Kinematics, Dynamics, and the Structure of Physical Theory[†]

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ABSTRACT

Every physical theory has (at least) two different forms of mathematical equations to represent its target systems: the dynamical (equations of motion) and the kinematical (kinematical constraints). Kinematical constraints are differentiated from equations of motion by the fact that their particular form is fixed once and for all, irrespective of the interactions the system enters into. By contrast, the particular form of a system's equations of motion depends essentially on the particular interaction the system enters into. All contemporary accounts of the structure and semantics of physical theory treat dynamics, *i.e.*, the equations of motion, as the most important feature of a theory for the purposes of its philosophical analysis. I argue to the contrary that it is the kinematical constraints that determine the structure and empirical content of a physical theory in the most important ways: they function as necessary preconditions for the appropriate application of the theory; they differentiate types of physical systems; they are necessary for the equations of motion to be well posed or even just cogent; and they guide the experimentalist in the design of tools for measurement and observation. It is thus satisfaction of the kinematical constraints that renders meaning to those terms representing a system's physical quantities in the first place, even before one can ask whether or not the system satisfies the theory's equations of motion.

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1 Introduction

Every physical theory has (at least) two different forms of mathematical equations to represent its target systems: the dynamical (equations of motion) and the kinematical (kinematical constraints). Since at least the seminal work of Suppes (1960, 1962), contemporary investigation and analysis of the structure and semantics of physical theories has focused on the character and role of a theory's equations of motion. In particular, the family of solutions to the equations of motion, and the models those solutions allow one to construct, have taken pride of place in determining the structure and semantics of a theory. This is true whether one hews to the semantic view of theories (Suppe 1974; Fraassen 1980) or the Best-Systems picture (Cohen and Callender 2009) or a semantics based on possible worlds (Lewis 1970), or one is a neo-Carnapian (Demopoulos 2013), or a structuralist (da Costa and French 2005), or a neo-Kantian (Friedman 2001), or an inferentialist Suárez (2004), or any other of the contemporary popular accounts of scientific theory. Only a fool or a philosopher would deny that the dynamics of a theory plays a central role of fundamental importance in a proper accounting of its structure and semantics. I believe, however, that focus on the dynamics to the exclusion of other fundamental structures theories possess can give at best only a partial picture of a theory, and in many if not most cases a distorted, misleading and even wildly inaccurate one.

I argue that it is exactly satisfaction of the kinematical constraints—fixed, unchanging relations of constraint among the possible values of a system's physical quantities—that ground the idea of the individual state of a system as represented by a given theory. If the individual quantities a theory attributes to a system do not stand in the minimal relations to each other required by the theory, then the idea of a state as representing that kind of system disintegrates, and without the idea of an individual state of a system, one can do nothing in the theory to try to represent the system. *A fortiori*, if the kinematical constraints are not satisfied, one has no grounds for believing that the system at hand is one of the type the theory treats. It is thus those constraints that differentiate types of physical systems, and not their dynamics. Kinematical constraints, therefore, also function as necessary preconditions for the appropriate application of the theory in the first place, before one can even ask whether a given system the theory purportedly treats satisfies its equations of motion. Indeed, they are necessary for the equations of motion to be well posed or even just cogent.

Finally, they, and not the equations of motion, guide the experimentalist in the design of tools for measurement and observation. It is thus satisfaction of the kinematical constraints that renders meaning to those terms representing a system's physical quantities in the first place, even before one can ask whether or not the system satisfies the theory's equations of motion.

2 Kinematics and Dynamics

It is often useful when contemplating a physical theory to distinguish its kinematical from its dynamical components. I begin with a general account of this.

The difference between the kinematic and the dynamic manifests itself first in the family of quantities a theory ascribes to a type of system. On the one hand, there are the quantities that can vary with time and place while the system remains otherwise individually the same; these are the dynamic quantities (position, velocity, angular momentum, shear-stress, electric current, ...). On the other, there are the quantities that one assumes, for the sake of argument and investigation, remain constant as the system dynamically evolves, on pain of the system's alteration *in specie*; these are the kinematic quantities (Hooke's constant, electrical resistance, shear viscosity, thermoconductivity, index of refraction, ...). This classification belongs to kinematics.

A state of a system is the aggregation of the values of its physically significant properties at an instant; it is represented by a proposition encapsulating all that can be known of the system physically, at least so far as the theoretical and experimental resources one relies on are concerned. If one can distinguish the values of the properties of the system at one time from those at another time by the available resources, then the system is in a state at the first time different from that at the second. A state, therefore, can be thought of as a set of the values of quantities that jointly suffice for the identification of the species of the system and for its individuation at a moment. As such, the state is the most fundamental unit of theoretical representation of a system *as* a unified system, rather than just as (say) a bunch of random, unrelated properties associated with a spatiotemporal region. The characterization of a system's state belongs to kinematics. Every known physical system has the property that at least some of its quantities almost always change in value as time passes, which is to say, the system in general occupies different states at different moments of time. The collection of states it serially occupies during an interval of time forms a *dynamical evolution* (or just 'possible evolution'). The characterization of possible evolutions belongs to dynamics.

Roughly speaking, then, kinematics comprises what one needs to know in order to fix the type of system at issue (is it a viscous fluid? an electromagnetic field?), and to give a complete description of its state at a single moment—complete, that is, with respect to the theory at issue, *i.e.*, a consistent ascription of values to all the quantities it bears that are treated by a model of it in the theory. Dynamics comprises what one needs to know in order to individuate a system and to describe its behavior over time, in order to conclude, for example, that one's model represents this system right here by the determination of the values that a particular set of its quantities respectively takes over the next 5 minutes, given both its state at the initial moment and the state of its environment (the forces, if any, it is subject to, or the interactions it enter into) at that moment and over the course of those 5 minutes.

Kinematics does more than classify the quantities of a type of physical system into the kinematic and the dynamic. It also imposes fixed, unchanging relations of constraint among their possible values, both constraints that must hold at a single instant and those that must hold over the course of any of the system's possible evolutions. More precisely, there are two kinds of kinematical constraints a theory may comprise, the *local* and the *global*. A local constraint involves only quantities that can be attributed to a single state of the system, such as position; a global one involves a quantity that cannot be attributed to any single state of the system, such as the period of an orbiting body.[1] Examples of kinematical constraints:

- Hooke's constant k has physical dimension $\frac{m}{t^2}$ (local)
- the shear-stress tensor is symmetric in Navier-Stokes theory, $\sigma_{ab} = \sigma_{(ab)}$ (local)
- Kepler's Harmonic Law, $\frac{a^3}{T^2} = M$ (global)
- stress-energy tensor is covariantly divergence-free in general relativity, $\nabla^n T_{na} = 0$ (local)
- the Heisenberg uncertainty principle, $\Delta x \, \Delta p \geq \frac{1}{2}\hbar$ (local)

I shall spend most of the rest of the paper discussing kinematics (and dynamics mostly by way of contrast). Although there is much more to say about the dynamical structure of a physical theory, for the purposes of this paper I must rest content with remarking that it includes in general a rich and deep lode of topological, geometrical, analytical and algebraic structures on the space of states that in particular encode relations among entire classes of dynamic evolutions; those relations often take in part the form of a set of partial-differential equations expressed in terms of the kinematic and dynamic quantities, the equations of motion, the solutions to which represent the totality of the system's dynamical evolutions starting from all kinematically possible initial states. The canonical example is Newton's Second Law: a Newtonian body accelerates in direct, fixed proportion to the net total force applied to it, the ratio of the acceleration to the total force being the kinematic quantity known as the body's inertial mass.

[1] There is a subtlety here. Any kinematical constraints that involve derivatives depend, strictly speaking, on values of quantities at more than one state, even for local constraints; some global constraints, moreover, can be formulated by laying down conditions that must hold at individual states (*e.g.*, that a Newtonian orbit be an ellipse can be formulated as a constraint on the value of the spatial derivative at every point of the orbit, or on the sum of the distances from the foci at each point); this seems superficially similar to some local ones, *e.g.*, conservation of angular momentum, which can also be formulated as a relation among derivatives at a point. Whether a constraint, then, is global or local, depends in part on whether one can formulate the condition over arbitrarily short periods of a possible evolution, which one can for conservation of angular momentum (the system satisfies angular momentum, say, during one part of an evolution but not another), but not for whether a planetary orbit is an ellipse (where, by definition, one must wait an entire orbital period before one can say the condition is satisfied or not).

3 Kinematical Constraints

Kinematical constraints are differentiated from equations of motion by the fact that the particular, concrete form of a kinematical constraint is fixed once and for all, irrespective of the interactions the system may enter into with other systems (such as a measuring apparatus in the laboratory). By contrast, the particular, concrete form of a system's equations of motion depends essentially on the particular interaction (if any) the system enters into with another system in its environment—*e.g.*, what external forces, if any, act on the system.

The difference between a kinematical constraint and an equation of motion comes out clearly in Newton's Second Law, written out explicitly as two coupled first-order differential equations.

$$\dot{\mathbf{x}} = \mathbf{v}$$

(always the same: kinematical constraint)

versus

$$\dot{\mathbf{v}} = \mathbf{F}/m$$

(the concrete form of **F** depends on environment, forces: equation of motion)

For a more interesting example, consider the Maxwell equations. According to this characterization, the first two,

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{B} &= 0 \\ \dot{\mathbf{B}} &= -\boldsymbol{\nabla} \times \mathbf{E} \end{aligned} \tag{3.1}$$

those governing the magnetic components **B** of the electromagnetic field, are both local kinematical constraints. They are kinematical constraints and not equations of motion because neither changes form no matter the environment the electromagnetic field evolves in (ignoring the possibility of magnetic monopoles). Indeed, even though one of the equations includes the time-derivative of another quantity, making it look like an equation of motion, I claim that from a physical point of view one must think of them both as kinematical constraints. The crux of the matter is that the electromagnetic field couples with other systems only by way of their manifestation of electric charge ρ or current $\mathbf{j}$, but those quantities when present change the form only of the other two Maxwell equations,

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{E} &= \rho \\ \dot{\mathbf{E}} &= \mathbf{j} - \boldsymbol{\nabla} \times \mathbf{B} \end{aligned} \tag{3.2}$$

those governing the electric components **E** of the electromagnetic field. In effect, the difference between the two pairs of relations shows that, in a precise sense, the magnetic field couples directly with no physical quantity of any other system in that the presence of electric charges and currents does not alter the form of its two defining equations. (The magnetic field does couple to electric current "to second order" by way of the second of equations (3.2), whence Ampère's Law.) Thus the form of equations (3.1) does not depend on the particular dynamical evolution the system manifests at any given time. Nonetheless, not just any old thing counts as a magnetic field no matter how it evolves and no matter what relations hold among its quantities at different points; only those things that behave like magnetic fields can be magnetic fields, which in this case means the identical satisfaction of the first two Maxwell equations.

4 Roles in Theory

Theories do not predict kinematical constraints; they demand them. I take a prediction to be something that a theory, while appropriately modeling a system, can still get wrong. Newtonian mechanics, then, does not predict that the kinematical velocity of a Newtonian body equal the temporal rate of change of its position; rather it requires it as a precondition for its own applicability. It can't "get it wrong". If the kinematical constraints demanded by a theory do not hold for a family of phenomena, that theory cannot treat it, for the system is of a type beyond the theory's scope. By contrast, if the equations of motion are not satisfied, that may tell one only that one has not taken all ambient forces on the system (couplings with its environment) into account; it need not imply that one is dealing with an entirely different form of system. Even in principle, one can never entirely rule out the mere possibility that the equations of motion are inaccurate only because there is a force one does not know how to account for, not because the system is not accurately treated by those equations of motion. This can never happen with a kinematical constraint. It is either satisfied, to the appropriate and required level of accuracy given the measuring techniques available and the state of the system and its environment, or it is not. This is a serious difference in physical significance among the types of proposition a theory contains, which, among other things, should be reflected in the way an account of semantics assigns significance to the theory's structural elements.

Indeed, satisfaction of kinematical constraints is required for the equations of motion of a theory to be well posed or even just cogent. The initial-value formulation of the Navier-Stokes equations, for example, is well set (in the sense of Hadamard) only if the shear-stress tensor is symmetric and the heat flux is orthogonal to fluid flow, both kinematical constraints (Lamb 1932; Landau and Lifschitz 1975). One cannot even formulate Newton's Second Law if velocity is not the first temporal derivative of position. More generally, in a sense one can make precise (Curiel 2014), if the kinematical constraints of Lagrangian mechanics are not satisfied ($\mathbf{v} = \dot{\mathbf{q}}$), then one cannot formulate the Euler-Lagrange equation; and similarly, if the kinematical constraints of Hamiltonian mechanics are not satisfied (the **p**s and **q**s do not satisfy the canonical Poisson-bracket relations[2]), then one cannot formulate Hamilton's equation. Thus satisfaction of the kinematical constraints is

[2] (q_i, p_j) satisfy the canonical Poisson-backet relations if

$$\begin{aligned} \{q_i, q_j\} &= 0 \\ \{q_i, p_j\} &= \delta_{ij} \\ \{p_i, p_j\} &= 0 \end{aligned} \tag{4.3}$$

where δ_{ij} is the Kronecker delta symbol, which equals 1 for $i = j$ and 0 otherwise.

required as a precondition for the appropriate application of a theory in modeling a kind of system, and so the kinematical constraints in fact function in that precise sense as *a priori* constitutive components of a physical theory.

This is not true of the dynamical relations the theory posits. A theory may appropriately treat a family of phenomena even when it does not model the dynamical behavior of all members of the family to any prescribed degree of accuracy, *i.e.*, even when the equations of motion are not satisfied in any reasonable sense (and thus when, according to the standard conception of semantics, the schematic representations of those phenomena cannot contribute to the semantic content of the terms occurring in those representations). A theory, however, can and does tell us much about the character and nature of physical systems for which it does not give accurate representations, systems, in other words, it cannot soundly represent in totality, cannot be true of, and so systems that, according to all the standard contemporary accounts of theory structure and semantics, the theory should have nothing to say about at all. If a system's behavior is not accurately captured by a theory's equations of motion, then that system cannot, *e.g.*, be represented by a Tarskian model constructed from a solution to the equations of motion; it is thus, according to the semantic view of theories, for instance, not even a candidate for contributing to the semantic content of the theory's theoretical terms, *inter alia*. In fact, though, such systems can still be appropriately represented by that theory in a precise and important sense, even though the equations of motion are not satisfied, so long as the kinematical constraints are.

Consider the example of a representation of a body of liquid as provided by the classical theory of fluid mechanics, Navier-Stokes theory. When the liquid is not too viscous, is in a state near hydrodynamical and thermodynamical equilibrium, and the level of precision and accuracy one demands of the representation is not at too fine a spatiotemporal scale, then the classical theory yields excellent models of the liquid's behavior over a wide range of states and environments. When the state of the liquid, say, begins to approach turbulence, the representation the theory provides begins to break down. It does so, however, in a subtle way, one that cannot be wholly accounted for by adverting merely to the fact that the theory becomes predictively inaccurate. In particular, there is a regime in which the theory's dynamical equations of motion no longer provide accurate predictions by any reasonable measure, and yet all the quantities the theory attributes to the liquid (*e.g.*, shear viscosity, mass density, hydrostatic pressure, shear-stress, *et al.*) will still be well defined, and all the kinematical constraints the theory jointly imposes on those quantities (*e.g.*, the constancy of shear viscosity, the continuity of mass-density, the conservation of energy, the symmetry of the shear tensor, *etc.*), will still be satisfied (Monin and Yaglom 1971). Call it *the regime of kinematical propriety*. In a strong sense, then, the theory can still provide a meaningful—and appropriate—model of the liquid even though that model is not adequately accurate in all its predictions. This sort of situation, where the theory's dynamics are no longer adequate but its kinematics are still appropriate, shapes and provides at least part of the physical meaning of terms like 'mass density' and 'shear'—physical meaning that *ipso facto* cannot be captured by a semantics that grounds meaning on the dynamics of the theory, and in particular by one that relies wholly or even in large on part on the family of solutions to the theory's equations of motion.

More precisely, then, a view about the structure and semantics of physical theory based ultimately on dynamics is inadequate for (at least) two reasons. First, it does not allow us, within the scope of the theory itself, to understand why such models are not sound even though all the quantities the theory attributes to the system are well defined and the values of those quantities jointly satisfy all kinematical constraints the theory requires. Second, we miss something fundamental about the meaning of various theoretical terms by rejecting such models out of hand merely on the grounds of their inaccuracy. It is surely part of the semantics of the term 'hydrostatic pressure', *e.g.*, that its definition as a physical quantity treated by classical fluid mechanics breaks down when the fluid approaches turbulence; because, however, the theory's equations of motion stop being accurate long before, in a precise sense, the quantity loses definition in the theory and long before the kinematical constraints of the theory stop being satisfied, any account of the structure of theories and their semantics that rejects the inaccurate models in which the term still is well defined will not be able to account for that part of the term's meaning. Thus, an adequate account of physical theory must be grounded on notions derived from relations in some sense prior to the theory's representations of the dynamical behavior of the physical systems it treats, relations that govern the propriety of the theory's representational resources for modeling the system at issue. These are the the theory's kinematical constraints.

One may think that this discussion about how, where and when theories breakdown more properly belongs to pragmatics (in the sense of semiotic theory) than to semantics. That is not so. A system of formal semantics that would ground itself in the family of possible physical systems for which it provides sound models cannot even get started until that family is demarcated. But that is exactly to require an investigation of the boundary of the theory's regime of kinematical propriety, which is thus logically and conceptually prior to any such system of semantics.

In order to be able to formulate and evaluate any kinematical constraint, of course, the quantities themselves in the terms of which the constraints are formulated must be well defined in the theory. For this to be the case, it is necessary that one be able to formulate the local kinematical constraints and verify that they hold. Without the satisfaction of the local kinematical constraints, the entire idea of the individual state of a system as represented by that theory disintegrates—individual quantities do not stand in the minimal relations to each other required by the theory—and without the idea of a state of a system, one can do nothing in the theory to try to treat the system.

More to the point, if the local kinematical constraints are not satisfied, one has no grounds for believing that the system at hand is one of the type the theory treats. Many different kinds of system, for example, have shear and stress—Navier-Stokes fluids, elastic solids, ionically charged plasmas, electromagnetic fields, *et al.* To say that a system has a quantity represented by a shear-stress tensor is not to have said very much. One must also know, among other things, whether the shear-stress tensor must be symmetric, or divergence-free, or stand in a fixed algebraic relation to another of the system's quantities such as heat flux, and so on. Each such possible condition is a kinematical constraint; and each different type of system that has a quantity appropriately represented by a shear-stress tensor will impose different constraints on that tensor. It is those constraints that differentiate types of physical systems, and not their dynamics. Think of all the kinds of systems whose dynamics obey the equation of a simple harmonic oscillator (pendulum, spring, vibrating string, electrical circuit, orbiting planet, trapped quantum particle, ...)—without question what differentiates them cannot be the form of their dynamics. It is only the form and content of the kinematical constraints one demands be obeyed by the quantities entering into the equations of motion. In this sense, then, the kinematical constraints are constitutive of the type of system the theory treats.

The same considerations show that kinematical constraints are, in a precise sense, analytic: they are made true solely by the meanings of the terms *in the context of the theory*. In that sense they are like L-sentences in a Carnapian framework (Carnap 1956). Unlike L-sentences, however, they have non-trivial semantic content, for the constraints they impose on physical system are non-trivial. Not all types of physical system will satisfy them, *viz.*, those systems not appropriately represented by the theory.

Finally, it is the kinematical constraints, not the equations of motion, that guide the experimentalist in the design of instruments for probing and measuring the quantities the theory attributes to the systems it treats. An instrument that is to measure velocity, for instance, must be sensitive to differences in spatial location at ever smaller measured temporal intervals. It does not care about how the system accelerates, *i.e.*, about its dynamics. Similarly, an instrument that would measure shear-stress of a Navier-Stokes fluid must conform to the equality of pressure and reversed sense of shear across imaginary surfaces in fluid that is represented by the symmetry of the shear-stress tensor. Again, the instrument need not care at all about the dynamics of the fluid to measure the shear-stress. In this way, they provide the foundation for the operationalization of the meaning of theoretical terms.[3]

To summarize, then, the roles that kinematical constraints play in physical theory:

1. they govern the propriety of theory in representing systems in the first place, *i.e.*, they serve as preconditions of applicability

2. they characterize the physical nature of systems the theory treats, *i.e.*, that constitutive of the kind of system the theory treats

3. they guarantee the cogency and good behavior of the dynamics, in so far as that can be guaranteed (by ensuring the well posedness of the initial-value formulation of the equations of motion, or by ensuring that the equations of motion are cogent as equations in the first place)

4. they provide guidance in the design of tools for measurement and observation, and so provide the empirical ground for the meaning of theoretical terms

The equations of motion play none of these roles.

[3] If one likes, one can take this as a way to make precise the sense in which experiments are "theory laden", and why that is irrelevant for the capacity of experiments to provide independent confirmation and refutation of theories: the equations of motion in general play no role in the design of experimental instruments, but it is, in general, only the equations of motion we test in experiments.

Before concluding the paper in the next section, it will be instructive to compare the way I have characterized kinematical constraints with the manifestly (and superficially) similar ideas in Neo-Kantian accounts of the structure and semantics of physical theory, such as those of Reichenbach (1965) and Friedman (2001). They postulate a relativized *a priori*, which also is in some sense constitutive of the kinds of systems treated by a theory, and which function in some sense as preconditions for the applicability of a theory. Kinematical constraints, on my conception, do have some similarities to that idea, but they have deep differences as well.

1. First and foremost, kinematical constraints are part of the theory itself, not supra-theoretical principles.

2. *Contra* several of the Reichenbachian examples of relativized *a priori* principles, such as that of genidentity (Padovani 2011), kinematical constraints have true physical content, not just formal character, in the sense that direct measurement can verify whether they hold or not of a given system.

3. One needs the satisfaction of kinematical constraints, *as experimentally verified*, in order to apply the theory appropriately in the most full-blooded sense, that of characterizing systems and making predictions about them.

With regard to the last point, Friedman (2001, p. 71) does say, "The role of constitutively *a priori* principles is to provide the necessary framework in which the testing of properly empirical laws is possible." Nonetheless, *a priori* principles on his conception are not amenable to direct experimental verification in the same way as kinematical constraints. Kinematical constraints, as opposed to the kind of *a priori* principles he characterizes, appear already as part of the theory itself, rigorously and precisely formulated—and so amenable to direct experimental testing—not as imprecise, loose and supra-theoretic adjuncts to the theory.

5 Bearing on Semantics

To accept a theory is, at a minimum, to accept its analytic or *a priori* propositions as true—as necessarily true in the context of the framework. To accept Newtonian mechanics is to accept that $\mathbf{v} = \dot{\mathbf{x}}$ and that $m\dot{\mathbf{v}} = \mathbf{F}$. It is not to accept that the gravitational force is $G\frac{m_1 m_2}{r^2}$, nor to accept that the net force on this body right here, right now, is 5 Newtons. One requires a semantics of frameworks that allows one to demarcate that class of propositions, the ones necessarily true in the context of the framework. One cannot know them as analytic if given the framework only as a formal structure, or if one uses a semantics such as a Tarskian one that treats all propositions as semantically on par with each other. The apriority of the propositions must come as part of the semantic interpretation of the framework itself.

One may say that a theory has *propriety of representation* for a system when the system satisfies its kinematical constraints, for their satisfaction is semantically prior to the satisfaction of the equations of motion (§4). It therefore seems promising to attempt to base a semantics for physical theory on this idea:

> We know the meaning of a theory when we know the conditions under which the kinematical constraints hold, *i.e.*, when the the theory has propriety in representation.

To know the meaning of a theory, therefore, cannot be to know the set of "possible worlds" the solutions to the theory's equations of motion represents. It is rather to know the conditions under which it is sensical to investigate the formulation of possible conditions of the theory's truth, *i.e.*, the satisfaction of its equations of motion, for this can be done only in so far as one already knows what systems the theory represents with propriety.

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