TÉCNICO LISBOA

BASICS OF ELECTROMECHANICAL POWER CONVERSION

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February 2014

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1. Basic Design and Operation of Electric Motors and Generators

1.1 Electromagnetic Forces

The electrical machines that are used in power applications, as motors or generators, develop electromagnetic forces or electromagnetic torques which are caused by the interaction of electric moving charges with an induction magnetic field, *B*. These forces are known as Lorentz forces and their density is determined by the expressions (1), where ρ is the charge density, that is, the electric charges that move with the velocity v, and are inside a small and unitary volume.

$$\overline{f} = \rho(\overline{v} \times \overline{B}) = \overline{J} \times \overline{B} \tag{1}$$

In the design of electrical machinery there are limits for the value of the induction magnetic field that come as a result of the saturation of ferromagnetic materials. The limits in the value of current density *J*, are a consequence of constraints in Joule losses due to heat dissipation capability in the equipment. To optimize the materials in electric machinery, the current density and the induction magnetic field are put orthogonally.

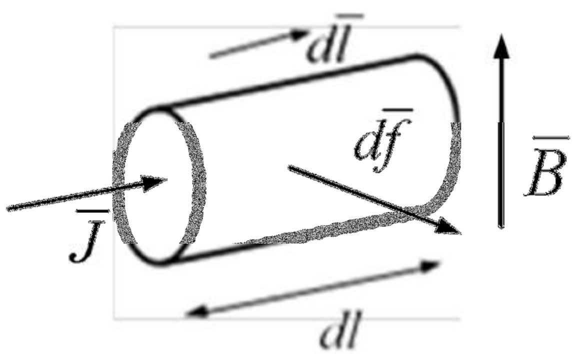

Figure 1 – Lorentz forces in a wire conductor.

Figure 1 shows a small length of a wire conductor with a current *i* resulting from an uniform and constant distribution of the current density in the conductor's section. The force developed in the small length of wire is calculated by the following relations

$$d\overline{f} = (\overline{J} \times \overline{B})Adl = \frac{i}{A}(\overline{n} \times \overline{B})Adl = i(d\overline{l} \times \overline{B})\ Nm^{-1}$$

(2)

Summing up for all the length of the wire and considering the orthogonally of *J* and *B*, one obtains:

$$F_w = Bli$$

(3)

The orthogonally between the current density and the induction magnetic field is assured by an adequate design of the electric circuit and the magnetic circuit.

1. Problem

Consider the device shown on Figure 2 that turns around axis Z. There are two quadrature rotating coils. Each coil has two active conductors with length l along Z axis. These two conductors are separated by distance d. The position of coil aa' is at the axis whose angular position is measured by the angle θ. Suppose that the device is on a place where there is a magnetic field whose flux density has a constant value given by $B_x \bar{e}_x + B_y \bar{e}_y$.

a) When coil aa' carries a constant current i_a and coil bb' carries a constant current i_b determine the electromagnetic force developed in each conductor of the two coils for different angular positions of the rotor.
b) Calculate the resulting force that is applied to the device.
c) Calculate the resulting torque that is generated.

Hints: Initially, consider $B_x = 0$ and $i_b = 0$ to simplify calculations. Integrate the value given by relation (2) in all conductors of the device. Remember that the torque is calculated by $\overline{T} = \bar{r} \times \overline{F}$.

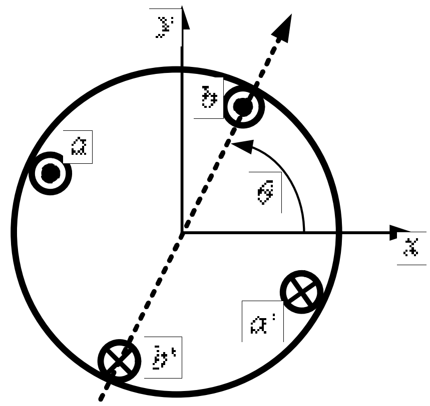

Figure 2 – Two quadrature coils on a cylindrical rotor with length l and diameter d.

1.2 An Elementary Generator

Consider a very simple system that is represented in Figure 3. A single wire conductor can move inside an induction magnetic field with a single position degree of freedom x_s.

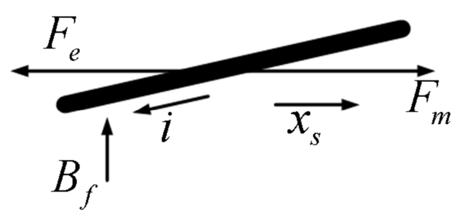

Figure 3 – A single wire conductor that moves inside an induction magnetic field.

When a driving force F_m is applied to the wire, this one starts to move with increasing speed. The electric charges inside the wire are moving inside a magnetic field. So they are under the action of an electric forces:

$$d\bar{f} = \rho_f(\bar{v} \times \bar{B})dV = \rho_f \bar{E} dV \tag{4}$$

or an electric field is induced field $\bar{E} = \bar{v} \times \bar{B}$. If the circuit is closed, this electric field generates an electric current in the wire. The existence of the current originates Lorentz forces that oppose the movement of the wire: $F_e = Bli$. The movement of the wire is described by equation (5) where the mass of the wire has the value M,

$$M\frac{dv}{dt} = F_m - Bli \tag{5}$$

Multiplying by the speed both terms of equation (5), one finds a relation between powers. Some of the mechanical power supplied to the system is converted into kinetic energy and some is converted to electric energy.

$$F_m v = \frac{dK_e}{dt} + F_e v \qquad K_e = \frac{1}{2}Mv^2 \tag{6}$$

In the case of the electric energy one writes the balance equation (7). Some of the received electric energy increases the stored magnetic energy dW_m / dt, some of it is converted to heat by Joule effect in the resistance, Ri^2, and the rest of it can be supplied to external loads, ui.

$$P_e = F_e v = \frac{dW_m}{dt} + Ri^2 + ui \qquad W_m = \frac{1}{2}Li^2 + W_{mf} \qquad F_e = Bli \tag{7}$$

Dividing both terms of equation (7) by the current intensity we obtain equation (8), the equation of an electrical circuit, as in Figure 4, where the electrical components of this elementary generator are represented.

$$E = L\frac{di}{dt} + Ri + u \qquad E = Blv \tag{8}$$

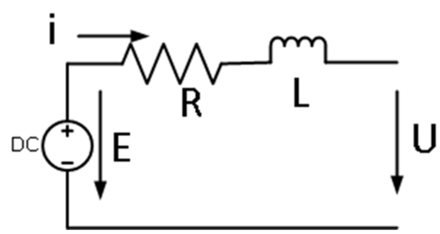

Figure 4 – Equivalent circuit of elementary generator.

The dynamic behavior of the generator is represented by the following set of differential equations:

$$\begin{cases} u = -L\dfrac{di}{dt} - Ri + Blv \\ F_m = Bli + M\,\dfrac{dv}{dt} \end{cases} \tag{9}$$

Conductors moving within a magnetic field develop an induced *emf.* This phenomenon can be used to design electric power sources.

2. Problem

Consider that the device presented in Problem 1 rotates at constant speed of N_r *rpm* (rotation per minute).

a) Verify that an *emf* is induced in each of the two coils.
b) Plot the evolution of the *emf* in each coil.

Hints: Remember that $\bar{v} = \frac{d\bar{r}}{dt}$. Consider relation (4) in each of the active conductors of each coil and use this result to find the energy necessary to maintain electric charge moving along each one of the active conductors.

1.3 Basic Design Guidelines for Linear and Rotating Generators

In electrical machinery it is important that the current density and the induction magnetic field are orthogonal. This situation is achieved in the small air- gap by the use of specific materials for the conductor wires where the charges move and the ferromagnetic material used to strap the induction magnetic field. These materials constitute respectively the electric and magnetic circuits of the machine, which must be closed – remember Maxwell equations $\nabla \bullet \bar{B} = 0, \nabla \bullet \bar{J} = 0$. To comply with these properties the wire conductors are wound forming a coil with the active conductors spaced by a distance designated by pole pitch. Figure 5 shows a coil in a linear and in a rotating machine. The two active conductors of the coil have inverse currents in both machines, but the induction magnetic field has different directions in the linear machine and the same direction in the rotating machine. This happens in order to ensure that forces or torques generated in different conductors of the coil push in the same direction.

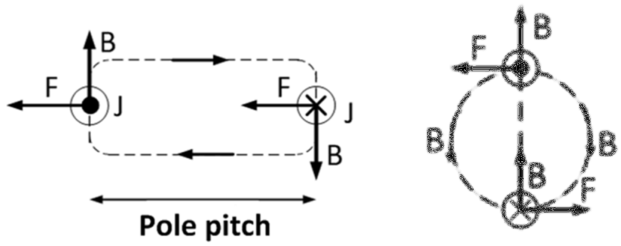

Figure 5 – Pole pitch of a coil in a linear and in a rotating machine.

To take advantage of all available space on the periphery of the machine several coils are used with a distribution like those presented in Figure 6

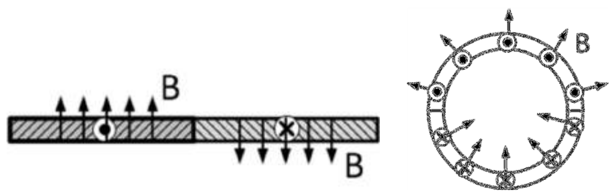

Figure 6 – Contribution of several coils.

Suppose that, due to the electromagnetic forces, the conductors – the electric currents – move relative to the induction magnetic field distribution and the two distributions have the relative positions presented in Figure 7. It is not difficult to verify that the distributions on the right side of the figure generate a null torque. We can conclude that the two distributions must maintain the synchronism to maintain a constant torque during steady state operation.

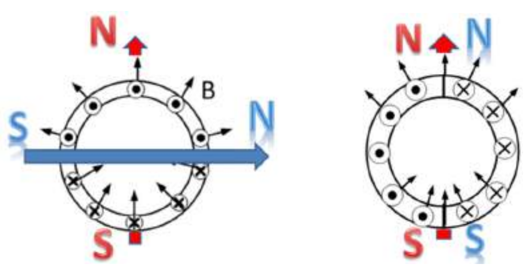

Figure 7 – Two relative positions of the distributions generate different values of the torque.

In order to maintain the synchronism, the two distributions must maintain its relative position like a compass that tries to align but the field rotates changing its direction.

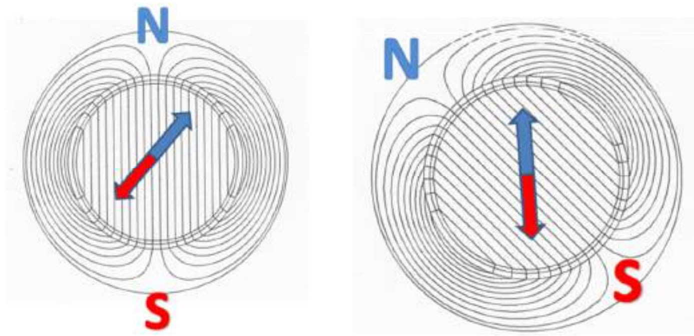

Figure 8 – Rotating fields

All the electric machines – motors or generators – work under these principles. The different performances are obtained according to the different ways used to generate the two distributions and how they maintain the synchronism.

In the precedent examples only two pole structures were presented, however other structures are used with larger number of pole pairs, p, as the one presented in Figure 9 with four poles, $p = 2$.

Figure 9 – A two poles pair structure.

Note that if the structure rotates with angular speed Ω, the rotor angular speed, the electric frequency is equal to $\omega = p\Omega$.

Typical rotating machines have two main components: one fixed in space, the stator, and other that can rotate, the rotor. Usually the rotor is inside as in the machine presented in Figure 10, but there are other cases where the rotor is at the external side, as occurs in recently developed permanent magnet machines for off shore wind applications.

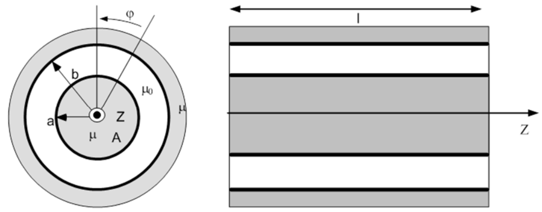

Figure 10 – Axial and longitudinal cut of a rotating electric machine.

The stator and the rotor are separated by a small air gap, which in the picture is represented much larger than in the real case. In the internal part of the stator and in the external part of the rotor there are conductors to carry electric currents.

3. Problem

Consider a device like the one described in Problem 1, but having only the coil aa'.

a) Show that when the device is rotating at constant speed and carrying a constant current inside the coil, no effective energy conversion is realized.
b) Switch current to invert it directions in such way to obtain not null average torque in a complete turn. Plot the current evolution and angular position of the rotor.

Hints: Remember that mechanical power is given by $p_m = \omega_r T$ and find $< T >_{av}$.

1.4 Producing Rotating Fields

An electric current flowing inside a coil generates an induction magnetic field distribution whose form depends on the distribution of the turns of the coil and also on the geometry of the magnetic circuit. An adequate distribution of the conductors enables us to obtain a sinusoidal wave form. If the coil rotates then the distribution maintains its form but changes the position. We say in this case that the rotating filed is obtained by a constant current inside a mobile coil. Of course, a similar result can be obtained using rotating permanent magnets.

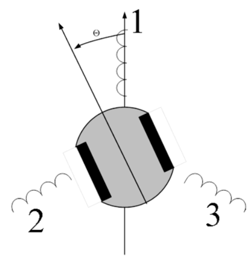

Figure 11 – Schematic representation of a three phase synchronous machine.

Figure 11 presents a schematic representation of a three phase synchronous machine, the electrical machine most used for production of electricity. This generator has a field winding in the rotor carrying a DC current. This current generates a distribution of induction magnetic field that changes its position according to the rotor position. The rotor angular speed is equal to the angular speed of the rotating field.

The three phase synchronous machine has other windings in the stator also shown in Figure 11. These windings are equal, with the only difference between them being the angular position they occupy around the stator. As Figure 11 represents, these windings are distributed in a regular way.

When one of those windings, for instance winding 1, carries a constant current, it is generated in the air gap of the machine a sinusoidal waveform distribution[1] of induced magnetic field whose radial component is represented by equation (10).

$$B_1 = A\cos(p\alpha)i_1 \tag{10}$$

Windings 2 and 3 generate a similar waveform distribution that is represented by equations (11)

$$B_2 = A\cos(p\alpha - \frac{2\pi}{3})i_2 \qquad B_3 = A\cos(p\alpha - \frac{4\pi}{3})i_3 \tag{11}$$

Consider now that the windings carry a balanced and symmetrical three phase current system (12).

$$i_1 = I_M \cos(\omega t) \qquad i_2 = I_M \cos(\omega t - \frac{2\pi}{3}) \qquad i_3 = I_M \cos(\omega t - \frac{4\pi}{3}) \tag{12}$$

[1] This distribution waveform is produced by a specific distribution of active conductors of the coils.

The resultant waveform distribution is described by equation (13).

$$B = \frac{3}{2} A I_M \cos(p\alpha - \omega t) \tag{13}$$

This equation shows that the resultant distribution is a rotating distribution with an angular speed equal to

$$\dot{\alpha} = \frac{\omega}{p} \tag{14}$$

We conclude that a rotating field can also be produced using stationary windings carrying variable currents.

4. Problem

a) Show that a single coil that generates a magnetic field distribution like the one represented by relation (10) produces a stationary sinusoidal waveform when the current is also a sinusoidal waveform on time. That is: the magnetic field distribution is a pulsating wave.

b) Show that a pulsating distribution may be described by the composition of two rotating field distributions similar to the one represented by the relation (13).