# Introduction to Digital communications

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2009

# Typical communication media

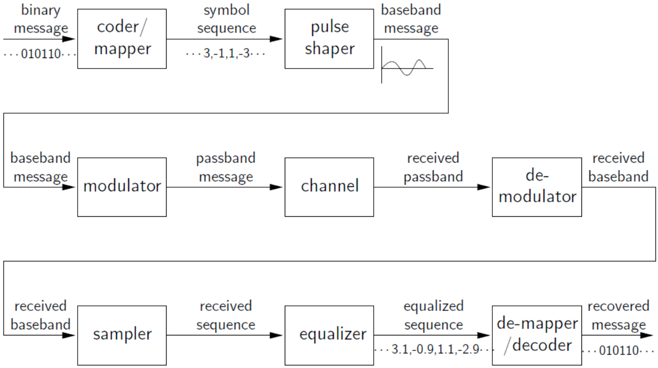
| | | |
|---|---|---|
| twisted pair wire | (e.g., telephone$_A$) | |
| coaxial cable | (e.g., TV$_{A,D}$, data$_D$) | |
| fiber optic cable | (e.g., ethernet$_D$) | |
| EM waves | (e.g., cellular phones$_{A,D}$, WiFi$_D$, TV$_{A,D}$) | |
| water waves | (e.g., underwater network$_{A,D}$) | |
| power lines$_{A,D}$ | | |
| compact disc$_D$ | | |
| hard drive$_D$ | | |
| magnetic tape$_{A,D}$ | | |

# Analog Communications


CSQ :

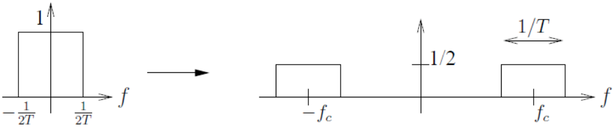
Perfect recovery in the presence of noise is not possible

# Digital communications




Shannon theory / coding, redundancy = perfect transmission is possible, at finite SNR.

# Modulator : from a ’baseband’ or lowpass signal, to a ’passband’ signal



# widely used freq. ranges

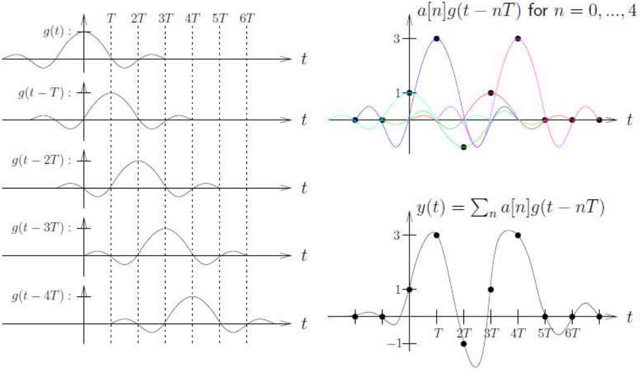
| system | transmission band | $\lambda/10$ |
|---|---|---|
| VHF (TV) | 30–300 MHz | 1–0.1 m |
| UHF (TV) | 0.3–3 GHz | 10–1 cm |
| cellular | 824–960 MHz | 3 cm |
| WiFi | 2.4 GHz | 1 cm |

# Mapper example

| bits | symbol |
|---|---|
| 00 | 3 |
| 01 | -1 |
| 10 | 1 |
| 11 | -3 |

| letter | ASCII code | | | | symbol sequence | | | |
|---|---|---|---|---|---|---|---|---|
| a | 01 | 10 | 00 | 01 | -1 | 1 | -3 | -1 |
| b | 01 | 10 | 00 | 10 | -1 | 1 | -3 | 1 |
| c | 01 | 10 | 00 | 11 | -1 | 1 | -3 | 3 |
| d | 01 | 10 | 01 | 00 | -1 | 1 | -1 | -3 |
| ⋮ | ⋮ | | | | ⋮ | | | |

# Linear modulator example

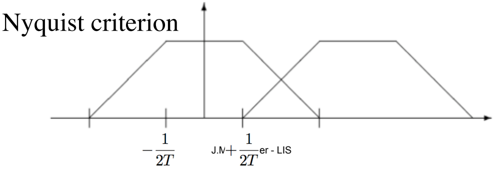



$a[n] = 1, 3, -1, 1, 3....$ -> ISI, synchronisation, channel response......

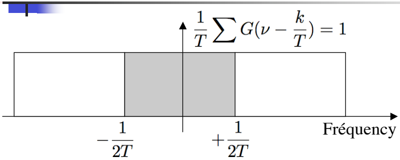
# Intersymbol Interference : problem, constraints

- Two facts:
  - Perfection ( $g(t) = \delta_0$ ) means infinite bandwidth
  - Requirement for zero ISI : $g_k = \delta_k$
- The aim:
  - Perfect discrete channel based on perfect finite bandwidth channel

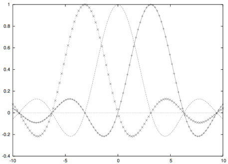
$$g(t)\sum \delta(t-kT) = \delta(t) \leftrightarrow G(\nu) * \frac{1}{T}\sum \delta(\nu - \frac{k}{T}) = 1 \longrightarrow \frac{1}{T}\sum G(\nu - \frac{k}{T}) = 1$$



# Ideal sinc solution



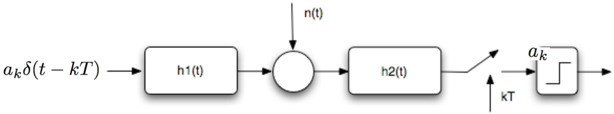

$[-\frac{1}{2T}, +\frac{1}{2T}]$ is the minimum bandwidth for interference free transmission




Strictly rectangular spectrum hard to implement:
- slowly decaying, infinitely long, noncausal waveform
- sensitivity to timing errors
- truncation errors (Gibbs phenomenon)

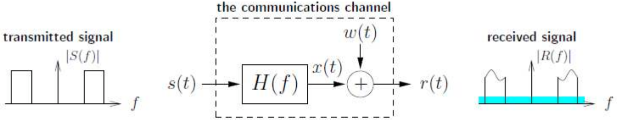
$$y_k = g_0 a_k + \sum_{n \neq k} a_n g_{k-n} + w_k$$

# Accounting for the (linear) channel dispersion (next sections)




where $(h_1 \star h_2)(t) = h(t)$ must satisfy Nyquist criterion !

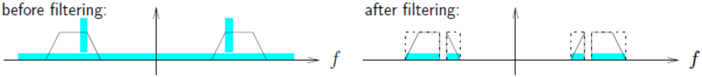
# Linear model of signal propagation



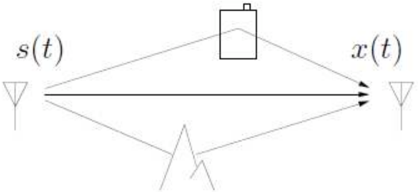

Dispersion (phase distortion), selective attenuation, multipath ....
noise = electronic, multi-access interference, co-channel interference....

$w(n)$ is Additive, white, Gaussian (AWGN)

# Selective filtering for SNR improvment



# Multipath filter model (no fading)

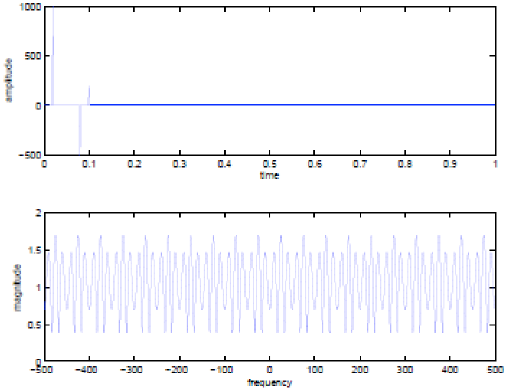



$$x(t) = \sum_{n_{path}} \alpha_n s(t - \tau_n) + n(t)$$

$$\tilde{x}(\nu) = \sum_{n_{path}} \alpha_n \mathrm{e}^{-j2\pi\nu\tau_n} \tilde{s}(\nu) + \tilde{n}(\nu)$$

$$\Rightarrow h(t) = \sum_{n_{path}} \alpha_n \delta(t - \tau_n)$$

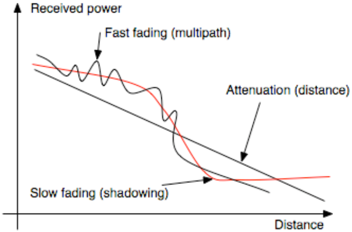
# Multipath filter model example



# Fading scales

- Distance
  - outdoor, indoor

$1/d^n$ with $n \approx 3$ (indoor) $n \approx 4$ (outdoor)




- Slow fading
  - Log-normal

6-10dB, 5 (indoor)-20m (outdoor)

- Fast fading
  - Multipath propagation

# Rayleigh model

- Path: cluster a micropaths:

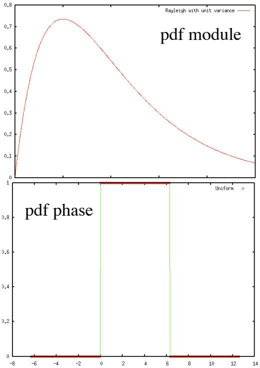
$$\alpha_p(t) = \rho_p(t)e^{i\phi_p(t)} = \sum_n \rho_{p,n}(t)e^{i\phi_{p,n}(t)}$$

- NLOS (No Line of Sight, urban) : CLT:

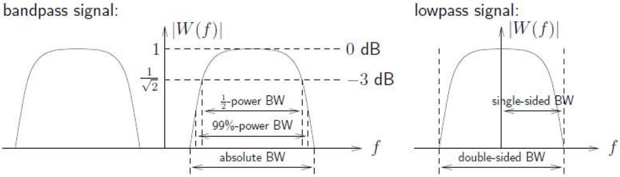
- $\Re e\,[\alpha_p(t)]$ and $\Im m\,[\alpha_p(t)]$ are uncorrelated gaussian with variance $\sigma^2_{\alpha_p}$
- The module $\rho_p(t)$ is Rayleigh :

$$p(\rho) = \frac{\rho}{\sigma^2_\alpha} exp\left(-\frac{\rho^2}{2\sigma^2_\alpha}\right) \text{ for } \rho > 0$$

- The phase $\phi_p(t)$ is uniform over $[0, 2\pi)$

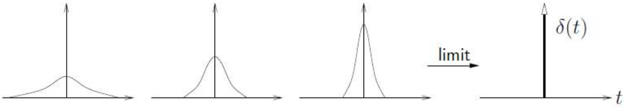


# Real Bandpass signals, bandwidth



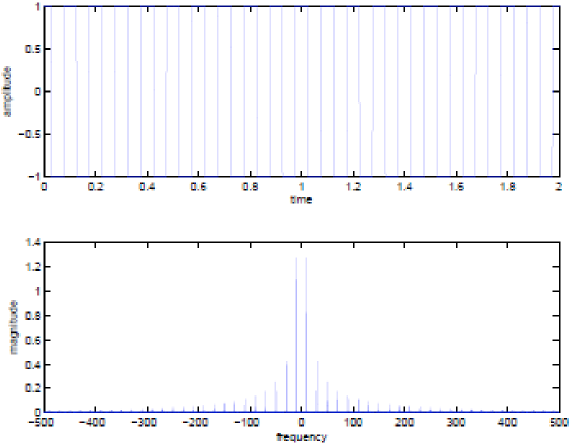
# physical constraints

- $$\Delta t \Delta \nu \geq \frac{1}{4\pi}$$
  => Small $T$ implies large freq!
  BUT Digital Comm. : seeks for narrow pulses and small freq. bandwidth !!!
- Physical 'Dirac' pulse

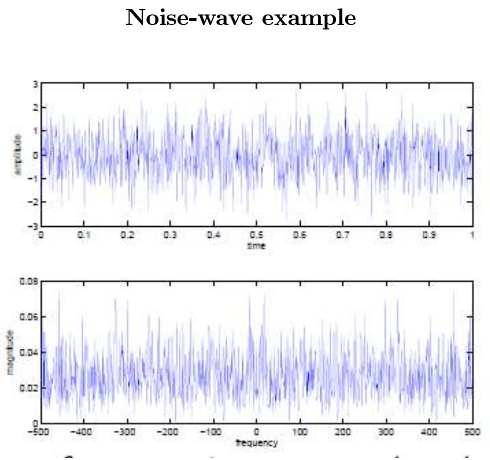


# Fourier spectrum of a periodic deterministic signal

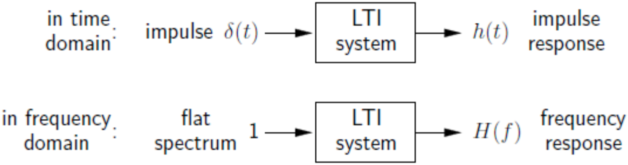
**Square-wave example:**

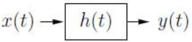


# Fourier spectrum of a random noise (estd)



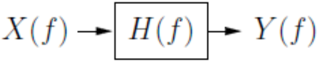
# LTI systems



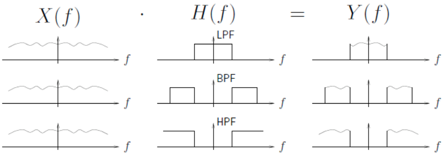




$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

# LTI in frequency domain



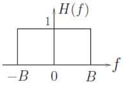

$$Y(f) = H(f)X(f)$$

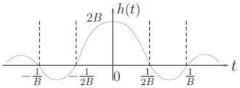


# Examples

## Ideal LPF :

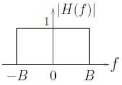
$$H(f) = \begin{cases} 1 & |f| \leq B \\ 0 & |f| > B \end{cases} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad h(t) = 2B\,\mathrm{sinc}(2Bt)$$

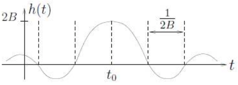


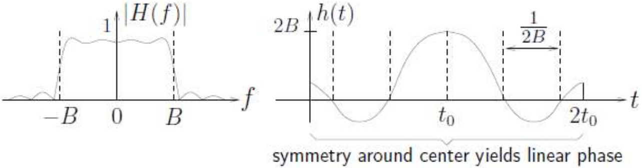

## Ideal delayed LPF :

$$H(f) = \begin{cases} e^{-j2\pi f t_0} & |f| \leq B \\ 0 & |f| > B \end{cases} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad h(t) = 2B\,\mathrm{sinc}\left(2B(t - t_0)\right)$$



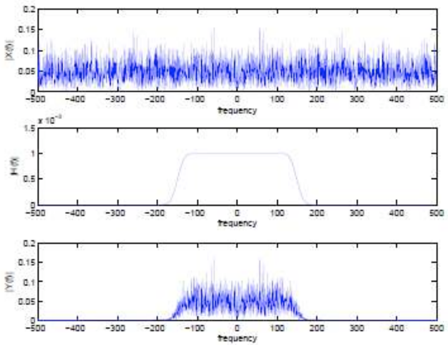



# Real causal linear phase LPF, with group delay $t_0$




We can do better : see linear phase FIR filter design...(last year lecture)

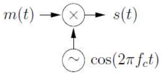
# filtering noise :




$$S_y(\nu) = |H(\nu)|^2 S_x(\nu)$$

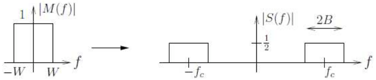
# A briel review of AM modulation

AM with suppressed carrier :



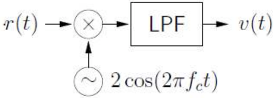

$s(t) = m(t)\cos(2\pi f_c t)$,

$f_c$ = carrier freq.




Rk : if $m(t)$ is real, AM transmitted spectrum is redundant : motivation for QAM !

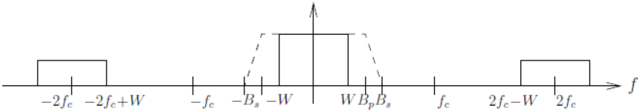
Demodulation (if trivial channel, and $f_c$ is known, and perfect synchro is assessed)




$v(t) = \text{LPF}\{r(t) \cdot 2\cos(2\pi f_c t)\}$.

# continued...

LPF has passband cutoff freq $B_p$, stopband cutoff $B_s$ s.t. $B_p \leq W$ and $B_s < 2f_c - W$ :



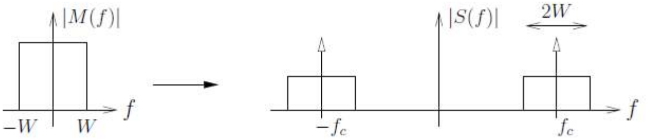

When the receiver local oscillator has freq. and phase offset $\{\delta f; \delta\phi\}$, then

$$v(t) = m(t)\cos(2\pi\delta f t + \delta\phi)$$

(time varying attenuation : left as exercice)

# AM whith carrier

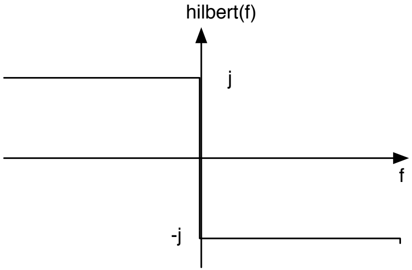
$$s(t) = (m(t) + A)\cos(2\pi f_c t)$$




modern systems : $A << max(m(t))$
Large carrier AM : $A > max(m(t))$, allow enveloppe detection based receivers.

Rk : Carrier - AM transmitted spectrum is redundant, consumes energy (carrier)

# Hilbert transform




Hilbert filter :

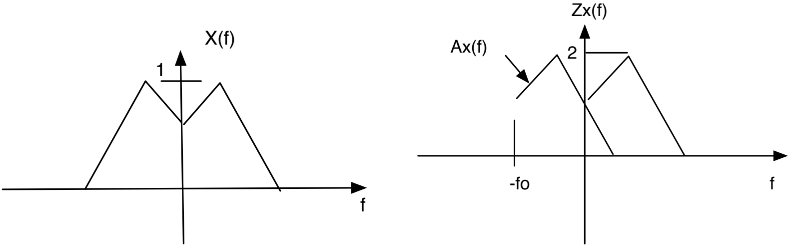
$$\mathcal{H}(\nu) = -j.sign(\nu)$$

Analytic transform of $x$

$$z_x(t) = x(t) + j.\mathcal{H}[x](t)$$

# Complex enveloppe

$$a_x(t) = z_x(t)\mathrm{e}^{-\mathrm{j}2\pi\nu_0 \mathrm{t}}$$




Rk: $x(t)$ real signal, but $z_x(t)$, $a_x(t)$ complex-valued signals.

# Consequences of previous definitions

$\mathcal{H}[x](t) \in \mathbb{R}$, $x(t) \in \mathbb{R} \Rightarrow \ x(t) = Re[z_x(t)]$
and

$$x(t) = Re[a_x(t)e^{j2\pi\nu_0 t}] \qquad \mathcal{H}[x](t) = Im[a_x(t)e^{j2\pi\nu_0 t}]$$

As $a_x(t) \in \mathbf{C} \Rightarrow \qquad a_x(t) = p_x(t) + j.q_x(t)$

$$\begin{aligned} x(t) &= p_x(t)\cos(2\pi\nu_0 t) - j.q_x(t)\sin(2\pi\nu_0 t) \\ \mathcal{H}[x](t) &= p_x(t)\sin(2\pi\nu_0 t) + j.q_x(t)\cos(2\pi\nu_0 t) \end{aligned}$$

"$a_x(t)$ : Baseband equivalent signal , relative to $\nu_0$"

# alternate formulation

$$\begin{bmatrix} x(t) \\ \mathcal{H}[x](t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & -\sin\omega_0 t \\ \sin\omega_0 t & \cos\omega_0 t \end{bmatrix} \begin{bmatrix} p_x(t) \\ q_x(t) \end{bmatrix}$$

thus

$$\begin{bmatrix} p_x(t) \\ q_x(t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t \\ -\sin\omega_0 t & \cos\omega_0 t \end{bmatrix} \begin{bmatrix} x(t) \\ \mathcal{H}[x](t) \end{bmatrix}$$

# Bedrossian's theorem

Let $f(t), g(t)$ such that :

- $F(\nu).G(\nu) = 0 \quad \forall\nu$
- $\left\{ \begin{array}{l} f(t) \text{ is LF}(\Delta F) \\ g(t) \text{is HF} \end{array} \right.$ such that $min[G(\nu)] >> 2\Delta F$

then

$$\mathcal{H}[f.g](t) = f(t).\mathcal{H}[g](t)$$

# Example

Let $x(t) = m(t)\cos(2\pi\nu_0 t + \phi)$, with $\Delta M(\nu) << \nu_0$

The complex enveloppe (relatively to $\nu_0$) of $x$ is

$$a_x(t) = m(t)e^{j\phi}$$

# Introduction to Digital communications -Lecture 2-

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# Bedrossian's theorem

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# Example

Let $x(t) = m(t)\cos(2\pi\nu_0 t + \phi)$, with $\Delta M(\nu) << \nu_0$

The complex envelope (relatively to $\nu_0$) of $x$ is

$$a_x(t) = m(t)e^{j\phi}$$

Actually for e.g. AM, transmitted signal is
$x(t) = m(t)\cos(2\pi\nu_0 t + \phi) = m(t)\cos(\phi)\cos(2\pi\nu_0 t) - m(t)\sin(\phi)\sin(2\pi\nu_0 t)$.

As $\mathcal{H}[\cos(2\pi\nu_0 t + \phi)] = \sin(2\pi\nu_0 t + \phi)$, it comes $z_x(t) = e^{2\pi\nu_0 t + \phi}$ , the complex envelope is $a_x(t)$, and

$$\begin{cases} m(t)e^{j\phi} = p_x(t) + jq_x(t) \\ x(t) = p_x(t)\cos(2\pi\nu_0 t + \phi) - q_x(t)\sin(2\pi\nu_0 t + \phi) \end{cases}$$

# Application to real valued passband signals

Def : $x(t)$ is a deterministic real passband signal if $\exists B \in \mathbb{R}^+$ s.t.

$$\begin{cases} X^+(\nu) & = X(\nu) \quad \text{if} \quad \nu > B \\ X^-(\nu) & = X(\nu) \quad \text{if} \quad \nu < B \\ X^-(\nu) = X^{+^*}(-\nu) & \end{cases}$$

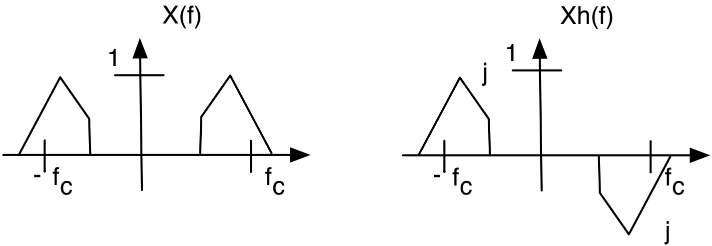
therefore
$$\boxed{\begin{cases} Z_x(\nu) & = 2X^+(\nu) \\ A_x(\nu) & = 2X^+(\nu+\nu_0) \end{cases}}$$

# Equalities for real passband signals

Exercice : As $x(t) = p_x(t)\cos(2\pi\nu_0 t) - q_x(t)\sin(2\pi\nu_0 t)$, prove that

- $$\begin{aligned} X^+(\nu) &= \tfrac{1}{2}[P_x(\nu-\nu_0) + jQ_x(\nu-\nu_0)] \\ X^-(\nu) &= \tfrac{1}{2}[P_x(\nu+\nu_0) - jQ_x(\nu+\nu_0)] \end{aligned}$$
- $A_x(\nu) = 2X^+(\nu+\nu_0) = P_x(\nu) + jQ_x(\nu)$
- $$\begin{aligned} P_x(\nu) &= X^+(\nu+\nu_0) + X^-(\nu-\nu_0) \\ Q_x(\nu) &= \tfrac{1}{j}[X^+(\nu+\nu_0) - X^-(\nu-\nu_0)] \end{aligned}$$

# Spectral interpretation, real passband signals -1-

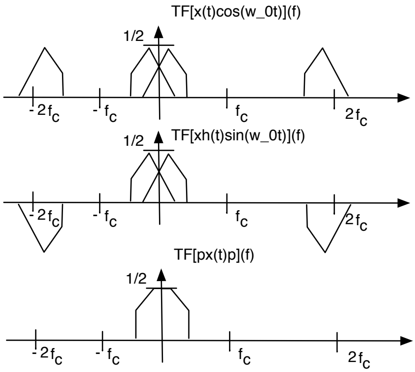



where

$$X_h(\nu) = TF[\mathcal{H}[x(t)](\nu)$$

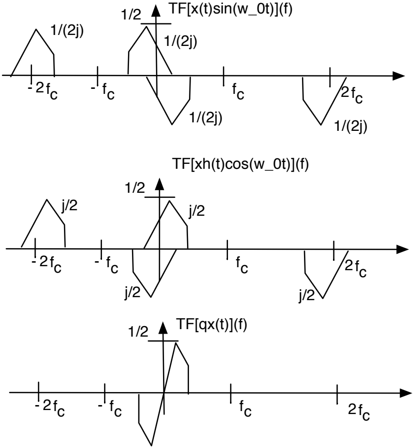
# Spectral interpretation, real passband signals -2-

$$p_x(t) = x(t)\cos(2\pi\nu_0 t) + x_h(t)\sin(2\pi\nu_0 t)$$



# Spectral interpretation, real passband signals -3-

$$q_x(t) = -x(t)\sin(2\pi\nu_0 t) + x_h(t)\cos(2\pi\nu_0 t)$$



# Typical application involving passband signals -1-

Let $x(t)$, $y(t)$ real valued bandpass signals, expressed in terms of their respective in-phase and quadrature components (rel. to $\nu_0$)

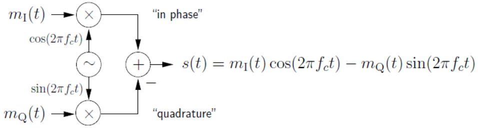
$$\begin{cases} x(t) & = p_x(t)\cos(2\pi\nu_0 t) - q_x(t)\sin(2\pi\nu_0 t) \\ y(t) & = p_y(t)\cos(2\pi\nu_0 t) - q_y(t)\sin(2\pi\nu_0 t) \end{cases}$$

then

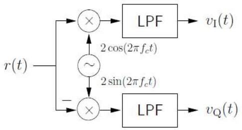
$$\boxed{\begin{aligned} s(t) &= x(t).y(t) \\ &= \frac{1}{2}\left[p_x(t).p_y(t) + q_x(t).q_y(t)\right] + \text{ HF terms around } 2\nu_0 \end{aligned}}$$

# Typical application involving passband signals -2- : QAM

Quadrature Amplitude Modulation




(TRIVIAL CHANNEL)




where $\begin{cases} v_I(t) = m_I(t) \\ v_Q(t) = m_Q(t) \end{cases}$ if perfect synchronization.

# Exercice

Replace $2\cos(2\pi\nu_0 t)$ (resp. sin() by $\cos(2\pi\nu_0 t + \phi)$ to account for lack of phase synchronization. Prove that

$$\begin{cases} v_I(t) & = m_I(t)\cos(\phi) + m_Q(t)\sin(\phi) \\ v_Q(t) & = -m_I(t)\sin(\phi) + m_Q(t)\cos(\phi) \end{cases}$$

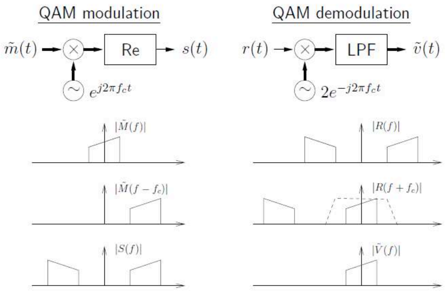
Show that $\phi \neq 0$ leads to some coupling between the in-phase and quadrature components, and to attenuation of both.

# Complex baseband representation of QAM

Writing the complex baseband form

$$\begin{aligned} \tilde{m}(t) &= m_I(t) + jm_Q(t) \\ \tilde{v}(t) &= v_I(t) + jv_Q(t) \end{aligned}$$

yields the much simpler representation of QAM




where if $r(t) = s(t)$, $\tilde{v}(t) = \tilde{m}(t)$

# Rice representation of random processes

Let $x(t)$ be a real-valued a second order stationary, zero-mean random process.

$$\begin{cases} z_x(t) &= x(t) + j.\mathcal{H}[x](t) \\ a_x(t) &= z_X \mathrm{e}^{-2j\pi\nu_0 t} \\ a_x(t) &= p_x(t) + j.q_x(t) = \rho_x(t)\mathrm{e}^{j\phi_x(t)} \\ x(t) &= p_x(t)\cos(2\pi\nu_0 t) - q_x(t)\sin(2\pi\nu_0 t) \end{cases}$$

Expressing $R_{xx}(\tau) = \mathrm{E}[x(t)x^*(t-\tau)]$ as a function of $p_x$, $q_x$, yields

$$x(t) \text{ wide sense stationnary} \Rightarrow \begin{cases} R_{pp}(\tau) = R_{qq}(\tau) \\ R_{pq}(\tau) = -R_{qp}(\tau) = -R_{pq}(-\tau) \\ \mathrm{E}[|a_x(t)|^2] = 2R_{pp}(0) \\ \mathrm{E}[x(t)] = \mathrm{cst} \Rightarrow R_{pq}(0) = 0 \end{cases}$$

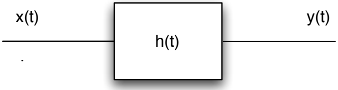
# Narrow band random processes

Def : $x(t)$ is wide sense stationary narrowband random process if its PSD $\gamma_x(\nu)$ is narrowband.
Let $a_x(t) = p_x(t) + j.q_x(t)$ be the complex envelope of $x$, relative to $\nu_0$, then

$a(t)$ is a complex random process, verifying

$$\boxed{\begin{aligned} \gamma_a(\nu) &= 4\gamma^+(\nu + \nu_0) \\ \gamma_p(\nu) &= \gamma_q(\nu) = \frac{1}{4}[\gamma_a(\nu) + \gamma_a(-\nu)] \\ \gamma_{pq}(\nu) &= \frac{1}{4j}[\gamma_a(-\nu) - \gamma_a(\nu)] \end{aligned}}$$

# Complex envelope transformation through filtering (narrowband)




$$Y(\nu) = H(\nu).X(\nu) \quad \Rightarrow Z_y(\nu) = H^+(\nu)Z_x(\nu) = 2H^+(\nu)X^+(\nu)$$
$$A_x(\nu) = 2X^+(\nu+\nu_0) \quad \Rightarrow 2Z_y(\nu+\nu_0) = 2H^+(\nu+\nu_0)Z_x(\nu+\nu_0)$$

yielding

$$\boxed{A_y(\nu) = H^+(\nu+\nu_0)A_x(\nu) = H_{eq}(\nu)A_x(\nu)}$$

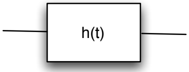
$H_{eq}(\nu) \neq A_h(\nu)$ BUT $H_{eq}(\nu) = \frac{1}{2}A_h(\nu)$
$H_{eq}(\nu)$ is LF shifted version of $H(\nu)$, without correcting factor 2!

# Time domain filter input-output relations for complex baseband signals

$$
\begin{aligned}
a_y(t) &= [H_{eq} \otimes a_x](t) \\
&= \tfrac{1}{2}[(p_h + j.q_h) \otimes (p_x + j.q_x)](t) \\
&= \ldots. \\
p_y(t) &= \tfrac{1}{2}[p_h \otimes p_x](t) - \tfrac{1}{2}[q_h \otimes q_x](t) \\
q_y(t) &= \tfrac{1}{2}[q_h \otimes p_x](t) + \tfrac{1}{2}[p_h \otimes q_x](t)
\end{aligned}
$$

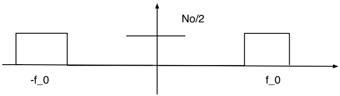
# Baseband formulation of interference formula

$\gamma_b(\nu)$




$\gamma_s(\nu) = |H(\nu)|^2 \gamma_b(\nu)$

where $\gamma_b(\nu)$ :




Applying previous formula leads to

$$
\begin{aligned}
\gamma_{a_b} &= 4\gamma_b^+(\nu + \nu 0) = 2N_0 \\
\gamma_{a_s} &= |H_{eq}(\nu)|^2 2N_0
\end{aligned}
$$

# continued

furthermore

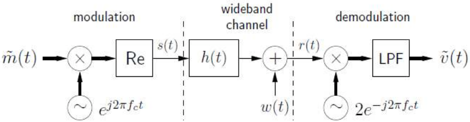
$$\begin{aligned}\gamma_{p_s}(\nu) &= \gamma_{q_s}(\nu) = \frac{1}{4}[\gamma_{a_s} + \gamma_{a_s}(-\nu)] \\ &= \frac{N_0}{2}[|H_{eq}(\nu)|^2 + |H_{eq}(-\nu)|^2]\end{aligned}$$

and

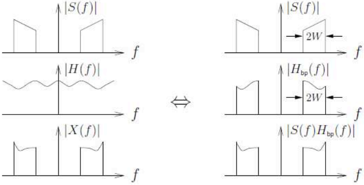
$$\gamma_{pq_s}(\nu) = \frac{N_0}{2j}[|H_{eq}(\nu)|^2 - |H_{eq}(-\nu)|^2]$$

- $\gamma_{pq_s}(\nu) = 0$ if $H_{eq}(\nu)$ is symetric (i.e. if $H^+(\nu)$ is symetric around $\nu_0$)
- In-phase and quadrature component have identical variances

$$\sigma^2 = N_0 \int_{-\infty}^{\infty} |H_{eq}(-\nu)|^2 d\nu$$

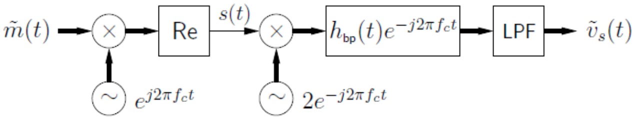



...because filtering $s(t)$ with $h(t)$ is equivalent to filtering $s(t)$ with $h_{bp}(t)$:

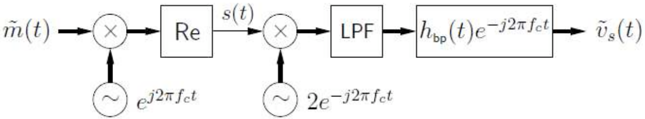


# contd...

as
$s(t) \otimes h_{bp}(t)]2\mathrm{e}^{-j2\pi f_c t} = [s(t)2\mathrm{e}^{-j2\pi f_c t}] \otimes [h_{bp}(t)\mathrm{e}^{-j2\pi f_c t}]$ :

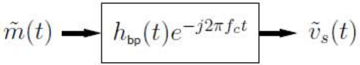



then reversing the order of the LTI systems :

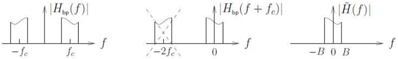


# Consequences :

- mod/demod are transparent (with synch oscillators) :

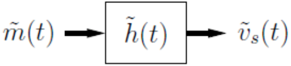



- $TF[h_{bp}(t)\mathrm{e}^{-j2\pi f_c t}]$ :

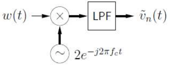


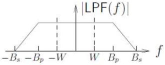

$H_{eq}(\nu) = \tilde{H}(\nu)$!

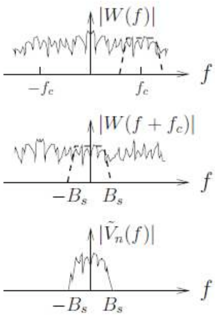
- finally, for the noiseless complex baseband channel :



# Noisy channel (additive)

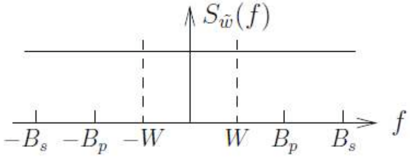


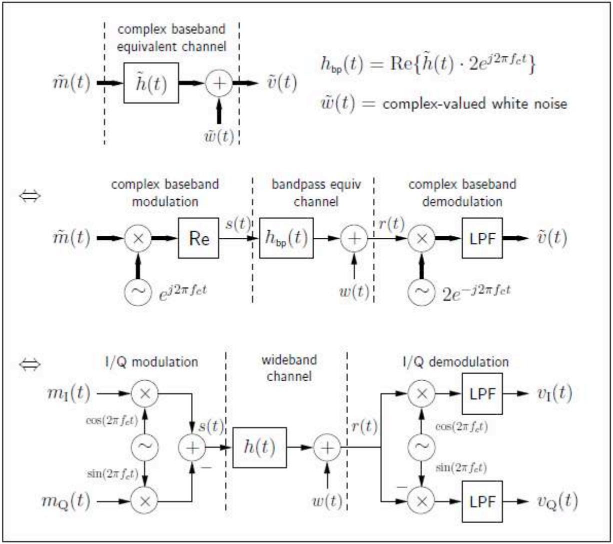



# Noisy channel (contd)

- from previous studies on complex baseband random process,
- if noise PSD is CONSTANT ( $\frac{N_0}{2}$ ) over the freq-range of interest,
- as all contribution outside $[-W, W]$ will be filtered out, considering that $\tilde{H}(\nu) = 1 \forall \nu \in [-W, W]$ the noise is modeled by a white (cst PSD $2N_0$) noise



# Summary


complex baseband
equivalent channel
$\tilde{m}(t)$
$\tilde{h}(t)$
$\tilde{v}(t)$
$\tilde{w}(t)$
$h_{bp}(t) = \mathrm{Re}\{\tilde{h}(t) \cdot 2e^{j2\pi f_c t}\}$
$\tilde{w}(t)$ = complex-valued white noise
⇔
complex baseband
modulation
bandpass equiv
channel
complex baseband
demodulation
Re
$s(t)$
$h_{bp}(t)$
$r(t)$
LPF
$e^{j2\pi f_c t}$
$w(t)$
$2e^{-j2\pi f_c t}$
⇔
I/Q modulation
wideband
channel
I/Q demodulation
$m_I(t)$
$\cos(2\pi f_c t)$
$\sin(2\pi f_c t)$
$m_Q(t)$
$h(t)$
$v_I(t)$
$v_Q(t)$

# Introduction to Digital communications -Lecture 3-

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2009

# Digital modulation definition

Discrete symbols $a_n \in \Omega$ → Continuous time series $\tilde{m}(t)$

$$\begin{cases} a_n & \simeq a(nT) \\ |\Omega| & = M \\ \frac{1}{T} & = R_s \qquad \text{"symbol rate"} \end{cases}$$

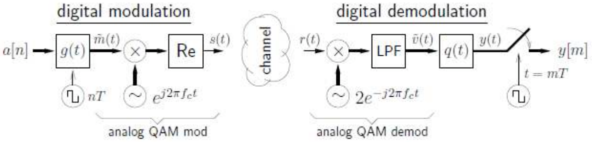
where

Channel coding : $u_k \leftrightarrow$ sequence of bits
therefore

$$R_b = \frac{1}{T} \log_2 M \qquad \text{bits/sec}$$

# Digital communication system

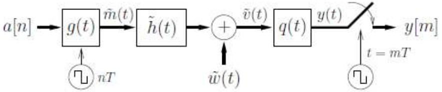
Transmitter : -pulse shaping : $\tilde{m}(t) = \sum_n a[n]g(t - nT)$
-modulation : $s(t) = Re\{\tilde{m}(t)e^{j2\pi f_c t}\}$




Receiver : -demodulation : $\tilde{v} = \mathrm{LPF}\{2r(t)e^{j2\pi f_c t}\}$
-filtering : $y(t) = \tilde{v}(t) \otimes q(t)$
-sampling : $y[m] = y(mT)$

# Digital communication system, contd

**C**-baseband :

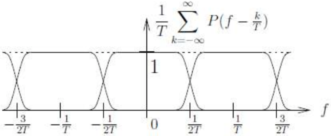



where for the noiseless channel

$$g(t) \otimes \tilde{h}(t) \otimes q(t) = p(t)$$

verifies NYQUIST ISI supression criterion

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} P\left(f-\frac{k}{T}\right) = 1.$$

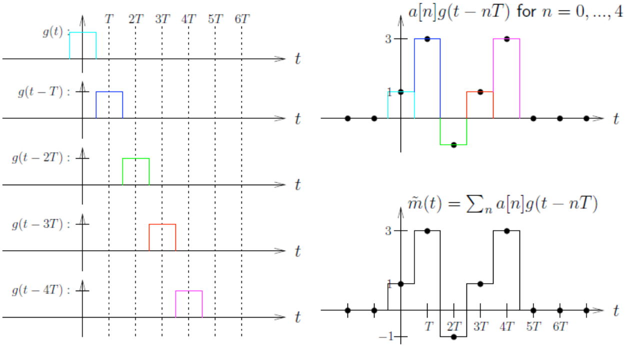


# Linear digital modulations

baseband message :

$$\tilde{m}(t) = \sum_n a[n]g(t - nT)$$

example for $a[n] = [1, 3, -1, 1, 3]$, with non-realistic $g(t)$ :



# PSD of linear digital comm. signal

$$\tilde{m}(t) = \sum_n a[n]g(t - nT)$$

$$\begin{aligned}\Gamma_m(t,\tau) &= \mathrm{E}[m(t)m^*(t-\tau)] \\ &= \sum_k \sum_{k'} \mathrm{E}[a[k]a^*[k']]g(t-kT)g^*(t-k'T)\end{aligned}$$

Assuming that $(a[k])$ is a wide sense stationary process :

$$\mathrm{E}[a[k]a^*[k']] = \Gamma_a(k-k')$$

$$\Rightarrow \Gamma_m(t,\tau) = \sum_l \Gamma_a(l) \sum_k \underbrace{g(t-kT)g^*(t-\tau-(k-l)T)}_{\text{Depends upon both } t \text{ and } \tau\text{ !}}$$

BUT

$$\Gamma_m(t+T,\tau) = \Gamma_m(t,\tau) \Rightarrow \text{ CYCLOSTATIONARITY}$$

# PSD of linear digital comm. signal, contd

$$\overline{\Gamma}_m(\tau) = \frac{1}{T}\int_0^T \Gamma_m(t,\tau)dt$$

if $g(t) \in \mathbb{R}$,

$$\boxed{\overline{\gamma}(f) = \frac{|G(f)|^2}{T}\sum_l \Gamma_a(l)\mathrm{e}^{-j2\pi flT}}$$

Letting $\begin{cases} m_a = \mathrm{E}[a]; \quad \sigma_a^2 = \mathrm{E}[a^2 - \mathrm{E}[a]] \\ \tilde{a} = \frac{a-\mathrm{E}[a]}{\sigma_a} \end{cases}$

$$\boxed{\overline{\gamma}(f) = \frac{|G(f)|^2}{T}\sum_l \left(\sigma_a^2\Gamma_{\tilde{a}}(l) + |m_a^2|\right)\mathrm{e}^{-j2\pi flT}}$$

# PSD of linear digital comm. signal, contd

As $\Gamma_{\tilde{a}}(l) = \Gamma^*_{\tilde{a}}(-l)$,

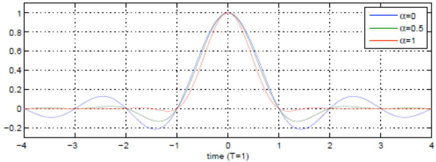
$$\begin{aligned}
\overline{\gamma}(f) \quad &= 2\sigma_a^2 \frac{|G(f)|^2}{T} \sum_{l=1}^{\infty} Re\left(\Gamma_{\tilde{a}}(l)\mathrm{e}^{-j2\pi flT}\right) && (1)\\
&+\sigma_a^2 \frac{|G(f)|^2}{T} && (2)\\
&+\frac{|m_a^2|}{T^2} \sum_k \left|G(\tfrac{k}{T})\right|^2 \delta(f-\tfrac{k}{T}) && (3)
\end{aligned}$$

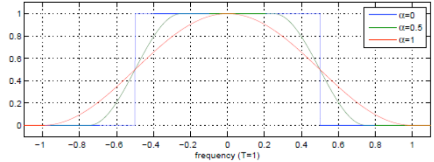
- (1)(2) : continuous part of the PSD; (3) : discrete part
- $(3) = 0$ if $|m_a| = 0$
- $G(0) = 0 \Rightarrow \overline{\gamma}_e(0) = 0$
- (1) is an ordinary function of $f$ if $\Gamma_a(l) \xrightarrow{l\to\infty} 0$ quickly enough

# Most popular pulse shape

$$g(t) = \frac{\cos(\frac{\pi\alpha t}{T})}{1-(\frac{2\alpha t}{T})^2}\mathrm{sinc}(\tfrac{t}{T}), \qquad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$G(f) = \begin{cases} T & |f| \le \frac{(1-\alpha)}{2T} \\ T\cos^2\left(\frac{\pi T}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right) & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0 & \frac{1+\alpha}{2T} \le |f| \end{cases}$$




$\alpha$ = roll-off parameter
larger $\alpha \Rightarrow$ less time-spread
more freq-spread

# General representations of Linear digital modulations

General expression of baseband message :

$$\tilde{m}(t) = \sum_n a_p[n]g_p(t - nT) + j.a_q[n]g_q(t - nT)$$

and

$$m(t) = \sum_n a_p[n]g_p(t - nT)\cos(2\pi f_c t) - a_q[n]g_q(t - nT)\sin(2\pi f_c t)$$

Example : Pulse Amplitude Modulation (PAM)

$$\boxed{\tilde{m}(t) = \sum_n a_p[n]g_p(t - nT), \qquad a[n] = (2k - 1 - M), k \in \{1, 2, \ldots, M\}}$$

i.e. $g_p(t) = g(t); g_q(t) = 0$

# Signal space dimension

Definition : Let $m(t) = \sum_n s(t - nT, a_n)$. The dimension $N$ of the signal space is the dimension of the real-valued functional space spanned by the signals $s(t, a)$.

- PAM : $\tilde{m}(t) = \sum_n a[n]g(t - nT)$,
where $a[n] \in \mathbb{R}$ and $g(t) \in \mathbb{R} \Rightarrow \tilde{m}(t) \in \mathbb{R}$, thus $N_{PAM} = 1$

- QAM : $\tilde{m}(t) = \sum_n g(t - nT)[a_p[n] + j.a_q[n]])$ where $(a_p, a_q) \in \mathbb{R}^2$,
and $g(t) \in \mathbb{R} \Rightarrow \tilde{m}(t) \in \mathbf{C}$, thus $N_{QAM} = 2$

*Rk : For OFDM or FSK, $N > 2$*

# Energy

Definition : Let $m(t) = \sum_n s(t - nT, a_n)$. The energy requested for transmitting a single symbol is $\mathcal{E}(a) = ||s(a)||^2$ ([1])

The average energy spend per symbol is, for $|\Omega| = M$ and uniform probability of all symbols :

$$\mathcal{E}_s = \frac{1}{M} \sum_{a \in \Omega} \mathcal{E}(a)$$

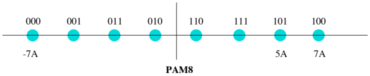
The average energy per bit is then

$$\mathcal{E}_b = \frac{1}{\log_2(M)} \mathcal{E}_s$$

[1] $||s(a_n)||^2 = \int_{(n-1)T}^{T} s^2(t, a_n) dt$

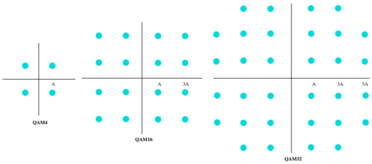
# Signal space examples

PAM :




$$\mathcal{E}_{s,PAM} = \frac{M^2-1}{3} A^2 (^2)$$

QAM :



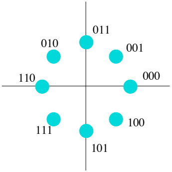

$$\mathcal{E}_{s,PAM} = \frac{2(M-1)}{3} A^2, \; (if \exists k / M = k^2)$$

---

$^2 1^2 + 2^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

PSK :

$$\begin{cases} \tilde{m}(t) = A\sum_n g(t-nT)\mathrm{e}^{j\phi(a[n])} \\ m(t) = A\sum_n g(t-nT)\cos(2\pi f_c t + \phi(a[n])) \end{cases}$$




PSK8

$$\mathcal{E}_{s,PSK} = A^2$$

# Differential modulations

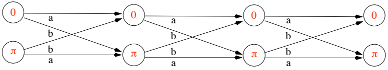
Perfect phase locking of the receiver : impossible

- phase rotation in PSK or QAMq $\rightarrow$ errors in symbol detection

$\Rightarrow$ Encode phase jumps, resulting in rotation invariant modulations

- phase rotation invariance, 'infinite modulation memory' to encode initial phase
- demod : $y_k = e^{j(\phi_k - \theta)}, < y_k, y_{k-1} >= e^{j(\phi_k - \phi_{k-1})} = e^{ja[k]}$

Example : trellis representation of a Differential Binary PSK ($M = 2$)




state changes are associated to phase jumps of $\pi$.

# Offset modulation

Pb : phase jumps $\Rightarrow$

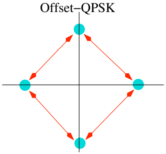
$$\begin{cases} \text{freq spread} \\ \text{high amplitude fluctuations, out of linear range of HF amplifiers} \end{cases}$$

Solution : $\pi$ phase jumps are not allowed !

Example : Offset QPSK :

$$g_p(t) = g(t - \tfrac{T}{2}) \qquad \text{and } g_q(t) = g(t)$$
$$m(t) = \sum_n a_p(t) g(t - nT - \tfrac{T}{2}) - a_q(t) q(t - nT)$$

$\Rightarrow$ $Re(\tilde{m}(t)$ and $Im(\tilde{m}(t)$ do not change simultaneously :



# Constant envelope modulation

Motivation : bounding the amplitude fluctuation (to ensure linear range operation of the HF electronic devices)
$\rightarrow$ Frequency modulation (FSK) with continuous phase (CPM).

FSK : $x(t) = \cos\left(2\pi f_c t + 2\pi f_d \int_{-\infty}^{t} m(\tau)d\tau\right)$, $m(\tau) = \sum_n a_n g_d(\tau - nT)$

Instantaneous frequency : $f_c + f_d m(t)$

# FSK example

Let $g_d(t) = \Pi_T(t)$ and $f_d m(t) = a[n]\frac{h}{2T}$

- $\Rightarrow$ phase of **C**-envelope ($\tilde{x}$) is $\int_{-\infty}^{t} f_d m(\tau) d\tau$ is piecewise linear.
- $\Rightarrow$ if $f_d$ is PAM (as often), then frequencies are separated by $\frac{h}{T}$ :
  $\boxed{h : \text{modulation index of FSK}}$
- phase jump between 2 consecutives 'symbols' = 0 $\rightarrow (f_1 - f_2)T$ is $\frac{1}{2}$-integer (then $h$ is $\frac{1}{2}$ integer).
  where

$$\begin{cases} x(t) = \cos(2\pi \left(f_0 t + \int_{t_0}^{t} m(\tau) d\tau\right) \\ m(\tau) : \frac{1}{4T} \sum_n a[n] g_d(t - kT), \qquad a[n] \in \{-1; +1\} \end{cases}$$

- this is Min Shift Keying (MSK) for a BPSK.

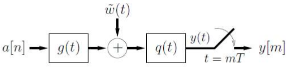
# Modulation standard examples

| Standard | Modulation type |
| --- | --- |
| DECT | GFSK[3] |
| GSM | GMSK |
| UMTS | QPSK |
| Modem v34kbit/s | QAM-1664 |

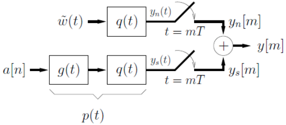
[3] GMSK = MSK where the binary data flow is pre-filtered (before F-modulation), to reduce sideband power.

# Performances

First consider the transmission of a unique symbol :
baseband representations of the communication system :




$\Leftrightarrow$




where $\mathrm{E}[\tilde{w}(t)\tilde{w}^*(t)] = N_0\delta(t)$ (complex white noise)

Ideal channel, no ISI $\Rightarrow \begin{cases} y_s[m] & = \sum_n a[n]p((m-n)T) = a[m]p(0) \\ p(0) & = \int_{-\infty}^{\infty} q(\tau)g(-\tau)d\tau \\ \mathcal{E}_s & = \mathrm{E}[|y_s[m]|^2] = \sigma_a^2 p(0)^2 \end{cases}$

# Performances, contd

as $y_n[m] = y_n(mT) = \int_{-\infty}^{\infty} q(\tau)\tilde{w}(mT-\tau)d\tau$,

$$\mathcal{E}_n = \mathrm{E}[|y_n[m]|^2] = N_0 \int_{-\infty}^{\infty} |q(\tau)|^2 d\tau$$

Then , the pdf of the observation is

$$f(y[m]|a[m]) = \frac{1}{\sqrt{2\pi\mathcal{E}_n}} \mathrm{e}^{-\frac{(y[m]-a[m]p(0))^2}{2\mathcal{E}_n}}$$

# Performances, contd

Considering e.g. PAM2 : $a[m] \in \{-1, +1\}$ , and a simple threshold detector (threshold $\eta$)

$$\begin{cases} P_{FA} = \Pr(y[m] > \eta | a[m] = -1) = \int_{\eta}^{\infty} f(y|a = -1) dy \\ P_{M} = \Pr(y[m] < \eta | a[m] = +1) = \int_{-\infty}^{\eta} f(y|a = +1) dy \end{cases}$$

Letting $\mathrm{erf}(x) = \frac{1}{\sqrt{x}} \int_0^x \mathrm{e}^{-t^2} dt$,

$$\begin{cases} P_{FA}(\eta) = \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{\eta + p(0)}{\sqrt{2\mathcal{E}_n}}\right) \\ P_{M}(\eta) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\eta - p(0)}{\sqrt{2\mathcal{E}_n}}\right) \end{cases}$$

and $$\boxed{P_{err}(\eta) = \Pr(a[m] = -1) P_{FA}(\eta) + \Pr(a[m] = +1) P_{M}(\eta)}$$

# Performances, contd

$P_{err}(\eta) = \Pr(a[m] = -1)P_{FA} + \Pr(a[m] = +1)P_M \Rightarrow$ choosing $\eta$ to minize $P_{err}$ ?

$$\boxed{\frac{\partial P_{err}}{\partial \eta} = 0 \Rightarrow \eta_{opt} = \frac{\mathcal{E}_n}{2p(0)} \log\left(\frac{p_0}{p_1}\right)}$$

where $p_0 = \Pr(a[m] = -1)$ and $p_1 = \Pr(a[m] = +1)$

# Optimizing the receiver (ideal channel)

Let $\rho = \frac{p(0)}{\sqrt{2\mathcal{E}_n}} \simeq \mathrm{SNR}$, and let $k = \frac{1}{4}\log\left(\frac{p_0}{p_1}\right)$
then

$$P_{err}(\eta_{opt}) = \frac{p_0}{2}\mathrm{erfc}(\rho + \frac{k}{\rho}) + \frac{p_1}{2}\mathrm{erfc}(\rho - \frac{k}{\rho})$$

As minimizing $P_{err} \Leftrightarrow$ maximizing $\rho$, and reexprssing $\rho$ :

$$\rho = \frac{p(0)}{\sqrt{2\mathcal{E}_n}} \propto \frac{\int_{-\infty}^{\infty} q(\tau)g(-\tau)d\tau)}{\sqrt{N_0}\left[\int_{-\infty}^{\infty} |q(\tau)|^2 d\tau\right]^{\frac{1}{2}}}$$

By Cauchy-Schwartz inequality, $\rho$ is maximum if $\exists\lambda$ such that

$$q(t) = \lambda g^*(-t)$$

# Optimizing the receiver (contd)

$q(t) = \lambda g^*(-t)$ : MATCHED FILTER EQUATION
Then by properly choosing $\lambda$,

$$\rho_{opt} = \sqrt{\frac{E_g}{N_0}}$$

and if $p_0 = p_1$ (then $k = 0$),

$$P_{err} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_g}{N_0}}\right)$$

finally for Best performance of the ISI free channel :

-$G(f)Q(f) = P(f)$ must satisfy the Nyquist criterion
-$Q(f) = G*(f)\mathrm{e}^{j2\pi f t_0}$

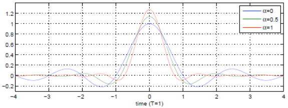
# Optimizing the receiver (contd)

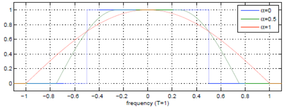
From preceding equation :
$|G(f)|^2$ must satisfy the Nyquist criterion. One option is

$$G(f) = \sqrt{P_{rc}(f)}$$

as $P_{rc}(f)$, the raised cosine filter, is Nyquist. The 'square-root raised cosine pulse' is

$$g_{srrc}(t) = \frac{(1-\alpha)\mathrm{sinc}(\frac{t}{T}(1-\alpha))}{1-(4\alpha\frac{t}{T})^2} + \frac{4\alpha\cos(\pi\frac{t}{T}(1+\alpha))}{\pi(1-(4\alpha\frac{t}{T})^2)}$$



# Introduction to Digital communications -Lecture 4-

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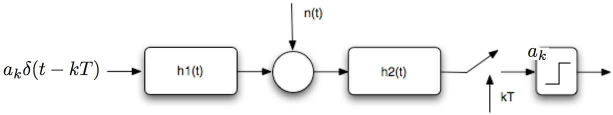
2009

# Real channel, facts :

- Channel impulse response $\neq \delta(t-\tau)$ (*Except on restricted band : case of multiuser freq. multiplexing*)
- Baseband signal PSD has infinite freq. support $\Rightarrow$ multi-user interferences : coder output MUST be filtered.
- Physical channel introduces attenuation, dispersin; e.g. coax. cable or paired wires ($\sqrt{f}$ attenuation).
- Channel selectivity, dur to multiple paths ... (for modulations with carriers)

# No ISI condition :

Accounting for the (linear) channel dispersion :




where

$(h_1 \star h_2)(t) = h(t)$ must satisfy Nyquist criterion :

$$h(t)\sum_k \delta(t-kT) = \delta(t) \quad \Leftrightarrow H(\nu) \otimes \frac{1}{T}\sum_k \delta(\nu - \frac{k}{T}) = 1$$
$$\Leftrightarrow \frac{1}{T}\sum_k H(\nu - \frac{k}{T}) = 1$$

- This warrant the existence of a unique $t_0$ over each time interval $T$
- In general :
  $y(t_0 + nT) = a_n r(t_0) + \sum_{k' \neq 0} a_{n-k'} r(t_0 + k'T) + w(t_0 + nT)$
  where $(n-k) = k'$, $r(t_0) = g \otimes h_1 \otimes h_2)(t)$
- $\sum_{k' \neq 0} a_{n-k'} r(t_0 + k'T) =$ ISI TERM

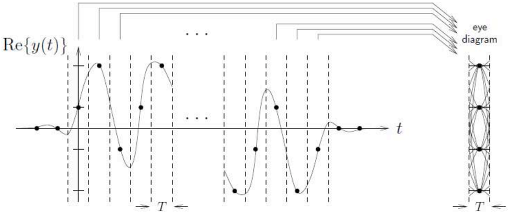
# EYE diagram

Definition :

$$z(t) = \sum_m r(t - mT)$$

Definition : Eye diagram : set of all possible trajectories of $z(t)$ over a time interval $T$

Csq : if $r(t) \neq O$ over $[t_0 - L_1 T, t_0 + L_2 T]$, then $\exists (L_1 + L_2 + 1)$ different sample segments of $z(t)$



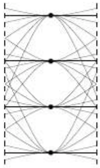

(No ISI here at $t_0 = \frac{T}{2}$)

# EYE Diagram, contd

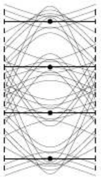
- if $r(t)$ satisfies Nyquist criterion, not $r(t) + w(t)$.
- Underlying hypothesis of perfectly synchronized system doest not generally hold
- In general, Nyquist criterion is not strictly met

Consequences :

When the eye is "open," decisions will be reliable:



As the eye "closes," decisions get more unreliable:



# EYE Diagram, contd

The eye diagram accounts for ALL possible segments of $z(t) \Rightarrow$ it is T-periodic Important remarks :

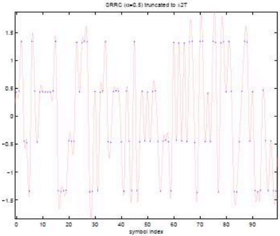
- The eye diagram accounts for ALL possible segments of $z(t) \Rightarrow$ it is T-periodic
- Satisfying Nyquist criterion IMPOSES

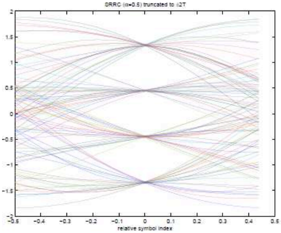
  $\boxed{\text{Freq support } (R(\nu)) \geq \frac{1}{T}}$

  "One cannot send a sequence of symbol at a rate of $\frac{1}{T}$ over a frequency bandwith smaller than $\frac{1}{T}$"

# EYE Diagram, contd

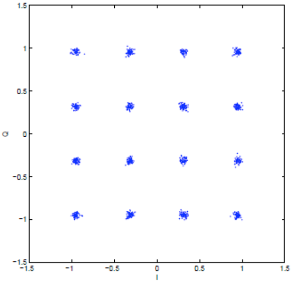
Example for the square root raised cosine ($r(t)$) :

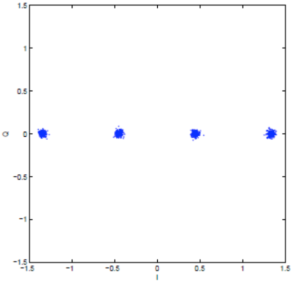


# Constellation diagrams

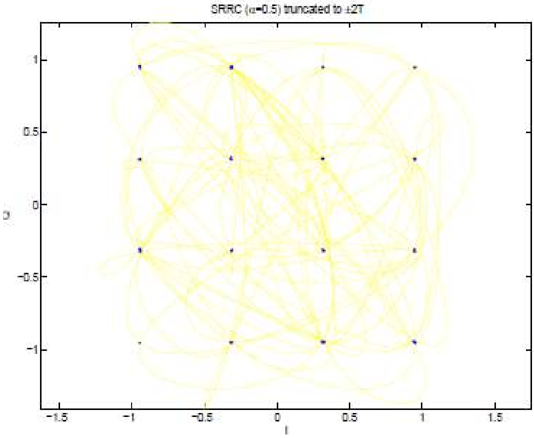
This is the plot $Im[y(n)]$ vs $Re[y(n)]$ for many integers $n$.
(Reminder : $y(n)$ is the complex baseband representation of the received signal, $a(n) \in \mathbf{C}$)
if everything works well, eq for QAM16 or PAM4 :



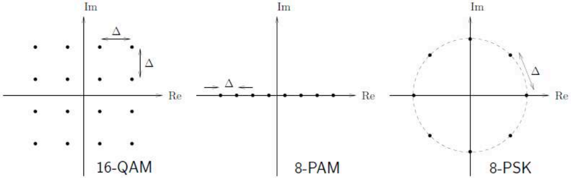
# Constellation diagrams, contd

Complex trajectory of the received signal : This is the plot $Im[y(t)]$ vs $Re[y(t)]$ for all possible sequences (here QAM16, SRRC)



# Decision regions

Remind the most popular modulations , and associated variance (uniform proba over the alphabet)




| alphabet | $M^2$-QAM | $M$-PAM | $M$-PSK |
|---|---|---|---|
| ${\sigma_a}^2$ | $\frac{\Delta^2}{6}\left(M^2-1\right)$ | $\frac{\Delta^2}{12}\left(M^2-1\right)$ | $\frac{\Delta^2}{4sin^2(\pi/M)}$ |

# Decision regions

Decision rule (DR) :

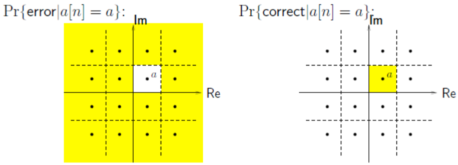
$$y(n) \overset{\text{Nearest Neighbor mapping}}{\longrightarrow} a(n) \in \Omega$$

- Consequence :
  Decision regions = Voronoi diagram of the constellation.
- Definition : Symbol Error Rate (SER) :
  $Proba[DR[Y(n)] \neq a | a(n) = a]$
- for M-PAM, and gaussian noise (zero-mean, $\sigma_n^2$)

$$SER_{M_PAM} = \left(\frac{M-1}{M}\right) \operatorname{erfc}\left(\sqrt{\frac{3\sigma_a^2}{2(M^2-1)\sigma_n^2}}\right)$$

# Decision regions, contd

- SER for $M^2$-QAM, circular white gaussian noise (zero-mean, $\sigma_n^2$)
  - additive noise variance $\frac{\sigma_n^2}{2}$ on $Im[r]$ and on $Re[r]$
  - integration on **C**-plane
  - 4 corner points, $4(M-2)$ edge points, $M^2-4M+4$ interior points.



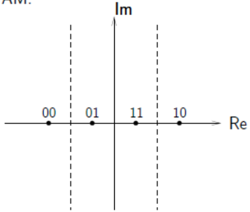

calculate $Proba[Error|a[n]=a] = 1 - Proba[correct|a[n]=a]$, (simpler).

$$SER_{M^2-QAM} = 1 - \left[1 - \frac{(M-1)}{M}\operatorname{erfc}\left(\sqrt{\frac{3\sigma_a^2}{2(M^2-1)\sigma_n^2}}\right)\right]^2$$

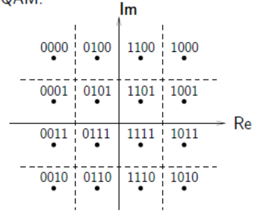
# SER and Bit Error Rate : Gray coding

if $|\Omega| = M$ 1 symbol error causes potentially $M$ bit errors! Gray coding allow to impose BER $\simeq$ SER

4-PAM:




16-QAM:



# Motivations, goals

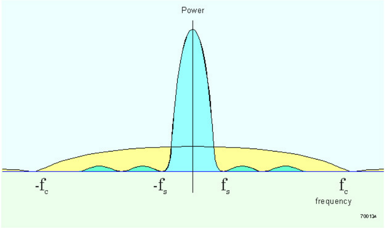
Originally :
- Provide robustness wrt jamers (military or secured communications)
- Lower probability of interception by lowering PSD of emitted signals

Modern applications :
- Robustness wrt echoes (multipaths), multi-users interferences
- CDMA, FDMA

# Spread spectrum

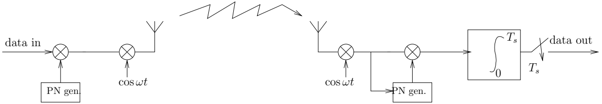
> *"Spread spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread is accomplished by means of a code which is independent of the data, and a synchronized reception with the code at the receiver is used for despreading and subsequent data recovery"*. [1]

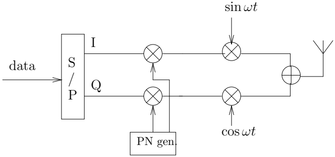
R.L. Pickholtz, D.L. Schilling a. L.B. Milstein, "Theory of Spread-Spectrum Communications-A Tutorial", IEEE Transactions on Communications, vol. Com30, no. 5, May 1982, pp. 855-884

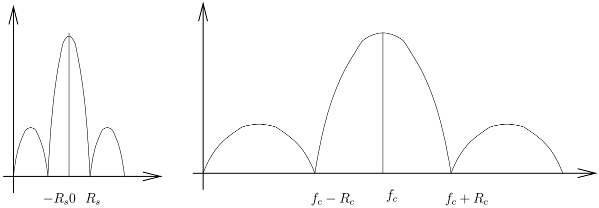


# DSSS principle

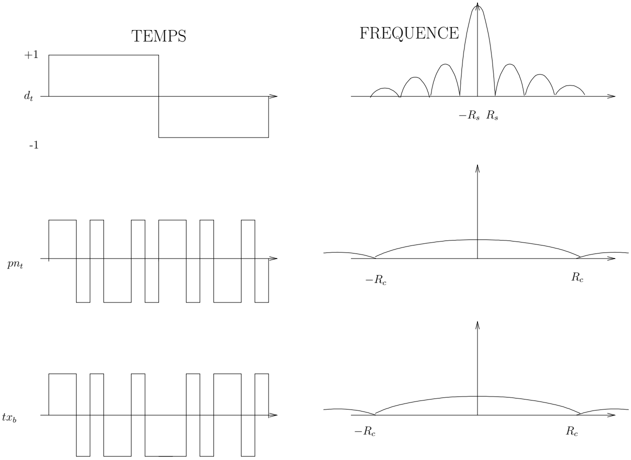
Increasing artificially the data rate ⇔ spreading the spectrum



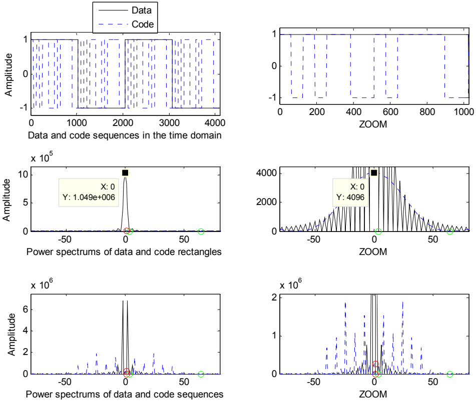



# DSSS principle



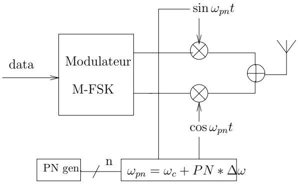
# DSSS principle

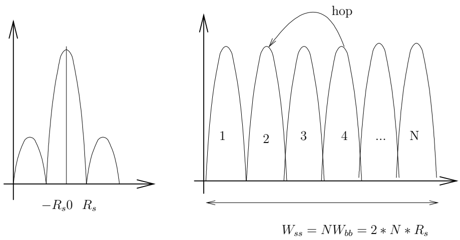
Example : chip rate= 16*(bit rate)



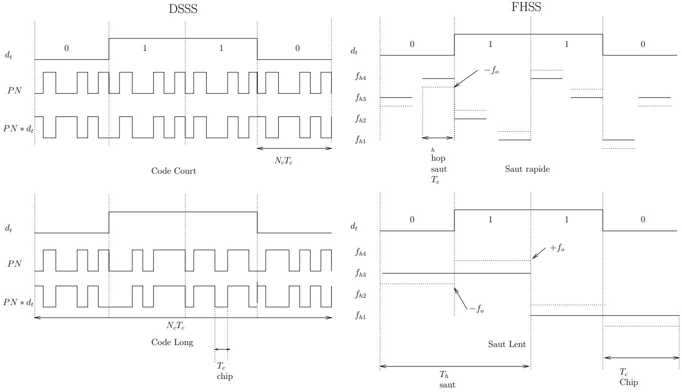
# FHSS principle

PN generator drives instantaneous frequency :



# Main types of DSSS and FHSS



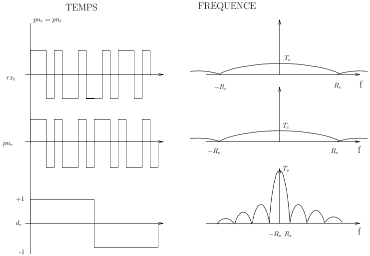

Definition :

$$\text{Spreading Factor} = \frac{W_{ss}}{W_d} = \frac{R_c}{R_d} = SF$$

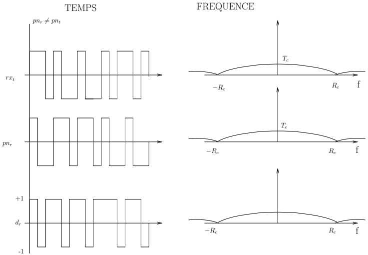
Where $W$ = bandwidth, $R_c$ =Chip rate, $R_d$ =symbol rate

# DSSS : despreading

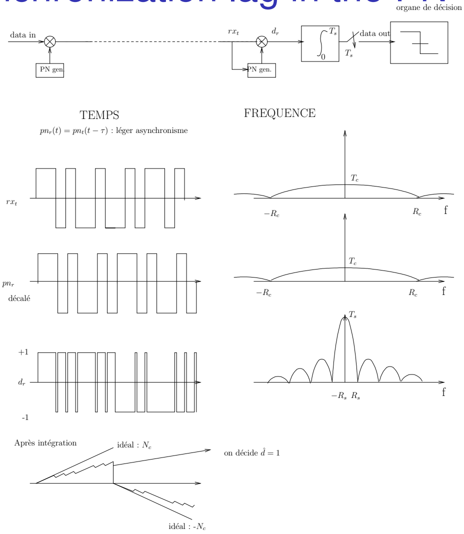
Correct decoder PN sequence



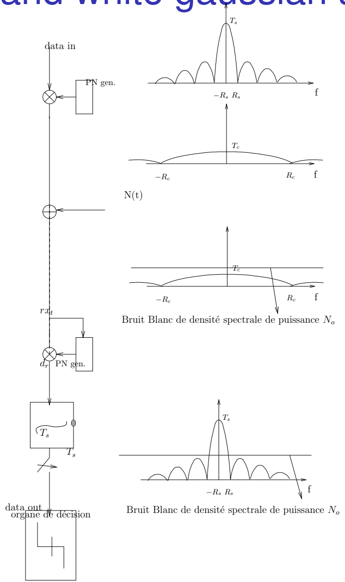

Different decoder PN sequence



# Effect of a synchronization lag in the PN Code at the receiver

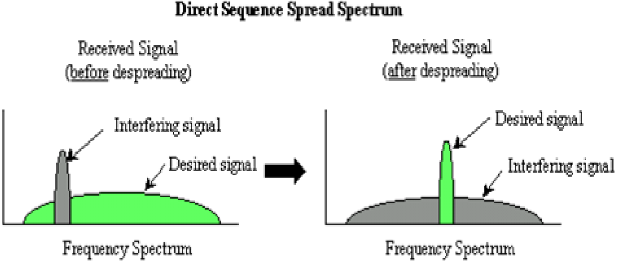


# Effect of a DSSS and white gaussian additive noise



# Interference rejection

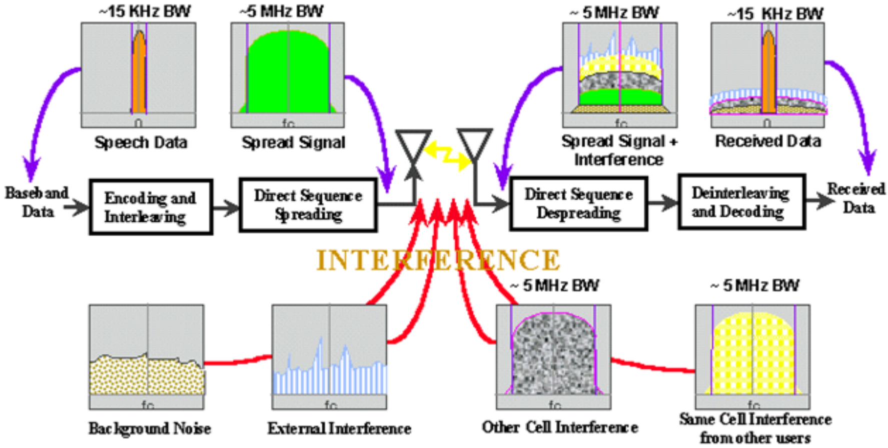
Assume the interferer has constant PSD $I_o$ over its bandwidth $W_{interf}$ :



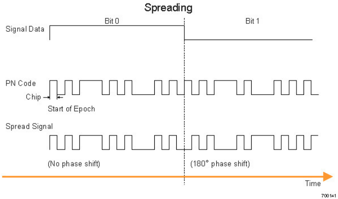

$$SNR_{ss} = \frac{S}{I_o W_{interf}} \qquad SNR_{despread} = \frac{S}{I_o W_d}$$

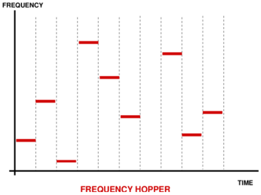
$$\boxed{\text{SNR gain} = SF}$$

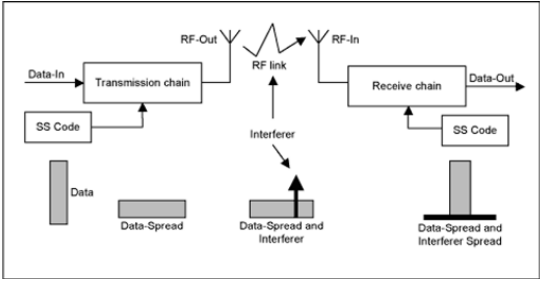
# Various Distorsions through SS transmitter systems



# SS summary


Spreading
Bit 0
Bit 1
Signal Data
PN Code
Chip
Start of Epoch
Spread Signal
(No phase shift)
(180° phase shift)
Time
700141


FREQUENCY
TIME
FREQUENCY HOPPER


RF-Out
RF link
RF-In
Data-In
Transmission chain
Receive chain
Data-Out
SS Code
Interferer
SS Code
Data
Data-Spread
Data-Spread and Interferer
Data-Spread and Interferer Spread

# Summary of SS methods benefits

- Interference rejection (immunity to multipath fading, jamming resistance).
- Energy density reduction (low probability of intercept)
- Fine time resolution (ranging, position determination, accurate universal timing).
- Multiple access (resource sharing, selective addressing).

One question : DSSS or FHSS ?
Near-Far Effect : Emitter B much closer to Receiver than Emitter A => Received power from B (even with orthogobal DSSS PN sequence) may mask signal from A

$\Rightarrow$ FHSS prefered (e.g. GSM)

# Pseudo random sequence

Important to notice : Pseudo random sequence behaves like noise, although it is fully deterministic.
Main properties

- Balanced code : number of $+1 \simeq$ number of $-1 \Rightarrow$ code mean $\simeq 0$
- autocorrelation : $R_{PN}(\tau) = \int_{-N_cT_c/2}^{N_cT_c/2} PN(t)PN(t-\tau)dt$ should be as close as possible to $\delta(t)$
- crosscorrelation : $R_{PN_iPN_j}(\tau) = \int_{-N_cT_c/2}^{N_cT_c/2} PN_i(t)PN_j(t-\tau)dt \simeq 0 \forall \tau \rightarrow$
  - 'orthogonality' between PN sequences if $R_{PN_iPN_j}(0) = 0$
  - More interesting : $R_{PN_iPN_j}(\tau) \simeq 0 \; \forall \tau$

# Examples

- Balanced code :
  $PN = \ +1 \quad +1 \quad -1 \quad +1 \quad -1 \quad -1 \quad -1 \ \rightarrow \Sigma = 0$

- Auto-correlation

$$
\begin{array}{lccccccccc}
PN(0) = & & +1 & +1 & -1 & +1 & -1 & -1 & -1 & \\
PN(0) = & & +1 & +1 & -1 & +1 & -1 & -1 & -1 & \rightarrow \\
------- & & --- & --- & --- & --- & --- & --- & --- & \\
 & & +1 & +1 & +1 & +1 & +1 & +1 & +1 &
\end{array}
$$

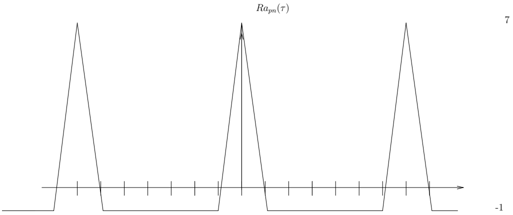
  $R_{PN}(0) = 7$

- Cyclic auto-corr

$$
\begin{array}{lccccccccc}
PN(1) = & & +1 & -1 & +1 & -1 & -1 & -1 & +1 & \\
PN(0) = & & +1 & +1 & -1 & +1 & -1 & -1 & -1 & \rightarrow \\
------- & & --- & --- & --- & --- & --- & --- & --- & \\
 & & +1 & +1 & -1 & -1 & -1 & +1 & -1 &
\end{array}
$$

  $R_{PN}(1) = -1$

# Examples, contd




$R_{PN}(\tau)$ :

- Cross-correlation

$$\begin{array}{cccccccc} +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ -- & -- & -- & -- & -- & -- & -- & -- \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \end{array} \rightarrow$$

$R_{PN_iPN_j}(0) = 0$

# Applications of orthogonality in PN sequences

- Orthogonal codes do not 'interfer' in despreading process => mutli-user capabilities
- orthogonal codes often do not enjoy good auto / cross-correlation properties for $\tau \neq 0$
  $\Rightarrow$
  - design short orthogonal code sequences (allow to separate users)
  - design long code sequences (with good cross and auto corr properties (good transmission properties)
  - Multiply the sequences to built a code with both properties

# Introduction to Digital communications -Lecture 5-

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# Motivations

Case of a unique carrier transmission system :

- Symbol rate $R_s = \frac{1}{T_s}$
- Echoes, multipaths, diffusion, difraction $\Rightarrow$ time leakage of a given symbol over $\simeq MT_s$, $M$ large
- Channel equalization complex
- the EYE diagram is almost closed $\Rightarrow$ detection problems, decision errors...

# Pb related to unique carrier transmission : example

- Assume
  - an optical path delay $\Delta l = 100m$, radio waves,
  - $R_s = 100 Msymbol/s$
- then delay $\tau = 300ns$, equivalently $\tau = 30$ symbols
- Equalization FIR filter of order $N \simeq 2\tau = 60$
- Compuational load $= 60/T_s$ ($C$-values) = 240 multiplications/additions per symbol = 24 Gops/s !!!

# Multicarrier solution

- Transmit $N_c$ in parallel (using $N_c$ sub-channels), each with duration $T_c = N_c T_s$
- Each channel has carrier frequency $f_i = f_0 + \Delta f$, of width $W_c = \frac{W_s}{N_c}$ where $W_s$ = spectral width in the mono-carrier case.
- Equalization of each sub-channel is much simpler as
  - $W_c << W_s \Rightarrow$ less fluctuations over $W_c$
  - delay is constant in time, but much mower as expressed in symbols $\Rightarrow$ lower order FIR equalizer
  - If $N_c >> 1$, each equalizer involves only one multiplication !
- Requires $T_c >> \tau$, i.e. large $N_c$
- Channel Coherence width $W_b \simeq \frac{1}{\tau}$, then $W_c = \frac{W_s}{N_c} << W_b \Rightarrow N_c >> 1$

# Multicarrier signal expression

$$s(t) = \sum_k \left( \sum_{m=0}^{N_c-1} d_{m,k} \psi_m(t - kT_c) \right)$$

Major requirement : avoid inter sub-channel interferences

- separate the sub-channel spectral bandwidth $\rightarrow$ low global spectral efficiency
- involves complex / expensive mixing an modulator devices

# OFDM solution

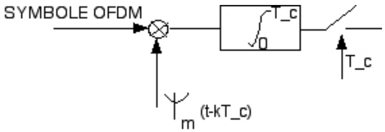
Allow overlapping frequency bands, but no interences, then carriers signals must verify

$$\int_0^{T_c} \psi_m(t - kT_c)\psi^{\star}_{m'}(t - kT_c)dt = \delta_{m,m'}$$

Classical simple solution :

$$\begin{cases} \psi_m(t) = \frac{1}{\sqrt{T_c}} \exp(j2\pi f_m t) & \text{si} \quad t \in [0, T_c[ \\ 0 \quad \text{sinon} & \end{cases}$$

where $f_m = f_0 + m\delta f = f_0 + m\frac{W_s}{N_c}$, $f_0$ being the first sub-channel central freq.



# OFDM solution, contd

as

$$\int_{kT_c}^{(k+1)T_c} \psi_m(t-kT_c)\psi^\star_{m'}(t-kT_c)dt$$
$$= \int_0^{T_c} \frac{1}{T_c} \exp(j2\pi(f_m - f'_m)(t))dt$$
$$= \frac{\sin(\pi(f_m - f'_m)T_c)}{\pi(f_m - f'_m)T_c}$$

orthogonality is met if $(f_m - f'_m)T_c = l$, $l \in \mathcal{Z}$, or equivalently

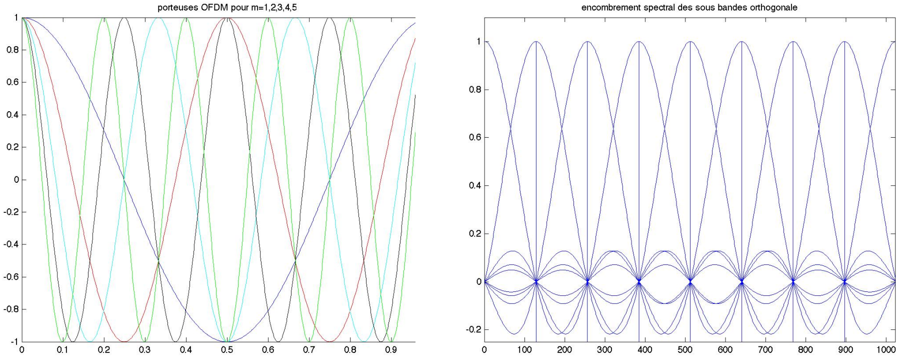
$$(f_m - f'_m)T'_c = (m - m')T_c\frac{W_s}{N_c} = (m - m')N_cT_s\frac{W_s}{N_c}$$

this implies

$$T_sW_s = 1$$

# Pulse shapes

$\psi_m$ puse shapes and assocoated spectral representations



# OFDM implementation

Using complex orthogonal exponentials leads to

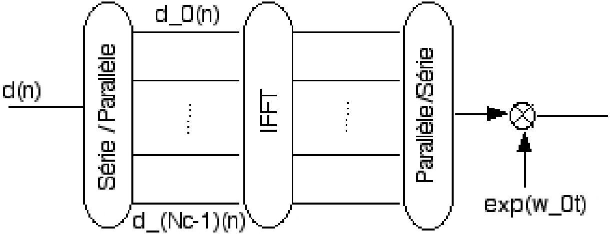
$$s(nT_c) = \sum_k \left( \sum_{m=0}^{N_c-1} d_{m,k} \frac{1}{\sqrt{T_c}} \exp(j2\pi f_m (n-k) T_c) \right)$$

$$= \underbrace{\sum_k \frac{\exp(j2\pi f_0 (n-k) T_c)}{\sqrt{T_c}}}_{} \underbrace{\left( \sum_{m=0}^{N_c-1} d_{m,k} \exp(j2\pi \frac{mn}{N_c}) \right)}_{}$$

Delay of OFDM symbols of duration $T_c$

IFFT of $d_{k,m}$ sequences, of length $T_c$
Requires $N_c \log_2 N_c$ ops (Cooley Tuckey)

# FT based OFDM modulation system



# OFDM performances

- For WGN additive channel, same perf. as single carrier modulation
- Perf. degrades for freq. selective channels : attenuated sub-channels will have high SER/BER (as high as 0.5!)
- Makes error correcting codes compulsory to reach singe carrier equiv. perf., with lower implementation cost

examples : WiFi 802.11*, WLAN, ADSL