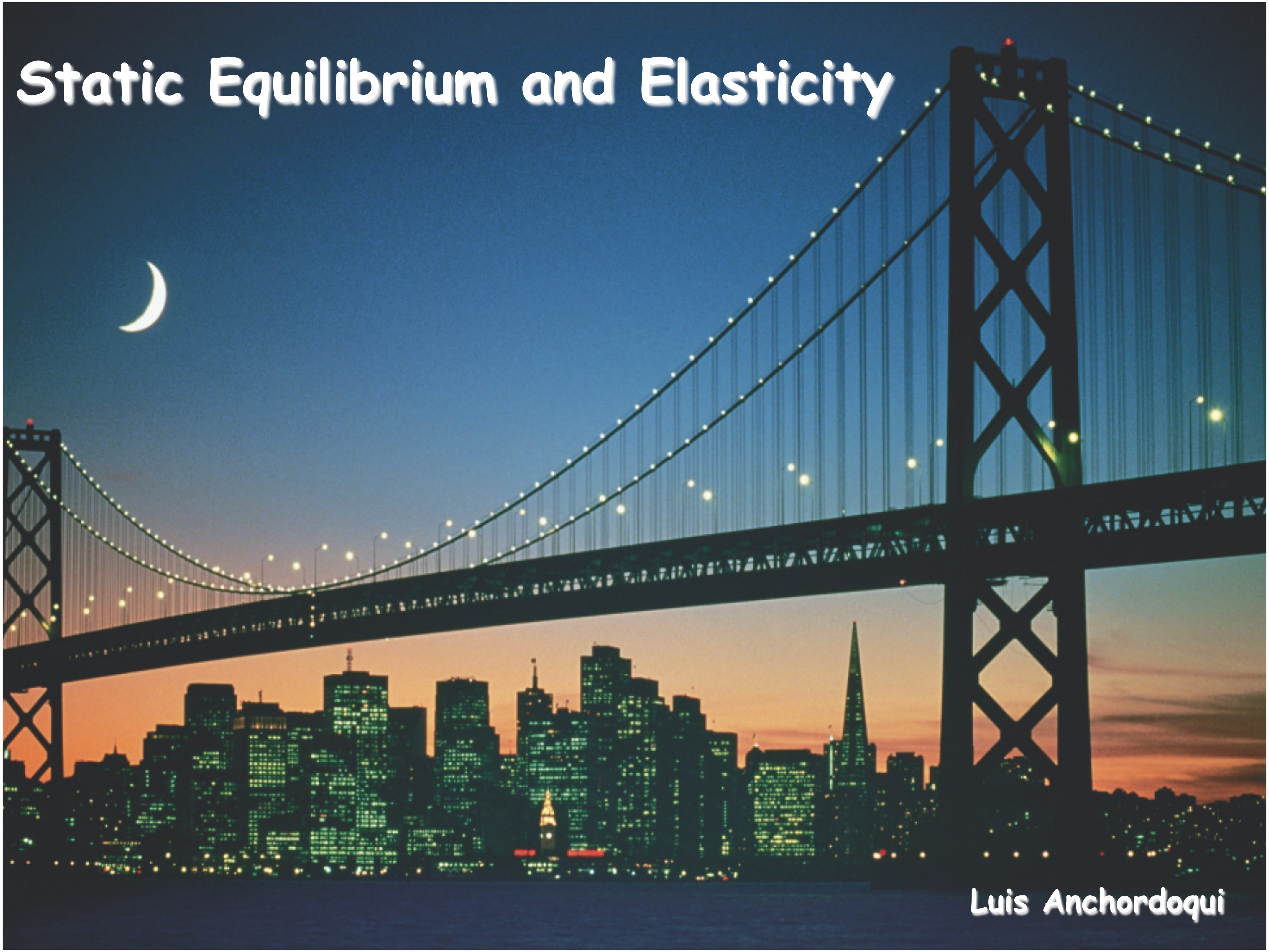
# Static Equilibrium and Elasticity



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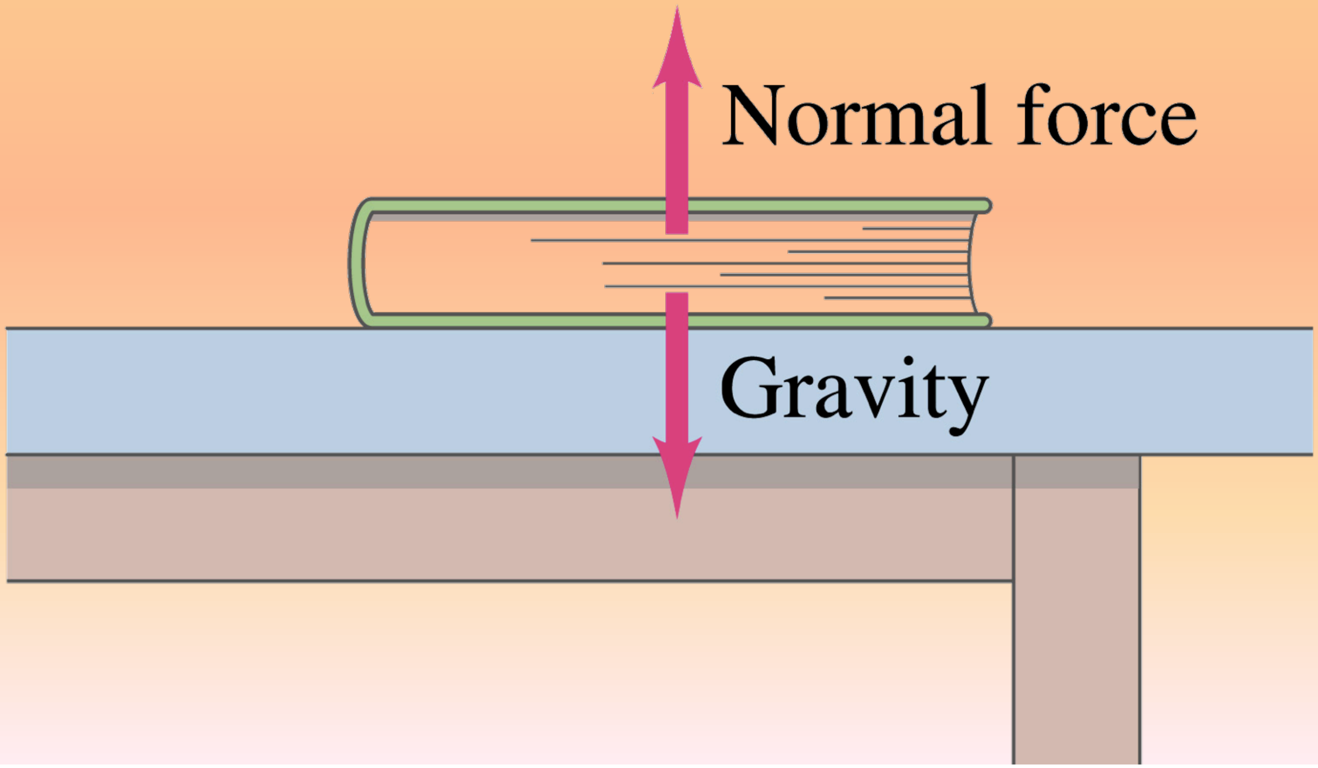
# The Conditions for Equilibrium

An object with forces acting on it, but that is not moving, is said to be in equilibrium.

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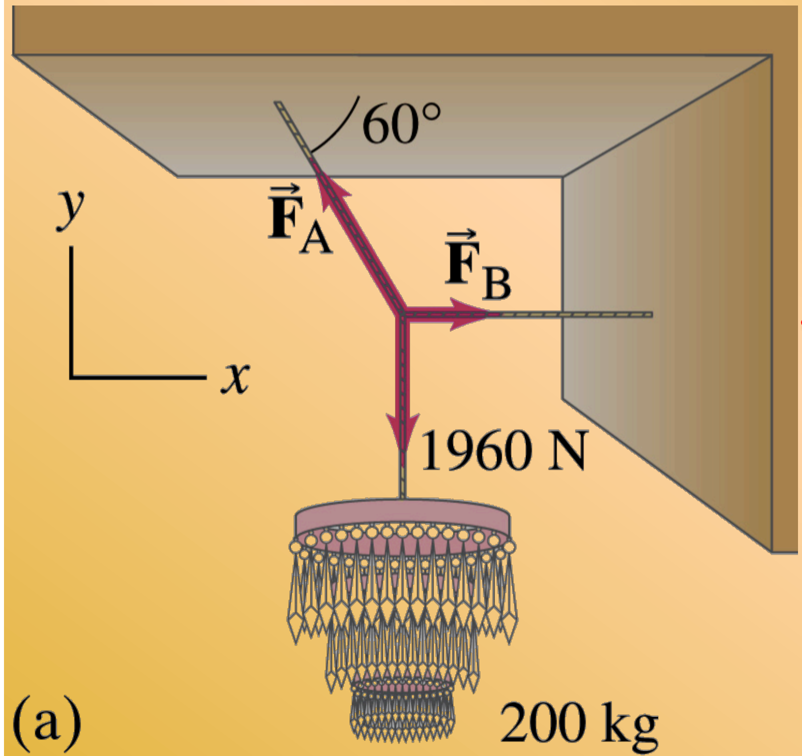
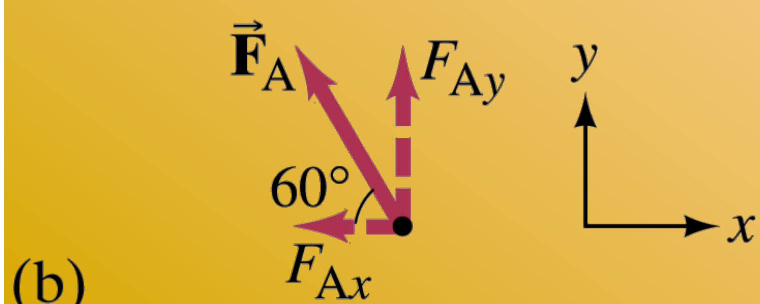
# The Conditions for Equilibrium (cont'd)




The first condition for equilibrium is that the forces along each coordinate axis add to zero.

$$\Sigma F_y = F_A \sin 60° - 200\text{kg}\ g = 0$$

$$F_A = 2260 \text{ N}$$

$$\Sigma F_x = F_B - F_A \cos 60° = 0$$

$$F_B = 1130 \text{ N}$$

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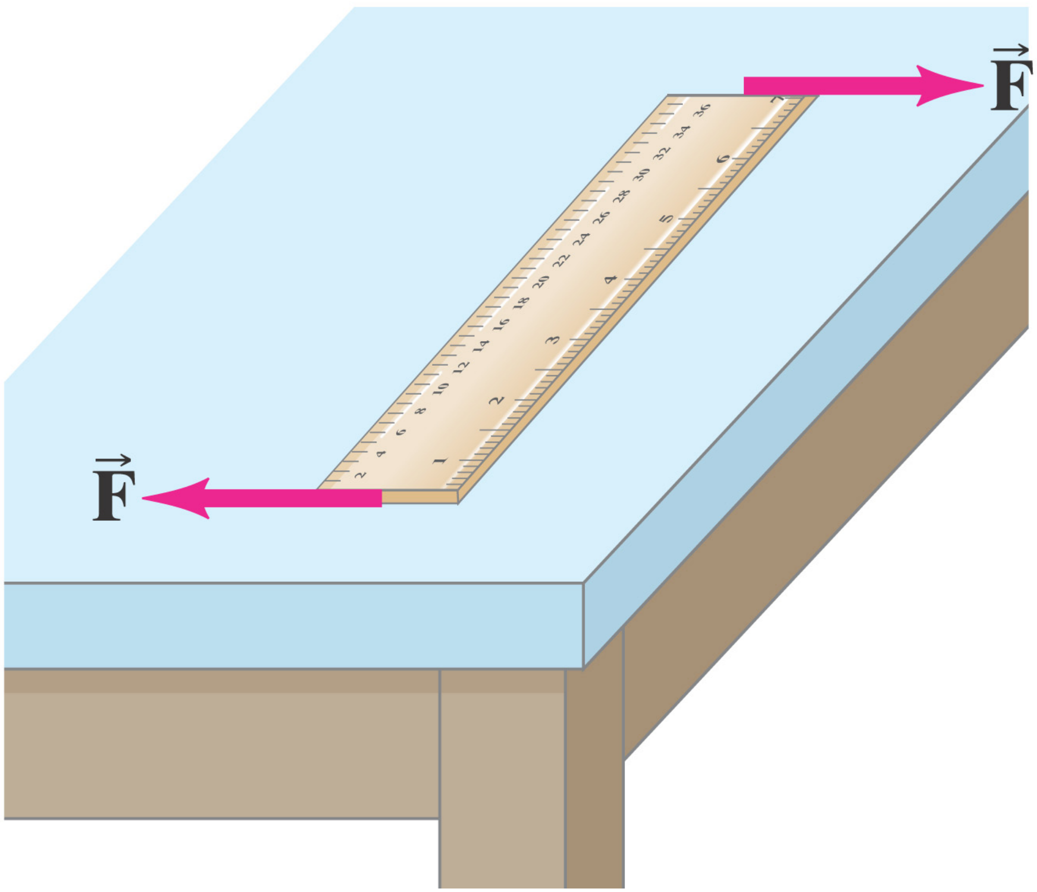
# The Conditions for Equilibrium (cont'd)

The second condition of equilibrium is that there be no torque around any axis; the choice of axis is arbitrary.

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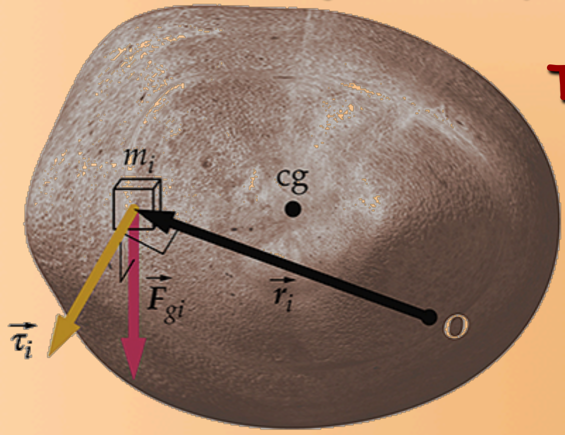
# Center of gravity

Consider an object composed of many small mass elements in static equilibrium




The force of gravity on the ith small element is $\vec{F}_{gi}$ and the total force of gravity on the object is

$$\vec{F} = \sum \vec{F}_{gi}$$

If $\vec{r}_i$ is the position vector of the ith particle relative to the origin $O$

⇓

$\vec{\tau}_i = \vec{r}_i \times \vec{F}_{gi}$ is the torque due to $\vec{F}_{gi}$ about $O$

The net gravitational torque about $O$ is then

$$\vec{\tau}_{net} = \sum ( \vec{r}_i \times \vec{F}_{gi} )$$

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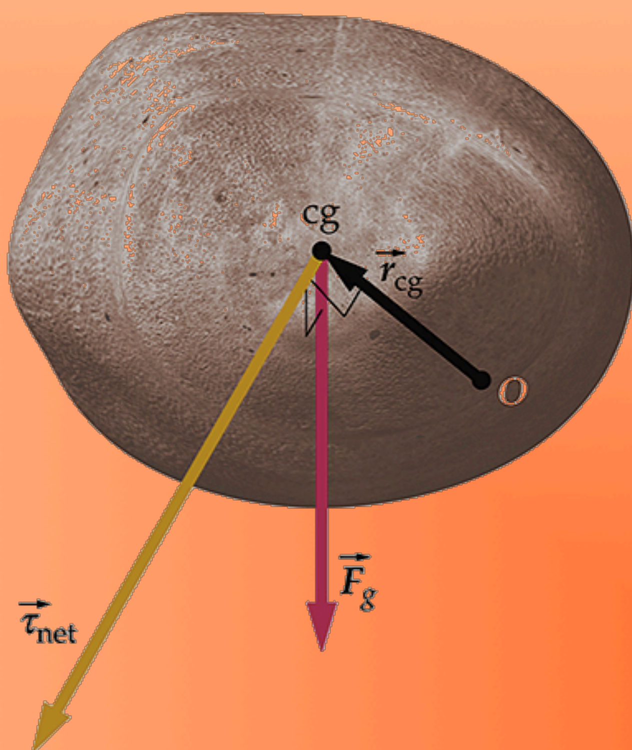
# Center of gravity (cont'd)

The net torque due to gravity about a point can be calculated as if the entire force of gravity $\vec{F}_g$ were applied at a single point known as the center of gravity




$$\tau_{net} = \vec{r}_{cg} \times \vec{F}_g$$

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# Center of gravity and center of mass

If the gravitational field $\vec{g}$ is uniform over the object

$$\vec{F}_{gi} = m_i \vec{g}$$

Summing on both sides leads to

$$\vec{F}_g = M \vec{g}$$

$$M = \sum m_i$$

$$\tau_{net} = \sum_i (\vec{r}_i \times \vec{F}_{gi}) = \sum_i (\vec{r}_i \times m_i \vec{g}) = \sum m_i \vec{r}_i \times \vec{g}$$

Using the definition of the center of mass

$$M\vec{r}_{cm} = \sum m_i \vec{r}_i$$

$$\vec{\tau}_{net} = M\vec{r}_{cm} \times \vec{g} = \vec{r}_{cm} \times M\vec{g} = \vec{r}_{cm} \times \vec{F}_g$$

The center of gravity and the center of mass coincide if the object is in a uniform gravitational field

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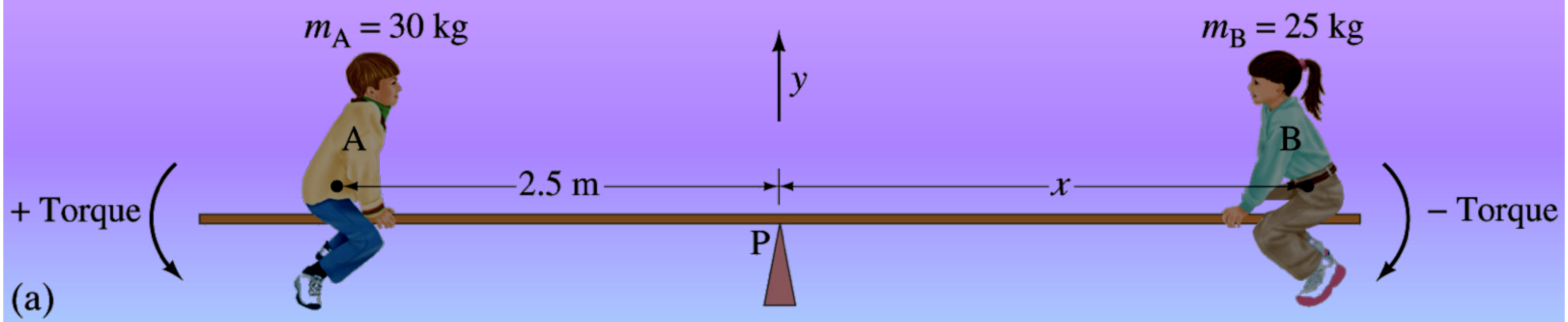
# Balancing a seesaw

A board of mass M = 2 kg serves as a seesaw for two children. Child A h as a mass of 30 kg and sits 2.5 m from the pivot point P (his CM gravity is 2.5 m from the pivot).
At what distance x from the pivot must child B of mass 25 kg, place herself to balance the seesaw?
Assume the board is uniform and centered over the pivot.

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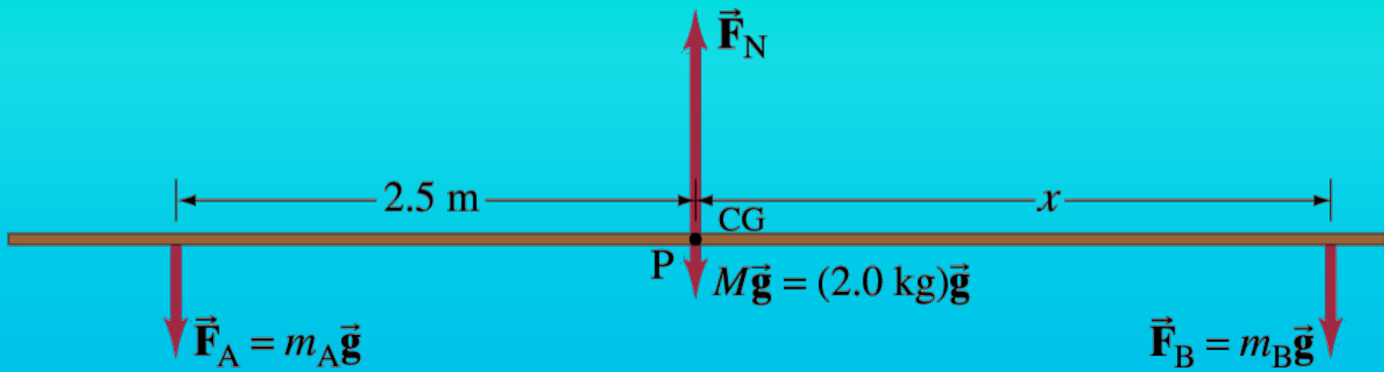
# Balancing a seesaw (cont'd)




$$\Sigma\tau = m_A\, g\ (2.5\text{m}) - mgx + MG\ (0\text{m}) + F_N\ (0\text{m}) = 0$$

$$x = 3\text{ m}$$

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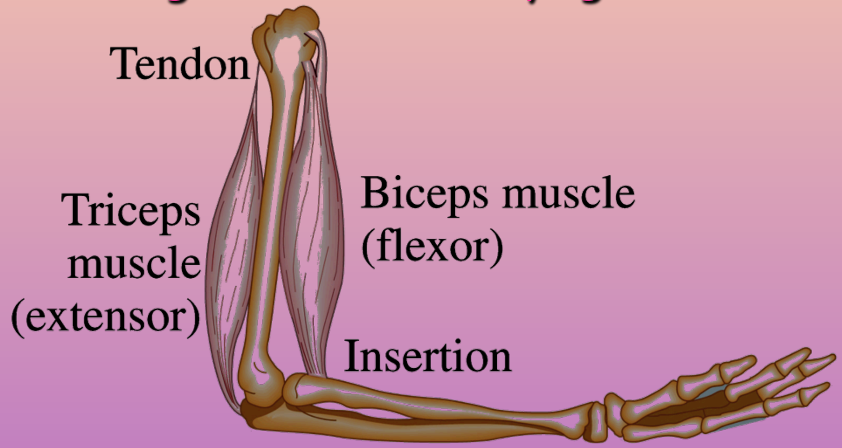
# Applications to muscles and joints

The techniques discussed for describing objects in equilibrium can readily be applied to the human body and can be of great use in studying forces on muscles, bones, and joints




The muscles are attached via tendons to two different bones

The points of attachment are called insertions

Two bones are flexibly connected at a joint

A muscle exerts a pull when its fibers contract under stimulation by a nerve but a muscle cannot exert a push

Muscles that tend to bring two limbs closer together , such as the biceps muscle in the upper arm, are called flexors

Muscles that act to extend a limb outward, such as the triceps muscle, are called extensors

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# Force exerted by the biceps muscle

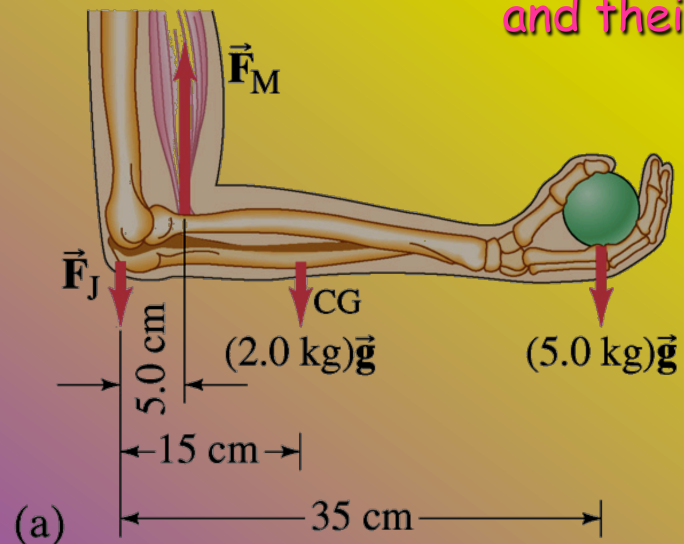
How much force must the biceps muscle exert when a 5 kg mass is held in the hand

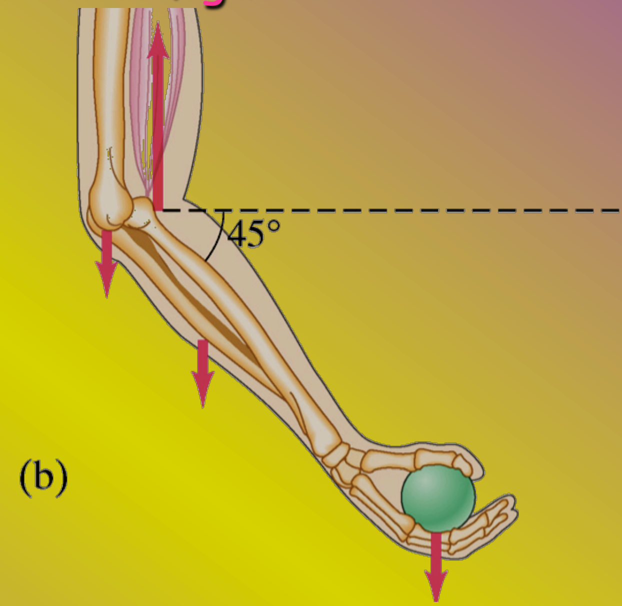
(a) with the arm horizontal

(b) when the arm is at 45 degrees angle?

Assume that the mass of the forearm and hand together is 2 kg

and their CM is as shown in the figure




$F_M$ = 400 N

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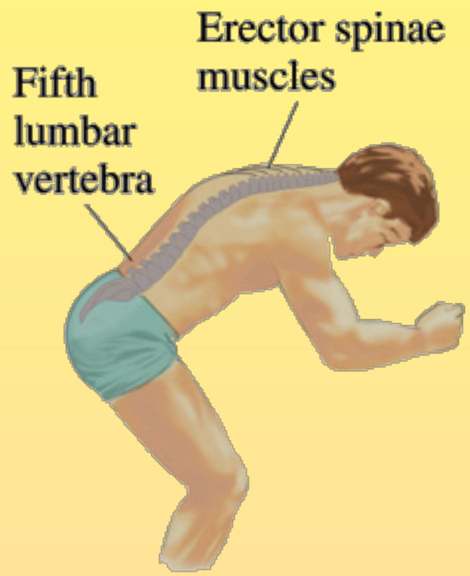
# Forces on your back

## Consider the muscles used to support the trunk when a person bends foward




The lowest vertebra on the spinal column (fifth lumbar vertebra) acts as a fulcrum for this bending position

The ¨erector spinae¨ muscles in the back that support the trunk acts at an effective angle about 12° to the axis of the spinae

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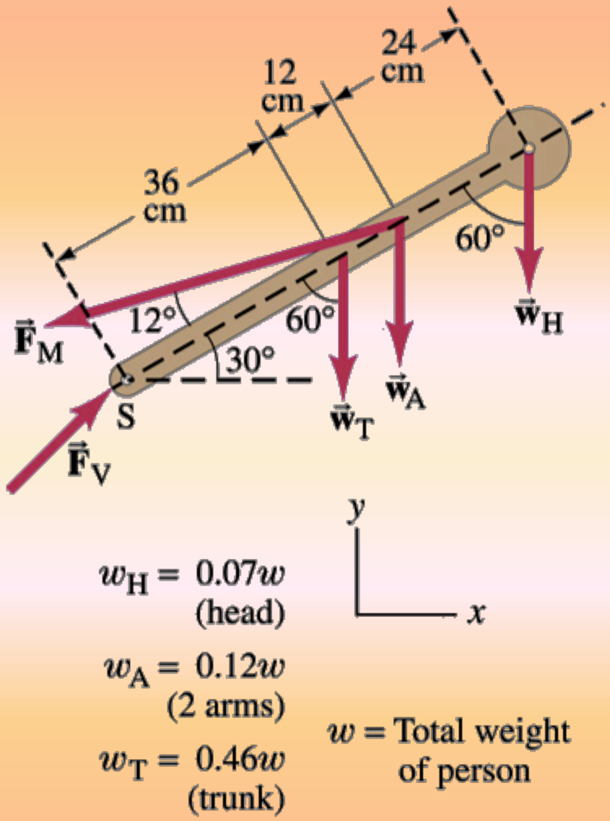
# Forces on your back (cont'd)

This figure is a simplified schematic drawing showing the forces on the upper body

We assume the trunk makes an angle of 30° with the horizontal




The force exerted by the back muscles is represented by $\vec{F}_M$, the force exerted on the base of the spine at the lowest vertebra is $\vec{F}_V$, and $\vec{w}_H$, $\vec{w}_A$, $\vec{w}_T$ represents weights of the head, freely arms, and trunk, respectively

The distances in cm refer to a person 180 cm tall, but are approximately in the same ratio 1:2:3 for an average person of any weight, and so the result of our estimate is independent of the height of the person

We calculate $\vec{F}_M$ using the torque equation taking the axis at the base of the spine ( point S)

$$\Sigma\tau = 0 \Rightarrow F_M = 2.37w$$

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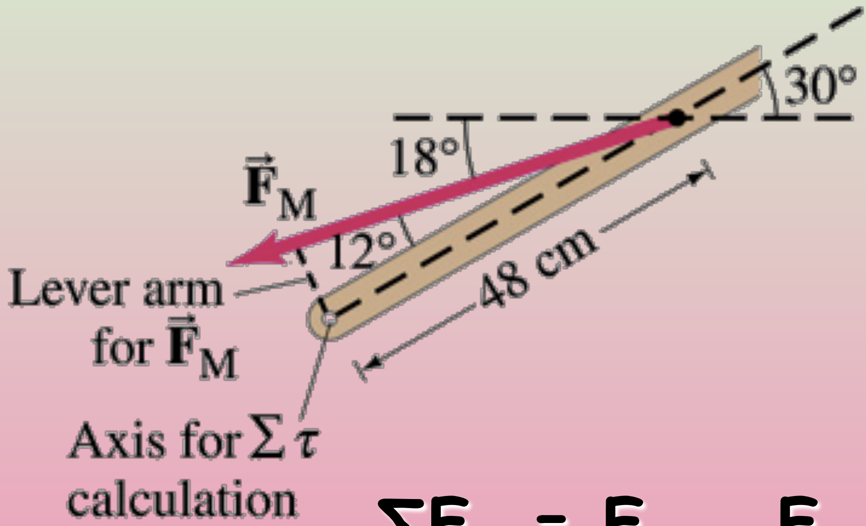
# Forces on your back (cont'd)




$$\Sigma F_y = F_{Vy} - F_M \sin 18° - w_H - w_A - w_T = 0$$

$$F_{Vy} = 1.38w$$

$$\Sigma F_x = F_{Vx} - F_M \cos 18° = 0$$

$$F_{Vx} = 2.25w$$

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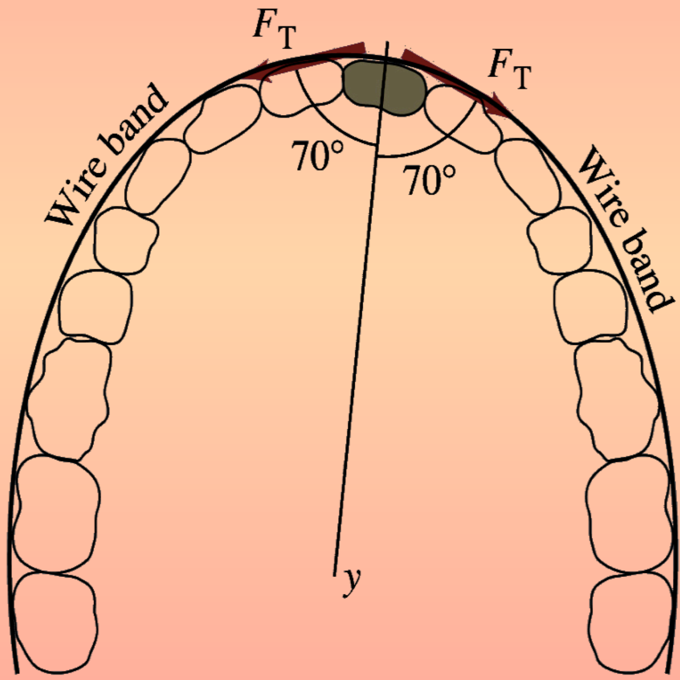
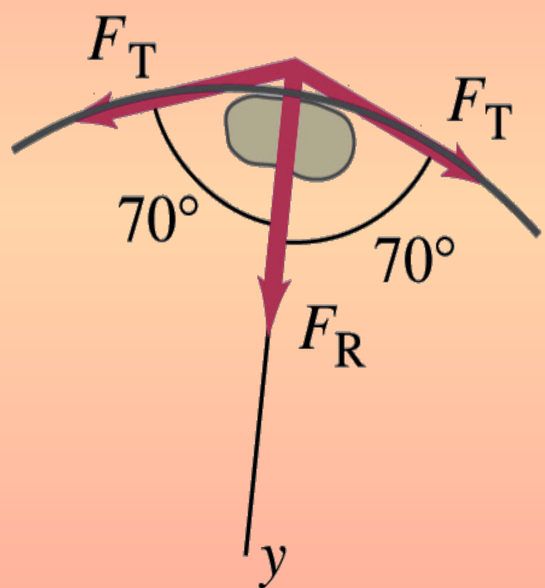
# Straightening teeth

The wire band shown in the figure has a tension $F_T$ = 2 N along it. It therefore exerts forces of 2 N on the highlighted tooth (to which it is attached) in the two directions shown. Calculate the resultant force on the tooth due to the wire




$$F_{Ry} = 2\ F_T \cos 70 = 1.37\ \text{N}$$

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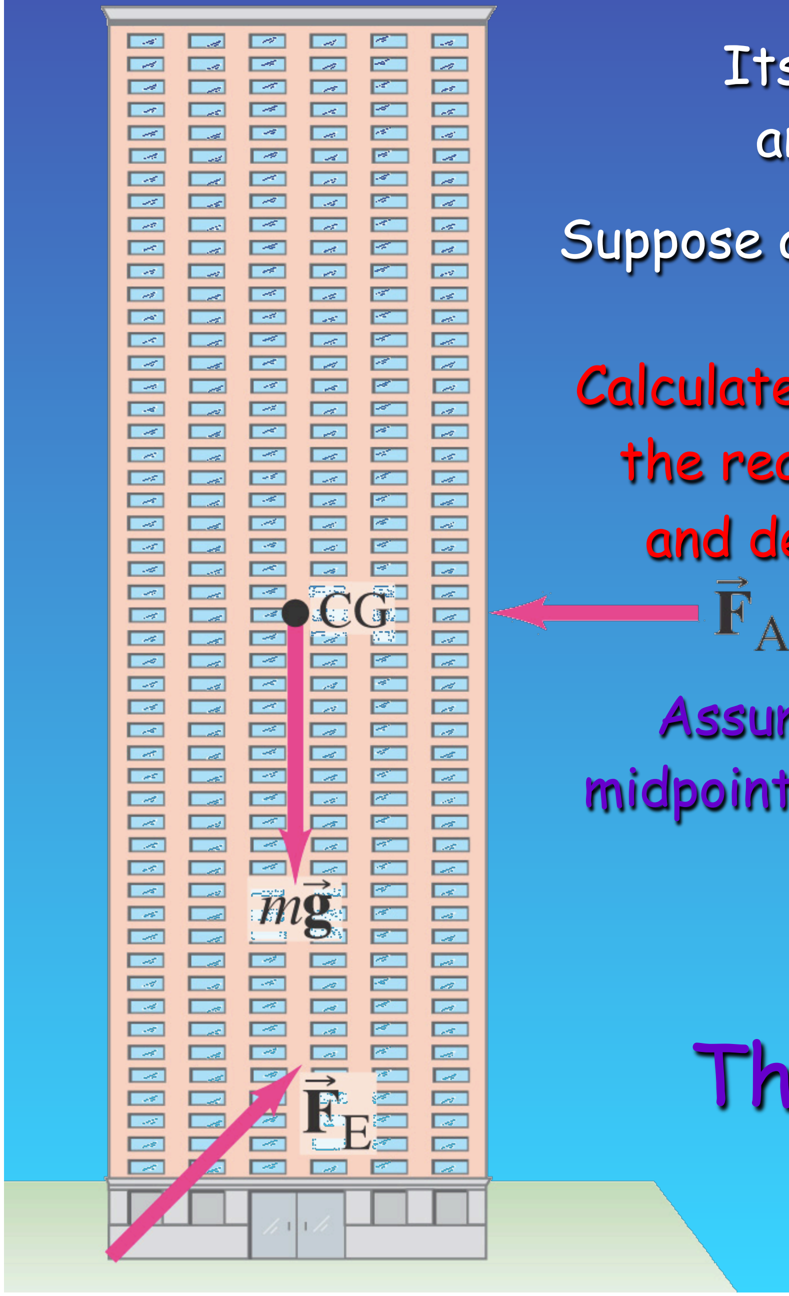
A 50-story building is being planned.

It is to be 200 m high with a base of 40 m by 70 m.




Its total mass will be about $1.8 \times 10^7$ kg, and its weight therefore $1.8 \times 10^8$ N.

Suppose a 200 km/h wind exerts a force of 950 N/m$^2$ over the 70 m-wide face.

Calculate the torque about the potential pivot point, the rear edge of the building (where $\vec{F}_E$ acts in) and determine whether the building will topple.

Assume the total force of the wind acts at the midpoint of the building's face, and that the building is not anchored in bedrock.

# The building will not topple

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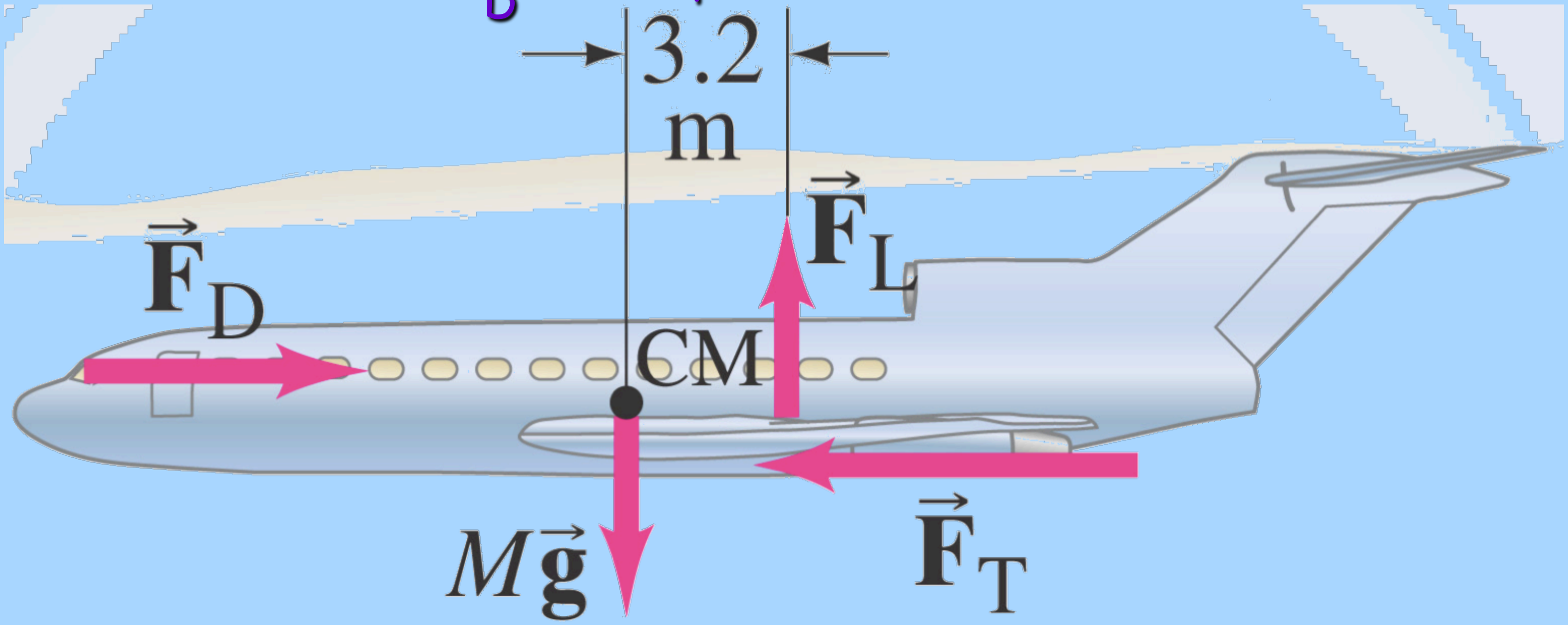
The forces acting on a 67,000 kg aircraft flying at constant velocity are shown in the figure.

The engine thrust, $F_T = 5 \times 10^5$ N, acts on a line 1.6 m below the CM.

Determine the drag force $F_D$ and the distance above the CM that its acts.

Assume $\vec{F}_D$ and $\vec{F}_T$ are horizontal.




$F_D = 5 \times 10^5$ N

$F_L = 6.6 \times 10^5$ N

$h_1 = 2.6$ m

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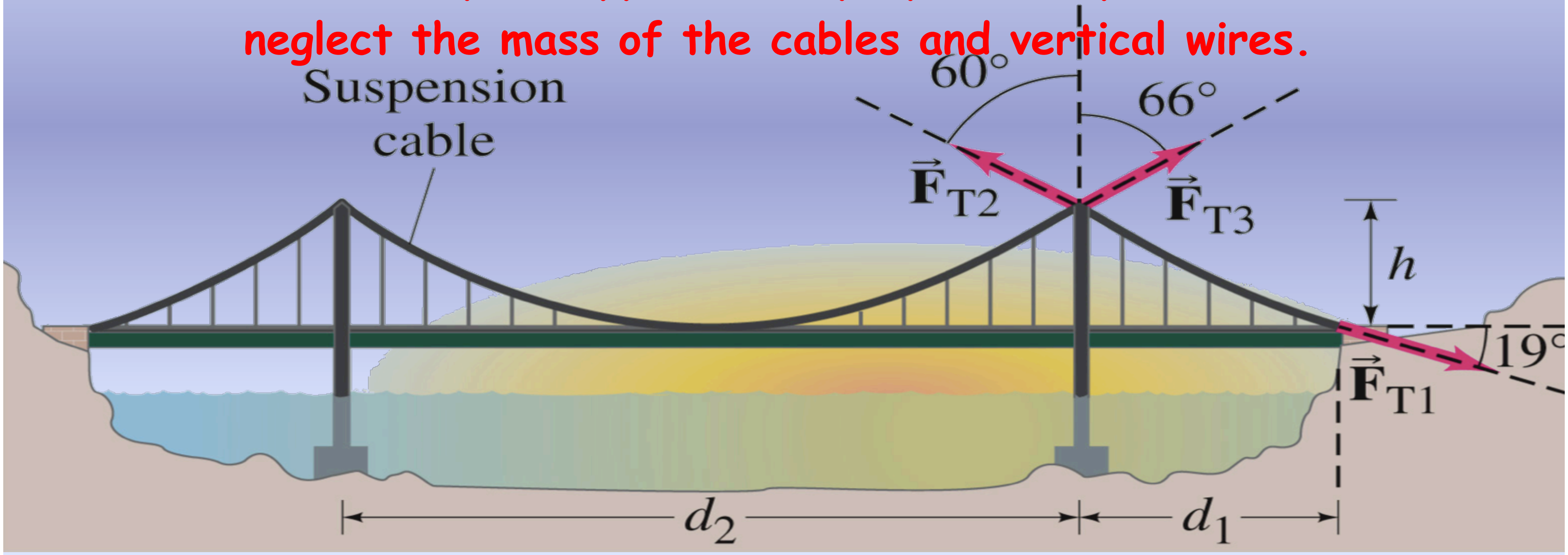
Consider the right-hand (northernmost) section of the Golden Gate bridge, which has a length $d_1$ = 343 m.

Assume the CG of this span halfway between the tower and achor.

Determine $F_{T1}$ and $F_{T2}$ (which act on the northermost cable) in terms of mg, the weight of the northernmost span, and calculate the height h needed for the equilibrium.

Assume the roadway is supported only by the suspension cables, and neglect the mass of the cables and vertical wires.




$F_{T1}$ = 4.5 mg   $F_{T2}$ = 5 mg   h = 158 m

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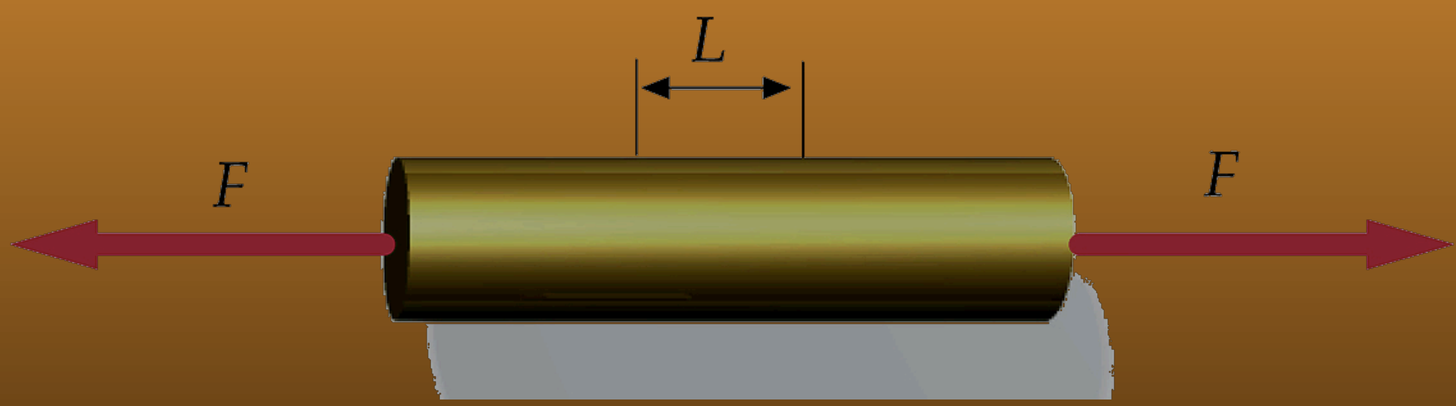
# Stress and strain

If a solid object is subjected to forces
that tend to stretch, shear or compress the object
its shape changes

If the object returns to its original shape when forces are removed
it is said to be elastic




A solid bar subject to stretching forces of magnitude F acting on each other

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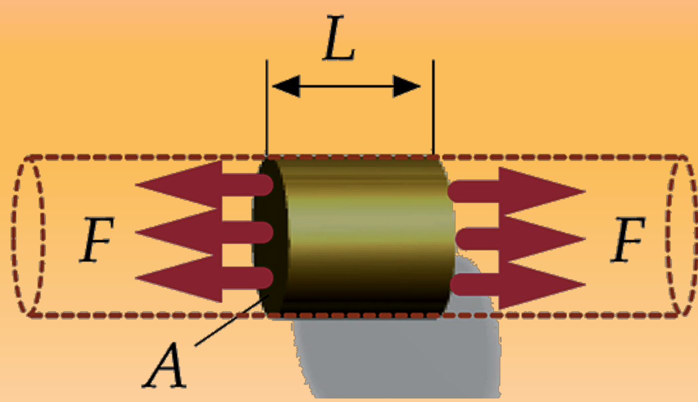
# Stress and strain (cont'd)




A small section of the bar of length L

The fractional change in the length of a segment of the bar is called the strain

$$\text{Strain} = \Delta L/L$$

The forces are distributed uniformely over the cross sectional area

The force per unit area is the stress

$$\text{Stress} = F/A$$

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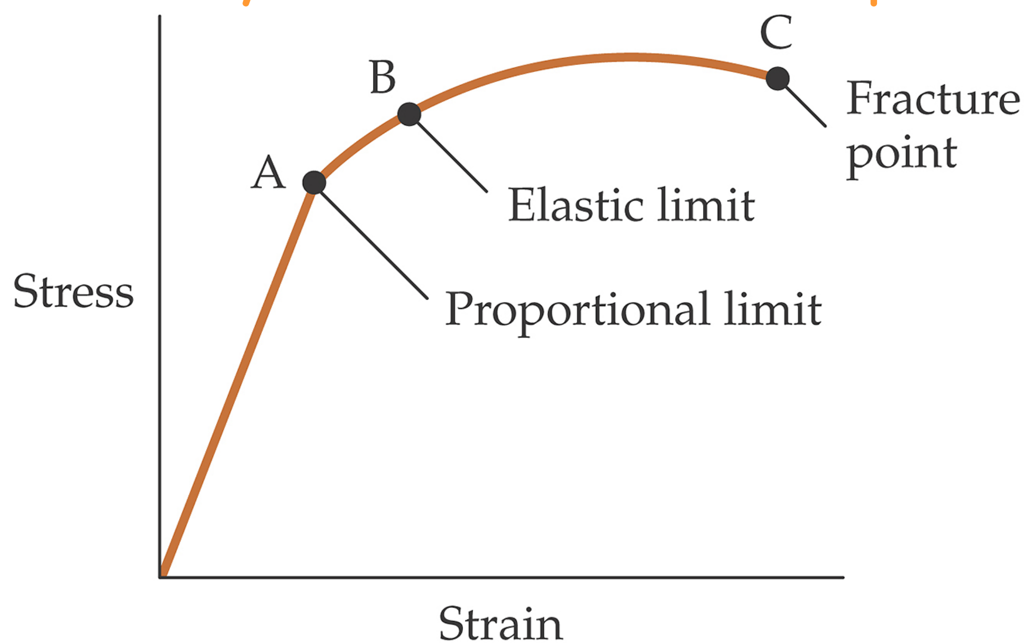
# Young's Modulus

Most objects are elastic for forces up to certain maximum called the elastic limit

If the object is stretched beyond the elastic limit it is permanently deformed




If an even greater stress is applied the material eventually breaks

The ratio of stress to strain in the linear region of the graph is a constant called the Young's modulus

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

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# Young's Modulus Y and Strengths of Various Materials†

| Material | Y, GN/m²‡ | Tensile strength, MN/m² | Compressive strength, MN/m² |
|---|---|---|---|
| Aluminum | 70 | 90 | |
| Bone | | | |
| Tensile | 16 | 200 | |
| Compressive | 9 | | 270 |
| Brass | 90 | 370 | |
| Concrete | 23 | 2 | 17 |
| Cooper | 110 | 230 | |
| Iron (wrought) | 190 | 390 | |
| Lead | 16 | 12 | |
| Steel | 200 | 520 | 520 |

† These values are representative. Actual values for particular samples may differ

‡ 1 GN = $10^3$ MN = 1 x $10^9$ N

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# Elevator safety

While working with an engineering company during the summer, you are assigned to check the safety of a new elevator system in the Prudential building. The elevator has a maximum load of 1000 kg, including its own mass, and is supported by a steel cable 3 cm in diameter and 300 m long at full extension.

There will be safety concerns if the steel stretches more than 3 cm. Your job is to determine whether or not the elevator is safe as planned, given a maximum acceleration of the system of 1.5 $m/s^2$

$$\Delta L = 2.4 \text{ cm}$$

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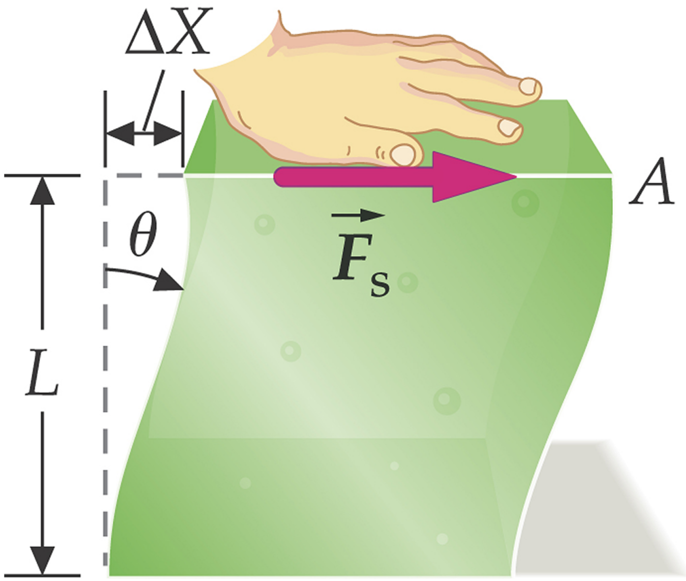
# Shear stress

A force $F_s$ applied tangentially to the top of a block of jello is known as a shear force

The ratio of the shear force to the area is called a shear stress




$$\text{Shear Stress} = F_s / A$$

A shear stress tends to deform an object

$$\text{Shear Strain} = \Delta X / L = \tan\theta$$

Shear Modulus

$$M_S = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F_s / A}{\Delta x / L} = \frac{F_s / A}{\tan\theta}$$

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# Bulk Modulus

Pressure is defined as the force per unit area $P = F/A$

If the pressure of an object increases

the ratio of the increase in pressure to the fractional decrease in volume is called the bulk modulus

$$B = \frac{\text{Stress}}{\text{Strain}} = -\frac{\Delta P}{\Delta V/V}$$

All stable materials decrease in volume when subjected to an external increase in external pressurethe negative sign means that B is always positive

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# Approximate Values of the Shear Modulus and the Bulk Modulus of Various Materials

| Material | $M_s$ GN/m² | B GN/m² |
|---|---|---|
| Aluminum | 30 | 70 |
| Brass | 36 | 61 |
| Copper | 42 | 140 |
| Iron | 70 | 90 |
| Lead | 5.6 | 8 |
| Steel | 84 | 160 |
| Tungsten | 150 | 200 |

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