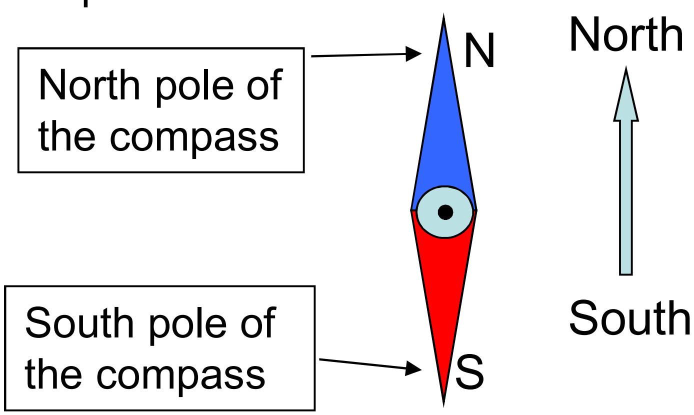
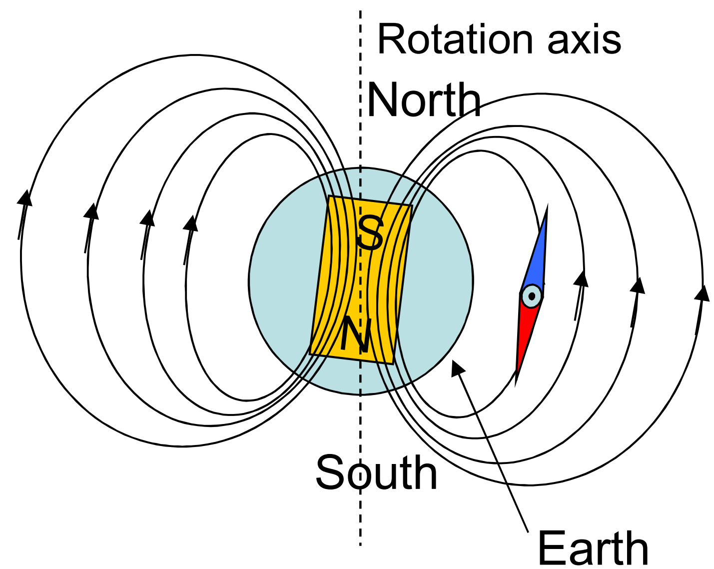
5 – Magnets and electromagnetism

Magnetism

In our modern everyday life, the phenomenon of magnetism is associated with iron that is attracted by permanent magnets that can also be made of iron compounds. Historically, magnetism was observed even before the mankind started producing metals. Naturally occuring magnetite, one of iron oxides, showed magnetic properties and it was used to build a navigational device compass. Compass is a small magnet that points to the North with this north pole and to the South with its south pole.

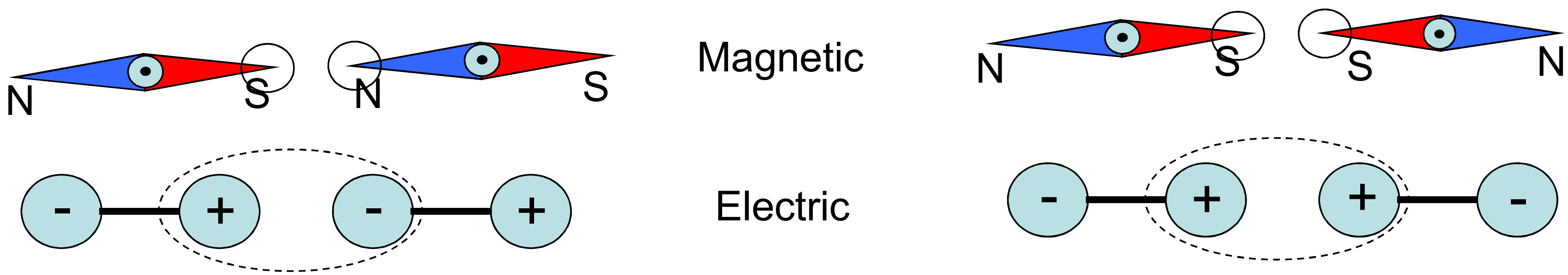




One can speak about the <u>magnetic filed lines</u> that go from the South to the North and make the compass orientate along them. The Earth itself is a magnet that creates this magnetic field acting on a compass.

Compass is an example of a magnetic dipole, that is, two magnetic poles at a distance from each other

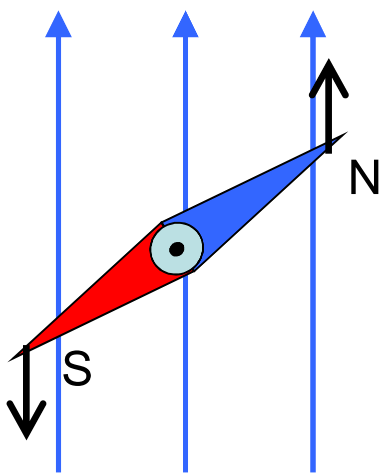
Analogy between magnetic and electric dipoles



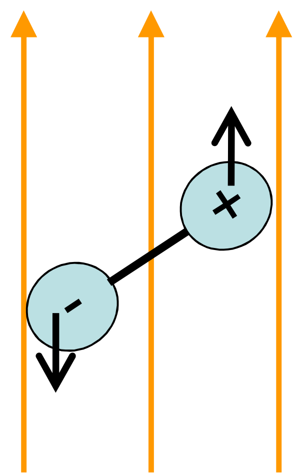
Two unlike poles attract each other

Two like poles repel each other

Torque on magnetic and electric dipoles:



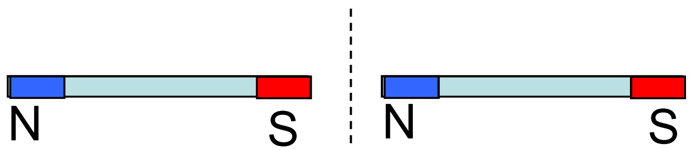
Magnetic field B



Electric field E

Magnetic poles interact similarly to electric poles (charges). Coulomb first established his Coulomb's law for magnetic poles that were located at the ends of long and thin magnetized stabs: In was easier to experiment of magnetic poles then on electric charges.

Analogy between electrostatics and magnetism is incomplete because one cannot isolate magnetic poles. Magnetic monopoles have not been found in nature. Cutting the long magnetized stab does not help since new South and North poles arise on both sides of the cut:



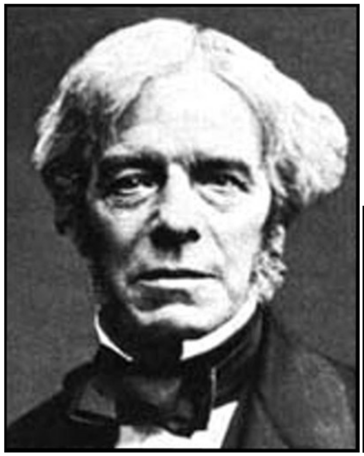
The only origin for magnetism is electric current, that is, moving electric charges.

Electromagnetism - historical

Alessandro Volta
(1745 - 1827)
Invented electric battery that gave possibility to experiment with electric currents

Hans Christian Ørsted
(Oersted)
(1777 - 1851)
Discovered magnetic field produced by electric currents

Andre Marie Ampere
(1775-1836)
Elucidated forces acting between wires carrying electric currents



Michael Faraday
(1791-1867)
Discovered electromagnetic induction

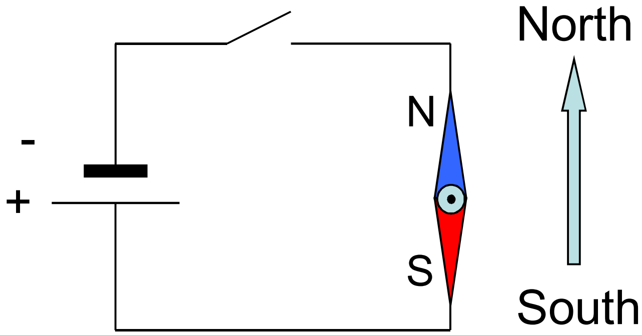
Heinrich Lenz
(1804-1865)
Studied electromagnetic induction, formulated Lenz law

Joseph Henry
(1797-1878)
Discovered self-induction

Magnetic field of electric currents

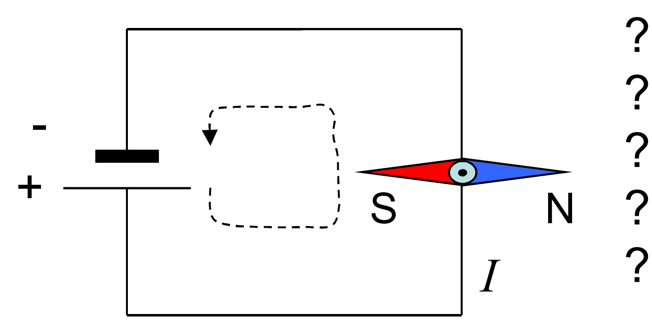
In 1800 Volta invented the electric battery that gave a possibility to create stable electric currents and experiment on them.

In 1819 Oersted discovered during his lecture that electric currents create magnetic field that can be detected by a compass.



$I = 0$

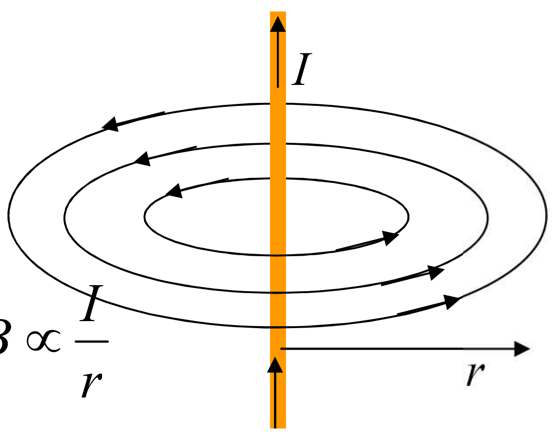
Here the compass experiences the magnetic field of the Earth only and thus it points to the North

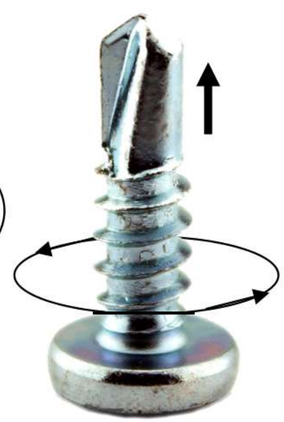


$I \neq 0$ (Compass above the wire)

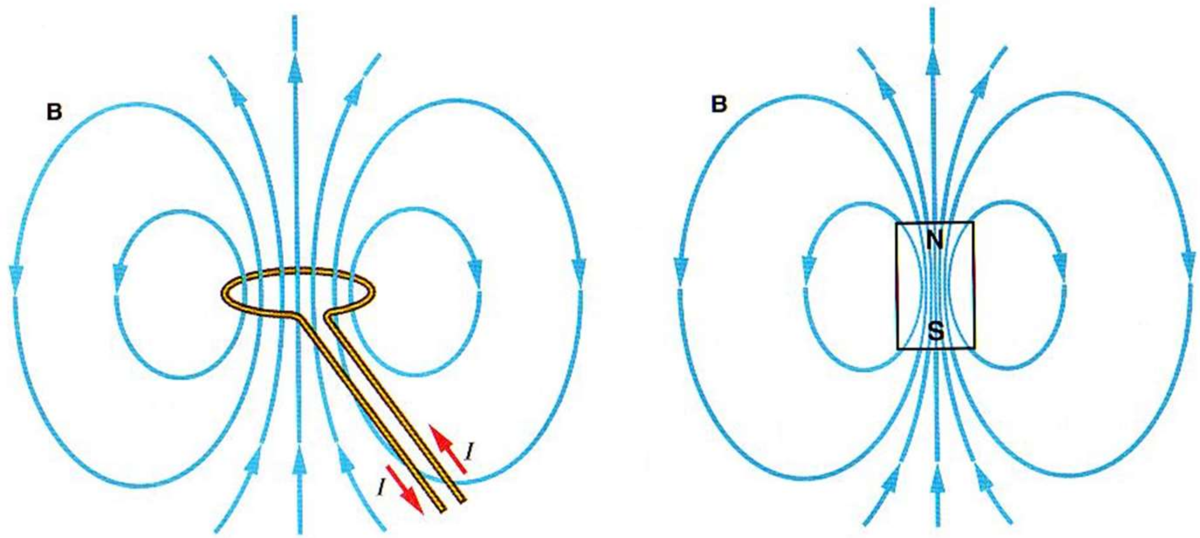
Here the current flowing through the wire creates magnetic field that is directed perpendicularly to the wire (from left to right above the wire). This magnetic field can exceed that of the Earth, and then the compass points perpendicularly to the wire.

$$B \propto \frac{I}{r}$$



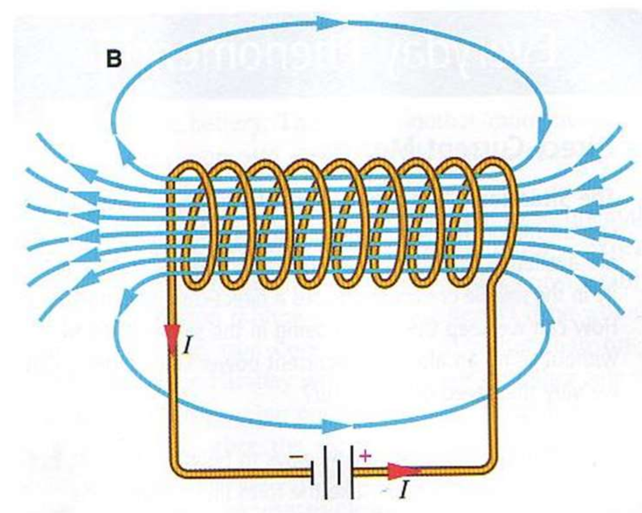


Strength of the magnetic field (magnetic induction) is proportional to the current and inversely proportional to the distance from the wire. Its direction can be memorized with the help of the screw rule.



The magnetic field produced by a current loop is identical to that produced by a short bar magnet

A current-carrying coil of wire produces a magnetic field greater than a single loop, and its strenth is proportional to the number of loops in the wire. This magnetic field is similar to one produced by a long bar magnet.

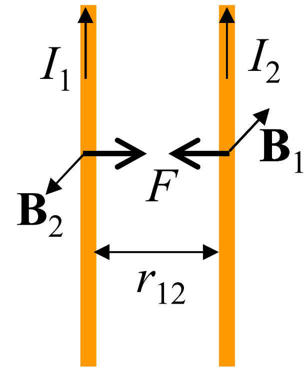


Comparison of the magnetic fields produced by permanent magnets and by currents suggests that magnetic fields from permanent magnets are produced by a special kind of current flowing on the atomic/molecular level, the so-called <u>molecular currents</u>. For a long bar magnet, molecular currents must flow on its surface around its main axis.

Forces between electric currents

Around 1820 Ampere found that two parallel wires attract if currents are flowing in the same direction and repel if currents are flowing in opposite directions. This force is given by

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r_{12}} l, \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \bullet m/A}$$



where l is the length of the wires, and r_{12} is the distance between the wires ($r \ll l$). If we define the magnetic field induction B as the proportionality coefficient between the current in a wire and the force per its unit length, that is,

$$\boxed{F = lIB}$$

(Particular formula for $\mathbf{I}$ and $\mathbf{B}$ perpendicular)

Unit of the magnetic field induction B is Tesla, 1T = 1 N/(A m)

A comparison of the two formulas above yields the expressions for the magnetic fields created by each wire:

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{r_{12}} \qquad B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{r_{12}} \qquad \Rightarrow \quad \boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

We see that magnetic field not only is produced by electric currents but also it acts on electric currents. This suggests that magnetic dipoles that create magnetic field (such as the Earth) and those that feel the magnetic field (such as compasses) are in fact current loops. These currents can be so-called “molecular currents” that flow on the atomic level (compasses).

General vector formula for the magnetic force acting on a current

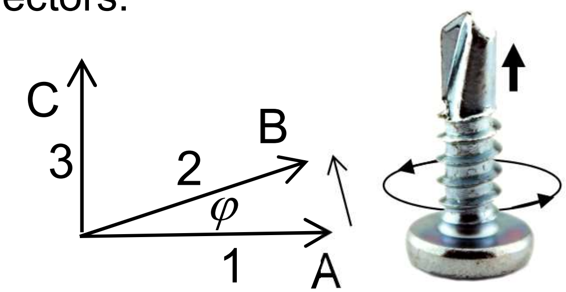
$$\mathbf{F} = l[\mathbf{I} \times \mathbf{B}]$$

Cross-product of two vectors:

$$\mathbf{A} \times \mathbf{B} = \mathbf{C}$$

1 2 3

$$AB \sin\varphi = C$$



Rotation 1→2 defines the direction of 3 via screw rule

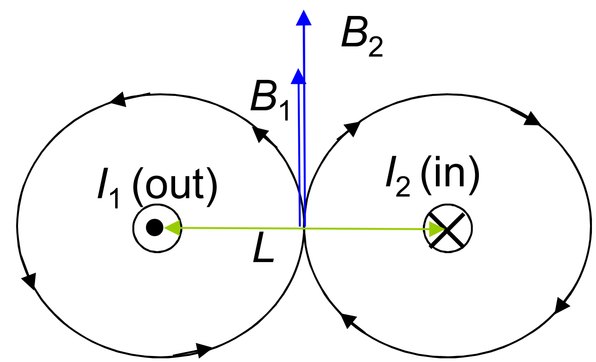
Somewhere, they are teaching the “left-hand rule” instead the screw rule. Whereas the screw rule shows the principle that is working in mathematics and physics, the left-hand rule is just mnemonics that helps memorizing the stuff without actual understanding. Using the latter is not recommended in this course.

Problem

Magnetic field between two wires

Formulation: Two parallel straight wires L = 10 cm apart carry currents I_1 = 5 A and I_2 = 7 A in opposite directions (see figure). Find the magnetic field halfway between the two wires.

Solution: According to the screw rule the magnetic fields are directed as shown in the figure and they have the same direction at the point of interest. The total field is



$$B = B_1 + B_2 = \frac{\mu_0}{2\pi}\frac{I_1}{r_1} + \frac{\mu_0}{2\pi}\frac{I_2}{r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{x} + \frac{I_2}{L-x}\right)$$

where, generalizing the problem, we have chosen the point of interest at the distance $r_1=x$ from the first wire and $r_2=L\text{-}x$ from the second wire. In our case $r_1=r_2=L/2$, thus

$$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{L/2} + \frac{I_2}{L/2}\right) = \frac{\mu_0}{2\pi}\frac{I_1+I_2}{L/2} = \frac{4\pi\times10^{-7}}{2\pi}\frac{5+7}{0.1/2} = 4.8\times10^{-7}\ \mathrm{T}$$

..Continued

The total magnetic field has a minimum at some point between the two wires that can be found by minimizing $B(x)$ with respect to x:

$$0 = \frac{dB}{dx} \sim \frac{d}{dx}\left(\frac{I_1}{x} + \frac{I_2}{L-x}\right) = -\frac{I_1}{x^2} + \frac{I_2}{(L-x)^2} = \frac{-I_1(L-x)^2 + I_2 x^2}{x^2(L-x)^2}$$

Thus one obtains a quadratic equation

$$0 = -I_1(L-x)^2 + I_2 x^2 = (I_2 - I_1)x^2 + 2I_1 Lx - I_1 L^2$$

That has a solution

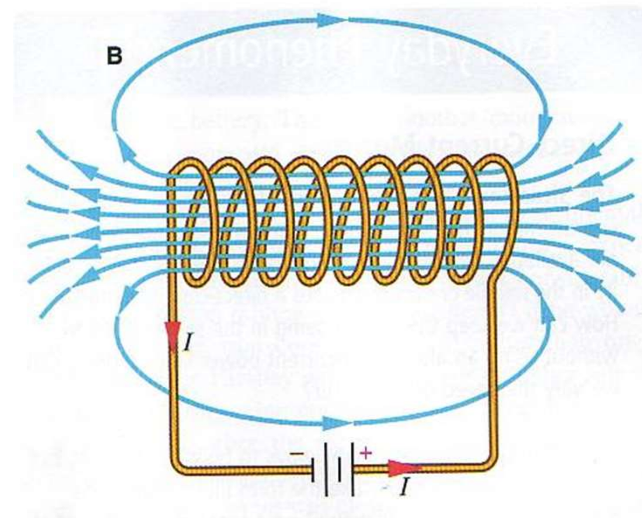
$$x = \frac{-I_1 L + \sqrt{(I_1 L)^2 + (I_2 - I_1)I_1 L^2}}{I_2 - I_1} = \frac{-I_1 L + L\sqrt{I_1 I_2}}{I_2 - I_1}$$

$$= L\frac{\sqrt{I_1}\left(\sqrt{I_2} - \sqrt{I_1}\right)}{\left(\sqrt{I_2} + \sqrt{I_1}\right)\left(\sqrt{I_2} - \sqrt{I_1}\right)} = L\frac{\sqrt{I_1}}{\sqrt{I_2} + \sqrt{I_1}}$$

This point is closer to the wire with a weaker current. For $I_1 = I_2$ this point is in the middle between the wires, of course.

Magnetic field in a solenoid

Solenoid is a long coil of tightly-wound wire (tighter than in the figure!). The cross-sectional shape of a solenoid does not matter, it can be a circle, a rectangle, etc. The cross-sectional shape and area *S* should not change along of the solenoid's length. Then not too close to the solenoid's edges the magnetic field *B* inside the solenoid is close to uniform and it can be calculated in an easy way from Ampere's law. The result has the form



$$B = \frac{\mu_0 I N}{l} = \mu_0 I n$$

where N is the number of wire turns in the solenoid and l is its length. One can see that B depends on the number of turns per length, $n = N/l$. The applicability condition for this formula is $R \ll l$, where R is the radius of the solenoid.

At the solenoid's edges on the symmetry axis, B is two times smaller then B inside given by the formula above. The size of the region near the ends where the magnetic field is non-uniform, is about R.

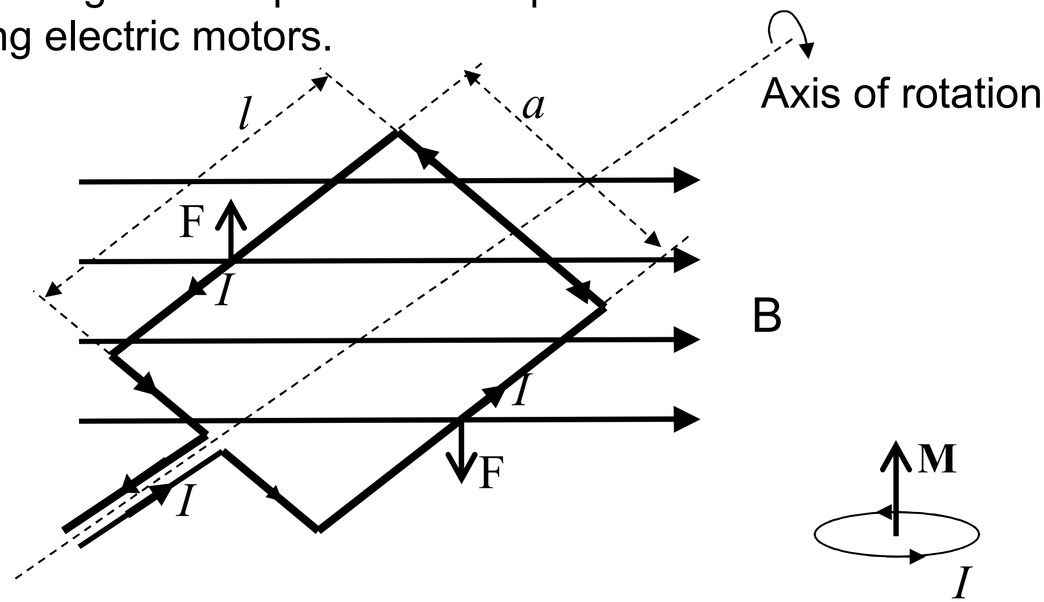
Solenoids (and, more general, coils) are used in electro-engineering, for instance, smooth the direct current. In circuit diagrams it is shown as

Magnetic torque on a current loop

If a wire with electric current forming a closed loop is placed in a uniform magnetic field, the net magnetic force acting on the wire is zero because the currents in different parts of the loop are directed differently in space and the corresponding magnetic forces average to zero. On the other hand, usually there is a magnetic torque on the loop of wire that allows important applications such as constructing electric motors.

In this case the wire will rotate under the influence of the torque until it comes to the vertical orientation, then the torque becomes zero. If the plane of the wire loop is parallel to the field (in our case horizontal) then the torque is maximal and equal to

$$\tau = Fa/2 + Fa/2 = Fa = lIBa = MB$$



where $\boxed{M = IS}$ is the magnetic moment of the current loop and $S = al$ is the surface of the loop.

General formula for the torque: $\boxed{\vec{\tau} = [\mathbf{M} \times \mathbf{B}]}$ (Torque is a vector and its direction is the axis around which the loop will rotate, $\mathbf{M}=I\mathbf{S}$)

As the result of the rotation of the loop, it comes to the orientation in which $\mathbf{M}$ is collinear with $\mathbf{B}$

Energy of a magnetic moment in a field:

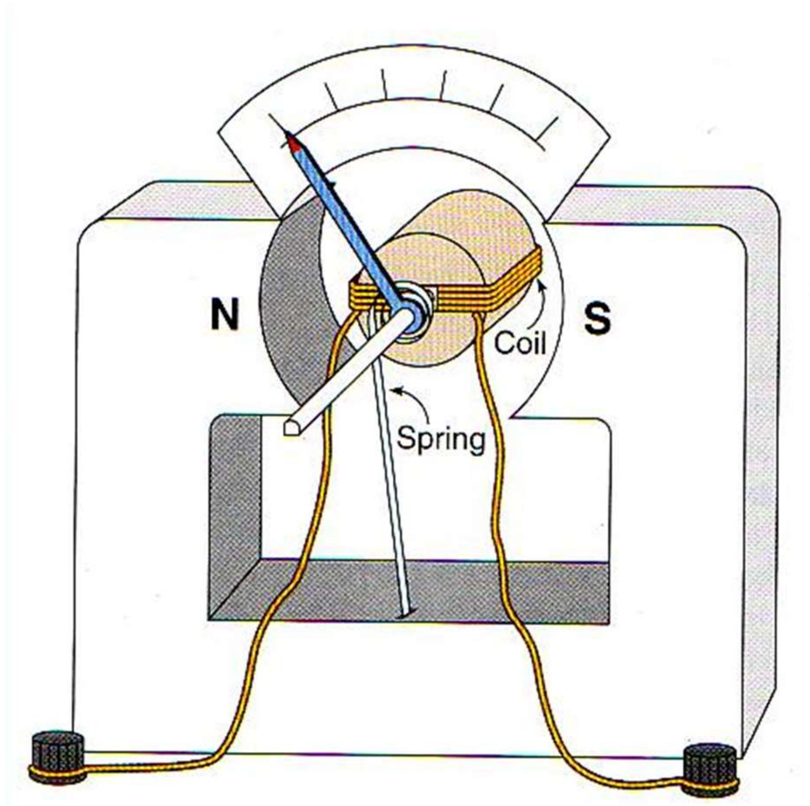
$$U = -\mathbf{M} \cdot \mathbf{B}$$

$\mathbf{M}$ collinear to $\mathbf{B}$ – stable equilibrium (lowest energy). $\mathbf{M}$ opposite to $\mathbf{B}$ – unstable equilibrium (highest energy).

Applications

1. If this loop of wire is connected to a battery with a sliding contact that changes the direction of the current as the wire rotates around its axis, the wire will rotate endlessly in one of the two possible directions, as the simplest realization of an electric motor.

2. Magnetic torque on a current loop is used to measure electric currents with ammeters (galvanometers):



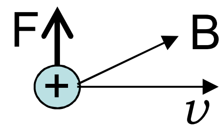
Force on moving charges

We know that the current in metals is due to the moving electric charges. The electric current is given by

$$I = qnvS \quad (q = e < 0)$$

where q is the charge of a single current carrier (electron), n is the concentration of the carriers, v is the average velocity of the carriers and S is the cross-section of the wire. Comparison of this formula with $F = lIB$ suggests that the force on each charge carrier is

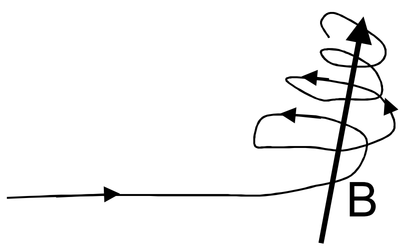
$$\boxed{F = qvB} \longrightarrow \mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

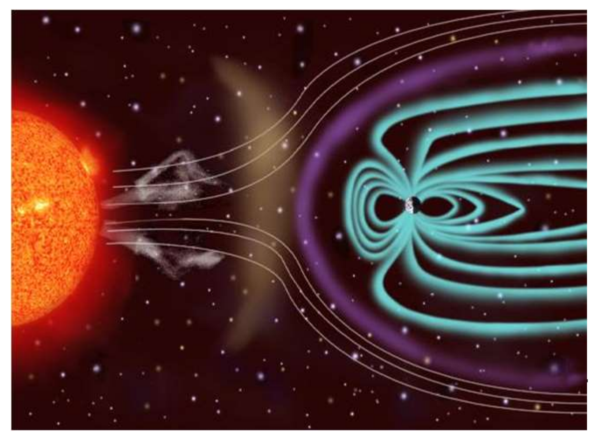


Since the total number of charge carriers in a wire is $N = lSn$, the force acting on the wire is

$$F = NqvB = lSnqvB = lIB, \qquad \text{OK}$$

The force acting on magnetic charges is perpendicular to their velocity, thus it does not produce work and does not change the kinetic energy of charged particles flying in a free space. The role of the magnetic force is that it changes the direction of particles' velocities and makes them perform circles. The magnetic force plays the role of a centripetal force.





Example: Magnetic field of the Earth deflecting the solar wind. Without the magnetic field of the Earth, we would be killed by the high-energy particles from the sun.

Problem

Formulation: Electron is accelerated from the rest by a potential difference 100 V and then it travels in the magnetic field of the Earth, 0.5×10^{-4} T. What are (a) Velocity of the electron; (b) Radius of the orbit of the electron in the magnetic field; (c) Period of its revolution?

Given: V = 100 V, $q = -e = -1.6\times10^{-19}$ C, $m = 0.911\times10^{-30}$ kg, $B = 0.5\times10^{-4}$ T

To find: (a) v; (b) R; (c) T

Solution: Electron's speed after crossing the potential difference V can be found from the energy conservation law:

$$qV = \frac{mv^2}{2} \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{2qV}{m}}} \qquad v = \frac{1}{1}\sqrt{\frac{2\times1.6\times10^{-19}\times100}{0.911\times10^{-30}}} = 5.92\times10^{6}\,\mathrm{m/s}$$

In the magnetic field there is the magnetic force on the electron that is perpendicular to its velocity

$$F = qvB$$

This force does not perform work on the electron and thus conserves its energy. It plays the role of the centripetal force making the electron to make circles. The second Newton's law for the circular motion reads

$$F = ma_c = \frac{mv^2}{R} \quad \Rightarrow \quad qvB = \frac{mv^2}{R} \quad \Rightarrow \quad \boxed{R = \frac{mv}{qB}} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \boxed{\frac{1}{B}\sqrt{\frac{2Vm}{q}} = R}$$

$$R = \frac{1}{0.5\times10^{-4}}\sqrt{\frac{2\times100\times0.911\times10^{-30}}{1.6\times10^{-19}}} = 0.674\,\mathrm{m}$$

The period of revolution is

$$T = \frac{2\pi R}{v} = \frac{2\pi mv}{qBv} = \frac{2\pi m}{qB} = \frac{2\pi\times0.911\times10^{-30}}{1.6\times10^{-19}\times0.5\times10^{-4}} = 7.15\times10^{-7}\,\mathrm{s}; \qquad \omega = \frac{2\pi}{T} = \frac{qB}{m} \text{ - cyclotron frequency}$$