A Model of the Beginnings of Coinage in Antiquity*

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Abstract: This is the first attempt to model the beginnings of coinage in Ionia, Lydia and Greece before the fifth century B.C. Apart from bringing together all of the influences on the essential choices facing the government and the private sector within a coherent whole, the effort yields one important result. Contrary to popular assumption, early coinage was not highly profitable. The Lydian government and the Ionian and Greek city-states provided an extremely wide array of denominations of coins in a single precious metal at considerable cost. Their willingness to bear this cost must have reflected a political strategy of promoting coinage. Such a political strategy would also be easy to explain. In addition, the paper examines the fact that the early Ionian and Lydian coins were composed of electrum, a subject of considerable interest and importance in itself.

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At the turn of the century, archaeologists and numismatists, with the aid of metallurgists, cracked the problem of explaining the origins of coinage around 630 B.C. in Ionia or Lydia or both. The fact that the earliest coins were made of electrum had been a puzzle since the metal is an alloy of gold and silver with traces of lesser metals. Electrum in its natural state contains anywhere from 65 to 85 % gold but neither weighting nor the touchstone can tell which it is with precision, partly because of the third elements, the lesser metals (Konuk and Lorber 2012). The parting of gold and silver also posed a technological problem that was only solved later, much closer to 550 B.C., as we found out fairly recently. Maybe the parting could have been done earlier; but if so it had been judged too costly; this is a moot point. It has also been known for a long time that the inventors of coinage added silver to the unadulterated metal and thereby reduced the gold content of the coins. But we have only fairly recently discovered that they were also able to control the gold content closely. The royal coins of the mint at Sardis, the capital of Lydia, that have been studied with advanced non-destructive methods contain 55% gold, plus or minus 2%, which could only be so voluntarily.[1] Holloway (1978) and Wallace (1987) had largely guessed the whole picture beforehand. But it is now clear.

Prior to coinage, gold and silver in a variety of physical forms had served as media of exchange for probably over a millennium in the Near East and at least for many centuries by the Phoenicians in the Levant to the South. Because of the abundance of natural supplies of electrum in the Western Anatolian region, at some point Lydia and the Greek city-states at its Western border in Ionia had adopted electrum as a medium of exchange in spite of its weaknesses. It can easily be imagined then that the idea would occur to someone of producing electrum pieces of identical weight and fineness with an insignia identifying the producer and signalling the exact content. It would have been a reasonable gamble, if the person was well-

[1]Duyrat and Blet-Lemarquand (2012) (still unpublished), as reported in Velde (2013). For earlier evidence, see Cowell et al (1998) and Cowell and Hyne (2000). Parting gold and silver poses a greater problem than parting either metal from other ones or mixing them together (see Ramage and Craddock 2000). The general assumption that electrum flowed directly from Mount Tmolus via the Pactolus river right next to Sardis has been recently proven wrong in another unpublished (yet available) article presented at the same 2012 conference as the Duyrat-Blet-Lemarquand paper: Cahill et al. (2015). The authors show that the flow consisted of pure gold. Thus, as they explain, the electrum must have come from other parts of the Lydian empire, in northwest Anatolia. Because so many discoveries about early coinage in the Near East, Asia Minor and the Mediterranean date recently, I rely strictly for all such dating on the most recent publications, all of them post-2000, preferably post-2010.

connected and respected, that when he offered them in his payments he would receive enough for the pieces to compensate him for the trouble, depending obviously on the costs, since the signed issues would provide a clear advantage over the rest. This could easily work on trust. In addition, however, the state would be in an excellent position to step in and increase the profitability of the activity by declaring the coins legal tender and accepting them in payment for taxes and fines. If so, of course, the government might undertake to produce the coins itself, as was the case. Whether private initiative really came first, as in this recital, cannot be proven though it is possible since among the 300 or so separate issues that have been identified, many of them are private (see van Alfen 2014, including references). Kroll (2001a) offers an excellent statement of the argument (with greater stress on the role of the state).

As an important part of the story, the invention hardly took the world by storm. It required about 80 years, from around 630 to around 550 B.C., for coinage to occur at all outside of Lydia and the nearby Greek colonies. Around 550 B.C. King Croesus of Lydia decided to introduce separate silver and gold coins (the investment in parting had then occurred). At about the same time, we do not know whether it was shortly before or after, some Greek city-states on the mainland and offshore islands, started to coin in silver (Aegina was the first). Within 30 to 50 years or so, silver coinage “caught on like wildfire” (Schaps 2014, p. 36; see likewise van Alfen 2006, p. IX) in Greece, yet nowhere else. After Lydia fell to the Persian King Cyrus in 547 B.C., the latter retained the gold and silver coinage Croesus had introduced not long before and his second-in-line successor Darius I substituted the gold Darics and the silver sigloi around 520 B.C. But the advance of coinage in the Persian (Achaemenid) empire only took place in the Western part, not in the more highly monetized and commercially sophisticated East. The Phoenicians who traded far and wide started to coin only in the middle of the fifth century and their Carthaginian outposts somewhat later at the end of the fifth century. It was clearly the conquests of Alexander the Great in the last third of the fourth century B. C. and the subsequent political expansion of Rome in the next four centuries that led to the wide spread of coinage in the ancient world outside of China (which coined independently) and

India (which also coined independently though more heavily after Alexander).[2] Of note, coinage only swept Egypt, a bastion of early civilization, in late fourth century B.C. North and central Italy and most of Europe also saw coinage arrive only under Roman influence. Rome itself started to coin late, around 300 B.C., and coinage really only took off there with its military advances in the third century, that is, about two and a half centuries after coinage had covered most of Greece (Burnett 2012).

In this article, I propose to model the beginnings of coinage in full awareness of its long take-off. The exercise has the basic merit of bringing together all of the influences on the essential choices facing the government and the private sector within a coherent whole. It also yields one important result: contrary to popular assumption, early coinage was probably not highly profitable. It is "usually understood [that] the electrum coins were highly overvalued," say Cahill and Kroll (2004, p. 613) (with minor rephrasing), by which they clearly mean highly profitable (see also Kroll 2008, p. 18, and 2012, p. 39). To the contrary, I will argue that highly profitable coinage is more of a hindrance than a help in interpreting early coinage. As a basic consideration, the innovation took place in an environment of well entrenched monetary usage of the precious metals and large metallurgical skill. Had coinage been very lucrative, the use of the scales would have presented a big hurdle and competition could have been expected to bring down the profits. Moreover, the early Lydian, Ionian and Greek coinage came in a wide range of denominations in a single precious metal, going from one to 96 and more. Yet even producing as few as 10 coins with the same metallic content as a single large one would have cost notably more as a percentage of market value. The lowest denomination must have been unprofitable. Otherwise, the highest denomination would have yielded massive profits. The lower denominations were almost surely subsidized and this was most likely a reflection of state policy. Such policy would also be easy to explain. As repeatedly stressed in the literature (Kraay 1964, pp. 89-90, 1976, pp. 322-323, Crawford 1970, Lo Cascio 1981, Howgego 1990, Scheidel 2008, 2009), the government itself had much to gain from the spread of coin-

[2] In the case of China, the evidence of the coinage dates at least a century later, maybe two, than its appearance in Greece (and took place in notably different forms; see Kakinuma 2014). In the case of India, it also probably dates over a century later (Schaps, 2006 and Grierson 1975, pp. 44-46) though the dating is much murkier (see Dhavalikar 1975). In light of the independence of the origins in both cases, one can make light of this.

age in managing its budgetary affairs because of its numerous payments and receipts of bullion in small individual lots. If it subsidized the activity, the government's savings in transaction costs from the spread of coinage could then easily offset its associated production losses.

That is the interpretation that I will propose. Perhaps the willingness of the kings of Lydia and the Greek poleis to subsidize small denominations also partly reflects concern with the welfare of the population. In the case of Greek poleis, official behavior could also reflect civic pride, as has been mentioned persuasively (in connection with coinage generally rather than low denominations in particular: for example, Grierson 1975, p. 14, Howgego 1995, p. 41, Wallace 2001, p. 131). However, I prefer to stick to the collateral benefits to the state from the spread of coinage in running its own affairs, if only to simplify. The central point is clear: the state's promotion of coinage can be explained independently of production profits.

All of this draws heavily on the rich archaeological, numismatist and classical discussion. The argument is also highly indebted to Sargent and Velde (2002). In this important work, the authors (hereafter SV) call attention to the recurrent problem of shortages of low-denomination coins in earlier times, using examples from Europe since the Middle Ages up through the 19th century. Bresson (2006) and van Alfen (2015) have picked up the application of SV's argument to early coinage in Ionia, Lydia and Greece. Interestingly, in fact, Lo Cascio (1981, pp. 80-83) had largely anticipated the whole argument beforehand in discussing ancient Rome. I will develop the argument further and also keep the focus more narrowly on the state's economic interests. My only deviation from the literature concerns the supposed high profitability of coinage, particularly prominent in the discussion of Lydia.

Since a good deal of the analytical argument follows independently of the problem of the special costs of producing lower denominations, I shall first overlook this aspect. Accordingly, the next section presents a simplified version of the model with uniform costs of producing different denominations. In this model, money consists strictly of precious metal and coin, the total quantity of money is given, and the issue is simply the decomposition of the stock between bullion and coin. There is a single metal but it is a precious one, whether gold, silver or electrum. No lesser metals, not even copper/bronze, are allowed since even copper/bronze coinage often (not always) tended toward token status (viz., ancient China and 4th century B.C. Greece, though not ancient Rome, where the bronze coins remained essentially full-bodied) and token coinage clearly raises distinct issues. Importantly, I assume the government is able to set the market premium on a weight of bullion in the form of coin by deciding on the rate at which it will accept bullion in this form in payment. This assumption is common in the literature and, I believe, reasonable for Lydia, Ionia and the Greek poleis. But it is a simplification, whose vulnerability in other contexts I will stress. Section III discusses the relevance of this simplified version. Section IV next presents the full model. There I will develop the probability that coins provided low profits. Section V adds reflections about the complete model. This section is where I will stress the limited application of the assumption of the government's ability to set the market premium on coins outside the Lydian, Ionian and Greek examples. A short concluding section follows.

II. A simplified version of the model

Consider the stock of money M as consisting partly of coins M_o, partly of bullion M_1. M_o is counted by tale; M_1 is weighted and checked for fineness. All money is priced in staters (a Lydian and Greek unit of account) and there are many denominations of coins. Aggregate M depends on supply and demand. Together the two determine the general price level of goods and services and M. However, M and the price level are both given and the issue is strictly the division of M between coin and bullion, M_o and M_1. This division depends, on the one hand, on the costs to producers of using bullion to produce coins and, on the other hand, the benefits to money-holders of holding coins rather than bullion. In equilibrium, producers must be satisfied with their marginal return, while holders must be satisfied with their holdings of coins relative to bullion. Otherwise shifts would take place between the two. The benefits of coinage to the state enter the model separately. These benefits are two-part: first, in so far as the government engages in production, there are possible profits, and second, there are economies in transaction costs, quite independently. The model contains six equations, of which the last three relate strictly to the government's interests. One of the equations, but only one, depends on the metal serving as M, electrum or either silver or gold. The six equations follow.

(1) $M_o + M_1 = M$

(2) $1 + b + s = 1 + m$ or (2a) $1 + b + c + s = 1 + m + \sigma$

(3) $m = m\ (M_o, gov, ageM_1)$

$m'(M_o) < 0 \quad m''(M_o) > 0 \quad m'(gov) > 0 \quad m'(ageM_1) < 0$

(4) $S = g\ M_o\ s \qquad 0 \leq g \leq 1$

(5) $g = g\ (K, s, pol) \qquad g'(K) > 0 \quad g'(s) < 0 \quad g''(s) > 0$

(6) $G = n\ M_o$

Endogenous variables: M_o, M_1, m, S, g, G.

Government policy instruments: *s, gov, pol*

Eq. (1) is obvious. Eqs. (2) and (2a) are alternatives, the first depending on gold or silver, the second on electrum. It is best to discuss the two separately. In the case of eq. (2), the equation compares the market price the producer charges for coining an additional stater of M_1 in gold or silver with the value to the holder of the additional coinage. b represents the costs of producing the coins or the brassage. Since these costs per stater of bullion are independent of volume and denomination (one stater, one-half, etc.), b is a constant. It is also given. s is the rate of profit, also a constant. If negative, production is at a loss. I will refer to s as the rate of seignorage even though in ordinary usage this rate includes b and applies strictly to earnings by the government. I also assume that the government can set s for the market as a whole by deciding on the premium over and above the price of bullion it will offer on payment in coins of identical weight ($b + s$ per stater of bullion). On the right hand side of eq. (2), m is the marginal saving in transaction costs to the holder on holding one extra stater of bullion in the form of coin (this is one variable that has been missing in the discussion). It is what the holder would be willing to pay for the advantage. I will term m the liquidity yield. It must be positive, as otherwise the coins would be simply treated as bullion. Therefore s cannot be lower than $-b$. If the left hand side is below the right, it is profitable to convert M_1 into M_o either through trades or by bringing extra bullion to producers for coinage. Any extra coins issued by producers would circulate at the current market price. If the left hand side is higher than the right, coins could not circulate at this price and people would want to go out of coins into bullion. Eq. (2) is therefore an equilibrium condition.

In the case of eq. (2a), things are more complicated. Roughly speaking, the left hand side is the market price of one additional stater of natural electrum in the form of coins, and the right hand side the value of the corresponding coins to the holder. But this is very roughly speaking since natural electrum is an uncertain composition between 65% and 85% gold and the rest in silver, and the electrum contained in coins is an exact composition, let us say, in light of the facts, 50% gold and 50% silver. Therefore, the right comparison is between the price of an additional stater of artificial electrum on the left after coinage and the value of the corresponding coins to the holder on the right. (Since silver is the cheaper metal, artificial electrum worth one stater *weighs* more than one stater of natural electrum.) Evidently, b, as defined previously is then only part of the cost of production: the cost of converting the artificial metal into coin. However, the producer also must be compensated for converting the natural metal into the artificial one of steady mix in the first place. c is this extra cost. The market price of one stater of artificial electrum after coinage over and above $1 + b + c$ staters then is the rate of seignorage s. On the right hand side, the liquidity yield m is as before the advantage at the margin of possessing coins rather than the metallic content of the coins (the artificial electrum) in the form of bullion. But there is now an extra gain to the holder: the benefit of possessing an assured composition of the metal. This is an insurance premium, σ, for which the holder is willing to pay. The whole explanation of the origin of coinage in Ionia and Lydia at the start of the paper evidently hinges on the importance of σ relative to c. If the left hand side of eq. (2a) is lower than the right, people would be willing to hold any additional coins that the issuers would provide at the current price. As people wish to increase their money stock in the form of coins, they also wish to trade money consisting of natural electrum for coins at the margin. In the opposite case, people would refuse fresh coins that producers might want to add to the stock at the current price and they would wish to trade coins in favor of natural electrum. Thus, eq. (2a) too is an equilibrium condition.

I will assume that equilibrium condition (2) or (2a), whichever one holds, is satisfied: that is, coin production meets any demand above the existing stock and no excess supply is in circulation. This is a strong assumption; but it is important to know where it leads and to be aware of any conclusion that depends on its violation.

The rest of the model can be stated identically for either gold and silver or electrum. Eq. (3) specifies the determinants of the liquidity yield *m*. Any rise in M_o lowers *m* for the same reason that any rise in *M* lowers the liquidity yield on money generally to the holder. Since the issue is the emergence of coinage at a time when the monetary habit with bullion is already well engrained, this negative relation makes eminent sense. The other two variables in the function separate two different groups of influences on *m*. *gov* stands for all measures government may take to increase the convenience of coins rather than bullion as a medium of exchange. This includes declaring coins legal tender in meeting debts and insisting on paying its own staff and military personnel in coins. Tax policy (encompassing tributes, fines and fees) also enters quite apart from its use in fixing *s*. For whatever the price the government may set on coins in collection, it can change the percentage of tax receipts that it requires in coins rather than *in kind* – that is, for example, in grain or services (historically relevant) – and any rise in this percentage will raise *m*. $ageM_1$ is an umbrella term for the strength of the monetary habits with M_1. A gross but inadequate measure of this strength of habit is the length of time since M_1 was adopted.

The last 3 equations concern the government's benefits from coinage. Eq. (4) states the government's receipts of seignorage *S* (already defined net of production costs) as a fraction *g* of the total sM_o, where *g* can be anything from 0 to 1.[3] Eq. (5) determines *g*. *K* is the necessary capital to produce M_o, for example, the mint. The rental rate on this capital inhibits private production more than production by government because private parties have less capacity to finance, if nothing else.[4] Of course, the presence of this rental rate means that *s* is not really pure profit. The next influence in the equation, *s*, refers to the attraction of production to the private sector and lowers *g*. *pol* refers to governmental measures affecting *g*, for example,

[3] As a further deviation from general usage, seignorage thus includes past as well as current revenues.
[4] This lesser capacity to finance by the private sector becomes much more important after the introduction of the highly mechanized forms of producing coins that followed many centuries later.

declaring private production illegal and severely punishable. I do not attribute a particular sign to *pol* because it may include measures admitting and possibly favoring private production, such as have occurred in ancient and modern China (e.g., Thierry 2003, pp. 21-27) and also in Europe, say, in 700-1500 (e.g., Bloch 1954, pp. 21-22). Eq. (6) next specifies the economies that the government obtains from the spread of the coinage habit in the population in its own monetary affairs. These economies are a fraction *n* of the total private demand for coins M_o, $n > 0$. I will refer to nM_o or G as collateral benefits. It is essential to keep in mind that G refers strictly to benefits to the state, not the general public.

There are 6 endogenous variables and 3 government instruments in this 6-equation model. All are listed below the equations. Given the government's decisions about *gov* and *pol*, consider the value of *s* (the third instrument) that the government would set if it were simply maximizing its total benefits from coinage, $S + G$. The solution follows:[5]

$$(7)\quad \frac{\partial(S+G)}{\partial s} = gM_o + sM_o\frac{\partial g}{\partial s} + sg\left(\frac{\partial m}{\partial M_o}\right)^{-1} + n\left(\frac{\partial m}{\partial M_o}\right)^{-1} = 0$$

(The second derivatives $m''(M_o)$ and $g''(s)$ attached to eqs. (3) and (5) have the appropriate signs for an optimum.) The first term on the right in eq. (7) is positive, but at any positive *s* or seignorage the next two terms are negative. The last term is negative independently of *s*. Suppose a rise in *s* ($ds > 0$). The government directly increases its seignorage *S* by a value depending on its part *g* in the issue of M_o (first right hand term). However, it reduces its seignorage by attracting private producers, possibly outlaws (second term). It also reduces its seignorage by raising the marginal cost of coins relative to bullion to holders, thus, in turn, encouraging people to economize on coins (in order to raise the marginal benefit on coins *m* to the appropriate degree) (third term).[6] This last effect has one additional consequence: by lowering M_o, it deprives the government of some collateral benefits *G* (last term).

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[5] As regards the last two terms on the right hand side of eq. (7), given eq. (2) (where *b* is independent of *s*, or in case of eq. (2a), *b*, *c*, and σ are independent of *s*), $\partial m/\partial s$ equals one. Therefore $\frac{\partial M_o}{\partial s} = \frac{\partial m}{\partial s}\frac{\partial M_o}{\partial m} = \frac{\partial M_o}{\partial m}$. (Eq. (3) does not serve to determine *m* but M_o.)

[6] In case of a negative *s*, the second and third terms on the right would reduce the government's seignorage losses instead (and therefore be positive) by reducing its coinage.

III. Discussion of the simplified version

I will now argue the application of the model in its preliminary form to early coinage in Lydia, Ionia and Greece. As regards eq. (1), it is clearly legitimate to aggregate all of Lydian and early Greek coins together in a single total M_o without worrying about separate market prices of different denominations. This is not always the case under metallic money but holds for Lydia up to the introduction of bimetallism around 550 B.C. and Greece up to the similar introduction of bimetallism via bronze coinage in the second half of the fifth century B.C. Both Lydia and the early Greeks were careful about exact weights and metallic content of lower denominations of their coins. The half staters were really halves in every sense, and so forth. The purity of the silver used by the Greeks for coinage has been widely commented. On these points, see, e.g., Nicolet-Pierre (2002, p. 115), Kim (2001, p. 18), Wallace (1989) and Velde (2014).

Next, as regards electrum, eq. (2a), I have assumed that the relevant Lydian and Ionian population knew the true metallic composition of the coins. This is obviously an approximation, but a reasonable one. Had the population been duped about the lower gold content of the electrum coins than natural electrum, greater profits would have been possible, the coinage would have been larger, there would have been more private production, and inflation would have resulted. This may all sound entirely plausible; but it is difficult to think that such a situation would have lasted for 80 years. The variability of the gold content of electrum coins by time and place only heightens the difficulty. Many issues of electrum coins contained less gold than the royal ones (see, for example, Wallace 1989). Did these issues yield a higher rate of profit and if not, why not? And if the royal issues were highly profitable, why did not better or cheaper products appear? High profits would have brought gainful opportunities to supply electrum coins of equal weight with higher gold content than the royal ones, advertise and invite detection. In a commercially astute population, could this have gone unnoticed? Yet we have no evidence of the sort. And why did Croesus ultimately abandon electrum in favor of bimetallism? Had electrum coins become unprofitable? And if they had, why is it that issues of electrum coins persisted 150 years later in some Greek city-states in Ionia and to the North (such as Mytilene, Phocaia and Cyzicus) and remained popular around the Black Sea (see

Wallace 2001, pp. 130-31 and Bresson 2009), to say nothing of the fact that Carthage adopted electrum coinage around the turn of the 4th century B.C.? In order to model the beginnings of coinage, some assumption must be made about the accuracy of popular beliefs concerning the content of electrum coins, and it is much simpler and straightforward to take as a point of reference a situation where deception is out (even if the assumption is not perfectly right), electrum coins of notably different content trade simultaneously in the same place at different prices and diverse considerations enter to explain the choice of varying ratios of gold in the production of new electrum coins at different times and places (including differences and movements in the optimal size coins because of changes in market prices of goods and services and varying weight standards, wear of the older coins, and changes in the relative price of gold and silver). The idea is not that the assumption is accurate but that the alternative of supposing the possibility of steady profits over decades because of misperceptions poses greater problems.

The mark-up on coins over bullion ($b + s$ or $b + c + s$) also deserves a separate word. There is a coordination problem, to which I referred in the opening section under the name of trust, since all media of exchange depend on confidence that others will accept them in payment. In this regard, bullion has a clear advantage over coin as money. In the case of bullion, the marginal value of the good in monetary usage and in non-monetary usage is the same. Thus, if the good's monetary usage disappears, as long as its commodity value stays the same (which is quite possible if the monetary usage is regional and the commodity usage far wider), the holder will bear no exchange loss. But this ceases to be true with coinage. As generally recognized and eqs. (2) and (2a) say, the coin must earn a premium over its commodity value because of the extra cost of production. If others then abandon the monetary usage of the good, the holder bears a loss. A genuine welfare role emerges for the government. Government is in a better position than any other party to help to coordinate beliefs at home. By promising to accept coins at a premium relative to their commodity value, it can then promote the adoption of the coins as media of exchange by enhancing the confidence that permits people to count the coins rather than weigh and test them for fineness.

The relevance of the variables $ageM_1$ and G in the model reflects factors in the early spread of coinage that come up repeatedly in the discussion. In regard to $ageM_1$, there is even some evidence that the coinage by Lydia and Ionia made particularly little impression in the places in their trading range where commercial activity was most sophisticated. Thus, the Phoenicians who had been active in international trade since Homeric times (centuries before Homer) were slow to adopt coinage, and so were the commercially sophisticated societies in Mesopotamia. Egypt, hardly a commercial backwater, did not begin to coin until the late fourth century B.C. (Lorber 2012) and yet it had no difficulties trading based on a well-engrained unit of account (the deben) and system of price quotations without media of exchange as well as with them (see Manning 2008) in its well organized economy, with large-scale redistribution. On these many points, see Le Rider (2002, pp. 1-100), Gitin and Golani (2001), Van de Mierop (2014) and Kroll (2008), among others. As regards G, it is a standard theme in the literature that military spending, in particular, had a large role in the penetration of coinage in antiquity. Crawford (1970) and Scheidel (2008, 2009) are prime examples. However, following Kraay (1964, pp. 89-90) and (1976, pp. 32-323), both Lo Cascio (1981) and Howgego (1990) stress the importance of large government projects and financing requirements independently of strictly military spending. All of these authors clearly see the government's benefits from the spread of coinage as coming largely, if not principally, from internal savings in budget management in accordance with eq. (6).[7]

What does the model say about seignorage and the profitability of coinage to the state? I will argue that the model would suggest that seignorage was low. We really know next to nothing about the costs of coinage at the time, which depend on contemporary labor costs and other prices, as well as contemporary technology, and the composition and the appearance of the coins. The one hard piece of evidence we have, from the Athens Coinage Decree of the 5th century, refers to a mint charge of 3 or 5 %, depending on interpretation, for converting silver into coin (Figueira 1998, p. 360 and Kroll 2001a, p. 205). The model says that choosing a high rate of profit on coins would encourage private competition with the government in its

[7] However, Lo Cascio and Howgego are also particularly keen to admit the state's concern with the welfare of the population, the former for Republican and Imperial Rome, the latter more generally for the ancient world.

own production and reduce the demand for coins (an issue of elasticity). Both of these effects would then limit the state's advantage: the first by lowering its production profits, the second by raising its transaction costs in its other revenue-seeking and trading activities. One aspect of this argument has particular resonance in world monetary history. There is extensive evidence that profitable coinage encourages private entry, often illegal. Counterfeiting was an enormous problem prior to the introduction of highly sophisticated and mechanized methods of producing coins both in China since the Han dynasty and Europe since medieval times (Peng 1994, *passim*, Thierry 2003, p. 21, SV (2002), pp. 64-68, 180, 223, 247, 265, 270-271, Spufford 1988, pp. 171-175). Yet we know of no corresponding problem of counterfeiting in Lydia and the early Greek *poleis*. To be sure, counterfeits crop up but the problem seems contained. Velde (2014, note 25) estimates "the counterfeits found in the numismatic trade" of Lydian electrum coins as "1 to 2% of the total number of coins." Of the Greek coins, Wallace (2001, p. 131) says: "It is a fact that the alloys of Greek coins were very carefully calibrated and scarcely ever adulterated". The simplest general explanation is low profitability. The model can even easily explain negative values of *s*.

I will now go on to show that once we repair the outstanding flaw of the model – the assumption of an identical percentage cost of production for low-denomination and high-denomination coins – the conclusion of the low profitability of coinage is almost irresistible.

IV. The full model

The Lydian and ancient Greek coins share one arresting feature in common: they offer a system of denominations that covers a wide range of trade values even though it is based on a single metal. Both the royal Lydian coins and those of Athens did so from the start (for Lydia, see, for example, Velde 2014, and for Athens, van Alfen 2012). Lydian electrum coins and those of the nearby Greek colonies in Ionia had 9 different convenient denominations going from one stater down to 1/192. The early Greek silver coins likewise had denominations going down from a tetradrachma at the top (4 drachmas) to one obol (1/6 of a drachma) to 1/8 of an obol (1/192 of a tetradrachma) in about 9 steps. Quite significantly, even the lowest coin did not necessarily provide small change. Indeed, the smallest Lydian coin, weighing around 0.08 of a gram clearly did not do so and it is doubtful that the lowest Greek one did so either, though it weighted about the same and was composed strictly of silver, a lesser metal than electrum.[8] However, the range of trade values of the denominations was extremely wide. Even as wide a range as 1 to 24, to say nothing of 1 to 48 or 96, with the top coin worth quite a lot – indeed, over a week's wages for an independent worker, as was the case – is a remarkable feature of the early Lydian and Greek examples. Other early coinage systems only achieved the same result with at least two metals or in the Chinese case, by incorporating other materials besides metals into the monetary system like hemp and silk cloth or, as often true since the late 10th century A.D., paper money.

The point is worth elaborating. When Rome started to coin circa 300 B.C., small change came at once with the lowest denominations of coins made of bronze, a much cheaper metal (often 1/100th as cheap) than silver, and the system only achieved anything resembling the range of purchasing power in Lydia or ancient Greece by combining bronze with two or three denominations of silver coins (Burnett 2012 and Woytek 2012).[9] Medieval Europe, later, had virtually only a single, light coin for five centuries beginning in the 7th, composed of silver: variously the penny, denier, denaro or pfennig (Spufford 1988, p. 27), with weights that differed immensely according to time and place, at least from 1.7 to 0.1 or 0.2 grams (Spufford, p. 225) and a penny weighing a gram possibly worth one-third to a day's wages.[10] Any range of denominations to speak of only began in 1201, with the appearance of the grosso in Venice, another silver coin, worth 24 denari (soon imitated with gros tournois, groats, groschen). To quote Grierson (1975, p. 27): "it was only between the beginning of the 13th and the mid-15th century... [that] Latin christendom came to have at its disposal a wide range of denominations,

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[8] In line with Le Rider, 2002, pp. 68-69, Velde (2014) estimates that 1/192d of a stater was worth over one day's wages in the Lydian case. In a careful study of the issue, not yet published, van Alfen (2015) calculates that the lowest Greek coin probably would not have qualified as small change either. Yet this coin and the smallest Lydian one would be miniscule and inconveniently small. Compare Howgego (1995, p. 1).

[9] The silver component appeared about the same time with the adoption of the silver didrachm. The silver denarius came next in 211/212 B. C. It took over two centuries before gold arrived, with Cesar (first the aureus), to widen the range of values of individual coins much further.

[10] "Typically, the daily wage [of unskilled labor] represented 1 to 3 silver coins, and thus daily necessities required smaller coins" (SV, 2002, p. 48).

reverting thus in some measure to the monetary pattern of antiquity."[11] In China, to which reference has been made, bronze/copper coins provided strictly a range of lower denominations. Higher ones depended on a different material, largely silks (of set sizes and quality). They also varied by states, with gold strictly important in Chu; and quite often higher denominations were simply lacking (Peng 1994, pp. 106, 208-213, Thierry 2003). Thus, the Ionian, Lydian and ancient Greek examples are truly exceptional. They lasted in Ionia and Lydia until Croesus introduced bimetallism around 550 B.C. and in ancient Greece until bronze coins provided small change in the second half of the 5th century, though Athens resisted the move until the middle of the 4th century (see Grandjean 2006).

The issue is obvious. To produce a low-value coin costs more as a percentage of market value than to produce a high-value coin of the same material with the same methods and differing only in size. SV provide a table (2002, p. 51) showing brassage costs as a percentage of value for silver and partly gold, drawn from various sources and covering 9 examples for different parts of late Medieval Europe. If we focus on large differences in denominations (say above 12 to 1), we find a clustering of readings of denominations differing by around 20 to 30 to 1, and for those readings (6 of them), the costs of brassage are about 5 to 9 times higher on the lower denomination than the higher one. Thus, a brassage cost of 1% on a "stater" (any unit) means 5% to 9% on a coin of 1/20 to 1/30 of a stater. Furthermore, the technology "was little changed from Greek and Roman times" (SV, p. 50). Governments therefore needed to offer mints special inducements to produce the lower denominations (see Spufford 1988, pp. 361-362, as well as SV). Notwithstanding, there often resulted a scarcity of small change in Europe, to which Cipolla (1956) had already called attention in a sparkling lecture titled "the big problem of petty cash" and SV relabelled "the big problem of small change." Adequate quantities of small denominations would not be minted largely because people simply would not accept the degree of debasement (incorporation of baser metals) of the coin that was needed to make the coinage profitable. The problem of small change was only fully solved recently in

[11] Some important steps in broadening the range of denominations in Europe were the appearance of the gold florin in 1252, and the arrival of higher-denominations of silver coins (thalers, silver écus, crowns) much later, in the 15th and 16th centuries.

world history, since the 19th century, with the advent of strictly token money that was convertible on demand in any quantities at their nominal price by the government.[12]

From all this evidence, it is apparent that the willingness of the Lydian government and the Greek city-states to absorb the cost of producing an extremely wide array of denominations of coins in a single precious metal must have reflected a political strategy of promoting coinage. Production profits could not be the sole or probably the outstanding motive. I shall now propose a corresponding modification of the model. To do so, I will borrow from SV and assume only two coins: a large one, the stater, and a small one, at least 1/24th the size (SV used the dollar and the penny). The required changes are so simple that I will relegate the equations to the appendix and home in directly on the central point.

Instead of the previous variable b, for brassage cost, we now have either b_H for the high denomination or b_L for the low denomination, with b_L greater than b_H. Correspondingly, the seignorage rate on the high denomination, s_H, exceeds the one on the low denomination, s_L, by $b_L - b_H$ (that is, $s_H = s_L + b_L - b_H$ regardless of gold/silver or electrum). Thus, either the government sets s_H and accepts the consequences for s_L, or it sets s_L and accepts them for s_H. It is clear that the ground for any positive seignorage whatever has shrunk dramatically. If the government sets s_L at zero, there are seignorage revenues $b_L - b_H$ per stater on high-denomination coins. But given what we know about $b_L - b_H$, this alternative is almost unimaginable. Large coins could hardly ever have gotten off the ground: the demand for them would have dropped to zero. If the government sets s_H at zero (or s_L at $b_H - b_L$), costs of production on the large coins are met. There are then production losses $b_L - b_H$ per stater of small coins (i.e., for each

[12]In China, the problem of small change was probably lesser because of concentration of coinage in low denominations resulting from the use of bronze/copper. In addition, China used casting, which requires less skilled labor than striking; the country produced coins of poorer designs and occasionally tolerated free coinage of subsidiary coins in accordance with government specifications (Peng 1994, pp. XXVII, 554, 659, 670, Thierry 1997, p. 166, and Cowell et al. 2005). Still the Chinese encountered the problem of small change recurrently after rises in copper prices over decades, as the values of coins would go up above the prices of the metallic content, and melting and shortage of small change would take place (Thierry, 2001, Von Glahn 1996, pp. 49-50). Interestingly, China also encountered problems of shortages of large denominations stemming from insufficiently developed banking and the high costs of transport of small coins for large payments prior to the late adoption of silver coins on anything like a national scale only in the 1800s (despite some significant use of silver bullion as money as early as the 13th century if not before; Peng 1994, pp. 676-706, von Glahn 2003). These shortages explain the repeated Chinese experiments with paper money from the late 10th century A.D. to the time of the Republic of China in 1912 (with major interruption in the so-called "silver century" of 1550-1650) (Peng 1994, pp. 368, 537-42, 707-731, Kranister, 1989, pp. 138-155).

lot of 24 of them or more). Yet the collateral benefits on the total coins M_o could outweigh the seignorage losses on the small coins. They could do so all the more if small coins constitute a small percentage of M_o, as we may suppose since small coins are only important as a percentage of market value in small transactions. (The point carries over with 8 or 9 denominations instead of two.) The authorities could also seek a compromise between s_L equal zero and s_H equal zero. But finding room for *any* positive seignorage S at all becomes a challenge. We would need to experiment with low enough collateral benefits per stater of coins (low enough values of n), low enough elasticity of the demand for coins, and strong ability to stave off private competition, and success would almost surely depend on admitting an intense desire by the public to stick closely to the current division of coin and bullion in the face of price incentives to move away. On the other hand, a negative value of S is easy to contemplate. The authorities would simply need to choose a rate of profit on output that breaks even on the large (one-stater) coins or makes a minor profit on them and accept the consequent production losses on the small coins in return for the collateral benefits on total coinage. The facts do not in any way impede this last interpretation. That is the fundamental argument for it.[13]

V. Further reflections

Some general reflections about the complete model are important. There are large omissions

[13] The idea of a high profitability of the Lydian coin issues rests heavily on the fact that when King Croesus introduced bimetallism, he issued new gold staters 25% lighter than the previous electrum staters. In addition, shortly thereafter (it is thought) he began issuing instead gold staters 25% lighter still, or of about the same market value than the earlier electrum staters. See Cahill and Kroll (2004), pp. 609-614, writing in the aftermath of Le Rider (2002), pp. 94-95 (and also Kroll 2008, p. 18). But on these supposed facts, we can still infer nothing about the profitability of the earlier electrum issues since we know nothing about the rate at which holders of the electrum staters were offered new gold staters at the mint. Here are two alternative scenarios that are both consistent with the same facts and the hypothesis of zero profits on the electrum coins. In both cases, the Lydian authorities have decided to produce new issues of gold staters that are considerably lighter than the earlier electrum counterparts in order simply to contain the value of the new coins (which would otherwise be a lot higher than their earlier counterparts even if the earlier coins had contained, say, as much as 75% gold). According to the first scenario, in full awareness of their ignorance of the rate at which to offer to replace the electrum coins, the authorities decide to experiment and issue new gold staters without replacing the electrum ones, watch the rate of which the new and old coins trade, and once they have a clear view of the market exchange rate, issue new gold coins of the right weight to permit them to offer to trade new staters for old ones at a one to one rate at the mint. This weight is 25% lighter than the initial issue (remember that the old electrum coins actually contained only about 55% gold). Second scenario: the authorities offer to trade the electrum coins for the new gold staters at once. But either they make an unfavorable offer of gold coins for electrum ones (too far below one gold coin for one electrum coin) or people adopt a wait and see attitude and too few electrum coins come to the mint. Once again, the authorities observe the market exchange rate, and after feeling properly informed, offer to trade gold staters at a certain downwardly adjusted weight for electrum ones at a rate of one to one. The downward adjustment is 25%.

because of the decision to abstract from the aggregate supply and demand for money. On the supply side, the model ignores the impact of existing mines, new discoveries of mines, conquests of mines and war booty (see Howgego 1990 and Schedel 2006 on these subjects). On the demand side, the omissions include inflation, balance-of-payments flows of precious metals, and money substitutes (for emphasis on money substitutes, see Cohen 2008 and Harris 2008). The essential justification for these neglects is that the omissions do not necessarily regard the choice between coin and bullion. Therefore the omissions need not help us to understand why coins took hold. A further justification, less important but still of note, is a desire to simplify.

Of greater concern, in my opinion, is the assumption that the government can control the rate of profit on coins. Though universal in the discussion, this assumption has its limits even for the monometallic coinage in Lydia, Ionia and Greece. In the case of the early electrum coins, the assumption best applies to the royal issues of Alyattes and Croesus of Lydia. I doubt that we would want to reason that the Greek colonies of Miletus, Mytilene, Ephesus, Samos and Phocaea, etc., could set the market premium on their issues at will, certainly not independently of the Lydian King's decision. In the case of the Ionian application, the electrum issues of the Greek colonists could even be treated like private issues in the analysis. This is a choice. As for mainland, offshore and Mediterranean Greece, it is widely estimated that perhaps around 100 poleis coined by the middle of the fifth century B.C. However, there is also widespread emphasis on the fact that the issues did not travel far except (at various times) for the Aeginetan, Athenian and Corinthian issues. This local circulation helps to justify the idea that there was official control since it implies a lack of competition. Here again, though, we would not wish to abuse the assumption and pretend that the most prominent issues – those of Aegina, Athens and Corinth, Athens most of all – did not limit the premium the other poleis could set on coins. In some cases, we might also prefer to apply the assumption to a group of neighboring poleis, since some specialized in particular denominations and expected neighboring coins of different denominations to circulate freely at home (see, e.g., Mackil and van Alfen 2006). The idea that the governments subsidized the lower denominations helps here too.

There is clearly more room for control over a premium if the premium represents a concession and therefore does not attract competition.

If we take a broader historical perspective, the merit of the assumption of official control of the rate of profit on coins in Ionia, Lydia and Greece owes a lot to the presence of monometallism and the absence of competing foreign coinages (with the reservations that I have emphasized). Once we get away from monometallism or else we admit high substitutability of foreign coins, the ground for exogenous control of the profit margin recedes. Bimetallism in gold and silver (for example, in the Western Achaemenid Empire after Cyrus) or in silver and bronze/copper (for example, in Republican Rome beginning in the third century B.C.) complicates matters considerably. No government has ever been able to control the relative price of gold, silver and copper at home. With bimetallism, governments can easily lose control over the value of the domestic stock of coins. How much they lose is always relevant.[14] Wide domestic holdings of foreign coins clearly undermine the assumption of domestic government control. The assumption therefore has only limited application in most of medieval Europe, where silver coins of many origins and of different weights and fineness often circulated side by side even as far back as the ninth century (Spufford 1988, ch. 1, Cipolla 1956, lecture 2, Glassman and Redish 1988, p. 78).

Some incidents of European history since the 13th century can also give a totally misleading impression of government's ability to control the price of the coinage. Once coinage becomes well established, it is often true that the high convenience of petty coins for small purchases (in other than diminutive sizes) admits production of coins in lesser metals (including alloys of silver and copper together or with lesser metals) at trade values far above that of their metallic content locally or nationally but not internationally. In these cases, the governments may even be able to "cry up" (*surhaussement*) or "cry down" the exchange value of some coins without changing their metallic content. There have indeed been incidents in European monetary history since 1000 where governments were able to raise or lower the exchange value of

[14] See Bransbourg (2011) for similar emphasis with respect to Rome.

some coins relative to others by little more than declaration. But these incidents simply illustrate the general principle that government is sometimes able to solve a coordination problem. The examples are typically ones where domestic coins were previously circulating above or below their face value in the market or not circulating at all and losing their moneyness because bargains could not be struck. The government could then correct the problem. (See SV, pp. 199-202, 233, Glassman and Redish, 1988, and for a Roman example dating 85 B.C., Lo Cascio 1981, p. 77).[15] SV (2002) provide a useful analysis of the limits of government ability to alter the relative prices of different types of coins in the absence of 100% token coinage.

There is one specific aspect of the early electrum coinage that may deserve further emphasis even though it has already emerged: the fact that large denominations preceded petty cash. I believe it is correct to say that this peculiarity of early coinage can be ascribed to the deficiency of electrum in its natural state as money for big-ticket items. This deficiency made special room for large coins. The Chinese record is clear. First came coins of modest value, fitting for small purchases: the early hoe- and knife-coins of copper/bronze. Typically smaller, round coins followed after the Qin unification in 221 B.C. if not earlier. With the exception of Chu gold, high-value coins in significant volume awaited modern history in China, coming perhaps for the first time with the inflow of imported foreign coins in the 18th century. The Roman experience is less telling but still to the point. When Rome began coinage around 300 B.C., it did so with bronze coins of small enough values to be fitting as small change (bronze had already served earlier as money). Yet Rome also simultaneously introduced higher-value silver coins (perhaps under Greek influence near-by, in Southern Italy). The European experience

[15]As these examples would also show, Gresham's Law can be misleading. When the nominal value of a coin is overstated relative to the nominal value of another, the undervalued coin can continue circulating at a premium above its face value or the overvalued money may only be able to circulate at a discount below its face value or both (Rolnick and Weber, 1986). Thus, rather than "bad money drives out good money," the principle should really be understood as "money that is overvalued in the market drives out undervalued money." The eminent historian and numismatist Bolin (1958) provides a striking example of the pitfalls of undue reliance on nominal values. Based on a careful analysis of the metallic content of large samples of ancient Roman coins of identical nominal face value and sophisticated inferences about minting and melting points resting on the statistical distribution of the weights, he arrives at the conclusion that gold coins were overvalued on average by 20% at the mint in Roman times, silver coins by 25% and copper coins by 33%. As regards gold and silver, this is almost a *reductio ad absurdum*. But rather than call into question his assumption that nominal values equal trade values (in other words, that all gold/silver coins of the same denomination in his comparisons really traded 1 to 1, or could have done so, at any time and place in Roman times), he refers to a basic principle of overvaluation of coinage by the Romans.

after the fall of the Roman empire resembles more the Chinese than the Roman example. In Europe the only coin that remained in usage in the 7th century was the silver penny that traded mostly below a day's wages and in subsequent times often provided small change via debasement and reduced weights. Higher denominations suitable for large transactions followed only in the 13th century.[16] Generally, besides Ionia and Lydia, our only clear examples of the circulation of valuable coins before petty cash relate to ancient Greece and the Achaemenid empire after the conquest of Lydia and the direct influence of the two. But both examples plainly trace back to the Ionian and Lydian experiment. Admittedly, the Indian experience does not necessarily fit. If coinage came there independently in the fifth or fourth century B.C., India might represent another instance of the arrival of big coins before small ones suitable for daily purchases though the point remains obscure.

The analysis leaves open two basic questions: (1) Why is it that the Greeks alone flock to coinage in the second half of the sixth century? (2) If the introduction of coinage is of such marginal interest as the model allows and the literature stresses, why is it that the scarcity of coinage causes so much pain and disruption once coinage is adopted? We have considerable evidence of the cost and disruption resulting from coin shortage both from China in 300 B.C. to 1800 (Peng 1994) and Europe in 1000 to 1700 (see Spufford 1988). On the first question, Schaps (2001, pp. 93-103, and 2004, pp. 16-17, 108-110) offers the interesting hypothesis that the Greeks were more commercially backward than many others in the region, or more precisely, that they had only acquired advanced monetary habits more recently, and therefore found greater allure in the innovation of coins during a period of demographic and economic expansion. Seaford (2004) appeals, instead, to Greek intelligence, capacity for abstract thought, "the Greek mind." Obviously Schaps' suggestion fits better with the analysis here. But the issue is open.[17] On the second question, the answer is clearer than the first.

Once coinage advances far enough, metallic money ceases to be a close substitute except for

[16] Even thereafter weighted ingots of precious metals and bills of exchange went on serving for wholesale and foreign commerce (to say nothing of transfer banking), as they had before (see Spufford, 1988, p. 378 and ch. 9, titled "ingots for large payments"). Of course, much the same can be said for Greece and Rome in antiquity after the arrival of petty cash, especially as regards ingots.

[17] Kroll (2001b, pp. 77-91) and (2008, pp. 14-17) notably takes issue with Schaps' thesis.

very large transactions, where precious metals in various forms prove to be highly persistent, quite apart from credit transfers. If so, the best alternatives to coins in regular, small transactions can be much costlier: namely, barter, gift exchange, and simple forms of credit avoiding the cash economy. In interpreting situations where coins disappear or become very scarce, we may tend to think of conditions of sudden monetary breakdowns since 1914, like hyperinflations, prisoner of war camps, Germany after WWII between the Reich mark and the Deutschmark, or Russia in 1990 following the immanent break-up of the Soviet Union. But the examples that Spufford (1988) summons from medieval Europe are probably far more to the point. He shows in rich detail how the switch to money, by which he means coinage, from a "minor role" to a situation where "it was the measure of all things" (p. 243) in 13th century medieval Europe (much as Schaps and Seaford describe Aristotle's Greece) meant a change in contractual relations between landlord and tenant, hired laborer and employer, and political authority and subject, which drew people into different sorts of social obligations to one another and made them vulnerable to different shocks. Therefore, once an adjustment to scarcity of coinage became necessary progressively over many decades, as for example in the fifteenth century until around 1465, the retreat from coinage to more time-consuming forms of trade did indeed call for profound changes (p. 376 and chs. 15 and 16 in general).

VI. Conclusion

This is the first attempt to model early coinage in Ionia, Lydia and Greece before the fifth century B.C. The analysis centres on the choice of money between coins and bullion in the same precious metal. One general advantage of the exercise is to bring together the varying influences that occur in the discussion. The joint treatment of the benefits and the costs of coinage is probably the essential step forward. As a result, some factors that have been ignored in the verbal yet sometimes highly analytical discussion of early coinage in antiquity make their first appearance here. This includes the impact of the cost of a coin on the quantity demanded, and the impact of the stock of coins on the marginal benefit. Generally, the liquidity yield on coins, which had been heretofore only latent at best, comes fully into light. The competition between private and public issues of coin plays a major role as well in the analysis. High profits to the state in producing coins invite private competition, both legal and illegal.

From the standpoint of the historical interpretation of early coinage, the most important contribution regards the behavior of the state. In the relevant historical examples, coinage took place in the same metal that had served before as money in the form of nuggets, dust, ingots, bars, broken jewellery, etc. In these circumstances, the benefit of the coins over those bits and pieces hinged critically on trust, which evidently the state was in a better position to provide than any private party. However, the relevant states also displayed peculiar behavior in the historical instances. They provided a range of denominations of coins that took centuries to emerge elsewhere (especially if we ignore the Roman republic and empire), and they did so despite the higher costs of producing the lower than the higher denominations. I argue, as others have, Bresson and van Alfen in particular, that governments must have had a political strategy of promoting coinage. The model attributes this strategy to the state's interest in the spread of the coinage habit stemming from its own heavy spending and receipts of money in many, many individual lots. On this view, the state's interest in coinage was mostly independent of seignorage. Indeed, as the analysis shows, its best interests probably lay in the sacrifice of seignorage in order to promote its collateral benefits from the wider use of coins.

The subsidiary role of seignorage in the beginnings of coinage is the one respect in which the present discussion is at variance with the literature, in particular, concerning Lydia. Otherwise, the coherence is high and the main ideas are largely borrowed. There is a sense, quite widespread in the literature, that the Lydian kings, Alyattes (610-560 B.C.) and Croesus (560-547 B.C.), found the production of coinage a very profitable activity. This sense has always been at odds with the idea, also conspicuous in the same literature, that these kings ruled in an area of sophisticated monetary usage of metals and high metallurgical skills. Subsequent monetary history tells us that after coinage takes hold, seignorage can be a source of steady income, but even then as a rule will only reap high profits at times of unexpected monetary expansion and only in an initial phase before adjustment takes place. Bloch (1954), p. 58; Spufford (1988), pp. 312-318; SV (2002), pp. 97, 230; Glassman and Redish (1988). Could the Lydian kings have gotten steady windfalls from coinage in the relevant historical setting?

In this article, I argue that the suggestion is unnecessary and difficult to reconcile with the evidence about the range of denominations.

Appendix

The complete model

The complete model is as follows.

(8) $M = M_o + M_1$

(9) $M_o = M_o^L + M_o^H$

(10) $\frac{M_o^L}{M_o^H} = x \qquad 0 < x < 1$

(11) $1 + b_H + s_H = 1 + m$ or (11a) $1 + b_H + c + s_H = 1 + m + \sigma \qquad m > 0$

(12) $b_H + s_H = b_L + s_L \qquad b_L > b_H$

(13) $m = m\,(M_o^H,\ gov, ageM_1)$

$m'(M_o^H) < 0 \quad m''(M_o^H) > 0 \quad m'(gov) > 0 \quad m'(age\ M_1) < 0$

(14) $S = g\, s_H\, M_o^H + s_L\, M_o^L \qquad 0 < g \leq 1$

(15) $g = g\,(K, s_H, pol) \qquad g'(K) > 0 \quad g'(s_H) < 0 \quad g''(s_H) > 0$

(16) $G = n\, M_o$

Endogenous variables: M_o, M_1, M_o^L, M_o^H, m, s_L, S, g, G.

Government policy instruments: s_H, gov, pol

M is exogenous as in the simplified version before. M_o^L and M_o^H refer, respectively, to coins in the low denomination and the high denomination. Eq. (10) defines the desired ratio of low- to high-denomination coins. This ratio is also exogenous. Eqs. (11) and (11a) repeat the earlier equilibrium condition (eq. 2 or 2a) on holding high-denomination coins as opposed to bullion. The two equations have been written in terms of the high denomination, but they could have been written in terms of the low denomination instead. Given satisfaction of eq. (10), it does not matter since the right hand sides of eqs. (11) and (11a) must be the same on both denominations. In the case eq. (11a) or electrum, c is also the same for small and large coin alike (since this cost precedes coinage). Therefore, eq. (12) follows for gold/silver or electrum alike, and it contains the essential change: b_L exceeds b_H. Consequently, the seignorage rate on producing a high-denomination coin s_H exceeds the one on producing a low-denomination coin s_L

by the positive difference in the brassage cost $b_L - b_H$. Eq. (13), next, is essentially the same as eq. (3) before. Once again, it would not matter if M_o^L served instead of M_o^H : the determination of *m* is the same. Eq. (14) defines the seignorage earned by the government, *S*, on coins. In this case, I assume that only the government produces the low denomination since others (including counterfeiters) would concentrate on producing the higher denomination. Eq. (15), in turn, determines the share of the government in the production of the large-denomination coins. Eq. (16) is the same as the earlier eq. (6).

There are 3 more equations than before and 3 new endogenous variables. Two of the new variables are M_o^L and M_o^H . The third one is either s_L or s_H but in light of eq. (12), not both. With 9 equations in 9 unknowns, under the usual assumptions of continuity there is a solution.

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