Chapter 1

# Foundations of Engineering Economy


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The need for engineering economy is primarily motivated by the work that engineers do in performing analysis, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of *making decisions.* These decisions involve the fundamental elements of *cash flows of money, time,* and *interest rates.* This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

# Objectives

**Purpose: Understand the fundamental concepts of engineering economy.**

| | | |
|---|---|---|
| 1. | Determine the role of engineering economy in the decision-making process. | **Definition and role** |
| 2. | Identify what is needed to successfully perform an engineering economy study. | **Study approach and terms** |
| 3. | Perform calculations about interest rate and rate of return. | **Interest rate** |
| 4. | Understand what equivalence means in economic terms. | **Equivalence** |
| 5. | Calculate simple interest and compound interest for one or more interest periods. | **Simple and compound interest** |
| 6. | Identify and use engineering economy terminology and symbols. | **Symbols** |
| 7. | Understand cash flows, their estimation, and how to graphically represent them. | **Cash flows** |
| 8. | Use the rule of 72 to estimate a compound interest rate or number of years for an amount of money to double in size. | **Doubling time** |
| 9. | Formulate Excel© functions used in engineering economy. | **Spreadsheet functions** |

## 1.1 WHAT IS ENGINEERING ECONOMY?

Before we begin to develop the fundamental concepts upon which engineering economy is based, it would be appropriate to define what is meant by engineering economy. In the simplest of terms, *engineering economy* is a collection of techniques that simplify comparisons of alternatives on an *economic* basis. In defining what engineering economy is, it might also be helpful to define what it is not. Engineering economy is not a method or process for determining what the alternatives are. On the contrary, engineering economy begins only after the alternatives have been identified. If the best alternative is actually one that the engineer has not even recognized as an alternative, then all of the engineering economic analysis tools in this book will not result in its selection.

While economics will be the sole criterion for selecting the best alternatives in this book, real-world decisions usually include many other factors in the decision-making process. For example, in determining whether to build a nuclear-powered, gas-fired, or coal-fired power plant, factors such as safety, air pollution, public acceptance, water demand, waste disposal, global warming, and many others would be considered in identifying the best alternative. The inclusion of other factors (besides economics) in the decision-marking process is called multiple attribute analysis. This topic is introduced in Appendix C.

## 1.2 PERFORMING AN ENGINEERING ECONOMY STUDY

In order to apply economic analysis techniques, it is necessary to understand the basic terminology and fundamental concepts that form the foundation for engineering-economy studies. Some of these terms and concepts are described below.

### 1.2.1 Alternatives

An *alternative* is a stand-alone solution for a given situation. We are faced with alternatives in virtually everything we do, from selecting the method of transportation we use to get to work every day to deciding between buying a house or renting one. Similarly, in engineering practice, there are always several ways of accomplishing a given task, and it is necessary to be able to compare them in a rational manner so that the most economical alternative can be selected. The alternatives in engineering considerations usually involve such items as purchase cost (first cost), anticipated useful life, yearly costs of maintaining assets (annual maintenance and operating costs), anticipated resale value (salvage value), and the interest rate. After the facts and all the relevant estimates have been collected, an engineering economy analysis can be conducted to determine which is best from an economic point of view.

### 1.2.2 Cash Flows

The estimated inflows (revenues) and outflows (costs) of money are called cash flows. These estimates are truly the heart of an engineering economic analysis.

They also represent the weakest part of the analysis, because most of the numbers are judgments about what is going to happen in the *future*. After all, who can accurately predict the price of oil next week, much less next month, next year, or next decade? Thus, no matter how sophisticated the analysis technique, the end result is only as reliable as the data that it is based on.

### 1.2.3 Alternative Selection

Every situation has at least two alternatives. In addition to the one or more formulated alternatives, there is always the alternative of inaction, called the *do-nothing (DN)* alternative. This is the *as-is* or *status quo* condition. In any situation, when one consciously or subconsciously does not take any action, he or she is actually selecting the DN alternative. Of course, if the status quo alternative *is* selected, the decision-making process should indicate that doing nothing is the most favorable economic outcome at the time the evaluation is made. The procedures developed in this book will enable you to consciously identify the alternative(s) that is (are) economically the best.

### 1.2.4 Evaluation Criteria

Whether we are aware of it or not, we use criteria every day to choose between alternatives. For example, when you drive to campus, you decide to take the "best" route. But how did you define *best?* Was the best route the safest, shortest, fastest, cheapest, most scenic, or what? Obviously, depending upon which criterion or combination of criteria is used to identify the best, a different route might be selected each time. In economic analysis, *financial units* (dollars or other currency) are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

### 1.2.5 Intangible Factors

In many cases, alternatives have noneconomic or intangible factors that are difficult to quantify. When the alternatives under consideration are hard to distinguish economically, intangible factors may tilt the decision in the direction of one of the alternatives. A few examples of noneconomic factors are goodwill, convenience, friendship, and morale.

### 1.2.6 Time Value of Money

It is often said that money makes money. The statement is indeed true, for if we elect to invest money today, we inherently expect to have more money in the future. If a person or company borrows money today, by tomorrow more than the original loan principal will be owed. This fact is also explained by the time value of money.

**The change in the amount of money over a given time period is called the *time value of money;* it is the most important concept in engineering economy.**

The time value of money can be taken into account by several methods in an economy study, as we will learn. The method's final output is a *measure of worth,* for example, rate of return. This measure is used to accept/reject an alternative.

## 1.3 INTEREST RATE, RATE OF RETURN, AND MARR

*Interest* is the manifestation of the time value of money, and it essentially represents "rent" paid for use of the money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest—interest paid and interest earned. Interest is *paid* when a person or organization borrows money (obtains a loan) and repays a larger amount. Interest is *earned* when a person or organization saves, invests, or lends money and obtains a return of a larger amount. The computations and numerical values are essentially the same for both perspectives, but they are interpreted differently.

Interest paid or earned is determined by using the relation

$$\text{Interest} = \text{end amount} - \text{original amount} \qquad \textbf{[1.1]}$$

When interest over a *specific time unit* is expressed as a percentage of the original amount (principal), the result is called the *interest rate* or *rate of return (ROR).*

$$\textbf{Interest rate or rate of return} = \frac{\textbf{interest accrued per time unit}}{\textbf{original amount}} \times \textbf{100\%} \qquad \textbf{[1.2]}$$

The time unit of the interest rate is called the *interest period.* By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as, 1% per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.

The term *return on investment (ROI)* is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs. The term *interest rate paid* is more appropriate for the borrower's perspective, while *rate of return earned* is better from the investor's perspective.

**EXAMPLE 1.1**

An employee at LaserKinetics.com borrows $10,000 on May 1 and must repay a total of $10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

**Solution**

The perspective here is that of the borrower since $10,700 repays a loan. Apply Equation [1.1] to determine the interest paid.

$$\text{Interest paid} = \$10{,}700 - 10{,}000 = \$700$$

Equation [1.2] determines the interest rate paid for 1 year.

$$\text{Percent interest rate} = \frac{\$700}{\$10{,}000} \times 100\% = 7\% \text{ per year}$$

**EXAMPLE 1.2**

**a.** Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.
**b.** Calculate the amount of interest earned during this time period.

**Solution**

**a.** The total amount accrued ($1000) is the sum of the original deposit and the earned interest. If $X$ is the original deposit,

$$\text{Total accrued} = \text{original amount} + \text{original amount (interest rate)}$$
$$\$1000 = X + X(0.05) = X(1 + 0.05) = 1.05X$$

The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

**b.** Apply Equation [1.1] to determine interest earned.

$$\text{Interest} = \$1000 - 952.38 = \$47.62$$

In Examples 1.1 and 1.2 the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved (e.g., if we wanted the amount of interest owed after 3 years in Example 1.2), it is necessary to state whether the interest is accrued on a *simple* or *compound* basis from one period to the next. Simple and compound interest will be discussed in Section 1.5.

Engineering alternatives are evaluated upon the prognosis that a reasonable rate of return (ROR) can be realized. A reasonable rate must be established so that the accept/reject decision can be made. The reasonable rate, called the *minimum attractive rate of return* (MARR), must be higher than the cost of money used to finance the alternative, as well as higher than the rate that would be expected from a bank or safe (minimal risk) investment. Figure 1-1 indicates the relations between different rates of return. In the United States, the current U.S. Treasury bill rate of return is sometimes used as the benchmark safe rate.

For a corporation, the MARR is always set above its *cost of capital*, that is, the interest rate a company must pay for capital funds needed to finance projects. For exam-

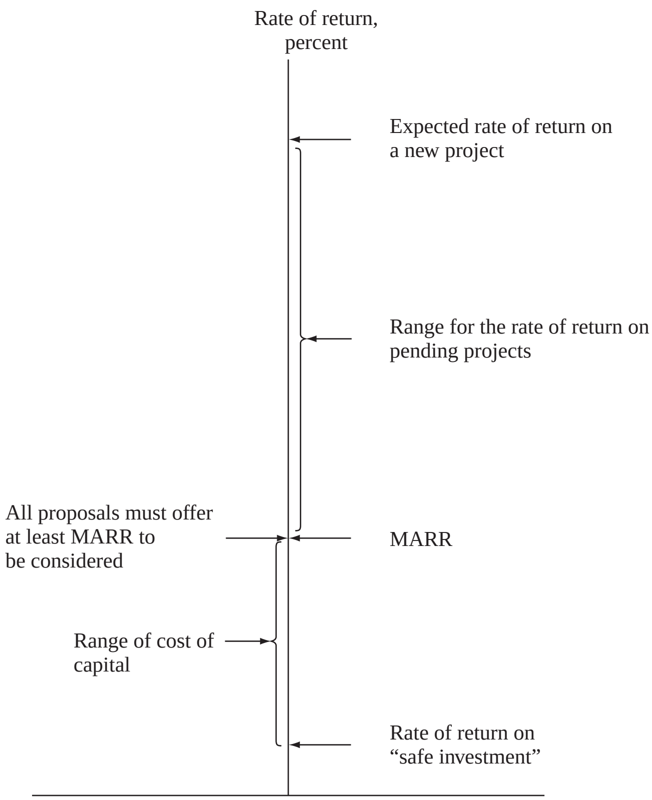


**FIGURE 1.1** MARR relative to cost of capital and other rate of return values.

ple, if a corporation can borrow capital funds at an average of 5% per year and expects to clear at least 6% per year on a project, the minimum MARR will be 11% per year.

The MARR is also referred to as the *hurdle rate;* that is, a financially viable project's expected ROR must meet or exceed the hurdle rate. Note that the MARR is not a rate calculated like the ROR; MARR is established by financial managers and is used as a criterion for accept/reject decisions. The following inequality must be correct for any accepted project.

$$\text{ROR} \geq \text{MARR} > \text{cost of capital}$$

Descriptions and problems in the following chapters use stated MARR values with the assumption that they are set correctly relative to the cost of capital and the expected rate of return. If more understanding of capital funds and the establishment of the MARR is required, refer to Section 13.5 for further detail.

An additional economic consideration for any engineering economy study is *inflation*. In simple terms, bank interest rates reflect two things: a so-called real rate of return *plus* the expected inflation rate. The safest investments (such as U.S. government bonds) typically have a 3% to 4% real rate of return built into their overall interest rates. Thus, an interest rate of, say, 9% per year on a U.S. government bond means that investors expect the inflation rate to be in the range of 5% to 6% per year. Clearly, then, inflation causes interest rates to rise. Inflation is discussed in detail in Chapter 10.

## 1.4 EQUIVALENCE

Equivalent terms are used often in the transfer between scales and units. For example, 1000 meters is equal to (or equivalent to) 1 kilometer, 12 inches equals 1 foot, and 1 quart equals 2 pints or 0.946 liter.

In engineering economy, when considered together, the time value of money and the interest rate help develop the concept of *economic equivalence*, which means that different sums of money at different times would be equal in economic value. For example, if the interest rate is 6% per year, \$100 today (present time) is equivalent to \$106 one year from today.

$$\text{Amount in one year} = 100 + 100(0.06) = 100(1 + 0.06) = \$106$$

So, if someone offered you a gift of \$100 today or \$106 one year from today, it would make no difference which offer you accepted from an economic perspective. In either case you have \$106 one year from today. However, the two sums of money are equivalent to each other *only* when the interest rate is 6% per year. At a higher or lower interest rate, \$100 today is not equivalent to \$106 one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of \$100 now is equivalent to \$100/1.06 = \$94.34 one year ago at an interest rate of 6% per year. From these illustrations, we can state the following: \$94.34 last year, \$100 now, and \$106 one year from now are equivalent at an interest rate of 6% per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1-year interest periods.

$$\frac{\$6}{\$100} \times 100\% = 6\% \text{ per year}$$

and

$$\frac{\$5.66}{\$94.34} \times 100\% = 6\% \text{ per year}$$

Figure 1.2 indicates the amount of interest each year necessary to make these three different amounts equivalent at 6% per year.

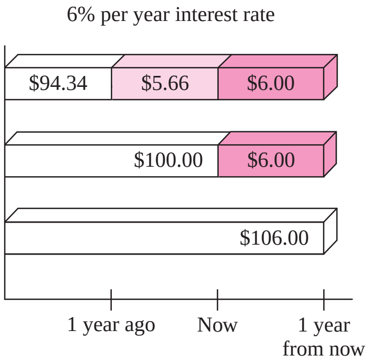


**FIGURE 1.2**
Equivalence of three amounts at a 6% per year interest rate.

**EXAMPLE 1.3**

AC-Delco makes auto batteries available to General Motors dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Delco facility. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

**a.** The amount of \$98 now is equivalent to a cost of \$105.60 one year from now.
**b.** A truck battery cost of \$200 one year ago is equivalent to \$205 now.
**c.** A \$38 cost now is equivalent to \$39.90 one year from now.
**d.** A \$3000 cost now is equivalent to \$2887.14 one year ago.
**e.** The carrying charge accumulated in 1 year on an investment of \$2000 worth of batteries is \$100.

**Solution**

**a.** Total amount accrued $= 98(1.05) = \$102.90 \neq \$105.60$; therefore, it is false. Another way to solve this is as follows: Required original cost is $105.60/1.05 = \$100.57 \neq \$98$.
**b.** Required old cost is $205.00/1.05 = \$195.24 \neq \$200$; therefore, it is false.
**c.** The cost 1 year from now is $\$38(1.05) = \$39.90$; true.
**d.** Cost now is $2887.14(1.05) = \$3031.50 \neq \$3000$; false.
**e.** The charge is 5% per year interest, or $\$2000(0.05) = \$100$; true.

## 1.5 SIMPLE AND COMPOUND INTEREST

The terms *interest, interest period*, and *interest rate* were introduced in Section 1.3 for calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms *simple interest* and *compound interest* become important.

*Simple interest* is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

$$\text{Interest} = (\text{principal})(\text{number of periods})(\text{interest rate}) \qquad [1.3]$$

where the interest rate is expressed in decimal form.

**EXAMPLE 1.4**

HP borrowed money to do rapid prototyping for a new ruggedized computer that targets desert oilfield conditions. The loan is \$1 million for 3 years at 5% per year simple interest. How much money will HP repay at the end of 3 years? Tabulate the results in \$1000 units.

**Solution**

The interest for each of the 3 years in \$1000 units is

$$\text{Interest per year} = 1000(0.05) = \$50$$

Total interest for 3 years from Equation [1.3] is

$$\text{Total interest} = 1000(3)(0.05) = \$150$$

The amount due after 3 years in \$1000 units is

$$\text{Total due} = \$1000 + 150 = \$1150$$

The \$50,000 interest accrued in the first year and the \$50,000 accrued in the second year do not earn interest. The interest due each year is calculated only on the \$1,000,000 principal.

The details of this loan repayment are tabulated in Table 1.1 from the perspective of the borrower. The year zero represents the present, that is, when the money is borrowed. No payment is made until the end of year 3. The amount owed each year increases uniformly by \$50,000, since simple interest is figured on only the loan principal.

**TABLE 1.1 Simple Interest Computations (in \$1000 units)**

| (1) End of Year | (2) Amount Borrowed | (3) Interest | (4) Amount Owed | (5) Amount Paid |
|---|---|---|---|---|
| 0 | \$1000 | | | |
| 1 | — | \$50 | \$1050 | \$ 0 |
| 2 | — | 50 | 1100 | 0 |
| 3 | — | 50 | 1150 | 1150 |

For *compound interest*, the interest accrued for each interest period is calculated on the *principal plus* the *total amount of interest accumulated in all previous periods.* Thus, compound interest means interest on top of interest. Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as

$$\textbf{Interest} = \textbf{(principal + all accrued interest)(interest rate)} \qquad \textbf{[1.4]}$$

**EXAMPLE 1.5** If HP borrows \$1,000,000 from a different source at 5% per year compound interest, compute the total amount due after 3 years. Compare the results of this and the previous example.

**TABLE 1.2 Compound Interest Computations (in $1000 units), Example 1.5**

| (1) End of Year | (2) Amount Borrowed | (3) Interest | (4) Amount Owed | (5) Amount Paid |
|---|---|---|---|---|
| 0 | $1000 | | | |
| 1 | — | $50.00 | $1050.00 | $ 0 |
| 2 | — | 52.50 | 1102.50 | 0 |
| 3 | — | 55.13 | 1157.63 | 1157.63 |

**Solution**

The interest and total amount due each year are computed separately using Equation [1.4]. In $1000 units,

| | |
|---|---|
| Year 1 interest: | $1000(0.05) = $50.00 |
| Total amount due after year 1: | $1000 + 50.00 = $1050.00 |
| Year 2 interest: | $1050(0.05) = $52.50 |
| Total amount due after year 2: | $1050 + 52.50 = $1102.50 |
| Year 3 interest: | $1102.50(0.05) = $55.13 |
| Total amount due after year 3: | $1102.50 + 55.13 = $1157.63 |

The details are shown in Table 1.2. The repayment plan is the same as that for the simple interest example—no payment until the principal plus accrued interest is due at the end of year 3. An extra $1,157,630 − 1,150,000 = $7,630 of interest is paid compared to simple interest over the 3-year period.

**Comment:** The difference between simple and compound interest grows significantly each year. If the computations are continued for more years, for example, 10 years, the difference is $128,894; after 20 years compound interest is $653,298 more than simple interest.

Another and shorter way to calculate the total amount due after 3 years in Example 1.5 is to combine calculations rather than perform them on a year-by-year basis. The total due each year is as follows:

$$\text{Year 1:} \quad \$1000(1.05)^1 = \$1050.00$$
$$\text{Year 2:} \quad \$1000(1.05)^2 = \$1102.50$$
$$\text{Year 3:} \quad \$1000(1.05)^3 = \$1157.63$$

The year 3 total is calculated directly; it does not require the year 2 total. In general formula form,

$$\textbf{Total due after a number of years} = \textbf{principal}(1 + \textbf{interest rate})^{\textbf{number of years}}$$

This fundamental relation is used many times in upcoming chapters.

We combine the concepts of interest rate, simple interest, compound interest, and equivalence to demonstrate that different loan repayment plans may be equivalent, but they may differ substantially in monetary amounts from one year to another. This also shows that there are many ways to take into account the time value of money. The following example illustrates equivalence for five different loan repayment plans.

## EXAMPLE 1.6

**a.** Demonstrate the concept of equivalence using the different loan repayment plans described below. Each plan repays a \$5000 loan in 5 years at 8% interest per year.

- **Plan 1: Simple interest, pay all at end.** No interest or principal is paid until the end of year 5. Interest accumulates each year on the *principal only.*
- **Plan 2: Compound interest, pay all at end.** No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principal *and* all accrued interest.
- **Plan 3: Simple interest paid annually, principal repaid at end.** The accrued interest is paid each year, and the entire principal is repaid at the end of year 5.
- **Plan 4: Compound interest and portion of principal repaid annually.** The accrued interest and one-fifth of the principal (or \$1000) is repaid each year. The outstanding loan balance decreases each year, so the interest for each year decreases.
- **Plan 5: Equal payments of compound interest and principal made annually.** Equal payments are made each year with a portion going toward principal repayment and the remainder covering the accrued interest. Since the loan balance decreases at a rate slower than that in plan 4 due to the equal end-of-year payments, the interest decreases, but at a slower rate.

**b.** Make a statement about the equivalence of each plan at 8% simple or compound interest, as appropriate.

### Solution

**a.** Table 1.3 presents the interest, payment amount, total owed at the end of each year, and total amount paid over the 5-year period (column 4 totals). The amounts of interest (column 2) are determined as follows:

**Plan 1** Simple interest = (original principal)(0.08)
**Plan 2** Compound interest = (total owed previous year)(0.08)
**Plan 3** Simple interest = (original principal)(0.08)
**Plan 4** Compound interest = (total owed previous year)(0.08)
**Plan 5** Compound interest = (total owed previous year)(0.08)

Note that the amounts of the annual payments are different for each repayment schedule and that the total amounts repaid for most plans are different, even though each repayment plan requires exactly 5 years. The difference in the total amounts repaid can be explained (1) by the time value of money, (2) by simple or compound interest, and (3) by the partial repayment of principal prior to year 5.

**TABLE 1.3 Different Repayment Schedules Over 5 Years for $5000 at 8% Per Year Interest**

| (1) End of Year | (2) Interest Owed for Year | (3) Total Owed at End of Year | (4) End-of-Year Payment | (5) Total Owed after Payment |
|---|---|---|---|---|
| *Plan 1: Simple Interest, Pay All at End* | | | | |
| 0 | | | | $5000.00 |
| 1 | $400.00 | $5400.00 | — | 5400.00 |
| 2 | 400.00 | 5800.00 | — | 5800.00 |
| 3 | 400.00 | 6200.00 | — | 6200.00 |
| 4 | 400.00 | 6600.00 | — | 6600.00 |
| 5 | 400.00 | 7000.00 | $7000.00 | |
| Totals | | | $7000.00 | |
| *Plan 2: Compound Interest, Pay All at End* | | | | |
| 0 | | | | $5000.00 |
| 1 | $400.00 | $5400.00 | — | 5400.00 |
| 2 | 432.00 | 5832.00 | — | 5832.00 |
| 3 | 466.56 | 6298.56 | — | 6298.56 |
| 4 | 503.88 | 6802.44 | — | 6802.44 |
| 5 | 544.20 | 7346.64 | $7346.64 | |
| Totals | | | $7346.64 | |
| *Plan 3: Simple Interest Paid Annually; Principal Repaid at End* | | | | |
| 0 | | | | $5000.00 |
| 1 | $400.00 | $5400.00 | $ 400.00 | 5000.00 |
| 2 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 3 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 4 | 400.00 | 5400.00 | 400.00 | 5000.00 |
| 5 | 400.00 | 5400.00 | 5400.00 | |
| Totals | | | $7000.00 | |
| *Plan 4: Compound Interest and Portion of Principal Repaid Annually* | | | | |
| 0 | | | | $5000.00 |
| 1 | $400.00 | $5400.00 | $1400.00 | 4000.00 |
| 2 | 320.00 | 4320.00 | 1320.00 | 3000.00 |
| 3 | 240.00 | 3240.00 | 1240.00 | 2000.00 |
| 4 | 160.00 | 2160.00 | 1160.00 | 1000.00 |
| 5 | 80.00 | 1080.00 | 1080.00 | |
| Totals | | | $6200.00 | |

**TABLE 1.3 (Continued)**

| (1) End of Year | (2) Interest Owed for Year | (3) Total Owed at End of Year | (4) End-of-Year Payment | (5) Total Owed after Payment |
|---|---|---|---|---|
| *Plan 5: Equal Annual Payments of Compound Interest and Principal* | | | | |
| 0 | | | | $5000.00 |
| 1 | $400.00 | $5400.00 | $1252.28 | 4147.72 |
| 2 | 331.82 | 4479.54 | 1252.28 | 3227.25 |
| 3 | 258.18 | 3485.43 | 1252.28 | 2233.15 |
| 4 | 178.65 | 2411.80 | 1252.28 | 1159.52 |
| 5 | 92.76 | 1252.28 | 1252.28 | |
| Totals | | | $6261.41 | |

**b.** Table 1.3 shows that $5000 at time 0 is equivalent to each of the following:

**Plan 1** $7000 at the end of year 5 at 8% simple interest.
**Plan 2** $7346.64 at the end of year 5 at 8% compound interest.
**Plan 3** $400 per year for 4 years and $5400 at the end of year 5 at 8% simple interest.
**Plan 4** Decreasing payments of interest and partial principal in years 1 ($1400) through 5 ($1080) at 8% compound interest.
**Plan 5** $1252.28 per year for 5 years at 8% compound interest.

Beginning in Chapter 2, we will make many calculations like plan 5, where interest is compounded and a constant amount is paid each period. This amount covers accrued interest and a partial principal repayment.

## 1.6 TERMINOLOGY AND SYMBOLS

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.

$P$ = value or amount of money at a time designated as the present or time 0. Also, $P$ is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); dollars

$F$ = value or amount of money at some future time. Also, $F$ is called future worth (FW) and future value (FV); dollars

$A$ = series of consecutive, equal, end-of-period amounts of money. Also, $A$ is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, dollars per month

$n$ = number of interest periods; years, months, days

$i$ = interest rate or rate of return per time period; percent per year, percent per month, percent per day

$t$ = time, stated in periods; years, months, days

The symbols $P$ and $F$ represent one-time occurrences: $A$ occurs with the same value each interest period for a specified number of periods. It should be clear that a present value $P$ represents a single sum of money at some time prior to a future value $F$ or prior to the first occurrence of an equivalent series amount $A$.

It is important to note that the symbol $A$ always represents a uniform amount (i.e., the same amount each period) that extends through *consecutive* interest periods. Both conditions must exist before the series can be represented by $A$.

The interest rate $i$ is assumed to be a compound rate, unless specifically stated as simple interest. The rate $i$ is expressed in percent per interest period, for example, 12% per year. Unless stated otherwise, assume that the rate applies throughout the entire $n$ years or interest periods. The decimal equivalent for $i$ is always used in engineering economy computations.

All engineering economy problems involve the element of time $n$ and interest rate $i$. In general, every problem will involve at least four of the symbols $P$, $F$, $A$, $n$, and $i$, with at least three of them estimated or known.

**EXAMPLE 1.7**

A new college graduate has a job with Boeing Aerospace. She plans to borrow \$10,000 now to help in buying a car. She has arranged to repay the entire principal plus 8% per year interest after 5 years. Identify the engineering economy symbols involved and their values for the total owed after 5 years.

**Solution**

In this case, $P$ and $F$ are involved, since all amounts are single payments, as well as $n$ and $i$. Time is expressed in years.

$P$ = \$10,000 $\quad$ $i$ = 8% per year $\quad$ $n$ = 5 years $\quad$ $F$ = ?

The future amount $F$ is unknown.

**EXAMPLE 1.8**

Assume you borrow \$2000 now at 7% per year for 10 years and must repay the loan in equal yearly payments. Determine the symbols involved and their values.

**Solution**

Time is in years.

$P$ = \$2000

$A$ = ? per year for 5 years

$i$ = 7% per year

$n$ = 10 years

In Examples 1.7 and 1.8, the $P$ value is a receipt *to* the borrower, and $F$ or $A$ is a disbursement *from* the borrower. It is equally correct to use these symbols in the reverse roles.

**EXAMPLE 1.9** On July 1, 2008, your new employer Ford Motor Company deposits \$5000 into your money market account, as part of your employment bonus. The account pays interest at 5% per year. You expect to withdraw an equal annual amount each year for the following 10 years. Identify the symbols and their values.

**Solution**

Time is in years.

$$
\begin{aligned}
P &= \$5000 \\
A &= \text{? per year} \\
i &= 5\% \text{ per year} \\
n &= 10 \text{ years}
\end{aligned}
$$

**EXAMPLE 1.10** You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

**Solution**

Time is expressed in years.

$$
\begin{aligned}
P &= \$5000 \\
A &= \$1000 \text{ per year for 5 years} \\
F &= \text{? at end of year 6} \\
i &= 6\% \text{ per year} \\
n &= 5 \text{ years for the } A \text{ series and 6 for the } F \text{ value}
\end{aligned}
$$

## 1.7 CASH FLOWS: THEIR ESTIMATION AND DIAGRAMMING

Cash flows are inflows and outflows of money. These cash flows may be estimates or observed values. Every person or company has cash receipts—revenue and income (inflows); and cash disbursements—expenses, and costs (outflows). These receipts and disbursements are the cash flows, with a plus sign representing cash inflows and a minus sign representing cash outflows. Cash flows occur during specified periods of time, such as 1 month or 1 year.

Of all the elements of an engineering economy study, cash flow estimation is likely the most difficult and inexact. Cash flow estimates are just that—estimates about an uncertain future. Once estimated, the techniques of this book guide the decision-making process. But the time-proven accuracy of an alternative's estimated cash inflows and outflows clearly dictates the quality of the economic analysis and conclusion.

*Cash inflows,* or receipts, may be comprised of the following, depending upon the nature of the proposed activity and the type of business involved.

**Samples of Cash Inflow Estimates**

Revenues (from sales and contracts)

Operating cost reductions (resulting from an alternative)

Salvage value

Construction and facility cost savings

Receipt of loan principal

Income tax savings

Receipts from stock and bond sales

*Cash outflows,* or disbursements, may be comprised of the following, again depending upon the nature of the activity and type of business.

**Samples of Cash Outflow Estimates**

First cost of assets

Engineering design costs

Operating costs (annual and incremental)

Periodic maintenance and rebuild costs

Loan interest and principal payments

Major expected/unexpected upgrade costs

Income taxes

Background information for estimates may be available in departments such as accounting, finance, marketing, sales, engineering, design, manufacturing, production, field services, and computer services. The accuracy of estimates is largely dependent upon the experiences of the person making the estimate with similar situations. Usually *point estimates* are made; that is, a single-value estimate is developed for each economic element of an alternative. If a statistical approach to the engineering economy study is undertaken, a *range estimate* or *distribution estimate* may be developed. Though more involved computationally, a statistical study provides more complete results when key estimates are expected to vary widely. We will use point estimates throughout most of this book.

Once the cash inflow and outflow estimates are developed, the net cash flow can be determined.

$$\begin{aligned}\textbf{Net cash flow} &= \textbf{receipts} - \textbf{disbursements} \\ &= \textbf{cash inflows} - \textbf{cash outflows}\end{aligned} \tag{1.5}$$

Since cash flows normally take place at varying times within an interest period, a simplifying end-of-period assumption is made.

The *end-of-period convention* means that all cash flows are assumed to occur at the end of an interest period. When several receipts and disbursements occur within a given interest period, the *net* cash flow is assumed to occur at the *end* of the interest period.

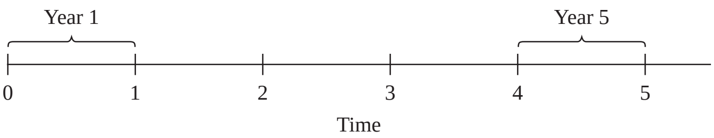


**FIGURE 1.3**
A typical cash flow time scale for 5 years.

However, it should be understood that, although $F$ or $A$ amounts are located at the end of the interest period by convention, the end of the period is not necessarily December 31. In Example 1.9 the deposit took place on July 1, 2008, and the withdrawals will take place on July 1 of each succeeding year for 10 years. *Thus, end of the period means end of interest period, not end of calendar year.*

The *cash flow diagram* is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on a time scale. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time $t = 0$ is the present, and $t = 1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Figure 1.3 is set up for 5 years. Since the end-of-year convention places cash flows at the ends of years, the "1" marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, you will probably avoid errors if you make a neat diagram to approximate scale for both time and relative cash flow magnitudes.

The direction of the arrows on the cash flow diagram is important. A vertical arrow pointing up indicates a positive cash flow. Conversely, an arrow pointing down indicates a negative cash flow. Figure 1.4 illustrates a receipt (cash inflow) at the end of year 1 and equal disbursements (cash outflows) at the end of years 2 and 3.

The perspective or vantage point must be determined prior to placing a sign on each cash flow and diagramming it. As an illustration, if you borrow $2500 to buy a $2000 used Harley-Davidson for cash, and you use the remaining $500 for a new paint job, there may be several different perspectives taken. Possible perspectives, cash flow signs, and amounts are as follows.

| **Perspective** | **Cash Flow, $** |
|---|---|
| Credit union | −2500 |
| You as borrower | +2500 |
| You as purchaser, | −2000 |
| and as paint customer | −500 |
| Used cycle dealer | +2000 |
| Paint shop owner | +500 |

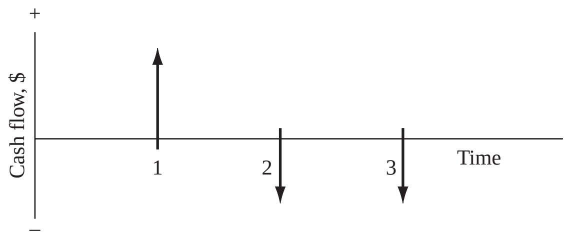


**FIGURE 1.4** Example of positive and negative cash flows.

## EXAMPLE 1.11

Reread Example 1.7, where $P = \$10{,}000$ is borrowed at 8% per year and $F$ is sought after 5 years. Construct the cash flow diagram.

### Solution

Figure 1.5 presents the cash flow diagram from the vantage point of the borrower. The present sum $P$ is a cash inflow of the loan principal at year 0, and the future sum $F$ is the cash outflow of the repayment at the end of year 5. The interest rate should be indicated on the diagram.

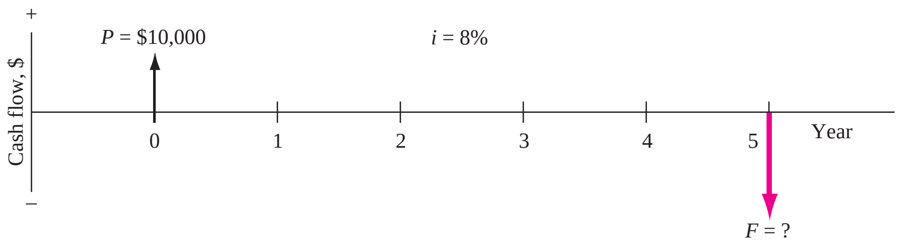


**FIGURE 1.5** Cash flow diagram, Example 1.11.

## EXAMPLE 1.12

Each year Exxon-Mobil expends large amounts of funds for mechanical safety features throughout its worldwide operations. Carla Ramos, a lead engineer for Mexico and Central American operations, plans expenditures of $1 million now and each of the next 4 years just for the improvement of field-based pressure-release valves. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

### Solution

Figure 1.6 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown $F$ value (positive cash flow equivalent) at exactly the same time as the fifth expenditure. Since the expenditures start immediately, the first \$1 million is shown at time 0, not time 1. Therefore, the last negative cash flow occurs at the end of the fourth year, when $F$ also occurs. To make this diagram appear similar to that of Figure 1.5 with a full 5 years on the time scale, the addition of the year $-1$ prior to year 0 completes the diagram for a full 5 years. This addition demonstrates that year 0 is the end-of-period point for the year $-1$.

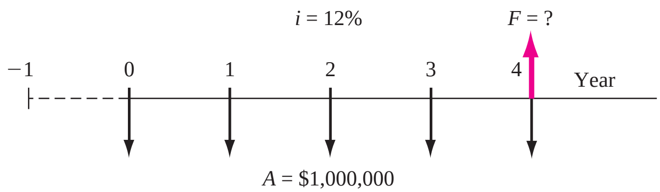


**FIGURE 1.6** Cash flow diagram, Example 1.12.

**EXAMPLE 1.13** A father wants to deposit an unknown lump-sum amount into an investment opportunity 2 years from now that is large enough to withdraw \$4000 per year for state university tuition for 5 years starting 3 years from now. If the rate of return is estimated to be 15.5% per year, construct the cash flow diagram.

**Solution**

Figure 1.7 presents the cash flows from the father's perspective. The present value $P$ is a cash outflow 2 years hence and is to be determined ($P = ?$). Note that this present value does not occur at time $t = 0$, but it does occur one period prior to the first $A$ value of \$4000, which is the cash inflow to the father.

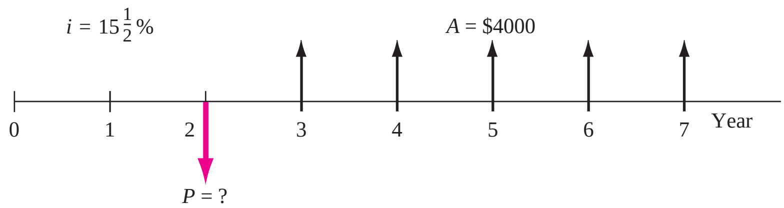


**FIGURE 1.7** Cash flow diagram, Example 1.13.

## 1.8 THE RULE OF 72

The *rule of 72* can estimate either the number of years $n$ or the compound interest rate (or rate of return) $i$ required for a single amount of money to double in size (2×). The rule is simple; the time required for doubling in size with a compound rate is approximately equal to 72 divided by the rate in percent.

$$\text{Approximate } n = 72/i \qquad [1.6]$$

**TABLE 1.4 Number of Years Required for Money to Double**

| | Time to Double, Years | |
|---|---|---|
| Compound Rate, % per year | Rule-of-72 Result | Actual |
| 5 | 14.4 | 14.2 |
| 8 | 9.0 | 9.0 |
| 10 | 7.2 | 7.3 |
| 12 | 6.0 | 6.1 |
| 24 | 3.0 | 3.2 |

For example, at 5% per year, it takes approximately $72/5 = 14.4$ years for a current amount to double. (The actual time is 14.2 years.) Table 1.4 compares rule-of-72 results to the actual times required using time value of money formulas discussed in Chapter 2.

Solving Equation [1.6] for $i$ approximates the compound rate per year for doubling in $n$ years.

$$\text{Approximate } i = 72/n \qquad \textbf{[1.7]}$$

For example, if the cost of gasoline doubles in 6 years, the compound rate is approximately $72/6 = 12\%$ per year. (The exact rate is 12.25% per year.) The approximate number of years or compound rate for a current amount to quadruple (4×) is twice the answer obtained from the rule of 72 for doubling the amount.

For simple interest, the rule of 100 is used with the same equation formats as above, but the $n$ or $i$ value is exact. For example, money doubles in exactly 12 years at a simple rate of $100/12 = 8.33\%$ per year. And, at 10% simple interest, it takes exactly $100/10 = 10$ years to double.