1 Mechanics and Materials

If a contractor builds a house for a man, and does not build it strong enough, and the house he built collapses and causes the death of the house owner, the contractor shall be put to death.

—Code of King Hammurabi of Babylon (ca. 1600 BC)

Objective: This chapter will provide the motivation and background for integrating the study of mechanics and materials in structural design.

What the student will learn in this chapter:

- Why we must consider both mechanics and materials in the design of structures
- An overview of structures, including loads, structural types, and analysis methods
- An overview of materials, including atomic structure, bonding, and properties
- An example of a mechanics $\Leftrightarrow$ materials link: probabilistic structural analysis

1.1 Introduction

The design of structures crosses many disciplines and has a long history. Some of the earliest structures were civil structures, mostly shelters such as tents and huts. Simple mechanical structures also were in early use, such as levers, and supports for processing and cooking game. Later, marine structures such as rafts and canoes came into existence. Other structures followed, including carts, construction equipment, and weapons. Then aerospace structures, such as balloons and parachutes, appeared.

Today, the fields of aerospace, civil, marine, and mechanical engineering are all involved in structural design. On a cursory level, we can define a *primary structure* as any physical body that must carry loads (other than its own weight), and hence develops stresses and strains. Often these stresses and strains are trivial, and the body can be considered a *secondary structure*. However, in many cases, inadequate design for carrying loads can lead to significant, even catastrophic, failure in primary structures. Unfortunately, examples of such failure are readily found, including the Tacoma Narrows Bridge disaster, the Kansas City Hyatt Regency skywalk collapse, and the Space Shuttle Challenger explosion. From early on, as in King Hammurabi's days, adequate structural design was recognized as critically important.

This text seeks to present structural design in a very broad and integrated context, what we call *total structural design*. This encompasses not just mechanics issues, such as the analysis of stress and strain, but materials issues and manufacture/assembly issues as well. Hence we want to think of a structure as an integrated material/structural system, realized with cradle-to-grave design.

In this text, you will learn about:

- Fundamental mechanics or macroscopic concepts such as stress, strain, and stiffness
- Fundamental materials or microscopic concepts such as atomic structure and bonding, defect theory, and the physics of material properties
- The intimate linkage between macroscopic and microscopic concepts, and their role in the design of structures
- How to apply a *generalized design template* in the design of structures
- How to manipulate *design degrees of freedom* to achieve an optimal structural design
- The different classes of structures and their special design considerations
- Simultaneous engineering of materials and structures

1.2 Motivational Examples

As performance expectations for products rise, the need for removing separate consideration of structure and material increases. At high levels of performance, we no longer have the luxury of first designing a structural "form," and then searching for an available material to fill in the space. We must think in terms of *material/structural systems*, which are designed in an integrated fashion. Numerous examples can be provided.

A skyscraper leaves very little room for inefficiency in structural design. Consequently, glass walls cannot be designed simply for covering and viewing purposes, but must be part of the load-carrying system as well. Coverings that also carry loads are called *monocoque*.

Another example is the development of ultralightweight structures for space applications. In areas such as imaging, solar propulsion, and communications, increased collecting area provides significant gains in performance. Since launch vehicle capacity is limited in terms of size and lift, the options for stowing, launching, and deploying very large precision structures in space is limited. (The International Space Station is an example of a large mechanically erectable structure—but it is not a precision structure like an antenna.)

One of the concepts being widely investigated at the current time is that of membrane/inflatable "gossamer" structures. These are made from thin, flexible films, unfurled or inflated on orbit, with some components possibly rigidized afterward. Extremely low areal densities must be developed, in order to keep total launch mass below specified limits. (*Areal density* is the total system mass divided by the collecting area.) Consider that the areal density will have to be diminished from that of the current Hubble space telescope (at $\sim 100\,\mathrm{kg/m^2}$), to $0.1\,\mathrm{kg/m^2}$—a decrease of three orders of magnitude! Such increases in system performance will require close cooperation between structural engineers and materials scientists, who each have a good understanding of the other's discipline. A good example of a gossamer spacecraft is shown in Figure 1-1.

Another example of a membrane structure is a tension fabric structure. In civil structures, where large areas need to be covered with clear spans, the "tent" roof provides an impressive solution. The roof of the Denver International Airport shown in Figures 1-2 and 1-3 is a wonderful example. Again, the close coupling of materials, mechanics, and design is crucial for membrane structures.

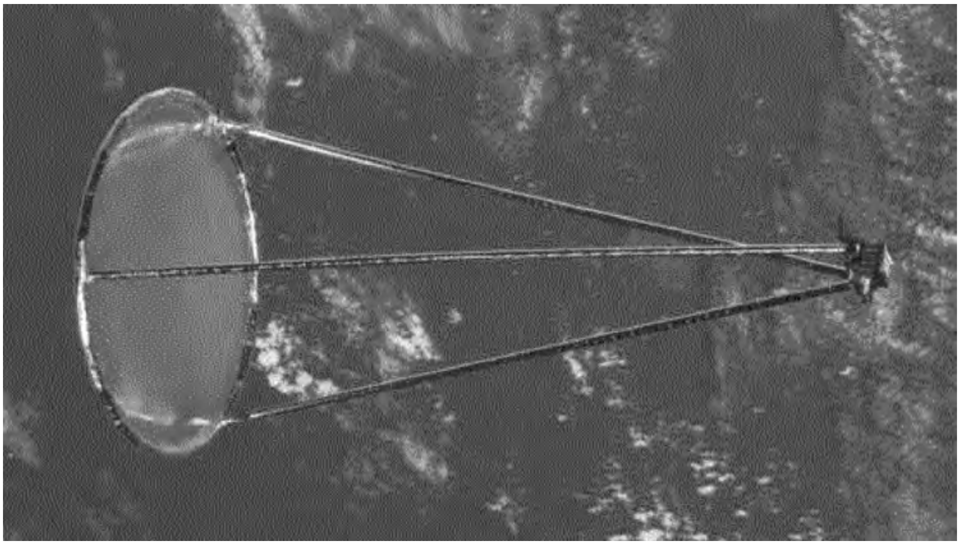


FIGURE 1-1 *Inflatable Antenna Experiment launched in 1996. (Courtesy of NASA.)*

FIGURE 1-2 *Tension fabric roof at the Denver International Airport.*

As still another example, consider the linkage system on a General Electric aircraft turbine engine as seen in Figures 1-4 and 1-5.

This system shows a richness of structural diversity. The system is composed of rods, beams, shafts, and plates, among others. The links (rods) may be under tension or compression, the beams and plates may undergo bending, and the shafts may be placed in torsion. Clamping

FIGURE 1-3 *Tension fabric roof detail at Denver International Airport.*

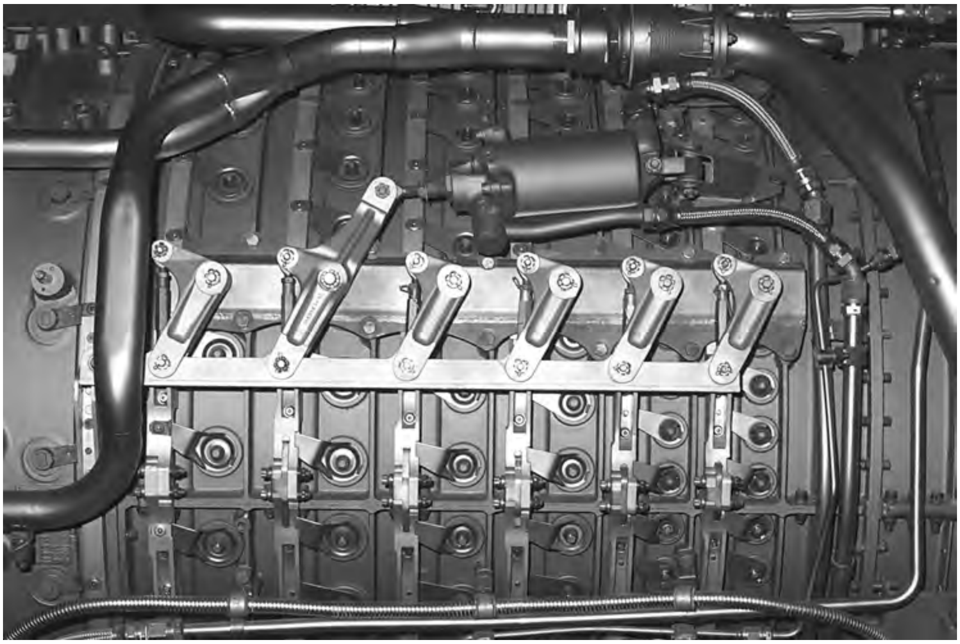


FIGURE 1-4 *Variable stator linkage system on a GE Aircraft Turbine Engine. (Courtesy of Mr. Andrew Johnson, GE Aircraft Engine Co.)*

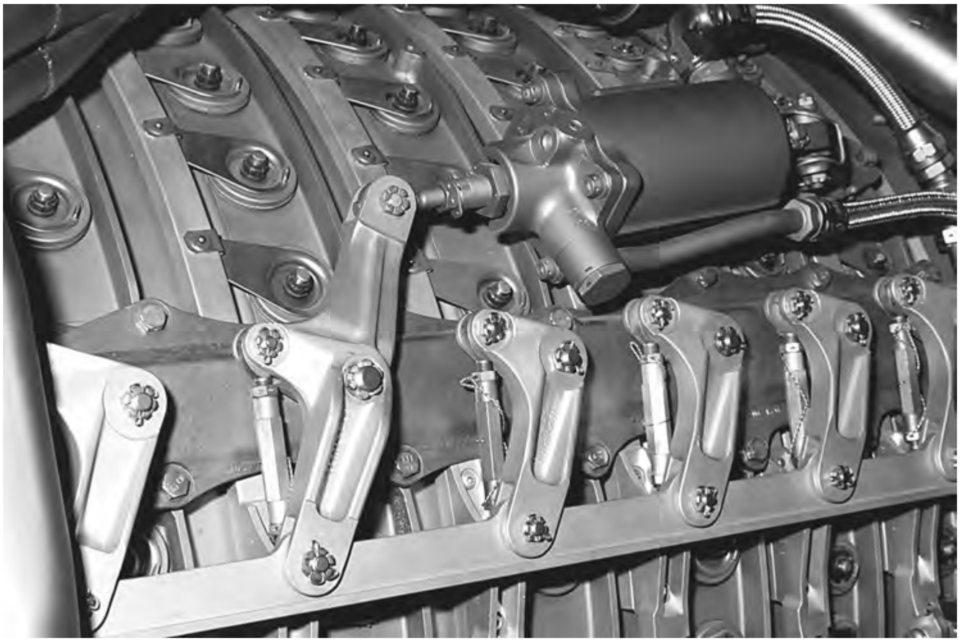


FIGURE 1-5 *Close up of the variable stator linkage system. (Courtesy of Mr. Andrew Johnson, GE Aircraft Engine Co.)*

forces are also apparent in a number of places. Everything has to work under an extreme environment (such as high temperature), and it's got to work right! Again, mechanics–materials–design must all come together in an integrative fashion.

1.3 Linkage between Mechanics and Materials Science in Structural Design

Applied mechanics and materials science are broad fields of human endeavor with long histories. *Mechanics* deals with the theoretical and experimental analysis of forces on material bodies, and the resultant motions and deformation that follow. *Materials science* is concerned with the atomic structure of materials and the properties resulting there from. For our purposes here, we offer a working, albeit oversimplified, definition: mechanics is physics, materials science is chemistry.

The early "royal philosophers" such as Da Vinci, Galileo, Hooke, and Young were truly renaissance scholars, in that they brought to their investigations a broad background in chemistry, physics, mathematics, and classical studies. The development of analytical mathematics and mechanics in the 18th century increasingly fostered specialists, as contrasted with these earlier generalists. At the same time, chemistry and physics were developing themselves, and with the expanded use of metals in the industrial 19th century, each began to focus into specialized areas (metallurgy, for example). As specialized bodies of knowledge developed, it became increasingly difficult to be inclusive and bridge across various disciplines.

This divergence continues into modern times. Consider, for example, Seely's text *Resistance of Materials* (1925), which contained a 55-page chapter on "Mechanical Properties of

Structural Materials"; by the printing of the 4th edition in 1955, nearly all mention of material properties had disappeared, save for a discussion on fatigue. One can find today very few "mechanics of materials" texts that combine mechanics and materials science (not to mention design) in an integrated and cogent manner (for an exception, see Roylance, 1996).

Today, examples abound that show the need for engineers and scientists who have an integrated, interdisciplinary background bridging mechanics and materials science. Consider, for example, the important and active area of high-performance composite materials. Here, an intimate knowledge of structure–property relations is demanded for technological advancement. Bulk response can be predicted in an averaged sense using a mechanics approach, which is necessary to design a real composite structure; but only knowledge of the fine-scale (micro- to nanoscale) structure–property relations and interactions among the constituents can lead to an optimal "engineering" of these materials for an intended application.

Other topics of current interest include computational modeling of materials with evolving microstructures; advanced manufacturing and processing challenges to mechanics and materials; mechanics and statistical physics of particulate materials; mechanics and materials science of contact; and processing and mechanics of nanoscale, multilayered materials. We will show in what follows that every structural design should be an integration of mechanics and materials technology.

1.4 Overview of Structures

1.4.1 Form and Function

Engineers would typically argue that the purpose (function) of an artifact should determine its appearance (form), at least in a general way. On the other hand, artists might consider the form of an object to be the primary thing!

In engineering design, functionality drives the form of a product. However, the number and variety of possible forms that ultimately embody the design may be astounding. As an example, consider the design of a paper clip. The functional requirements might be listed as:

- Provide a clamping force to hold two or more sheets of paper together
- Easily engaged/removed
- Lightweight
- Reusable
- Inexpensive

Consider now the possible forms the clip might take (Figure 1-6). Simply put,
***Design* is a process that determines the form of an artifact that (optimally) satisfies the functional requirements**.

1.4.2 Structural Loads

In structural design, we use the term *loads* to mean forces and moments applied to the structure, either externally on the boundaries (*surface loads*), or within the structure (*body loads*).

Loads are further considered to be either static or dynamic. *Static loads* are loads that do not depend on time, that is, they are of constant magnitude, direction, and location. Although it might seem that certain structures are static, for example, a civil structure such as a building, this is rarely the case. "Live loads" from occupant activity, wind loads, seismic (earthquake)

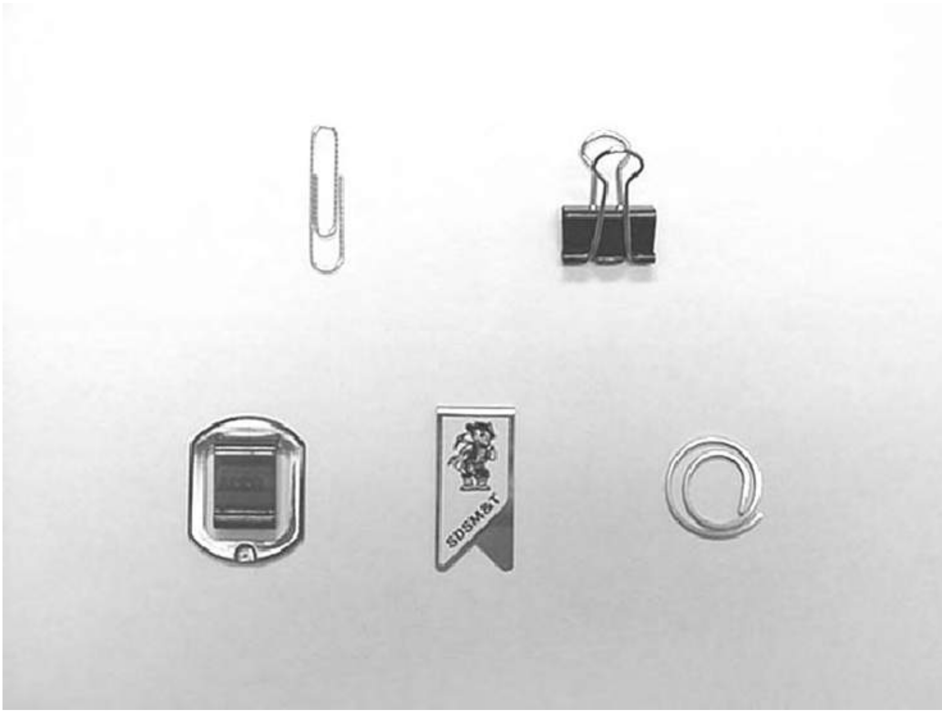


FIGURE 1-6 *Possible forms for a paper clip.*

loads, thermal cycling, and so on, all may give rise to a dynamic load environment. However, if the loads vary 'slowly' with time, they are often considered *quasi-static* and taken as static loads. ("Slowly" usually means that structure inertia forces due to accelerations may be neglected with respect to the difference of the externally applied forces and the internal resistance forces.)

Dynamic loads are divided into two main categories:

1. *Steady-state loads* are loads that maintain the same character (frequency, amplitude, etc.) over the long term.
2. *Transient loads* are loads that change their character (e.g., they may decay) with time.

Common structural loads are summarized in Table 1-1.

The orientation of the load on a structural member is also important. Although this issue will be discussed in more detail in later chapters, a brief summary is given in Figure 1-7.

1.4.3 A Taxonomy of Structures

Humans have always tried to understand complex systems by decomposing them into a number of simpler, more manageable parts. The hope is that when this compartmentalized knowledge is put back together (synthesized), an accurate representation of the whole system results. While this approach has worked well in countless human enterprises, it is based on the linear assumption of superposition, which fails as systems become nonlinear and more complex.

TABLE 1-1 Summary of Common Structural Loads

Surface loads (Common name)	*Units SI (US)*	*Body loads*	*Units SI (US)*
Concentrated force ("Point load")[a]	N (lb)	Gravitational force ("Gravity load" or "weight")	N (lb)
Distributed force ("Line load")[a] ("Pressure")	N/m (lb/in) N/m^2 (psi)	Thermal stress ("Thermal load")	N/m^2 (psi)
Moment or couple ("Bending moment" or "torsion moment")	N-m (lb-in)	Magnetic forces ("Magnetic moment")	$N\text{-}m/m^3$(lb-in/in^3)

[a]In principle, "point" and "line" loads cannot exist, since any load will act over a finite spatial area, no matter how small; however, if that area is small relative to the structure size, then for practical purposes the load may be considered a "point" load or a "line" load.

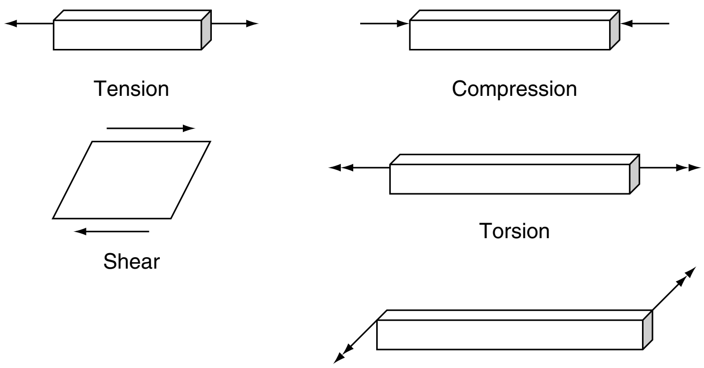


FIGURE 1-7 *Simple load orientations. Single-headed arrows are forces; double-headed arrows are moments (that follow the right-hand rule).*

Forewarned by this knowledge, we will attempt here a *taxonomy* (classification) *of structures* (complex systems) by decomposing them into their structural elements (simpler parts). Most real structures are comprised of a number of different types of structural elements, any one of which may assume a variety of different roles. The primary characteristic used to classify a structural element is how it carries loads.

Carrying loads is the primary function of a structure, and it is this characteristic that largely determines the form of the structural elements. Loads carried include tensile and compressive axial loads; shear loads; torsion moment loads; bending moment loads; distributed loads; gravity loads; and thermal loads.

Structural forms fall into two major categories: *line-forming* and *surface-forming*. Surface-forming elements may be further subdivided into *area-forming* and *volume-forming* elements (see, e.g., Schodek, 2001). A taxonomy of structural elements is given in Table 1-2.

A review of Table 1-2 reveals that beams, plates, and shells are structural elements fully capable of carrying all types of loads. Specialty elements such as cables, rods, and membranes

TABLE 1-2 A Taxonomy of Structural Elements

Function ⇓	*Form*						
	Line-forming			*Surface-forming*			
				Area-forming		*Volume-forming*	
Loads carried	*Cable*	*Rod*	*Beam*	*Plane membrane*	*Plate*	*Curved membrane*	*Shell*
Tensile axial	✓	✓	✓	✓	✓	✓	✓
Compressive axial	–	✓	✓	–	✓	–	✓
Direct shear	–	–	✓	–	✓	–	✓
Torsion moment	–	✓	✓	–	✓	–	✓
Bending moment	–	–	✓	–	✓	–	✓
Distributed force	✓	–	✓	✓	✓	✓	✓
Thermal	✓	✓	✓	✓	✓	✓	✓

carry limited types of loads. (Note: *finite element analysis* codes used in structural analysis have libraries of structural elements based upon a similar taxonomy.) Schematics of the elements are shown in Figure 1-8.

Line-forming elements (LFEs) are slender structures having one spatial dimension (length) significantly greater than any other dimension (width, height, thickness, etc.). Schematically, for analysis purposes, LFEs may be represented as lines, either straight or curved as required. LFEs may carry loads in tension, compression, torsion, bending, or some combination of these, depending on the nature of the applied loads, structural geometry, material properties, and boundary conditions. LFEs may form axial, torsional, or bending structures such as rods, cables, and beams.

1. *Rods* carry concentrated axial forces in either tension or compression (no moments or transverse forces). Thus they are *axial structures*. Only *pinned* (simply supported) boundary conditions are required. Cross-sectional geometries may take any shape, but simple shapes are most common, such as circular and rectangular. The cross-section may be solid or hollow.
2. *Cables* carry concentrated axial tensile forces (but not compressive axial forces—you cannot push a rope!), as well as concentrated or distributed transverse forces (but not moments!). In the latter case, cables deform under the action of these transverse loads in such a way that they remain *axial* structures (specifically, no-compression axial structures). (Of course, in the former case of axial loads, cables are by definition axial structures.) As with rods, only pinned boundary conditions are required. Cross-sectional geometries may be of any shape, but solid circular cross-sections are common.
3. *Beams* are the most complete LFEs since they can carry axial compressive or tensile forces as in rods or cables, as well as transverse concentrated or distributed forces as in cables; moreover, they can carry torsion and bending moments. In cases where moments are applied, at least one boundary condition must be able to support moment reactions (e.g., a "fixed" or "built-in" condition). Cross-sectional shapes may be of any geometry, solid or hollow.

Surface-forming elements (SFEs) are thin structures having two spatial dimensions (length and width) significantly greater than the third dimension (thickness). Schematically, for

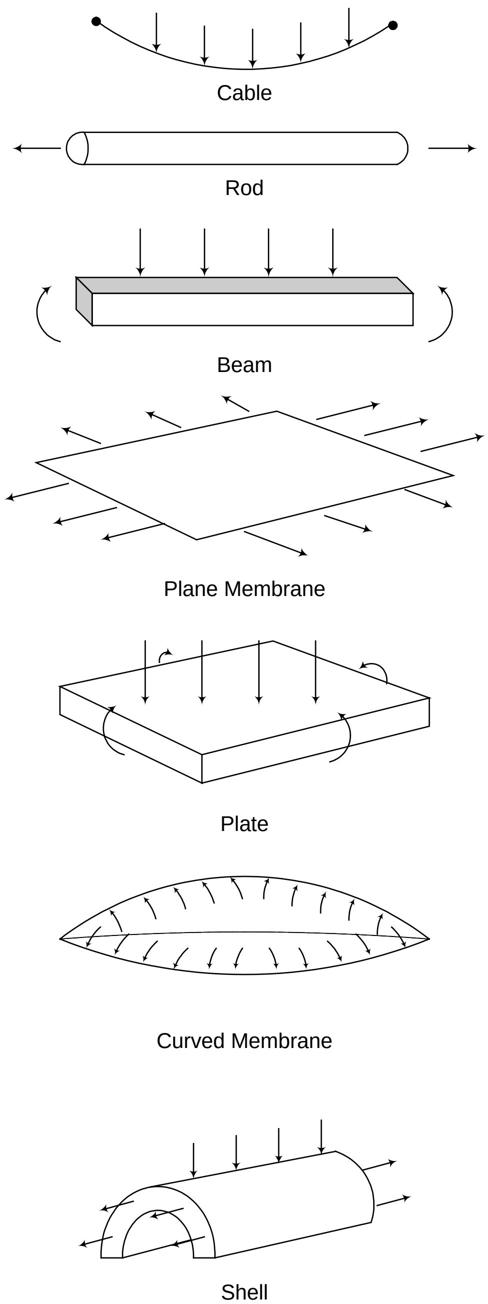


FIGURE 1-8 *Schematic of basic structural elements (not all loads shown).*

analysis purposes, SFEs may be represented as surfaces, either straight or curved as required. SFEs may carry loads in tension, compression, torsion, bending, or some combination of these, depending on the nature of the applied loads, structural geometry, material properties, and boundary conditions. SFEs may form axial, torsional, or bending structures such as *plates* and

TABLE 1-3 Common Boundary Conditions[a]

Name (alternate names)	*Translation constraint*	*Rotation constraint*
Free—2D or 3D	None	None
Fixed (clamped)—2D or 3D	No translation ($u = v = w = 0$)	No rotation ($u' = v' = w' = 0$)
Ball and socket (swivel)—3D	No translation ($u = v = w = 0$)	None
Simple (pinned)—2D	No translation ($u = v = 0$)	None
Simple (roller)—2D	No translation $\perp$ to roller surface (e.g., $v = 0$)	None

[a]The prime or $'$ symbol indicates differentiation with respect to a spatial coordinate, i.e., a gradient or slope. u, v, and w are translations in the x-, y-, and z-directions, respectively.

shells. Being more complicated structures, SFEs are outside the scope of this text and will not be described further.

1.4.4 Boundary Conditions

It should be obvious that if a structure is not tied down somewhere, it will not be able to carry loads in many situations. Imagine trying to hang a weight from a hook not connected to the ceiling! Thus we see that the conditions at the structure boundaries, that is, the *boundary conditions*, are critically important in structural design and analysis.

A number of idealized boundary conditions can be defined. (We say "idealized" since boundary conditions on real structures rarely meet these definitions exactly.) In general, boundary conditions either resist translation or rotation, or both, in one or more directions. In order to do this, they must generate resisting forces (translations) or moments (rotations). In other words, translation and/or rotation are constrained at the boundary by the generation of either a force and/or moment, respectively. (Mathematically, boundary conditions provide additional *equations of constraint*.)

If there are $\left\{\begin{matrix}\text{fewer}\\ \text{more}\end{matrix}\right\}$ constraints than the minimum required, the structure is $\left\{\begin{matrix}\text{under constrained}\\ \text{constrained}\end{matrix}\right\}$, i.e., the structure is *statically indeterminate* in that there are $\left\{\begin{matrix}\text{more}\\ \text{fewer}\end{matrix}\right\}$ independent equations available than unknowns to solve for. A number of common boundary conditions are provided in Table 1-3 and Figure 1-9.

An example of a real boundary condition can be seen in the pin joint in the truss shown in Figure 1-10.

1.4.5 Strength and Stiffness

When we think of a structure carrying loads, two primary questions must be asked:

- Is it strong enough?
- Is it stiff enough?

In other words, we wish to know how well the primary functional requirement of a given structure—carrying loads—is performed; this is the primary role of *structural analysis*.

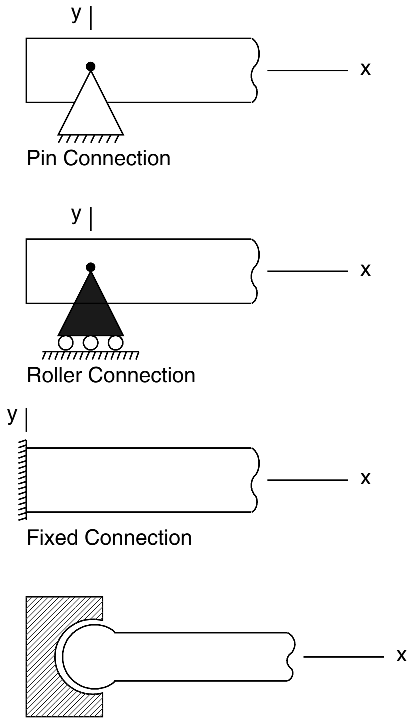


FIGURE 1-9 *Some examples of boundary conditions listed in Table 1-3.*

(As we'll see in Chapter 2, a number of other questions must be asked as well, such as: Is it light enough? Cheap enough? Repairable enough?)

Strength and stiffness are independent quantities (they are, we will find, *design degrees of freedom*, a concept to be discussed in Chapter 2) that depend on the constituents of the material in different ways. We'll come to understand these concepts more in later sections as well. At this point, we just mention that strength considerations are usually of greater importance in structures than stiffness. If you carry a load with a large rubber band, you are usually not too worried if it stretches quite a bit, as long as it doesn't break (i.e., as long as it is strong enough). However, if the band stretches too much, your arms may not be long enough to keep the load from interfering with the ground, and now stiffness (or lack of it!) does become a concern.

An important issue we cannot avoid is: "How much is enough?" Unfortunately, the answer is a somewhat unsatisfying "It depends." It depends on the application, the failure mode, how catastrophic would be the failure (e.g., loss or injury of human life), the past history of similar designs, how well known are the loads and material properties, and so on. In any case, we must always compare what we have (stress, strain, deflection—the load effects) to what we can allow (strength, stiffness—the resistance). This comparison is done generally in one of two ways (a third being a combination of these two): *resistance factor design* and *load factor design* (the third being *load and resistance factor design).*

By way of illustration, one common, elementary approach to determining "enough" is to define a *factor of safety* (FS) (some might say a factor of ignorance!) that tries to account for the effect of many of the issues described above into one quantity. The factor of safety


FIGURE 1-10 *(A) Pin joints in a truss. (B) Pin joint detail.*

(FS $\geq$ 1) is applied to the resistance side of the comparison, and as such creates a *knockdown factor* resulting in an *allowable resistance*. For example, if the resistance considered is strength, then we have:

$$\frac{S_{\text{failure}}}{\text{FS}} = \sigma_{\max} \tag{1.1}$$

where

$S_{failure}$ = designated failure strength (tensile yield, tensile ultimate, compressive yield, etc.)

σ_{max} = maximum stress (at the critical section) associated with the appropriate failure mode (shear, tension, etc.).

The most general form of the comparison is through *load and resistance factor design* (LRFD):

$$\alpha R = \sum_{i=1}^{I} \beta_i Q_i \tag{1.2}$$

where

α = resistance factor (≤ 1)

R = nominal resistance

β_i = ith load factor (usually > 1)

Q_i = ith nominal load effect, associated with the ith nominal applied load, I total loads being applied.

We will not pursue this later approach in this text. Organizations such as the American Institute for Steel Construction (AISC) provide tables of load and resistance factors for various applications (AISC, 2000).

1.4.6 Load Path

Since structures carry loads, a useful tool in structural analysis is the concept of *load path*, that is, the path by which the load is carried. It helps if one can imagine a "flow" of stress or load along the path. In many cases, this is trivial and obvious, particularly when there is only a single, unique path. For example, the load from a sign is carried (literally to ground in this case) by the cantilevered arm and the beam-column (post) as in Figure 1-11.

A sign supported by a truss presents a more complicated load path in Figure 1-12.

To continue the discussion, imagine that in Figure 1-11, the vertical beam-column is attached to ground through a bolted flange connection (Figure 1-13 is a detailed section of the lower portion of Figure 1-11). What is the load path to ground now?

We will find as we proceed that the load sharing among multiple paths apportions itself in large part according to the path stiffness. As a simple example, consider a load carried by two parallel springs, one considerably stiffer than the other. It is not hard to imagine that the stiffer spring carries a greater portion of the load. Developing an intuition for load paths will prove to be a very useful asset for the structural engineer.

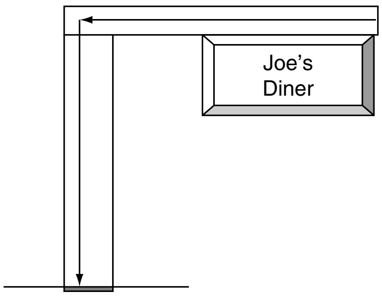


FIGURE 1-11 *Load path for a simple sign, arm, and post.*

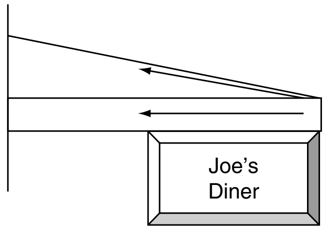


FIGURE 1-12 *Load paths in truss-supported sign.*

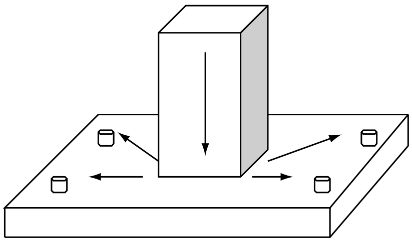


FIGURE 1-13 *Load paths at the base of the beam-column in Figure 1-11.*

EXAMPLE 1-1: Reactions—Exact and Approximate

Consider the following problem of an overconstrained (Why?!) beam of length L subjected to a uniformly distributed load p_0 (Figure E1-1).

1. The exact solution for the reaction loads at A, B, and C (by methods found in Chapter 5) is:

$$R_B = \frac{5}{8}\frac{p_0 L}{2}, \; R_A = R_C = \frac{3}{16}\frac{p_0 L}{2}$$

In fact:

$$\frac{R_B}{R_A} = \frac{5/8}{3/16} = 3.33$$

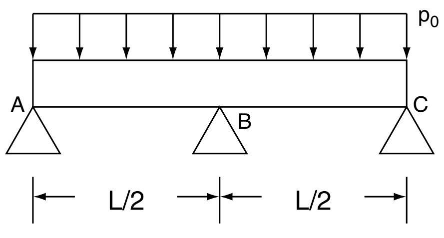


FIGURE E1-1 *Configuration sketch.*

Continued

EXAMPLE 1-1: Reactions—Exact and Approximate–*Cont'd*

2. As a first guess without knowing better, an approximate solution might be simply to take $R_A^* = R_B^* = R_C^* = p_0L/3$. In that case, the error in (say) R_B is

$$\frac{R_B - R_B^*}{R_B} \times 100 = \frac{0.625 - 0.333}{0.625} \times 100 = 47\% \text{ and } \frac{R_B^*}{R_A^*} = 1.00$$

Not very good! Our estimate is only about one-half the true answer and the reaction at B does not equal the reaction at A.

3. Let's use the concept of load path to come up with another approximate solution. We imagine each of the boundary supports as "collection points" of load arriving there by different load paths. For example, a reasonable choice might be as shown in Figure E1-2.

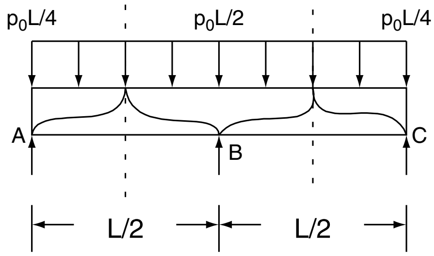


FIGURE E1-2 *Estimate of load paths.*

Now it can be seen by inspection that

$$\tilde{R}_B = \frac{p_0L}{2}, \ \tilde{R}_A = \tilde{R}_C = \frac{p_0L}{4}$$

and

$$\frac{R_B - \tilde{R}_B}{R_B} \times 100 = \frac{0.625 - 0.50}{0.625} \times 100 = 20\% \text{ and } \frac{\tilde{R}_B}{\tilde{R}_A} = 2.00$$

(A much better approximation than in 2!)

1.4.7 Characteristic Tasks in Structural Analysis

Structural analysis supports structural design. Structural analysis can include a variety of tasks aimed at understanding the complex response of structures, specifically the resultant stress, strain, and displacement to applied loads, which answer the questions about strength and stiffness. A sample of such tasks is given below:

- Loads analysis
- Strength analysis
- Stiffness analysis

- Natural frequency analysis
- Dynamic response analysis
- Damping analysis
- Thermal (heat transfer) analysis
- Thermoelastic (thermal deformation) analysis
- Structural life (fracture, fatigue) analysis
- Mass property (weight, center of gravity) analysis
- Precision (shape and position accuracy) analysis
- Sensitivity (of the design to perturbations or small changes) analysis

1.4.8 Methods of Structural Analysis

Structural analysis consists of three fundamental parts:

1. *Equilibrium*. Here we consider forces, moments, and the application of Newtonian mechanics (or the potential and kinetic energies and the application of Lagrangian mechanics), and stress. If the body is in global equilibrium, then every local particle of the body is also in equilibrium. Equilibrium implies negligible accelerations (inertia forces), and hence implies static analysis. (Dynamic analysis would consider the motion of the body, requiring inertia forces in the full equations of motion.)
2. *Deformation*. Here we consider the geometry of material particle displacement and the concept of strain. We assume that any material is continuous and fully populated with material particles (the *continuum hypothesis)*. We further assume in this text that deformations are small enough that only a linear analysis is required.
3. *Constitution*. Stress and strain are dual quantities that are intimately related within a given material/structural system. These are *stress–strain relations*.

The methods of structural analysis are mathematical analysis and experimental analysis. Mathematical analysis may result in closed-form solutions, series solutions, asymptotic solutions, numerical solutions, and so on. Except for relatively simple structural problems, numerical solutions are usually required. The direct stiffness method, and its descendant the finite element method, are the most ubiquitous and powerful numerical methods for structural analysis today.

Experimental analysis involves the testing of real or prototypical structures and uses various techniques to assess strain and/or displacement. Some of these experimental techniques are:

- Strain gages
- Stress coatings
- Optical interferometers
- Extensometers
- Videography
- Thermography

Quite often, combinations of mathematical and experimental methods are used.

In every case of analyzing real structures, certain assumptions about their behavior or character must be made. This leads to an idealized structure model that will be divorced from the real structure to some greater or lesser degree. **The engineer must always be aware of this discrepancy**.

1.5 Overview of Materials

In the past two decades, industries such as computers, electronics, aerospace, machinery, automotive, metals, energy, chemical, telecommunications, and biomaterials have witnessed major improvements in the performance and reliability of their products. The principal reasons are the development of new materials, novel use of existing materials, better understanding of the structure–property relationships, and incorporation of materials science in the total design of such structures. The listed industries have benefited by billions of dollars every year from the development and use of new materials, and have significantly contributed to the growth, prosperity, security, and quality of human life. Table 1-4 illustrates the types of materials used in these representative industries.

It may be noted that a wide variety of materials are used among the different industries, though many of the materials are common to most of the industries. However, the usage and required properties vary widely. Table 1-5 lists the typical properties required of materials used in different industries.

Thus it is apparent that development and novel use of structural, electronic, and biological materials are the backbone of many industries. It is essential for scientists and engineers to not only understand the science of materials, but also know how to apply that knowledge to the design and creation of new and reliable structural, electronic, and biological products. The properties of structural materials such as toughness, strength, stiffness, and weight, which are determined by the interaction of the atoms and their arrangement in molecules, crystalline and noncrystalline arrays, and defects at the atomic or macroscopic level (see, e.g., Shackelford, 1996). Consequently, the ability to predict and control the material's structure, at both the atomic and macroscopic level, is essential to developing materials that can achieve the

TABLE 1-4 Typical Materials Used in Various Industries

Electronics	*Aerospace*	*Automotive*	*Machinery*
Semiconductors	Metals	Metals	Metals
Polymer composites	Polymer composites	Polymer composites	Polymer composites
Metals	Metal alloys	Metal alloys	Metal alloys
Plastics	Plastics	Plastics	Plastics
Ceramics	Ceramics	Ceramics	Ceramics
Single crystals	Metal-matrix composites	Elastomers	Fluids
	Superalloys		
	Elastomers		
	Single crystals		
Energy	*Chemical*	*Biomaterials*	
Metals	Metals	Metals	
Polymer composites	Polymer composites	Water-soluble polymers	
Metal alloys	Metal alloys	Biopolymers	
Plastics	Plastics	Polymers	
Ceramics	Ceramics	Ceramics	
Metal-matrix composites	Metal-matrix composites	Nanomaterials	
Superalloys	Fluids	Biotissue	
Fluids		Artificial skin	

TABLE 1-5 Desired Material Properties for Representative Industries

	Industry						
Desired properties	*Elect.*	*Aero.*	*Auto.*	*Mach.*	*Energy*	*Chem.*	*Biomat.*
Weight		•	•	•			•
Strength		•	•	•	•	•	•
Stiffness	•	•	•	•	•	•	•
Fracture toughness	•	•	•	•	•	•	•
High temperature resistance	•	•	•		•	•	
Corrosion resistance		•	•	•	•	•	•
Durability	•	•	•	•	•	•	•
Shape formability		•	•	•			•
Reactivity		•			•	•	•
Recyclability			•	•	•	•	

level of performance required in, for example, airplanes, automobiles, or bridges. For example, the launch of space vehicles requires low structural weight. This requires new materials with high strength-to-weight ratios and the ability to withstand a harsh environment such as high temperature, ultraviolet radiation, and abrasive dust. In this regard, some of the materials used are composites, thin films and layered structures, ablative materials, and ceramics. Thus, knowing the fundamental mechanisms of failure such as fracture, fatigue, creep, wear, and corrosion, the goal is to achieve the appropriate material with the desired structure.

Macroscopically, materials may be classified in a number of different ways. One useful choice is to divide them into two classes—isotropic and anisotropic. *Isotropic* materials have identical properties in all directions, whereas *anisotropic* materials have properties that depend on the direction along which the property is measured. The most common materials and their characteristics are listed in Figure 1-14.

For example, aluminum (metal) is an isotropic material because its mechanical properties, such as Young's modulus, do not depend on the direction in which a tensile test specimen is cut out from a large plate. The plate consists of crystals oriented at random. Each direction will thus give a value of about 70 GPa (10×10^6 psi). However, if a single crystal of aluminum was tested in the [100] crystal direction (this will be discussed in detail in Chapter 4), the Young's modulus obtained will be about 64 GPa (9.2×10^6 psi). This is due to the differences in the atomic arrangement in the planes and directions within a crystal. Hence, a single crystal behaves as an anisotropic material, whereas a regular aluminum plate consisting of randomly oriented crystals behaves as an isotropic material.

The common crystal structure of materials and the types of bonds formed in these materials is shown in Figure 1-15. The interactions of the electrons with each other and with the nucleus are responsible for the binding of the atoms into molecules and crystals. The mutual attraction between atoms results in four types of bonds.

The ionic, covalent, and metallic bonds are called *primary bonds*, while the van der Waals bonds are called *secondary bonds*. The primary bonds are strong and stiff and are mainly responsible for the physical and mechanical properties of metals and ceramics, such as high melting points and high Young's modulus. The secondary bonds are relatively weak and are primarily found in polymers. (A more detailed discussion of atomic bonding is given in Review Module 1.)

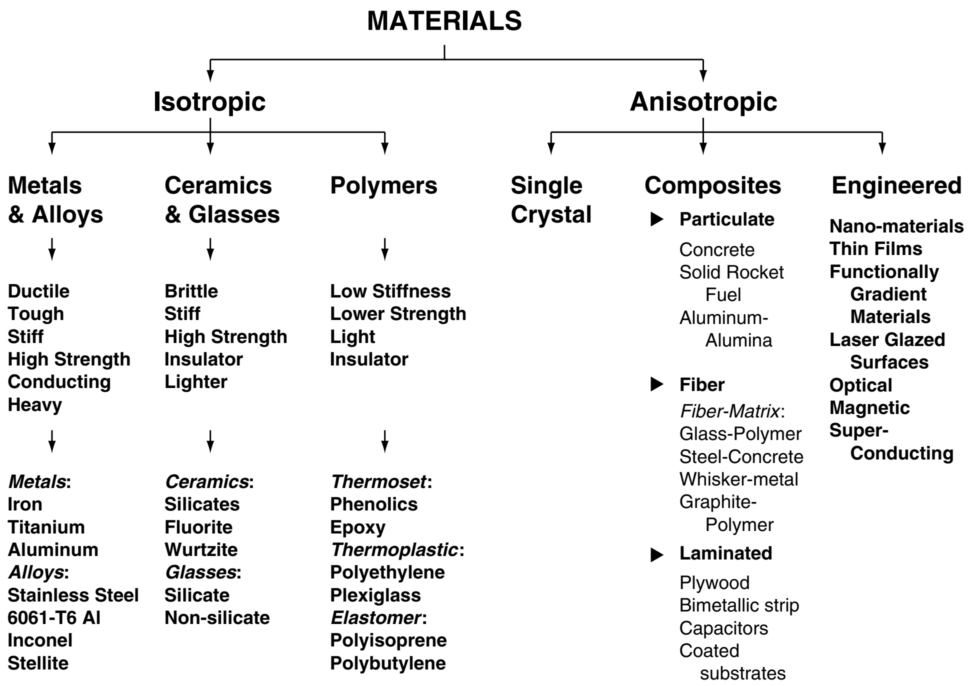


FIGURE 1-14 *Common materials and their characteristics.*

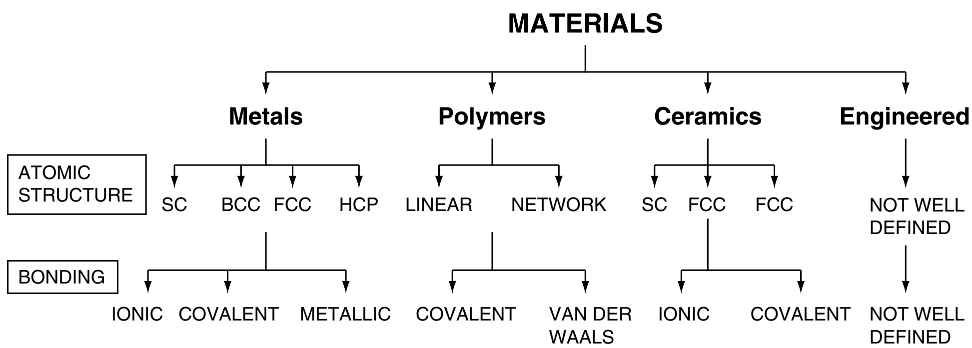
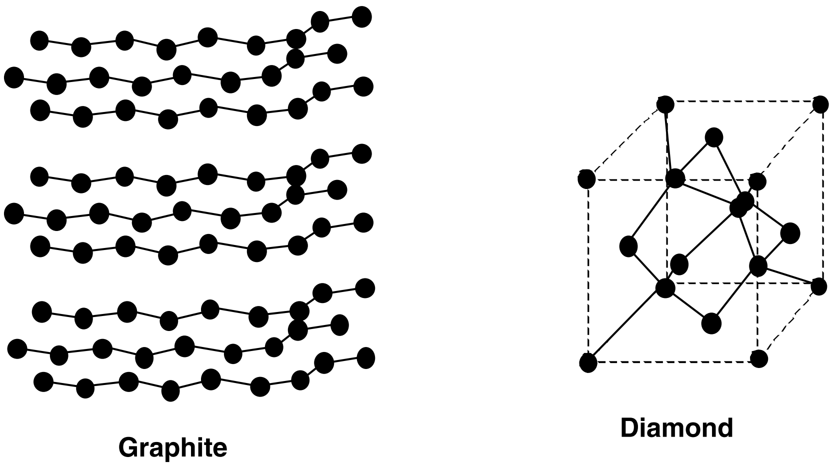


FIGURE 1-15 *Common atomic and molecular structures and bonds.*

The four main crystal structures are *simple cubic* (SC), *body-centered cubic* (BCC), *face-centered cubic* (FCC), and *hexagonal close-packed* (HCP). The deformation response of these crystal structures to applied load is different and accounts for the difference in the properties such as ductility and strength. Materials with the SC and HCP structure are brittle; examples are beryllium, magnesium, zirconium, and cobalt. The FCC materials, for example, aluminum, gold, silver, lead, copper, iron, and platinum, have low strength but high ductility. The BCC materials have high strength but relatively lower ductility. Examples of BCC materials are titanium, tungsten, vanadium, iron, chromium, and zirconium. It may be noted that some materials exist



Physical Properties

Property	***Graphite***	***Diamond***
Hardness	Very Low	Very High
Color	Black	Colorless
Density	High	Low
Thermal Conductivity	Good	Excellent

FIGURE 1-16 *Comparison of graphite and diamond property and structure.*

in more than one crystal structure depending on the conditions. For example, at low temperature iron has the BCC structure, but at a high temperature it transforms to an FCC structure.

It is now apparent that atomic and molecular arrangements play an important role in determining the microstructure and mechanical behavior of materials. The rules of bonding then control the resulting properties of materials. For example, let us consider the structure–property relationship in graphite and diamond, both of which are crystalline modifications of carbon. The structure of graphite is shown in Figure 1-16. It is made up of planes of carbon atoms, which are covalently bonded in planar hexagonal arrays, with weak secondary bonds between the planes. Within these planes the atoms are closer together than in the diamond crystal, whereas the binding force is less between the planes, which are more widely separated than in diamond. The diamond crystal is also illustrated in Figure 1-16. In diamond, each carbon atom is surrounded by four other carbon atoms symmetrically arranged at the corners of a tetrahedron, with the surrounded atom at the center. These carbon atoms are bonded by covalent bonds. The difference in bonding and arrangement of carbon atoms results in different structures, which in turn result in very significant differences in the properties, as illustrated in Figure 1-16.

1.6 Mechanics ⇔ Materials Link: Probabilistic Structural Analysis

Although we do our best to determine exactly the magnitudes of loads acting on a structure, or accurately determine material property values, there will always be some uncertainty associated with our knowledge of these quantities. For example, we will never know for certain that the wind load on a structure will have a fixed magnitude, frequency, and duration. If we measure the tensile yield strength of a dozen samples of a given material, we will see scatter in our data.

Thus we can consider that the structural property data we seek has some distribution of values. For our purposes here, we can consider this distribution to be Gaussian or normally distributed about a mean value. (Other distributions, such as Weibull, are possible, but the actual form of the distribution is not that important for our present discussion.) For example, consider the yield strength S_y data for a steel bar (Figure 1-17).

The combined effects of uncertainty in loads and geometry will manifest themselves in uncertainties in the calculated stress. For example, the distribution of stress values σ might look like Figure 1-18.

A deterministic analysis (using only the mean values) of the factor of safety, without reference to the uncertainty in the strength and stiffness values as shown by their distributions, would show by Eq. (1.1) that:

$$\mathrm{FS} = \frac{< S_y >}{< \sigma >} = \frac{6}{3} = 2$$

That sounds like a good margin! This would be terribly misleading, however, since a more compete, probabilistic analysis would show that there is a region of overlap of values such that $\sigma > S_y$! Figure 1-19 shows Figures 1-17 and 1-18 plotted side-by-side.

What this means is that there is some probability that a stress value will exist that exceeds a possible strength value—with ensuing failure! (More details of this subject will not be provided here, but see, e.g., Shigley and Mischke, 2001.) The important point to keep in mind is that although for demonstration, calculation, or design purposes we may use a single value to represent the magnitude of a structural property, in reality **a range of values exists for that property**.

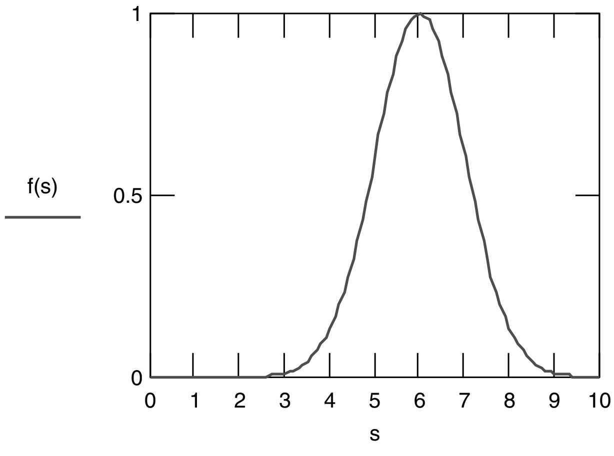


FIGURE 1-17 *In this figure,* $s = S_y$*, and* f(s) *is the distribution of values of the yield strength. The mean value of* S_y *is* $<S_y> = 6$ *(*$\times 10^5$ *psi, perhaps).*

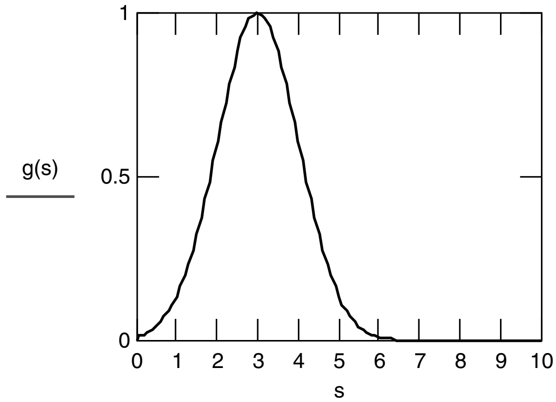


FIGURE 1-18 *Here* s = σ, g(s) *is the distribution of stress values, and the mean value of stress is* $<\sigma>=3$ *(* $\times 10^5$ *psi, say).*

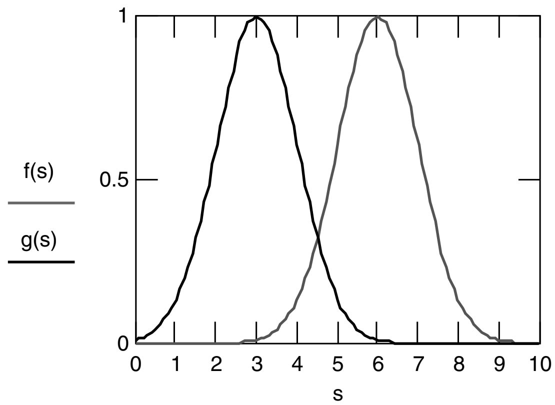


FIGURE 1-19 *The region of overlap between stress and strength is clearly seen in this figure.*

Key Points to Remember

- Structures are physical artifacts that carry loads other than their own weight.
- Every structure is a material as well. Hence the mechanics of a structure is intimately linked with the properties of the structural material.
- Critical elements of structural mechanics include geometry, loads, boundary conditions, and load paths.
- Critical elements of structural materials include atomic bond type and strength, crystal structure, and material properties.
- It should always be kept in mind that mechanics and materials quantities are distributed random variables, even though in many cases we operate with their mean values.

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Problems

1-1. What is a structure? Give examples. Give examples of physical artifacts that are *not* structures.

1-2. What is the difference between a primary and a secondary structure?

1-3. Give two examples for each of the following:

	Aerospace	*Civil*	*Marine*	*Mechanical*
Early structure	1.	1.	1.	1.
	2.	2.	2.	2.
Modern structure	1.	1.	1.	1.
	2.	2.	2.	2.

1-4. Is the skin of an airplane an example of a monocoque structure? Of a car? Of a human?

1-5. Give a simple example of how physics and chemistry provide different points of view of the same phenomenon.

1-6. In the design of a paperclip, how does the requirement for being reusable relate to mechanics issues? Materials issues?

1-7. Give an example of a dynamic load whose magnitude remains constant but whose direction of application changes with time.

1-8. Through what mechanisms do concentrated forces act? Gravity forces? Electromagnetic forces?

1-9. What are the different types of loads acting on the pole due to a steady wind blowing, at points A, B, and C in Figure P1-9?

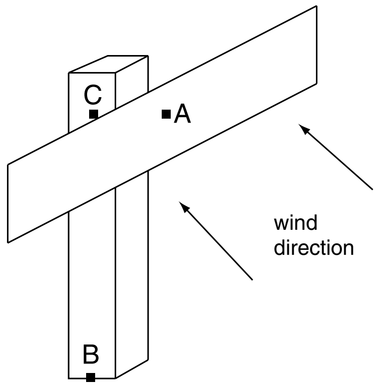

FIGURE P1-9

1-10. What is the type of loading in each of the members A to D in Figure P1-10?

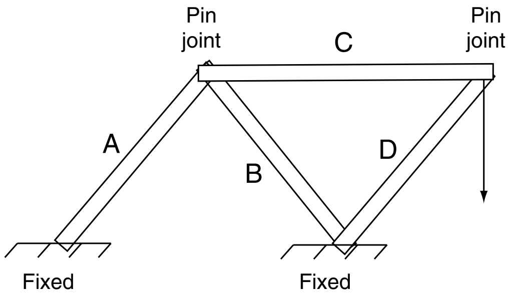

FIGURE P1-10

1-11. What is the loading at points A and B in Figure P1-11?

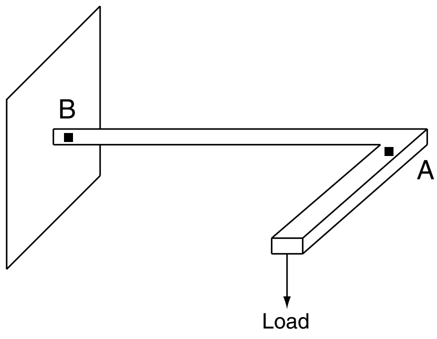

FIGURE P1-11

1-12. Give two examples of structural designs/products that were inspired by new materials, or the novel use of existing materials. (You may wish to conduct a search on the Internet or in the library.)

1-13. Draw the load path in the specimen, shown in Figure P1-13, subjected to a tensile load. Will the specimen bend under the load or remain straight?

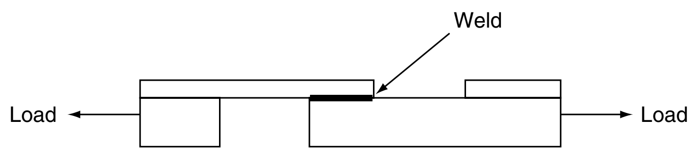

FIGURE P1-13

1-14. Sketch the load paths for the following structures:

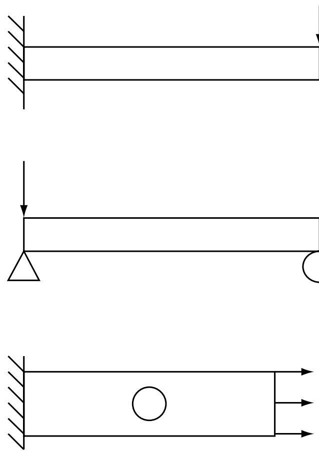

FIGURE P1-14

1-15. Give two examples each from everyday life of real structures undergoing:

- Tension
- Compression
- Shear
- Torsion
- Bending

1-16. Give an example from your local environment for each of the structural elements listed in Table 1-2/Figure 1-8.

1-17. When does a beam become a plate and vice versa?

1-18. Give an example from everyday life for each of the boundary conditions shown in Figure 1-9.

1-19. Sketch beams with the following combinations of boundary conditions:

- Free-clamped
- Pinned-roller
- Pinned-clamped

Draw a correct and complete free body diagram for each end of each beam.

1-20. Which is more conservative, an FS of 2 or 3? Why?

1-21. (a) Define isotropic and anisotropic.
(b) Give two or three examples of anisotropic materials.
(c) In common engineering materials, what is the type of relationship between crystal structure and mechanical properties?

1-22. From Table 1-4, select one material for each application and give a brief description of how it is used in each case.

1-23. Give names of at least five elements that have the BCC, FCC, and HCP structures.

1-24. Which of the elements named in Problem 1-23 are ductile and which are brittle? What are their elastic moduli?

1-25. The energy of interaction of two atoms separated by a distance r can be written as

$$U(r) = -\frac{\alpha}{r} + \frac{\beta}{r^8}$$

where α and β are constants.

(a) State which of the terms is 'attractive' and which is 'repulsive'.

(b) Show that for particles to be in equilibrium $r = r_0 = \left(\frac{8\beta}{\alpha}\right)^{\frac{1}{7}}$.

(c) In stable equilibrium show that the energy of attraction is 8 times that of the repulsion.

(For this problem, the student should refer to Review Module 1.)

1-26. For the data supplied in Table P1-26A:

TABLE P1-26A Data on the Ultimate Tensile Strength of Copper (listed in order of decreasing magnitude)

Sample number	*Strength (MPa)*	*Sample number*	*Strength (MPa)*
1	1175	21	1215
2	1185	22	1215
3	1190	23	1217
4	1193	24	1218
5	1196	25	1219
6	1198	26	1220
7	1201	27	1220
8	1203	28	1222
9	1203	29	1223
10	1206	30	1223
11	1206	31	1226
12	1207	32	1226
13	1210	33	1228
14	1210	34	1228
15	1211	35	1230
16	1211	36	1230
17	1212	37	1236
18	1213	38	1239
19	1214	39	1242
20	1215	40	1252

TABLE P1-26B Frequency Distribution of Ultimate Tensile Strength

Group intervals (MPa)	*Observations in the group*	*Relative frequency*	*Cumulative frequency*
1166-1178	1	0.025	0.025
1179-1192	2	0.050	0.075
1193-1206			
1207-1220			
1221-1234	9	0.225	0.900
1235-1248			
1249-1261	1		
Total	40		

- Calculate the mean strength, and the one, two, and three standard deviation values
- Complete Table P1-26B
- Plot relative and cumulative frequencies versus ultimate tensile strength