# SOIL MECHANICS
# COURSE CODE 3120
# UNIT: 2
# PERMEABILITY

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## Course Objectives

1. To develop an appreciation of soil as a vital construction material, so that it may subsequently be used in the design and construction of foundation for civil engineering structures.

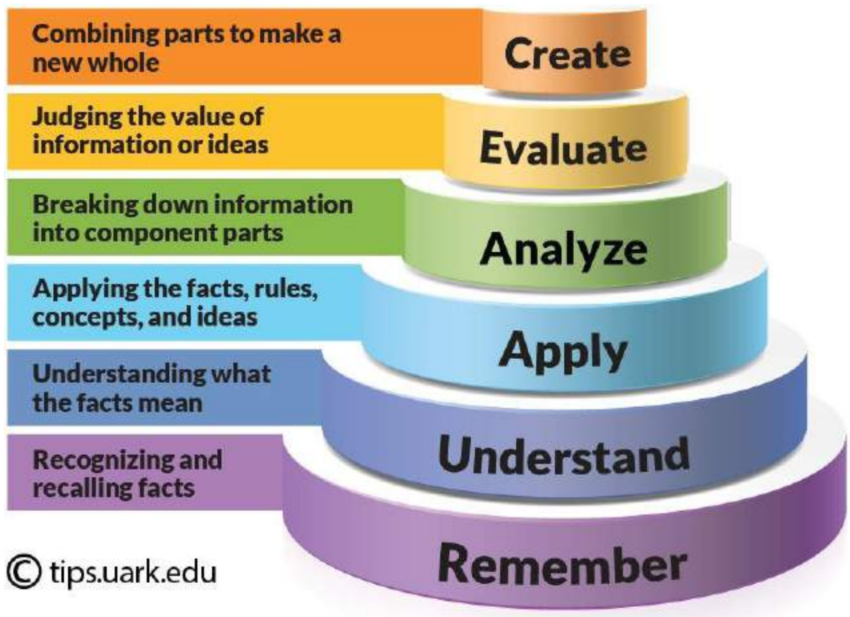
2. To develop an understanding of the relationships between physical characteristics and mechanical properties of soils.

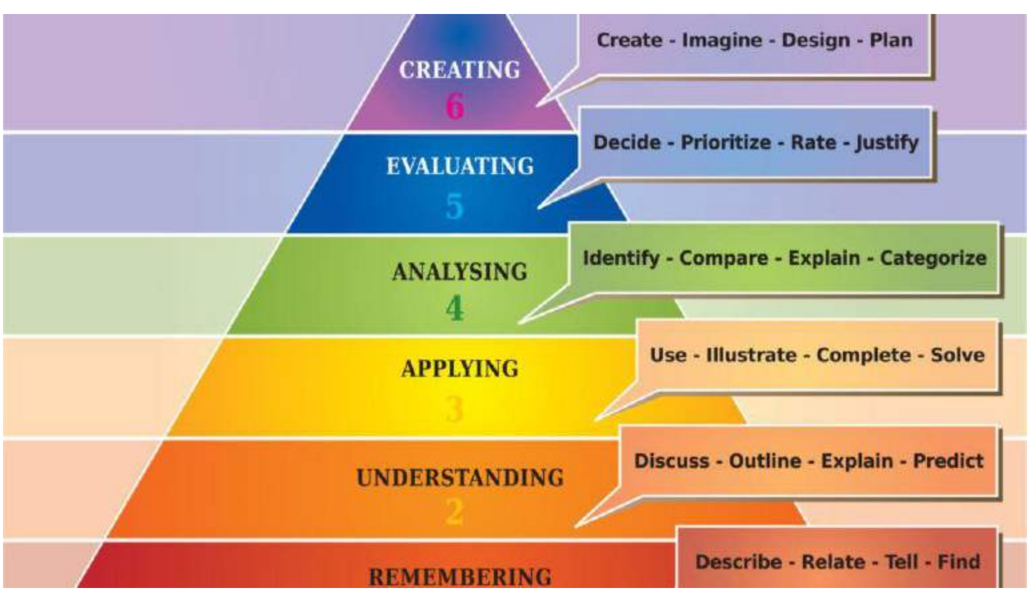
3. To inculcate the basic knowledge of soil such as its identification and classification, determination of various engineering properties and its suitability as a foundation/subgrade material.

4. To understand and experience experimental measurement of the physical and mechanical soil properties commonly used in engineering practice.

5. To develop good technical reporting and data presentation skills.

Learning Levels
Blooms's Taxonomy



# Course Outcomes

Upon successful completion of this course, it is expected that students will be able to:

1. Memorize and understand laws of soil mechanics, soil classification, engineering properties of soils, their significance, and determination methods.
2. Learn to apply laws of Mathematics, Mechanics and Hydraulics to address Geotechnical problems.
3. Develop soft skills and apply the fundamental soil mechanics principles and software to analyze common geotechnical problems.
4. Develop skills to evaluate design options and be able to device optimal (safe, economical, sustainable, and feasible) design solution.
5. Develop the understanding and skills to use various BIS and ISO standards for design practices.

# Syllabus

**Unit: Permeability:**

- Soil water
- Permeability
- Factors affecting permeability
- Darcy’s law
- Laboratory determination of permeability
- Permeability of stratified soils
- Effective and neutral Stresses
- Quick sand conditions and liquefaction of soil

**Unit: Shear Strength of Soil:**

- State of stress at a point, Mohr’s stress circle. Shear strength of soil. Mohr
- and Mohr-Coulomb failure envelop.
- Shear Strength Tests: Direct, Triaxial, Unconfined and Vane shear tests,
- Principles of drained and undrained tests,
- stress path

# Books

1. Gopal Ranjan and A.S.R.Rao, “Basic and Applied Soil Mechanics”, New Age International (P) Ltd, New Delhi.
2. Alam Singh, “Soil Engineering in Theory and Practice”, Asia Publishing House,New Delhi.
3. V. N. S.Murty, “Soil Mechanics and Foundation Engineering”, Sai Kripa Technical Consultants, Banglore.
4. Das B. M., “Principles of Geotechnical Engineering”, Cengage Learning, USA
5. Muni Budhu, Soil Mechanics And Foundations, John Wiley & Sons, Inc. USA
6. M. J. Tomlinson, “Foundation design and construction”
7. Web links to e-learning: nptel

## Geotechnical Engineering— A Historical Perspective

Recorded history tells us that ancient civilizations flourished along the banks of rivers, such as the Nile (Egypt), the Tigris and Euphrates (Mesopotamia), the Huang Ho (Yellow River, China), and the Indus (India).

Dykes dating back to about 2000 B.C. were built in the basin of the Indus to protect the town of Mohenjo Dara (in what became Pakistan after 1947).

During the Chan dynasty in China (1120 B.C. to 249 B.C.) many dykes were built for irrigation purposes.
Ancient Greek civilization used isolated pad footings and strip-and-raft foundations for building structures.

Beginning around 2750 B.C., the five most important pyramids were built in Egypt in a period of less than a century (Saqqarah, Meidum, Dahshur South and North, and Cheops).

This posed formidable challenges regarding foundations, stability of slopes, and construction of underground chambers.

With the arrival of Buddhism in China during the Eastern Han dynasty in 68 A.D., thousands of pagodas were built. Many of these structures were constructed on silt and soft clay layers.

In some cases the foundation pressure exceeded the load-bearing capacity of the soil and thereby caused extensive structural damage.

Pagodas

Leaning Tower Pisa

One of the most famous examples of problems related to soil-bearing capacity in the construction of structures prior to the 18th century is the Leaning Tower of Pisa in Italy

Construction of the tower began in 1173 A.D. when the Republic of Pisa was flourishing and continued in various stages for over 200 years.

The structure weighs about 15,700 metric tons and is supported by a circular base having a diameter of 20 m.

The tower has tilted in the past to the east, north, west and, finally, to the south.
Recent investigations showed that a weak clay layer exists at a depth of about 11 m (36 ft) below the ground surface compression, which caused the tower to tilt.

It became more than 5 m out of plumb with the 54 m height.

The tower was closed in 1990 because it was feared that it would either fall over or collapse. It recently has been stabilized by excavating soil from under the north side of the tower. About 70 metric tons of earth were removed in 41 separate extractions that spanned the width of the tower.

Based on the emphasis and the nature of study in the area of geotechnical engineering, the time span extending from 1700 to 1927 can be divided into four major periods (Skempton, 1985):

1. Pre-classical (1700 to 1776 A.D.)
2. Classical soil mechanics—Phase I (1776 to 1856 A.D.)
3. Classical soil mechanics—Phase II (1856 to 1910 A.D.)
4. Modern soil mechanics (1910 to 1927 A.D.)

**Pre-classical (1700 to 1776 A.D.)**

This period concentrated on studies relating to natural slope and unit weights of various types of soils, as well as the semi-empirical earth pressure theories.

Henri *Gautier* (1660–1737)
Bernard Forest de Belidor (1671–1761)
Francois Gadroy (1705–1759)

**Classical soil mechanics—Phase I (1776 to 1856 A.D.)**

In the pre-classical period, practically all theoretical considerations used in calculating lateral earth pressure on retaining walls were based on an arbitrarily based failure surface in soil.

Charles Augustin Coulomb (1736–1806)
Jacques Frederic Francais (1775–1833)
Claude Louis Marie Henri Navier (1785–1836)
William John Macquorn Rankine (1820–1872)

**Classical soil mechanics—Phase II (1856 to 1910 A.D.)**

Several experimental results from laboratory tests on sand appeared in the literature in this phase.

Philibert Gaspard Darcy (1803–1858)
Joseph Valentin Boussinesq (1842–1929)
Osborne Reynolds (1842–1912)

**Modern soil mechanics (1910 to 1927 A.D.)**

In this period, results of research conducted on clays were published in which the fundamental properties and parameters of clay were established.
Albert Mauritz Atterberg (1846–1916)
Jean Fontard (1884–1962)
Arthur Langley Bell (1874–1956)
Karl Terzaghi (1883–1963)

Publication of *Erdbaumechanik auf Bodenphysikalisher Grundlage* by Karl Terzaghi in 1925 gave birth to a new era in the development of soil mechanics.

Karl Terzaghi is known as the father of modern soil mechanics

## Modern Geotechnical Engineer

**Arthur Casagrande**

**A. W. Skempton**

**A. W. Bishop**

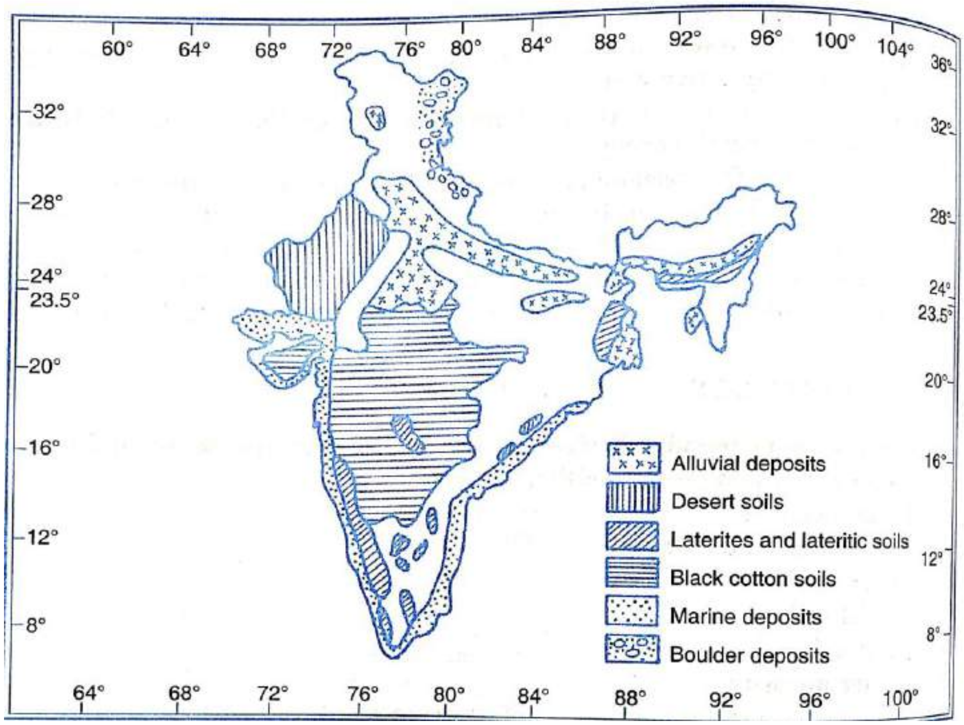
**B. J. Henkel**

**Ralph B. Peck**

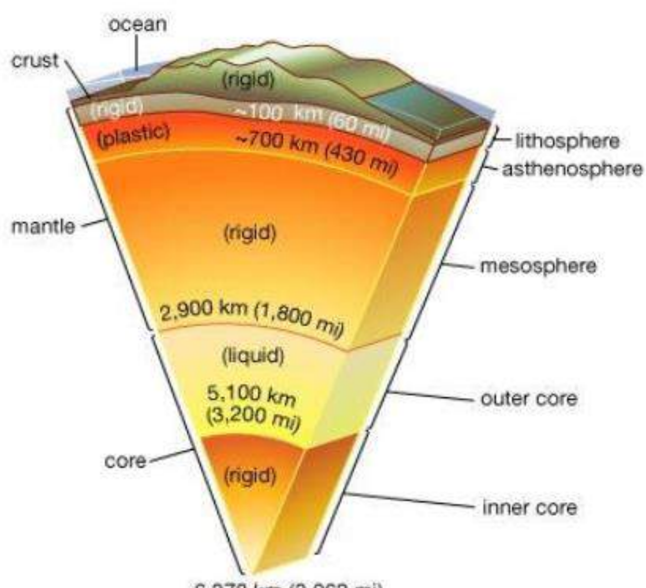
**Braja M. Das**

**Gopal Ranjan**

**V N S Murthy**




# Earth Strata Thickness



## Subsurface Water

• It is the water found beneath the earth surface.

• The main source of this water is rainfall (that percolated down to fill up the void spaces of subsoil mass).

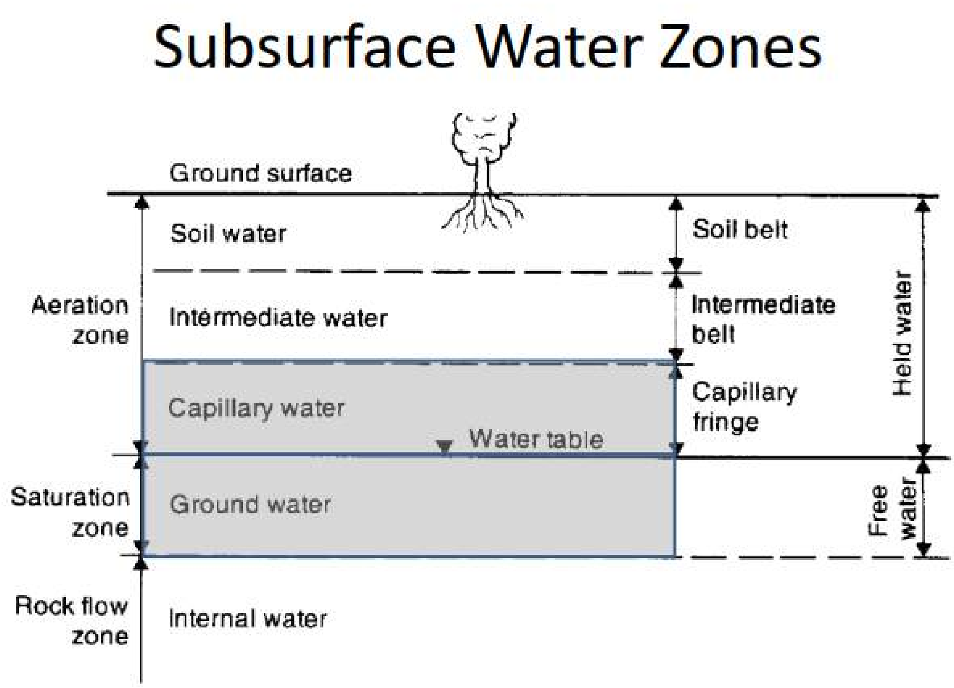
• This water can exist up to an estimated depth of 12000m (12km) below ground surface [Smith, 1998]. Below which due to large pressures and plastic flow of rocks, it cannot exist in free state.

• Subsurface water can be classified into two distinct zones:

– **Saturation Zone**

– **Aeration Zone**

Subsurfac



# Free Water Zone

.This is the zone within which all the pores and fissures are filled up with water under hydrostatic condition. This water is free to move under the influence of gravity. It may be evaporated by oven-drying at 105-110°C

• The upper level of this zone is termed as Water Table (WT), Phreatic Surface or Ground Water Level (GWL). The WT may also be defined as the level of water in the ground.
• The GWL tends to replicate the ground surface topography.

• At GWL the hydrostatic pressure is zero.

• The GWL varies based on factors like rainfall, atmospheric pressure, temperature, etc.

• In coastal regions, it is governed by tidal forces.

• It can also reach up to the ground surface. In which case, springs, lakes, swamps and other similar features are formed

# Held Water Zone

• It is a fine layer of water that sticks electrostatically to the surface of clay particle. The presence of adsorbed water influences the physical and mechanical properties of fine-grained soils (Budhu, 2007).

• The surface of the clay minerals carry negative charge (anions), hence it attracts positively charged (cations) water molecules from the surroundings.

• In addition to the positively charged water molecules, other exchangeable cation like, calcium, sodium, or potassium ALSO get attracted to the surface of the soil particle.

• This results in the formation of a thin layer termed as **diffused double layer**. The cations in the double layer in turn attract more water molecules from the surrounding.

.The water in the diffuse double layer is either directly attracted to the soil particle or through the cations.

• The layer of water that is strongly held or bonded to the mineral surface is termed as adsorbed water.

• In most of the soils, the adsorbed water doesn't evaporate when drying (with the exception of gypsum) in an oven at a temperature of 105 to 110°C.

• This adsorbed-water layer also influences the behaviour of soil

| Soils | Height of capillary rise |
|---|---|
| Coarse sand | 0.03-0.15 m |
| Medium sand | 0.12- 1.10 m |
| Fine sand | 0.30- 3.5 m |
| Silt | 1.5- 12m |
| Clay | > 10 m |

## Questions to guide learning…

- What causes flow of water through soils?
- What law describes the flow of water through soils?
- What is permeability and how is it determined?
- What are the values of permeability or hydraulic conductivity for coarse-grained and fine-grained soils?
- What are the potential applications of hydraulic conductivity

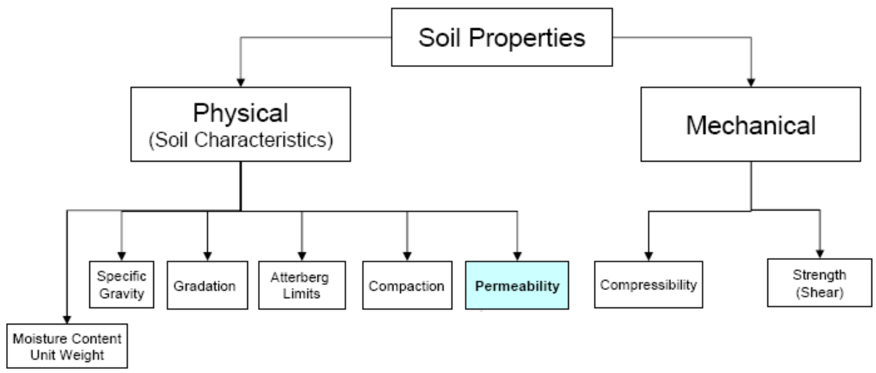
## Soil Permeability- Definition

- It is a property of soil that allows the flow of fluid through its interconnected void space

OR

- It is a measure of how easily a fluid (e.g. water) can pass through the soil
  - Different soil has different permeability's.

## Soil Permeability- Definition

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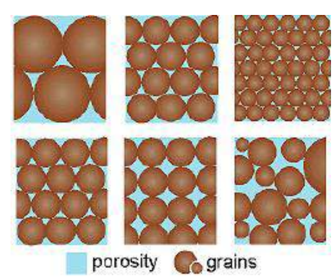
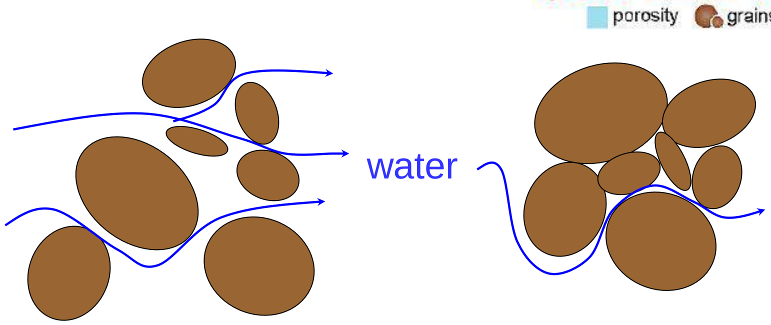
## Soil Permeability -Definition

Soils consists of solid particles with interconnected voids where water can flow from a point of **high energy** to a point of **low energy**

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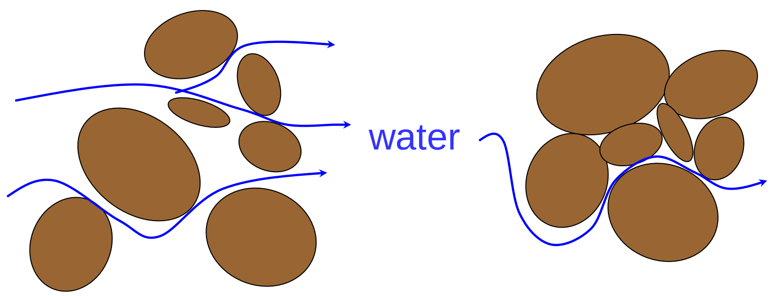
## What is Permeability?

- Permeability is the measure of the soil's ability to permit water to flow through its pores or voids




**Loose soil**

- easy to flow

- **high** permeability

**Dense soil**

- difficult to flow

- **low** permeability

# Application of Permeability

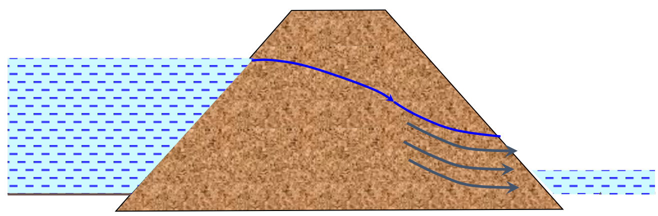
The permeability of soils is important in the problems related to:

- Stability of foundations,
- Seepage loss through embankments of reservoirs,
- Drainage of subgrades,
- Excavation of open cuts in water bearing stratum,
- Rate of flow of water into wells,
- Dewatering foundations pits before and during construction,
- Consolidation of soils, and
- Similar others.

## Importance of Permeability

The following applications illustrate the importance of permeability in geotechnical design:

- The design of earth dams is very much based upon the permeability of the soils used.
- The stability of slopes and retaining structures can be greatly affected by the permeability of the soils involved.

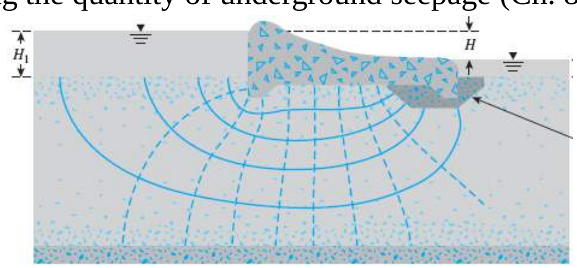


- Filters made of soils are designed based upon their permeability.

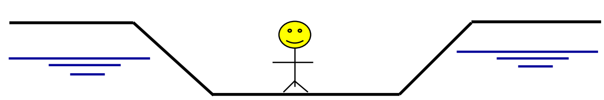


## Importance of Permeability

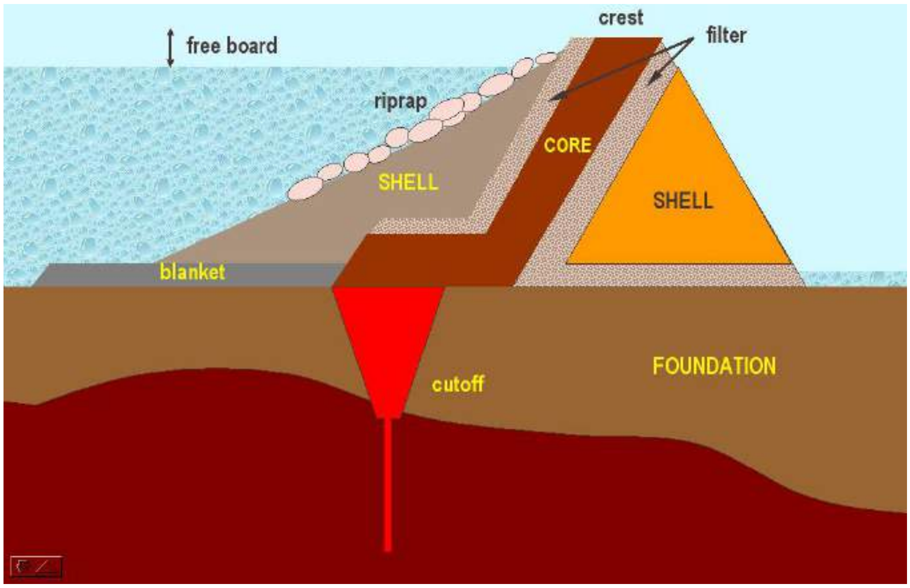
- Estimating the quantity of underground seepage (Ch. 8)




- Solving problems involving pumping seepage water from construction excavation



## Importance of Permeability

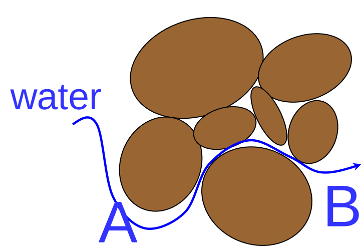





## Why does water flow?

If flow is from A to B, **<u>the energy</u>** is higher at A than at B.

Energy is dissipated in overcoming the soil resistance and hence is the head loss.



## Bernoulli's Equation

The energy of fluid comprise of:

1. Potential energy

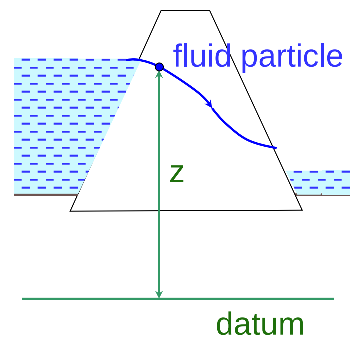
   - due to **elevation** (z) with respect to a datum

2. Pressure energy

   - due to **pressure**

3. Kinetic energy

   - due to **velocity**

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## Bernoulli's Equation

Then at any point in the fluid, the total energy is equal to

Total Energy = Potential energy + Pressure energy + Kinetic energy

$= \rho_w gh + P + \frac{1}{2}\rho_w v^2$

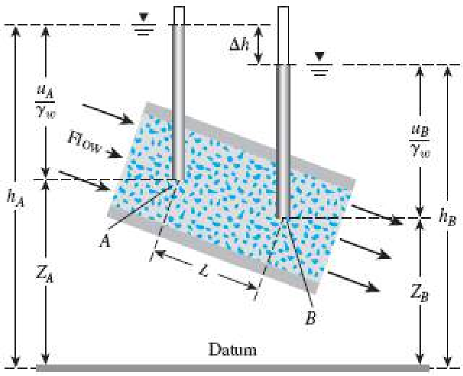
Expressing the total energy as head (units of length)

Total Head = Elevation Head + Pressure Head + Velocity Head

$$h = Z + \frac{u}{\gamma_w} + \cancel{\frac{v^2}{2g}} \rightarrow 0$$

**For flow through soils, velocity (and thus velocity head) is very small. Therefore, $v^2/2g$ = zero.**

# Bernoulli's Equation




At any point

$$h = Z + \frac{u}{\gamma_w}$$

The head loss between A and B

$$\Delta h = h_A - h_B = \left( Z_A + \frac{u_A}{\gamma_w} \right) - \left( Z_B + \frac{u_B}{\gamma_w} \right)$$

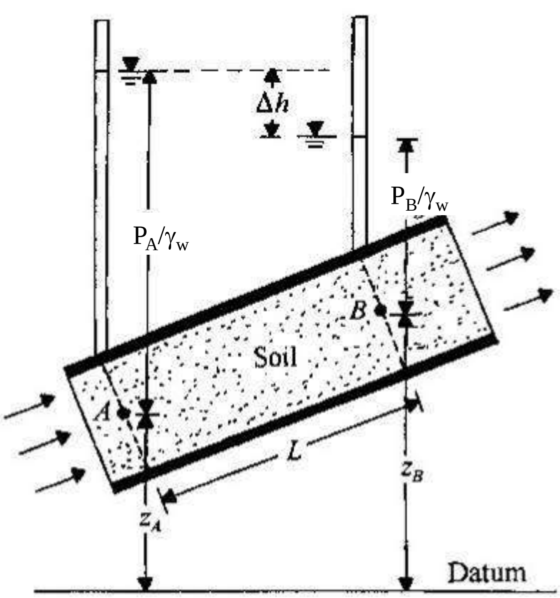
Head loss in **non-dimensional** form

$$i = \frac{\Delta h}{L}$$

Hydraulic gradient

Difference in total head

Distance between points A and B

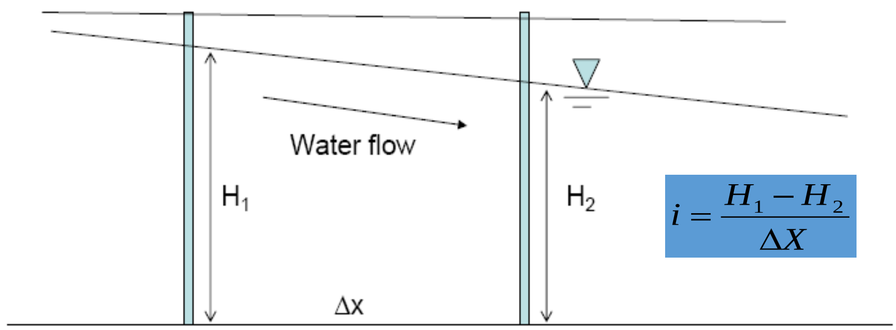



Tricky case!!

Remember always to look at total head

## Hydraulic Gradient

- In the field, the gradient of the head is the head difference over the distance separating the 2 wells.




$$i = \frac{H_1 - H_2}{\Delta X}$$

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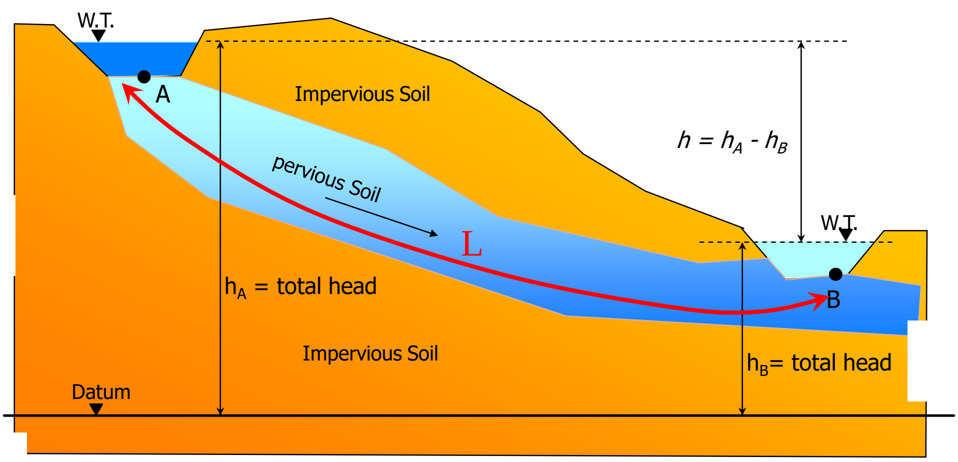
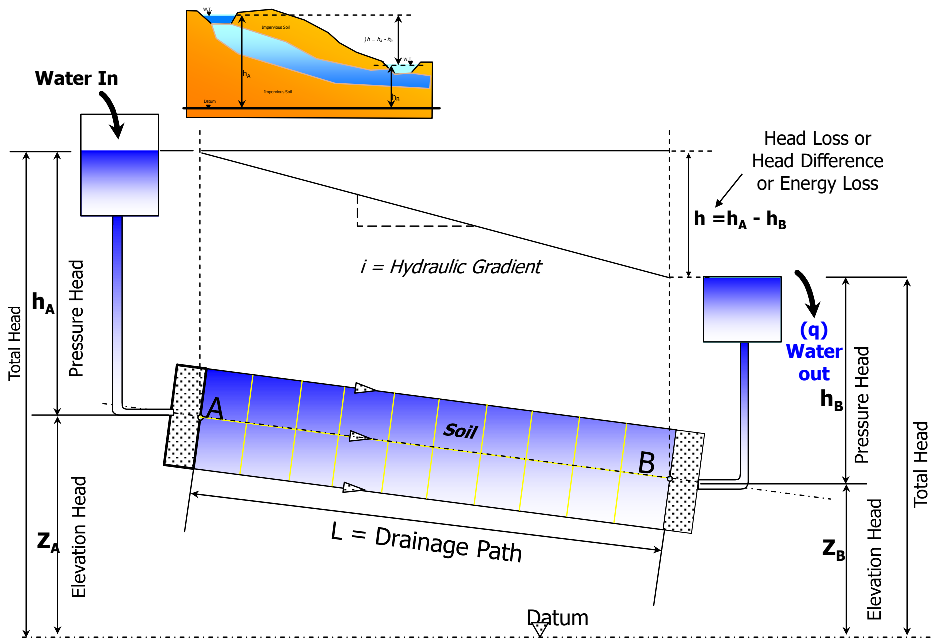
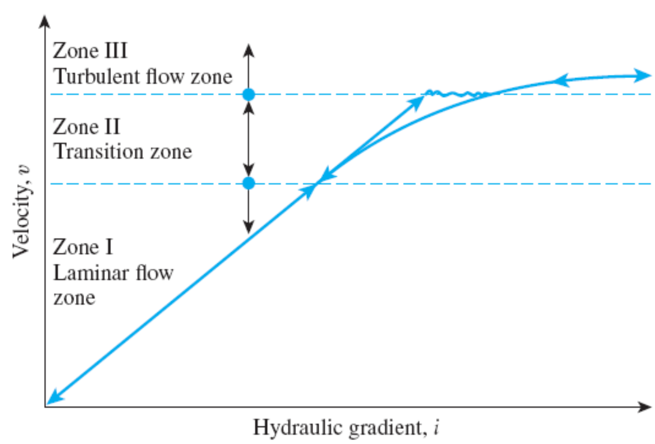
## Hydraulic Gradient




$$i = \frac{(h_A - h_B)}{L} = \frac{\Delta h}{L}$$




## Darcy's Law:




Since velocity in soil is small, flow can be considered laminar

$$v \propto i$$

v = discharge velocity =

i = hydraulic gradient

v = k i  k = coefficient of permeability

## Darcy's Law:

Since velocity in soil is small, flow can be considered laminar

$$v = k.i$$

Where:

v = discharge velocity which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow.

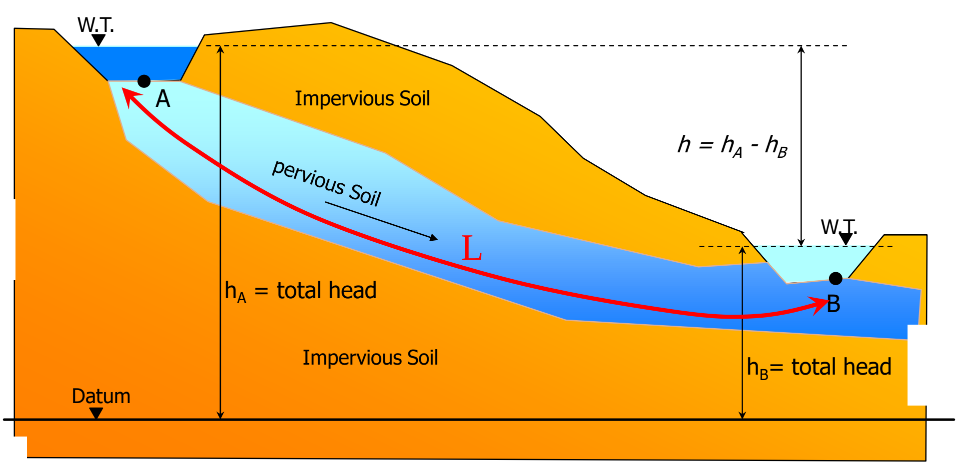
$k$ = hydraulic conductivity (has units of L/T)

$i$ = hydraulic gradient = h/L

Then the quantity of water flowing through the soil per unit time is

Discharge = Q = v. A = k (h/L). A

## Flow in Soil




$$i = \frac{(h_A - h_B)}{L} = \frac{\Delta h}{L} \Rightarrow q = k\ i\ A = k\ \frac{\Delta h}{L}\ A$$

To determine the quantity of flow, two parameters are needed

* $k$ = hydraulic conductivity **(how permeable the soil medium)**
* $i$ = hydraulic gradient **(how large is the driving head)**

**k can be determined using**

1- Laboratory Testing → [constant head test & falling head test]
2- Field Testing → [pumping from wells]
3- Empirical Equations

**i can be determined**

1- from the head loss and geometry
2- flow net

## Hydraulic Conductivity

- The hydraulic conductivity k is a measure of how easy the water can flow through the soil.
- The hydraulic conductivity is expressed in the units of velocity (such as cm/sec and m/sec).

**Table 7.1** Typical Values of Hydraulic Conductivity of Saturated Soils

| Soil type | $k$ (cm/sec) | $k$ (ft/min) |
|---|---|---|
| Clean gravel | 100−1.0 | 200−2.0 |
| Coarse sand | 1.0−0.01 | 2.0−0.02 |
| Fine sand | 0.01−0.001 | 0.02−0.002 |
| Silty clay | 0.001−0.00001 | 0.002−0.00002 |
| Clay | <0.000001 | <0.000002 |

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# Hydraulic Conductivity

- Hydraulic conductivity of soils depends on several factors:
  - Fluid viscosity ($\eta$): as the viscosity increases, the hydraulic conductivity decreases
  - Pore size distribution
  - Temperature
  - Grain size distribution
  - Degree of soil saturation

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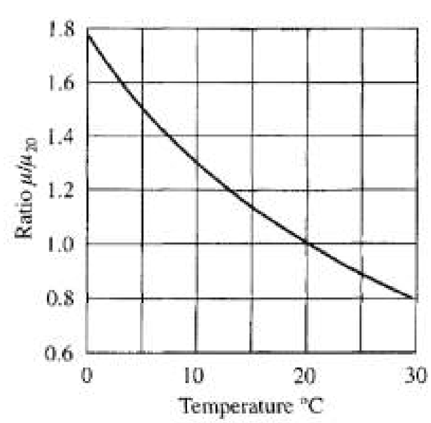
It is inversely proportional to the viscosity of water which decreases with increasing temperature as shown in the Figure.
Therefore, permeability measurements at laboratory temperatures should be corrected with the aid of Figure before application to field temperature conditions by means of the equation

$$k_F = k_T . \frac{\mu_T}{\mu_F}$$

Where, $k_F$ and $k_T$ are the permeability values corresponding to the field and test temperatures respectively and $\mu_F$ and $\mu_T$ are the corresponding viscosity values.
It is customary to report the values of $k_T$ at a standard temperature of 20°C. The equation is:

$$k_{20°C} = k_{T°C} \frac{\mu_{T°C}}{\mu_{20°C}}$$



# Hydraulic Conductivity, *k*

- Typical Values

<table>
<tr><th></th><th colspan="12">Coefficient of Permeability $k$ in cm per sec (log scale)</th></tr>
<tr><th></th><th>$10^2$</th><th>$10^1$</th><th>1.0</th><th>$10^{-1}$</th><th>$10^{-2}$</th><th>$10^{-3}$</th><th>$10^{-4}$</th><th>$10^{-5}$</th><th>$10^{-6}$</th><th>$10^{-7}$</th><th>$10^{-8}$</th><th>$10^{-9}$</th></tr>
<tr><td>Drainage</td><td colspan="6">Good</td><td colspan="2">Poor</td><td colspan="4">Practically Impervious</td></tr>
<tr><td rowspan="2">Soil types</td><td colspan="2" rowspan="2">Clean gravel</td><td colspan="3">Clean sands, clean sand and gravel mixtures</td><td colspan="4">Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.</td><td colspan="3" rowspan="2">"Impervious" soils, e.g., homogeneous clays below zone of weathering</td></tr>
<tr><td></td><td colspan="6">"Impervious" soils modified by effects of vegetation and weathering</td></tr>
<tr><td rowspan="2">Direct determination of $k$</td><td colspan="6">Direct testing of soil in its original position—pumping tests. Reliable if properly conducted. Considerable experience required</td><td colspan="6"></td></tr>
<tr><td colspan="5">Constant-head permeameter. Little experience required</td><td colspan="7"></td></tr>
<tr><td rowspan="2">Indirect determination of $k$</td><td colspan="2"></td><td colspan="3">Falling-head permeameter. Reliable. Little experience required</td><td colspan="3">Falling-head permeameter. Unreliable. Much experience required</td><td colspan="4">Falling-head permeameter. Fairly reliable. Considerable experience necessary</td></tr>
<tr><td colspan="5">Computation from grain-size distribution. Applicable only to clean cohesionless sands and gravels</td><td colspan="4"></td><td colspan="3">Computation based on results of consolidation tests. Reliable. Considerable experience required</td></tr>
</table>

After A. Casagrande and R. E. Fadum

**FIG. 2.9** Permeability and drainage characteristics of soils. (Table 6, p. 48, of *Soil Mechanics in Engineering Practice*, K. Terzaghi and R. B. Peck, Wiley, New York, 1948.)

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# Factors Affecting Permeability

**Soil type**: Coarse-grained soils have higher hydraulic conductivities than fine-grained soils. The water in the double layer in fine-grained soils significantly reduces the seepage pore space.

**Particle size**: Hydraulic conductivity depends on the square of effective size $D_{10}$ for coarse-grained soils.

**Pore fluid properties**, Governed by the viscosity of permeant. Higher the viscosity permeants decrease permeability: $k_1 : k_2 \approx \mu_1 : \mu_2$, where $\mu$ is the viscosity and the subscripts 1 and 2 denote two types of pore fluids in a given soil.

**Void ratio**: : $k_1 : k_2 \approx e_1^2 : e_1^2$ where subscripts 1 and 2 denote two types of soil fabric for coarse grained soils. This ratio is useful in comparing the hydraulic conductivities of similar soils with different void ratios.

# Factors Affecting Permeability

**Pore size**: The greater the pore size, the higher the hydraulic conductivity.

**Homogeneity, layering, and fissuring**: Water tends to seep quickly through loose layers, through fissures and along the interface of layered soils. Catastrophic failures can occur from such seepage.

**Entrapped gases**: Entrapped gases tend to reduce the hydraulic conductivity. It is often very difficult to get gas-free soils. Even soils that are under groundwater level and are assumed to be saturated may still have some entrapped gases.

## Laboratory Testing of Hydraulic Conductivity

Two standard laboratory tests are used to determine the hydraulic conductivity of soil

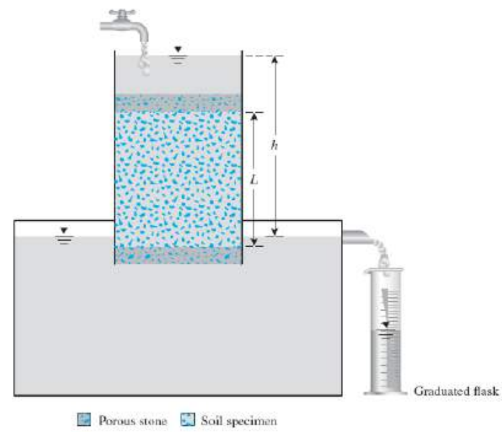
- The constant-head test
- The falling-head test.

## Constant Head Test

- The constant head test is used primarily for coarse-grained soils.
- It can be applied to fine-grained soils.
- But, in highly impervious soils the velocity of flow is small- time consuming
- This test is based on the assumption of laminar flow (Darcy's Law apply)

The hydraulic head causing flow is maintained constant (i.e. steady state of flow)

• The quantity of water (Q) flowing through a soil specimen of known cross-sectional area (A) and Length in a given time (t) is measured. Rate of flow (q) is calculated using Darcy's Law, as:

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## Constant Head Test

**From Darcy's Law**

$$Q = k \cdot i \cdot A \cdot t = k \cdot \frac{h}{L} \cdot A \cdot t$$

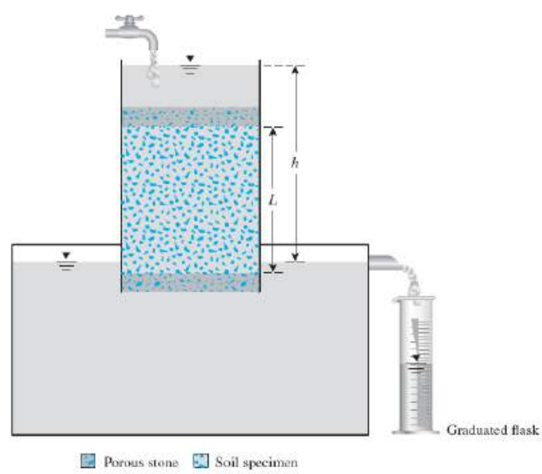
Where:
Q = volume of water collection
A = cross section area of soil specimen
T = duration of water collection

Rearranging the above equation

$$k = \frac{Q \cdot L}{h \cdot A \cdot t}$$

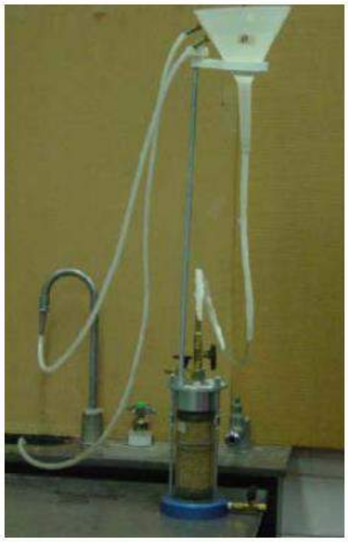
Then compute:For Temperature Correction:

$$k_{20^0 C} = k_{T^0 C} \frac{\eta_{T^0 C}}{\eta_{20^0 C}}$$



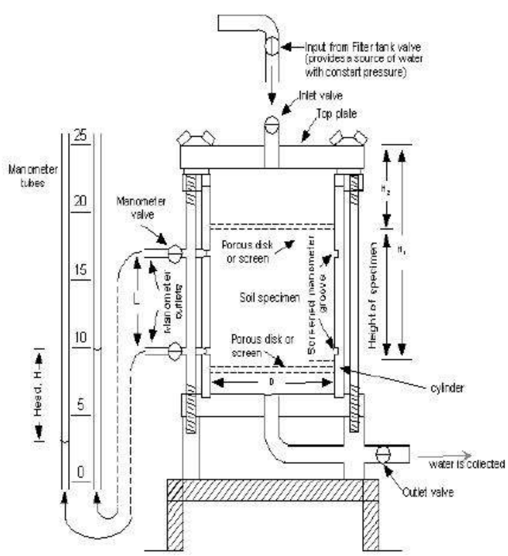

54

# Constant Head Test



55

# Constant Head Test




Constant-Head Permeameter

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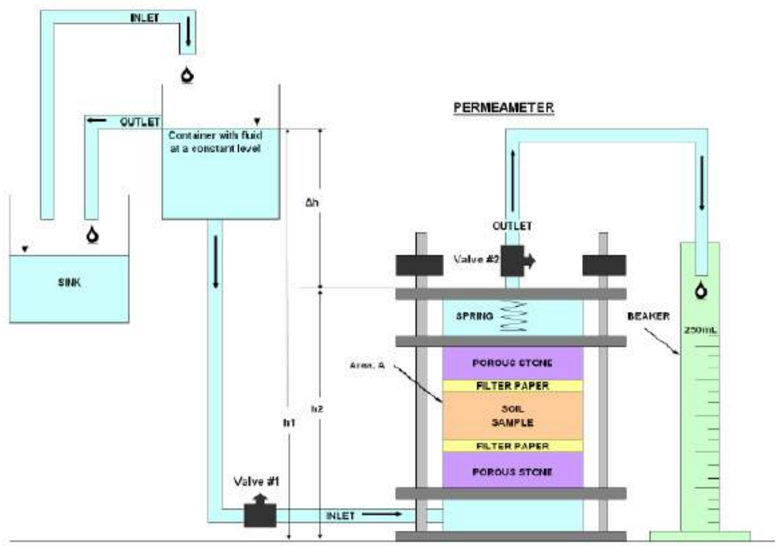
## Constant Head Test



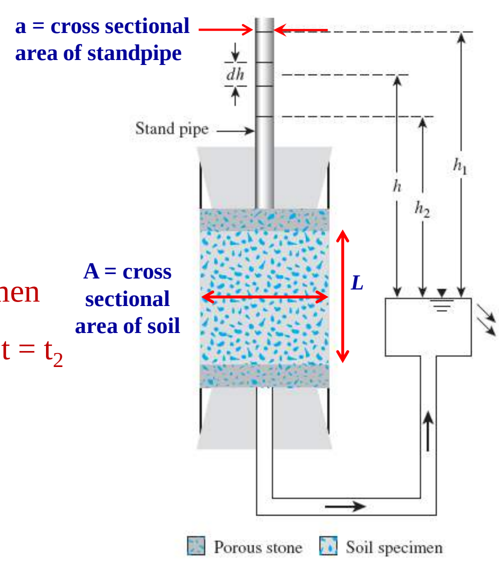

57

## Falling Head Test

- The falling head test is mainly for fine-grained soils.

Simplified Procedure:

- Record initial head difference, $h_1$ at $t_1 = 0$
- Allow water to flow through the soil specimen
- Record the final head difference, $h_2$ at time $t = t_2$

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# Falling Head Test

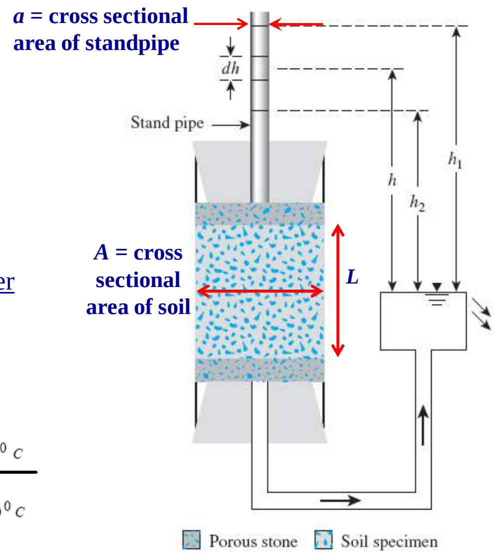
## Calculations:

$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

Where:
$A$ = cross sectional area of the soil
$a$ = cross sectional area of the standpipe
$h_1$ = distance to bottom of the beaker before the test
$h_2$= distance to bottom of the beaker after the test
$L$ = length of the sample
$t$ = $t_2$-$t_1$

Then compute:

$$k_{20^0 C} = k_{T^0 C}\frac{\eta_{T^0 C}}{\eta_{20^0 C}}$$





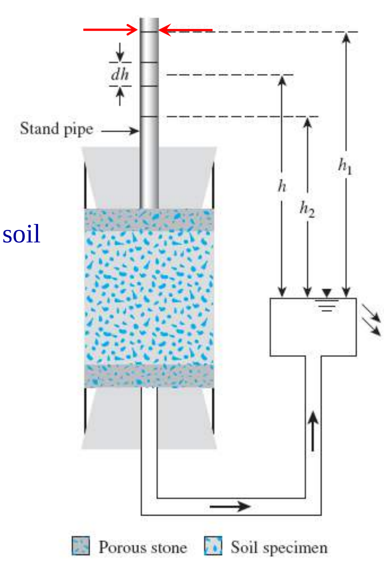

# Falling Head Test

## Calculations:

$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

The above equation is derived assuming:

The flow through the standpipe = flow through the soil



# Falling Head Test

## Derivation of Falling Head equation

The rate of flow of the water through the specimen at any time $t$ can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt} \tag{7.20}$$

where $q$ = flow rate
$a$ = cross-sectional area of the standpipe
$A$ = cross-sectional area of the soil specimen

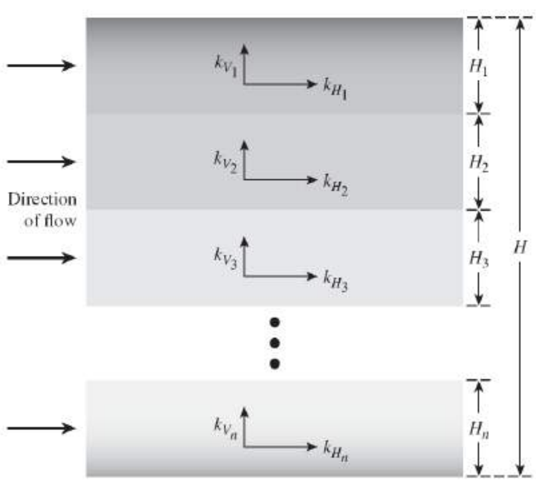
Rearrangement of Eq. (7.20) gives

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right) \tag{7.21}$$

Integration of the left side of Eq. (7.21) with limits of time from 0 to $t$ and the right side with limits of head difference from $h_1$ to $h_2$ gives

$$t = \frac{aL}{Ak}\log_e\frac{h_1}{h_2}$$

# Equivalent Hydraulic Conductivity on Stratified Soils




- Horizontal flow
- Constant hydraulic gradient conditions
- Analogous to resistors in series

$$k_{H(\text{eq})} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n)$$

For a flow that is parallel to soil stratification, such as the one shown in Figure 2a, the head loss $h_L$ over the same flow path length $L$ will be the same for each layer. Thus, $i = i_1 = i_2 = \cdots = i_n$. The flow rate through a layered system (with width = 1 unit) is:

$$q = k_1\, i_1\, H_1 \times 1 + k_2\, i_2\, H_2 \times 1 + k_3\, i_3\, H_3 \times 1 + \cdots + k_n\, i_n\, H_n \times 1$$

$$q = (k_1\, H_1 + k_2\, H_2 + k_3\, H_3 + \cdots + k_n\, H_n)i$$

For the equivalent system as shown in Figure 2b:

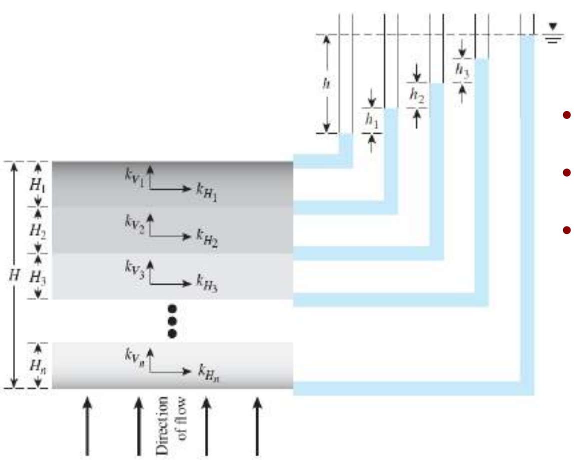
$$q = k_{eq}\, i\, H$$

Equating 1 and 2, we obtain the equivalent, or average coefficient of permeability $k_{eq}$

$$k_{eq}\, i\, H = (k_1\, H_1 + k_2\, H_2 + k_3\, H_3 + \cdots + k_n\, H_n)i$$

$$\boldsymbol{k_{eq} = \frac{k_1\, H_1 + k_2\, H_2 + k_3\, H_3 + \cdots + k_n\, H_n}{H}}$$

## Equivalent Hydraulic Conductivity on Stratified Soils




- Vertical flow
- Constant velocity
- Analogous to resistors in parallel

$$k_{V(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)}$$

# Permeability of Stratified Soil Deposits

### Flow Perpendicular to the bedding plane

Consider a stratified soil having horizontal layers of thickness $H_1, H_2, H_3, \dots, H_n$ with coefficients of permeability $k_1, k_2, k_3, \dots, k_n$ as shown in Figure 1a. For flow perpendicular to soil stratification, the flow rate $q$ through area $A$ of each layer is the same.

From Darcy's Law

$$q = k\, i\, A$$

From Figure 1b:

$$q = \frac{k_{eq} h_L\, A}{H} \quad \text{or} \quad h_L = \frac{H\, q}{k_{eq} A}$$

Therefore, the head loss across the $n$ layers is given as:

$$h_L = \frac{H_1 q}{k_1 A} + \frac{H_2 q}{k_2 A} + \frac{H_3 q}{k_3 A} + \dots + \frac{H_n q}{k_n A}$$

$$h_L = \left(\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \dots + \frac{H_n}{k_n}\right)\frac{q}{A} \qquad \text{....... Equation 1}$$

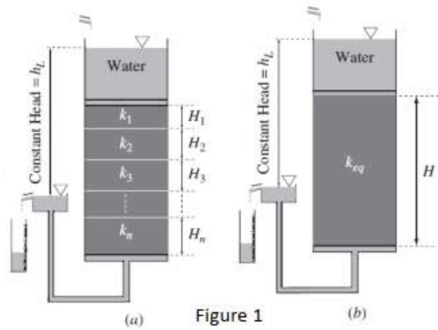



Figure 1

Figure 1b shows an "equivalent" soil layer that can replace the stratified system shown in Figure 1a.

The thickness of the equivalent soil layer is $H$ ($= H_1 + H_2 + H_3 + \cdot \cdot \cdot + H_n$) and its permeability is $k_{eq}$. For this equivalent system, using Darcy's law we can write

$$h_L = \frac{qH}{Ak_{eq}} \qquad \text{.......... Equation 2}$$

Comparing Equation 1 and Equation 2

$$\frac{qH}{Ak_{eq}} = \left(\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \dots + \frac{H_n}{k_n}\right)\frac{q}{A}$$

$$k_{eq} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \dots + \frac{H_n}{k_n}}$$

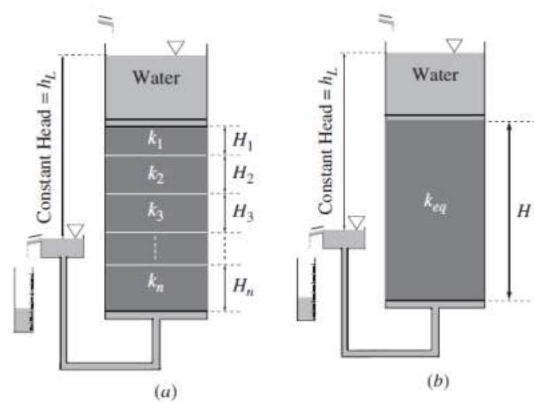



Figure 1

# Limitations of Laboratory tests for Hydraulic Conductivity

i. It is generally hard to duplicate in-situ soil conditions (such as stratification).

ii. The structure of in-situ soils may be disturbed because of sampling and test preparation.

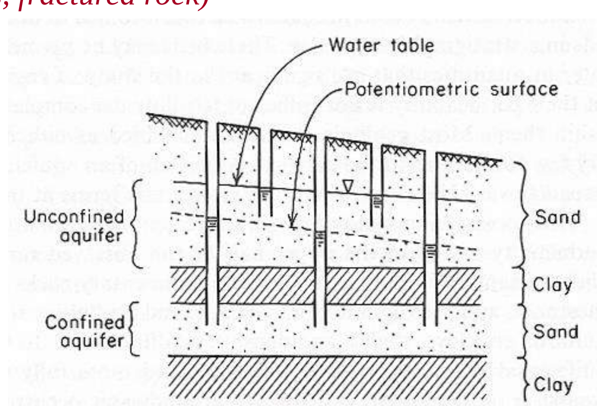
iii. Small size of laboratory samples lead to effects of boundary conditions.

# Pumping Well with Observation holes

Definitions

- Aquifer: Soil or rock forming stratum that is saturated and permeable enough to yield significant quantities of water

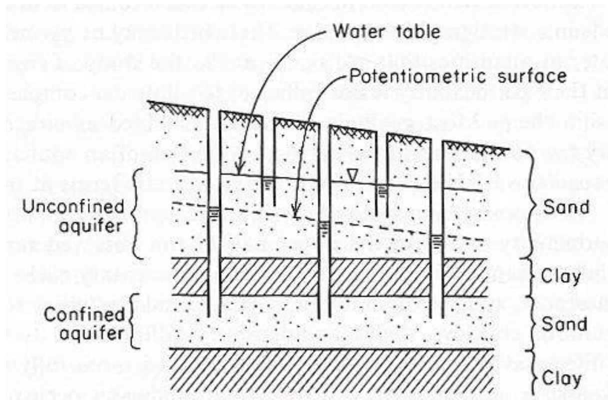
  *(e.g. sands, gravels, fractured rock)*



# Pumping Well with Observation holes

## Definitions (cont.)

- Unconfined Aquifer (water table aquifer) is an aquifer in which the water table forms the upper boundary.
- Confined Aquifer is an aquifer confined between two impervious layers (e.g. clay).




# Permeability Tests in the field using Pumping Wells

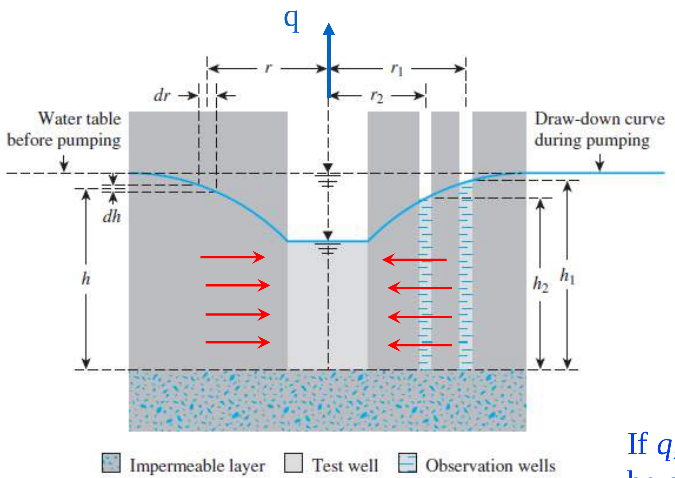
- Used to determine the hydraulic conductivity of soil in the field.
- During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached.
- The steady state is established when the water level in the test and observation wells becomes constant.
- The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as

$$q = k\left(\frac{dh}{dr}\right)2\pi rh$$

$$\int_{r_2}^{r_1}\frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

# Pumping Well with Observation holes

## Pumping Well in an Unconfined Aquifer




$$k = \frac{2.303 \cdot q \,.\log_{10} \dfrac{r_2}{r_1}}{\pi(h_2^2 - h_1^2)}$$

OR

$$k = \frac{q\,.\ln \dfrac{r_2}{r_1}}{\pi(h_2^2 - h_1^2)}$$

If $q$, $h_1$, $h_2$, $r_1$, $r_2$ are known , $k$ can be calculated

# Pumping Well with Observation holes

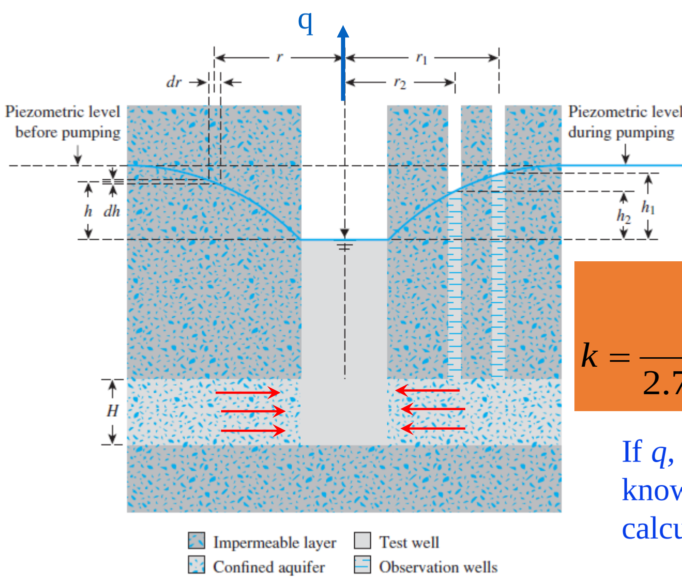
## Pumping Well in an Unconfined Aquifer

$$k(cm/\sec) = \frac{2.303 \cdot q \,.\log_{10} \dfrac{r_2}{r_1}}{14.7\pi(h_2^2 - h_1^2)}$$

Where q is in gpm and $h_1$ and $h_2$ are in ft

Pumping Well with Observation holes

Pumping Well in a Confined Aquifer

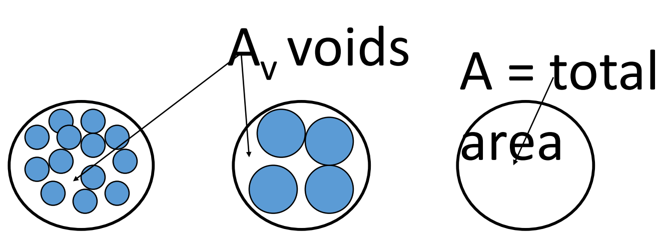



$$k = \frac{q}{2.727 \cdot H} \frac{\log_{10} \frac{r_2}{r_1}}{(h_2 - h_1)}$$

If $q$, $h_1$, $h_2$, $r_1$, $r_2$ are known , $k$ can be calculated

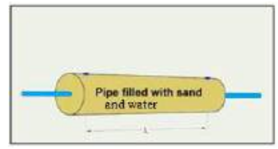
# Darcy & Seepage Velocity

- Darcy velocity $V_D$ is a fictitious velocity since it assumes that flow occurs across the entire cross-section of the sediment sample. Flow actually takes place only through interconnected pore channels (voids), at the seepage velocity $V_S$.



# Darcy & Seepage Velocities

- From the Continuity Eqn. Q = constant
- "Pipe running full" means "Inputs = Outputs"
- $Q = A\ V_D = A_V\ V_s$
  - Where:



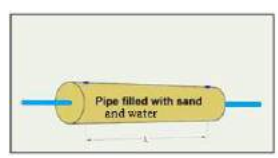

$Q$ = flow rate
$A$ = total cross-sectional area material
$A_V$ = area of voids
$V_s$ = seepage velocity
$V_D$ = Darcy velocity

Since A > $A_V$ *, and Q = constant,* $V_S > V_D$

***Pinch hose, reduce area, water goes faster***


# Darcy & Seepage Velocities

- From the Continuity Eqn. Q = constant
- "Pipe running full" means "Inputs = Outputs"
- $Q = A\ V_D = A_V\ V_s$
  - Where:




$Q$ = flow rate
$A$ = total cross-sectional area material
$A_V$ = area of voids
$V_s$ = seepage velocity
$V_D$ = Darcy velocity

Since A > $A_V$ *, and Q = constant,* $V_S > V_D$

***Pinch hose, reduce area, water goes faster***

## Darcy & Seepage Velocity: Porosity

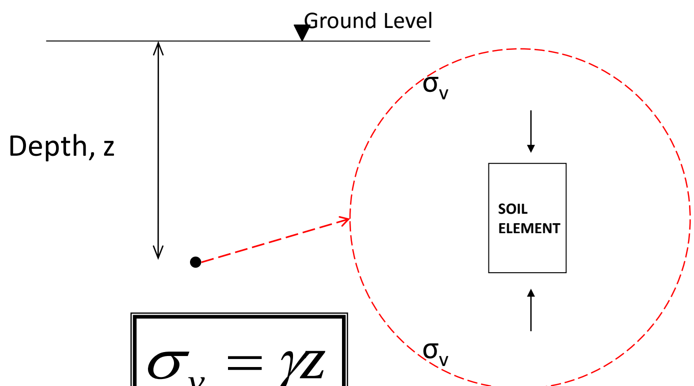
- $Q = A\ V_D = A_V\ V_S$ , therefore $V_S = V_D\ (A/A_V)$
- Multiplying both sides by the length of the medium ($L$) divided by itself, L / L = 1
- $V_S = V_D\ (AL / A_V L) = V_D\ (Vol_T / Vol_V)$ *we get volumes*
- Where:
  $Vol_T$ = total volume
  $Vol_V$ = void volume
- By definition, ***$Vol_V / Vol_T = n$*, the sediment porosity**

So the actual velocity: $\boxed{V_S = V_D / n}$

## Soil Stress and Pore Water Pressure

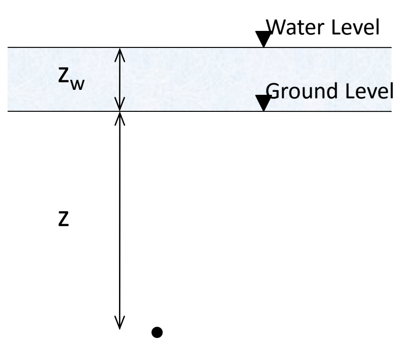
- The **total vertical stress ($\sigma_v$)** acting at a point below the ground surface is due to the weight of everything lying above i.e. soil, water, and surface loading
- Total vertical stresses are calculated from the unit weight of the soil
- Any change in total vertical stress ($\sigma_v$) may also result in a change in the **horizontal** total stress ($\sigma_h$) at the same point
- The relationships between vertical and horizontal stress are complex ($\Delta\sigma_v \neq \Delta\sigma_h$)

## TOTAL VERTICAL STRESS in homogeneous soil



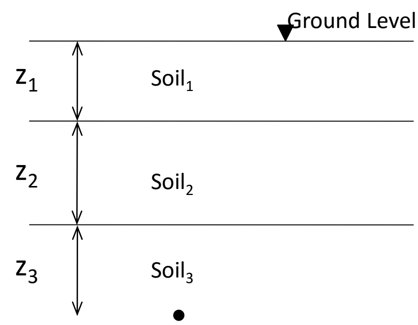

$$\boxed{\sigma_v = \gamma z}$$

## TOTAL VERTICAL STRESS below a river or lake



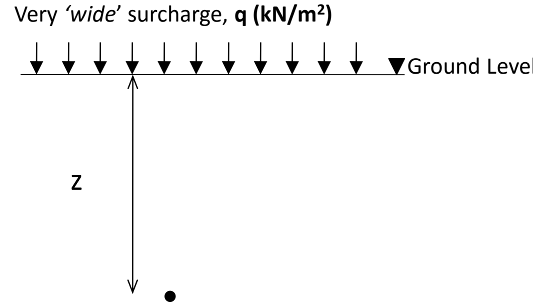

$$\boxed{\sigma_v = \gamma z + \gamma_w z_w}$$

# TOTAL VERTICAL STRESS in multi-layered soil




$$\boxed{\sigma_v = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3}$$

# TOTAL VERTICAL STRESS with a surface surcharge load

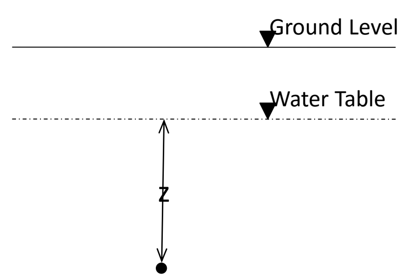



$$\boxed{\sigma_v = \gamma z + q}$$

# PORE WATER PRESSURE

- The water in the pores of a soil is called **pore water**.
- The pressure within this pore water is called **pore water pressure (u)**
- It is also called **neutral stress** because it acts on all sides of the particles, but doesn't cause the soil particle to press against adjacent particles.
- It has no shear component.
- The magnitude of pore water pressure depends on:
    a) the depth below the water table
    b) the conditions of seepage flow

# PORE WATER PRESSURE under hydrostatic conditions (no water flow)




$$\boxed{u = \gamma_w Z}$$

# EFFECTIVE STRESS CONCEPT
(Terzaghi, 1923)

$$\sigma = \sigma' + u$$

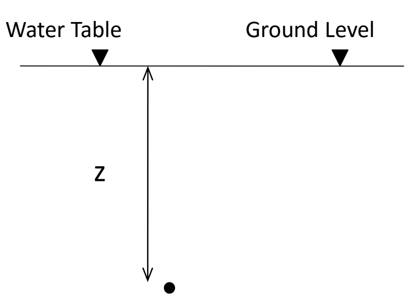
where

$\sigma$ = Total Vertical Stress

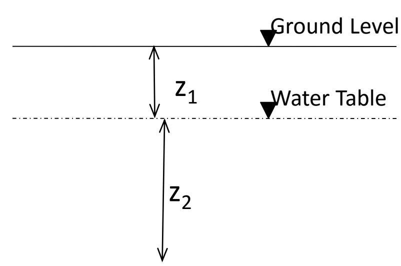
$\sigma'$ = Effective Stress

$u$ = Pore Water Pressure

# VERTICAL EFFECTIVE STRESSES



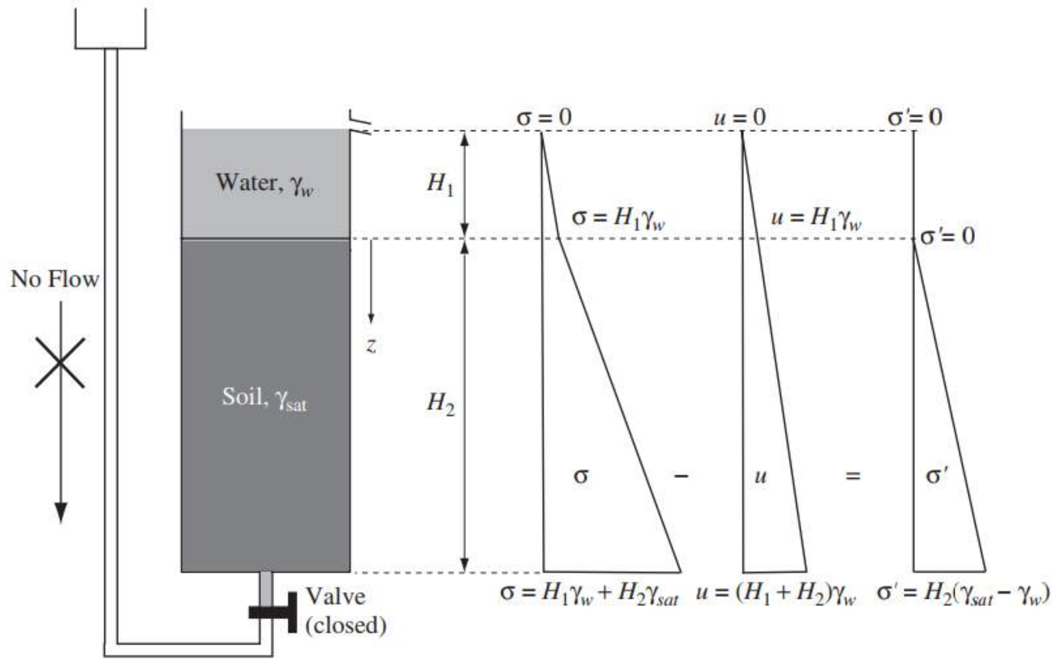

$$\sigma' = \sigma - u$$




$$\sigma' = \sigma - u$$

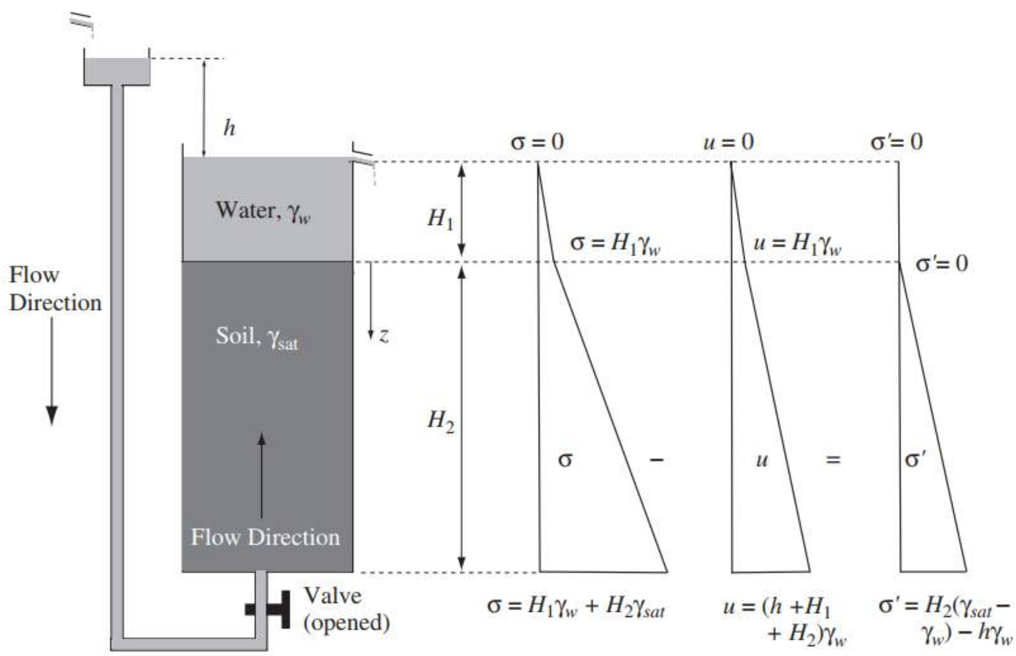
$$\sigma' = \gamma_d z_1 + \gamma_{sat} z_2 - \gamma_w z_2$$

$$\sigma' = \gamma_d z_1 + \gamma_{sub} z_2$$



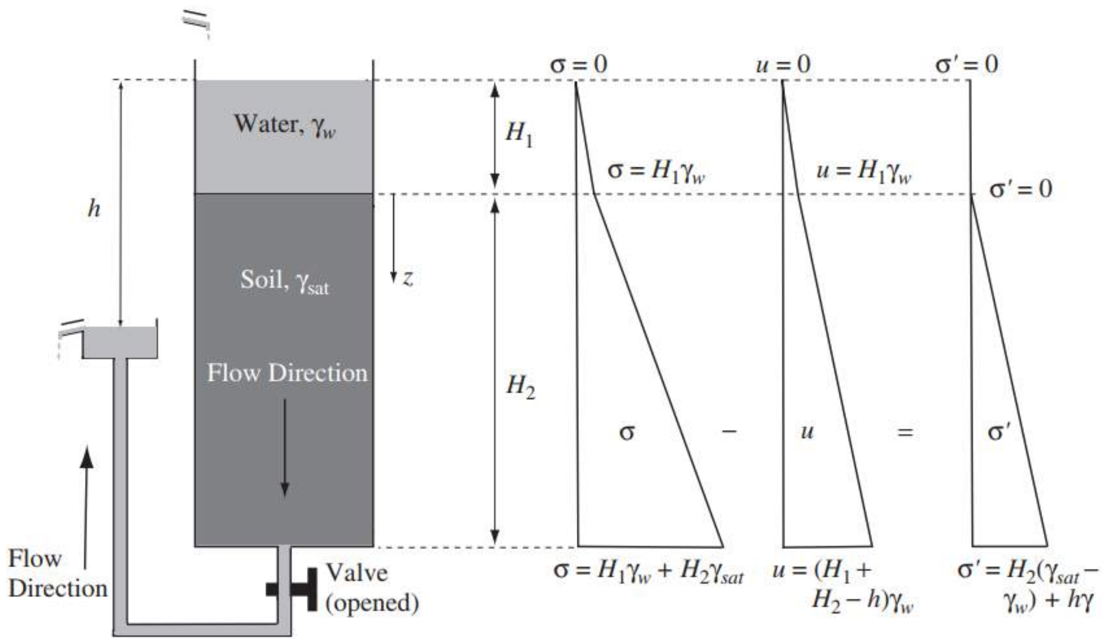
σ = 0
u = 0
σ′ = 0
Water, $\gamma_w$
$H_1$
σ = $H_1\gamma_w$
u = $H_1\gamma_w$
σ′ = 0
No Flow
z
Soil, $\gamma_{sat}$
$H_2$
σ − u = σ′
Valve (closed)
σ = $H_1\gamma_w + H_2\gamma_{sat}$
u = $(H_1 + H_2)\gamma_w$
σ′ = $H_2(\gamma_{sat} - \gamma_w)$


(a)



h
σ = 0
u = 0
σ′ = 0
Water, $\gamma_w$
$H_1$
σ = $H_1\gamma_w$
u = $H_1\gamma_w$
σ′ = 0
Flow Direction
Soil, $\gamma_{sat}$
z
$H_2$
σ − u = σ′
Flow Direction
Valve (opened)
σ = $H_1\gamma_w + H_2\gamma_{sat}$
u = $(h + H_1 + H_2)\gamma_w$
σ′ = $H_2(\gamma_{sat} - \gamma_w) - h\gamma_w$


(b)




(c)

In reference to Figure above, the soil specimen has the same total vertical stress in the three conditions. The total vertical stresses, pore water pressures, and effective stresses at depth *z* are:

No-flow condition:

**Where, $\gamma' = \gamma_{sub}$**

$$\sigma_v = \gamma_w H_1 + \gamma_{sat} z, \qquad u = \gamma_w (H_1 + z), \qquad \sigma'_v = \gamma' z$$

Upward-flow condition:

$$\sigma_v = \gamma_w H_1 + \gamma_{sat} z, \qquad u = \gamma_w (H_1 + z) + iz\gamma_w, \qquad \sigma'_v = \gamma' z - iz\gamma_w$$

Downward-flow condition:

$$\sigma_v = \gamma_w H_1 + \gamma_{sat} z, \qquad u = \gamma_w (H_1 + z) - iz\gamma_w, \qquad \sigma'_v = \gamma' z + iz\gamma_w$$

In the upward-flow condition the pore water pressure increases and the effective stress decreases with depth. On the other hand, when the flow is downward, the pore water pressure decreases and the effective stress increases.

In reference to upward-flow condition, if the hydraulic gradient induced by the head difference is large enough, the upward seepage force will cause the effective stress within the soil to become zero, thus causing a sudden loss of soil strength in accordance with the effective stress principle.

$$\sigma' = (\gamma_{sat} - \gamma_w)H_2 - \gamma_w h$$

$$\sigma' = \gamma_{sub}H_2 - \gamma_w iH_2$$

If $\sigma' = 0$, $i = i_{cr}$

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

**Where, $\gamma' = \gamma_{sub}$**

It can be shown (from the phase diagram) that

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

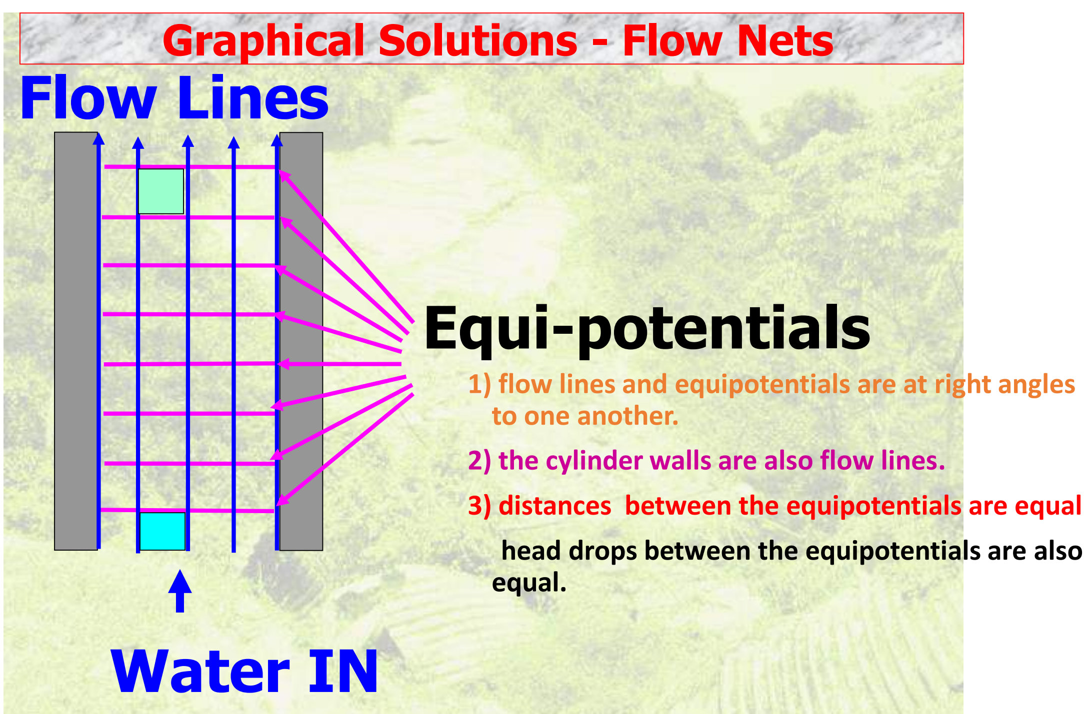
The critical gradient of natural granular soil deposits can be calculated if the void ratios of the deposits are known. For all practical purposes the specific gravity of granular materials can be assumed as equal to 2.65

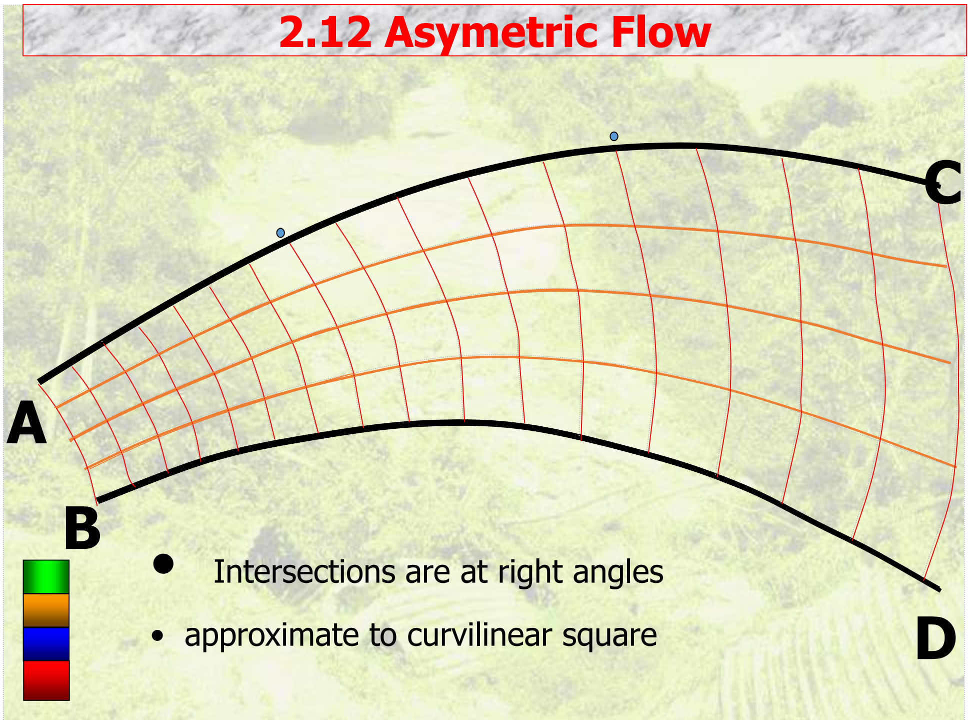
Critical hydraulic gradients of granular soils

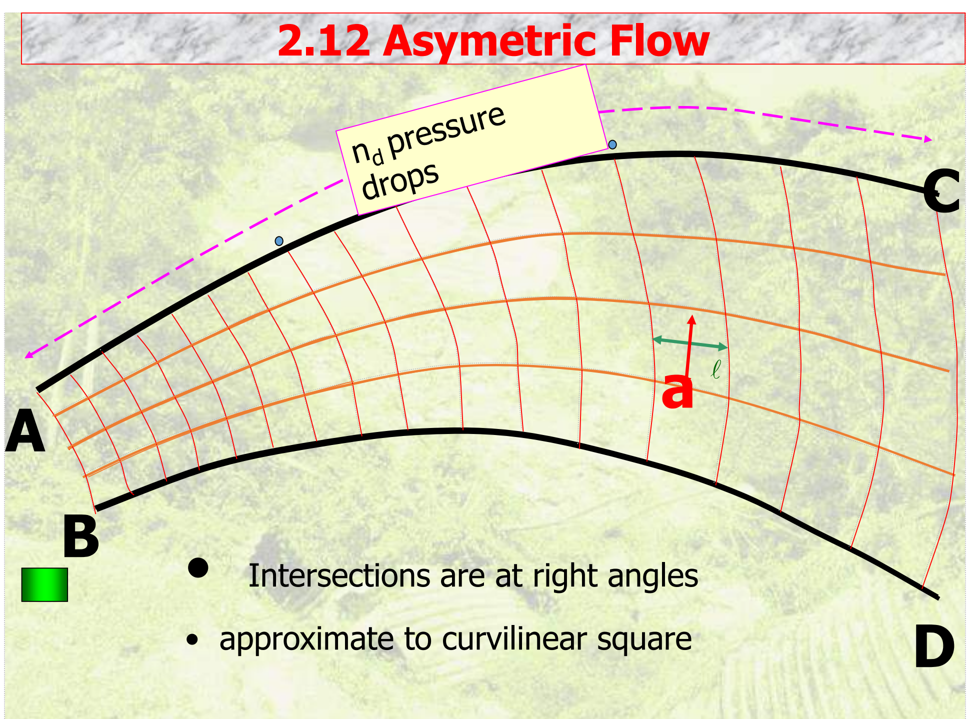
| Soil No. | Void ratio | $i_c$ |
|---|---|---|
| 1 | 0.5 | 1.10 |
| 2 | 0.6 | 1.03 |
| 3 | 0.7 | 0.97 |
| 4 | 0.8 | 0.92 |
| 5 | 1.0 | 0.83 |

- When upward flow takes place at critical hydraulic gradient , a soil such as sand loses all its
- shearing strength and it cannot support any load.
- The soil is said to have become ‘**quick**’ or ‘alive’ and boiling will occur.
- The popular name of this phenomenon is **quicksand**
- Hence quicksand is not a type of sand but only a hydraulic condition.

- **Soil liquefaction** can be defined as the phenomenon by which the strength of the soil is lost either due to dynamic loading.
- Most of the earthquake forces are the major causes of soil liquefaction. The vibrations of earthquake shockwaves in water-saturated soils trigger the phenomenon.
- Liquefaction occurs in saturated soils, that is, soils in which the space between individual particles is completely filled with water.
- This water exerts a pressure on the soil particles that influences how tightly the particles
- themselves are pressed together.
- Prior to an earthquake, the water pressure is relatively low.
- However, earthquake shaking can cause the water pressure to increase to the point where the soil particles can readily move with respect to each other.




2.12 Asymetric Flow
C
A
B
D
Intersections are at right angles
approximate to curvilinear square


2.12 Asymetric Flow
$n_d$ pressure drops
C
A
B
D
a
$\ell$
Intersections are at right angles
approximate to curvilinear square

# 2.12 Asymetric Flow (continued)

pressure drop between AB and CD is **H** and let there be $n_d$ pressure drops and $n_f$ flow lines.

$$\longrightarrow i = \frac{h}{\ell} = \frac{H}{n_d \ell}$$

$$\text{By Darcy's Law}: \quad v = ki = \frac{kH}{n_d \ell}$$

$$q_f = kia = \frac{kHa}{n_d \ell}$$

$$\text{and} \quad a = \ell$$

where $q_f$ is the flow per unit cross-section and **a x 1** is the cross- section between flow lines.

$$q_f = \frac{kH}{n_d}$$

the total seepage = $n_f q_f = \dfrac{n_f kH}{n_d}$

## Summary of Flow Nets

Solutions are relatively straightforward.

1) draw the appropriate flow net
2) count the number of pressure drops in the flow net (over the relevant distance)
3) count the number of flow lines
4) do a simple calculation

- work out total flow
- work out pressure at any given point
- etc.

# Flow Net Theory

1. Streamlines $\Psi$ and Equip. lines $\phi$ are $\perp$.
2. Streamlines $\Psi$ are parallel to no flow boundaries.
3. Grids are curvilinear squares, where diagonals cross at right angles.
4. Each stream tube carries the same flow.

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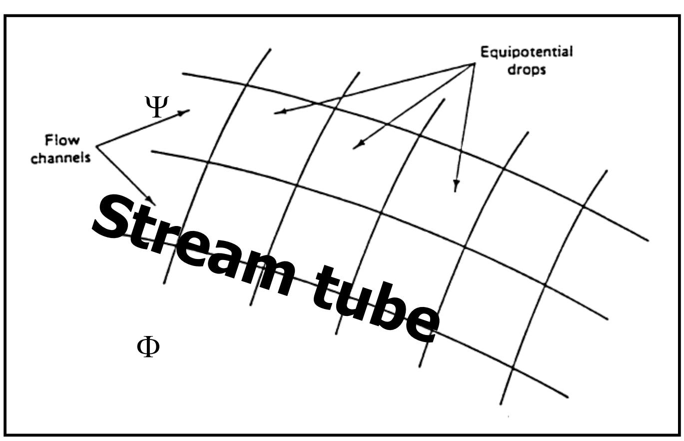
# Flow Net in Isotropic Soil

Portion of a flow net is shown below

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# Flow Net in Isotropic Soil

- The equation for flow nets originates from Darcy's Law.

- Flow Net solution is equivalent to solving the governing equations of flow for a uniform isotropic aquifer with well-defined boundary conditions.



# Flow Net in Isotropic Soil

- Since the flow net is drawn with squares, then $d_m \approx dl$, and:

$$q = (m/n)KH \quad [L^2T^{-1}]$$

where:

- $q$ = rate of flow or seepage per unit width
- $m$= number of flow channels
- $n$= number of equipotential drops
- $h$ = total head loss in flow system
- $K$ = hydraulic conductivity

# Drawing Method:

1. Draw to a convenient scale the cross sections of the structure, water elevations, and aquifer profiles.

2. Establish boundary conditions and draw one or two flow lines Y and equipotential lines F near the boundaries.

3. Sketch intermediate flow lines and equipotential lines by smooth curves adhering to right-angle intersections and square grids. Where flow direction is a straight line, flow lines are an equal distance apart and parallel.



# Method:

4. Continue sketching until a problem develops. Each problem will indicate changes to be made in the entire net. Successive trials will result in a reasonably consistent flow net.
5. In most cases, 5 to 10 flow lines are usually sufficient. Depending on the no. of flow lines selected, the number of equipotential lines will automatically be fixed by geometry and grid layout.

6. Equivalent to solving the governing equations of GW flow in 2-dimensions.

- ***Protective Filters***: This topic has been covered through Class Assignment under **Assignment-I.**
- **Tutorial Sheet-1**: Provided in the class, is for numerical practice of the theories and hypothesis discussed above.

- BEST OF LUCK FOR EXAM

- Dr. Md Rehan Sadique