ELECTRICITY

D. C. Circuits, Kirclihoff's laws, network analysis and circuits. Galvanometers, A. C. circuits, inductance, capacitance, impedance and admittance, rims and peak values, power. RLC circuits, Q-factor, resonance, circuit theorems, filters, electronics vacuum diode, triodes, pentode, small signal equivalent circuits. Rectifiers and amplifiers, semi conductors, pn-junction, field effect transistors, bipolar transistors, feedback oscillators.

Basic Physical Concepts

Charge: Charge and the motion of charges give rise to all electrical and electronic effects. There are both positive and negative charges; the net or total charge of an object is given by the algebraic sum of all positive and negative charges in the object. Charge can be detected because a charge brought near a second charge experiences a force due to the second charge. This electrostatic force is give by coulomb's law:

$$F = \frac{kq_1q_2}{r^2} \ldots\ldots (1)$$

q_1 being the 1st charge, q_2 the 2nd charge, r the distance between the two objects and k is a constant whose value is given by 9.0 x 109N i.e. $(1/4\pi\varepsilon_o)$.

ε_o = Permittivity of free space

The S.I unit of charge is coulomb's.

Electric currents: An electric current is electric charge in motion. Specifically, the number of coulomb's passing through some surface area per unit time is the current flowing through the surface i.e.

$$I = \frac{\triangle Q}{\triangle t} \ldots\ldots (2)$$

When the current is being carried by particles having charge q_1 the current is given by $I = Nq$…...(3), where N is the number of these particles crossing the surface per unit time for a wire having a cross-sectional area of A (in square meters) a density free electrons given by η (number per cubic meter), and a drift velocity of Vd (meters per second), the current is given by

$$I = \eta e V_d A \text{……(4)}$$

The S.I unit is Amperes

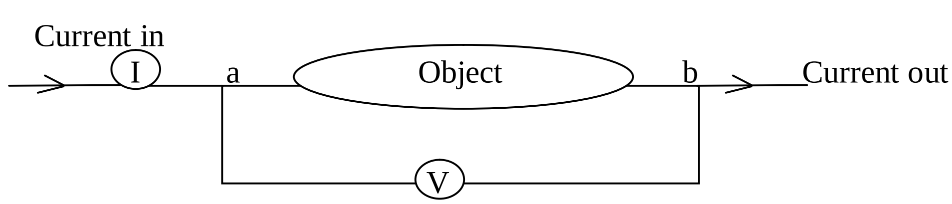
Voltage: As an electrically charged object is moved about in space, it is necessary to do either positive or negative work on it to overcome the electrostatic forces on it due to other charges in space. If Wab is the work done in moving an object having charge Q from point a to point b, then the electric potential different between points a and b, Vab is define as

$$V_{ab} = \frac{W_{ab}}{Q} \text{……(5)}$$

The unit is given in joule per coulombs also known as volts (v)

The nature of the electrostatic force is such that the work done on a charged object is moving around a closed path is zero

Resistance: Consider the fig below



The points across which the voltage is being measured are labeled a and ab. The resistance of the object is defined as $R = \frac{V_{ab}}{I}$ ……. (6)

The unit is ohms (Ω)

Sometimes it is more convenient to deal with the reciprocal of resistance, called the conductance. It has the dimension of current per voltage measured in Mhos or Siemens. If the graph of voltage is plotted against current (I) and the line passes through the origin then the resistance of the object is independent of the voltage applied to the device and it shows that object obeys ohms law.

OHM'S LAW: Ohm's law is a statement of voltage – current relationship for an object whose resistance is constant. It is usually written as

$$V = IR \ldots\ldots. (7)$$

Although ohm's law looks the same as the definition of resistance in equation (6) it is not really the same. The definition of resistasnce is a general statement. Ohms law only applies to those situations for which the resistance of the object is independent of the current flowing through it. The Resistance in eqn (7) is a constant independent of the voltage and current, whereas the Resistance in eqn (6) may vary as a function of current or voltage.

Power: The workdone to move a charge Q through a voltage drop of V per unit time i.e.

$$\text{Power} = \frac{\text{workdone}}{\text{Time}} = \frac{QV}{t} = IV \text{ in watts} \ldots\ldots (8)$$

Since Vab = Wab/Q

For a resistor having resistance R, a voltage V applied across the resistor, and a current I flowing through U, Ohm's law and eqn (8) can be combined to yield power dissipated in a resistor

$$IV = \frac{V^2}{R} \quad I^2R \ldots\ldots (9)$$

Conductors, Insulators and Resistivity: The Resistance of a piece of wire depends on the length of the wire, the site of the wire, and the material from which the wire is made.

The resistance of an object having a uniform cross section made out of an ohmic materials is given by $R \alpha \frac{L}{A}$, $R = \frac{\rho L}{A}$ …… (10)

ρ is the resistivity of the material.

Materials that have high conductivities, such as copper and silver, are called conductors while matierals having small conductivities, such as glass, are called insulators.

Basic Concepts of Circuit Analysis

Circuit Elements: The circuit to be considered in the next few paragraph will be the combination of the following four elements.

i. Wire
ii. Resistors
iii. Batteries
iv. Power supplies

Wires: The wires we shall be discussing in this paragraph are assumed to be ideal wires with no resistance and no other imperfections.

Resistors: are components that obey ohm's law that is their resistance is independent of the current flowing through them. Resistors are the most frequently used circuit elements in electronics. For now, allo resistors are assumed to be perfect, that is, their resistance does not changed and they have no other imperfections.

The S.I unit of resistance is the ohms.

Besides fixed resistors, there are also variable resistors – resistors whose resistance can be changed by twisting a knob etc.

Batteries: They are sources of electromotive force (emf). The are circuit elements that maintain a more or less constant voltage between their terminals. Inside the battery, a chemical rnx takes place to maintain the potential different between the terminals. The chemical rnx overcomes the electrostatic tendency for positive charge to move from the positive to the negative terminal through the battery. As current flows through the battery, the chemical rnx moves positive charges from the negative to the positive.

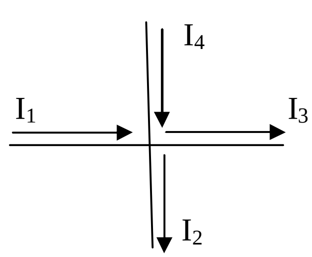
Power Supplies: A power supply is an electronic equivalent of a battery. Power supplies are generally used in the laboratory for reasons of simplicity, capacity and economy.

Schematic Diagrams: Provide a method of representing the electrical properties of a circuit. There are standard symbols for all electronic components in current use. With these standard symbols, most of the information needed to understand and build an electronic circuit can be represented conveniently on paper. The use of a standard set of symbols obviously facilitates communications between people in the field.

Basic Circuit Laws: The basic for all circuit analysis is Kirchoff's two laws. One of these laws deals with the currents flowing into a node, whereas the other deals with the sum of the voltage changes around a closed circuit. Currents flowing into a node are positive and currents flowing out of a node are negative.

Kirchoff's current law (kcl): States that the algebraic sum of the currents flowing into a node is zero. Symbolically, this is written as $\Sigma I = O$ ……. (11)

e.g



For the situation in this four-wire node kirchoff's current gives $I_1 - I_2 - I_3 + I_4 = 0$

Kirchoff's voltage law (kvl): States that the algebraic sum of all the potential changes around any close circuit (or closed loop) is zero. This is sometimes stated as follows: The sum of all potential rises equals the sum of all the potential drops around a circuit. Symbolically, written as

$$\Sigma V = O \text{ ……. } (12)$$

Rules for signs

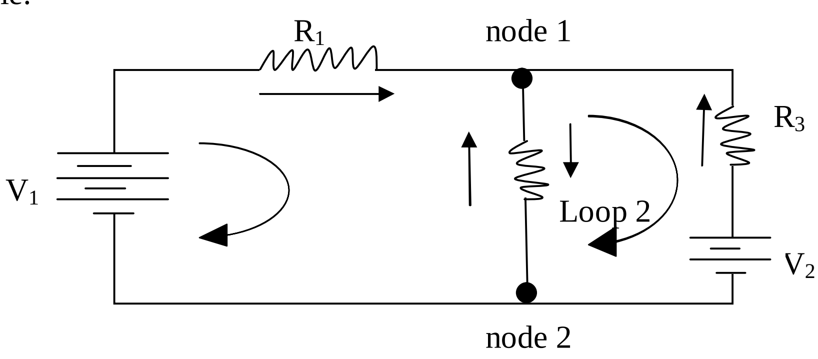
Kcl: 1. Currents flowing into a junction are positive

2. Current flowing out of a junction are negative.

KVL:

i. If the imaginary loop around the circuit enters a source of e.m.f (battery, power supply, etc) at the negative terminal and leaves at the positive terminal, use a +ve voltage. If the loop enters at the +ve terminal, use a negative voltage.
ii. If the loop going through a resistor is in the same direction as the current, use – IR. If the loop does in the opposite direction as the current, use +IR
iii. If there are two (or more) currents idenfied in a resistor, use the sum of the voltage drops with the sign given as in item 2 above.

Example:



Applying kcl to node 1, yields

$$I_1 + I_2 - I_3 + I_4 = 0$$

In a similar way, applying Kcl to node 2 yields

$$-I_1 - I_2 - I_3 + I_4 = 0$$

Applying Kvl to loop I, yields

$$V_1 - I_1 R_1 - I_2 R_2 - I_3 R_2 = 0$$

Likewise, applying Kcl to loop 2 gives

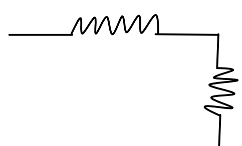
$$-V_2 - I_2 R_2 - I_3 R_2 - I_4 R_3 = 0$$

SOME SIMPLE CIRCUITS

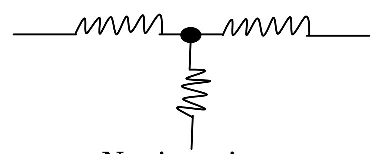
Definition of Series and Parallel Circuit Elements

By definition components are in series when there are no branching nodes between them. Branching nodes are nodes where three or more conductors join, i.e nodes at which the current divides or has a choice where to go. For components in series, all the current that flows through the component must flow through all the others. See figure below:

Series

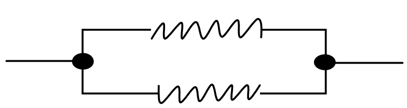


Series

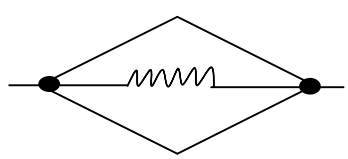


Not in series

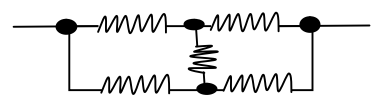
Components are in parallel when they are connected between the same nodes. This means that the current flowing between the nodes need only go through one of the components. See figure below:



Parallel



Parallel



Not parallel

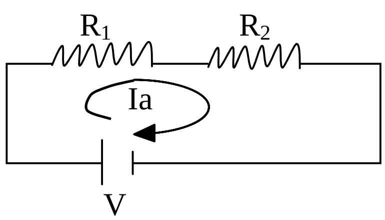
The Equivalent of Resistors in Series

It is possible to substitute one resistor for two or more resistors in series without causing changes in the voltage and currents in the remaining circuit, the single resistor is said to be the equivalent of the resistors in series.

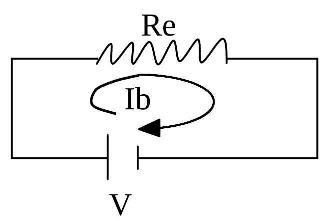
If n resistors are connected in series, the resistance of the equivalent resistor is given by

$$Re = R_1 + R_2 + \ldots\ldots\ldots + R_n \ldots\ldots. (13)$$

This result will be derived now for the case of two resistors in series. This is shown in the fig. below.



(a) The 2 resistors in series



(b) The efuwalet resistor

The two resistors in series one to be replaced with one resistor such that the current flowing in the battery does not change.

KVL applied to the circuit in part (a) of the fig. yields

$$V - Ia\, R_1 - Ia\, R_2 = 0$$

$$\text{or} \quad Ia = \frac{V}{R_1 + R_2} \ldots\ldots. (*)$$

For the circuit in (b)

$$Ia = \frac{V}{Re} \ldots\ldots. (**)$$

Since the two currents are to be the same (nothing else in the circuit is to change),

then Ia = Ib. (***)

Substituting equations (*) and (**) into (***) gives

$$\frac{V}{R_1 + R_2} = \frac{V}{Re}$$

or

$$\cancel{V}\,(Re) = (\cancel{V})\ [R_1 + R_2)$$

$$\therefore\ Re = R_1 + R_2$$ qed. QED

Which is just the equation given in eqn (13) when it is applied to the case of two resistors in series.

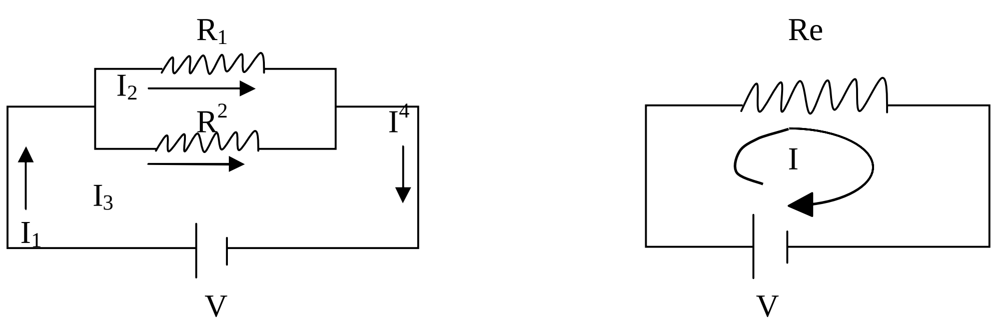
It is obvious that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors

The Equivalent of Resistors in Parallel

It is also possible to replace two or more resistance in parallel by one equivalent resistor without producing changes in the voltage and currents in the circuits. If there are n resistors in parallel, the resistance of the equivalent resistor Re is given by

$$\frac{1}{Re} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{Rn} \quad \ldots\ldots (14)$$

This result will be derived for two resistors in parallel. The situation is shown in the fig. below.



In (a) of the fig. the current through each part of the circuit has been drawn and named applying kcl and kvl to the circuit.

First it is clear that I, and I_4 are the same current: $I_1 = I_4$

Kcl applied to either of the two nodes in (a) gives $I_1 - I_2 - I_3 = 0$ or

$$I_1 = I_2 + I_3 \text{ …….. } (15)$$

KVL applied to the loop including the battery and R_1 gives $V - I_2 R_1 = 0$

$$I_2 = \frac{V}{R_1} \text{ …… } (16)$$

Finally, applying KVL to the loop including the battery and R_2 yields

$$V - I_3 R_2 = 0$$

$$I_3 = \frac{V}{R_2} \text{ …… } (16)$$

Substituting eqn (16) and eqn (17) into (15) gives

$$I_1 = \frac{V}{R_1} + \frac{V}{R_2} \quad (*)$$

For fig. (b), it is clear that $I = \frac{V}{Re}$ ……(18)

The two currents are to be equal:

$$I = I_1 \text{ …… } (19)$$

Substituting eqns (*) and (18) into eqn (19) gives

$$\frac{V}{Re} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Which gives $\frac{V}{Re} = \frac{1}{R_1} + \frac{1}{R_2}$ …… (20)

Which is time for the particular case of two resistors in parallel for the special case of two resistors in parallel, it is often easier to use the expression.

$$Re = \frac{R_1R_2}{R_1 + R_2} \quad \ldots\ldots. (21)$$

This expression is only true for two resistors in parallel for more resistors in parallel, the general expression must be used.

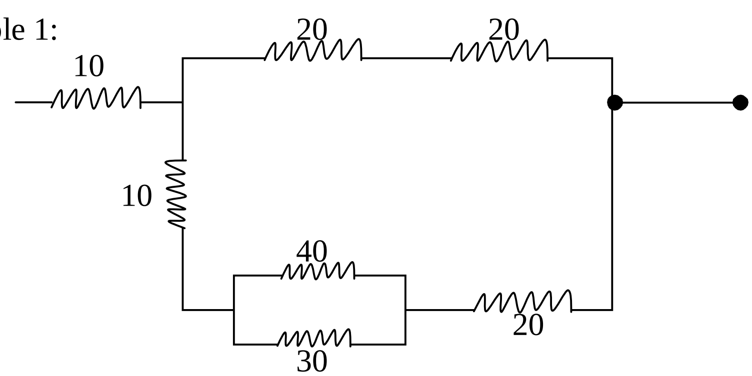
The equivalent resistor is smaller than any of the individual resistors; in particular, it is smaller than the smallest of the resistors in the parallel combination. This is often a useful check of a numerical result.

More complex Networks

A common type of problem is to present a great mass of resistors arranged in some complex looking network and asked to reduce to a single equivalent resistor. In practice, such situation rarely occurs; most networks can not be reduced this way because they are neither series nor parallel. However such problems do provide lots of practice in recognizing and replacing series and parallel combinations with their equivalents.

The procedure for solving such problems is to look for two or more resistors that are in series or in parallel and replace them with their equivalents, then redraw the circuit in its reduced form and repeat until no further reduction can be made.

Example 1:

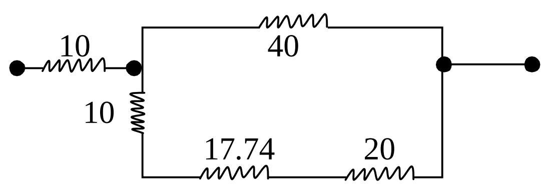


To help in the process, the resistors to be replaced at each step will be enclosed in dashed lines.

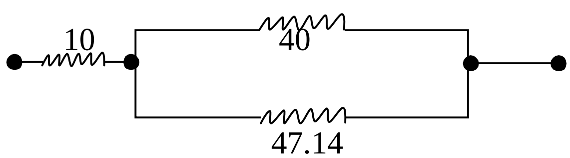
Goes thus:- there are two 20Ω resistors in series, theser replaced by one 40Ω resistor. Furthermore, there is a 40Ω resistor in parallel with 30Ω, the equivalent of these is 17.14Ω i.e.

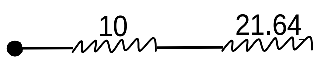
$$\frac{1}{Re} = \frac{1}{R_1} + \frac{1}{R_2} = 17.14\Omega \equiv$$

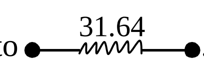
$$Re = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, 

Then these are in series and we have Re = 47.14Ω . Also



Reduces to 

and since in series it finally reduces to 

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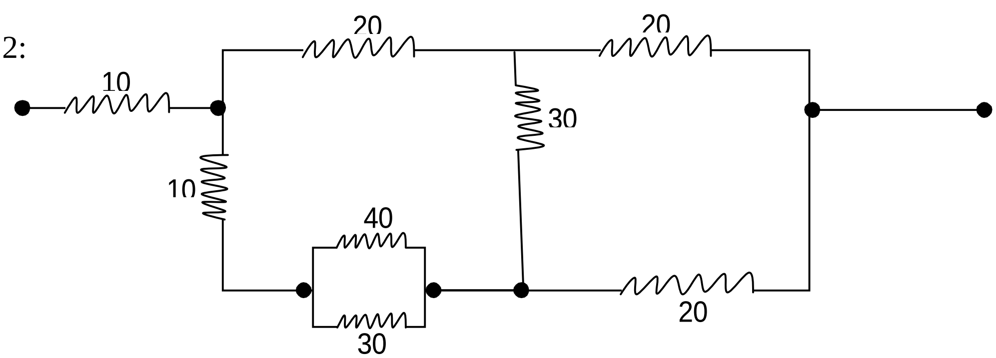
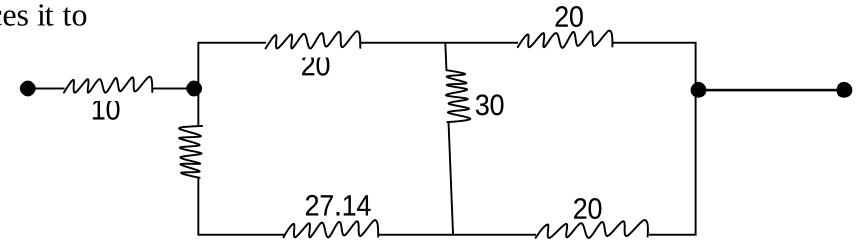
Example 2:



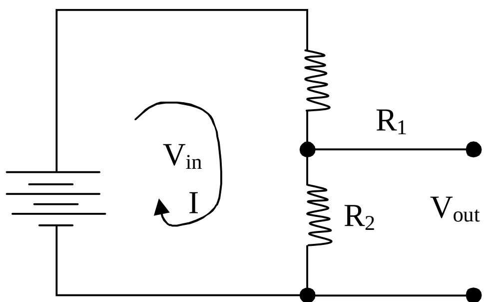
The 40 and 30Ω resistors in parallel being replaced 17.14Ω. The 10Ω resistor in series with 17.14Ω reduces it to



The remaining combinations are neither series nor parallel. This circuit can not be reduced any further using only the techniques introduced here later on an efficiency to find this equivalent resistor will found since the circuit contains only resistors.

VOLTAGE DIVIDER

It consists of only two resistors despite the apparent simplicity of the circuit. It is used so frequently. The circuit is shown below:



A Voltage divider

The derivation of a voltage divider in quite simple. It is assumed that no current flows from the node in the direction of vout.

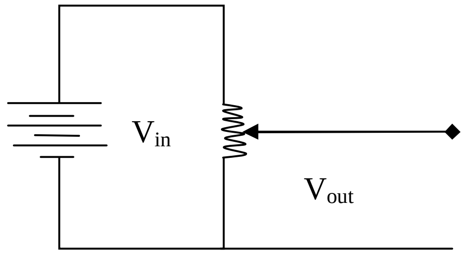
Ohm's law gives $V_{out} = IR_2$ …… (22)

and we know that $I = \frac{V_{in}}{Re} = \frac{V_{in}}{R_1 + R_2}$ ……. (23) …… (24)

Substituting eqn (23) into eqn (22) give (24)

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2} \quad \text{…….. (24)}$$

It is easy to see how the circuit gets its name. The input voltage is divided into two parts; one part appears across R_1 and the other across R_2. The part across R_2 is called the output. The output voltage is a fraction of the input voltage, the fraction being determined by the two resistors. R_1 and R_2 could be replaced by the two halves of a potentiometer, with V_{out} being from the slider as show below.



A voltage divider made with a potentiometer. The most familiar use of this arrangement is the volume control on your radio or tape player.

Finally, as a generalization of this circuit, if there are n resistor in series, the voltage across resistor X is given by

$$V_x = V_{in} \frac{R_x}{R_1 + R_2 + \dots + R_n} \quad \dots\dots\dots (*25)$$

The analysis of the voltage divider is only correct as long as no current flows into (or out of) the output leads.

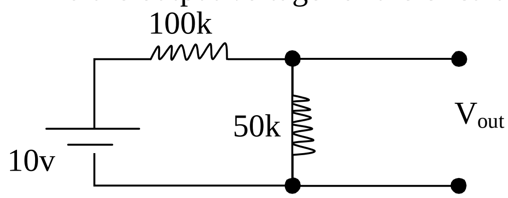
Current Divider

The two-resistor circuits shown below are called a current divider. It is easy to show that

$$I_1 = I_t \frac{R_s}{R_1 + R_s} \quad \dots\dots\dots (26)$$

Against it is easy to see how this circuit gets its name. The current is divided into two parts. One part goes through R_1 and the rest goes through R_s, sometimes called the shunt resistor. The fraction going through R_1 is determined by the value of the resistors. As the value of the shunt resistor is decreased, the fraction of the current going through R_1 is decreased. This is also a relatively common circuit one use of this circuit is described on meters later.

1. Find the output voltage for the circuit below



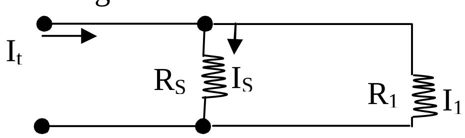
Solution:

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

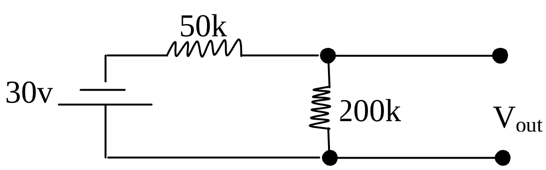
$$= 10 \frac{50k}{100k + 50k}$$

$$= \frac{500}{150} = 3.33v$$

Current divider diagram



2. Find the output voltage for the circuit below



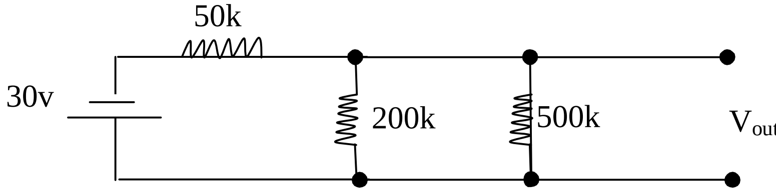
Solution:

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$= 30V \frac{200k}{50k + 200k}$$

$$= \underline{24V}$$

3. If a 500-kilodim resistor were placed across the output terminals in the fig. above, what would is output voltage be?

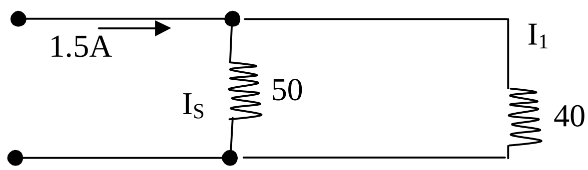


Solution: $Re = \frac{R_1 R_2}{R_1 + R_2} = \frac{100,000}{700} = 142.86\Omega$

$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2}\right) = 30\left(\frac{142.86}{192.86}\right)$$

$$= \underline{22.2V}$$

4. Find the current in each resistor in the fig. below.

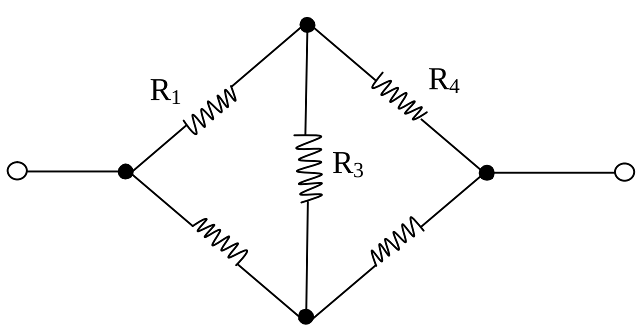


a. $I_1 = It\left(\frac{R_s}{R_s + R_1}\right) = 1.5\left(\frac{50}{90}\right) = 0.833A$

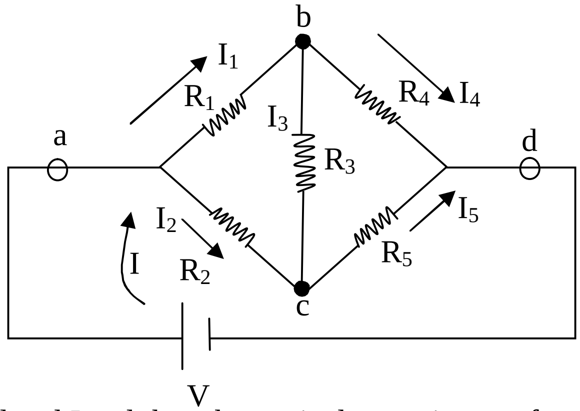
b. $I_s = It\left(\frac{R_1}{R_1 + R_s}\right) = 1.5\left(\frac{40}{90}\right) = 0.667A$

Mesh Equations: Although the basic laws of circuit analysis introduced earlier can be used to analyze essentially any circuits, a direct application of these laws often results in many equations and involves many unknowns. For a circuit of any degree of complexity, a strategy that minimizes the number of variables and equations is needed. One such method, called the mesh equation method, or Maxwell's method, is introduced. Everything in this section is completely general. Everything can be generalized by replacing the word "resistance" by impedance".

To illustrate how circuit problems can easily become very complex, a relatively simple circuit will be analyzed by a straight forward application of KVL and KCL. Later, this same example is done more directly using the mesh method.



The fig. above shows one of the simplest networks of resistors that cannot be reduced to a simpler circuit by combining series and parallel resistors. Yet since it consists only of resistors, it must be possible to replace the resistors with a single equivalent resistor. To find the equivalent resistance of this circuit, it is necessary to imagine a voltage applied to the circuit as shown.

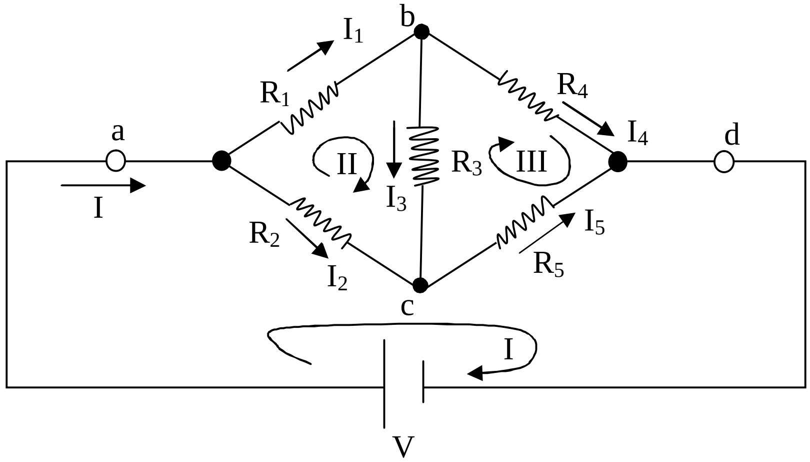


The current is calculated I and then the equivalent resistance form $Re = \frac{V}{I}$

Procedure:

1. Unique current through each component or sequence of components in series in the circuit is drawn, the current are named carefully to avoid giving two names to the same current.
2. Apply Kcl to get as many equations as possible using this theorem. If there are m currents and n nodes in a circuit, then n – 1 independent equation are obtained with Kcl.
3. Apply Kvl to get enough additional equations m – n – 1 additional equations can be obtained from Kvl.
4. Solve the resultant system of equations.

That is:



n – 1 equations are obtained applying Kcl to nodes a, b and d

$I - I_1 - I_2 = 0$
$I_1 - I_3 - I_4 = 0$
$I_4 - I_5 - I = 0$

Three more equations can be obtained by using Kvl (from $6 - 4 - 1$)
$m - n - 1$

Any three closed loops can be picked

$V - R_2 I_2 - I_5 R_5 = 0$ - loop 1

$- R_1 I_1 - I_3 R_3 - R_2 I_2 = 0$ - loop 2

$- R_4 I_4 - I_5 R_5 - I_3 R_3 = 0$ - loop 3

These 6 equations will be solved using the Crammer's rule

Firstly, the equations are re-written in standard form

I	$-I_1$	$-I_2$	.	.	.	.	=	0
.	I_1	.	$-I_3$	$-I_4$	.	.	=	0
$-I$	.	.	.	$+I_4$	$+I_5$		=	0
.	.	R_2I_2	.	.	$+R_5I_5$		=	V
.	$-R_1I_1$	$+R_2I_2$	$-R_3I_3$	.	.		=	0
.			R_3I_3	$-R_4I_4$	$+I_5R_5$		=	0

Solving for I by Crammer's rule gives

$$I = \begin{vmatrix} 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ V & 0 & V_2 & 0 & 0 & R_5 \\ 0 & -R_1 & R_2 & -R_3 & 0 & 0 \\ 0 & 0 & 0 & R_3 & -R_4 & R_5 \end{vmatrix}$$

Evaluating these two determinants is tedious but straight forward process.

$$I_e = \frac{V(R_1R_3 + R_1R_4 + R_1R_5 + R_2R_3 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5)}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}$$

So finally, the equivalent resistance is given by

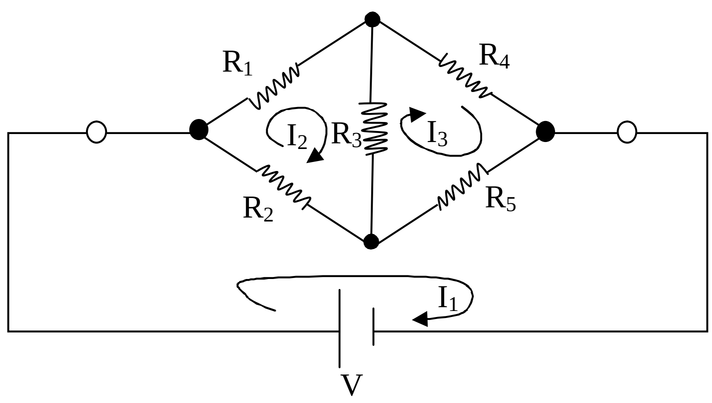
$$R_e = \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}{R_1R_3 + R_1R_4 + R_1R_5 + R_2R_3 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5}$$

Mesh Equations: This is a bit simple than the 1st method used.

The procedure for mesh method:

i. Pick closed current loops called mesh currents, or loop currents make sure no two different branches have the same current and each branch must have at least one current.
ii. Apply Kvl to each loop and
iii. Solve for the loop currents as best, a solution with minutes sign means that the current goes the opposite to the way it is drawn.

To illustrate these rules, solve the worked example using mesh equation method.



Apply KVL to these loops. The loops have been traversed in the same direction as the loop currents and the various terms collected.

Note that there are two currents flowing in R_2, R_3 and R_5.

$$V \;-\; I_1 (R_2 + R_5) \;+\; I_2 R_2 \;+\; I_2 R_2 \;+\; I_3 R_5 = 0$$

$$I_1 R_2 \;-\; I_2 (R_1 + R_2 + R_3) \;+\; I_3 R_3 = 0$$

$$I_1 R_5 \;+\; I_2 R_3 \;-\; I_3 (R_3 + R_4 + R_5) = 0$$

These equations are solved by Crammer's rule.

So re-write the equations in the standard form and changing all the signs on some of the equations to make the diagonal terms positive

$$I_1 (R_2 + R_5) \;-\; I_2 R_2 \;-\; I_3 R_5 = V$$

$$-I_1 R_2 \;+\; I_2 (R_1 + R_2 + R_3) \;-\; I_3 R_3 = 0$$

$$-I_1 R_5 \;-\; I_2 R_3 \;+\; I_3 (R_3 + R_4 + R_5) = 0$$

To find the equivalent resistance, it is only necessary to find I_1. This is given by

$$\begin{vmatrix} V & -R_2 & -R_5 \\ 0 & (R_1 + R_2 + R_3) & -R_3 \\ 0 & -R_3 & (R_3 + R_4 + R_5) \end{vmatrix}$$

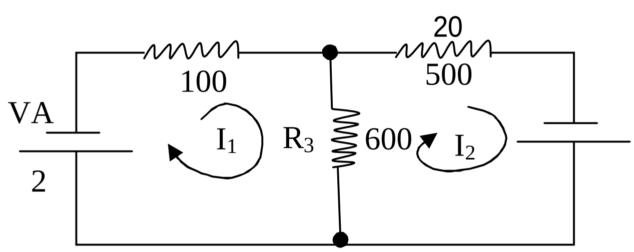
$$\text{IF} \quad \begin{vmatrix} (R_1 + R_5) & -R_2 & -R_5 \\ -R_2 & (R_1 + R_2 + R_3) & -R_3 \\ -R_5 & -R_3 & (R_3 + R_4 + R_5) \end{vmatrix}$$

Evaluating these determinants gives

$$I_1 = \frac{V (R_1R_3 + R_1R_4 + R_1R_5 + R_2R_3 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5)}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}$$

Which is same as above but easier

Example 2:



$$-V_A - I_1 (R_1 + R_3) + I_2 R_3 = 0$$

$$V_B + I_1 R_3 - I_2 (R_2 + R_3) = 0$$

Re-writing these in standard form gives

$$I_1 (R_1 + R_3) - I_2 R_3 = -V_A$$

$$-I_1 R_3 + I_2 (R_2 + R_3) = V_B$$

Hence $700 I_1 - 600 I_2 = -2$

$$-600 I_1 + 1100 I_2 = 4$$

$$I_1 = \frac{\begin{vmatrix} -2 & -600 \\ 4 & 1100 \end{vmatrix}}{\triangle} = \frac{\begin{vmatrix} -2 & -600 \\ 4 & 1100 \end{vmatrix}}{\begin{vmatrix} 700 & -600 \\ -600 & 1100 \end{vmatrix}} = \frac{200}{410,000}$$

$$= 0.488\text{mA}$$

$$I_2 = \frac{\begin{vmatrix} 700 & -2 \\ -600 & 4 \end{vmatrix}}{\triangle} = \frac{1600}{410,000} = 3.90\text{mA}$$

Thus the current through R1 is I1, and that through R2 is I2, whereas the current through R3 is the difference of I2 and I1, or 3.412mA upwards.

ADVANCED CIRCUIT ANALYSIS THEOREMS

THEVENIN'S THEOREM: States that any complex network of linear circuit elements (sources, resistors and impedances) having two terminals can be replaced by a single equivalent voltage source connected in series with a single resistor (impedance).

There are two ways of discovering the situation in which Thevenin's theorem is used

i. The theorem is useful to find how the current in some resistor in a circuit varies as that particular resistor is varied and the remaining circuit is unchanged.

ii. It is also useful to find how the output of some circuit changes as the output is loaded with a resistor.

The procedure for using Thevenin's theorem is as follows:

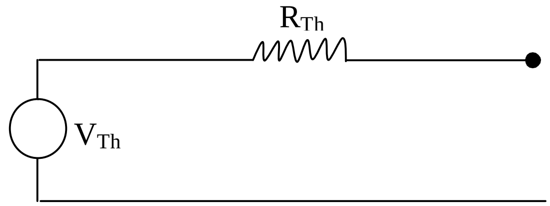
i. If the circuit to be replaced by the Thevenin's equivalent circuit already has two open terminals, label these terminals V_{in}. If there is a circuit element at the point where the circuit is to be studied, remove the element from the circuit, replace it with an open circuit, and label the terminals V_{in}.

ii. Compute V_{in}, the voltage at the open terminals.

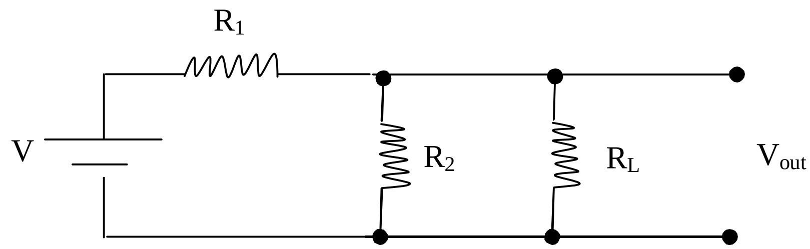
iii. Replace all the voltage sources in the circuit with short circuit and all the current sources in the circuit with open circuits.

iv. Compute R_{in}, the resistance of the circuit looking back into the output terminals after making these changes.

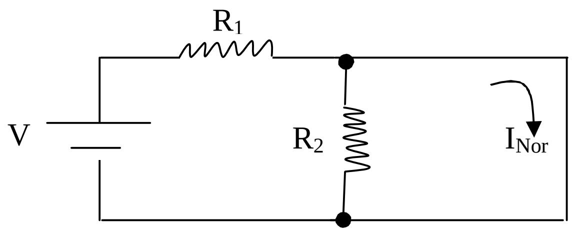
v. The network can then be replaced by the circuit shown below



Example:



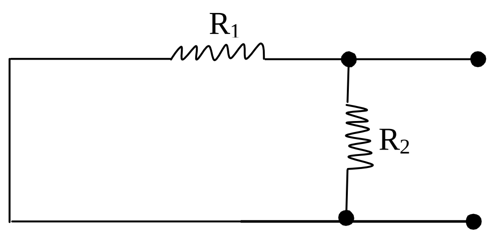
1. Remove R_L and lable the resulting terminals V_{Th} as shown below



2. Compute V_{Th}, in this case what is left is a voltage divider.

$$V_{Th} = \frac{V_{in} R_2}{R_1 + R_2}$$

3. Replace voltage source by a short circuit obtaining

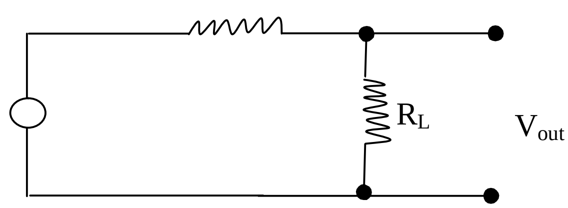


4. Compute R_{Th}, the resistance looking back into the output terminals; in this case. This is a parallel combination of two resistor.

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$$

5. Finally, the simplified circuit drawn

$$\frac{R_1 R_2}{R_1 + R_2} \qquad R_{Th}$$



Now the original question can be answered. The output of interest is another voltage divider:

$$Vout = \frac{VR_2}{\quad} \qquad \frac{R_L}{\qquad}$$

$$R_1 + R_2 \qquad R_L + \frac{R_1 R_2}{R_1 + R_2}$$

NORTON'S THEOREM: States that any complex network of sources and resistance (impedance) can be re-placed by a current source and a resistor (impedance) in parallel with it.

The procedure for using Norton's theorem is as follows:

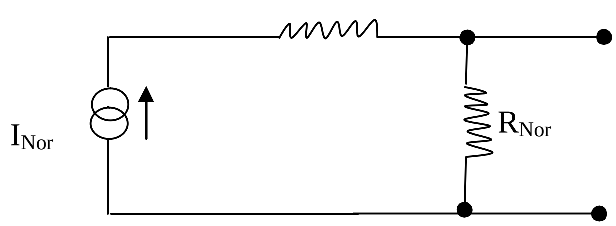
i. If the circuit has two terminals, connect these by a short circuit. If the current through some element is to be studied, replace it by a short circuit.

ii. Calculate II I_{Nor}, the current in this short circuit.

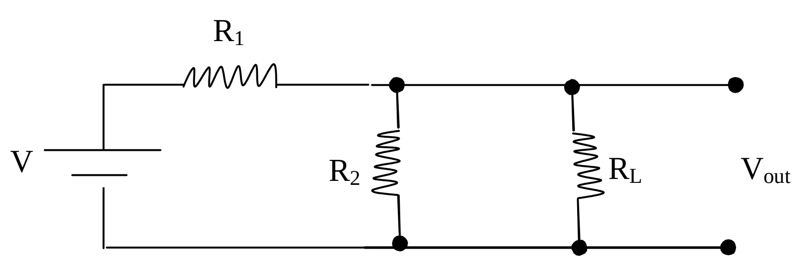
iii. Replace all voltage sources with short circuits and all current sources with open circuits.

iv. Complete the shunt resistance R_{Nor}, looking back into the circuit after making all these changes. Note that $R_{Nor} = R_{Th}$

v. Finally, replace the network by the current shown in the figure below.

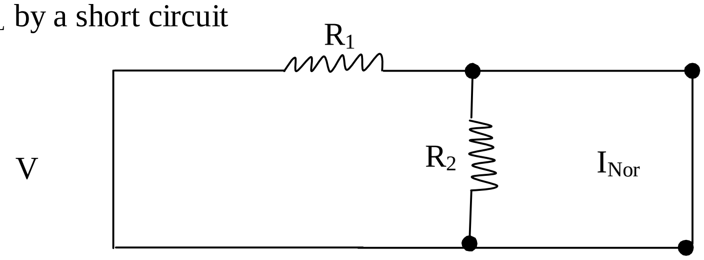


Example



The goal is to study how V_{out} varies as R_L is varied

i. Replace R_L by a short circuit



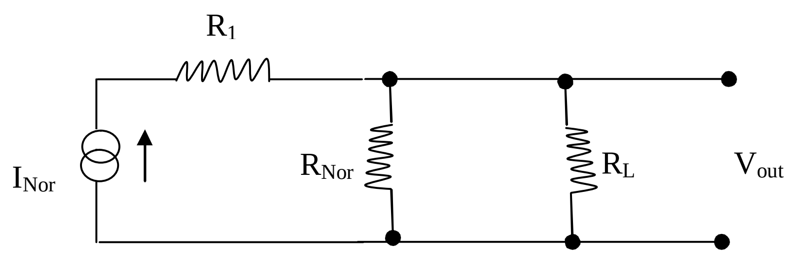
2. Compute $I_{Nor} = \frac{V}{R1}$

3. Replace the voltage sources by short circuits and the current sources by open circuits.

4. Looking back into the output of the circuit, calculate R_{Nor}. In this case, this is the parallel combination of two resistors.

$$R_{No} = \frac{R_1 R_2}{R_1 + R_2}$$

5. Finally, replace the originatl circuit with the result shown below.



Once again this has reduced the original problem to a simple one, that of a current divider. The answer can be written down reasonably directly.

The current through R_L is given by the current divider formular.

$$IR_L = I_{Nor} \frac{R_{Nor}}{R_{Nor} + R_L}$$

and the output voltage is

$$V_{out} = I_{R_L} R_L$$

$$V_{out} = \frac{V}{\cancel{R_1}} \cdot \frac{\cancel{R_1} R_2/(R_1 + R_2)}{R_1 R_2/(R_1 + R_2) + R_L} R_L$$

The result as that obtained by the case of the Thevenin's Theorem

There are relationships between Norton's and Thevenin's equivalents these aer

$$R_{Nor} = R_{Th} \quad \text{and}$$

$$V_{Th} = I_{Nor} R_{Nor}$$

There are two other general circuit analysis theorems that are used extensively in circuit analysis. These are:

i. The superposition theorem and

ii. The reciprocity theorem

The Superposition Theorem: States that the current in any branch of a linear circuits is equal to the sum of the currents produced separately by each source in the remainder of the circuit, with all the other sources set equal to zero.

The Reciprocity Theorem: States that the partial current in branch X of a linear, dc circuit produced by a voltage source in branch Y is the same as the partial current that would be produced in branch Y by the same source if it were placed in branch X.

ALTERNATIVE CURRENT

So far, we have not mentioned time in the circuit analysis, this is equivalent to the assumption that the voltages and currents under consideration do not change, it implies that the situation has no beginning or end, clearly this is only an approximation.

In real circuits, the currents and voltages change as functions of time. The nature of the voltages and currents in a circuit can be divided into three classes:

a. **DC or Steady State:** Cases where the voltages and currents are almost constant with respect to time and is not considered.
b. **A.C:** Where the voltages and currents are purely sinusoidal in time, this is sometimes called sinusoidal steady state or a.c steady state
c. **Others:** Cases where the voltages and currents vary in some more complex and less easily described manner than sinusoidal.

The current described by the equation below is plotted in the fig. above.

$$\iota(t) = I\rho \sin(wt) \ldots\ldots *$$

As it can be seen, in the fig, the current increases from 0 to Iρ, decreases back to 0, reverses its direction, and continues to decrease to a value of Iρ in the opposite direction and so on. Because of this reversals, or alternations, of direction, it is called an alternating current, or a.c.

$$v(t) = vp \sin(wt) \ldots\ldots *_2$$

this is an ac voltage.

The Basic Properties of the Sine Wave:

Consider a voltage having a pure sinusoidal change centered on zero:

The frequency of the ware f measured in cycles per second or Hertz (Hz) is the reciprocal of the period.

$$f = \frac{I}{T} \ldots\ldots *4$$

The frequency tells how many times per second the voltage goes through a complete cycle of values and returns to the same point in its cycle.

The angular frequency w: measured in radians, is used as an express in sine wave given by

$$w = \frac{2\pi f}{T} = 2\pi \ldots\ldots *5$$

Vp is the amplitude of the sine wave described by equation (*).

The extreme values of the voltage are +Vp and –Vp. The Φ is the phase of the voltage. It is related to the voltage at the time t = 0 … i.e. V (0) = Vp sinΦ …… *6

Consider 2 a.c. voltages both having the same amplitudes and frequencies but different phases. The 1st being

$$V_1 = Vp \sin (wt) \;\ldots\ldots\; *7$$

and the second V2 = Vp sin (wt + Φ)……. *8

If Φ is positive the 2nd voltage is said to lead the 1st by Φ. If Φ is negative, the 2nd voltage is said to lag the 1st by Φ as shown for example in the fig. below.

Measurements of Amplitudes: The amplitudes, Vp is the obvious choice as the most convenient parameter to describe how big the voltage is, when dealing with an algebraic expression for an a.c voltage.

The maximum potential between two successive extreme values is sometimes of interest and is called the peak-to-peak voltage. For the sine ware, this is

$$Vpp = Vp - (-V p) = 2Vp \;\ldots\ldots *9$$

For the sine wave the average value is zer i.e (Vp + C – Vp) = 0

However, the average of a sine wave over one half of a cycle or the average of the absolute value of a sine wave over a number of cycles is

Vav

It is frequently necessary to find the dc voltage that would generate the same amount of heat (power) in a resistor as does the ac voltage when averaged over one full cycle. So to do this we find the V_{rms} to find the equivalent dc voltage. To do this power is voltage times current i.e.

$$\rho = \nu\iota = \mathrm{v}\,\frac{\mathrm{v}}{R} = \frac{V^2}{R}$$

To find the average power, it is necessary to average the square of the voltage over one cycle and they take the square root to find the equivalent dc voltage.

This take the Root of the mean of the square, or the rms value.

The Cosine Wave:

This far, all voltages and currents have been written in terms of the sine wave:

$V = Vp \sin(wt + \Phi)$ but this is also the same as the form $V = Vp \cos(wt + \Phi)$ because the sine and cosine differ only by a phase factor.

That is $V = Vp \cos(wt) = Vp \sin(wt + \pi/2)$

The ac voltages can sometime be represented in the complex form $V = Vpe\ \iota\ (wt + \Phi)$...... (*11)

Example:

A sine wave has an amplitude of 25v peak to peak. What is its amplitude? Its average value? The average of its absolute value? It rms value?

Solution:

a. The amplitude from the Vpp is $Vpp = \frac{25v}{2} = \frac{12.5v}{2}$

b $V_{av} = \frac{vp + (-vp) = 0}{2}$

c $V_{av} = \frac{2vp}{\pi}$ or $vp \times 0.637 = 12.5 \times 0.637 = 7.96v$

d $V_{rms} = \frac{vp}{\sqrt{2}} = 0.707vp = 0.707 \times 12.5 = 8.84v$

Elements of a.c circuit analysis

Capacitor: In general, any two conducts insulated from each other constitute a capacitor shown below is a parallel plate capacitor.

The capacitors alwa

The capacitors always remain neutral as seen above but considering plates separately, they are taken to store charges. Because the plates are not neutral, there is an electric field between the plates. The potential different between the two plates is proportional to the charge stored on the plates. The constant of proportionality is the capacitance Cof the capacity

$$C = \frac{Q}{V} \quad \text{or} \quad Q = CV \text{ .... (*12)}$$

Capacitance is in farads (F). 1 farad = I C/IV

The capacity of a parallel-plate capacitor is given by

$$C = \varepsilon \frac{A}{d},$$

ε – di electric constant of the insulator between the plates

A = Area of one of the plates

d = The separation between the plates

Differentiating the definition of capacity gives

$$Q = CV$$

$$\frac{Q}{V} = \iota = \frac{dC}{dt} V + C \frac{dv}{dt}$$

If the capacity is fixed i.e. dC = 0 then

$$\iota = C \frac{dv}{dt} \text{ …….. } (*13)$$

If eqn *3 is integrated it gives

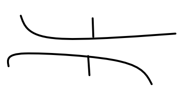
$$\int \iota \, dt = \int C \, dv$$

$$\int \iota \, dt = C \int dv \text{ ,… } VC = \int \iota \, dt$$

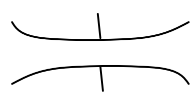
$$: V = \int \iota \, dt \text{ …… } (*14)$$

Equations (*13) and (*14) are the voltage – current characteristics for a character. They also show that the voltage across a capacitor cannot change instantaneously.

The symbol is



Fixed value



Variable value

Inductions: A wire carrying a current generate magmatic field in the space around the wire. As the current varies, the magnetic field varies, and the varying magnetic field induces a voltage in the wire that opposes the original changes in the current (faraday's law) under suitable conditions, this voltage is proportional to the change in the current. This defines the self inductance or simply the inductance L, of the piece or wire.

$$V(t) = L \frac{d\iota}{dt} \text{ } (*15) \text{ v}$$

Is the voltage caused by the change in the current. The definition of an inductance is also the voltage – current relationship for an inductor.

Integrating equation *5 give

$$\iota (t) = \frac{I}{L} \int v (t)\, dt \ldots.. *16$$

These are current – voltage relationship

Inductance is in Henry (H).

The symbol for inductors are shown below

Air core fixex   iron core fixd   Iron core variable

A Summary of the V – I Characteristics

For instance if a varying or sinusoidal current is passed through an element of the corresponding voltage across the element is given, e.g.

If $\iota = Im \sin (wt)$

Is flowing through a capacitor, the voltage across the capacitor is

$$V = \frac{I}{C} \int \iota\, dt = \frac{I}{C} \int Im \sin (wt)\, dt$$

$$= \frac{Im}{wc} \text{cost} (wt)$$

In a similar manner all at other elements in the 2 tables are calculated.

Calculation of voltage given currents

Components	General ι	ι = R Im sin (wt)
Resistor	V = R	V = R Im sin (wt)
Inductor	$V = L \frac{di}{dt}$	V = Lw Im cos (wt)
Capacitor	$V = \frac{I}{C} \int i \; dt$	$V = \frac{-Im}{Wc} \cos (wt)$

Calculation of current given voltage

Components	General ι	ι = R Im sin (wt)
Resistor	$\iota = \frac{V}{R}$	$\iota = \frac{V_m}{R} \sin (wt)$
Inductor	$\iota = \frac{I}{L} \int Vdt$	$\iota = - \frac{V_m}{wL} \cos (wt)$
Capacitor	$\iota = C \frac{dv}{dt}$	$\iota = w \, CV_m \cos (wt)$

Impedance

Considering the fig. above

An expression for the current in this sample circuit is

$$\iota = wcV_m \cos (wt) = wcV_m \sin (wt + 90^0) \quad (*17)$$

This shows that the magnitude of the current is (wc) times the magnitude of the voltage and the current leads the voltage by 90^0 ($\pi/2$) radiance.

The impedance Z of a circuit element is defined as the complex ratio

$$Z = \frac{V}{} \ldots\ldots \quad (*18)$$

ι

where the voltage across and current through the circuit element are described by complex numbers.

Impedance has the same dimension as the resistance, (voltage over current) and measured also in ohms, the reciprocal of impedance is the admittance. Considered again the simple circuit consisting of an ac generator a single capacitor shown below, this time, the voltage generator is described by a complex sinusoidal generator.

$$\iota = C\frac{dv}{dt} = CV_m jwe^{jwt} = jwcV \quad (*19)$$

Hence, the impedance Z of the capacitor C U

$$Zc = \frac{V}{\iota} = \frac{V}{jwcv} = \frac{I}{jwc} = \frac{-j}{wc} \ldots\ldots (*20)$$

and $j = \sqrt{-1}$

In same way $Z_L = jwL$ …….(*21) and that of a resistor $Z_R = R$ …… (*22) eqn (*19) shows that current through the capacitor is jwc times the applied voltage. Multiplying byj corresponds to rotating by 90^0 in the complex plane. Thus the eqn (*19) says that the current lead the voltage by 90^0 which is what eqn (*17) says.

All the theorems and methods used thus for remaining unchanged and equally true and useful when the word resistance is replaced by impedance.

Two simple examples

Consider the voltage divider below

The relationship between the input and output is

$$V_{out} = V_{in} \frac{Z_2}{Z_1 + Z_2} \text{ .... (*23)}$$

The equivalent impedance of several components in series is the sum of the individual impedances. Thus impedances in series

$Zeq = Z_1 + Z_2 + Z_3$ ef....... (*24)

Power: When a sinusoidal voltage is applied to a circuit, on the average, no power is dissipated in the capacitors or inductors. This is done to the 90^0 phase shift between the voltage and current.

The instantaneous power is defined as

$$P = v\iota$$

The average power is found by averaging over one full cycle. Thus

$$P_{av} = \int_0^{2\pi} V\iota \, dt$$

$$= \int_0^{2\pi} [V_m \sin (wt)] [wcv_m \cos (wt)]dt\iota \, dt$$

$$= 0$$

Reactance: Is defined as the magnitude of the impedance. Thus

This current is a simple voltage divider

It is called a high – pass filter. The goal here is to find expressions for the voltages drops and currents in this circuit.

At Low Frequencies: the capacitor will have a very high impedance (tending to infinity as frequency tends to zero. Thus at very low frequency the output voltage will be a small fraction of the input voltage.

At High Frequencies: the Zc will be small, thus at high frequency the output voltage will be equal to the input voltage.

:. the output voltage is $V_{out} = V_{in} \dfrac{Z_R}{Z_R + Zc}$

$$V_{out} = V_{in} \frac{R}{R - (j/wc)} \quad \ldots\ldots (*28)$$

If the circuit is not recognized as a voltage divider the easiest way to proceed is to we mesh equations, same result will be obtained. Thus, it is a one-loop circuit, the loop current is drawn – going around the loop in the same direction as the current gives.

$$Vin - \iota\, Zc - \iota\, Z_R = 0$$

$$\iota = \frac{V_{in}}{Z_R - Zc} = \frac{V_{in}}{R - (j/wc)} \quad R \quad \ldots\ldots (*29)$$

The output voltage is given by $V_{out} = \iota\, Z_R$

$$V_{out} = V_{in} \frac{R}{R - (j/wc)}$$

which is same as the previous result

Then the equivalent impedance Ze of the circuit can be found

$$Z_{eq} = \frac{V}{I} = \frac{V_{in}}{\frac{V_{in}}{R - (j/wc)}}$$

Which is what would have been obtained by direct application of equation

$$\text{i.e. } Z_{eq} = Z_1 + Z_2 + \ldots..$$

The gain of any circuit is the ratio of the output voltage to the input voltage. Thus for the voltage divider, the gain G is

$$G = \frac{V_{out}}{V_{in}} = \frac{V_{in} \frac{R}{R - (j/wc)}}{V_{in}}$$

$$G = \frac{R}{R - (j/wc)} \ldots\ldots (*32)$$

Written in rationalized form we have

$$G = \frac{R}{R - (j/wc)} \quad \frac{R + (j/wc)}{R + (j/wc)}$$

$$G = \frac{R^2 + j/wc)}{R^2 + [1/w^2c^2)]}$$

We normally write the gain in polar form i.e. considering its magnitude and its phase angle

$$/G/ = \frac{R^2}{R^2 + I/w^2c^2} + \frac{\frac{jR}{wc}}{R^2 + I/w^2c^2}$$

$$a \quad + \quad \iota\ b$$

$$/G/ = \sqrt{a^2 + b^2}$$

$$\left(\frac{R^2}{R^2 + I/w^2c^2}\right)^2 + \left(\frac{R/wc}{R^2 + I/w^2c^2}\right)^2$$

$$= \sqrt{\frac{R^4 + (R/wc)^2}{(R^2 + I/w^2c^2)^2}}$$

$$= \frac{R}{(R^2 + I/w^2c^2)^{½}} \quad …….. (33)$$

and the phase angle is $\Phi = \tan^{-1}(b/a)$

$$= \frac{\dfrac{R/wc}{R^2 + I/w^2c^2}}{\dfrac{R^2}{R^2 + I/w^2c^2}} = \frac{\dfrac{R}{wc}}{R^2} = \frac{\dfrac{R}{wc}}{R^2}$$

$$= \frac{R}{wc} \times \frac{1}{R^2} = \frac{1}{Rwc} \quad …..(*34)$$

Considering some limiting cases, it makes the equation more reasonable

At very low frequency $wc \approx 0$, $1/wc >> R$, then $1/Rwc >> 1$ so the gain is $/G/ \approx 0$ and the phase shift is $\Phi = \tan^{-1} \infty = 90^0 = \pi/2$ radius

At very high frequency $1/wc \approx 0$, $1/Rwc \approx 0$ so gara $/G/ \approx 1$, and phase shift is $\Phi = \tan^{-1} O = 0$

A Parallel Combination of Impedances

Considering some limiting cases,

At low frequencies that is as w tends to O the terms involving w in the numerator and then can be ignored and equation * 35 becomes

Z_{eq} = R (for low frequencies * (36)

At high frequencies, i.e. w becomes very large, the term involving only R in the numerator and Im then can be ignored.

$$Z_{eq} = \frac{-jwR^2c}{W^2c^2R^2}$$

$$= \frac{-j}{wc} \quad \text{(for high frequency)} \quad *37$$

Thus at low frequencies this combination acts like a pure resistor, at high frequencies, it acts like a pure capacitor. Only at intermediate frequencies does this circuits look like some combs of both resistor and a capacitor.

RESONANCE

The RcL circuit: The RcL circuit shown below is an example of a series resonance circuit.

It is possible to workout the expressions for the total impedance Z, the current ι, the output voltage V_{out}, and the gain G of this circuit. These are:

$$Z = R + jwL \frac{-j}{Wc} \quad \ldots\ldots(*38)$$

$$/Z/ = R^2 + \left(w^2L^2 + \frac{1}{w^2c^2} - 2\frac{L)}{C}\right)^{½} \ldots\ldots (*39)$$

$$\text{Since } /Z/ = \sqrt{R^2 + (Z^2 - Zc)^2} \equiv \sqrt{R^2 + (Z^2_L + Z^2_c - 2\, Z_L\, Z_c)}$$

$$R^2 + (jwL)^2 + \left(\frac{1}{jwc}\right)^2 - 2\,\frac{j\cancel{w}L}{j\cancel{w}c}$$

$$\iota = \frac{V}{Z} = \frac{V}{R + jwL - (j/wc)} \quad \ldots\ldots (*40)$$

$$V_{out} = \iota\, Zc = \frac{V}{R + jwL - (j/wc)} \quad \frac{- j/wc}{I}$$

$$\equiv \frac{- jV/wc}{R + jwL - (j/wc)} \quad \ldots\ldots. (*41)$$

$$\text{Gain } G = \left|\frac{V_{out}}{V_{in}}\right| = \frac{- j\,\cancel{v}/wc}{R + jwL - j/wc} \times \frac{1}{\cancel{V}}$$

$$= \frac{- j/wc}{R + jwL - j/wc}$$

$$= \frac{1}{\{R^2 + [wL - (1/wc)]^2\}½} \qquad = \frac{1}{wc\ /z/}$$

This equation looks two complex but however at resonance, things aer much simpler.

The resonant frequency f_0 is defined as that frequency at which $Z_L = Zc$

$$\text{i.e. } jw_0L = \frac{-j}{w_0c} \longrightarrow \frac{jw_0L}{1} = \frac{1}{jw_0c} \longrightarrow 1 = j^2w_0^2Lc$$

$$w_0^2 = \frac{1}{LC} \longrightarrow w_0 = \sqrt{\frac{1}{LC}} \ldots\ldots. (*44)$$

At resonance, the sum of the impedance of the capacitor and the inductor is zero, then all these general equations are simpler. (i.e. eqns *38 - *42) becomes

$$Z = R \ldots\ldots (*45)$$

$$\iota = \frac{V}{R} = \ldots\ldots (*46)$$

$$V_{out} = \frac{-jv}{\quad}$$

Rw_0C …….. (*47)

and $G = \dfrac{1}{Rw_0C}$

The plots of the general solution for the magnitude of the impedance and the voltage gain versus frequency are shown below.

Looking at the figures, there is a dip in the impedance, which becomes purely resistive a t resonance (i.e. Z = R). There is also a peak in the voltage gain at resonance.

- The most interesting thing of these plots is that the gain is greater than 1, i.e. the output voltage is greater than the input voltage.

This is caused by the phase differences in the voltages across the induction and capacitor. The voltage across the induction leads the current by 90^0, whereas the voltage across the capacitor lags the current by 90^0, despite the fact that the same current flows through the resistor, induction and the capacitor.

- At resonance, the two voltages have the same magnitude and add to zero i.e.

  $\text{Sin}(w0t + 90^0) + \text{Sin}(wot - 90^0) = 0$
- This means that the current is limited only by R, which can be made small allowing the current to be quite large.
- If V_{out} were taken across the resistor, the gain would peak at 1 at resonance.
- To get a voltage gain out of a series resonance circuit, the output must be taken across either the inductor or the capacitor. Since V_{out} = Vc in this case may be quite large even than V_{in}.
- At resonance, the current flowing in the circuit for a fixed input voltage V is determined by only R.
- The resonance frequencies is determined only by L and C (since $w0 = \sqrt{1/LC}$)
- The quality factor Q is the ratio of the inductive or capacitive reactance to the resistance,

  i.e. $Q = \frac{X_L}{R} = \frac{Xc}{R} = \frac{w_L}{R}$ ……. (*48)
- It determines the details of the shape of the voltage gain and impedance plots.
- The greater the Q of the circuit, the sharper is the peak in the voltage gain and the narrower is the dip in the impedance graph. Fig. ** and (***) above
- The Q factor of a circuit is also a measure of how long an oscillation will continue once it has been excited.
- The higher the Q, the longer the oscillation will continue i.e.

- L and C determine the resonance frequencies, whereas R and C determine the decay time constant for the circuit.
- Any time that there is the need to pick out a signal at one frequency and suppress nearby frequencies, resonant circuits and used

STEP FUNCTION ANALYSIS

Step function voltages are voltages that change suddenly from one steady value to another e.g fig. below.

They are neither ac nor dc, but some of the technique used to analyze both ac and dc circuits can be used in this case.

The application of the basic circuit analysis laws to situations involving step functions give rise to differential equations. These equations must be solved to analyze these circuits completely.

The Rc circuit – part 1

- The circuit consists of a battery, a resistor and a capacitor in series with a two – position switch that can be used to connect or disconnect the battery from the rest of the circuit.
- The initial condition of the circuit is shown in fig (a). It is assumed that the switch has been in down position for long i.e. no voltage across the capacitor, no charge on the capacitor, and no current is flowing in the circuit.
- At some instance of time, the switch is moved upwards i.e. at time t = o. This situation is shown in fig (b) above.
- The goal of the analysis is to find the current and voltages in the circuit as a function of time from the moment the switch was closed.
- At the instance that the switch is closed, there is no voltage drop across the capacitor because there is no charge on the capacitor this is due to the fact that the voltage across the capacitor cannot change instantaneously.
- KVL shows that the entire voltage must appear across the resistor.
- Thus, the instant the switch is closed, the current changes very rapidly (instantaneously) from zero to an initial value of V/R = ι
- As the current flows, charge builds up on the capacitor as shown in fig (b). This means that there is an increasing voltage drop across C and hence the voltage drop across R must decrease (according to KVL). At time goes on the current must decrease. Eventually, the voltage across the capacitor will build up to V; at this point, there will be no current flow and nothing will change thereafter.
- To get a complete solution to this problem, KVL is applied to the circuit in fig (b).

$$V - \iota R - \frac{q}{c} = 0 \ldots\ldots (*49)$$

$$\iota = \frac{dq}{dt} \ldots\ldots (*50)$$

$$V - \frac{Rdq}{} - \frac{q}{} = 0 \ldots\ldots (*51)$$

dt c

An alternative way is to use the voltage – current relationship for the capacitor,

$$q = \int ιdt \;\ldots\ldots(*52)$$

then equation (*49) becomes the integral equation

$$V - ιR - 1\int ιdt = 0 \; (*53)$$

To solve this differential equation (*51) we assumed a solution of the form

$$q = B + ke^{\alpha t} \;\ldots\ldots. \; (*54)$$

substituting into the differential eqn (*51) to have

$$V - Rk\alpha e^{\alpha t} - \frac{B}{C} - \frac{K}{C} e^{\alpha t} = 0 \;\ldots\ldots(*55)$$

This equation can only be satisfied at all times if the terms involving and those not having the ex ponential add up to zero separately. Thus.

$$V - \frac{B}{C} = 0 \;\ldots\ldots(*56)$$

and $$- RK\alpha - \frac{K}{C} \;\; 0 \;\ldots\ldots. \; (*57)$$

eqn *56 becomes $B = VC \;\ldots.. \; (*58)$

and eqn *57 becomes $\alpha = \frac{-1}{RC} = \frac{-1}{\gamma}$

Substituting eqn * 58 and (*59) into the assumed solution to give

$$q = CV + Ke^{-t/RC} \;\ldots\ldots\; (*60)$$

k is found from the initial conditions of the problem i.e. t = 0

$$q(0) = O = CV + Ke^{-O} \;\ldots\ldots\; (*61)$$

$$k = -CV \;\ldots\ldots\; (*61)$$

the solution of eqn (*51) is

$$q = CV(1 - e^{-t/RC}) = CV(1 - e^{-t/\gamma})$$

where $\gamma = RC$ called the time constant for the circuit and is the characteristic time unit for the problem.

The current is the derivative of the charge on the capacitor i.e.

$$\iota = \frac{dq}{dt} = \frac{V}{R} e^{-t/RC} \; (*64)$$

The solutions are plotted below

The RC – Circuit – PART 2 (DISCHARGING)

After the switch has been in the upper position for a very long time, it is back to lower position see fig. the time t = 0 is taken as the time at which the switch was move to the down position.

The initial condition is that of q (0) = Cv from the diagram, it is seen that the current is discharging the capacitor, so that

$$\iota = \frac{dq}{dt} \;\ldots\ldots\ldots\; (*65)$$

Applying KVL to the circuit to give

$$\iota R = \frac{q}{c} = 0 \; \ldots\ldots. \; (*66)$$

or $$\frac{Rdq}{dt} = -\frac{q}{c} \; \ldots\ldots. \; (*67)$$

assume a solution

$$q = C_{ve}^{-t/Rc} \ldots\ldots. \; (*68)$$

and $$\iota = -\frac{dq}{dt} = \frac{V}{R} \; e^{-t/Rc} \; \ldots\ldots. \; (*69)$$

The results are plotted below

THE RL – CIRCUIT

In the fig. below, the capacitor is replaced by an inductor.

- The switch S has been in the down position for long so that there is no initial current.
- As before, at time t = o, (by definition), the switch moved to the upper position,
- At that instant, the current is zero, and because there is an inductor in the circuit, the value of the current can not change instantaneously.
- Applying KVL to the circuit yields

$$V - L\frac{di}{dt} - \iota R = O \; \ldots\ldots. \; (*70)$$

- This is essentially the same differential equation as the previous only this time it is for ι, not q.
- Solving in similar manner as before, the solution is

$$\iota = \frac{V}{R}\left(1 - e^{-t/\gamma}\right) \ldots\ldots.(*71)$$

where the time constant is given by

$$\gamma = \frac{R}{L} \ldots\ldots (*72)$$

After a very long time, the switch is moved back to its lower position. (In the laboratory, it must be made certain that the current is never opened at anytime).

- This means using a “shorting” type switch).
- The solution for the current this time is

$$\iota = \iota_0 e^{-t/\gamma} = \frac{V}{R} e^{-t/\gamma} \ldots\ldots.(*73)$$

These solutions have the same form as those for the RC circuit, except that the currents and voltages have been interchanged.

The RCL Circuit

Again, after being in the down position for a very long time, the switch S is moved to its upper position.

Applying KVL to this situation yields

$$V - \iota R - \frac{q}{C} - L\frac{di}{dt} = 0 \ldots\ldots (*74)$$

or $$V - \iota R - \underline{I} \int \iota dt - L\,\underline{di} = 0 \ldots\ldots (*75)$$

C dt

Differentiating yields

$$\frac{Rd\iota}{dt} + \frac{\iota}{c} + \frac{Ld\iota^2}{dt^2} = 0 \text{ …… (*76)}$$

This is the equation of the damped harmonic oscillator. The solutions will not be given here only their general nature will be discussed, it is a general so equation in mechanics.

- It turns out that there are three classes of possible solutions, depending on the relative values of R, C, and L. These depend on whether the quantity.

$$\left(\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}\right)^{½} \text{ …… (*77)}$$

  is zero, real or imaginary

- If $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ …… (*78)

Is said to be over damped

- If $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ …… (*79)

Then the oscillator is critically damped

- If $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ …… (*80)

It is said to be under damped

Using the definition of Q – factor, these conditions become

Over damped: $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ or $Q < \frac{1}{2}$ …... (*81)

Critically damped: $\left(\frac{R}{\ }\right)^2 = \frac{1}{\ }$ or $Q = \frac{1}{\ }$ …... (*82)

2L LC 2

Under damped: $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ or $Q > \frac{1}{2}$ …... (*83)

READ UP GALVANOMETERS

ELECTRONICS

What is Electronics?

It is a field of engineering and applied physics dealing with the design and application of devices, usually electronic circuits, the operation of which depends on the flow of electrons for the generation, transmission, reception and storage of information.

VACUUM TUBE

A vacuum tube consists of an air – evacuated glass envelop that contains several metal electrodes. A simple two-element tube (Vacuum diode) consists of a cathode and an anode that is connected to the positive terminal of a power supply.

The cathode is a small metal tube heated by a filament and the anode is metal cylinder around the cathode (also called the plate).

Free electrons migrate to the anode from the cathode.

If an alternating voltage is applied to the anode, electrons will only flow to the anode during the positive half – cycle, and during the negative cycle of the alternating voltage, the anode repels the electrons and no current passes through the tube.

Diodes connected in such a way that only positive half – cycle of an alternating current (AC) are permitted to pass are called rectifier tubes, and these are used in the conversion of AC to DC.

By inserting a grid, consistency of a spiral of metal wire, between the cathode and the anode and applying a negative voltage to the grid, the flow of electrons can be controlled. When the grid is negative, it repels elections and only a fraction of the electrons emitted by the cathode can reach the anode.

n-Type and p-Type Semiconductors

The materials used in diodes and transistors are semiconductors, such as silicon and germanium. However, they are not pure materials, because small amounts of "impurity" atoms (about one part in a million) have been added to them so that either and abundance or a lack of electrons exists.

A semiconductor doped with an impurity that contributes mobile electrons is called an **n-type semiconductor,** since the mobile charge carriers have a negative charge.

A semiconductor doped with an impurity that introduces mobile positive holes is called **p-type semiconductor.**

Electrons from the n-type semiconductor and holes from the p-type semiconductor flow across the junction and combine. This process leaves the n-type material with a positive charge layer and the p-type material with a negative charge layer, as part c of the drawing indicates. The positive and negative charge layers on the two sides of the junction set up an electric field E, much like that in a parallel plate capacitor. This electric field tends to prevent and further movement of charge across the junction, and all charge flow quickly stops.

Suppose now that a battery is connected across the p-n junction, where the negative terminal of the battery is attached to the n-material, and the positive terminal is attached to the p-material. In this situation the junction is said to be in a condition of **forward bias**, and as we now been seen, a current exists in the circuit. The mobile electrons is in the n-material are repelled by the negative terminal of the battery and move toward the junction. Likewise, the positive holes is the p-material are repelled by the position terminal of the battery and also move toward the junction. At the junction the electrons fill the holes. In the meantime, the negative circuits, in which both halves of every cycle of the generator voltage drive current through the load resistor in the same direction.

Power supplies using diodes are also found in virtually all electronic appliances, such as televisions and microwaves ovens.

Transistor

A number of different kinds of transistors are in use today. One common type is the **bipolar junction transistor**, which consists of two p-n junction formed by three layers of doped semiconductors. As figure (g) indicates , there are pnp and npn transistors. In either case, the middle region is made very thin compared to the outer region.

A transistor is useful because it is capable of amplifying a smaller voltage into one that is much greater. In other words, a small change in the voltage applied as input to a transistor produces a large change in the output from the resistor.

Fig (h) shows a pnp transistor connected to two batteries, labeled V_E and V_C. The voltage V_E is applied in such a way that the p-n junction on the left has a forward bias, while the p-n applied in such a way that the p-n junction on the left has a forward bias while the p-n junction on the right has a reverse bias. Moreover, the voltage V_c is usually much larger than V_E for a reason to be discussed shortly. The drawing also shows the standard symbol and nomenclature for the three sections of the transistor, namely the emitter, the base and the collector. The arrowhead in the symbol points in the direction of the conventional current through the emitter.

The position terminal of V_E pushes the mobile positive holes in the p-type material of the emitter toward the emitter/base junction. And since this junction has a forward bias, the holes enter the base region readily. Once in the base region, the holes come under the strong influence of V_c and are attracted to its negative terminal. Since the base is to thin (about 10^{-6}m or so), approximately 98% of the holes are drawn directly through the base and on into the collector. The remaining 2% of the holes combine with free electrons in the base region, thereby giving rise to a small base current I_B. As the drawing shows, the moving holes in the emitter and collector constitute currents that are labeled I_E and I_C, respectively. From Kirchhoff's junction rule it follows that $I_C = I_E – I_B$.

Because the base current I_B is small, the collector current is determined primarily by current from the emitter ($I_C = I_E – I_B \approx I_E$). This means that a change in I_E will cause a change in I_C of nearly the same amount. Furthermore, a substantial change in I_E can be caused by only a small change in the forward bias voltage V_E. To see that this is the case, look back at fig (h) and notice how steep the current-versus-voltage curve is for a p-n junction; small changes in the forward bias voltage give rise to large changes in the current.

With the help of fig (h) we can now appreciate what was meant by the earlier statement that a small change in the voltage applied as input to a transistor leads to a large change in the output. This picture shows an ac generator connected in series with the battery V_E, and a resistance R connected in series with the collector. The generator voltage could originate from an electric guitar pickup or the phono cartridge of a turntable, while the resistance R could represent a loudspeaker. The generator introduces small voltage changes in the forward bias across the emitter/base junction and, thus, causes large corresponding changes in the current I_C leaving the collector and passing through the resistance R. As a result, the output voltage across R is an enlarged or amplified version of the input voltage of the generator.

The operation of an npn transistor is similar to that of a pnp transistor. The main difference is that the bias voltages (and current directions) are reversed, as fig (i) indicates.

It is important to realize that the increased power available at the output of a transistor amplifier does not come from the transistor itself. Rather, it comes from the power provided by the voltage source V_C. The transistor, acting like an automatic valve, merely allows the small, weak signals from the input generator to control the power taken from the voltage source V_C and delivered to the resistance R.

Another type of transistor is the field-effect-transistor (FET), such as transistor operates on the principle of reputation or attraction of charges due to a superimposed electric field. Amplification of currents is accomplished in a manner similar to the grid control of a vacuum tube field-effect-transistor operate more efficiently than bipolar types, because a large signal can be controlled by a very small amount of energy.

Rectifiers and Amplifiers

Amplifiers Circuit

Electronic amplifiers are used mainly to increase the voltage, current or power of a signal.

An ideal linear amplifier would provide signal amplification with no distortion, so that the output was proportional to the input.

In practice, however, some degree of distortion is always introduced.

Linear amplifiers are used for audio and video signals, whereas non-linear amplifiers find use of oscillators, power electronics, modulators, mixers, logic circuits, and other applications where an amplification cut-off is designed. Although vacuum tubes played a major role in amplifiers in the past, today discrete transistor or ICs are general used.

Rectifiers

Are devices used to convert a alternating currents to direct currents.

Feedback Oscillator

Oscillators generally consist of an amplifier and some type of feedback: the output signal is feedback to the input of the amplifier.

The frequency – determining elements may be a tuned inductance – capacitance circuit or a vibrating crystal.

Oscillators are used to produce audio and video signals for a wide variety of purpose. For example, simple audio-frequency oscillators are used in modern push-button to transmit data to the central telephone exchange when dialing.

Audio tones generally by oscillators are also found in alarm clocks, radios, electronic instruments computers and warning systems.

High – frequency oscillators are used in communications equipment to provide tuning and signal – detection functions.

Radio and television stations use precious high-frequency oscillators to produce transmitting frequencies.

Using a transistor to explain feedback oscillator

Small Signal Equivalent

When transistors are biased in the active region (i.e. the base emitter region) and used for amplification, it is often worth while to approximate their behavior conditions of small voltage variations at the base – emitter junction.

If these variations are smaller than the thermal voltage $V_t = K_B T/q$, it is possible to represent the transistor by a linear equivalent circuit.

This representation can be of great aid in the design of amplifying circuits. It is called the small-signal transistor model.

When a transistor is biased in the active mode, collector current is related to base-emitter voltage by the following equation:

$$I_C = Is\ e\ x\ p\left(\frac{qV_{BE}}{K_B T}\right) = Is\ e\alpha p\left(\frac{V_{BE}}{V_t}\right)$$

where I_S = Source Current,

V_{BE} = Junction Voltage (emitter)

If VBE varies incrementally, I_C will also vary

$$\frac{\partial I_C}{\partial V_{BE}} = \frac{I_S}{V_t}\ e\alpha p\left(\frac{V_{BE}}{V_t}\right) = \frac{I_C}{V_t}\ e\alpha p\ \ V_{BE}$$

Equivalent circuit representing small-signal active bias for a bipolar junction transistor

Low frequency small-signal equivalent circuit including base resistance

Equivalent Circuit of the Triode

The most important use of the triode is as an amplifier.

If the voltage applied to the grid is changed by a small amount, there will be a corresponding change in the anode current. If the anode is connected to its high tension source through a resistance R, the change in anode current will cause a change in the potential drop across R.

Equivalent circuit of a triode tube, considered is a voltage generator

Equivalent circuit of a triode tube, considered as a current generator

Small Signal Equivalent Circuits

Small signal means that the signal is small and that the transistor can be reduced to a linear circuit equivalent. For the transistor to work the active region i.e. the base-emitter region, there must be a P.difference between this region and the emitter current. ι_E and the collector current ι_C determining the amplification of the transistor.

Feedback Oscillators

Small signal equivalent circuits

Equivalent circuit of triode

Equivalent circuit representing small signal

Active bias tan bipolar junction transistors

Low frequency small – signal equivalent circuit including base resistance.