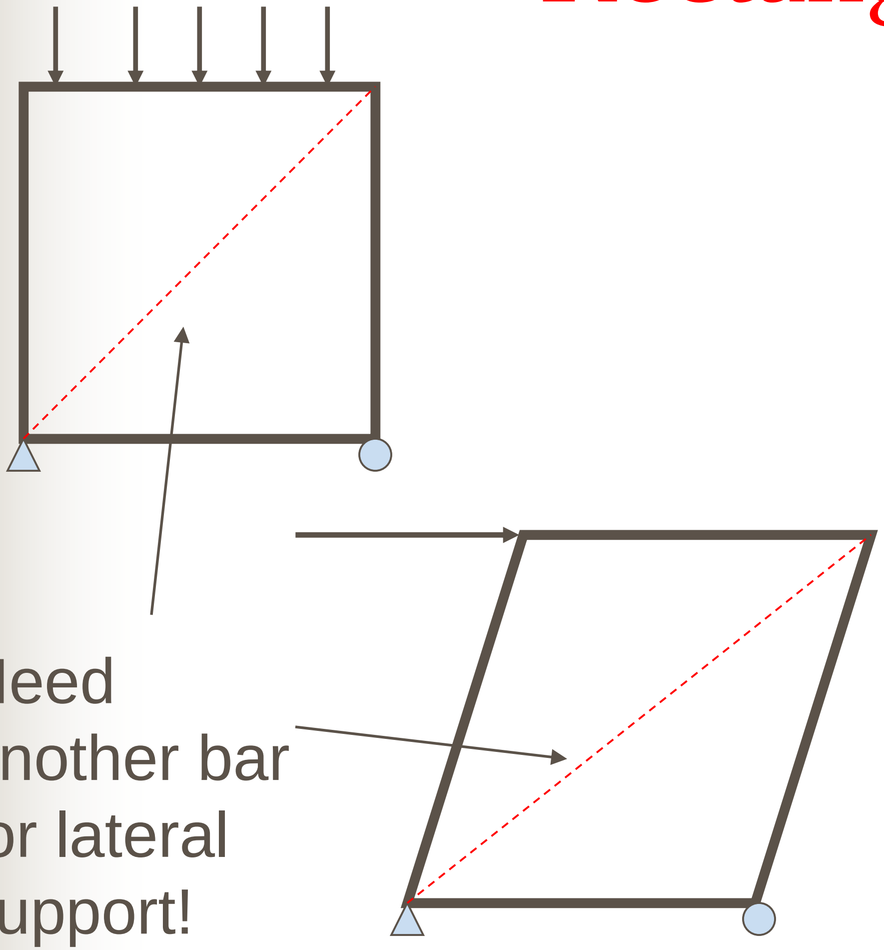
Triangle

- Advantages
  - Able to withstand lateral & vertical loading
  - Many triangular shapes available
- Disadvantage
  - Wide base





Rectangle



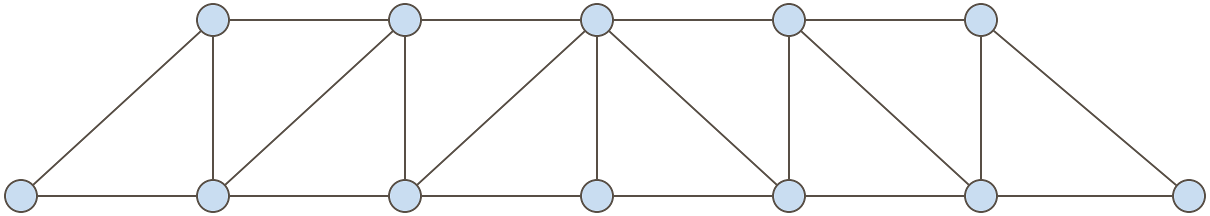
Need another bar for lateral support!

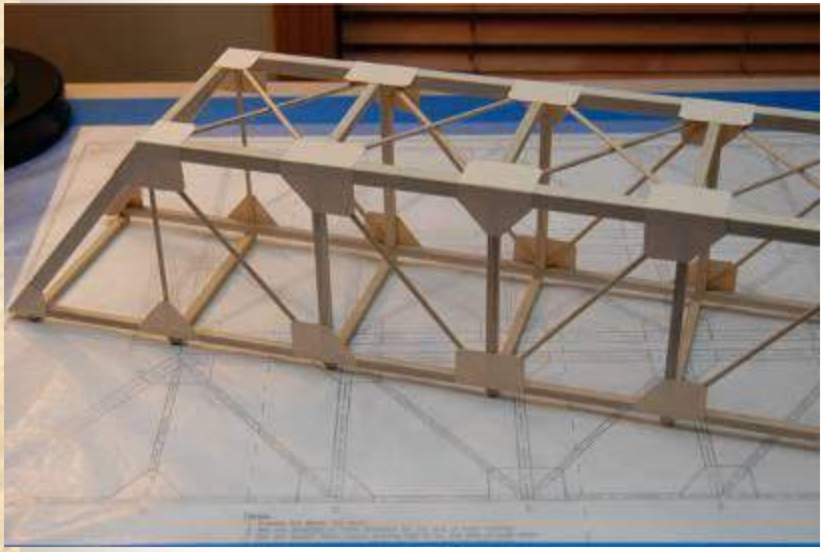
--BRACING--

- Advantages
  - Proficient in resisting vertical load.
- Disadvantages
  - No lateral (horizontal) load support

Truss

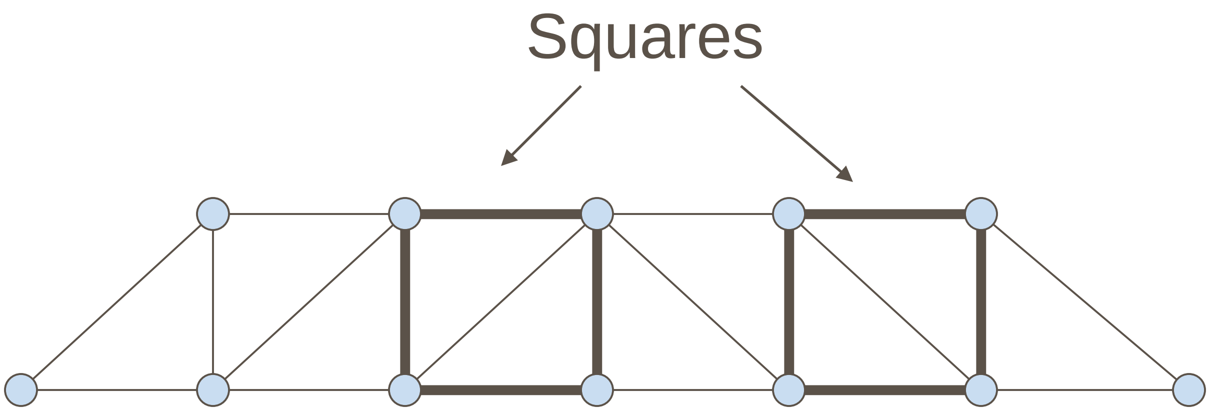
- Combination of square and triangle





Truss

- Combination of square and triangle



Truss

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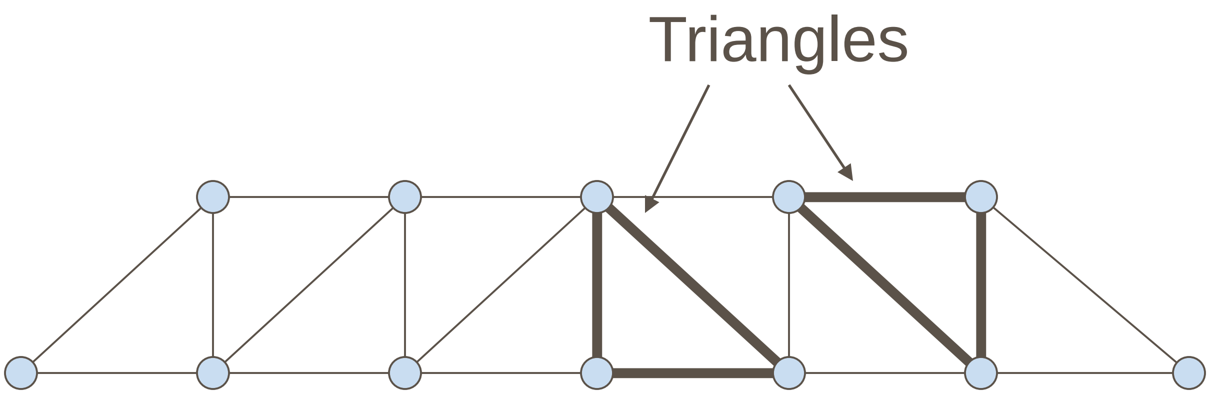
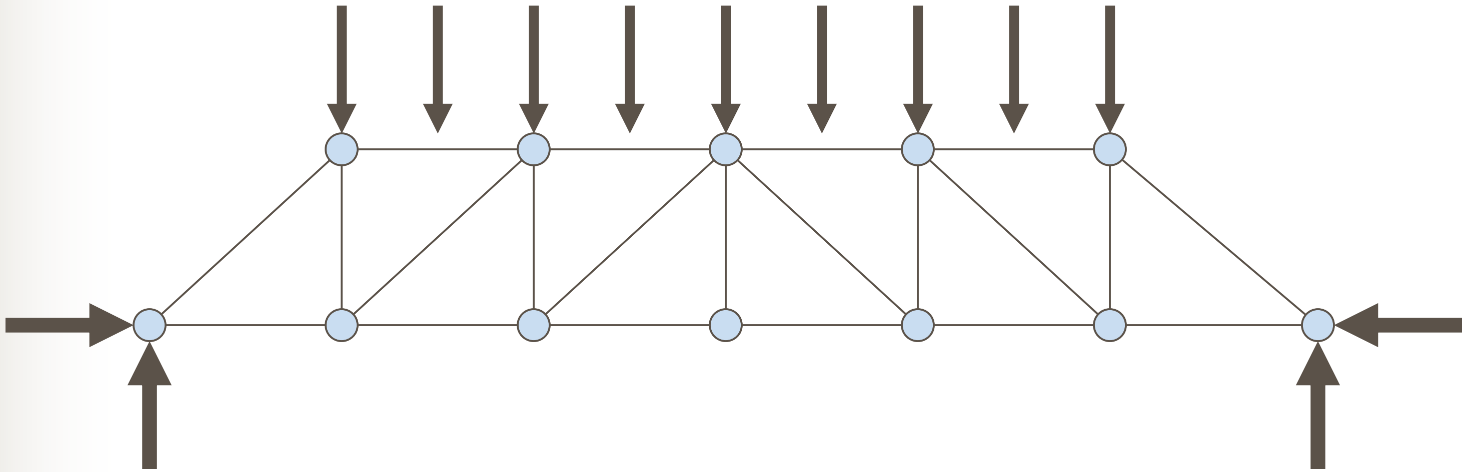


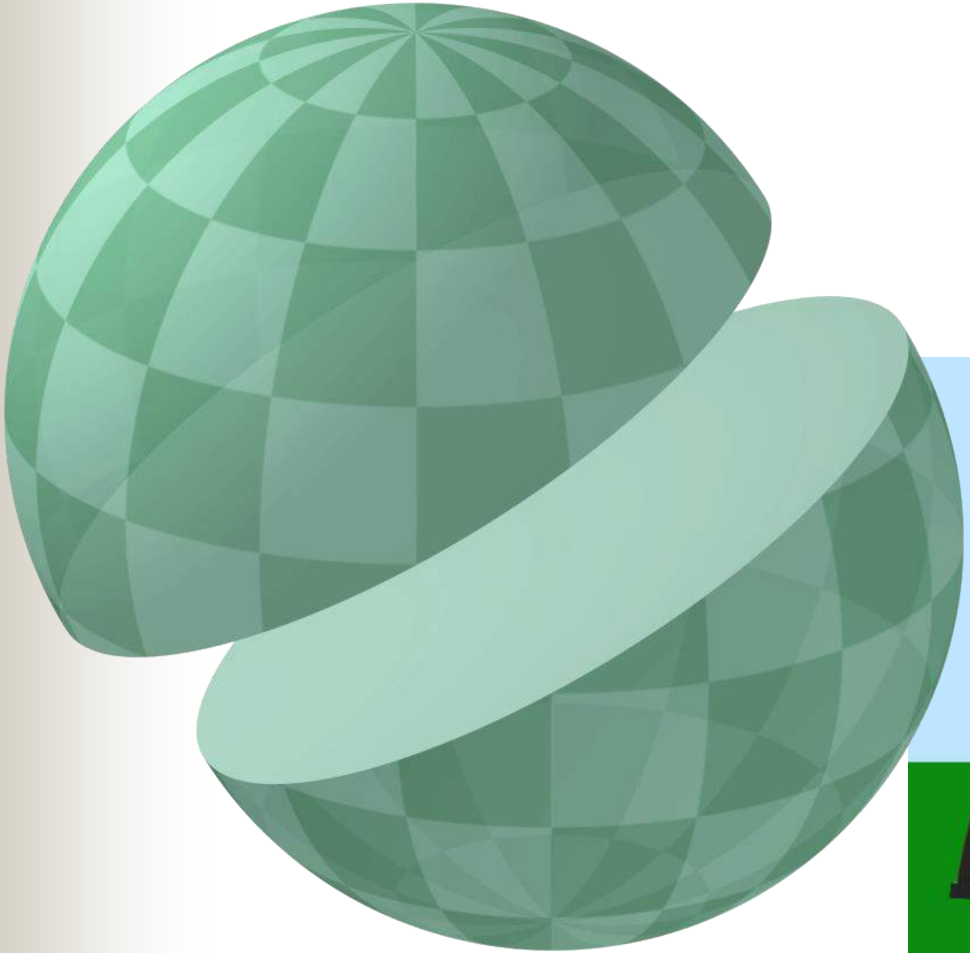
Truss

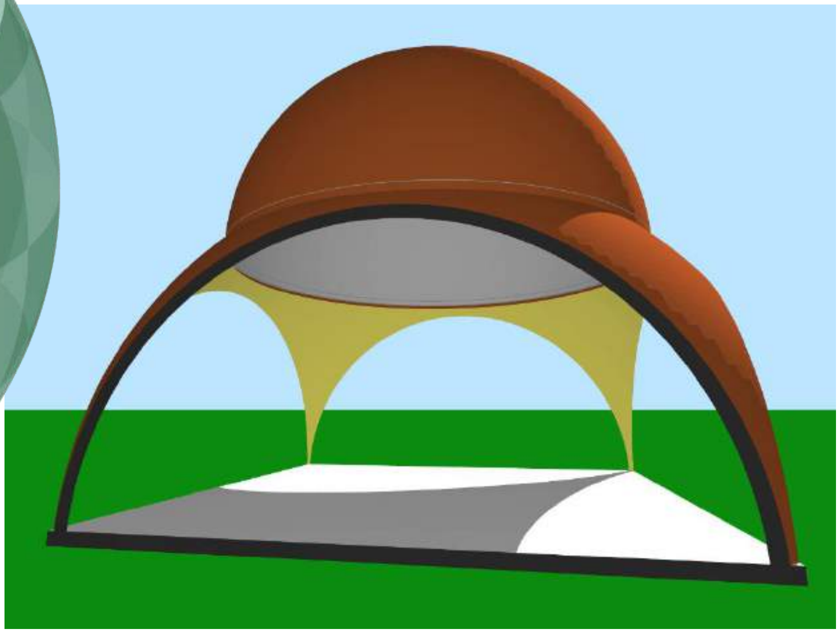
- Combination of square and triangle
  - Both vertical and lateral support



Geodesic Dome

Domes



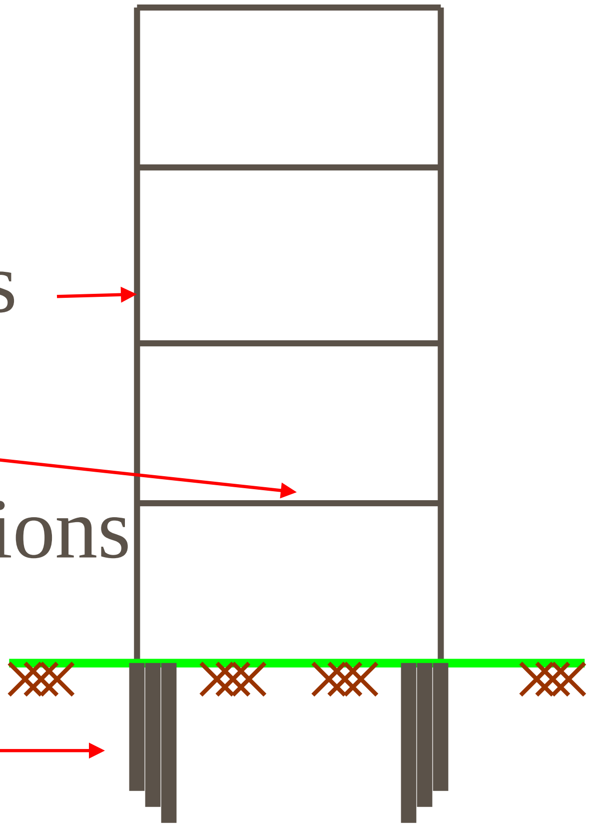


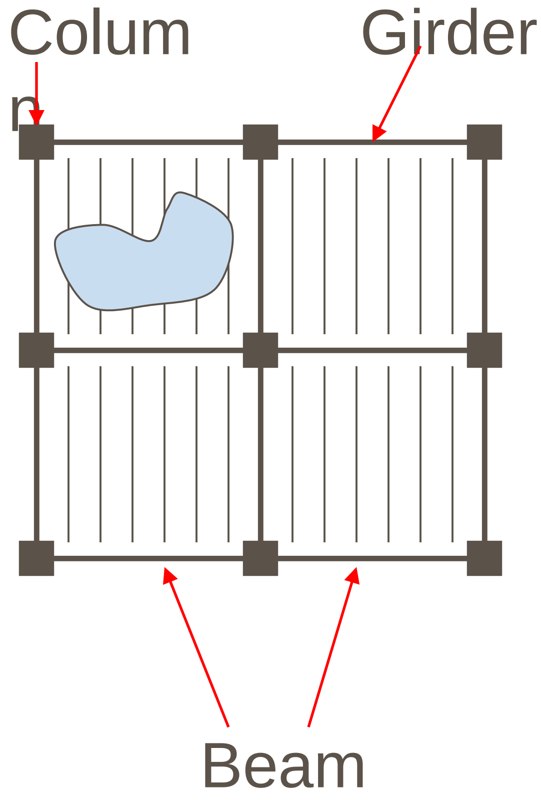
Domes

- Advantages
  - Very strong shape, gets strong as the dome size increases
  - Perfect load distribution
  - No need for structural supports
  - Great aerodynamic performance

Structural Components

- Beams
- Girders
- Columns
- Floors
- Foundations





Load Path

- Floor

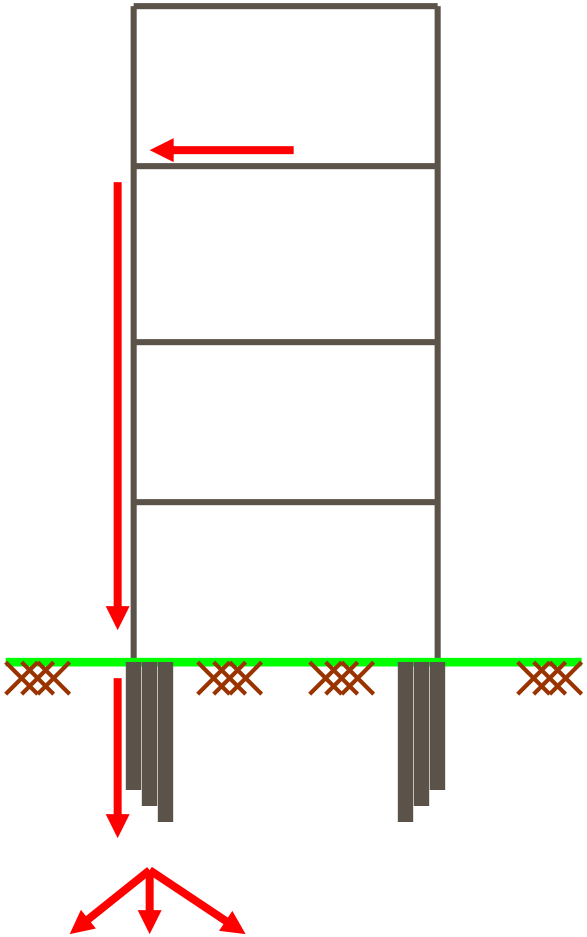
  ↓
- Beams

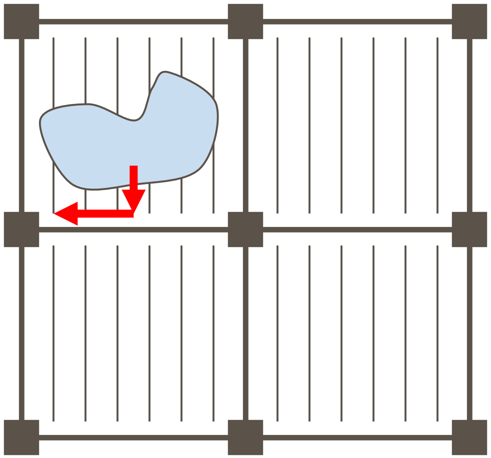
  ↓
- Girders

  ↓
- Columns

  ↓
- Foundation

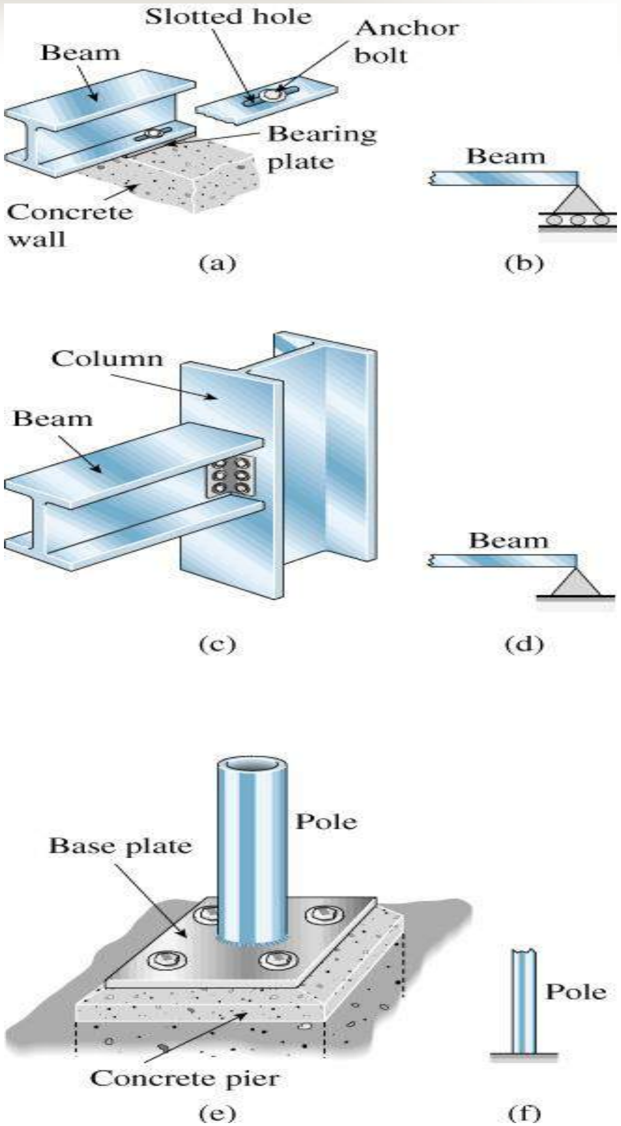
  ↓
- Soil/Bedrock





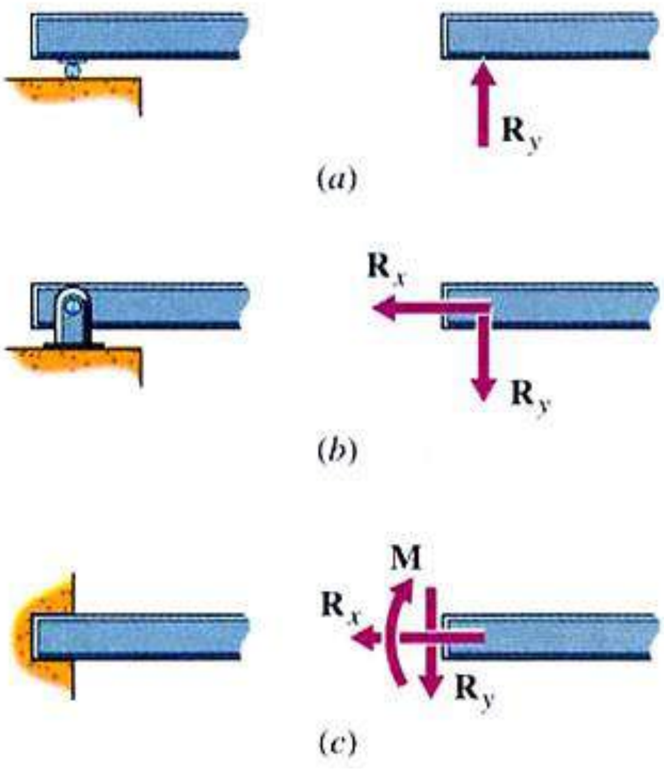
Support Connections

- Roller support (allows rotation/translation)
- Pin connection (allows rotation)
- Fixed joint (allows no rotation/translation)

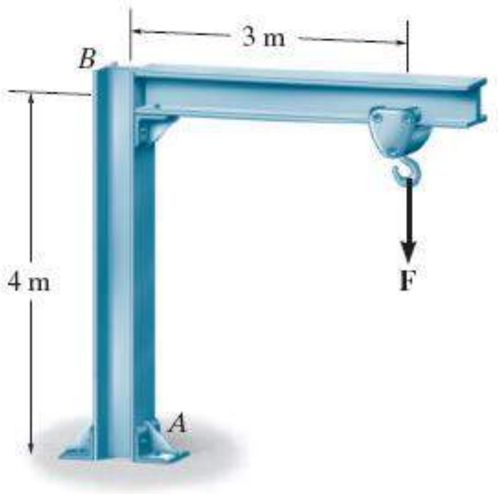


A beam have a variety of supports.

- roller (1-DOF)
- pinned (2-DOF)
- fixed (3-DOF)

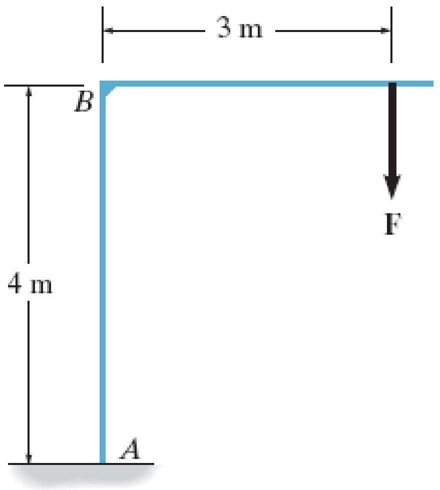


The process of defining an ideal structure from a real structure is called modeling. To carry out practical analysis it becomes necessary to idealize a structure.



actual structure

(a)

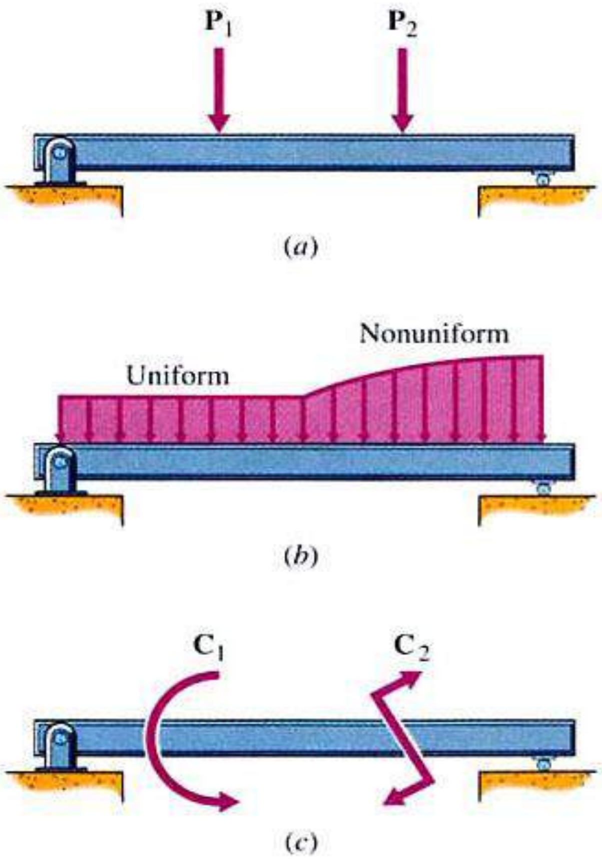


idealized structure

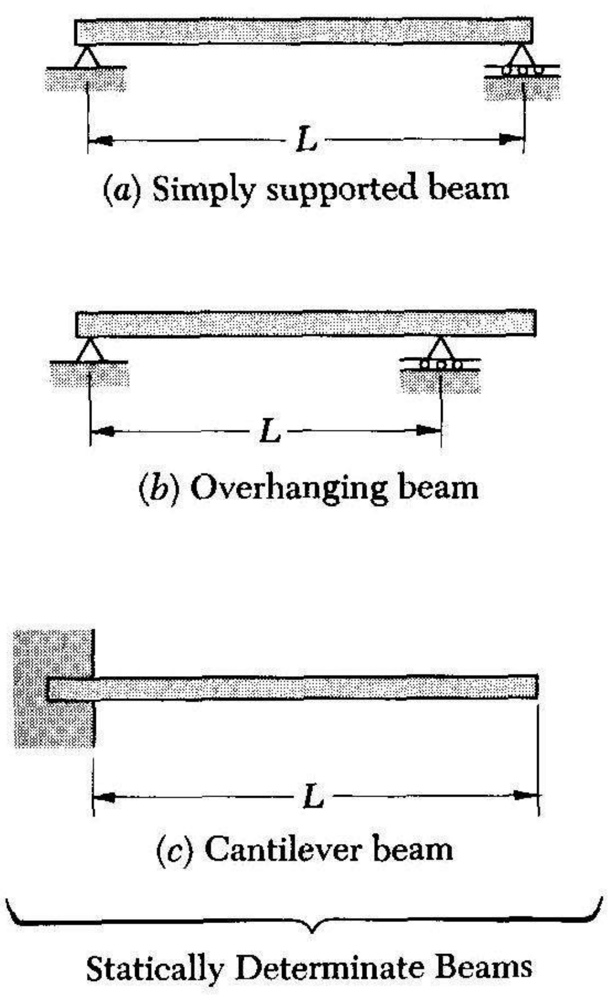
(b)

A beam have a variety of loads.

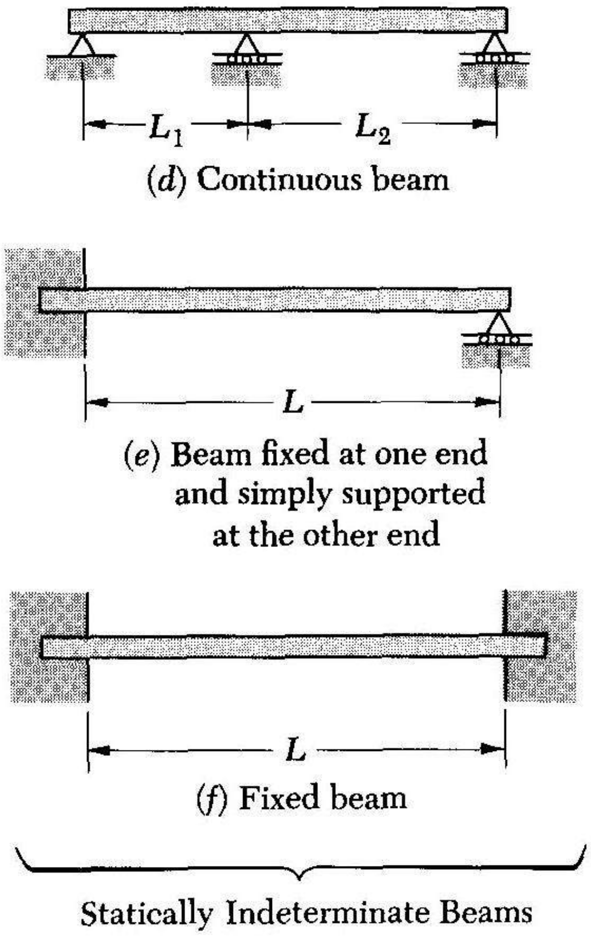
- point loads
- distributed loads
- applied moments



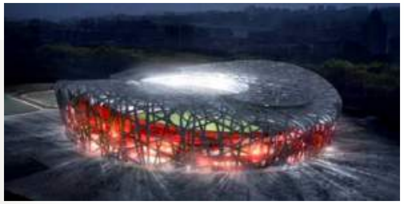
A beam can be classified as statically determinate beam, which means that it can be solved using equilibrium equations, or it is ...



A beam can be classified as statically indeterminate beam, which can not be solved with equilibrium equations. It requires a compatibility condition.



Structural Analysis



CASE STUDY

CASE STUDY:
Washington Monument

Analysis Process

Steps for structural analysis:

1) Structural Idealization
2) Applying Loads
3) Calculating Reactions
4) Calculating Internal Forces
5) Calculating Internal Stresses
6) Evaluating Safety and Efficiency

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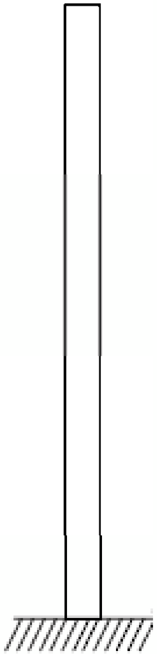
1. Structural Idealization = Structural Modeling

Analysis Process

≈



- How can I simplify geometry?

Assume an average cross-section

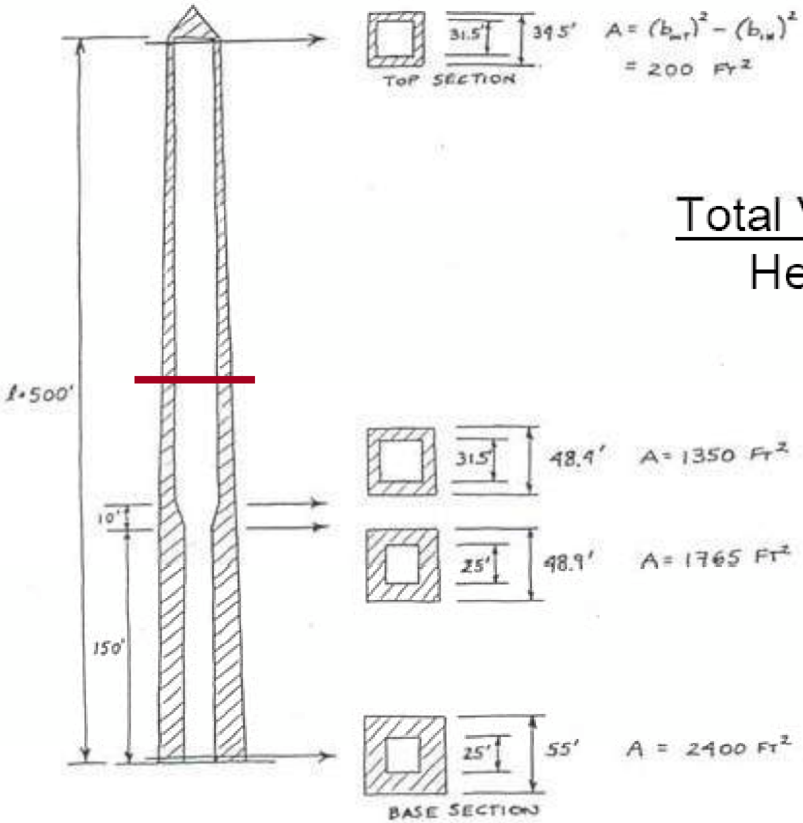
- How is it supported?

“Fixed” base

1. Structural Idealization = Structural Modeling

Analysis Process

Determing an average cross section:



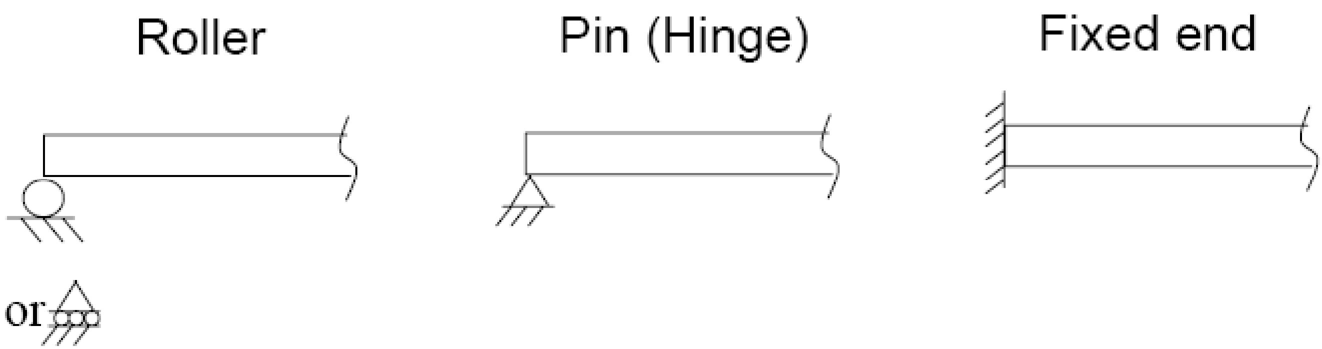
$$\frac{\text{Total Volume}}{\text{Height}} = \frac{580{,}000 \text{ ft}^3}{500 \text{ ft}}$$

$$\approx 1160 \text{ ft}^2$$

Q: What is the cross-section at mid-height? (Try calculating it. It should be about 1033 ft^2)

1. Structural Idealization = Structural Modeling

Structural supports *(and their idealizations):*

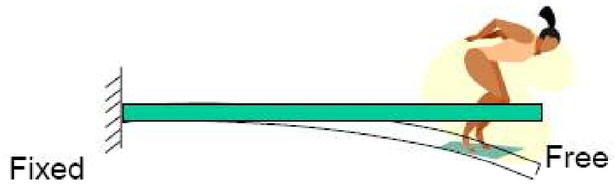


1. Structural Idealization = Structural Modeling

Four different types of end conditions:

Cannot translate vertically, can translate horizontally and can rotate

Cannot translate, but can rotate



Cannot translate or rotate

Free to translate and rotate

What do these supports do?

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2. Applying Loads

What loads act on this structure?

Dead Load: *self-weight*

Live Load: *people, wind*

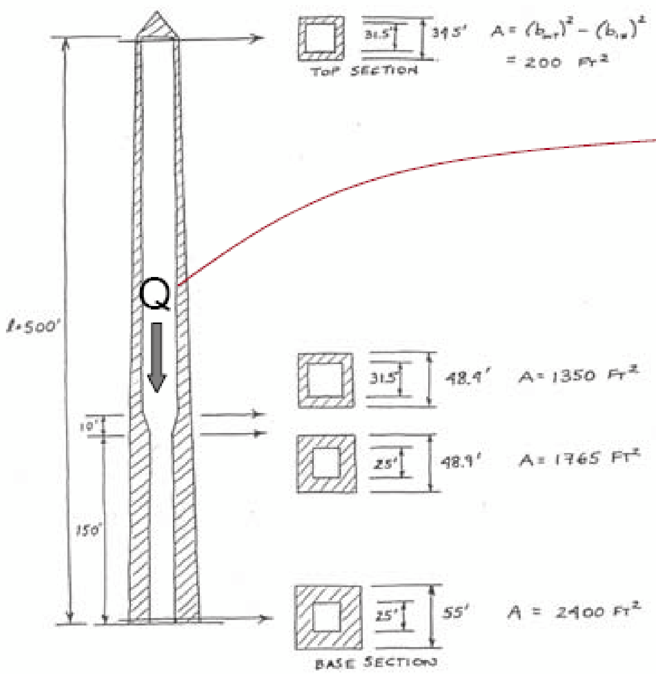
Negligible for this structure

Other live loads include snow, traffic, earthquakes

2. Applying Loads

Analysis Process

DEAD LOADS:



Given:

Total Volume = 580,000 ft^3

Masonry weighs 0.15 k/ft^3

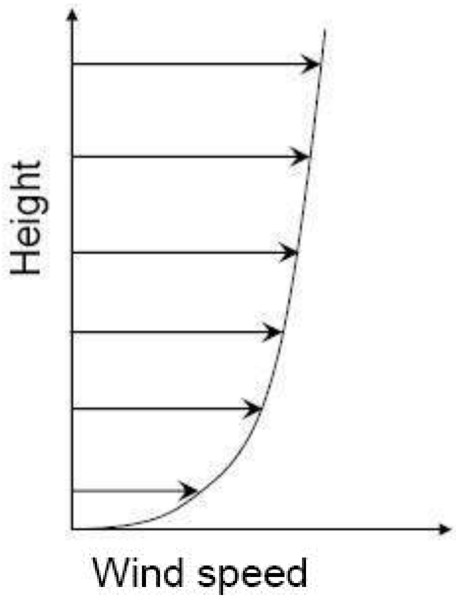
Elevators & stairs weigh 2000k

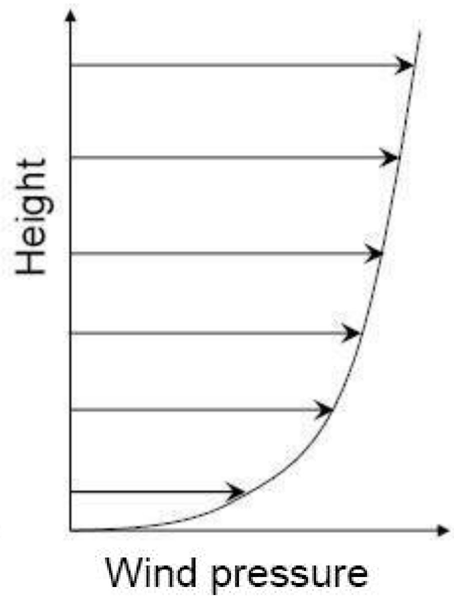
Dead Load = Q = 580,000 x 0.15 + 2000 = 89,000 kips

(can also describe Dead Load as a line load: q = 89,000k/500ft = 178k/ft)

2. Applying Loads

WIND LOAD:





NOTE: Wind pressure increases with height, but exposed surface area of monument decreases with height (tapers)

Pressure x area = Force

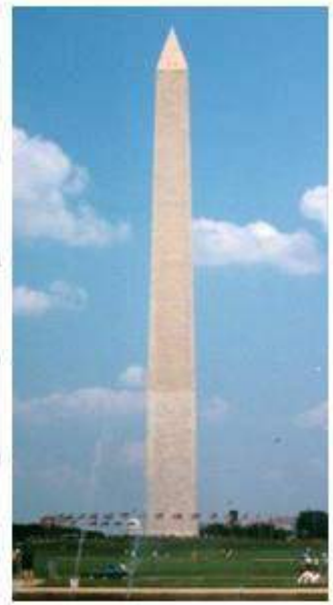
2. Applying Loads

WIND LOAD:

Divide tower into sections and see what the forces (pressure x area) are:

Forces:

209 k →
218 k →
218 k →
206 k →
163 k →



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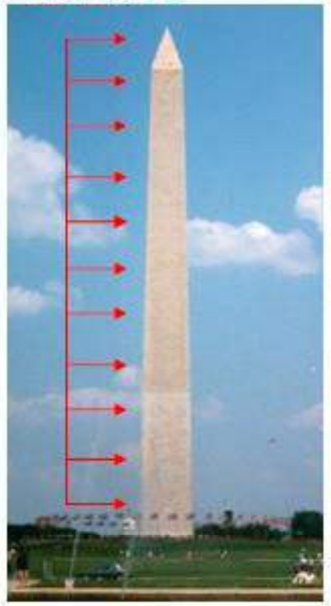
2.0 k/ft



Wind force = Σ Forces = 1014 k

Wind load along the height = (1014 k)/(500 ft) = 2.0 k/ft

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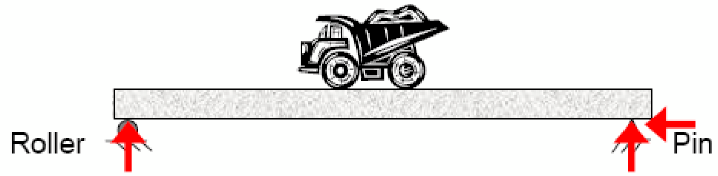
3. Calculating Reactions

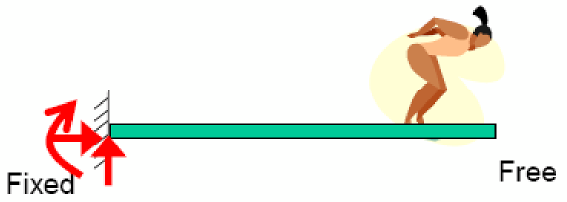
Analysis Process

Reactions resist applied loads

Reactions keep a structure in **equilibrium**

- Newton's 2nd Law: **F** = m **a**
- Want static equilibrium: **a** = 0
- Therefore the sum of all forces (F) must = 0





3. Calculating Reactions

Analysis Process

Sum of all forces must = 0

Consider structures in 2 dimensions

$\sum F_x = 0$

$\sum F_y = 0$ — No translation

$\sum M_p = 0$ — No rotation

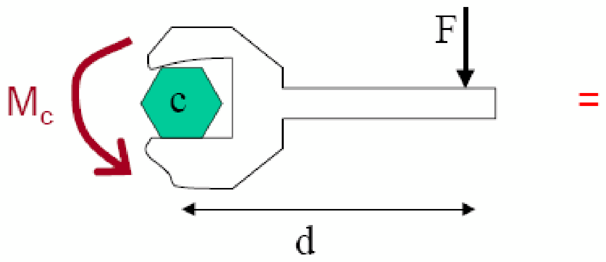
Calculate Reactions using these 3 equations

Forces

Forces are vectors

Bending Moments

A bending moment is a tendency to rotate

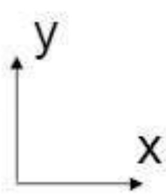


M = F x d about point c

M_c= Reaction

3. Calculating Reactions

Reactions in the Washington Monument (Dead)



$$\Sigma F_y = 0; \quad -Q + V = 0$$

$$\therefore \; V = Q$$

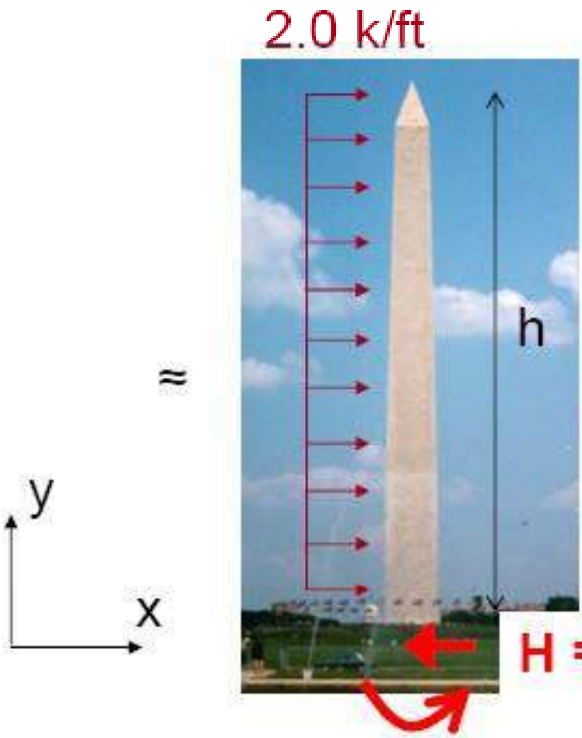
Q = Dead load (known) acting at the center of gravity, or centroid or the monument

V = vertical reaction (unknown)

3. Calculating Reactions

Analysis Process

Reactions in the Washington Monument (Wind)



$$\Sigma F_x = 0;\ \ 2.0(h) - H = 0$$

$$\therefore\ H = 2.0h$$

$$\Sigma M_{base} = 0;\ \ M - 2.0(h)(h/2) = 0$$

$$\therefore M = 2.0(h^2/2)$$

H = horizontal reaction (unknown)

M = bending moment (unknown)

CASE STUDY:
Washington Monument

Analysis Process

Steps for structural analysis:

1) Structural Idealization
2) Applying Loads
3) Calculating Reactions
4) Calculating Internal Forces
5) Calculating Internal Stresses
6) Evaluating Safety and Efficiency

4. Calculating Internal Forces

Analysis Process

Loads and reactions balance each other through the material in the structure. As a result, internal forces are created in the structure.

4. Calculating Internal Forces

There are three types of internal force we will discuss:

	<u>***Technical name***</u>
Axial Force:	**Compression & Tension**
Horizontal Force:	**Shear**
Bending Moment:	**Internal Bending Moment**

Note: "Horizontal" refers to the direction of the force in the Monument example. In a beam, a vertical force causes internal shear forces. For our studies, <u>shear forces are caused by the force that is perpendicular to the axis of the structure</u>.

4. Calculating Internal Forces

Internal forces are determined by using Free Body Diagrams (FBDs)

Free Body = Portion of structure (or a "cut")

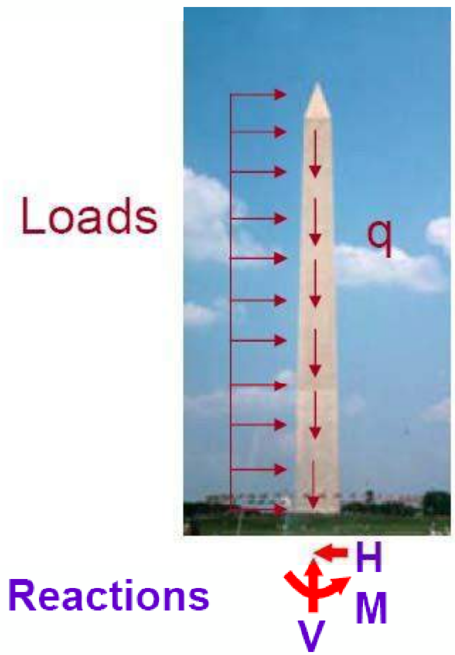
FBDs must be in equilibrium the same way a structural idealization must be in equilibrium.

What do you have to do to figure out the internal forces?

4. Calculating Internal Forces

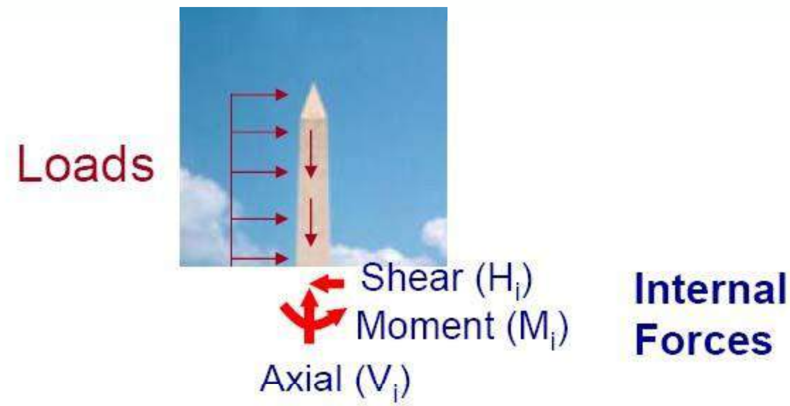
Analysis Process

Structure:



Loads and reactions must be in equilibrium

Free Body Diagram:



Loads and internal forces must be in equilibrium

➡ *Use equilibrium equations to determine unknown internal forces*

4. Calculating Internal Forces

Analysis Process

Internal forces are determined by using
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CASE STUDY:
Washington Monument

Analysis Process

Steps for structural analysis:

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5. Calculating Internal Stresses

A structure can carry applied forces if its material is strong enough and if there is enough of it.

Material Strength

Structure Geometry (cross-section)

Engineers use the term "stress" to relate the force in a structure and its size.

(Stress is similar in concept to "pressure")

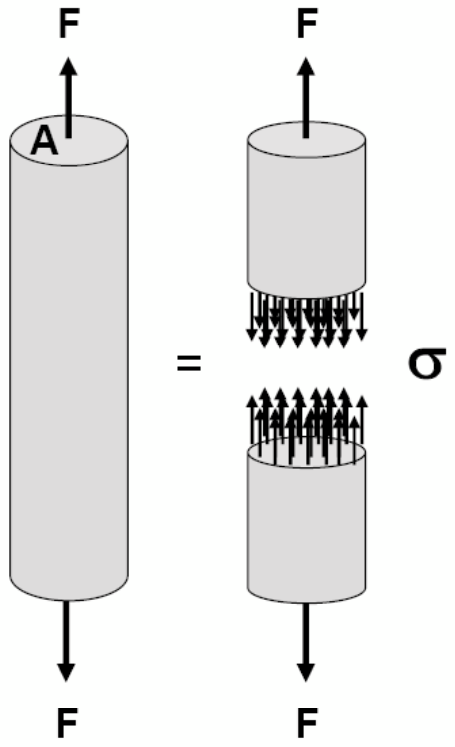
5. Calculating Internal Stresses

Analysis Process

Each internal force leads to a certain internal stress:

Axial:	**Tensile/Compressive stress**
Shear:	**Shear stress**
Bending Moment:	**Flexural stress**

Tensile and Compressive Stress (σ):



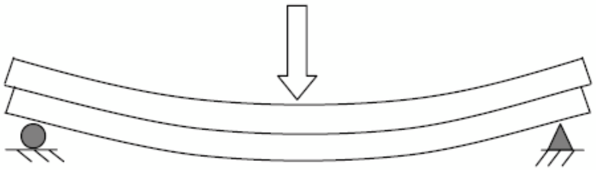
$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Note "F" here is the same as "V" in the Washington Monument example

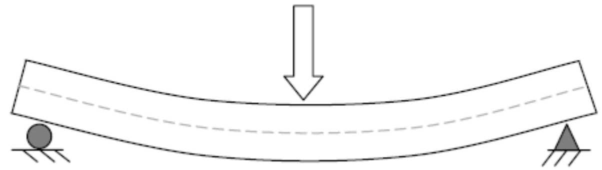
5. Calculating Internal Stresses

Analysis Process

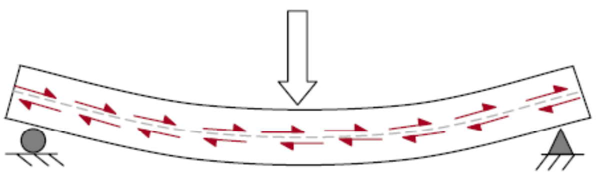
Shear Stress:



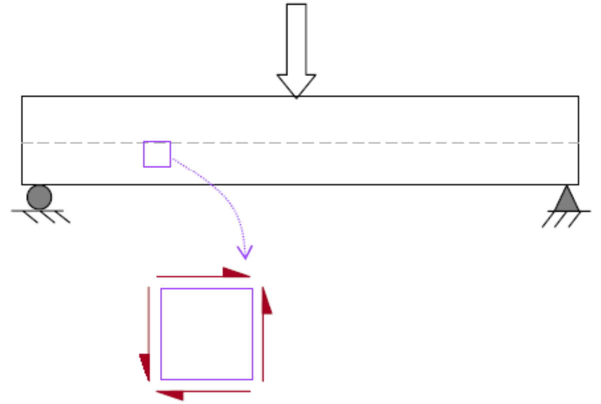
Two beams on top of each other, but not connected will slide past each other as they bend



In a homogeneous beam the same tendency for sliding exists



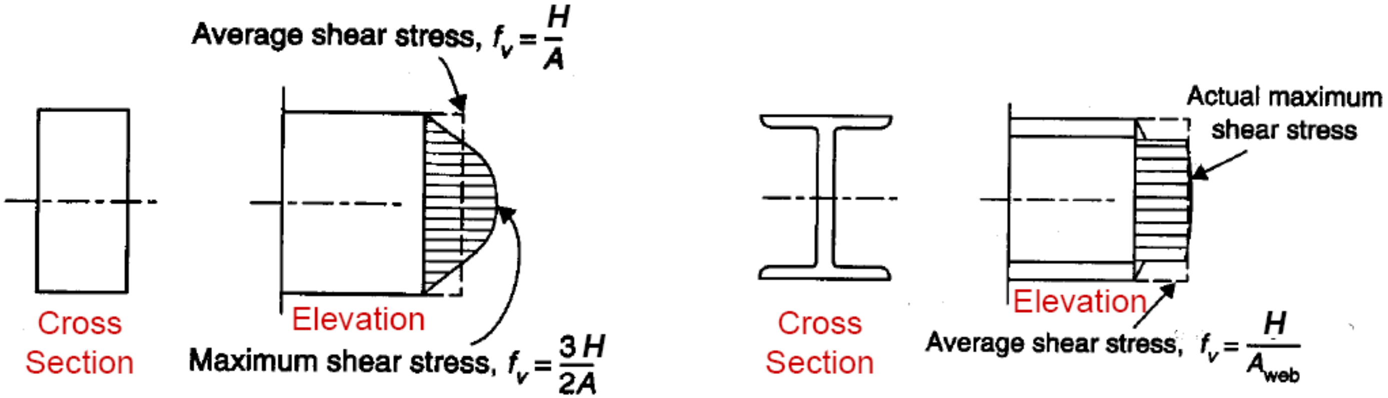
Shearing stresses develop as a result of the sliding tendency *(the same thing happens in the vertical direction too)*



5. Calculating Internal Stresses

Analysis Process

Shear Stress:

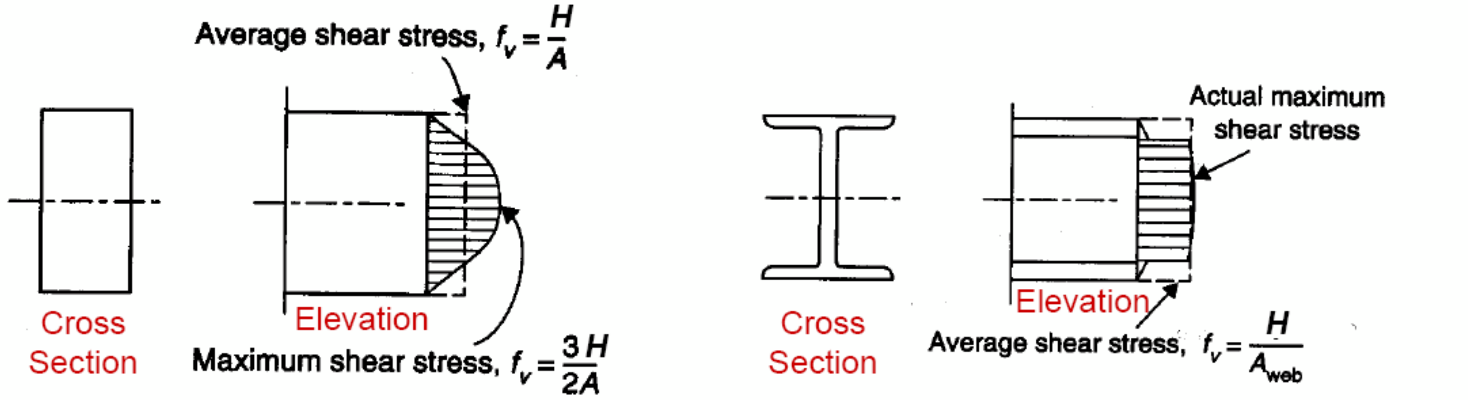


f_v calculated using:

$$\text{Shear stress} = \tau = \frac{H\,Q}{I\,b}$$

Q, I and b are properties based on the dimensions of the beam cross-section (for example, b is the width of the beam)

Shear Stress:



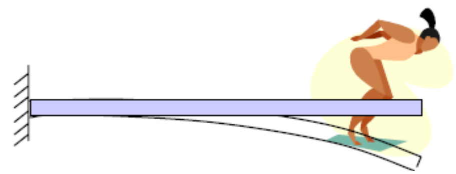
f_v *calculated using:* Shear stress = $\tau = \frac{H\,Q}{I\,b}$

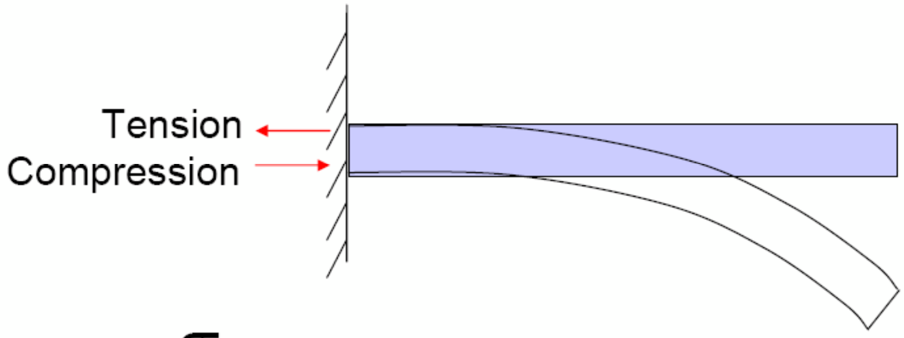
Q, I and b are properties based on the dimensions of the beam cross-section (for example, b is the width of the beam)

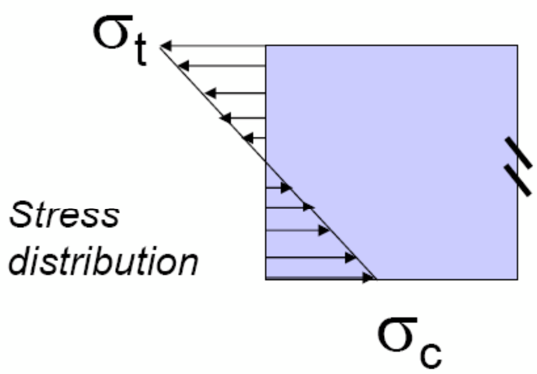
5. Calculating Internal Stresses

Analysis Process

Flexural Stress:







$$\sigma_{t,\,c} = \frac{M}{S} = \frac{\text{Moment}}{\text{“Section Modulus”}}$$

The section modulus of a beam is a function of the geometry of the beam cross section. It's units are (length)^3 such as in.^{3}

CASE STUDY:
Washington Monument

Analysis Process

Steps for structural analysis:

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6) Evaluating Safety and Efficiency

Safety is measured by safety factors

$$S.F. = \frac{\text{Load (or stress) that would cause failure}}{\text{Actual load (or stress)}}$$

Safety factors should be > 1

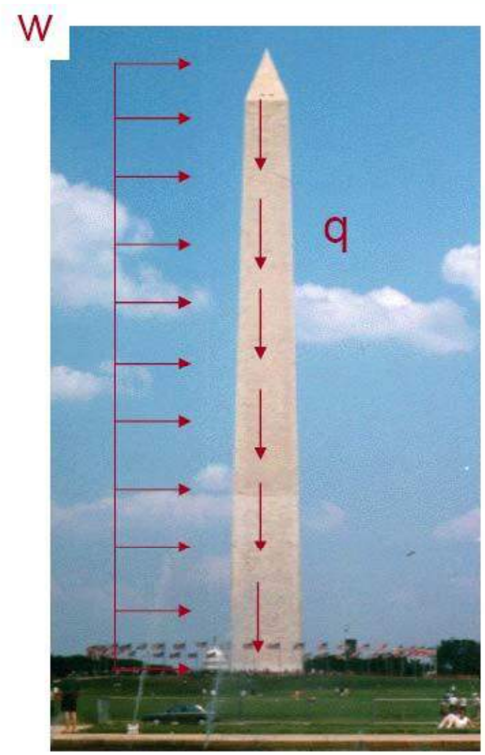
There can be many types of failure

For the Washington Monument, we can look at the safety against several failure modes:

6. Evaluating Safety and Efficiency

Analysis Process

Safety against **masonry crushing**:



Where will crushing occur under dead and live loads? *(where is the highest compressive stress?)*

You are given that the crushing strength (stress) of masonry is 3000 psi

Maximum compressive stress due to actual loads =

$$\frac{V}{A} + \frac{M}{S} = \underline{334 \text{ psi}}$$

Note: $S = 26{,}500 \text{ ft}^3$

Factor of Safety against crushing =

S.F. = 3000/334 = **9**