SATHYABAMA
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SCHOOL OF MECHANICAL

DEPARTMENT OF MECHATRONICS

UNIT – I Signals and Control Systems – SMRA1402

UNIT I - SYSTEM CONCEPTS

1.1 Basic Concept of Control System

System: A combination or arrangement of a number of different physical components to form a whole unit such that that combining unit performs to achieve a certain goal.

Control: The action to command, direct or regulate a system.

Plant or process: The part or component of a system that is required to be controlled.

Input: It is the signal or excitation supplied to a control system.

Output: It is the actual response obtained from the control system.

Controller: The part or component of a system that controls the plant.

Disturbances: The signal that has adverse effect on the performance of a control system.

Control system: A system that can command, direct or regulate itself or another system to achieve a certain goal.

Automation: The control of a process by automatic means

Control System: An interconnection of components forming a system configuration that will provide a desired response.

Actuator: It is the device that causes the process to provide the output. It is the device that provides the motive power to the process.

Design: The process of conceiving or inventing the forms, parts, and details of system to achieve a specified purpose.

Simulation: A model of a system that is used to investigate the behavior of a system by utilizing actual input signals**.**

Optimization: The adjustment of the parameters to achieve the most favorable or advantageous design.

Feedback Signal: A measure of the output of the system used for feedback to control the system.

Negative feedback: The output signal is feedback so that it subtracts from the input signal.

Block diagrams: Unidirectional, operational blocks that represent the transfer functions of the elements of the system.

Signal Flow Graph (SFG): A diagram that consists of nodes connected by several directed branches and that is a graphical representation of a set of linear relations.

Specifications: Statements that explicitly state what the device or product is to be and to do. It is also defined as a set of prescribed performance criteria.

Open-loop control system: A system that utilizes a device to control the process without using feedback. Thus the output has no effect upon the signal to the process.

Closed-loop feedback control system: A system that uses a measurement of the output and compares it with the desired output.

Regulator: The control system where the desired values of the controlled outputs are more or less fixed and the main problem is to reject disturbance effects.

Servo system: The control system where the outputs are mechanical quantities like acceleration, velocity or position.

Stability: It is a notion that describes whether the system will be able to follow the input command. In a non-rigorous sense, a system is said to be unstable if its output is out of control or increases without bound.

Multivariable Control System: A system with more than one input variable or more than one output variable.

1.2 Classification of Control Systems

Natural control system and Man-made control system:

Natural control system: It is a control system that is created by nature, i.e. solar system, digestive system of any animal, etc.

Man-made control system: It is a control system that is created by humans, i.e. automobile, power plants etc.

Automatic control system and Combinational control system:

Automatic control system: It is a control system that is made by using basic theories from mathematics and engineering. This system mainly has sensors, actuators and responders.

Combinational control system: It is a control system that is a combination of natural and man-made control systems, i.e. driving a car etc.

Time-variant control system and Time-invariant control system:

Time-variant control system: It is a control system where any one or moreparameters of the control system vary with time i.e. driving a vehicle.

Time-invariant control system: It is a control system where none of its parametersvary with time i.e. control system made up of inductors, capacitors and resistors only.

Linear control system and Non-linear control system:

Linear control system: It is a control system that satisfies properties of homogeneityand additive.

Non-linear control system: It is a control system that does not satisfy properties of homogeneity.

Continuous-Time control system and Discrete-Time control system:

Continuous-Time control system: It is a control system where performances of all of its parameters are function of time, i.e. armature type speed control of motor.

Discrete -Time control system: It is a control system where performances of all of its parameters are function of discrete time i.e. microprocessor type speed control of motor.

Deterministic control system and Stochastic control system:

Deterministic control system: It is a control system where its output is predictableor repetitive for certain input signal or disturbance signal.

Stochastic control system: It is a control system where its output is unpredictable ornon-repetitive for certain input signal or disturbance signal.

Single-input-single-output (SISO) control system and Multi-input-multi-output (MIMO) control system:

SISO control system: It is a control system that has only one input and one output.

MIMO control system: It is a control system that has only more than one input andmore than one output.

Open-loop control system and Closed-loop control system:

Open-loop control system: It is a control system where its control action onlydepends on input signal and does not depend on its output response.

Closed-loop control system: It is a control system where its control action depends on both of its input signal and output response.

1.3 Open-loop control system and Closed-loop control system

Open-loop control system:

It is a control system where its control action only depends on input signal and does not depend on its output response as shown in Fig.1.1.

Fig.1.1 An open-loop system

Examples: traffic signal, washing machine, bread toaster, etc.

Advantages:

- Simple design and easy to construct
- Economical
- Easy for maintenance
- Highly stable operation

Dis-advantages:

- Not accurate and reliable when input or system parameters are variable in nature
- Recalibration of the parameters are required time to time

Closed-loop control system:

It is a control system where its control action depends on both of its input signal and output response as shown in Fig.1.2.

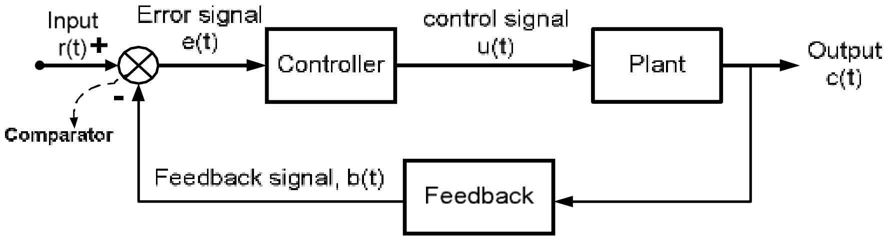


Fig.1.2. A closed-loop system

Examples: automatic electric iron, missile launcher, speed control of DC motor, etc.

Advantages:

- More accurate operation than that of open-loop control system
- Can operate efficiently when input or system parameters are variable innature
- Less nonlinearity effect of these systems on output response
- High bandwidth of operation
- There is facility of automation
- Time to time recalibration of the parameters are not required

Dis-advantages:

- Complex design and difficult to construct
- Expensive than that of open-loop control system
- Complicate for maintenance
- Less stable operation than that of open-loop control system

Comparison between Open-loop and Closed-loop control systems:

It is a control system where its control action depends on both of its input signal and output response.

Table 1. Comparison between Open-loop and Closed-loop control systems

Sl. No.	Open-loop control systems	Closed-loop control systems
1	No feedback is given to the control system	A feedback is given to the control system
2	Cannot be intelligent	Intelligent controlling action
3	There is no possibility of undesirable system oscillation(hunting)	Closed loop control introduces the possibility of undesirable system oscillation(hunting)
4	The output will not very for a constant input, provided the system parameters remain unaltered	In the system the output may vary for a constant input, depending upon the feedback
5	System output variation due to variation in parameters of the system is greater and the output very in an uncontrolled way	System output variation due to variation inparameters of the system is less.
6	Error detection is not present	Error detection is present
7	Small bandwidth	Large bandwidth
8	More stable	Less stable or prone to instability
9	Affected by non-linearities	Not affected by non-linearities
10	Very sensitive in nature	Less sensitive to disturbances
11	Simple design	Complex design
12	Cheap	Costly

1.3 Transfer Function

Definition: It is the ratio of Laplace transform of output signal to Laplace transform of inputsignal assuming all the initial conditions to be zero, i.e.
T(s) is the transfer function of the system. It can be mathematically represented as follows.

T(S)=C(S)/R(S) with zero initial conditions

Example 3.1: Determine the transfer function of the system shown below

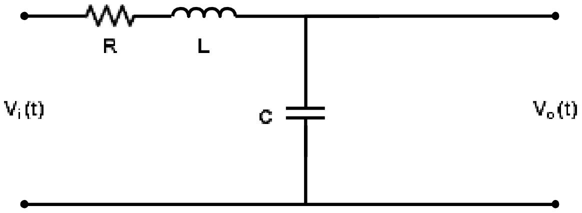


Fig.1.3 a system in time domain

Solution:

Fig.1.3 is redrawn in frequency domain as shown in Fig1.4

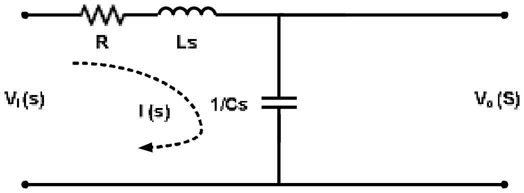


Fig.1.4. a system in frequency domain

Applying KVL to loop-1

$$V_i(s) = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

Applying KVL to loop-'

$$V_o(s) = \left(\frac{1}{Cs}\right) I(s)$$

From eq 2

$$I(s) = V_o(s) / \left(\frac{1}{Cs}\right) = CsV_o(s)$$

Now, using eq

$$V_i(s) = \left(R + Ls + \frac{1}{Cs}\right) CsV_o(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{\left(R + Ls + \frac{1}{Cs}\right)Cs} = \frac{1}{LCs^2 + RCs + 1}$$

Then transfer function of the given system is

$$G(s) = \frac{1}{LCs^2 + RCs + 1}$$

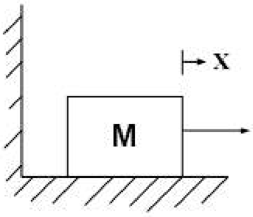
Description of physical system

Components of a mechanical system: Mechanical systems are of two types, i.e. (i) translational mechanical system and (ii) rotational mechanical system.

Translational mechanical system

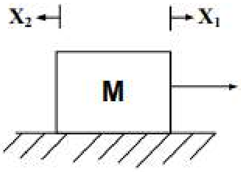
There are three basic elements in a translational mechanical system, i.e. (a) mass, (b) spring and (c) damper.

(a) **Mass:** A mass is denoted by M. If a force f is applied on it and it displays distance x, then $f = M\dfrac{d^2x}{dt^2}$ as shown in Fig.4.1.



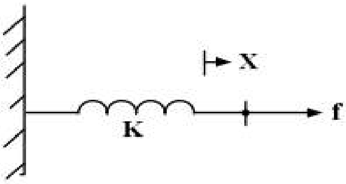
Force applied on a mass with displacement in one direction

If a force f is applied on a massM and it displays distance $x_1$in the direction of f and distance x_2 in the opposite direction, then $f = M\left(\dfrac{d^2x_1}{dt^2} - \dfrac{d^2x_2}{dt^2}\right)$ as shown in Fig.4.2.



Force applied on a mass with displacement two directions

(b) **Spring:** A spring is denoted by K. If a force f is applied on it and it displays distance x, then $f = Kx$ as shown in Fig.4.3.

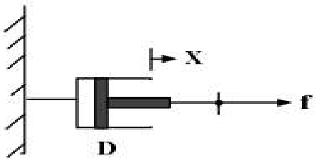


Force applied on a spring with displacement in one direction

If a force f is applied on a springK and it displays distance $x_1$in the direction of f and distance x_2 in the opposite direction, then $f = K(x_1 - x_2)$

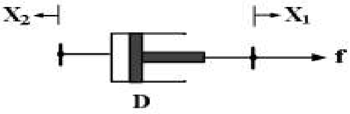
Force applied on a spring with displacement in two directions

(c) **Damper:** A damper is denoted by D. If a force *f* is applied on it and it displays distance *x*, then $f = D\frac{dx}{dt}$



Force applied on a damper with displacement in one direction

If a force *f* is applied on a damperD and it displays distance $x_1$in the direction of *f* and distance x_2 in the opposite direction, then $f = D\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$



Rotational mechanical system

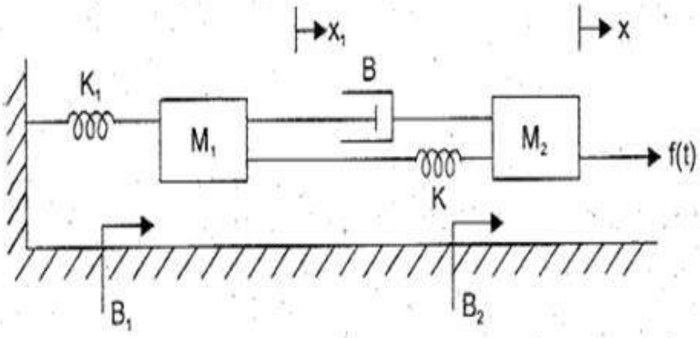
There are three basic elements in a Rotational mechanical system, i.e. (a) inertia, (b) spring and (c) damper.

(a) **Inertia:** A body with aninertia is denoted by *J*. If a torque*T* is applied on it and it displays distanceΘ, then $T = J\frac{d^2\theta}{dt^2}$. If a torque*T* is applied on a body with inertia *J* and it displays distance Θ_1 in the direction of *T* and distance Θ_2 in the opposite direction, then $T = J\left(\frac{d^2\theta_1}{dt^2} - \frac{d^2\theta_2}{dt^2}\right)$.

(b) **Spring:** A spring is denoted by K. If a torque*T* is applied on it and it displays distanceΘ, then $T = K\theta$. If a torque*T* is applied on a body with inertia *J* and it displays distance Θ_1 in the direction of *T* and distance Θ_2 in the opposite direction, then $T = K(\theta_1 - \theta_2)$.

(c) **Damper:** A damper is denoted by D. If a torque*T* is applied on it and it displays distanceΘ, then $T = D\frac{d\theta}{dt}$. If a torque*T* is applied on a body with inertia *J* and it displays distance Θ_1 in the direction of *T* and distance Θ_2 in the opposite direction, then $T = D\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$.

Write the differential equations governing the mechanical system shown in fig 1. and determine the transfer function.



SOLUTION

In the given system, applied force 'f(t)' is the input and displacement 'x' is the output.

Let, Laplace transform of f(t) = $\mathcal{L}\{f(t)\}$ = F(s)

Laplace transform of x = $\mathcal{L}\{x\}$ = X(s)

Let the displacement of mass M_1 be x_1. The free body diagram of mass M_1 is shown in fig The opposing forces acting on mass M_1 are marked as f_{m1}, f_{b1}, f_b, f_{k1} and f_k.

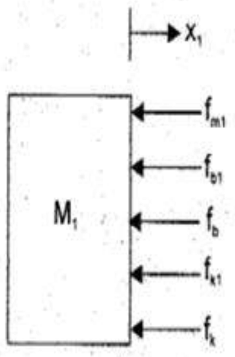


Fig : Free body diagram of mass M_1 (node 1).

$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}; \quad f_{b1} = B_1 \frac{dx_1}{dt}; \quad f_{k1} = K_1 x_1;$$

$$f_b = B\frac{d}{dt}(x_1 - x); \quad f_k = K(x_1 - x)$$

By Newton's second law,

$$f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$$

$$\therefore M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$$

On taking Laplace transform of above equation with zero initial conditions we get,

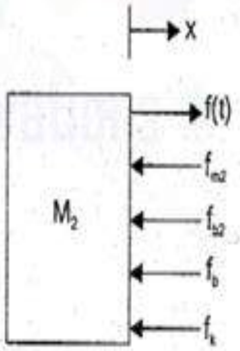
$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs\,[X_1(s) - X(s)] + K_1 X_1(s) + K\,[X_1(s) - X(s)] = 0$$

$$X_1(s)\,[M_1 s^2 + (B_1 + B)s + (K_1 + K)] - X(s)\,[Bs + K] = 0$$

$$X_1(s)\,[M_1 s^2 + (B_1 + B)s + (K_1 + K)] = X(s)\,[Bs + K]$$

$$\therefore X_1(s) = X(s)\,\frac{Bs + K}{M_1 s^2 + (B_1 + B)\,s + (K_1 + K)} \qquad \text{......(1)}$$

The free body diagram of mass M_2 is shown in fig The opposing forces acting on M_2 are marked as f_{m2}, f_{b2} and f_k.



Free body diagram of mass M_2 (node 2).

$$f_{m2}=M_2\frac{d^2x}{dt^2} \quad ; \quad f_{b2}=B_2\frac{dx}{dt}$$

$$f_b=B\frac{d}{dt}(x-x_1) \quad ; \quad f_k=K(x-x_1)$$

By Newton's second law,

$$f_{m2}+f_{b2}+f_b+f_k=f(t)$$

$$M_2\frac{d^2x}{dt^2}+B_2\frac{dx}{dt}+B\frac{d}{dt}(x-x_1)+K(x-x_1)=f(t)$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_2s^2X(s)+B_2sX(s)+Bs[X(s)-X_1(s)]+K[X(s)-X_1(s)]=F(s)$$

$$X(s)[M_2s^2+(B_2+B)s+K]-X_1(s)[Bs+K]=F(s) \qquad .....(2$$

Substituting for $X_1(s)$ from equation (1) in equation (2) we get,

$$X(s)[M_2s^2+(B_2+B)s+K]-X(s)\frac{(Bs+K)^2}{M_1s^2+(B_1+B)s+(K_1+K)}=F(s)$$

$$X(s)\left[\frac{[M_2s^2+(B_2+B)s+K][M_1s^2+(B_1+B)s+(K_1+K)]-(Bs+K)^2}{M_1s^2+(B_1+B)s+(K_1+K)}\right]=F(s)$$

$$\therefore \frac{X(s)}{F(s)}=\frac{M_1s^2+(B_1+B)s+(K_1+K)}{[M_1s^2+(B_1+B)s+(K_1+K)][M_2s^2+(B_2+B)s+K]-(Bs+K)^2}$$

RESULT

The differential equations governing the system are,

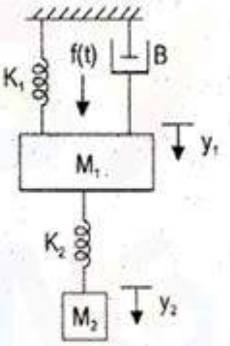
1. $M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+B\frac{d}{dt}(x_1-x)+K_1x_1+K(x_1-x)=0$

2. $M_2\frac{d^2x}{dt^2}+B_2\frac{dx}{dt}+B\frac{d}{dt}(x-x_1)+K(x-x_1)=f(t)$

The transfer function of the system is,

$$\frac{X(s)}{F(s)}=\frac{M_1s^2+(B_1+B)s+(K_1+K)}{\left[M_1s^2+(B_1+B)s+(K_1+K)\right]\left[M_2s^2+(B_2+B)s+K\right]-(Bs+K)^2}$$

Determine the transfer function $\frac{Y_2(s)}{F(s)}$ of the system shown in fig



SOLUTION

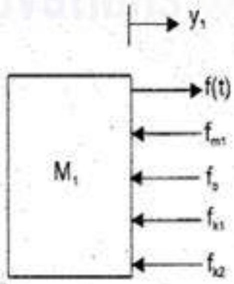
Let, Laplace transform of f(t) = $\mathcal{L}\{f(t)\} = F(s)$

Laplace transform of $y_1 = \mathcal{L}\{y_1\} = Y_1(s)$

Laplace transform of $y_2 = \mathcal{L}\{y_2\} = Y_2(s)$

The system has two nodes and they are mass M_1 and M_2. The differential equations governing the system are the force balance equations at these nodes.

The free body diagram of mass M_1 is shown in fig 2.



The opposing forces are marked as f_{m1}, f_b, f_{k1} and f_{k2}

$$f_{m1} = M_1\frac{d^2y_1}{dt^2} \;;\; f_b = B\frac{dy_1}{dt} \;;\; f_{k1} = K_1y_1 \;;\; f_{k2} = K_2(y_1 - y_2)$$

By Newton's second law, $f_{m1} + f_b + f_{k1} + f_{k2} = f(t)$

$$\therefore M_1\frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + K_1y_1 + K_2(y_1 - y_2) = f(t) \qquad .....(1)$$

On taking Laplace transform of equation (1) with zero initial condition we get,

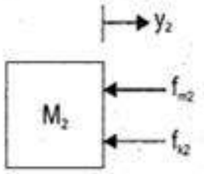
$$M_1s^2Y_1(s) + BsY_1(s) + K_1Y_1(s) + K_2[Y_1(s) - Y_2(s)] = F(s)$$

$$Y_1(s)[M_1s^2 + Bs + (K_1 + K_2)] - Y_2(s)K_2 = F(s) \qquad .....(2)$$

$$f_{m2} = M_2\frac{d^2y_2}{dt^2} \;; \qquad f_{k2} = K_2(y_2 - y_1)$$

By Newton's second law, $f_{m2} + f_{k2} = 0$

$$\therefore M_2\frac{d^2y_2}{dt^2} + K_2(y_2 - y_1) = 0$$



On taking Laplace transform of above equation we get,

$$M_2s^2Y_2(s) + K_2[Y_2(s) - Y_1(s)] = 0$$

$$Y_2(s)\,[M_2s^2 + K_2] - Y_1(s)\,K_2 = 0$$

$$\therefore Y_1(s) = Y_2(s)\,\frac{M_2s^2 + K_2}{K_2} \qquad .....(3)$$

Substituting for $Y_1(s)$ from equation (3) in equation (2) we get,

$$Y_2(s)\left[\frac{M_2s^2 + K_2}{K_2}\right]\left[M_1s^2 + Bs + (K_1 + K_2)\right] - Y_2(s)\,K_2 = F(s)$$

$$Y_2(s)\left[\frac{(M_2s^2 + K_2)\,[M_1s^2 + Bs + (K_1 + K_2)] - K_2^2}{K_2}\right] = F(s)$$

$$\therefore \frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[M_1s^2 + Bs + (K_1 + K_2)\right]\left[M_2s^2 + K_2\right] - K_2^2}$$

RESULT

The differential equations governing the system are,

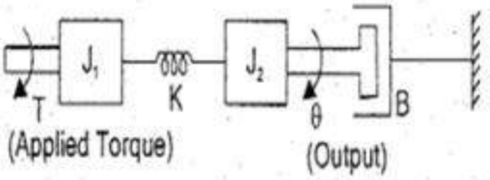
1. $M_1 \frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + K_1y_1 + K_2(y_1 - y_2) = f(t)$

2. $M_2 \frac{d^2y_2}{dt^2} + K_2(y_2 - y_1) = 0$

The transfer function of the system is,

$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[M_1s^2 + Bs + (K_1 + K_2)\right]\left[M_2s^2 + K_2\right] - K_2^2}$$

Write the differential equations governing the mechanical rotational system shown in fig 1. Obtain the transfer function of the system.



SOLUTION

In the given system, applied torque T is the input and angular displacement θ is the output.

Let, Laplace transform of T $= \mathcal{L}\{T\} = T(s)$

Laplace transform of θ $= \mathcal{L}\{\theta\} = \theta(s)$

Laplace transform of $\theta_1 = \mathcal{L}\{\theta_1\} = \theta_1(s)$

Hence the required transfer function is $\frac{\theta(s)}{T(s)}$

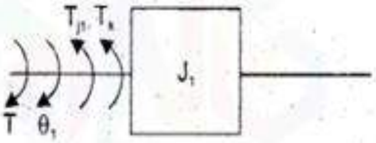
The system has two nodes and they are masses with moment of inertia J_1 and J_2. The differential equations governing the system are given by torque balance equations at these nodes.

$$J_2 s^2\theta(s) + B s \theta(s) + K\theta(s) - K\theta_1(s) = 0$$

Let the angular displacement of mass with moment of inertia J_1 be θ_1. The free body diagram of J_1 is shown in fig. The opposing torques acting on J_1 are marked as T_{j1} and T_k.

$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2} \quad ; \quad T_k = K(\theta_1 - \theta)$$

By Newton's second law, $T_{j1} + T_k = T$



Free body diagram of mass with moment of inertia J_1.

$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T \qquad .....(1)$$

On taking Laplace transform of equation (1) with zero initial conditions we get,

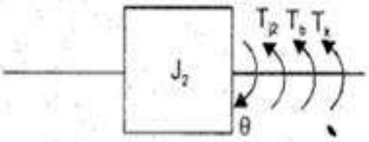
$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1 s^2 + K)\theta_1(s) - K\theta(s) = T(s) \qquad .....(2)$$

The free body diagram of mass with moment of inertia J_2 is shown in fig The opposing torques acting on J_2 are marked as T_{j2}, T_b and T_k.

$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} \quad ; \quad T_b = B\frac{d\theta}{dt} \quad ; \quad T_k = K(\theta - \theta_1)$$

By Newton's second law, $T_{j2} + T_b + T_k = 0$



Free body diagram of mass with moment of inertia J_2.

$$\therefore J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$(J_2 s^2 + Bs + K)\,\theta(s) - K\theta_1(s) = 0$$

$$\theta_1(s) = \frac{(J_2 s^2 + Bs + K)}{K}\,\theta(s) \qquad .....(3)$$

Substituting for $\theta_1(s)$ from equation (3) in equation (2) we get,

$$(J_1 s^2 + K)\,\frac{(J_2 s^2 + Bs + K)}{K}\,\theta(s) - K\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + K)\,(J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) = T(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)\,(J_2 s^2 + Bs + K) - K^2}$$

RESULT

The differential equations governing the system are,

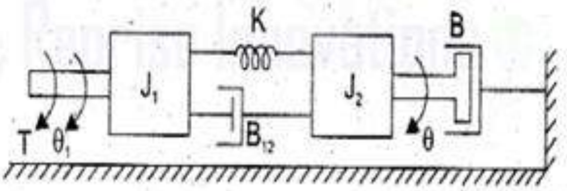
1. $J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T$
2. $J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$

The transfer function of the system is,

$$\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)\,(J_2 s^2 + Bs + K) - K^2}$$

Write the differential equations governing the mechanical rotational system shown in fig 1. and determine the transfer function $\theta(s)/T(s)$.

SOLUTION



In the given system, the torque T is the input and the angular displacement θ is the output.

Let, Laplace transform of $T = \mathcal{L}\{T\} = T(s)$

Laplace transform of $\theta = \mathcal{L}\{\theta\} = \theta(s)$

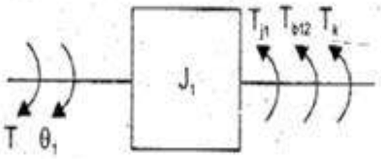
Laplace transform of $\theta_1 = \mathcal{L}\{\theta_1\} = \theta_1(s)$

Hence the required transfer function is $\frac{\theta(s)}{T(s)}$

The system has two nodes and they are masses with moment of inertia J_1 and J_2. The differential equations governing the system are given by torque balance equations at these nodes.

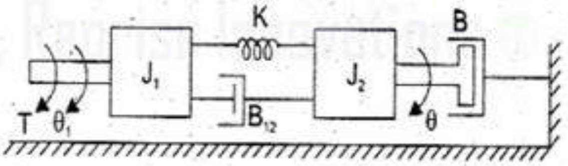
Let the angular displacement of mass with moment of inertia J_1 be θ_1. The free body diagram of J_1 is shown in fig 2. The opposing torques acting on J_1 are marked as T_{j1}, T_{b12} and T_k.

$$T_{j1} = J_1\frac{d^2\theta_1}{dt^2} \quad ; \quad T_{b12} = B_{12}\frac{d}{dt}(\theta_1-\theta) \quad ; \quad T_k = K(\theta_1-\theta)$$



Write the differential equations governing the mechanical rotational system shown in fig and determine the transfer function $\theta(s)/T(s)$.

SOLUTION



In the given system, the torque T is the input and the angular displacement θ is the output.

Let, Laplace transform of $T = \mathcal{L}\{T\} = T(s)$

Laplace transform of $\theta = \mathcal{L}\{\theta\} = \theta(s)$

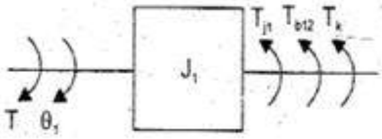
Laplace transform of $\theta_1 = \mathcal{L}\{\theta_1\} = \theta_1(s)$

Hence the required transfer function is $\frac{\theta(s)}{T(s)}$

The system has two nodes and they are masses with moment of inertia J_1 and J_2. The differential equations governing the system are given by torque balance equations at these nodes.

Let the angular displacement of mass with moment of inertia J_1 be θ_1. The free body diagram of J_1 is shown in fig 2. The opposing torques acting on J_1 are marked as T_{j1}, T_{b12} and T_k.

$$T_{j1} = J_1\frac{d^2\theta_1}{dt^2} \quad ; \quad T_{b12} = B_{12}\frac{d}{dt}(\theta_1-\theta) \quad ; \quad T_k = K(\theta_1-\theta)$$



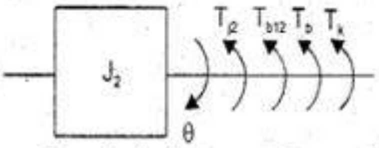
$$J_1 \frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta) = T$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$J_1 s^2\theta_1(s) + s\,B_{12}\,[\theta_1(s) - \theta(s)] + K\theta_1(s) - K\theta(s) = T(s)$$

$$\theta_1(s)\,[J_1 s^2 + sB_{12} + K] - \theta(s)\,[sB_{12} + K] = T(s) \qquad .....(1)$$

The free body diagram of mass with moment of inertia J_2 is shown in fig. The opposing torques are marked as T_{j2}, T_{b12} T_b and T_k.



Free body diagram of mass with moment of inertia J_2

$$T_{j2} = J_2\frac{d^2\theta}{dt^2} \quad ; \quad T_{b12} = B_{12}\frac{d}{dt}(\theta - \theta_1)$$

$$T_b = B\frac{d\theta}{dt} \quad ; \quad T_k = K(\theta - \theta_1)$$

By Newton's second law, $T_{j2} + T_{b12} + T_b + T_k = 0$

$$J_2\frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}(\theta - \theta_1) + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2\frac{d^2\theta}{dt^2} - B_{12}\frac{d\theta_1}{dt} + \frac{d\theta}{dt}(B_{12} + B) + K\theta - K\theta_1 = 0$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$J_2 s^2\theta(s) - B_{12}s\theta_1(s) + s\theta(s)\,[B_{12} + B] + K\theta(s) - K\theta_1(s) = 0$$

$$\theta(s)\,[s^2J_2 + s(B_{12} + B) + K] - \theta_1(s)\,[sB_{12} + K] = 0$$

$$\theta_1(s) = \frac{[s^2J_2 + s(B_{12} + B) + K]}{[sB_{12} + K]}\,\theta(s) \qquad .....(2)$$

Substituting for $\theta_1(s)$ from equation (2) in equation (1) we get,

$$[J_1 s^2 + sB_{12} + K]\,\frac{[J_2 s^2 + s(B_{12} + B) + K]\,\theta(s)}{(sB_{12} + K)} - (sB_{12} + K)\,\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + sB_{12} + K)\,[J_2 s^2 + s(B_{12} + B) + K] - (sB_{12} + K)^2}{(sB_{12} + K)}\right]\theta(s) = T(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{(sB_{12} + K)}{(J_1 s^2 + sB_{12} + K)\,[J_2 s^2 + s(B_{12} + B) + K] - (sB_{12} + K)^2}$$

RESULT

The differential equations governing the system are,

1. $J_1\frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta) = T$
2. $J_2\frac{d^2\theta}{dt^2} - B_{12}\frac{d\theta_1}{dt} + \frac{d\theta}{dt}(B_{12} + B) + K(\theta - \theta_1) = 0$

The transfer function of the system is,

$$\frac{\theta(s)}{T(s)} = \frac{(sB_{12} + K)}{(J_1 s^2 + sB_{12} + K)\,[J_2 s^2 + s(B_{12} + B) + K] - (sB_{12} + K)^2}$$

1.4 Signal Flow Graph

It is a pictorial representation of a system that graphically displays the signal transmission in it.

Mason's gain formula states the overall gain of the system

$$\text{Overall gain, } T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

where, T = $T(s)$ = Transfer function of the system

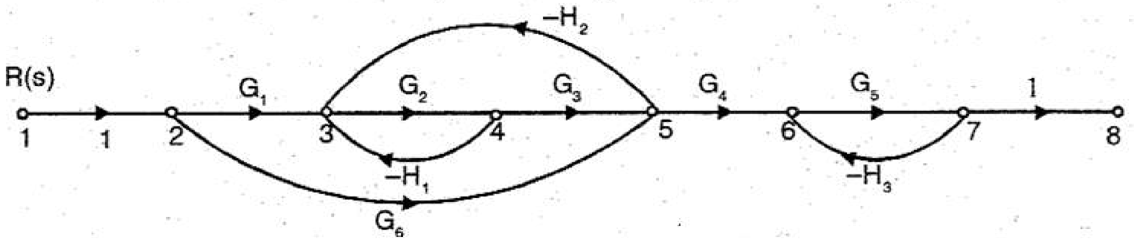
P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

$$\Delta = 1 - (\text{Sum of individual loop gains}) + \begin{pmatrix}\text{Sum of gain products of all possible} \\ \text{combinations of two non - touching loops}\end{pmatrix} - \begin{pmatrix}\text{Sum of gain products of all possible} \\ \text{combinations of three non - touching loops}\end{pmatrix} + \ldots\ldots\ldots\ldots$$

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Find the overall transfer function of the system whose signal flow graph is shown in fig 1.

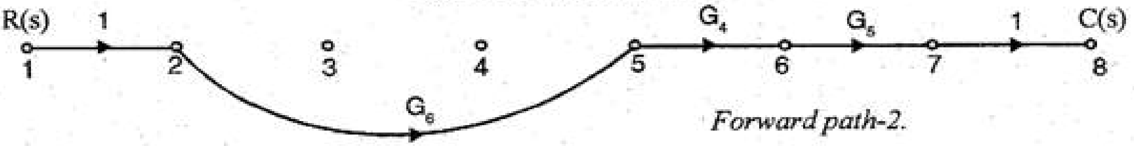


Forward Path Gains

There are two forward paths. ∴ K = 2

Let forward path gains be P_1 and P_2.

Fig 2 : Forward path-1.



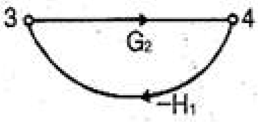
Forward path-2.

Gain of forward path-1, $P_1 = G_1\ G_2 G_3 G_4 G_5$

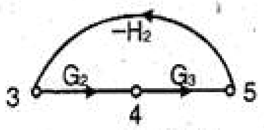
Gain of forward path-2, $P_2 = G_4 G_5 G_6$

Individual Loop Gain

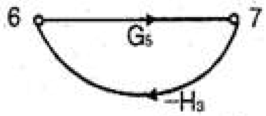
There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.



Loop-1.



Loop-2.



Loop-3.

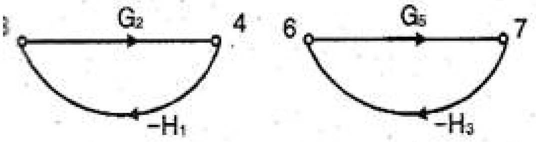
Loop gain of individual loop-1, $P_{11} = -G_2H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

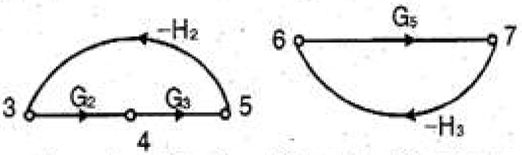
Loop gain of individual loop-3, $P_{31} = -G_5H_3$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be P_{12} and P_{22}.



First combination of 2 non-touching loops.



Second combination of 2 non-touching loops.

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

Calculation of Δ and Δ_K

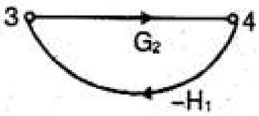
$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in fig 9.



$$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

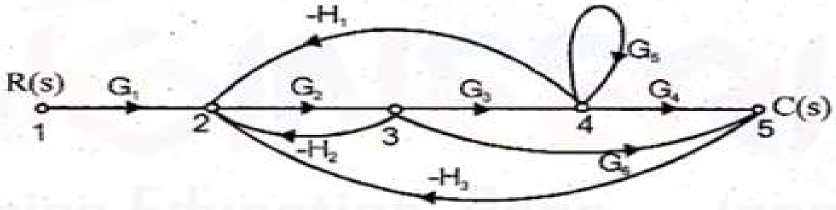
$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is 2 and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6 / G_2 + G_6H_1\right]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

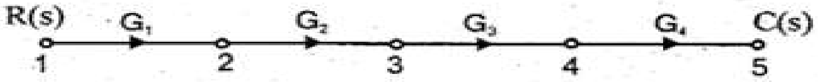
Find the overall gain C(s)/R(s) for the signal flow graph shown in fig .



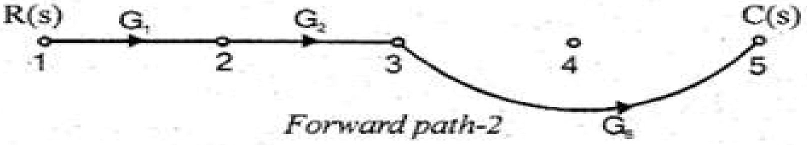
SOLUTION

I. Forward Path Gains

There are two forward paths. ∴ K = 2. Let the forward path gains be P_1 and P_2.



Forward path-1



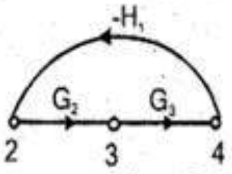
Forward path-2

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

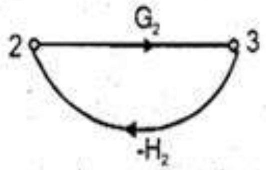
Gain of forward path-2, $P_2 = G_1G_2G_6$

ndividual Loop Gain

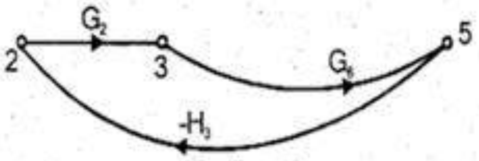
There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.



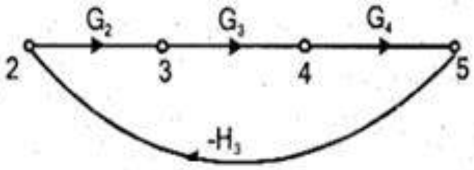
loop-1



loop-2



loop-3



loop-4

loop-5

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

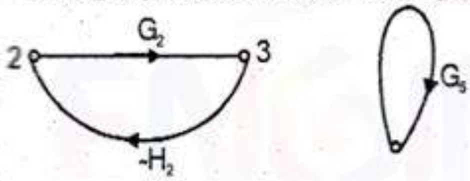
Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

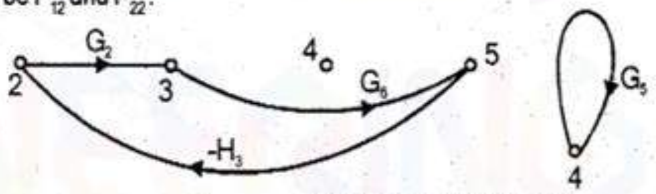
Loop gain of individual loop-5, $P_{41} = G_5$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.
Let the gain products of two non-touching loops be P_{12} and P_{22}.



First combination of two non-touching loops



Second combination of two non-touching loops

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = G_2G_5H_2$

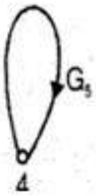
Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

Calculation of Δ and Δ_K

$$\begin{aligned}\Delta &= 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})\\ &= 1-(-G_2G_3H_1-H_2G_2-G_2G_3G_4H_3+G_5-G_2G_6H_3)\\ &\quad +(-G_2H_2G_5-G_2G_5G_6H_3)\end{aligned}$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig



$\therefore \Delta_2 = 1 - G_5$

Transfer Function, T

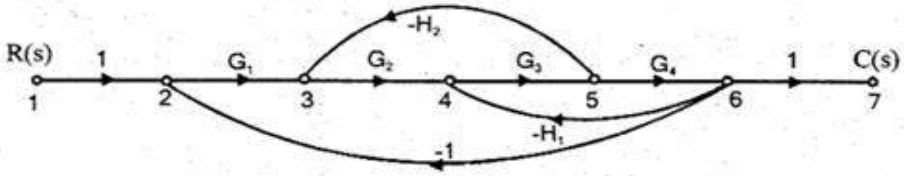
By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K$$

$$= \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right] = \frac{1}{\Delta}\left[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1-G_5)\right]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1+G_2G_3H_1+H_2G_2+G_2G_3G_4H_3-G_5+G_2G_6H_3-G_2H_2G_5-G_2G_5G_6H_3}$$

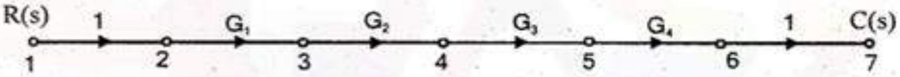
Find the transfer function for the given signal flow graph.



SOLUTION

I. Forward Path Gains

There is only one forward path. ∴ K = 1.
Let the forward path gain be P_1.

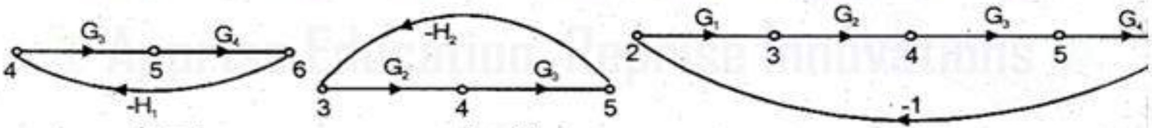


Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

II. Individual Loop Gain

There are three individual loops. Let the loop gains be P_{11}, P_{21}, P_{31}.



loop-1 *loop-2* *loop-3*

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$
Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$
Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31})$$
$$= 1-(-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)$$
$$= 1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4$$

Since no part of the graph is non-touching with forward path-1, $\Delta_1 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}P_1\Delta_1 \text{ (Number of forward path is 1 and so K = 1)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

1.5 Block Diagram Reduction Technique

Procedure for reduction of Block Diagram model:

Step 1: Reduce the cascade blocks.

Step 2: Reduce the parallel blocks.

Step 3: Reduce the internal feedback loops.

Step 4: Shift take-off points towards right and summing points towards left.

Step 5: Repeat step 1 to step 4 until the simple form is obtained.

Step 6: Find transfer function of whole system

Procedure for finding output of Block Diagram model with multiple inputs:

Step 1: Consider one input taking rest of the inputs zero, find output using the procedure described in section 4.3.

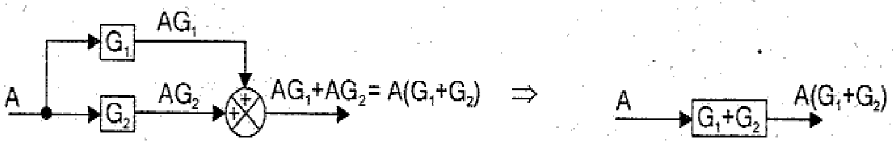
Step 2: Follow step 1 for each inputs of the given Block Diagram model and find their corresponding outputs.

Step 3: Find the resultant output by adding all individual outputs.

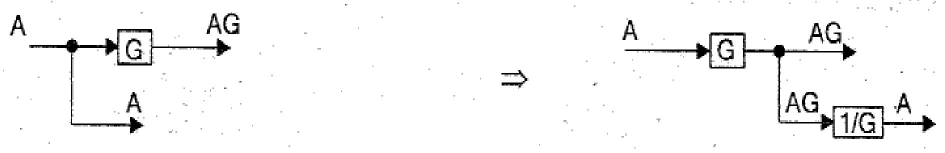
RULES OF BLOCK DIAGRAM ALGEBRA

***Rule-1 :* Combining the blocks in cascade**

***Rule-2 :* Combining Parallel blocks (or combining feed forward paths)**

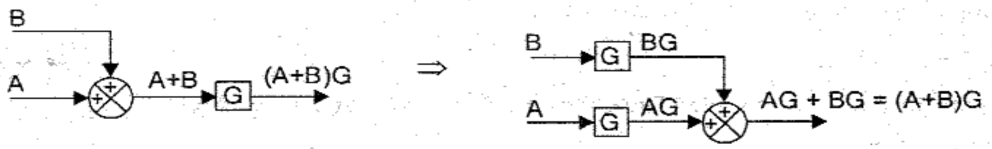


***Rule-3 :* Moving the branch point ahead of the block**

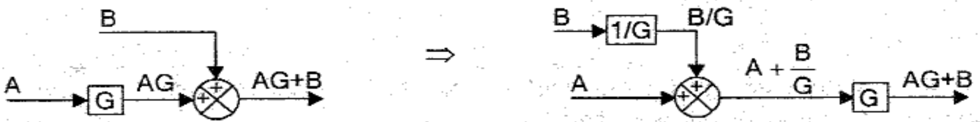


Rule-4 : ***Moving the branch point before the block***

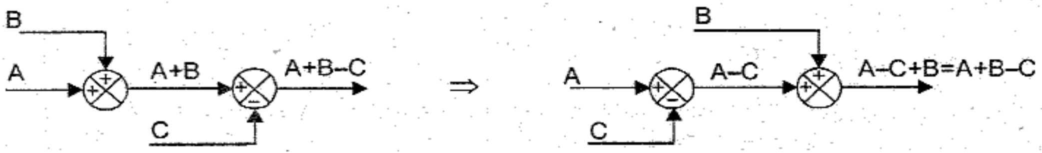
Rule-5 : ***Moving the summing point ahead of the block***



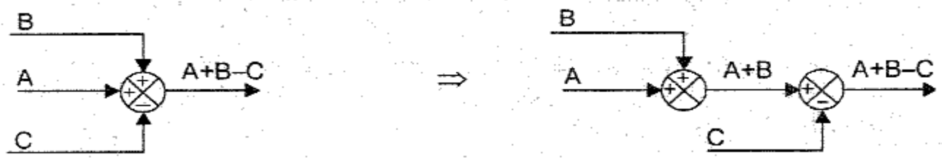
Rule-6 : ***Moving the summing point before the block***



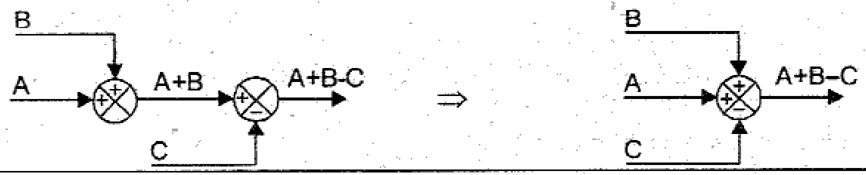
Rule-7 : ***Interchanging summing point***



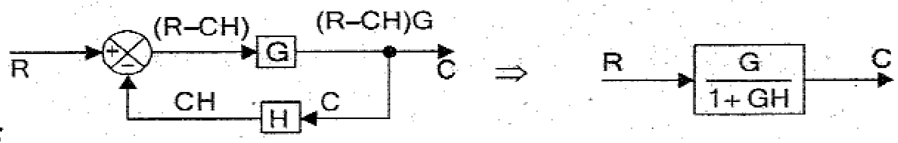
Rule-8 : ***Splitting summing points***



Rule-9 : ***Combining summing points***



Rule-10 : Elimination of (negative) feedback loop



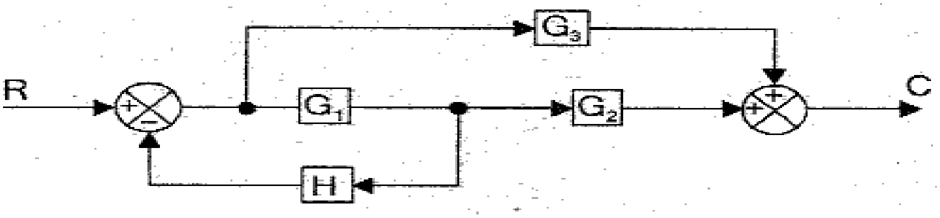
Proof:

$C = (R - CH)G \quad \Rightarrow \quad C = RG - CHG \quad \Rightarrow \quad C + CHG = RG$

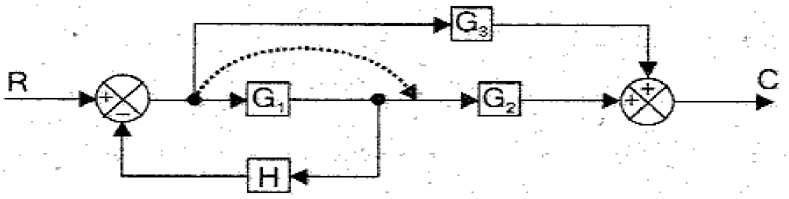
$\therefore C(1 + HG) = RG \quad \Rightarrow \quad \frac{C}{R} = \frac{G}{1 + GH}$

Rule-11 : Elimination of (positive) feedback loop

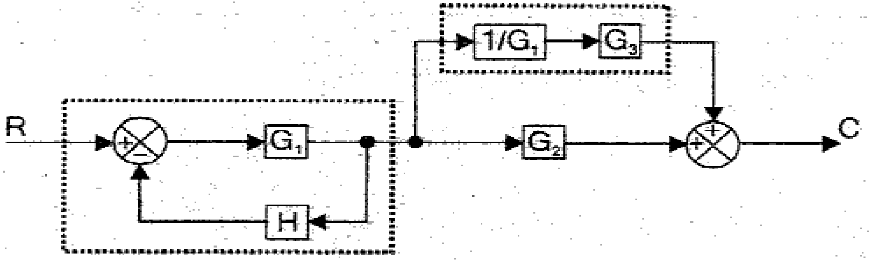
Reduce the block diagram shown in fig 1 and find C/R.



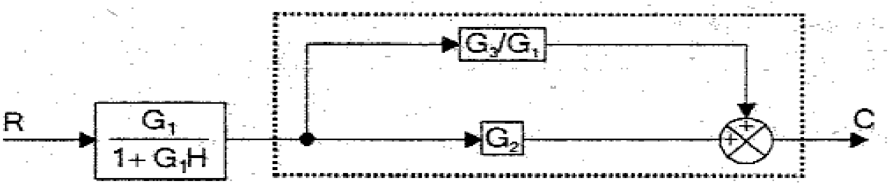
Step 1: Move the branch point after the block.



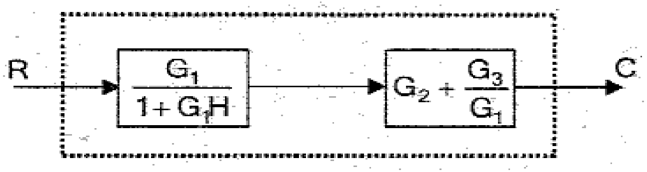
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining blocks in cascade



$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

The overall transfer function of the system, $\frac{C}{R}=\frac{G_1G_2+G_3}{1+G_1H}$

Using block diagram reduction technique find closed loop transfer function of the system whose block diagram in fig 1.

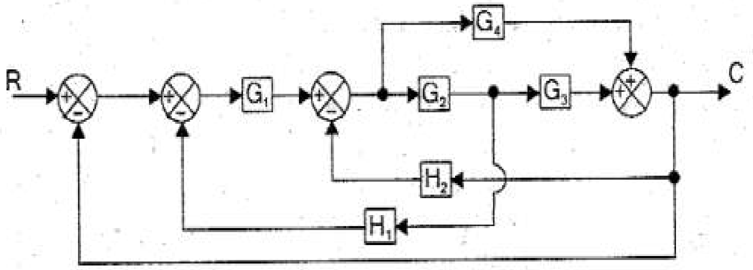
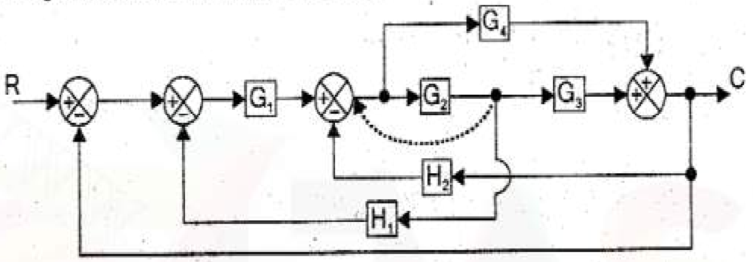


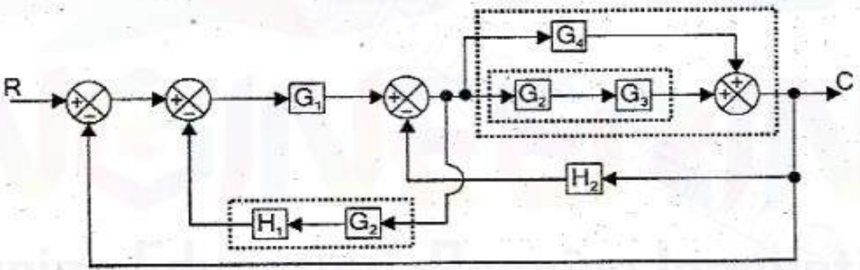
Fig 1.

SOLUTION

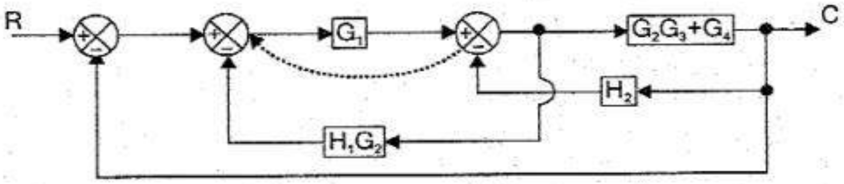
Step 1: Moving the branch point before the block



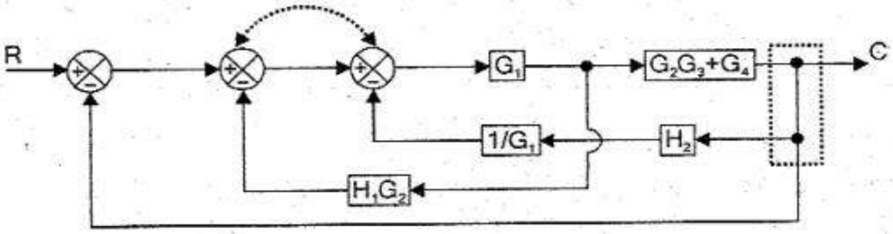
Step 2: Combining the blocks in cascade and eliminating parallel blocks



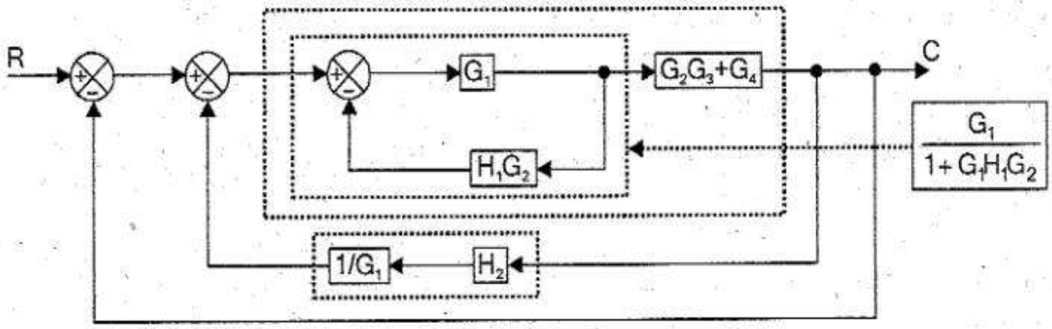
Step 3: Moving summing point before the block.



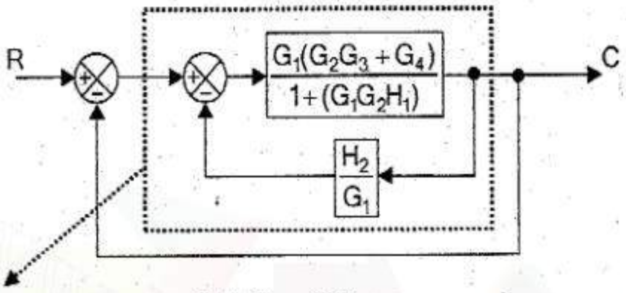
Step 4: Interchanging summing points and modifying branch points.



Step 5: Eliminating the feedback path and combining blocks in cascade

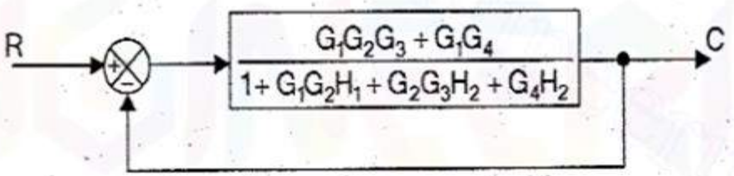


Step 6: Eliminating the feedback path



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\dfrac{H_2}{G_1}} \Rightarrow \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}} \Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminating the feedback path



$$\frac{C}{R}=\frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

RESULT

The overall transfer function is given by,

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

Convert the given block diagram to signal flow graph and determine C(s)/R(s).

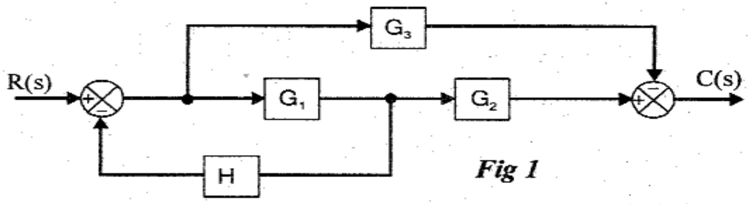


Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

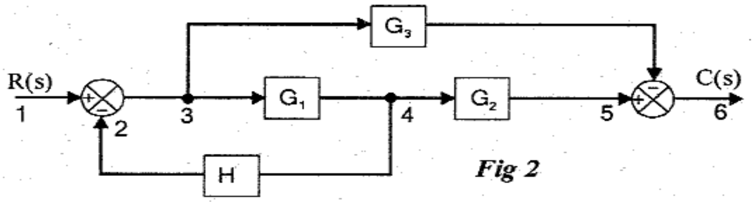


Fig 2

The signal flow graph of the above system is shown in fig 3.

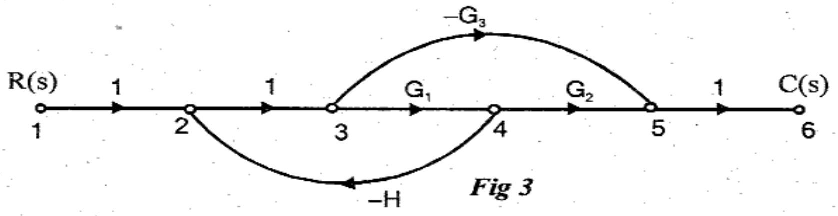


Fig 3

I. Forward Path Gains

There are two forward paths. ∴ K=2

Let the forward path gains be P_1 and P_2.

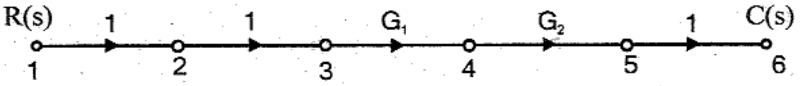


Fig 4 : Forward path-1

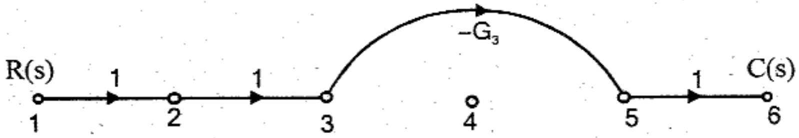


Fig 5 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2$

Gain of forward path-2, $P_2 = -G_3$

Individual Loop Gain

There is only one individual loop. Let the individual loop gain be P_{11}.

Loop gain of individual loop-I, $P_{11} = -G_1H$.

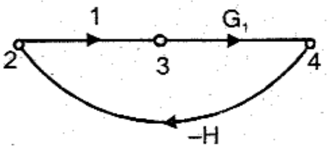


Fig 3 : loop-1

Gain Products of Two Non-touching Loops

There are no combinations of non-touching Loops.

Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}] = 1 + G_1H$$

Since there are no part of the graph which is non-touching with forward path-1 and 2,

$$\Delta_1 = \Delta_2 = 1$$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{G_1G_2 - G_3}{1+G_1H}$$

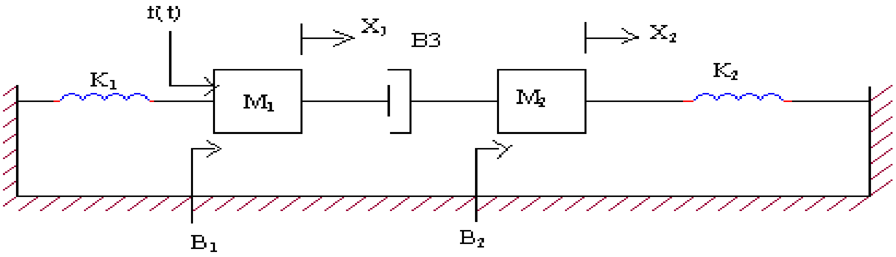
Question Bank

Part – A

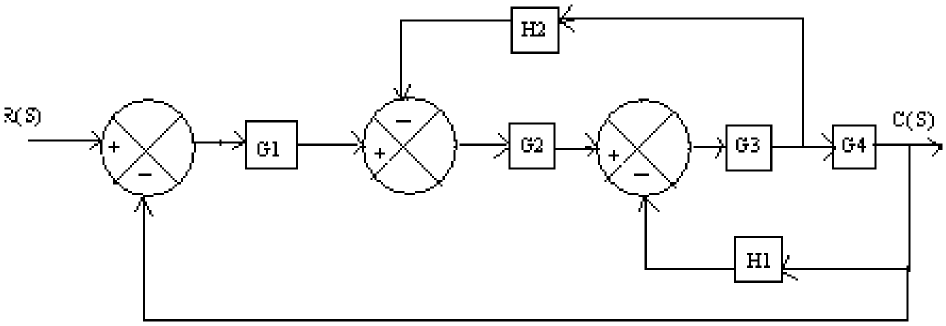
1. What is control system?
2. Define open loop control system.
3. Define closed loop control system.
4. Define transfer function.
5. What are the basic elements used for modelling mechanical rotational system?
6. Name two types of electrical analogous for mechanical system.
7. What is block diagram?
8. What is the basis for framing the rules of block diagram reduction technique?
9. What is a signal flow graph?
10. What is transmittance?
11. What is sink and source?
12. Define non- touching loop.
13. Write Masons Gain formula.
14. Write the analogous electrical elements in force voltage analogy for the elements of mechanical translational system.
15. Write the analogous electrical elements in force current analogy for the elements of mechanical translational system.
16. Write the force balance equation of m ideal mass element.
17. Write the force balance equation of ideal dashpot element.
18. Write the force balance equation of ideal spring element.
19. What is servomechanism?
20. Why is negative feedback invariably preferred in closed loop system?

Part B

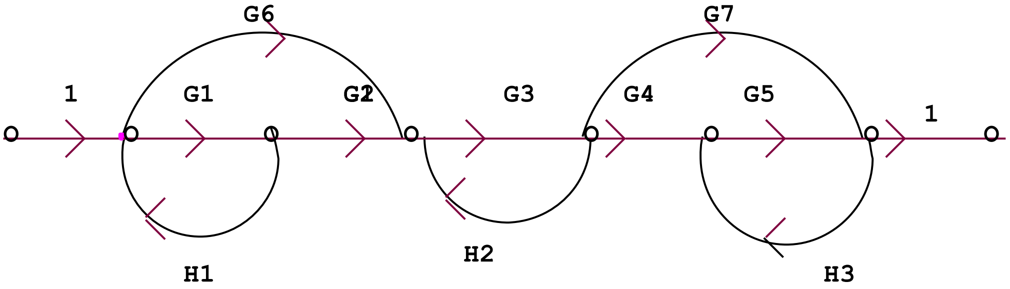
1. Explain open loop and closed loop control system with example.
2. Obtain the transfer function of translational mechanical system shown in Fig 1



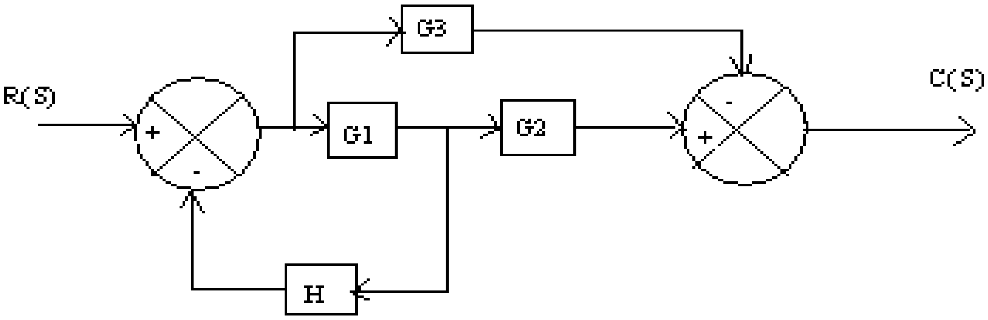
3. Simplify the block diagram and obtain the closed loop transfer function C(S)/R(S) for Figure shown below.



4. Obtain the transfer function C(S) / R(S) of the signal flow graph shown below.

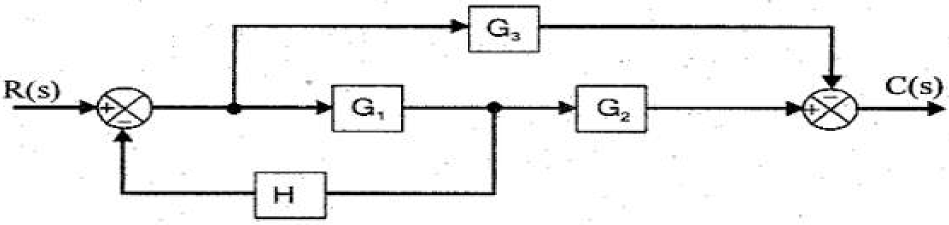


5. Draw the signal flow graph and determine the transfer function C/R for the Block diagram shown below.



6.

Convert the given block diagram to signal flow graph and determine C(s)/R(s).



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SCHOOL OF MECHANICAL

DEPARTMENT OF MECHATRONICS

UNIT – II Signals and Control Systems – SMRA1402

UNIT II - TIME RESPONSE ANALYSIS OF CONTROL SYSTEMS

2.1 Time Domain Analysis of Control Systems

Time response

Time response c(t)is the variation of output with respect to time. The part of time response that goes to zero after large interval of time is called transient response $c_{tr}(t)$. The part of time response that remains after transient response is called steady-state response $c_{ss}(t)$.

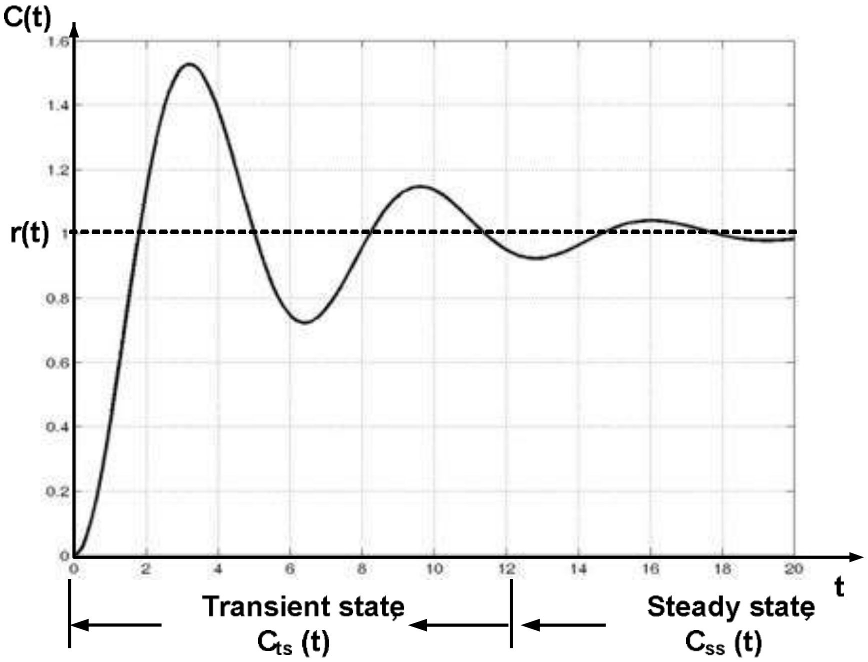


Fig.2.1. Time response of a system

2.2 Standard Test Signals

The standard input signals are

- Impulse
- Step
- Ramp
- Parabolic
- Sinusoidal

By using above standard test signals of control systems, analysis and design of control systems are carried out, defining certain performance measures for the system.

Impulse Signal

In below an impulse signal is shown in Fig.2.2

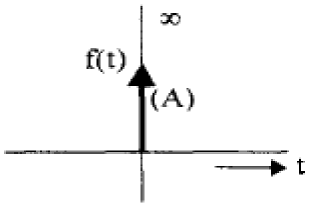


Fig. 2.2 Impulse signal

The impulse function is zero for all t not equal to 0 and it is infinity at $t = 0$. It rises to infinity at $t = 0^-$ and comes back to zero at $t = 0^+$ enclosing a finite area. If this area is A it is called as an impulse function of strength A. If A = 1 it is called a unit impulse function. Thus an impulse signal is denoted by $f(t)$ = A (t).

Step Signal

In below a step signal is shown in Fig.2.3

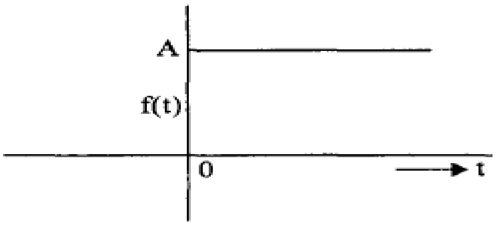


Fig.2.3 Step Signal

It is zero for $t < 0$ and suddenly rises to a value A at $t = 0$ and remains at this value for $t > 0$: It is denoted by $f(t) = Au(t)$. If A = 1, it is called a unit step function.

Ramp signal

In below a ramp signal is shown in Fig.2.4

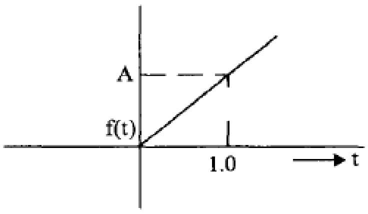


Fig.2.4 Ramp signal

It is zero for $t < 0$ and uniformly increases with a slope equal to A. It is denoted by $f(t) = At$.

If the slope is unity, then it is called a unit ramp signal.

Parabolic signal

In below a parabolic signal is shown in Fig.2.5

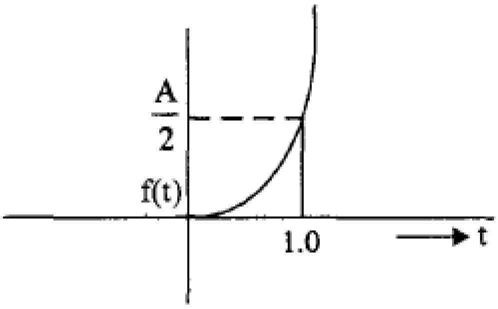


Fig.2.5 Parabolic signal

A parabolic signal is denoted by f (t) = At/2. If A is equal to unity then it is known as a unit parabolic signal.

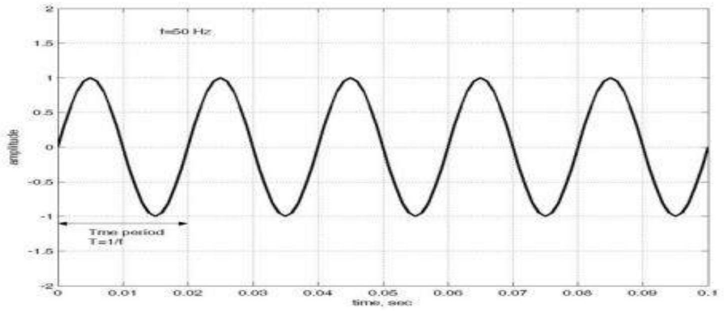
Sinusoidal Signal

A sinusoidal x(t) is mathematically defined as follows.

$$x(t) = \sin \omega t$$

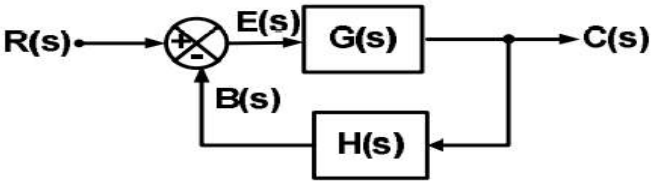
Laplace transform of sinusoidal signal is

$$X(s) = \int_0^\infty e^{-st} \sin \omega t \, dt = \frac{\omega}{s^2 + \omega^2}$$



Steady-state error:

A simple closed-loop control system with negative feedback is shown as follows.



A simple closed-loop control system with negative feedback

Here,

$$E(s) = R(s) - B(s)$$

$$B(s) = C(s)H(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = R(s) - C(s)H(s)$$

$$E(s) = R(s) - E(s)G(s)H(s)$$

$$\Rightarrow \left[1 + G(s)H(s)\right]E(s) = R(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Types of input and steady-state error are summarized as follows.

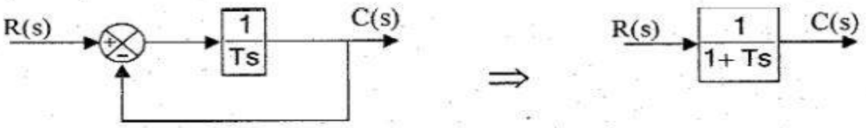
Error Constant	**Equation**	**Steady-state error (e_{ss})**
Position Error Constant (K_P)	$K_P = \lim_{s\to 0} G(s)H(s)$	$e_{ss} = \frac{A}{1+K_P}$
Velocity Error Constant (K_V)	$K_V = \lim_{s\to 0} sG(s)H(s)$	$e_{ss} = \frac{A}{K_V}$
Acceleration Error Constant (K_A)	$K_A = \lim_{s\to 0} s^2G(s)H(s)$	$e_{ss} = \frac{A}{K_A}$

Steady-state error and error constant for different types of input are summarized as follows.

Type	**Step input**		**Ramp input**		**Parabolic input**	
	K_P	e_{ss}	K_V	e_{ss}	K_A	e_{ss}
Type 0	K	$\frac{A}{1+K}$	0	∞	0	∞
Type 1	∞	0	K	$\frac{A}{K}$	0	∞
Type 2	∞	0	∞	0	K	$\frac{A}{K}$

2.3 Response of first order System for unit step input

The closed loop order system with unity feedback is



Closed loop for first order system.

The closed loop transfer function of first order system, $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

If the input is unit step then, $r(t) = 1$ and $R(s) = \frac{1}{s}$.

$$\therefore \text{The response in s-domain, } C(s) = R(s)\frac{1}{(1+Ts)} = \frac{1}{s}\frac{1}{(1+Ts)} = \frac{1}{sT\left(\frac{1}{T}+s\right)} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}$$

By partial fraction expansion,

$$C(s) = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{T}\right)}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times s\Bigg|_{s=0} = \frac{\frac{1}{T}}{s+\frac{1}{T}}\Bigg|_{s=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

B is obtained by multiplying C(s) by (s+1/T) and letting s = −1/T.

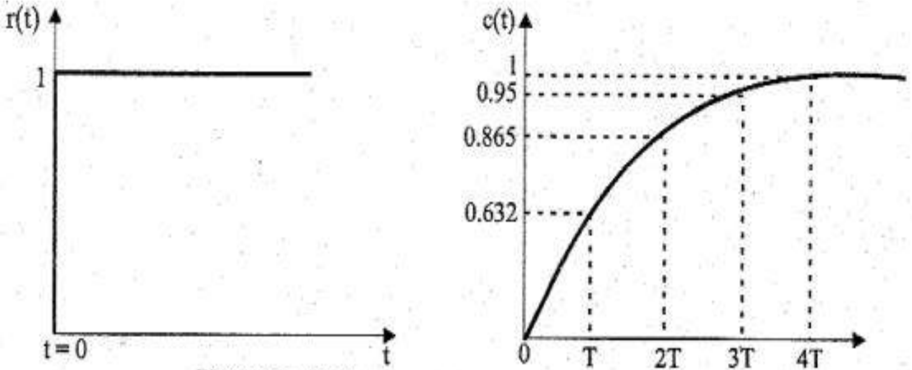
$$B = C(s) \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s}\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{\frac{-1}{T}} = -1$$

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\} = 1 - e^{-\frac{t}{T}}$$

When, $t = 0$, $c(t) = 1 - e^{0} = 0$
When, $t = 1T$, $c(t) = 1 - e^{-1} = 0.632$
When, $t = 2T$, $c(t) = 1 - e^{-2} = 0.865$
When, $t = 3T$, $c(t) = 1 - e^{-3} = 0.95$
When, $t = 4T$, $c(t) = 1 - e^{-4} = 0.9817$
When, $t = 5T$, $c(t) = 1 - e^{-5} = 0.993$
When, $t = \infty$, $c(t) = 1 - e^{-\infty} = 1$



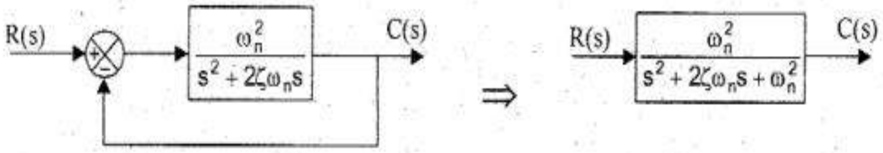
Unit step input.

Response for Unit step input.

Response of first order system to Unit step input.

SECOND ORDER SYSTEM

The closed loop second order system is



Closed loop for second order system.

The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where, ω_n = Undamped natural frequency, rad/sec.

ζ = Damping ratio.

The ***damping ratio*** is defined as the ratio of the actual damping to the critical damping. The response c(t) of second order system depends on the value of damping ratio. Depending on the value of ζ, the system can be classified into the following four cases,

Case 1 : Undamped system, $\zeta = 0$

Case 2 : Under damped system, $0 < \zeta < 1$

Case 3 : Critically damped system, $\zeta = 1$

Case 4 : Over damped system, $\zeta > 1$

RESPONSE OF UNDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For undamped system, z = 0.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

When the input is unit step, $r(t) = 1$ and $R(s) = \frac{1}{s}$.

$$\therefore \text{The response in s-domain, } C(s) = R(s)\,\frac{\omega_n^2}{s^2 + \omega_n^2} = \frac{1}{s}\,\frac{\omega_n^2}{s^2 + \omega_n^2}$$

By partial fraction expansion,

$$C(s) = \frac{\omega_n^2}{s\,(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \frac{\omega_n^2}{s\,(s^2 + \omega_n^2)} \times s\Bigg|_{s=0} = \frac{\omega_n^2}{s^2 + \omega_n^2}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

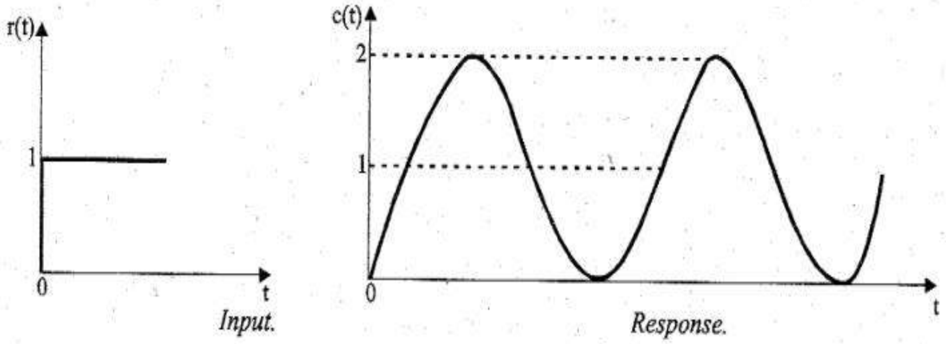
B is obtained by multiplying C(s) by $(s^2 + \omega_n^2)$ and letting $s^2 = -\omega_n^2$ or $s = j\omega_n$.

$$B = C(s) \times (s^2 + \omega_n^2)\Big|_{s=j\omega} = \frac{\omega_n^2}{s\,(s^2 + \omega_n^2)} \times (s^2 + \omega_n^2)\Bigg|_{s=j\omega} = \frac{\omega_n^2}{s}\Bigg|_{s=j\omega_n} = \frac{\omega_n^2}{j\omega_n} = -j\omega_n = -s$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2} = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

$\mathcal{L}\{1\} = \frac{1}{s}$	$\mathcal{L}\{\cos \omega t\} = \frac{s}{s^2 + \omega^2}$

$$\text{Time domain response, } c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{s^2 + \omega_n^2}\right\} = 1 - \cos \omega_n t$$



Input. *Response.*

Response of undamped second order system for unit step input.

RESPONSE OF UNDERDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

For underdamped system, $0<\zeta<1$ and roots of the denominator (characteristic equation) are complex conjugate.

The roots of the denominator are, $s=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$

Since $\zeta<1$, ζ^2 is also less then 1, and so $1-\zeta^2$ is always positive.

$$\therefore\ s=-\zeta\omega_n\pm\omega_n\sqrt{(-1)(1-\zeta^2)}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d=\omega_n\sqrt{1-\zeta^2}$

$$\therefore s=-\zeta\omega_n\pm j\omega_d$$

The response in s-domain, $C(s)=R(s)\,\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

For unit step input, r(t) = 1 and R(s) = 1/s.

$$\therefore C(s)=\frac{\omega_n^2}{s\,(s^2+2\zeta\omega_n s+\omega_n^2)}$$

By partial fraction expansion, $C(s)=\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$

A is obtained by multiplying C(s) by s and letting s = 0.

$$\therefore\ A = s\times C(s)\Big|_{s=0} = s\times\frac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2}=1$$

To solve for B and C, cross multiply

On cross multiplication

$$\omega_n^2 = s^2+2\zeta\omega_n s+\omega_n^2+(Bs+C)s$$
$$\omega_n^2 = s^2+2\zeta\omega_n s+\omega_n^2+Bs^2+Cs$$

Equating coefficients of s^2 we get, $0 = 1 + B$ $\quad\therefore B = -1$

Equating coefficient of s we get, $0 = 2\zeta\omega_n + C$ $\quad\therefore C = -2\zeta\omega_n$

$$\therefore\ C(s) = \frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Let us add and subtract $\zeta^2\omega_n{}^2$ to the denominator of second term

$$\therefore\ C(s) = \frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2+\zeta^2\omega_n^2-\zeta^2\omega_n^2} = \frac{1}{s}-\frac{s+2\zeta\omega_n}{(s^2+2\zeta\omega_n s+\zeta^2\omega_n^2)+(\omega_n^2-\zeta^2\omega_n^2)}$$

$$= \frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} = \frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \qquad \boxed{\omega_d=\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

Let us multiply and divide by ω_d in the third term of the equation

$$\therefore\ C(s) = \frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$\boxed{\mathcal{L}\{1\}=\frac{1}{s}}$$
$$\boxed{\mathcal{L}\{e^{-at}\sin\omega t\}=\frac{\omega}{(s+a)^2+\omega^2}}$$
$$\boxed{\mathcal{L}\{e^{-at}\cos\omega t\}=\frac{s+a}{(s+a)^2+\omega^2}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$= 1-e^{-\zeta\omega_n t}\cos\omega_d t-\frac{\zeta\omega_n}{\omega_d}\,e^{-\zeta\omega_n t}\sin\omega_d t = 1-e^{-\zeta\omega_n t}\left(\cos\omega_d t+\frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}}\,\sin\omega_d t\right) \qquad \boxed{\omega_d=\omega_n\sqrt{1-\zeta^2}}$$

$$= 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\ \cos\omega_d t+\zeta\ \sin\omega_d t\right) = 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin\omega_d t\times\zeta+\cos\omega_d t\times\sqrt{1-\zeta^2}\right)$$

Let us express c(t) in a standard form as shown below.

$$c(t) = 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\sin\omega_d t\times\cos\theta+\cos\omega_d t\times\sin\theta)$$

$$= 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\theta)$$

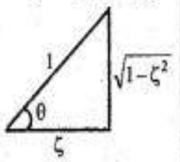
$$\text{where, }\left(\theta=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Note : On constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$, we get

$\sin\theta=\sqrt{1-\zeta^2}$

$\cos\theta=\zeta$

$\tan\theta=\dfrac{\sqrt{1-\zeta^2}}{\zeta}$



The equation   is the response of under damped closed loop second order system for unit step input. For step input of step value, A, the equation (2.28) should be multiplied by A.

$\therefore$ For closed loop under damped second order system,

$$\text{Unit step response } = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ; \ \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\text{Step response } = A\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ; \ \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

Using equation   the response of underdamped second order system for unit step input is sketched and observed that the response oscillates before settling to a final value. The oscillations depends on the value of damping ratio.

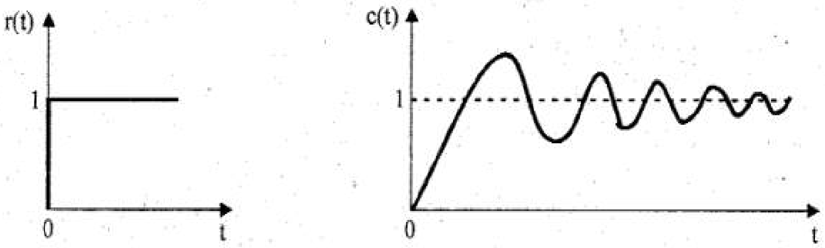


Fig 2.10.a : Input.   *Fig 2.10.b : Response.*

Fig 2.10 : Response of under damped second order system for unit step input.

RESPONSE OF CRITICALLY DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For critical damping $\zeta = 1$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

When input is unit step, r(t) = 1 and R(s) = 1/s.

$\therefore$ The response in s-domain,

$$C(s) = R(s)\frac{\omega_n^2}{(s+\omega_n)^2} = \frac{1}{s}\ \frac{\omega_n^2}{(s+\omega_n)^2} = \frac{\omega_n^2}{s\,(s+\omega_n)^2}$$

By partial fraction expansion, we can write,

$$C(s) = \frac{\omega_n^2}{s\,(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$A = s \times C(s)\Big|_{s=0} = \frac{\omega_n^2}{(s+\omega_n)^2}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$B = (s+\omega_n)^2 \times C(s)\Big|_{s=-\omega_n} = \frac{\omega_n^2}{s}\Bigg|_{s=-\omega_n} = -\omega_n$$

$$C = \frac{d}{ds}\left[(s+\omega_n)^2 \times C(s)\right]\Bigg|_{s=-\omega_n} = \frac{d}{ds}\left(\frac{\omega_n^2}{s}\right)\Bigg|_{s=-\omega_n} = \frac{-\omega_n^2}{s^2}\Bigg|_{s=-\omega_n} = -1$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n} = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

$\mathcal{L}\{1\} = \frac{1}{s}$	
$\mathcal{L}\left\{te^{-at}\right\} = \frac{1}{(s+a)^2}$	
$\mathcal{L}\left\{e^{-at}\right\} = \frac{1}{s+a}$	

The response in time domain,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}\right\}$$

$$c(t) = 1 - \omega_n t\, e^{-\omega_n t} - e^{-\omega_n t}$$

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

The equation is the response of critically damped closed loop second order system for unit ep input. For step input of step value, A, the equation should be multiplied by A.

$\therefore$ For closed loop critically damped second order system,

Unit step response $= 1 - e^{-\omega_n t}(1+\omega_n t)$

Step response $= A\left[1 - e^{-\omega_n t}(1+\omega_n t)\right]$

Using equation the response of critically damped second order system is sketched as shown fig 2.11 and observed that the response has no oscillations.

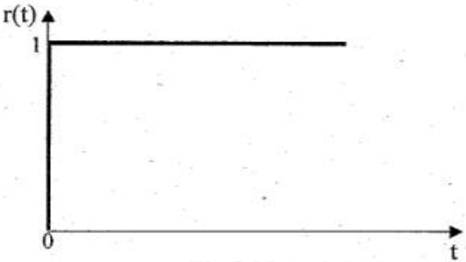


Fig 2.11.a : Input.

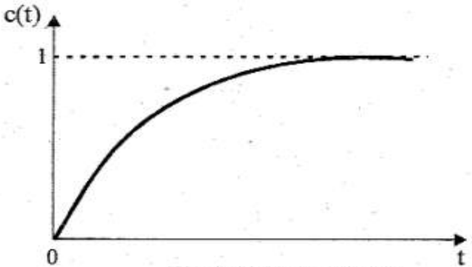


Fig 2.11.b : Response.

RESPONSE OF OVER DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For overdamped system $\zeta > 1$. The roots of the denominator of transfer function are real and istinct. Let the roots of the denominator be s_a, s_b.

$$s_a, s_b = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1} = -\left[\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}\right]$$

Let $s_1 = -s_2$ and $s_2 = -s_b$ $\quad \therefore s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$

$$s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$

The closed loop transfer function can be written in terms of s_1 and s_2 as shown below.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

For unit step input r(t) = 1 and R(s) = 1/s.

$$\therefore C(s) = R(s)\frac{\omega_n^2}{(s+s_1)\,(s+s_2)} = \frac{\omega_n^2}{s\,(s+s_1)\,(s+s_2)}$$

By partial fraction expansion we can write,

$$C(s) = \frac{\omega_n^2}{s\,(s+s_1)\,(s+s_2)} = \frac{A}{s} + \frac{B}{s+s_1} + \frac{C}{s+s_2}$$

$$A = s \times C(s)\big|_{s=0} = s \times \left.\frac{\omega_n^2}{s\,(s+s_1)\,(s+s_2)}\right|_{s=0} = \frac{\omega_n^2}{s_1 s_2}$$

$$= \frac{\omega_n^2}{\left[\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right]\left[\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right]} = \frac{\omega_n^2}{\zeta^2\omega_n^2 - \omega_n^2\left(\zeta^2-1\right)} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$B = (s+s_1) \times C(s)\big|_{s=-s_1} = \left.\frac{\omega_n^2}{s(s+s_2)}\right|_{s=-s_1} = \frac{\omega_n^2}{-s_1(-s_1+s_2)}$$

$$= \frac{-\omega_n^2}{s_1\left[-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1} + \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right]} = \frac{-\omega_n^2}{\left[2\omega_n\sqrt{\zeta^2-1}\right]s_1} = \frac{-\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}$$

$$C = C(s) \times (s+s_2)\big|_{s=-s_2} = \left.\frac{\omega_n^2}{s\,(s+s_1)}\right|_{s=-s_2} = \frac{\omega_n^2}{-s_2(-s_2+s_1)}$$

$$= \frac{\omega_n^2}{-s_2\left[-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1} + \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right]} = \frac{\omega_n^2}{\left[2\omega_n\sqrt{\zeta^2-1}\right]s_2} = \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_2}$$

The response in time domain, c(t) is given by,

$$c(t) = \mathcal{L}^{-1}\left\{ = \frac{1}{s} - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\,\frac{1}{(s+s_1)} + \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_2}\,\frac{1}{(s+s_2)}\right\}$$

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\,e^{-s_1 t} + \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_2}\,e^{-s_2 t}$$

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)$$

$$\text{where, } s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$$

$$s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$

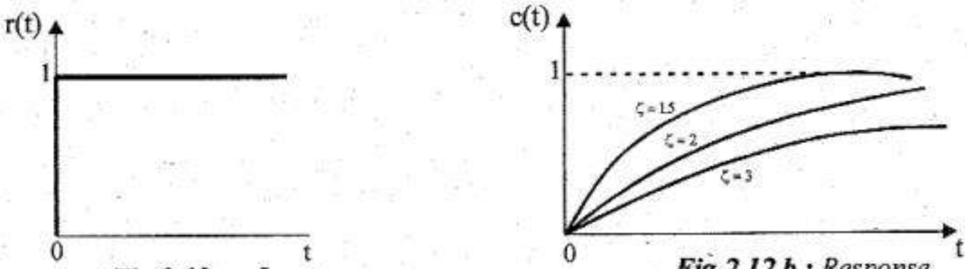


***Fig 2.12.a :** Input.*

***Fig 2.12.b :** Response.*

***Fig 2.12 :** Response of over damped second order system for unit step input.*

2.4 Time Domain Specifications

Control systems are generally designed with damping less than one, i.e., oscillatory step response. Higher order control systems usually have a pair of complex conjugate poles with damping less than unity that dominate over the other poles. Therefore, the time response of second- and higher-order control systems to a step input is generally of damped oscillatory nature as shown in Figure.

In specifying the transient-response characteristics of a control system to a unit step input, we usually specify the following:

1. Delay time,
2. Rise time,
3. Peak time,
4. Peak overshoot,
5. Settling time,
6. Steady-state error,

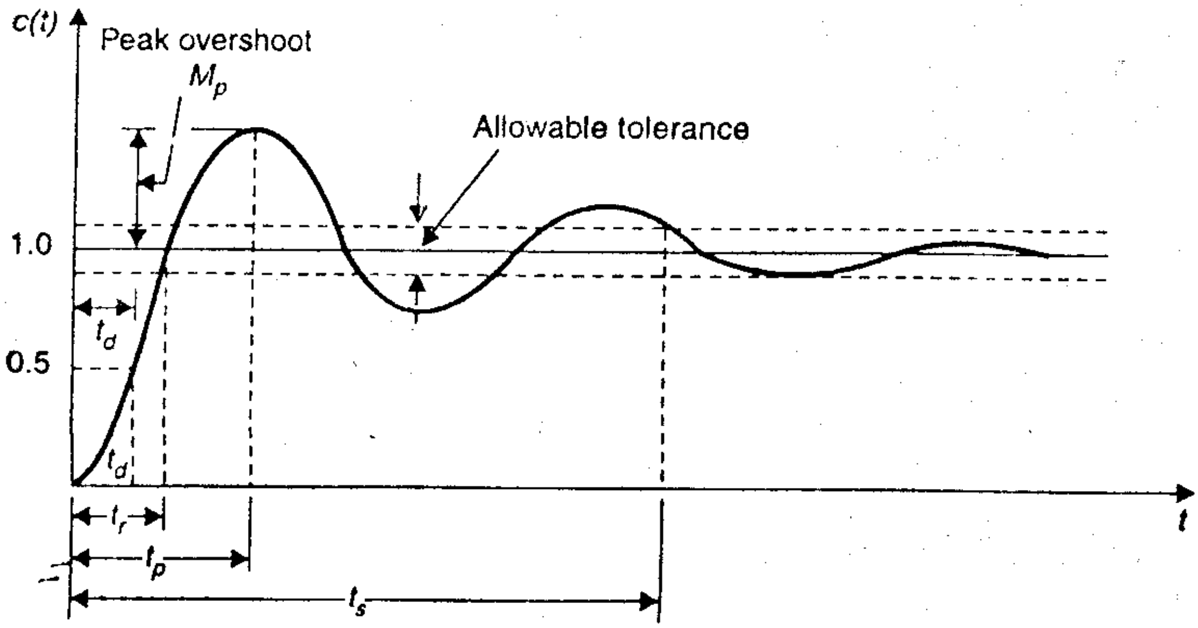


Fig.2.13 Time Domain Specifications

1. ***Delay time, :*** It is the time required for the response to reach 50% of the final value in first attempt.

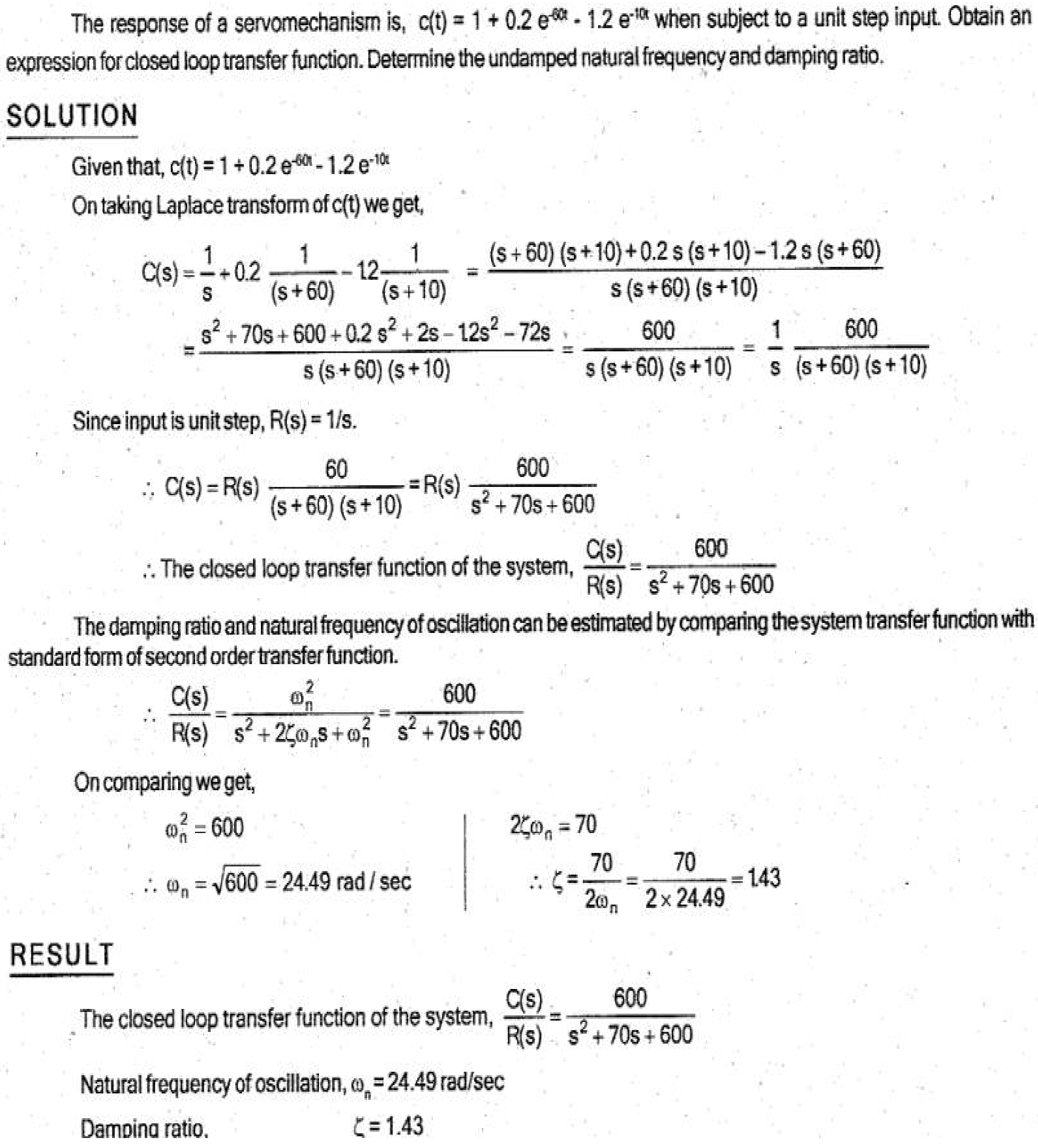
2. ***Rise time, :*** It is the time required for the response to rise from 0 to 100% of the final value for the underdamped system.

3. ***Peak time, :*** It is the time required for the response to reach the peak of time response or the peak overshoot.

4. ***Settling time, :*** It is the time required for the response to reach and stay within a specified tolerance band (2% or 5%) of its final value.

5. ***Peak overshoot, :*** It is the normalized difference between the time response peak and the steady output and is defined as,

6. ***Steady-state error, :*** It indicates the error between the actual output and desired output as 't' tends to infinity.



The response of a servomechanism is, $c(t) = 1 + 0.2\,e^{-60t} - 1.2\,e^{-10t}$ when subject to a unit step input. Obtain an expression for closed loop transfer function. Determine the undamped natural frequency and damping ratio.

SOLUTION

Given that, $c(t) = 1 + 0.2\,e^{-60t} - 1.2\,e^{-10t}$

On taking Laplace transform of c(t) we get,

$$C(s) = \frac{1}{s} + 0.2\,\frac{1}{(s+60)} - 1.2\frac{1}{(s+10)} = \frac{(s+60)\,(s+10) + 0.2\,s\,(s+10) - 1.2\,s\,(s+60)}{s\,(s+60)\,(s+10)}$$

$$= \frac{s^2 + 70s + 600 + 0.2\,s^2 + 2s - 1.2s^2 - 72s}{s\,(s+60)\,(s+10)} = \frac{600}{s\,(s+60)\,(s+10)} = \frac{1}{s}\;\frac{600}{(s+60)\,(s+10)}$$

Since input is unit step, R(s) = 1/s.

$$\therefore\; C(s) = R(s)\,\frac{60}{(s+60)\,(s+10)} = R(s)\,\frac{600}{s^2 + 70s + 600}$$

$\therefore$ The closed loop transfer function of the system, $\frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600}$

The damping ratio and natural frequency of oscillation can be estimated by comparing the system transfer function with standard form of second order transfer function.

$$\therefore\; \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{600}{s^2 + 70s + 600}$$

On comparing we get,

$$\omega_n^2 = 600 \qquad\qquad 2\zeta\omega_n = 70$$

$$\therefore\; \omega_n = \sqrt{600} = 24.49\ \text{rad / sec} \qquad\qquad \therefore\; \zeta = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.49} = 1.43$$

RESULT

The closed loop transfer function of the system, $\frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600}$

Natural frequency of oscillation, ω_n = 24.49 rad/sec

Damping ratio. ζ = 1.43

The unity feedback system is characterized by an open loop transfer function G(s) =K/s (s +10). Determine the gain K, so that the system will have a damping ratio of 0.5 for this value of K. Determine peak overshoot and time at peak overshoot for a unit step input.

SOLUTION

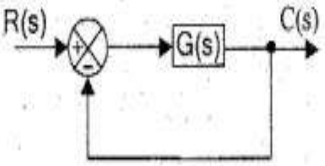


Fig 1 : Unity feedback system.

The unity feedback system is shown in fig 1.

The closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

Given that, G(s) = K/s (s +10)

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s\,(s+10)+K} = \frac{K}{s^2+10s+K}$$

The value of K can be evaluated by comparing the system transfer function with standard form of second order transfer function.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K}{s^2+10s+K}$$

On comparing we get,

$$\begin{array}{l|l|l} \omega_n^2 = K & 2\zeta\omega_n = 10 & K = 100 \\ \therefore \omega_n = \sqrt{K} & \text{Put } \zeta = 0.5 \text{ and } \omega_n = \sqrt{K} & \omega_n = 10 \text{ rad / sec} \\ & \therefore 2 \times 0.5 \times \sqrt{K} = 10 & \\ & \qquad \sqrt{K} = 10 & \end{array}$$

The value of gain, K=100.

Percentage peak overshoot, $\%M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$

$$= e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100 = 0.163 \times 100 = 16.3\%$$

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.5^2}} = 0.363$ sec

RESULT

The value of gain, $K = 100$

Percentage peak overshoot, $\%M_p = 16.3\%$

Peak time, $t_p = 0.363$ sec.

For a unity feedback control system the open loop transfer function, $G(s)=\dfrac{10(s+2)}{s^2(s+1)}$. Find

a) the position, velocity and acceleration error constants,

b) the steady state error when the input is R(s), where $R(s)=\dfrac{3}{s}-\dfrac{2}{s^2}+\dfrac{1}{3s^3}$

SOLUTION

a) To find static error constants

For a unity feedback system, H(s)=1

Position error constant, $K_p=\underset{s\to 0}{Lt}\,G(s)H(s)=\underset{s\to 0}{Lt}\,G(s)\ =\underset{s\to 0}{Lt}\,\dfrac{10(s+2)}{s^2(s+1)}=\infty$

Velocity error constant, $K_v=\underset{s\to 0}{Lt}\,s\,G(s)H(s)=\underset{s\to 0}{Lt}\,s\,G(s)\ =\underset{s\to 0}{Lt}\,s\dfrac{10(s+2)}{s^2(s+1)}\ =\infty$

Acceleration error constant, $K_a=\underset{s\to 0}{Lt}\,s^2G(s)H(s)=\underset{s\to 0}{Lt}\,s^2G(s)$

$$=\underset{s\to 0}{Lt}\,s^2\,\frac{10(s+2)}{s^2(s+1)}=\frac{10\times 2}{1}=20$$

b)

The error signal in s - domain, $E(s)=\dfrac{R(s)}{1+G(s)H(s)}$

Given that, $R(s)=\dfrac{3}{s}-\dfrac{2}{s^2}+\dfrac{1}{3s^3}$; $\quad G(s)=\dfrac{10(s+2)}{s^2(s+1)}$; $\quad H(s)=1$

$$\therefore E(s)=\frac{\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}}{1+\frac{10(s+2)}{s^2(s+1)}}=\frac{\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}}{\frac{s^2(s+1)+10(s+2)}{s^2(s+1)}}$$

$$=\frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]-\frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]+\frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

Steady state error, $e_{ss}=\underset{t\to\infty}{Lt}\,e(t)=\underset{s\to 0}{Lt}\,s\,E(s)$

$$\therefore e_{ss}=\underset{s\to 0}{Lt}\,s\left\{\frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]-\frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]+\frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]\right\}$$

$$=\underset{s\to 0}{Lt}\left\{\frac{3s^2(s+1)}{s^2(s+1)+10(s+2)}-\frac{2s(s+1)}{s^2(s+1)+10(s+2)}+\frac{(s+1)}{3s^2(s+1)+30\,(s+2)}\right\}\ =0-0+\frac{1}{60}$$

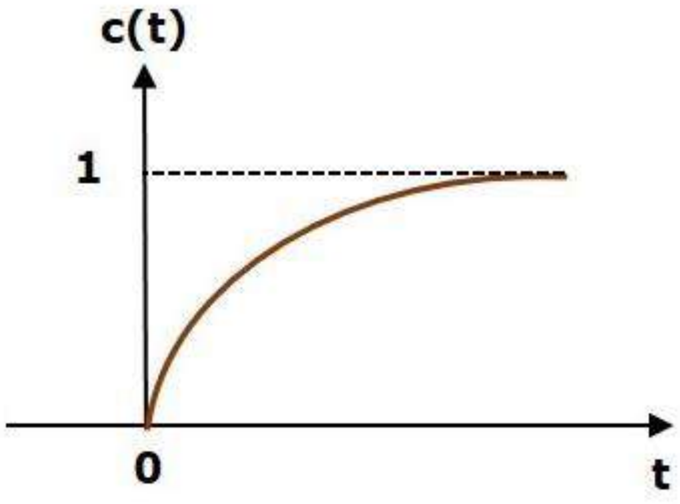
$$=\frac{1}{60}$$

2.5 Stability

Concept of stability Stability is a very important characteristic of the transient performance of a system. Any working system is designed considering its stability. Therefore, all instruments are stable with in a boundary of parameter variations.

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A **stable system** produces a bounded output for a given bounded input.

The following figure shows the response of a stable system.



This is the response of first order control system for unit step input. This response has the values between 0 and 1. So, it is bounded output. We know that the unit step signal has the value of one for all positive values of **t** including zero. So, it is bounded input. Therefore, the first order control system is stable since both the input and the output are bounded.

Types of Systems based on Stability

We can classify the systems based on stability as follows.

- Absolutely stable system
- Conditionally stable system
- Marginally stable system

Absolutely Stable System

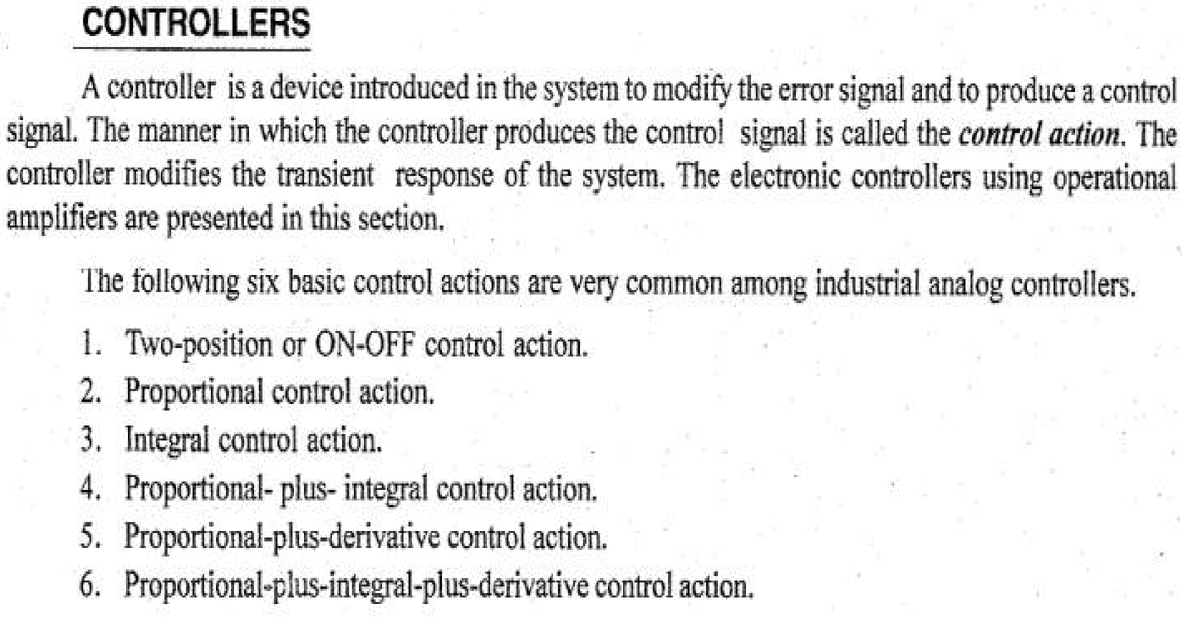
If the system is stable for all the range of system component values, then it is known as the **absolutely stable system**. The open loop control system is absolutely stable if all the poles of the open loop transfer function present in left half of **'s' plane**. Similarly, the closed loop control system is absolutely stable if all the poles of the closed loop transfer function present in the left half of the 's' plane.

Conditionally Stable System

If the system is stable for a certain range of system component values, then it is known as **conditionally stable system**.

Marginally Stable System

If the system is stable by producing an output signal with constant amplitude and constant frequency of oscillations for bounded input, then it is known as **marginally stable system**. The open loop control system is marginally stable if any two poles of the open loop transfer function is present on the imaginary axis. Similarly, the closed loop control system is marginally stable if any two poles of the closed loop transfer function is present on the imaginary axis.



CONTROLLERS

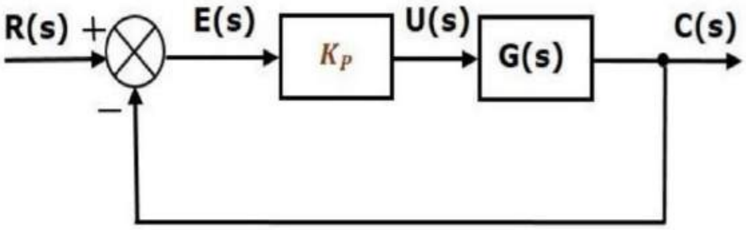
A controller is a device introduced in the system to modify the error signal and to produce a control signal. The manner in which the controller produces the control signal is called the ***control action***. The controller modifies the transient response of the system. The electronic controllers using operational amplifiers are presented in this section.

The following six basic control actions are very common among industrial analog controllers.

1. Two-position or ON-OFF control action.
2. Proportional control action.
3. Integral control action.
4. Proportional- plus- integral control action.
5. Proportional-plus-derivative control action.
6. Proportional-plus-integral-plus-derivative control action.

Proportional Controller

The proportional controller produces an output, which is proportional to error signal. Therefore, the transfer function of the proportional controller is Kp. Where, U(s) is the Laplace transform of the actuating signal u(t) E(s) is the Laplace transform of the error signal e(t) KP is the proportionality constant The block diagram of the unity negative feedback closed loop control system along with the proportional controller is shown in the following figure.



$$u(t) \propto e(t)$$

$$\Rightarrow u(t) = K_P e(t)$$

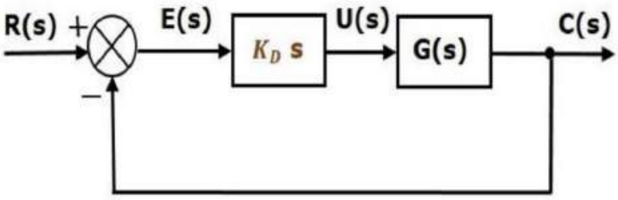
Apply Laplace transform on both the sides -

$$U(s) = K_P E(s)$$

$$\frac{U(s)}{E(s)} = K_P$$

Derivative Controller

The derivative controller produces an output, which is derivative of the error signal.



$$u(t) = K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

Apply Laplace transform on both sides.

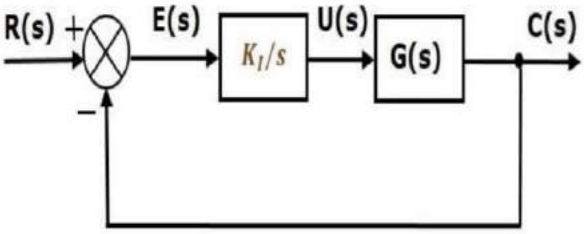
$$U(s) = K_D s E(s)$$

$$\frac{U(s)}{E(s)} = K_D s$$

Therefore, the transfer function of the derivative controller is KDs. Where, KD is the derivative constant. The block diagram of the unity negative feedback closed loop control system along with the derivative controller is shown in the above figure.

Integral Controller

The integral controller produces an output, which is integral of the error signal.



$$u(t) = K_I \int e(t)dt$$

Apply Laplace transform on both the sides -

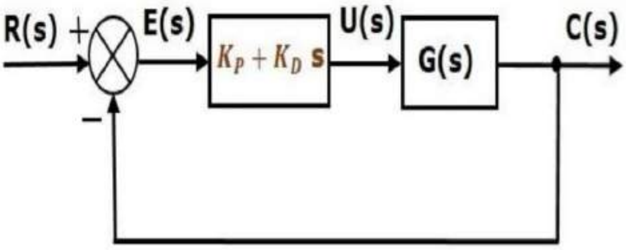
$$U(s) = \frac{K_I E(s)}{s}$$

$$\frac{U(s)}{E(s)} = \frac{K_I}{s}$$

Therefore, the transfer function of the integral controller is $\frac{K_I}{s}$

Proportional Derivative (PD) Controller

The proportional derivative controller produces an output, which is the combination of the outputs of proportional and derivative controllers. Therefore, the transfer function of the proportional derivative controller is KP+KDs. The block diagram of the unity negative feedback closed loop control system along with the proportional derivative controller is shown in the following figure. The proportional derivative controller is used to improve the stability of control system without affecting the steady state error.



$$u(t) = K_P e(t) + K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

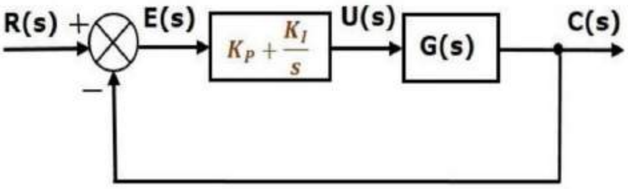
Apply Laplace transform on both sides -

$$U(s) = (K_P + K_D s)E(s)$$

$$\frac{U(s)}{E(s)} = K_P + K_D s$$

Proportional Integral (PI) Controller

The proportional integral controller produces an output, which is the combination of outputs of the proportional and integral controllers. The block diagram of the unity negative feedback closed loop control system along with the proportional integral controller is shown in the following figure.



$$u(t) = K_P e(t) + K_I \int e(t) dt$$

Apply Laplace transform on both sides -

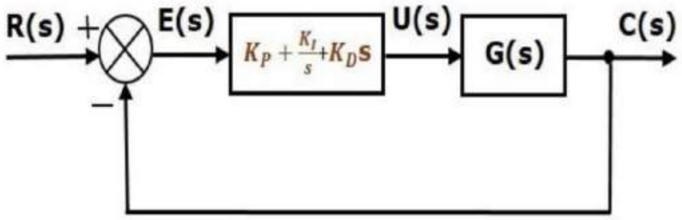
$$U(s) = \left(K_P + \frac{K_I}{s}\right) E(s)$$

$$\frac{U(s)}{E(s)} = K_P + \frac{K_I}{s}$$

Therefore, the transfer function of proportional integral controller is $K_P + \frac{K_I}{s}$.

Proportional Integral Derivative (PID) Controller

The proportional integral derivative controller produces an output, which is the combination of the outputs of proportional, integral and derivative controllers. The block diagram of the unity negative feedback closed loop control system along with the proportional integral derivative controller is shown in the following figure.



$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

Apply Laplace transform on both sides -

$$U(s) = \left(K_P + \frac{K_I}{s} + K_D s\right) E(s)$$

$$\frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$$

Therefore, the transfer function of the proportional integral derivative controller is $K_P + \frac{K_I}{s} + K_D s$.

Question Bank

Part – A

1. Name the standard test signals used in control system?
2. Define damping ratio.
3. Define delay time, rise time, peak time.
4. The damping ratio of a system is 0.5. Its natural frequency of oscillation is 10 rad /sec. Determine the rise time and the peak time.
5. What are the static error constants?
6. Give the correlation between static and dynamic error coefficient.
7. Define delay time, rise time, peak time.
8. What is transient and steady state response?
9. Give the closed loop transfer function of a second order system.
10. Determine the % of peak overshoot for a damping ratio of 0.6.
11. What is time response?
12. What is transient and steady state response?
13. What is test signal? What is its significance?
14. Name the standard test signals used in control system?
15. What is a step signal? Give its functional representation.
16. What is a ramp signal? Give its functional representation.
17. What is a parabolic signal? Give its functional representation.
18. What is a impulse signal? Give its functional representation.
19. Define the order of a system.
20. Define damping ratio.
21. Give the closed loop transfer function of a second order system.
22. How the second order system is classified based on the value of damping ratio?
23. What will be the nature of response of a second order system with different types of damping.
24. Sketch the response of a second order system under damped condition.
25. Compare the step responses of first order and second order system.
26. What is damped frequency of oscillation?
27. A second order system has damping ratio of 0.3 and natural frequency of oscillation 5 rad/ sec. Determine the damped frequency of oscillation.
28. The closed loop transfer function of a second order system is given as $20/(S^2+6S+10)$. Determine the damping ratio, natural frequency of oscillation and type of damping.

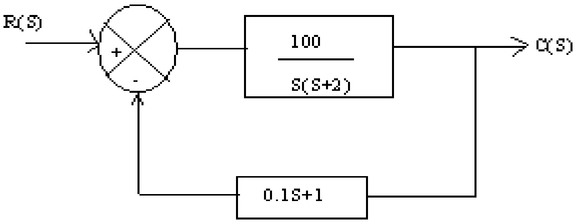
29. List the time domain specifications.

30. Determine the % of peak overshoot for a damping ratio of 0.6.

Part B

1a. Derive the expression for peak overshoot and settling time.

1b. A positional control system with velocity feedback is as shown. What is the response of the system for unit step input for Fig 5 shown below,



2. A closed loop transfer function is evaluated by the differential equation,

$\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64e$, where c is the displacement of output shaft displacement of input shaft and e = r - c, determine (i) Undamped natural frequency,

(ii) Damping ratio, (iii) Percentage maximum overshoot for unit step input.

3. What is the advantage of using generalized error coefficients. Derive from first principle, the expression for generalized error coefficients. How will you evaluate them mathematically?

4. Derive the expression for the response of second order system for underdamped condition, when input is unit step.

5a. Explain the time domain specifications.

5b. A unity feedback system is charactertised by an open loop transfer function G(S) = K / S(S+10). Determine the gain K, so that the system will have a damping ratio of 0.5 for this value of K. Determine settling time, peak overshoot and time to peak overshoot for a unit step input.

6. Explain the design procedure for PID controller.

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SCHOOL OF MECHANICAL

DEPARTMENT OF MECHATRONICS

UNIT – III Signals and Control Systems – SMRA1402

UNIT III - FREQUENCY RESPONSE AND STABILITY ANALYSIS OF CONTROL SYSTEMS

3.1 Stability

Stability is an important concept. In this chapter, let us discuss the stability of system and types of systems based on stability. What is Stability? A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A stable system produces a bounded output for a given bounded input. The following figure shows the response of a stable system.

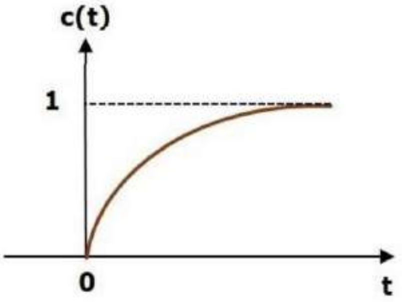


Fig.3.1 Response of a stable system

This is the response of first order control system for unit step input. This response has the values between 0 and 1. So, it is bounded output. We know that the unit step signal has the value of one for all positive values of t including zero. So, it is bounded input. Therefore, the first order control system is stable since both the input and the output are bounded. Types of Systems based on Stability We can classify the systems based on stability as follows.

- Absolutely stable system
- Conditionally stable system
- Marginally stable system

Absolutely Stable System

If the system is stable for all the range of system component values, then it is known as the absolutely stable system. The open loop control system is absolutely stable if all the poles of the open loop transfer function present in left half of ‘s’ plane. Similarly, the closed loop control system is absolutely stable if all the poles of the closed loop transfer function present in the left half of the ‘s’ plane.

Conditionally Stable System

If the system is stable for a certain range of system component values, then it is known as conditionally stable system.

Marginally Stable System

If the system is stable by producing an output signal with constant amplitude and constant frequency of oscillations for bounded input, then it is known as marginally stable system. The open loop control system is marginally stable if any two poles of the open loop transfer function is present on the imaginary axis. Similarly, the closed loop control system is marginally stable if any two poles of the closed loop transfer function is present on the imaginary axis. In this chapter, let us discuss the stability analysis in the ‘s’ domain using the Routh-Hurwitz stability criterion. In this criterion, we require the characteristic equation to find the stability of the closed loop control systems.

3.2 Routh-Hurwitz Stability Criterion

Routh-Hurwitz stability criterion is having one necessary condition and one sufficient condition for stability. If any control system doesn’t satisfy the necessary condition, then we can say that the control system is unstable. But, if the control system satisfies the necessary condition, then it may or may not be stable. So, the sufficient condition is helpful for knowing whether the control system is stable or not.

Necessary Condition for Routh-Hurwitz Stability

The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts. Consider the characteristic equation of the order ‘n’ is –

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s^1 + a_n s^0 = 0$$

Note that, there should not be any term missing in the nth order characteristic equation. This means that the nth order characteristic equation should not have any coefficient that is of zero value. Sufficient Condition for Routh-Hurwitz Stability The sufficient condition is that all the elements of the first column of the Routh array should have the same sign. This means that all the elements of the first column of the Routh array should be either positive or negative.

Routh Array Method

If all the roots of the characteristic equation exist to the left half of the ‘s’ plane, then the control system is stable. If at least one root of the characteristic equation exists to the right half of the ‘s’ plane, then the control system is unstable. So, we have to find the roots of the characteristic equation to know whether the control system is stable or unstable. But it is difficult to find the roots of the characteristic equation as order increases. So, to overcome this problem there we have the Routh array method.

In this method, there is no need to calculate the roots of the characteristic equation. First formulate the Routh table and find the number of the sign changes in the first column of the Routh table. The number of sign changes in the first column of the Routh table gives the number of roots of characteristic equation that exist in the right half of the ‘s’ plane and the control system is unstable.

Follow this procedure for forming the Routh table.

- Fill the first two rows of the Routh array with the coefficients of the characteristic polynomial as mentioned in the table below. Start with the coefficient of Sn and continue up to the coefficient of S0.
- Fill the remaining rows of the Routh array with the elements as mentioned in the table below. Continue this process till you get the first column element of row S0.

Note –

If any row elements of the Routh table have some common factor, then you can divide the row elements with that factor for the simplification will be easy. The following table shows the Routh array of the nth order characteristic polynomial.

$$a_0s^n + a_1s^{n-1} + a_2s^{n-2} + \ldots + a_{n-1}s^1 + a_ns^0$$

s^n	a_0	a_2	a_4	a_6	$\ldots$	$\ldots$
s^{n-1}	a_1	a_3	a_5	a_7	$\ldots$	$\ldots$
s^{n-2}	$b_1 = \frac{a_1a_2-a_3a_0}{a_1}$	$b_2 = \frac{a_1a_4-a_5a_0}{a_1}$	$b_3 = \frac{a_1a_6-a_7a_0}{a_1}$	$\ldots$	$\ldots$	$\ldots$
s^{n-3}	$c_1 = \frac{b_1a_3-b_2a_1}{b_1}$	$c_2 = \frac{b_1a_55-b_3a_1}{b_1}$	$\vdots$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
s^1	$\vdots$	$\vdots$				
s^0	a_n					

Using Routh criterion, determine the stability of the system represented by the characteristic equation, $s^4+8s^3+18s^2+16s+5=0$. Comment on the location of the roots of characteristic equation.

SOLUTION

The characteristic equation of the system is, $s^4+8s^3+18s^2+16s+5=0$.

The given characteristic equation is 4th order equation and so it has 4 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^4	:	1	18	5	.... Row-1
s^3	:	8	16		.... Row-2

The elements of s^3 row can be divided by 8 to simplify the computations.

s^4	:	1	18	5	.... Row-1
s^3	:	1	2		.... Row-2
s^2	:	16	5		.... Row-3
s^1	:	1.7			.... Row-4
s^0	:	5			.... Row-5

Column-1

$s^2 : \frac{1\times18-2\times1}{1} \quad \frac{1\times5-0\times1}{1}$

$s^2 : 16 \quad 5$

$s^1 : \frac{16\times2-5\times1}{16}$

$s^1 : 1.6875 \approx 1.7$

$s^0 : \frac{1.7\times5-0\times16}{17}$

$s^0 : 5$

On examining the elements of first column of routh array it is observed that all the elements are positive and there is no sign change. Hence all the roots are lying on the left half of s-plane and the system is stable.

RESULT

1. Stable system
2. All the four roots are lying on the left half of s-plane.

Construct Routh array and determine the stability of the system whose characterisitc equation is $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$. Also determine the number of roots lying on right half of s-plane, left half of s-plane and on imaginary axis.

SOLUTION

The characteristic equation of the system is, $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$.

The given characteristic polynomial is 6th order equation and so it has 6 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^6 : 1 8 20 16 .... Row-1
s^5 : 2 12 16 .... Row-2

The elements of s^5 row can be divided by 2 to simplify the calculations.

s^6 : 1 8 20 16 .... Row-1
s^5 : 1 6 8 ....Row-2
s^4 : 1 6 8 ....Row-4
s^3 : 0 0 ....Row-4
s^3 : 1 3 ....Row-4
s^2 : 3 8 ....Row-5
s^1 : 0.33 ....Row-6
s^0 : 8 ....Row-7

Column-1

On examining the elements of 1st column of routh array it is observed that there is no sign change. The row with all zeros indicate the possibility of roots on imaginary axis. Hence the system is limitedly or marginally stable.

The auxiliary polynomial is,

$$s^4+6s^2+8=0$$

Let, $s^2=x$

$$\therefore x^2+6x+8=0$$

The roots of quadratic are, $x=\dfrac{-6\pm\sqrt{6^2-4\times 8}}{2}$

$$=-3\pm 1=-2 \text{ or } -4$$

The roots of auxiliary polynomial is,

$$s=\pm\sqrt{x}=\pm\sqrt{-2} \text{ and } \pm\sqrt{-4}$$
$$=+j\sqrt{2},-j\sqrt{2},+j2 \text{ and } -j2$$

s^4 : $\dfrac{1\times 8-6\times 1}{1}$ $\dfrac{1\times 20-8\times 1}{1}$ $\dfrac{1\times 16-0\times 1}{1}$
s^4 : 2 12 16
divide by 2
s^4 : 1 6 8

s^3: $\dfrac{1\times 6-6\times 1}{1}$ $\dfrac{1\times 8-8\times 1}{1}$
s^3: 0 0

The auxiliary equation is, $A=s^4+6s^2+8$. On differentiating A with respect to s we get,

$$\frac{dA}{ds}=4s^3+12s$$

The coefficients of $\frac{dA}{ds}$ are used to form s^3 row.

s^3: 4 12
divide by 4
s^3: 1 3

s^2 : $\dfrac{1\times 6-3\times 1}{1}$ $\dfrac{1\times 8-0\times 1}{1}$
s^2 : 3 8

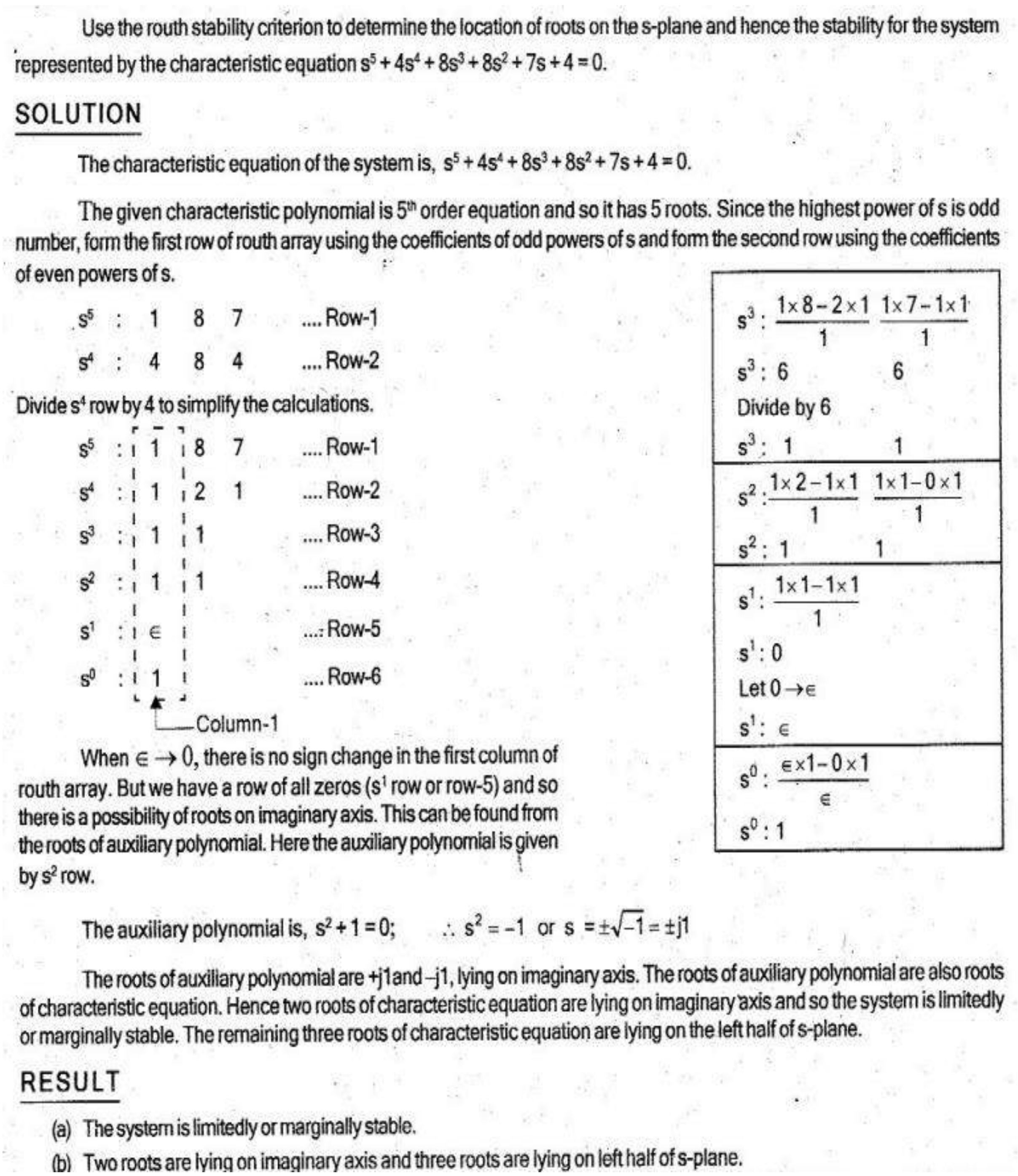
s^1 : $\dfrac{3\times 3-8\times 1}{3}$
s^1 : 0.33

s^0 : $\dfrac{0.33\times 8-0\times 3}{0.33}$
s^0 : 8

The roots of auxiliary polynomial are also roots of characteristic equation. Hence 4 roots are lying on imaginary axis and the remaining two roots are lying on the left half of s-plane.

RESULT

1. The system is limitedly or marginally stable.
2. Four roots are lying on imaginary axis and remaining two roots are lying on left half of s-plane.



Use the routh stability criterion to determine the location of roots on the s-plane and hence the stability for the system represented by the characteristic equation $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$.

SOLUTION

The characteristic equation of the system is, $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$.

The given characteristic polynomial is 5th order equation and so it has 5 roots. Since the highest power of s is odd number, form the first row of routh array using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s.

s^5 : 1 8 7 .... Row-1

s^4 : 4 8 4 .... Row-2

Divide s^4 row by 4 to simplify the calculations.

s^5 : 1 8 7 .... Row-1

s^4 : 1 2 1 .... Row-2

s^3 : 1 1 .... Row-3

s^2 : 1 1 .... Row-4

s^1 : $\in$ Row-5

s^0 : 1 .... Row-6

Column-1

$$s^3 : \frac{1\times 8-2\times 1}{1} \quad \frac{1\times 7-1\times 1}{1}$$

s^3 : 6 6

Divide by 6

s^3 : 1 1

$$s^2 : \frac{1\times 2-1\times 1}{1} \quad \frac{1\times 1-0\times 1}{1}$$

s^2 : 1 1

$$s^1 : \frac{1\times 1-1\times 1}{1}$$

s^1 : 0

Let $0 \to \in$

s^1 : $\in$

$$s^0 : \frac{\in\times 1-0\times 1}{\in}$$

s^0 : 1

When $\in \to 0$, there is no sign change in the first column of routh array. But we have a row of all zeros (s^1 row or row-5) and so there is a possibility of roots on imaginary axis. This can be found from the roots of auxiliary polynomial. Here the auxiliary polynomial is given by s^2 row.

The auxiliary polynomial is, $s^2 + 1 = 0$; $\quad \therefore\ s^2 = -1$ or $s = \pm\sqrt{-1} = \pm j1$

The roots of auxiliary polynomial are +j1and –j1, lying on imaginary axis. The roots of auxiliary polynomial are also roots of characteristic equation. Hence two roots of characteristic equation are lying on imaginary axis and so the system is limitedly or marginally stable. The remaining three roots of characteristic equation are lying on the left half of s-plane.

RESULT

(a) The system is limitedly or marginally stable.

(b) Two roots are lying on imaginary axis and three roots are lying on left half of s-plane.

3.3 Root Locus

The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Rules for Construction of Root Locus

Follow these rules for constructing a root locus.

Rule 1 − Locate the open loop poles and zeros in the ‘s’ plane.

Rule 2 − Find the number of root locus branches. We know that the root locus branches start at the open loop poles and end at open loop zeros. So, the number of root locus branches N is equal to the number of finite open loop poles P or the number of finite open loop zeros Z, whichever is greater.

Mathematically, we can write the number of root locus branches N as

$N=P$ if $P \geq Z$

$N=Z$ if PZ,

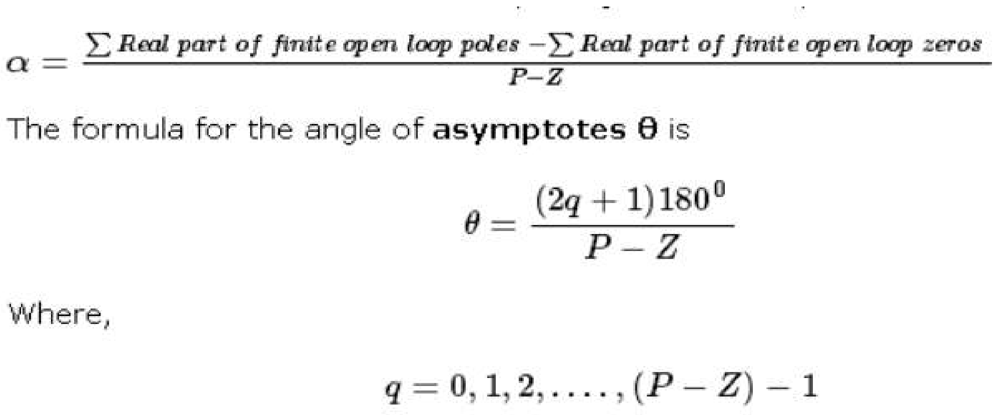
Rule 3 − Identify and draw the real axis root locus branches. If the angle of the open loop transfer function at a point is an odd multiple of 180^0, then that point is on the root locus. If odd number of the open loop poles and zeros exist to the left side of a point on the real axis, then that point is on the root locus branch. Therefore, the branch of points which satisfies this condition is the real axis of the root locus branch.

Rule 4 − Find the centroid and the angle of asymptotes.

- If $P=Z$, then all the root locus branches start at finite open loop poles and end at finite open loop zeros.
- If $P>Z$, then Z number of root locus branches start at finite open loop poles and end at finite open loop zeros and $P-Z$ number of root locus branches start at finite open loop poles and end at infinite open loop zeros.
- If $P<Z$, then P number of root locus branches start at finite open loop poles and end at finite open loop zeros and $Z-P$ number of root locus branches start at infinite open loop poles and end at finite open loop zeros.

So, some of the root locus branches approach infinity, when $P \neq Z$. Asymptotes give the direction of these root locus branches. The intersection point of asymptotes on the real axis is known as centroid.

We can calculate the centroid α by using this formula,



$$\alpha = \frac{\sum Real\ part\ of\ finite\ open\ loop\ poles - \sum Real\ part\ of\ finite\ open\ loop\ zeros}{P-Z}$$

The formula for the angle of **asymptotes θ** is

$$\theta = \frac{(2q+1)180^0}{P-Z}$$

Where,

$$q = 0, 1, 2, \ldots, (P-Z)-1$$

Rule 5 − Find the intersection points of root locus branches with an imaginary axis. We can calculate the point at which the root locus branch intersects the imaginary axis and the value of K at that point by using the Routh array method and special case (ii).

- If all elements of any row of the Routh array are zero, then the root locus branch intersects the imaginary axis and vice-versa.
- Identify the row in such a way that if we make the first element as zero, then the elements of the entire row are zero. Find the value of K for this combination.
- Substitute this K value in the auxiliary equation. You will get the intersection point of the root locus branch with an imaginary axis. Rule 6 − Find Break-away and Break-in points.
- If there exists a real axis root locus branch between two open loop poles, then there will be a break-away point in between these two open loop poles.
- If there exists a real axis root locus branch between two open loop zeros, then there will be a break-in point in between these two open loop zeros.

  Note − Break-away and break-in points exist only on the real axis root locus branches. Follow these steps to find break-away and break-in points.
- Write K in terms of s from the characteristic equation 1+G(s)H(s)=0.
- Differentiate K with respect to s and make it equal to zero. Substitute these values of ss in the above equation.
- The values of ss for which the K value is positive are the break points. Rule 7 − Find

the angle of departure and the angle of arrival. The Angle of departure and the angle of arrival can be calculated at complex conjugate open loop poles and complex conjugate open loop zeros respectively. The formula for the angle of departure ϕd is

$$\phi_d = 180^0 - \phi$$

The formula for the **angle of arrival** ϕ_a is

$$\phi_a = 180^0 + \phi$$

Where,

$$\phi = \sum \phi_P - \sum \phi_Z$$

A unity feedback control system has an open loop transfer function, $G(s) = \frac{K}{s\,(s^2+4s+13)}$. Sketch the root locus.

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s\,(s^2+4s+13)=0$.

The roots of the quadratic are, $s = \frac{-4 \pm \sqrt{4^2 - 4 \times 13}}{2} = -2 \pm j3$

$\therefore$ The poles are lying at $s = 0, -2+j3$ and $-2-j3$.

Let us denote the poles as P_1, P_2, and P_3.

Here, $P_1 = 0$, $P_2 = -2+j3$ and $P_3 = -2-j3$.

The poles are marked by X (cross) as shown in fig 4.22.1.

Step 2 : To find the root locus on real axis

There is only one pole on real axis at the origin. Hence if we choose any test point on the negative real axis then to the right of that point the total number of real poles and zeros is one, which is an odd number. Hence the entire negative real axis will be part of root locus. The root locus on real axis is shown as a bold line in fig 4.22.1.

Note : *For the given transfer function one root locus branch will start at the pole at the origin and meet the zero at infinity through the negative real axis.*

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. There is no finite zero. Hence all the three root locus branches ends at zeros at infinity. The number of asymptotes required are three.

$$\text{Angles of asymptotes} = \frac{\pm 180^{\circ}\,(2q+1)}{n-m} \quad ; \quad q = 0, 1, \ldots\ldots\ldots\ldots n-m$$

Here $n = 3$, and $m = 0$. $\quad \therefore q = 0, 1, 2, 3$.

When $q = 0$, $\quad \text{Angles} = \pm \frac{180^{\circ}}{3} = \pm 60^{\circ}$

When $q = 1$, $\quad \text{Angles} = \pm \frac{180^{\circ} \times 3}{3} = \pm 180^{\circ}$

When $q = 2$, $\quad \text{Angles} = \pm \frac{180^{\circ} \times 5}{3} = \pm 300^{\circ} = \mp 60^{\circ}$

When $q = 3$, $\quad \text{Angles} = \pm \frac{180^{\circ} \times 7}{3} = \pm 420^{\circ} = \pm 60^{\circ}$

Note : *It is enough if you calculate the required number of angles. Here it is given by first three values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0 - 2 + j3 - 2 - j3 - 0}{3} = \frac{-4}{3} = -1.33$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.22.1.

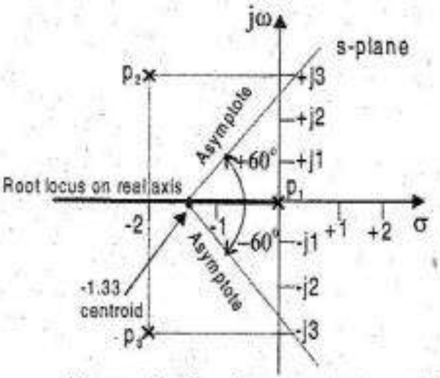


Figure showing the asymptote, root locus on real axis and location of poles and centroid

Step 4 : To find the breakaway and breakin points

The closed loop transfer function
$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{\dfrac{K}{s\,(s^2+4s+13)}}{1+\dfrac{K}{s\,(s^2+4s+13)}}=\frac{K}{s\,(s^2+4s+13)+K}$$

The characteristic equation is, $s(s^2+4s+13)+K=0$

$$\therefore\ s^3+4s^2+13s+K=0 \quad\Rightarrow\quad K=-s^3-4s^2-13s$$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds}=-(3s^2+8s+13)$$

Put $\dfrac{dK}{ds}=0$

$$\therefore\ -(3s^2+8s+13)=0 \qquad\Rightarrow\qquad (3s^2+8s+13)=0$$

$$\therefore s=\frac{-8\pm\sqrt{8^2-4\times13\times3}}{2\times3}=-1.33\pm j1.6$$

Check for K : When, $s=-1.33+j1.6$, the value of K is given by,

$$K=-(s^3+4s^2+13s)=-[(-1.33+j1.6)^3+4\,(-1.33+j1.6)^2+13\,(-1.33+j1.6)]$$

$\neq$ positive and real.

Also it can be shown that when $s=-1.33-j1.6$ the value of K is not equal to real and positive.

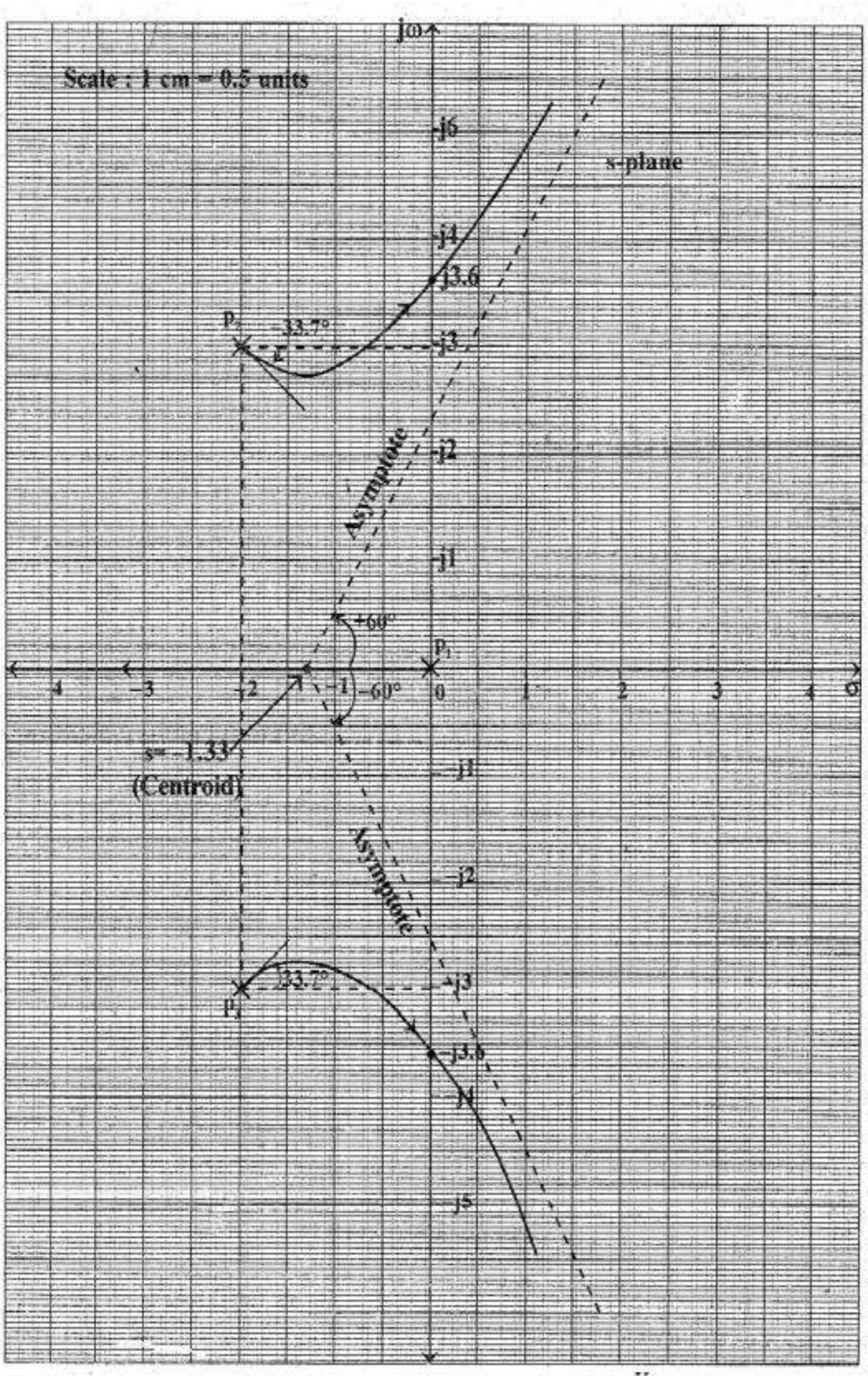
Since the values of K for, $s=-1.33\pm j1.6$, are not real and positive, these points are not an actual breakaway or breakin points. The root locus has neither breakaway nor breakin point.

Step 5 : To find the angle of departure

Let us consider the complex pole p_2 shown in fig 4.22.2. Draw vectors from all other poles to the pole p_2 as shown in fig Let the angles of these vectors be θ_1 and θ_2.

Here, $\theta_1=180°-\tan^{-1}(3/2)=123.7°$; $\theta_2=90°$

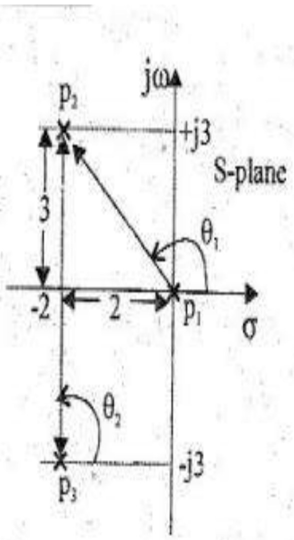
Angle of departure from the complex pole $p_2=180°-(\theta_1+\theta_2)$
$$=180°-(123.7°+90°)$$
$$=-33.7°$$


jω
Scale : 1 cm = 0.5 units
j6
s-plane
j4
j3.6
P2
-33.7°
j3
Asymptote
j2
j1
+60°
P1
-4
-3
-2
-1
-60°
0
1
2
3
4
σ
s= -1.33
(Centroid)
-j1
-j2
Asymptote
-j3
33.7°
P3
-j3.6
j4
-j5

The angle of departure at complex pole p_3 is negative of the angle of departure at complex pole A.

$\therefore$ Angle of departure at pole $p_3 = +33.7°$

Mark the angles of departure at complex poles using protractor.



Step 6 : To find the crossing point on imaginary axis

The characteristic equation is given by,

$$s^3 + 4s^2 + 13s + K = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0 \quad \Rightarrow -j\omega^3 - 4\omega^2 + 13j\omega + K = 0$$

On equating imaginary part to zero, we get,

$$-\omega^3 + 13\omega = 0$$

$$-\omega^3 = -13\omega$$

$$\omega^2 = 13 \Rightarrow \quad \omega = \pm\sqrt{13} = \pm 3.6$$

On equating real part to zero, we get,

$$-4\omega^2 + K = 0$$

$$K = 4\omega^2$$

$$= 4 \times 13 = 52$$

The crossing point of root locus is ± j3.6. The value of K at this crossing point is K = 52. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig . The roct locus has three branches one branch starts at the pole at origin and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at complex poles (along the angle of departure), crosses the imaginary axis at ± j3.6 and travel parallel to asymptotes to meet the zeros at infinity.

The open loop transfer function of a unity feedback system is given by, $G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$. Sketch the root locus of the system.

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equations, $(s^2 + 4s + 11) = 0$.

The roots of the quadratic are, $s = \frac{-4 \pm \sqrt{4^2 - 4 \times 11}}{2} = -2 \pm j2.64$

∴ The poles are lying at, $s = 0, -2 + j2.64, -2 - j2.64$

The zeros are lying at, $s = -9$ and infinity.

Let us denote the poles as p_1, p_2, p_3 finite zero by z_1.

Here, $p_1 = 0$, $p_2 = -2+j2.64$, $p_3 = -2-j2.64$ and $z_1 = -9$.

The poles are marked by X(cross) and zeros by "o" (circle) as shown in fig 4.24.1.

Step 2 : To find the root locus on real axis.

One pole and one zero lie on real axis.

Choose a test point to the left of s = 0, then to the right of this point, the total number of poles and zeros is one which is an odd number. Hence the portion of real axis from s = 0 to s = –9 will be a part of root locus.

If we choose a test point to the left of s = –9 then to the right of this point, the total number of poles and zeros is two, which is an even number. Hence the real axis from s = –9 to –∞ will not be a part of root locus.

The root locus on real axis is shown as a bold line in fig 4.24.1.

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. One root locus branch starts at the pole at origin and travel along negative real axis to meet the zero at s = –9. The other two root locus branches meet the zeros at infinity. The number of asymptotes required are two.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ\ (2q+1)}{n-m}; \qquad q = 0, 1, 2, \ldots\ldots n-m.$$

Here, n = 3 and m = 0. ∴ q = 0, 1, 2, 3 .

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180^\circ}{3} = \pm 60^\circ$$

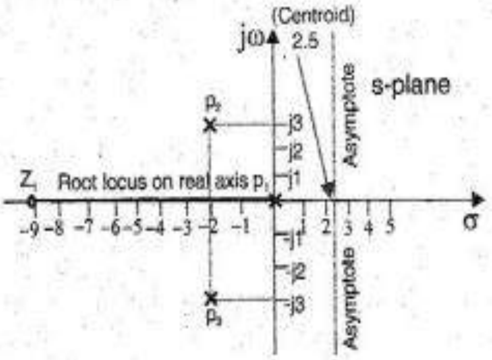
$$\text{When } q = 1, \quad \text{Angles} = \pm\frac{180^\circ \times 3}{2} = \pm 270^\circ = \mp 90^\circ$$

$$\text{When } q = 2, \quad \text{Angles} = \pm\frac{180^\circ \times 5}{2} = \pm 450^\circ = \pm 90^\circ$$

***Note** : It is enough if you calculate the required number of angles. Here it is given by first two values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0 - 2 + j2.64 - 2 - j2.64 - (-9)}{2} = 2.5$$

The centroid is marked and from the centroid, the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown 4.24.1.

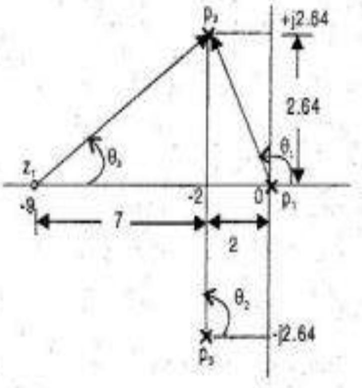


Step 4 : To find the breakaway and breakin points

From the location of poles and zero and from the knowledge of typical sketches of root locus, it can be concluded that there is no possibility of breakaway or breakin points.

Step 5 : To find the angle of departure

Let us consider the complex pole p_2 as shown in fig 4.24.2. Draw vectors from all other poles and zero to the pole p_2 as shown in fig 4.24.2. Let the angles of these vectors be θ_1, θ_2 and θ_3.



$$\text{Here, } \theta_1 = 180° - \tan^{-1}\frac{2.64}{2} = 127.1°$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{2.64}{7} = 20.7°$$

$$\left.\begin{matrix}\text{Angle of departure from}\\ \text{the complex pole } p_2\end{matrix}\right\} = 180° - (\theta_1 + \theta_2) + \theta_3$$

$$= 180° - (127.1° + 90°) + 20.7° = -16.4°$$

The angle of departure at the complex pole p_3 is negative of the angle of departure at complex pole p_3.

$$\therefore \text{Angle of departure at pole } p_3 = -(-16.4) = +16.4°$$

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point of imaginary axis

$$\left.\begin{matrix}\text{The closed loop}\\ \text{transfer function}\end{matrix}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K(s+9)}{s\,(s^2+4s+11)}}{1+\dfrac{K\,(s+9)}{s\,(s^2+4s+11)}} = \frac{K(s+9)}{s\,(s^2+4s+11)+K(s+9)}$$

The characteristic equation is the denominator polynomial of C(s)/R(s).

∴ The characteristic equation is,

$$s\,(s^2+4s+11) + K\,(s+9) = 0 \;\Rightarrow\; (s^3+4s^2+11s) + Ks + 9K = 0$$

put $s = j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 11(j\omega) + K(j\omega) + 9K = 0 \quad\Rightarrow\quad -j\omega^3 - 4\omega^2 + j11\omega + jK\omega + 9K = 0$$

On equating imaginary part to zero,

$$-j\omega^3 + j11\omega + jK\omega = 0 \;\Rightarrow\; -j\omega^3 = -j11\omega - jK\omega$$

$$\therefore\; \omega^2 = 11 + K$$

$$\text{Put } K = 8.8, \;\therefore \omega^2 = 11 + 8.8 = 19.8$$

$$\omega = \pm\sqrt{19.8} = \pm 4.4$$

On equating real part to zero,

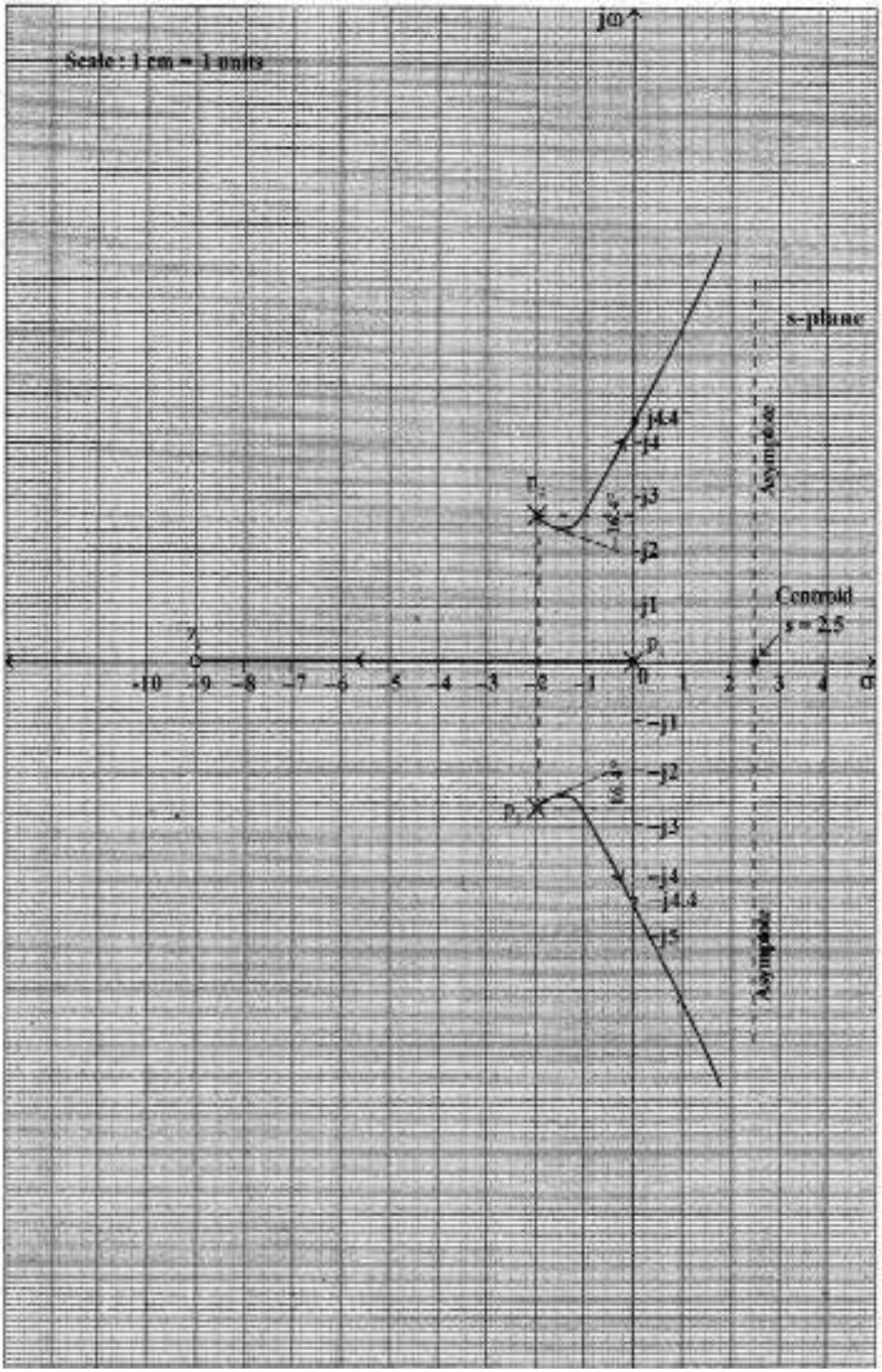
$$-4\omega^2 + 9K = 0 \;\Rightarrow\; 9K = 4\omega^2$$

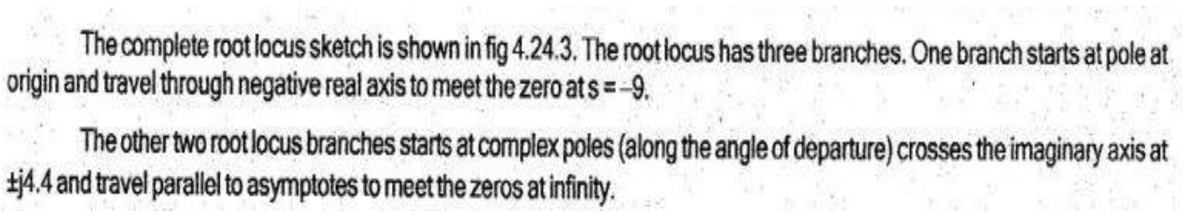
$$\text{Put, } \omega^2 = 11 + K \;\; \therefore 9K = 4(11+K) = 44 + 4K$$

$$\therefore 9K - 4K = 44$$

$$\therefore 5K = 44 \;\Rightarrow\; K = \frac{44}{5} = 8.8$$

The crossing point of root locus is ±j4.4. The value of K at this crossing point is K = 8.8 (This is the limiting value of K for the stability of the system).

Scale : 1 cm = 1 units
jω
s-plane
j4.4
j4
j3
j2
j1
Centroid
s = 2.5
Asymptote
p₂
p₁
p₃
z
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
σ
-j1
-j2
-j3
-j4
-j4.4
-j5
Asymptote



The complete root locus sketch is shown in fig 4.24.3. The root locus has three branches. One branch starts at pole at origin and travel through negative real axis to meet the zero at s = −9.

The other two root locus branches starts at complex poles (along the angle of departure) crosses the imaginary axis at ±j4.4 and travel parallel to asymptotes to meet the zeros at infinity.

Stability Analysis using Bode Plots

From the Bode plots, we can say whether the control system is stable, marginally stable or unstable based on the values of these parameters.

- Gain cross over frequency and phase cross over frequency
- Gain margin and phase margin

Phase Cross over Frequency

The frequency at which the phase plot is having the phase of -180^0 is known as phase cross over frequency. It is denoted by ωpc. The unit of phase cross over frequency is rad/sec.

Gain Cross over Frequency

The frequency at which the magnitude plot is having the magnitude of zero dB is known as gain cross over frequency. It is denoted by ωgc. The unit of gain cross over frequency is rad/sec.

The stability of the control system based on the relation between the phase cross over frequency and the gain cross over frequency is listed below.

- If the phase cross over frequency ωpc is greater than the gain cross over frequency ωgc, then the control system is stable.
- If the phase cross over frequency ωpc is equal to the gain cross over frequency ωgc, then the control system is marginally stable.
- If the phase cross over frequency ωpcis less than the gain crosses over frequency ωgc, then the control system is unstable.

Gain Margin

GM is equal to negative of the magnitude in dB at phase cross over frequency.

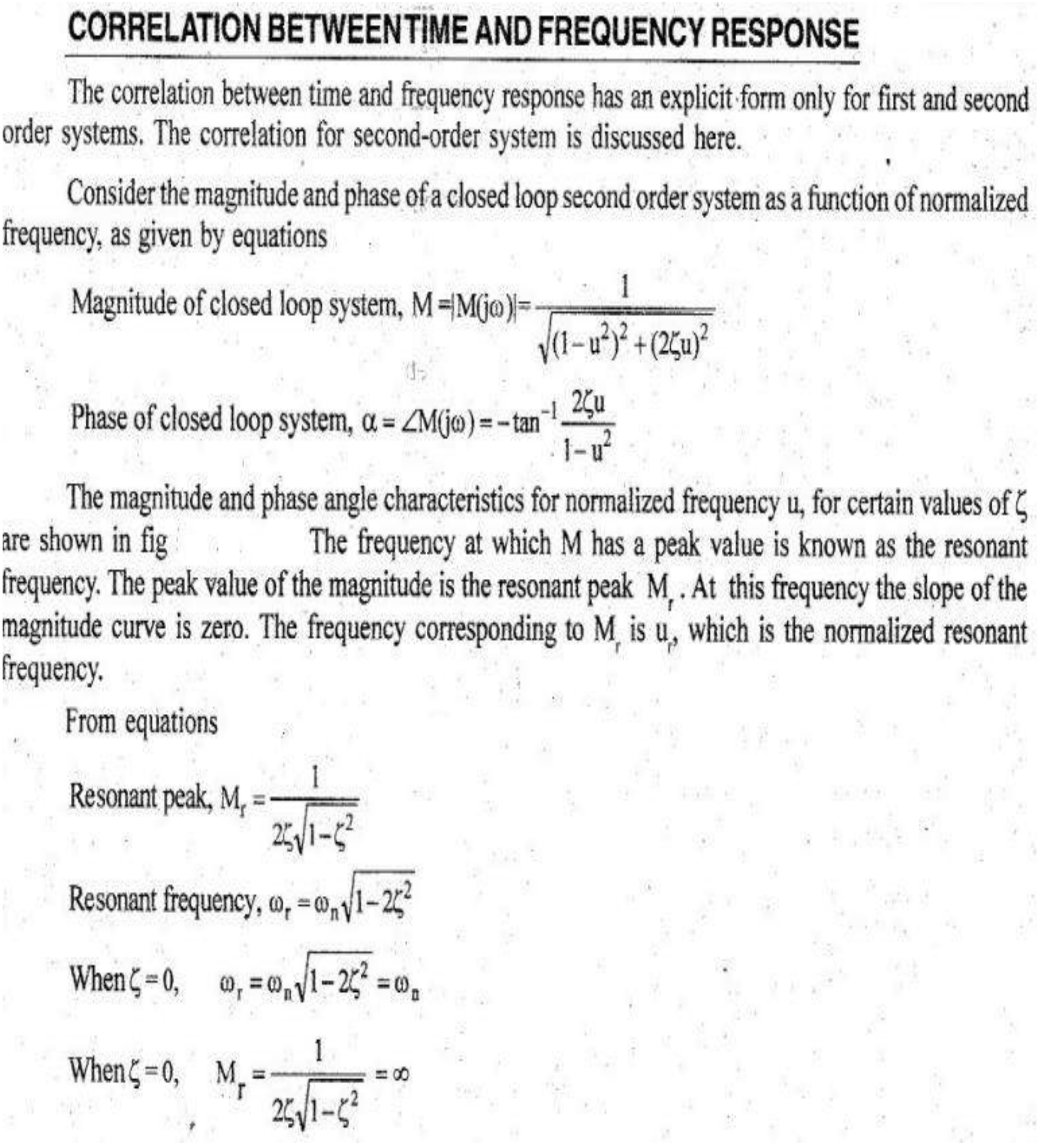
$$GM=20\log|G(j\omega)|\ \omega=\omega pc$$

The unit of gain margin (GM) is dB.

Phase Margin

The formula for phase margin, PM=1800+ϕgc

Where, ϕgc is the phase angle at gain cross over frequency. The unit of phase margin is degrees.



CORRELATION BETWEENTIME AND FREQUENCY RESPONSE

The correlation between time and frequency response has an explicit form only for first and second order systems. The correlation for second-order system is discussed here.

Consider the magnitude and phase of a closed loop second order system as a function of normalized frequency, as given by equations

Magnitude of closed loop system, $M = |M(j\omega)| = \dfrac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$

Phase of closed loop system, $\alpha = \angle M(j\omega) = -\tan^{-1}\dfrac{2\zeta u}{1-u^2}$

The magnitude and phase angle characteristics for normalized frequency u, for certain values of ζ are shown in fig The frequency at which M has a peak value is known as the resonant frequency. The peak value of the magnitude is the resonant peak M_r. At this frequency the slope of the magnitude curve is zero. The frequency corresponding to M_r is u_r, which is the normalized resonant frequency.

From equations

Resonant peak, $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

When $\zeta = 0$, $\omega_r = \omega_n\sqrt{1-2\zeta^2} = \omega_n$

When $\zeta = 0$, $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = \infty$

Sketch the bode plot for the following transfer function and determine phase margin and gain margin.

$$G(s) = \frac{75\,(1+0.2s)}{s\left(s^2+16s+100\right)}$$

SOLUTION

On comparing the quadratic factor in the denominator of G(s) with standard form of quadratic factor we can estimate ζ and ω_n.

$$\therefore s^2+16s+100 \;=\; s^2+2\zeta\omega_n s+\omega_n^2$$

On comparing we get,

$$\omega_n^2 = 100 \quad\Rightarrow\quad \omega_n = 10$$

$$2\zeta\omega_n = 16 \quad\Rightarrow\quad \zeta = \frac{16}{2\omega_n} = \frac{16}{2\times 10} = 0.8$$

Let us convert the given s-domain transfer function into bode form or time constant form.

$$\therefore G(s) = \frac{75(1+0.2s)}{s\left(s^2+16s+100\right)} = \frac{75\,(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)} = \frac{0.75\,(1+0.2s)}{s\left(1+0.01s^2+0.16s\right)}$$

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in G(s).

$$\therefore G(j\omega) = \frac{0.75\,(1+0.2\,j\omega)}{j\omega\left(1+0.01(j\omega)^2+0.16j\omega\right)} = \frac{0.75\,(1+j0.2\omega)}{j\omega\left(1-0.01\omega^2+j0.16\omega\right)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{0.2} = 5$ rad / sec and $\omega_{c2} = \omega_n = 10$ rad / sec

Note : For the quadratic factor the corner frequency is ω_n.

The various terms of G(jω) are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	–	-20	
$1+j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	20	$-20+20=0$
$\frac{1}{1-0.01\omega^2+j0.16\omega}$	$\omega_{c2} = \omega_n = 10$	- 40	$0-40=-40$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec and $\omega_h = 20$ rad/sec.

Let, $A = |G(j\omega)|$ in db .

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At, } \quad \omega = \omega_l, \; A = 20 \log \left| \frac{0.75}{j\omega} \right| = 20 \log \frac{0.75}{0.5} = 3.5 \text{ db}$$

$$\text{At, } \quad \omega = \omega_{c1}, \; A = 20 \log \left| \frac{0.75}{j\omega} \right| = 20 \log \frac{0.75}{5} = -16.5 \text{ db}$$

$$\text{At, } \quad \omega = \omega_{c2}, \; A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{at } \omega = \omega_{c1})}$$

$$= 0 \times \log \frac{10}{5} + (-16.5) = -16.5 \text{ db}$$

$$\text{At } \omega = \omega_h, \; A' = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{at } \omega = \omega_{c2})}$$

$$= -40 \times \log \frac{20}{10} + (-16.5) = -28.5 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 5 db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a,b,c and d on the graph. Join the points by straight lines and mark the slope on the respective region.

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90^\circ - \tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} \quad \text{for } \omega \leq \omega_n$$

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90^\circ - \left(\tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} + 180^\circ \right) \text{ for } \omega > \omega_n$$

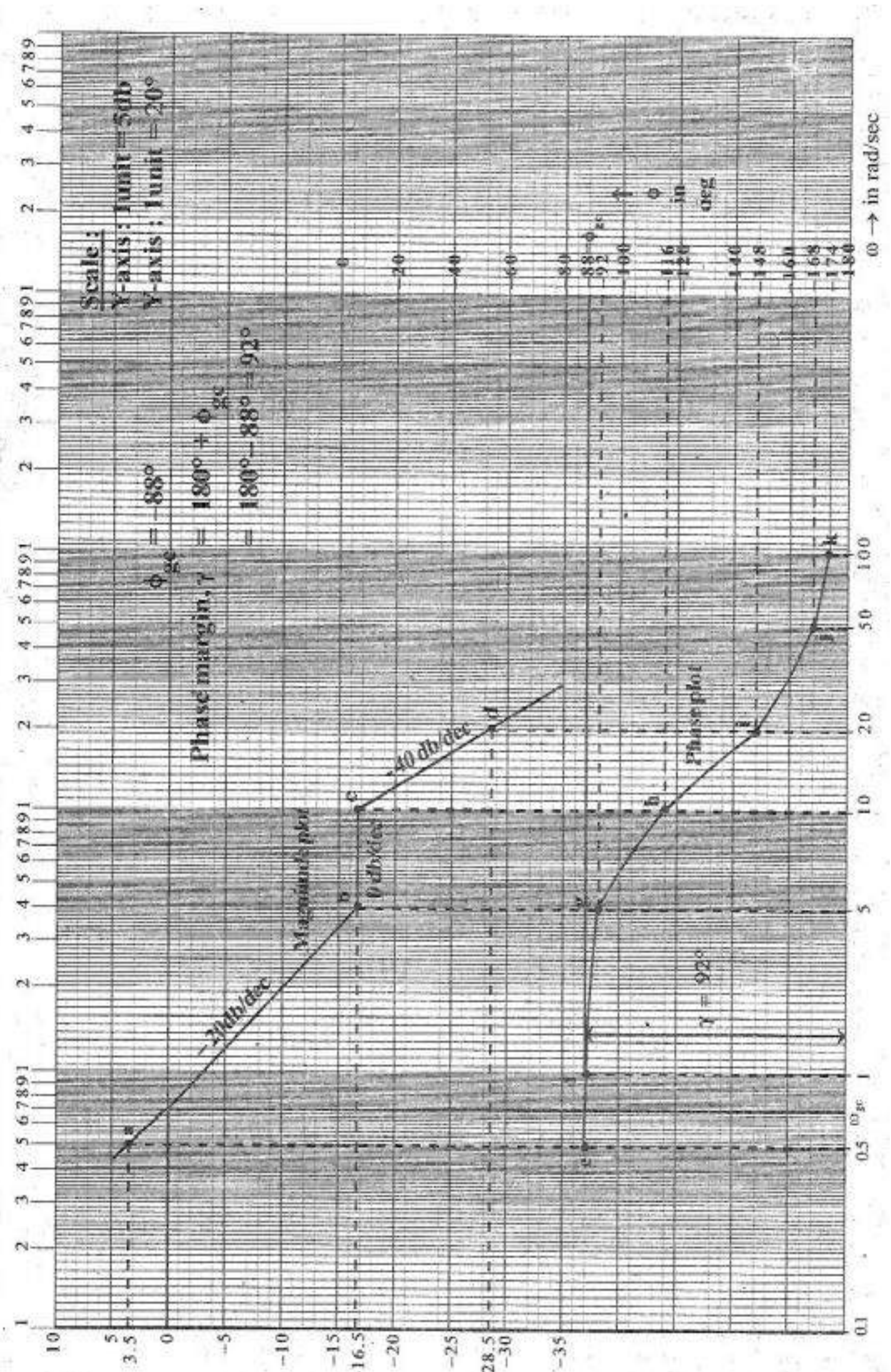
The phase angle of $G(j\omega)$ are calculated for various values of w and listed in Table-2.

Note : In quadratic factors the phase varies from 0° to 180°. But calculator calculates $\tan^{-1}$ only between 0° to 90°. Hence a correction of 180° should be added to phase after ω_n

TABLE-2

ω rad/sec	$\tan^{-1} 0.2\,\omega$ deg	$\tan^{-1} \frac{0.16\omega}{1-0.01\omega^2}$ deg	$\phi = \angle G(j\omega)$ deg	Points in phase plot
0.5	5.7	4.6	$-88.9 \approx -88$	e
1	11.3	9.2	$-87.9 \approx -88$	f
5	45	46.8	$-91.8 \approx -92$	g
10	63.4	90	$-116.6 \approx -116$	h
20	75.9	$-46.8+180 = 133.2$	$-147.3 \approx -148$	i
50	84.3	$-18.4+180 = 161.6$	$-167.3 \approx -168$	j
100	87.1	$-92+180 = 170.8$	$-173.7 \approx -174$	k

On the same semilog graph sheet choose a scale of 1unit = 20^0 on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.


Scale :
Y-axis : 1unit = 5db
Y-axis : 1unit = 20°
φgc = −88°
Phase margin, γ = 180° + φgc
= 180° − 88° = 92°
−20db/dec
0 db/dec
−40 db/dec
Magnitude plot
Phase plot
γ = 92°
a
b
c
d
e
f
g
h
i
j
k
ωgc
ω → in rad/sec
φ in deg
10
5
3.5
0
−5
−10
−15
−16.5
−20
−25
−28.5
−30
−35
0
20
40
60
80
88−φgc
92
100
116
120
140
148
160
168
174
180
0.1
0.5
1
5
10
20
50
100
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9

Let ϕ_{gc} be the phase of $G(j\omega)$ at gain cross-over frequency, ω_{gc}.

From the fig 3.2.1, we get, $\phi_{gc} = 88°$

$$\therefore \text{ Phase margin, } g = 180° + \phi_{gc} = 180° - 88° = 92°$$

The phase plot crosses –180° only at infinity. The $|G(j\omega)|$ at infinity is $-\infty$ db.

Hence gain margin is $+\infty$.

Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies.

$$G(s) = \frac{10}{s\,(1+0.4s)\,(1+0.1s)}$$

SOLUTION

The sinusoidal transfer function of $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given transfer function.

$$\therefore G(j\omega) = \frac{10}{j\omega\,(1+j0.4\omega)\,(1+j0.1\omega)}$$

MAGNITUDE PLOT

The corner frequencies are,

$$\omega_{c1} = \frac{1}{0.4} = 2.5 \text{ rad / sec and } \omega_{c2} = \frac{1}{0.1} = 10 \text{ rad / sec}$$

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{10}{j\omega}$	–	– 20	
$\frac{1}{1+j0.4\omega}$	$\omega_{c1} = \frac{1}{0.4} = 2.5$	– 20	$-20 - 20 = -40$
$\frac{1}{1+j0.1\omega}$	$\omega_{c2} = \frac{1}{0.1} = 10$	– 20	$-40 - 20 = -60$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, ω_l = 0.1 rad/sec, and ω_h = 50 rad/sec.

Let, $A = |G(j\omega)|$ in db.

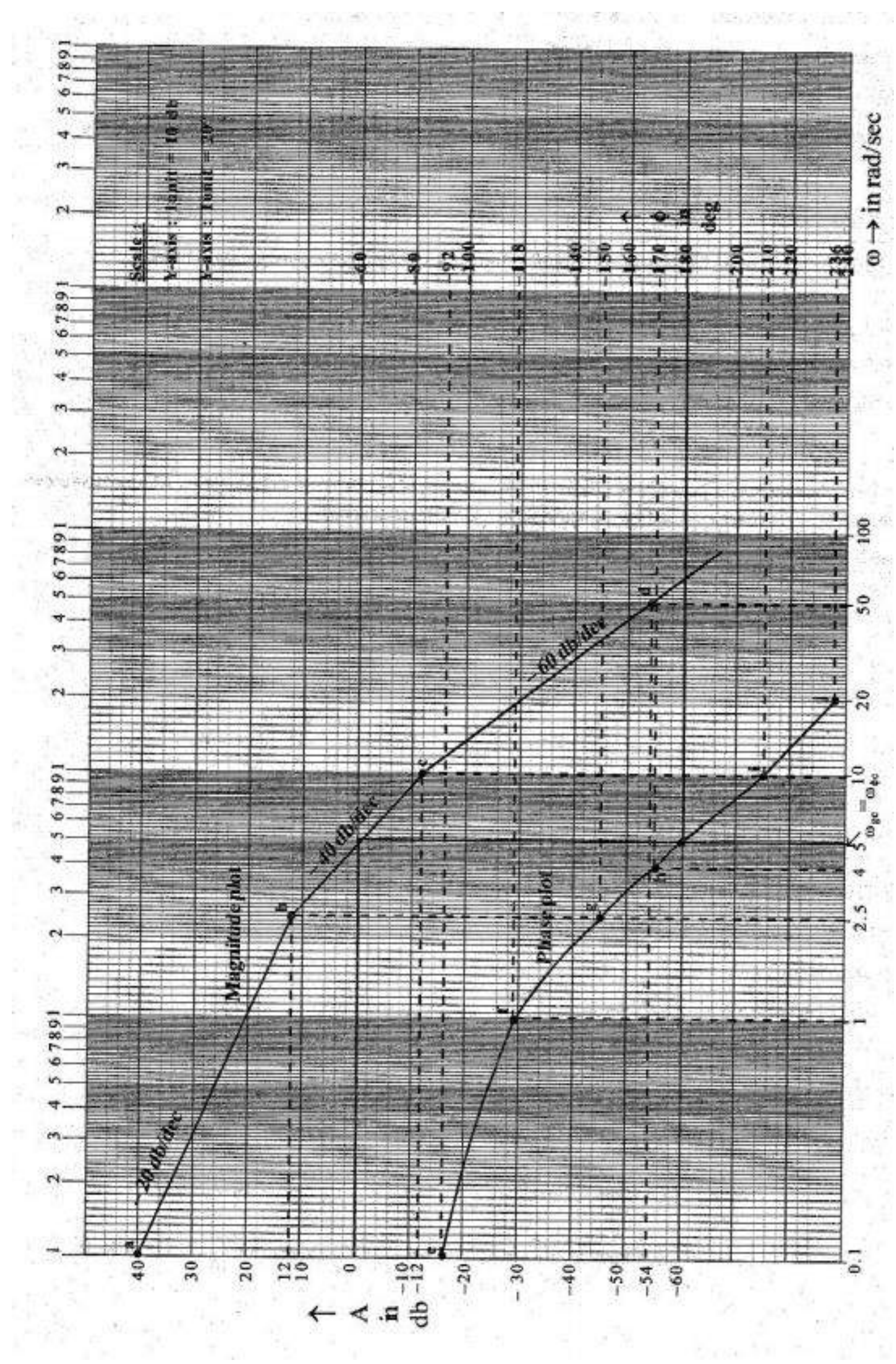
Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

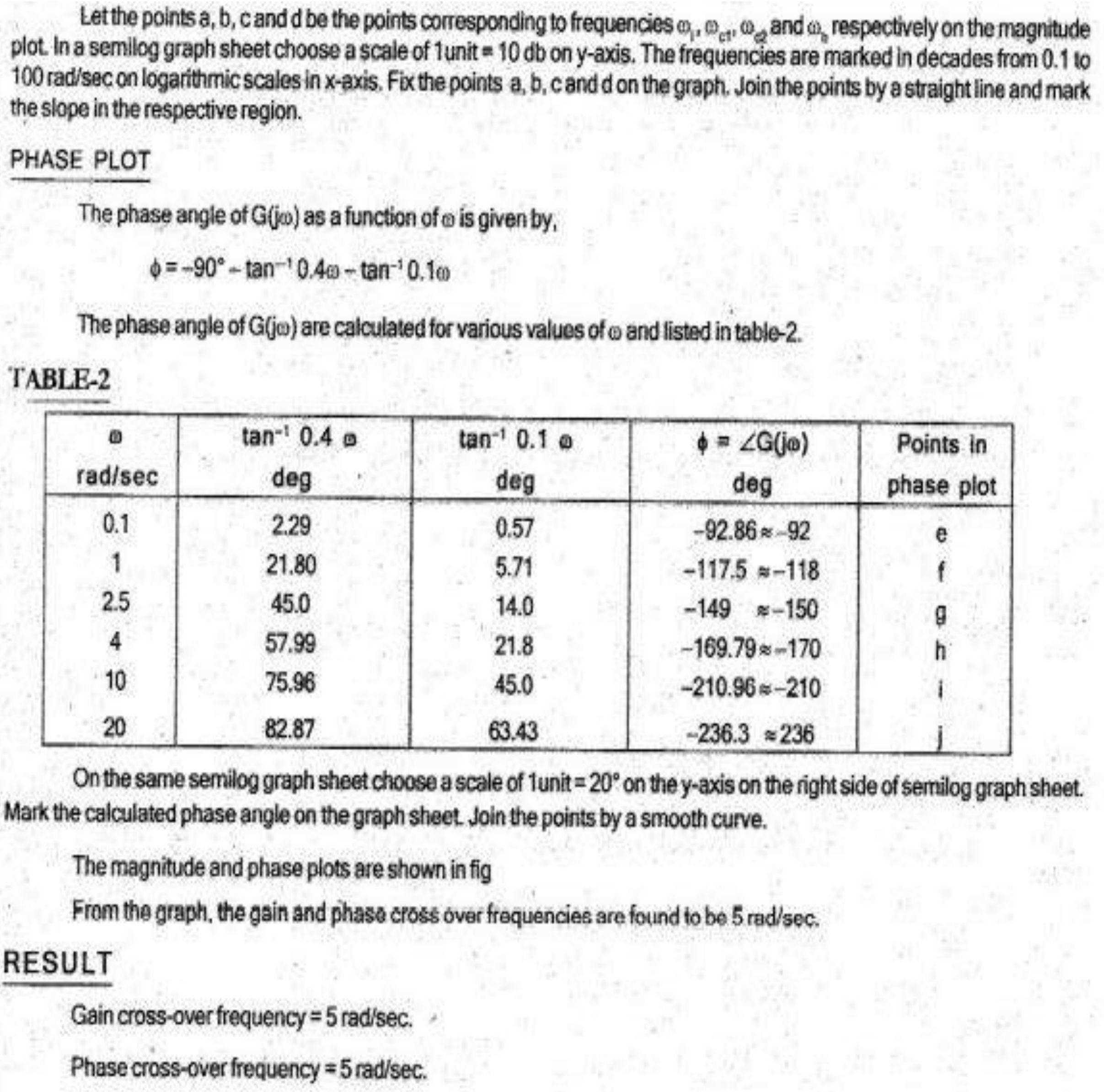
$$\text{At } \omega = \omega_l,\ A = 20 \log \left|\frac{10}{j\omega}\right| = 20 \log \frac{10}{0.1} = 40 \text{ db}$$

$$\text{At } \omega = \omega_{c1},\ A = 20 \log \left|\frac{10}{j\omega}\right| = 20 \log \frac{10}{2.5} = 12 \text{ db}$$

$$\text{At } \omega = \omega_{c2},\ A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})} = -40 \times \log \frac{10}{2.5} + 12 = -12 \text{ db}$$

$$\text{At } \omega = \omega_h,\ A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega = \omega_{c2})} = -60 \times \log \frac{50}{10} + (-12) = -54 \text{ db}$$


Scale :
Y-axis : 1unit = 10 db
Y-axis : 1unit = 20°
Magnitude plot
Phase plot
–20 db/dec
–40 db/dec
–60 db/dec
a
b
c
d
e
f
g
h
i
j
A in db ↑
φ in deg ↑
ω → in rad/sec
$\omega_{gc} = \omega_{pc}$
0.1
1
2.5
4
5
10
20
50
100
40
30
20
12
10
0
–10
–12
–20
– 30
–40
–50
–54
–60
–60
–80
–92
–100
–118
–140
–150
–160
–170
–180
–200
–210
–220
–236
–240
1 2 3 4 5 6 7 8 9 1



Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10 db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by a straight line and mark the slope in the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by,

$$\phi = -90° - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

The phase angle of G(jω) are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$\tan^{-1}$ 0.4 ω deg	$\tan^{-1}$ 0.1 ω deg	φ = ∠G(jω) deg	Points in phase plot
0.1	2.29	0.57	−92.86 ≈ −92	e
1	21.80	5.71	−117.5 ≈ −118	f
2.5	45.0	14.0	−149 ≈ −150	g
4	57.99	21.8	−169.79 ≈ −170	h
10	75.96	45.0	−210.96 ≈ −210	i
20	82.87	63.43	−236.3 ≈ 236	j

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

The magnitude and phase plots are shown in fig

From the graph, the gain and phase cross over frequencies are found to be 5 rad/sec.

RESULT

Gain cross-over frequency = 5 rad/sec.

Phase cross-over frequency = 5 rad/sec.

Question Bank

Part – A

1. What is the necessary and sufficient condition for stability?
2. Define ‘Bounded Input and Bounded Output’ stability.
3. What will be the nature of impulse response when the roots of the characteristic’s equation are lying on the imaginary axis?
4. What are asymptotes? How will you find the angle of asymptotes?
5. Give some typical sketches of root locus plots.
6. How to determine the angle of departure and angle of arrival?

7. What is break in and breakaway point?
8. What are the two special cases in applying routh Hurwitz criterion?
9. The characteristics equation of a system, S^2–5S+2 = 0, Whether it is stable, Unstable or limitedly stable.
10. What is characteristics equation?
11. How the roots of characteristics equation are related to stability?
12. What is the necessary condition for stability?
13. Define 'Bounded Input and Bounded Output' stability.
14. What is the requirement for BIBO stability?
15. What is impulse response?
16. What will be the nature of impulse response when the roots of the characteristic's equation are lying on the imaginary axis?
17. What will be the nature of impulse response if the roots of the characteristic's equation are lying on the right half of S-plane?
18. What is the necessary and sufficient condition for stability?
19. Explain the use of Routh's Criteria?
20. What are asymptotes? How will you find the angle of asymptotes?
21. What is break in and breakaway point?
22. What are frequency domain specification?
23. Define corner frequency.
24. What are the advantages of Bode plot?
25. Define gain margin and phase margin.
26. What are the advantages of frequency response analysis?
27. What is phase and gain cross over frequency.
28. What is frequency response?
29. What are the advantages of frequency response analysis?
30. What are frequency domain specification?
31. Define resonant peak, resonant frequency, Band width and cut off rate.
32. What is phase and gain cross over frequency.
33. Give the expression for resonant peak and resonant frequency.
34. What are the various graphical techniques available for frequency response analysis?
35. What is Bode plot?
36. Define corner frequency.
37. What are the advantages of Bode plot?

38. What is approximate Bode plot?
39. Sketch a bode plot showing positive gain margin and positive phase margin.
40. Sketch a bode plot showing negative gain margin and negative phase margin.

Part B

1. State and explain the rules for sketching the root locus for the system.
2. a. What should be the value of K whose characteristic equation is given by

   $s^3+3Ks^2+(k+2)+4 = 0$ to be stable?

   b. Write short notes on BIBO stability.

3. Use Routh's criteria and determine the stability of the following system

   Whose characteristics equation are i) $(s+1)(s+2)(s+10)+100=0$

   ii) $(s+4)(s+6)+12s=0$

4. Using root locus method find C(s)/R(s) for the system whose
   $G(S) = k(s+3)/s^2+2s+2$ and $H(s) = 1/s$ for the damping factor $\delta = 0.5$.
5. Plot the root loci for $G(s)H(s) = K/s(s+2)(s+4)$ And evaluate the value of K at the point when the root loci crosses the imaginary axis.
6. Sketch the root of the system whose characteristic equation is $s^2+s+10k(s+1) = 0$. After obtaining all the information to make the sketch also comment about the stability of the system.
7. Explain in detail the correlation between time and frequency response for a second order system.
8. Plot the bode diagram for the following transfer function and obtain the gain and phase crossover frequency $G(S) = 10/ S(0.5S+1)(1+0.1S)$
9. Sketch the bode plot for the given transfer function and determine the gain

   margin and phase margin over frequency. $G(S) = 1 / s(1+0.5s)(1+0.1s)$

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SCHOOL OF MECHANICAL

DEPARTMENT OF MECHATRONICS

UNIT – IV Signals and Control Systems – SMR1402

UNIT IV - CLASSIFICATION OF SIGNALS

Continuous time signals (CT signals) and Discrete time signals (DT signals) - Basic operations on signals elementary signals- Step, Ramp, Pulse, Impulse, Exponential - Classification of CT and DT signals - Periodic, aperiodic signals-Deterministic and Random signals-even and odd signals - Real and Complex signals - Energy and power signals.

Signal: Signals are represented mathematically as functions of one or more independent variables. It mainly focuses attention on signals involving a single independent variable. For convenience, this will generally refer to the independent variable as time**.** It is defined as physical quantities that carry information and changes with respect to time.

Ex: voice, television picture, telegraph.

Continuous Time signal – If the signal is defined over continuous-time, then the signal is a continuous- time signal.

Ex: Sinusoidal signal, Voice signal, Rectangular pulse function

Discrete Time signal – If the time t can only take discrete values, such as t=kTs is called Discrete Time signal

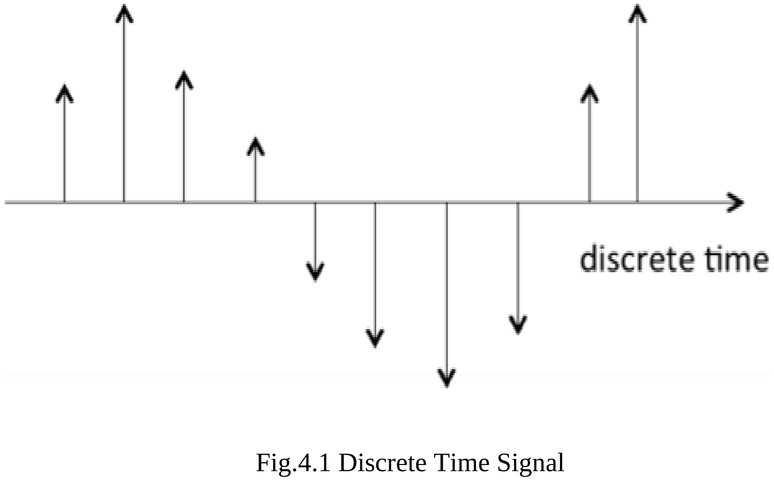


Fig.4.1 Discrete Time Signal

Unit Step Signal:

The Unit Step Signal u(t) is defined as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Graphically it is given by

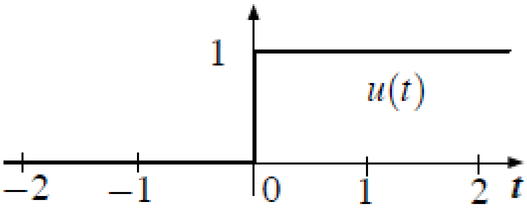


Fig.4.2 Unit Step Signal

Ramp Signal:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Graphically it is given by

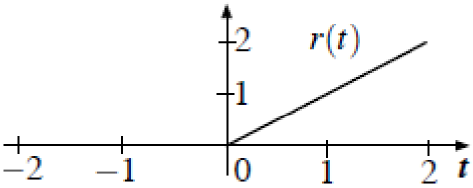


Fig.4.3 Ramp Signal

Pulse Signal:

A signal is having constant amplitude over a particular interval and for the remaining interval the amplitude is zero.

Impulse Signal:

$$\delta[n] \equiv \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Impulse Signal DT representation

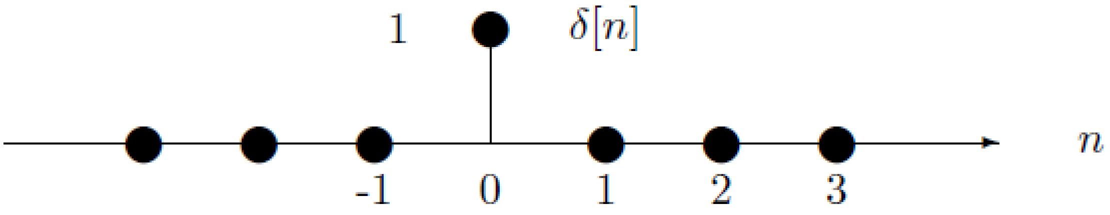


Fig.4.4 Impulse Signal DT Signal

Impulse Signal CT representation

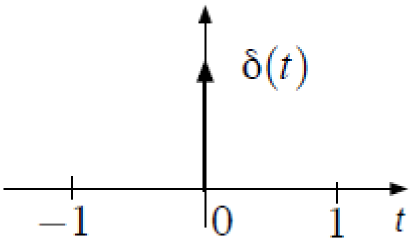


Fig.4.5 Impulse Signal CT Signal

Exponential Signal:

Exponential signal is of two types. These two types of signals are real exponential signal and complex exponential signal which are given below.

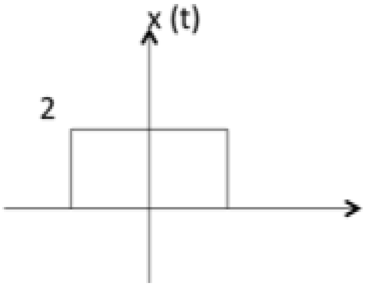
Real Exponential Signal: A real exponential signal is defined as $x(t)=Ae^{\sigma t}$

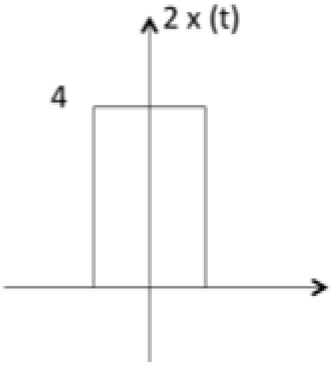
Complex exponential Signal: The complex exponential signal is given by $x(t)=Ae^{st}$ where $s=\sigma+j\omega$

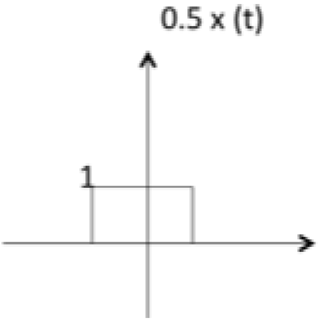
Basic Operations on signals:

Several basic operations by which new signals are formed from given signals are familiar from the algebra and calculus of functions.

1. Amplitude Scaling :y(t)= a x(t), where a is a real (or possibly complex) constant. C x(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.

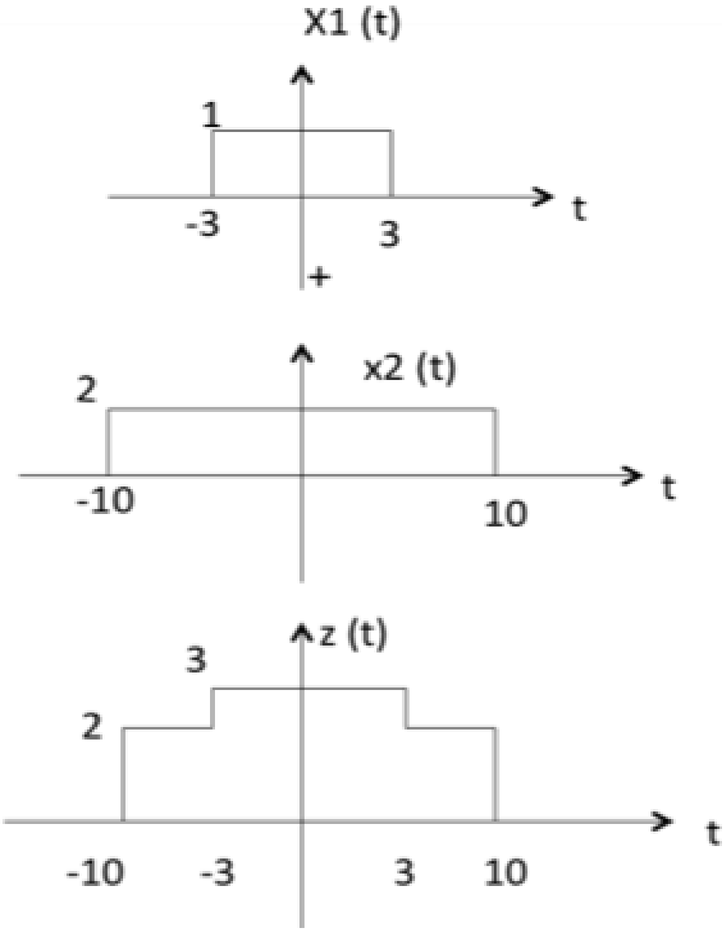






2. Amplitude Shift: y(t)= x(t)+ b, where b is a real (or possibly complex) constant

3. Signal Addition: y(t)= x1(t) + x2(t)



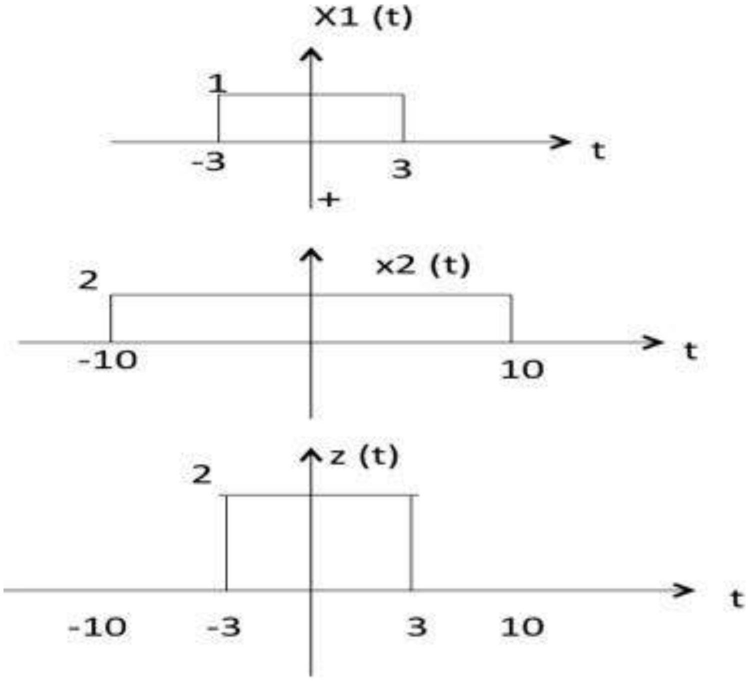
As seen from the diagram above,

-10 < t < -3 amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2

-3 < t < 3 amplitude of z(t) = x1(t) + x2(t) = 1 + 2 = 3

3 < t < 10 amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2

4. Signal Multiplication: y(t)= x1(t). x2(t)



As seen from the diagram above,

-10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

-3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2 3

< t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

5. Time Shift**:**

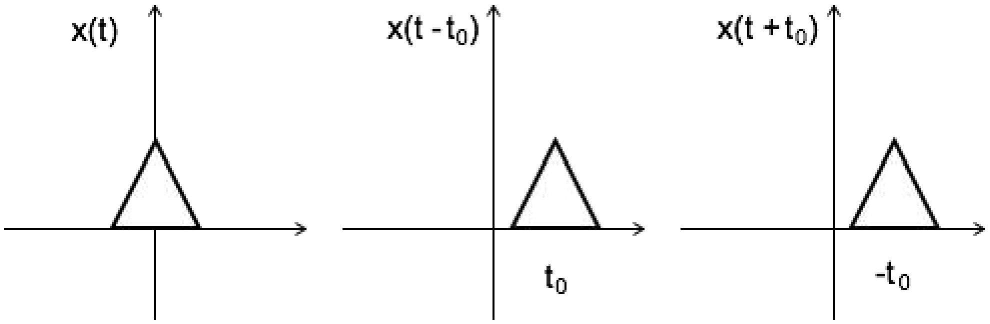
If x(t) is a continuous function, the time-shifted signal is defined as

$y(t) = x(t - t_0)$.

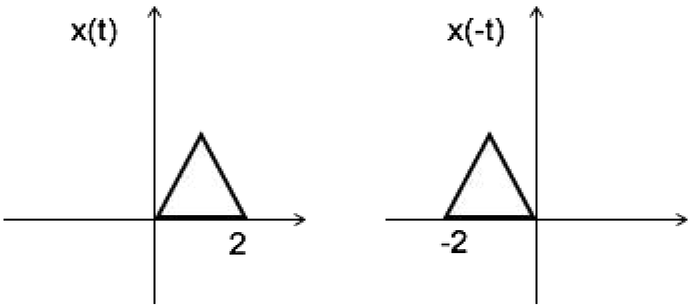
If $t_0 > 0$, the signal is shifted to the right, and if $t_0 < 0$, the signal is shifted to the left.

$x(t \pm t_0)$ is time shifted version of the signal $x(t)$.

$x(t + t_0)$ →→ negative shift and $x(t - t_0)$ →→ positive shift



6. Time Reversal: If $x(t)$ is a continuous function, the time-reversed signal is defined as $y(t) = x(-t)$. $x(-t)$ is the time reversal of the signal $x(t)$.



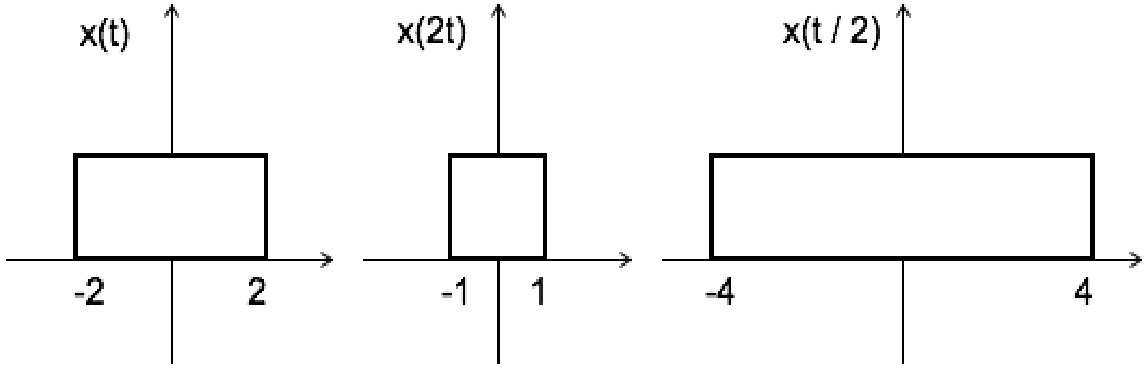
7. Time Scaling: If $x(t)$ is a continuous function, a time-scale version of this signal is defined as $y(t)= x(at)$. If $a>1$, the signal $y(t)$ is a compressed version of $x(t)$, i.e., the time interval is compressed to $1/a$.

   If $0<a< 1$, the signal $y(t)$ is a stretched version of $x(t)$, i.e., the time interval is stretched by $1/a$.

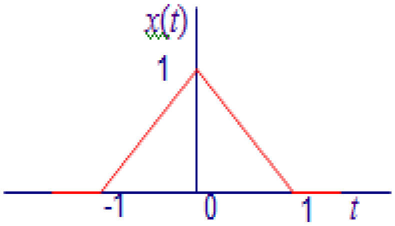
. When operating on signals, the time-shifting operation must be performed first, and then the time-scaling operation is performed. $x(At)$ is time scaled version of the signal $x(t)$. where, A is always positive.

$|A| > 1$ →→ Compression of the signal

$|A| < 1$ →→ Expansion of the signal



1. A triangular pulse signal x(t) is depicted below.



Sketch each of the following signals:

(a) $x(3t)$

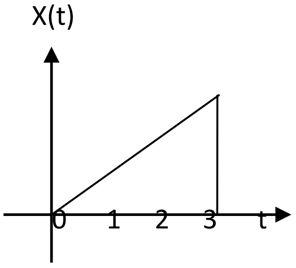
(b) $x(3t + 2)$

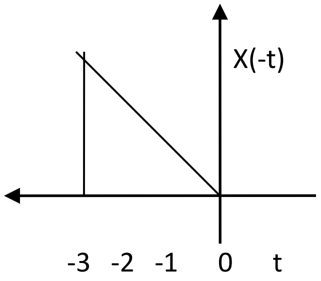
(c) $x(-2t -1)$

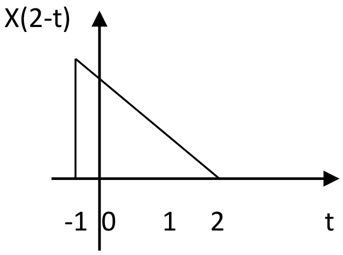
(d) $x(0.5t -1)$

2. Draw the waveform x(-t) and x(2-t) of the signal x(t) = t 0≤t≤3

0 t>3







Classification of DT and CT Signals:

1. Even and Odd signal
2. Deterministic and Random Signal
3. Periodic and Aperiodic signal
4. Energy and Power signal

Even and Odd Signal:

An even signal is any signal 'x' such that x(t) = x(-t). Odd signal is a signal 'x' for which x(t) = -x(-t).

The even and odd parts of a signal are given by

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Here Xe(t) denotes the even part of signal X(t) and Xo(t) denotes the odd part of signal X(t).

Deterministic Signal:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signals can be modeled exactly by a mathematical formula are known as deterministic signals.

Random (or) Nondeterministic Signals:

Non deterministic signals and events are either random or irregular. Random signals are also called non deterministic signals are those signals that take random values at any given time and must be characterized statistically. Random signals, on the other hand, cannot be described by a mathematical equation they are modeled in probabilistic terms.

Periodic signal:

A CT signal x(t) is said to be periodic if it satisfies the following property: x(t)=x(t+T) at all time t, where T=Fundamental Time Interval (T=2π/ω)

Ex:

1. x(t)=sin(4πt). It is periodic with period of 1/2
2. x(t)=cos(3πt). It is periodic with period of 2/3

Aperiodic Signal:

A CT signal x(t) is said to be periodic if it satisfies the following property: $x(t) \neq x(t+T)$ at all time t, where T=Fundamental Time Interval

Energy Signal:

The Energy in the signal is defined as

$$E = \int_{-\infty}^{\infty} |\, x(t)|^2 \, \mathrm{d}t \, .$$

Power Signal:

The Power in the signal is defined as

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |\, x(t)|^2 \, \mathrm{d}t$$

If $0<E<\infty$ then the signal x(t) is called as Energy signal. However there are signals where this condition is not satisfied. For such signals we consider the power. If $0<P<\infty$ then the signal is called a power signal. Note that the power for an energy signal is zero (P=0) and that the energy for a power signal is infinite ($E=\infty$). Some signals are neither energy nor power signals.

Real and Complex signals:

Exponential signal is of two types. These two type of signals are real exponential signal and complex exponential signal which are given below.

Real Exponential Signal:

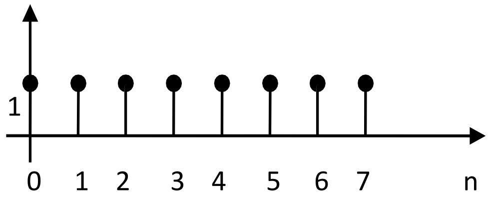
A real exponential signal is defined as $x(t)=Ae^{\sigma t}$

Complex exponential Signal:

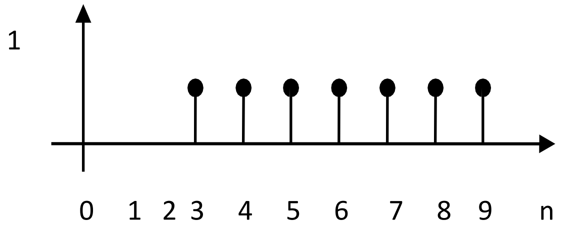
The complex exponential signal is given by $x(t)=Ae^{st}$ where $s=\sigma+j\omega$

1. Draw the signal x(n) = u(n) – u(n-3)

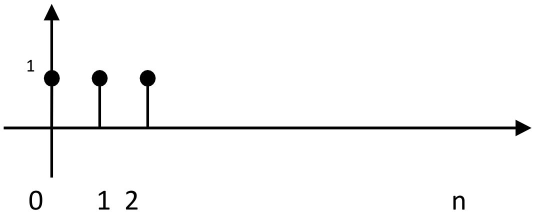
u(n)



u(n-3)



X(n)=u(n)-u(n-3)



2. What is the total energy of the discrete time signal x(n) which takes the value of unity at n= - 1,0,1?

Energy of the signal is given as,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-1}^{1} \| x(n)|^2$$

$$= |x(-1)|^2 + |x(0)|^2 + |x(1)|^2 = 3$$

3. Determine if the following signals are Energy signals, Power signals, or neither, and evaluate E and P for each signal.

$$E_a = \int_{-\infty}^{\infty} |a(t)|^2 \, dt = \int_{-\infty}^{\infty} |3\sin(2\pi t)|^2 \, dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}[1-\cos(4\pi t)]dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}dt - 9\int_{-\infty}^{\infty} \cos(4\pi t)dt$$

$$= \infty \quad \text{J}$$

$$P_a = \frac{1}{1}\int_{0}^{1} |a(t)|^2 \, dt = \int_{0}^{1} |3\sin(2\pi t)|^2 \, dt$$

$$= 9\int_{0}^{1} \frac{1}{2}[1-\cos(4\pi t)]dt$$

$$= 9\int_{0}^{0} \frac{1}{2}dt - 9\int_{0}^{1} \cos(4\pi t)dt$$

$$= \frac{9}{2} - \left[\frac{9}{4\pi}\sin(4\pi t)\right]_0^1$$

$$= \frac{9}{2} \quad \text{W}$$

So, the energy of that signal is infinite and its average power is finite (9/2). This means that it is a power signal as expected. It is a power signal.

4. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

(a) $x(t) = \cos\left(t + \frac{\pi}{4}\right)$

(b) $x(t) = \sin\frac{2\pi}{3}t$

(c) $x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t$

(d) $x(t) = \cos t + \sin\sqrt{2}t$

$(a)\quad x(t)=\cos\left(t+\frac{\pi}{4}\right)=\cos\left(\omega_0 t+\frac{\pi}{4}\right)\rightarrow\omega_0=1$

$x(t)$ is periodic with fundamental period $T_0=2\pi/\omega_0=2\pi$.

$(b)\quad x(t)=\sin\frac{2\pi}{3}t\rightarrow\omega_0=\frac{2\pi}{3}$

$x(t)$ is periodic with fundamental period $T_0=2\pi/\omega_0=3$.

$(c)\quad x(t)=\cos\frac{\pi}{3}t+\sin\frac{\pi}{4}t=x_1(t)+x_2(t)$

where $x_1(t)=\cos(\pi/3)t=\cos\omega_1 t$ is periodic with $T_1=2\pi/\omega_1=6$ and $x_2(t)=\sin(\pi/4)t=\sin\omega_2 t$ is periodic with $T_2=2\pi/\omega_2=8$. Since $T_1/T_2=\frac{6}{8}=\frac{3}{4}$ is a rational number, $x(t)$ is periodic with fundamental period $T_0=4T_1=3T_2=24$.

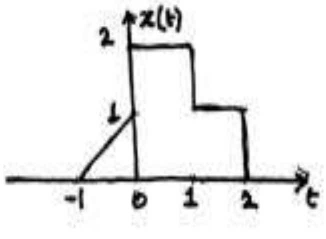
$(d)\quad x(t)=\cos t+\sin\sqrt{2}\,t=x_1(t)+x_2(t)$

where $x_1(t)=\cos t=\cos\omega_1 t$ is periodic with $T_1=2\pi/\omega_1=2\pi$ and $x_2(t)=\sin\sqrt{2}\,t=\sin\omega_2 t$ is periodic with $T_2=2\pi/\omega_2=\sqrt{2}\,\pi$. Since $T_1/T_2=\sqrt{2}$ is an irrational number, $x(t)$ is nonperiodic.

Questions for Practice

PART - A

1. For the signal shown in Fig. 1, find x (2t + 3).



2. Sketch the following signals

   i)$x(t)=4(t+3)$ ii)$x(t) = -2r(t)$

3. Define continuous time complex exponential signal.
4. Define unit impulse and unit step signal.
5. State the relationship between step, ramp and delta function (CT).

6. Define even and odd signal?

7. Determine whether the following signal is energy or power? $x(t) = e^{-2t} u(t)$

8. Find the fundamental period of the given signal $x(n)=\sin((6n\pi/7)+1)$.

9. Check whether the discrete time signal sin3n is periodic.

10. Define a random signal.

11. Determine the power and RMS value of the following signals $x(t)=10\cos5t\cos10t$.

12. Determine whether the following signal is energy or power? $x(n) = u(n)$

PART – B

1. Find the time period T of the following signal

   (i) $X(n) = \cos(n\pi/2) - \sin(n\pi/8) + 3 \cos\{ (n\pi/4) + (n/3) \}$

   (ii)Define and plot the following signals. Ramp, Step, Pulse, Impulse and Exponential signal.

2. (i) What is the periodicity of the signal $x(t) = \sin 100\pi t + \cos 150 \pi t$?

   (ii)What are the basic continuous time signals? Draw any four Waveforms and write their equations.

3. Determine the energy of the discrete time signal.

   $$X(n) = \{ \begin{matrix} (1/2)^n & , n\geq0 \\ 3^n & , n<0 \end{matrix}$$

4. Determine the even and odd component for the following signals.

   i)$x(t) = \cos t + \sin t + \cos t \sin t$

   ii)$x(n) = \{\underset{\uparrow}{-2}, 1, 2, -1, 3\}$

5. Determine whether the following signals are periodic or not.

   i) $x(t) = 2\cos(10t+1) - \sin(5t-1)$

   ii)$x(n) = 12\cos(20n)$

6. Identify which of the following signals are energy signals, power signals and neither power nor energy signals.

   i)$x(t) = e^{-3t}u(t)$

   ii)$x(t) = \cos t$

   iii)$x(t) = t\, u(t)$

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SCHOOL OF MECHANICAL

DEPARTMENT OF MECHATRONICS

UNIT – V Signals and Control Systems – SMRA1402

UNIT V – FOURIER TRANSFORM AND LAPLACE TRANSFORM

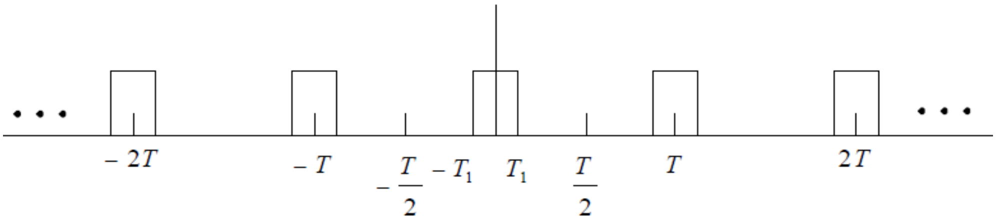
Continuous time Fourier Transform -Properties of CTFT-Inverse Fourier transform-unilateral and bilateral Laplace Transform analysis with examples - Basic properties - Parseval’s relation - Convolution in time and frequency domain-Inverse Laplace transform using partial fraction expansion method - Relation between Fourier transform and Laplace transform-Fourier series analysis.

Continuous Time Fourier Transform

Any continuous time periodic signal x(t) can be represented as a linear combination of complex exponentials and the Fourier coefficients (or spectrum) are discrete. The Fourier series can be applied to periodic signals only but the Fourier transform can also be applied to non-periodic functions like rectangular pulse, step functions, ramp function etc. The Fourier transform of Continuous Time signals can be obtained from Fourier series by applying appropriate conditions. The Fourier transform can be developed by finding Fourier series of a periodic function and the tending T to infinity.

Representation of aperiodic signals: Starting from the Fourier series representation for the continuous-time periodic square wave:

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases},$$



The Fourier coefficients a_k for this square wave are

$$a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}.$$

or alternatively

$$Ta_k = \frac{2\sin(\omega T_1)}{\omega}\bigg|_{\omega = k\omega_0}$$

Where, 2 sin(wT1) / w represent the envelope of *Ta$_k$*

When T increases or the fundamental frequency w $_0$ 2p / T decreases, the envelope is sampled with a closer and closer spacing. As T becomes arbitrarily large, the original periodic square wave approaches a rectangular pulse.

Ta$_k$ becomes more and more closely spaced samples of the envelope, as T →∞, the Fourier series coefficients approaches the envelope function.

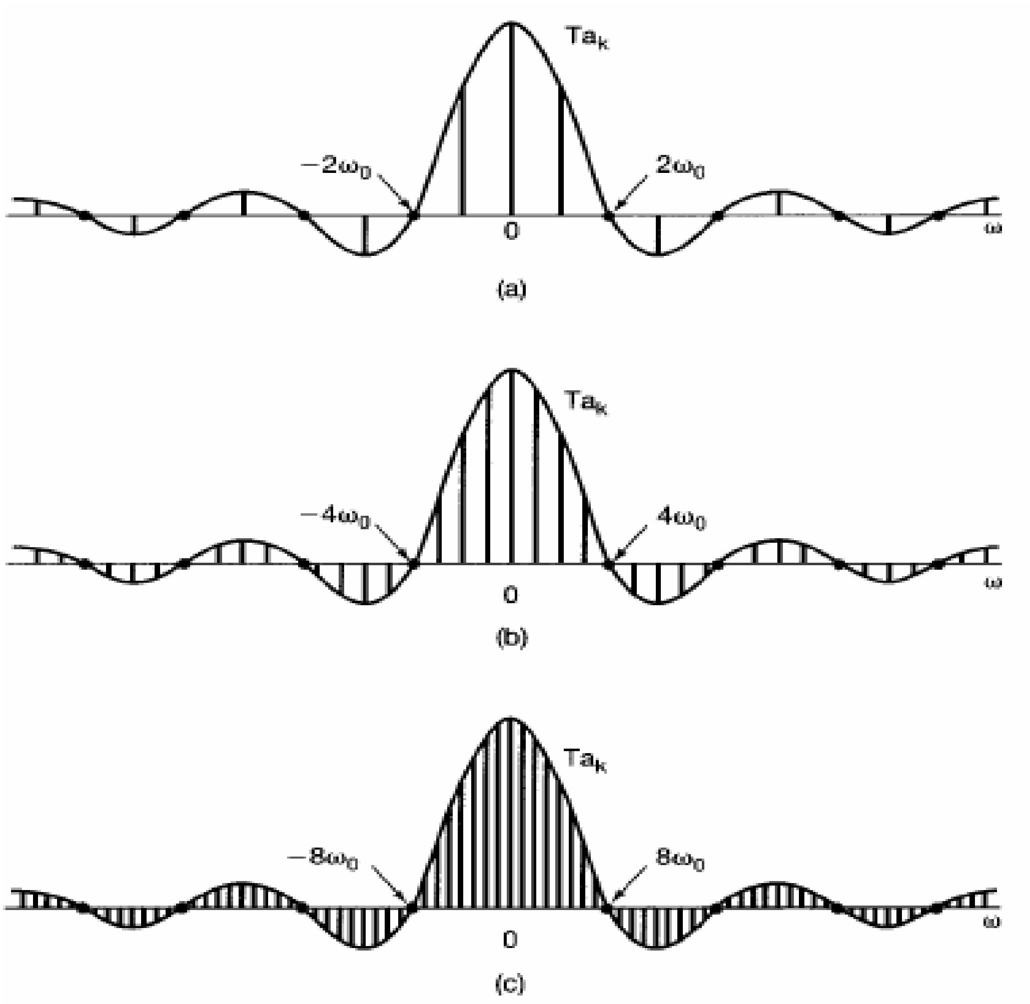


Fig. 5.1 Aperiodic Signals

This example illustrates the basic idea behind Fourier’s development of a representation for aperiodic signals. Based on this idea, we can derive the Fourier transform for aperiodic signals. From this aperiodic signal, we construct a periodic signal (t) , shown in the figure below.

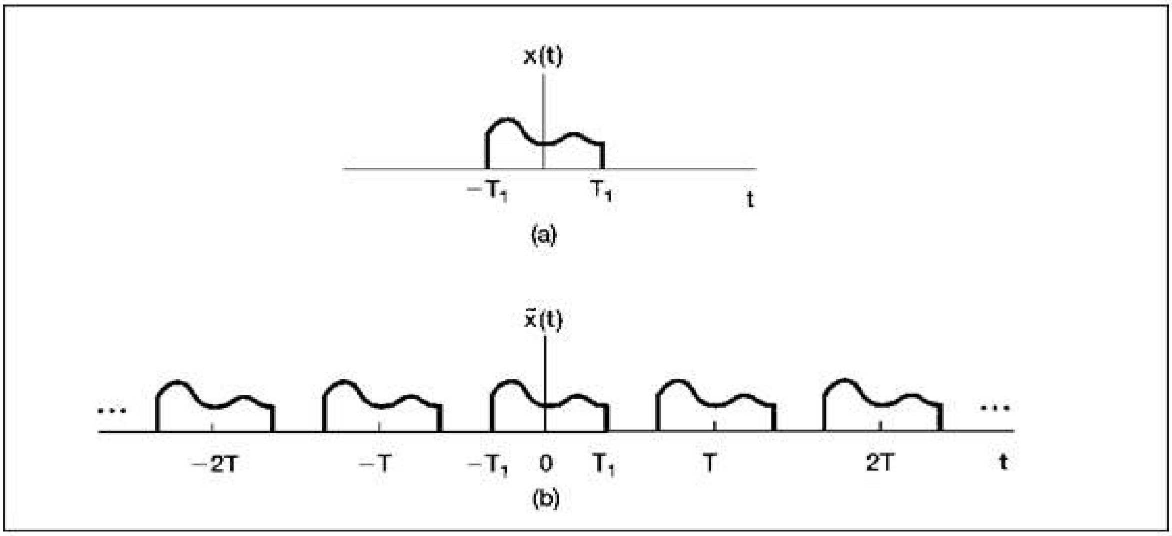


Fig. 5.2 Periodic Signals

As $T \to \infty$, $\tilde{x}(t) = x(t)$, for any infinite value of t

The Fourier series representation of $\tilde{x}(t)$ is

$$\tilde{x}(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} ,$$

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} \tilde{x}(t) e^{-jk\omega_0 t} dt .$$

Since $\tilde{x}(t) = x(t)$ for $|t| < T/2$, and also, since $x(t) = 0$ outside this interval, so we have

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T}\int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt .$$

Define the envelope X (jw) of Ta_k as,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt .$$

we have for the coefficients a_k,

$$a_k = \frac{1}{T} X(jk\omega_0)$$

Then $\tilde{x}(t)$ can be expressed in terms of $X(j\omega)$, that is

$$\tilde{x}(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} X(jk\omega_0)e^{jk\omega_0 t} = \frac{1}{2\pi}\sum_{k=-\infty}^{+\infty} X(jk\omega_0)e^{jk\omega_0 t}\omega_0 .$$

As $T \to \infty$, $\tilde{x}(t) = x(t)$ and consequently,

Equation 2.8 becomes representation of *x(t).* In addition the right hand side of equation becomes an integral.

This results in the following Fourier Transform.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \quad \textit{Inverse Fourier Transform}$$

and

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad \textit{Fourier Transform}$$

Convergence of Fourier Transform

If the signal x(t) has finite energy, that is, it is square integrable,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty ,$$

Then we guaranteed that X(jw) is finite or equation 2.10 converges. If $e(t) = \tilde{x}(t) - x(t)$,

We have

$$\int_{-\infty}^{\infty} |e(t)|^2 dt = 0 .$$

An alternative set of conditions that are sufficient to ensure the convergence:

Contition1: Over any period, x(t) must be absolutely integrable, that is

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty ,$$

Condition 2: In any finite interval of time, x(t) have a finite number of maxima and minima.

Condition 3:

In any finite interval of time, there are only a finite number of discontinuities. Furthermore, each of these discontinuities is finite.

Examples of Continuous-Time Fourier Transform

consider signal $x(t) = e^{-at}u(t)$, $a > 0$.

$$X(j\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt = -\frac{1}{a + j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty} = \frac{1}{a + j\omega}, \qquad a > 0$$

If a is complex rather than real, we get the same result if Re{a}>0

The Fourier transform can be plotted in terms of the magnitude and phase, as shown in the figure below.

$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}, \qquad \angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right).$$

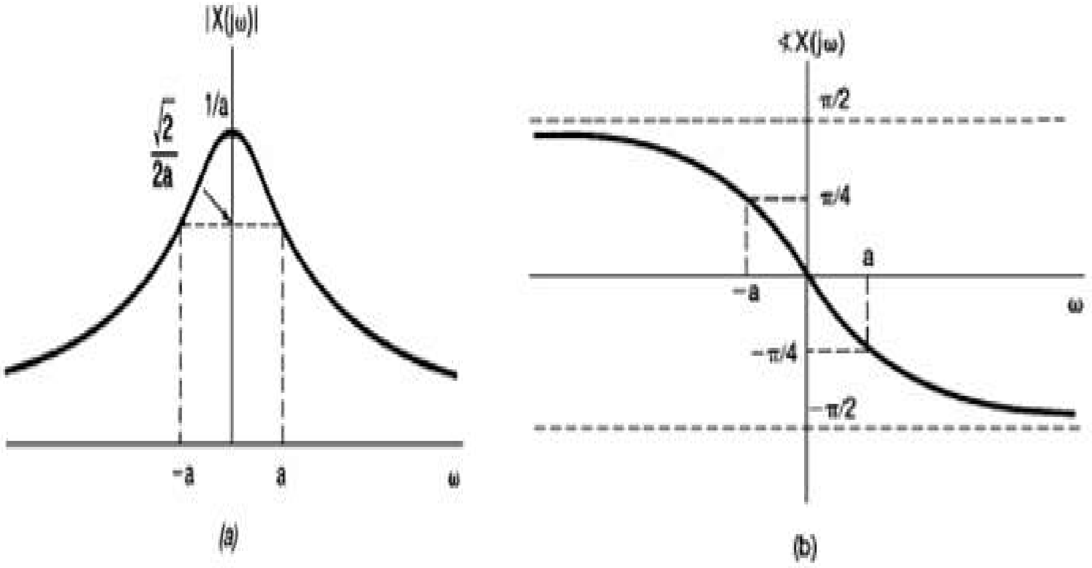
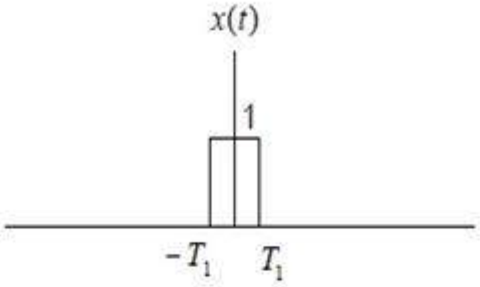


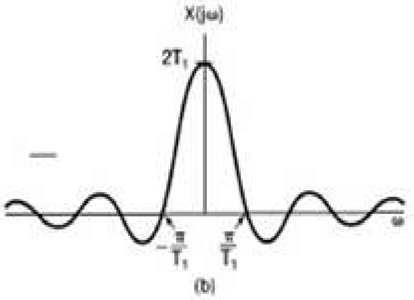
Fig. 5.3 Magnitude and Phase plot

Example:

Calculate the Fourier transform of the rectangular pulse signal

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}.$$





$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-T_1}^{T_1} 1e^{-j\omega t}dt = 2\frac{\sin \omega T_1}{\omega}.$$

The inverse Fourier Transform of the sinc function is

$$x(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}d\omega = \frac{\sin Wt}{\pi t}$$

Comparing the results we have,

Square wave $\underset{FT^{-1}}{\overset{FT}{\rightleftarrows}}$ Sine Function

This means a square wave in the time domain, its Fourier transform is a sine function. However, if the signal in the time domain is a sine function, then its Fourier transform is a square wave. This property is referred to as **Duality Property.**

We also note that when the width of $X(\,jw)$ increases, its inverse Fourier transform x(t) will be compressed. When W → ∞, $X(\,jw)$ converges to an impulse. The transform pair with several different values of W is shown in the figure below.

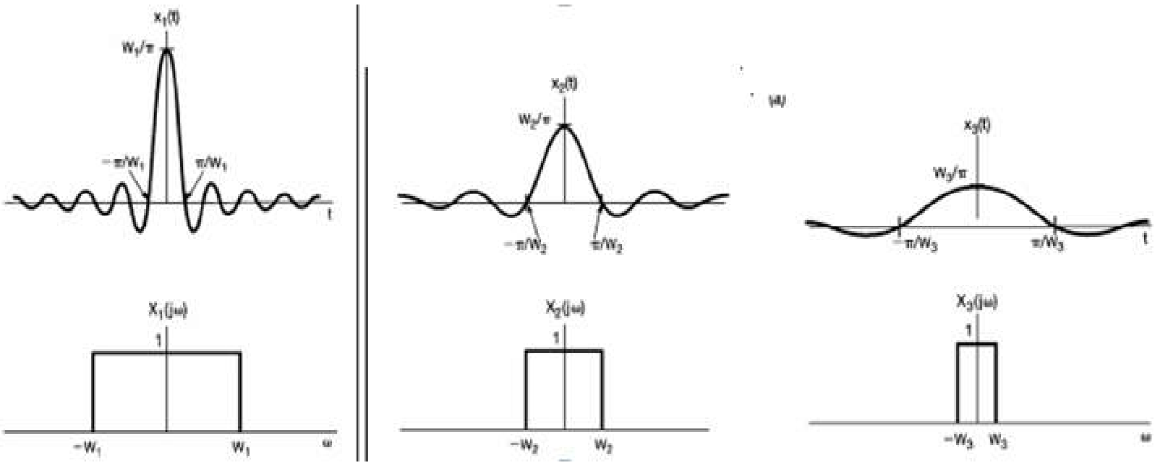


Fig. 5.4 Transform Pairs

The Fourier series representation of the signal $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} .$$

It's Fourier transform is

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) .$$

Properties of Fourier Transform

1. **Linearity**

If $x(t) \xleftrightarrow{F} X(jw)$ $y(t) \xleftrightarrow{F} Y(jw)$

then

$ax(t) + by(t) \xleftrightarrow{F} aX(jw) + bY(jw)$

2. **Time Shifting**

If $x(t) \xleftrightarrow{F} X(jw)$

Then

$x(t-t_0) \xleftrightarrow{F} e^{-jwt_0}X(jw)$

3. Conjugation and Conjugate Symmetry

If $x(t) \overset{F}{\longleftrightarrow} X(jw)$

Then

$x^*(t) \overset{F}{\longleftrightarrow} X^*(-jw)$

4. Differentiation and Integration

If $x(t) \overset{F}{\longleftrightarrow} X(jw)$ then $\frac{dx(t)}{dt} \overset{F}{\leftrightarrow} j\omega X(j\omega)$

$$\int_{-\infty}^{t} x(\tau)d\tau \overset{F}{\leftrightarrow} \frac{1}{j\omega} X(jw) + \pi X(0)\delta(\omega)$$

5. Time and Frequency Scaling

$x(t) \overset{F}{\leftrightarrow} X(j\omega)$

then,

$x(at) \overset{F}{\leftrightarrow} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$

From the equation we see that the signal is compressed in the time domain, the spectrum will be extended in the frequency domain. Conversely, if the signal is extended, the corresponding spectrum will be compressed.

If a = -1, we get from the above equation,

$X(-t) \overset{F}{\leftrightarrow} X(j\omega)$

That is reversing a signal in time also reverses its Fourier transform.

6. Duality

The duality of the Fourier Transform can be demonstrated using the following example.

$$x_1(t)= \begin{cases}1, & t < T_1\\ 0, & t > T_1\end{cases} \overset{F}{\leftrightarrow} X_1^{(j\omega)} = \frac{2\sin\omega T_1}{\omega}$$

$$x_2(t) = \frac{sinWT_1}{\pi t} \overset{F}{\leftrightarrow} X_2(j\omega) = \begin{cases}1, & |\omega| < W\\ 0, & |\omega| > W\end{cases}$$

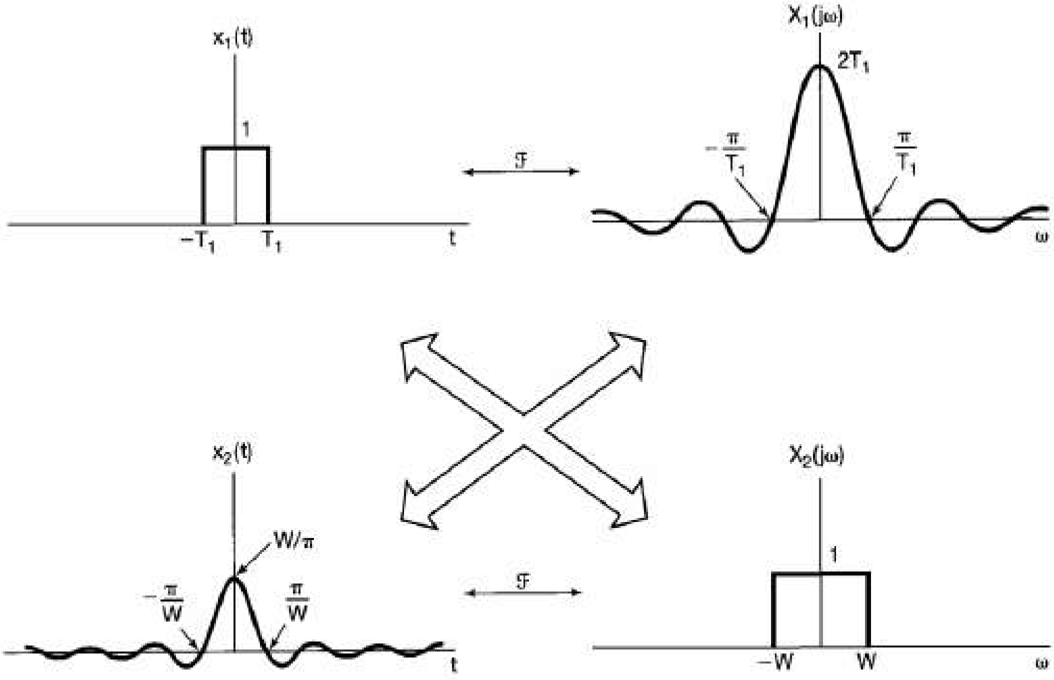


Fig. 5.5 Dual Pairs

For any transform pair, there is a dual pair with the time and frequency variables interchanged.

$$-jtx(t) \xleftrightarrow{F} \frac{dX(j\omega)}{d\omega}$$

$$e^{j\omega_0 t}x(t) \xleftrightarrow{F} X(j(\omega-\omega_0))$$

$$-\frac{1}{jt}x(t)+\pi x(0)\delta(t) \xleftrightarrow{F} \int_{-\infty}^{\omega} x(\eta)d\eta$$

Parseval's Relation

If $x(t) \overset{F}{\leftrightarrow} X(j\omega)$,

We have,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(jw)|^2\, d\omega$$

Parseval‟s relation states that the total energy may be determined either by computing the energy per unit time $x(t)\ ^{2}$ and integrating over all time or by computing the energy per unit frequency $x(j\omega)\ ^{2}/2\pi$ and integrating over all frequencies. For this reason, $(j\omega)\ ^{2}$ is often referred to as the energy density spectrum.

The Parseval's theorem states that the inner product between signals is preserved in going from time to the frequency domain. This is interpreted physically as “Energy calculated in the time domain is same as the energy calculated in the frequency domain”

The convolution properties

$$y(t) = h(t) * \boldsymbol{x}(t) \overset{\boldsymbol{F}}{\leftrightarrow} Y(j\omega) = H(j\omega)X(j\omega)$$

The equation shows that the Fourier transform maps the convolution of two signals into product of their Fourier transforms.

$H(j\omega)$, the transform of the impulse response is the frequency response of the LTI system, which also completely characterizes an LTI system.

Example

The frequency response of a differentiator.

$$y(t)=\frac{dx(t)}{dt}$$

From the differentiation property,

$Y(j\omega)= j\omega X\ j\omega$

The frequency response of the differentiator is

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=j\omega$$

The Multiplication Property

$$r(t) = s(t)p(t) \longleftrightarrow R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\theta)P(j(\omega-\theta))d\theta$$

Multiplication of one signal by another can be thought of as one signal to scale or modulate the amplitude of the other, and consequently, the multiplication of two signals is often referred to as amplitude modulation.

Laplace Transform

The Laplace Transform is the more generalized representation of CT complex exponential signals. The Laplace transform provide solutions to most of the signals and systems, which are not possible with Fourier method. The Laplace transform can be used to analyze most of the signals which are not absolutely integrable such as the impulse response of an unstable system. Laplace Transform is a powerful tool for analysis and design of Continuous Time signals and systems. The Laplace Transform differs from Fourier Transform because it covers a broader class of CT signals and systems which may or may not be stable.

Till now, we have seen the importance of Fourier analysis in solving many problems involving signals. Now, we shall deal with signals which do not have a Fourier transform. We note that the Fourier Transform only exists for signals which can absolutely integrated and have a finite energy. This observation leads to generalization of continuous-time Fourier transform by considering a broader class of signals using the powerful tool of "Laplace transform". With this introduction let us go on to formally defining both Laplace transform.

Definition of Laplace Transform

The Laplace transform of a function x(t) can be shown to be,

$$\mathbf{L\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt}$$

This equation is called the Bilateral or double sided Laplace transform equation.

$$x(t) = \int_{-\infty}^{\infty} X(s) e^{st} ds$$

This equation is called the Inverse Laplace Transform equation, x(t) being called the Inverse Laplace transform of X(s). The relationship between x(t) and X(s) is

$$x(t) \xrightarrow{LT} X(s)$$

Region of Convergence (ROC):

The range of values for which the expression described above is finite is called as the Region of Convergence (ROC).

Convergence of the Laplace transform

The bilateral Laplace Transform of a signal x(t) exists if

$$X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Substitute $s=\sigma + j\omega$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-\sigma t}e^{-j\omega t}\ dt$$

Relationship between Laplace Transform and Fourier Transform

The Fourier Transform for Continuous Time signals is in fact a special case of Laplace Transform. This fact and subsequent relation between LT and FT are explained below. Now we know that Laplace Transform of a signal 'x'(t)' is given by

Now we know that Laplace Transform of a signal 'x'(t)' is given by:

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

The s-complex variable is given by $s = \sigma + j\Omega$

But we consider $\sigma = 0$ and therefore "s" becomes completely imaginary. Thus we have $s=j\Omega$. This means that we are only considering the vertical strip at $\sigma = 0$.

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\Omega t}dt$$

From the above discussion it is clear that the LT reduces to FT when the complex variable only consists of the imaginary part. Thus LT reduces to FT along the jΩ axis. (imaginary axis).

Fourier Transform of x(t) = Laplace Transform of $x(t)\ _{s=j\Omega}$

Laplace transform becomes Fourier transform

if $\sigma=0$ and $s=j\omega$.

$$X(s)|_{s=j\omega} = FT\{x(t)\}$$

Example of Laplace Transform

Find the Laplace transform and ROC of x(t) = $e^{-at}u(t)$

We notice that by multiplying by the term u(t) we are effectively considering the unilateral Laplace Transform whereby the limits tend from 0 to +∞

Consider the Laplace transform of x(t) as shown below

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-(s+a)t} dt$$

$$= \frac{1}{s+a}\text{; for (s+a)} > 0$$

(1) Find the Laplace transform and ROC of x(t)=-$e^{-at}u(-t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{0} -e^{-at} e^{-st}\, dt$$

$$= \int_{-\infty}^{0} e^{-(s+a)t} dt$$

$$= \frac{1}{s+a}\text{; for (s+a)} < 0$$

If we consider the signals $e^{-at}u(t)$ and $-e^{-at}u(-t)$, we note that although the signals are differing, their Laplace Transforms are identical which is 1/(s+a). Thus we conclude that to distinguish L.T's uniquely their ROC's must be specified.

Properties of Laplace Transform

1. Linearity

If $x_1(t) \overset{L}{\leftrightarrow} X_1(s)$ with ROC R1 and $x_2(t) \overset{L}{\leftrightarrow} X_2(s)$ with ROC R2, then

$ax_1(t) + bx_2(t) \overset{L}{\leftrightarrow} aX_1(s) + bX_2(s)$ with ROC containing $R_1 \cap R_2$

The ROC of X(s) is at least the intersection of R_1 and R_2, which could be empty, in which case x(t) has no Laplace Transform.

2. Differentiation in the time domain

If $x(t) \overset{L}{\leftrightarrow} X(s)$ with ROC = R then $\frac{dx(t)}{dt} \overset{L}{\leftrightarrow} sX(s)$ with ROC = R.

This property follows by integration by parts.

Hence, $\frac{dx(t)}{dt} \overset{L}{\leftrightarrow} sX(s)$ The ROC of *sX(s)* includes the ROC of X(s) and may be larger.

3. Time Shift

If $x(t) \overset{L}{\leftrightarrow} X(s)$ with ROC = R then

$x(t-t_0) \overset{L}{\leftrightarrow} e^{-st_0}X(s)$ with ROC = R

4. Time Scaling

If $x(t) \overset{L}{\leftrightarrow} X(s)$ with ROC = R, then

$x(at) \overset{L}{\leftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right)$; ROC = $\frac{R}{a}$ ie., $\frac{s}{a} \epsilon R$

5. Multiplication

$x(t) \times y(t) \overset{L}{\leftrightarrow} \frac{1}{2\pi j} [X(S) * Y(S)]$

6. Time Reversal

When the signal x(t) is time reversed(180° Phase shift)

$X(-t) \overset{L}{\leftrightarrow} X(-s)$

7. Frequency Shifting

$$\boldsymbol{e^{s_0 t} x(t) \overset{L}{\leftrightarrow} X(s - s_0)}$$

8. Conjugation symmetry

$x^*(t) \overset{L}{\leftrightarrow} x^*(-s)$

9. Parseval's Relation of Continuous Signal

It states that the total average power in a periodic signal x(t) equals the sum of the average in individual harmonic components, which in turn equals to the squared magnitude of X(s) Laplace Transform.

$$\int_0^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} |X(S)|^2 ds$$

10. Differentiation in Frequency

When x(t) is differentiated with respect to frequency then,

$$-t\ x(t) \overset{L}{\leftrightarrow} \frac{dX(s)}{dS}$$

11. Integration Property

When a periodic signal x(t) is integrated, then the Laplace Transform becomes,

$$\int_{-\infty}^{t} x(t)dt \overset{L}{\leftrightarrow} \frac{1}{S}X(S) + \frac{\int_{-\infty}^{\infty} x(\tau)d\tau}{S}$$

12. Convolution Property

$$x(t) * y(t) \overset{L}{\leftrightarrow} X(s).Y(S)$$

13. Initial Value Theorem

The initial value theorem is used to calculate initial value $x(0^+)$ of the given sequence x(t) directly from the Laplace transform X(S). The initial value theorem does not apply to rational functions X(S) whose numerator polynomial order is greater than the denominator polynomial orders.

The initial value theorem states that,

$$\lim_{S\to\infty} SX\ S = X(0^+)$$

14. Final Value Theorem

It states that,

$$\lim_{S\to\infty} SX\ S = X(\infty)$$

Questions for Practice

Part A

1. Determine the Fourier transform of unit impulse signal.
2. Find the Fourier transform of signum function.
3. Find the Laplace transform of hyperbolic sine function.
4. What is Fourier series representation of a signal?
5. Write discrete time Fourier series pair.
6. List out the properties of Fourier series.
7. What is the need for transformation of signal?
8. Define Fourier Transform.
9. Define Laplace Transform.
10. Mention the properties of Laplace Transform.
11. How do we fine Fourier series coefficient for a given signal.
12. State time shifting property of CT Fourier series.
13. State time shifting property of DT Fourier series.
14. Determine the Fourier series coefficient of sinω_0n.
15. State conjugate symmetry of CT Fourier series.
16. Determine the Fourier series coefficient of cosω_0n.
17. Determine the Fourier Transform of x(t)=sin $\omega_0 t$.
18. Determine the Fourier Transform of x(t)=cos $\omega_0 t$.
19. Determine the Fourier Transform of Step signal
20. Find the Laplace transform of the signal x(t) = e^{-bt}

Part B

1. State Parseval's theorem for discrete time signal.
2. Find the FT of the following and sketch the magnitude and phase spectrum
   (i) x(t) = $\delta(t)$
   (ii) x(t) = $e^{-at} u(t)$
   (iii) x(t) = e^{-t}
   (iv) x(t) = $e^{2t} u\ t$

3. Find the Laplace transform

   of i) x(t) = $\delta\ t$

   (ii) x(t) = u(t)

   (iii) x(t) = cos $\Omega_0\, t$

   (iv) x(t) = sin $\omega_t u(t)$

4. Determine initial and final value of a signal x(t) = sin $2t\ u(t)$

5. Determine the initial and final value of signal x(t) whose unilateral Laplace Transform is, X(S) = (2S+5)/S(S+3)

6. State and prove the properties of Fourier Transform.

7. State and prove the properties of Laplace Transform.

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