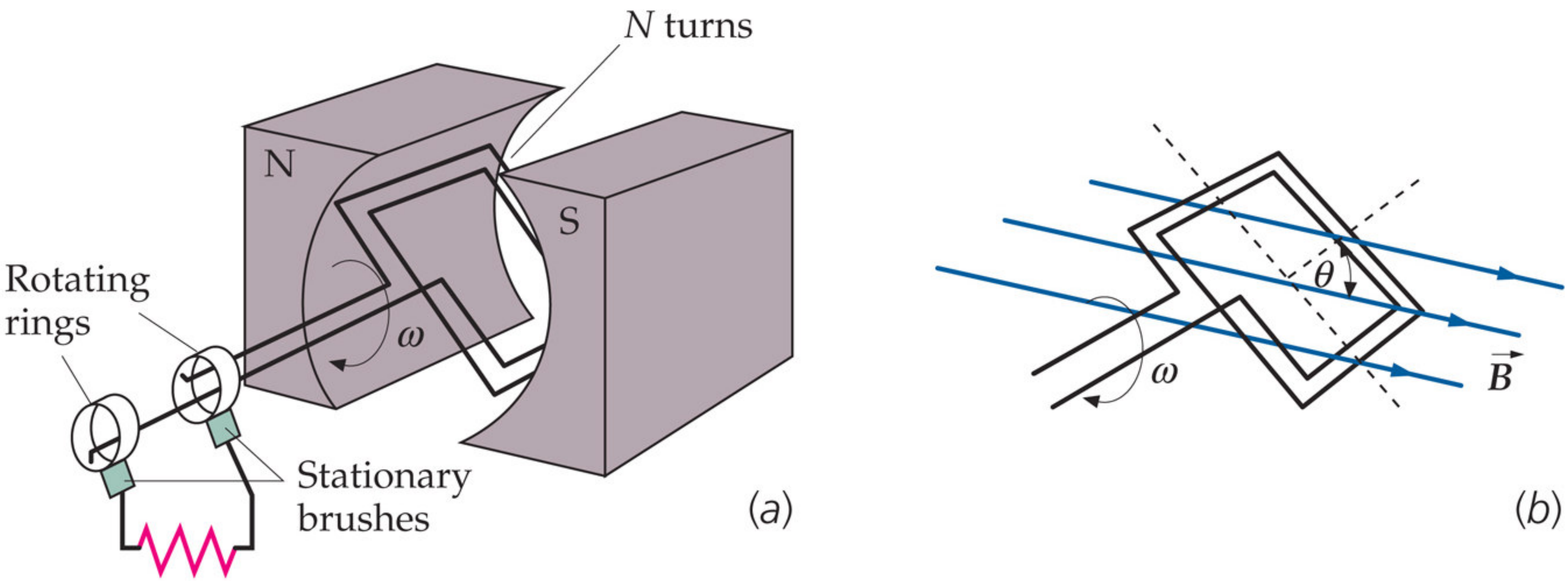
Chapter 31

Alternating Current Circuits

Alternating Current Circuits

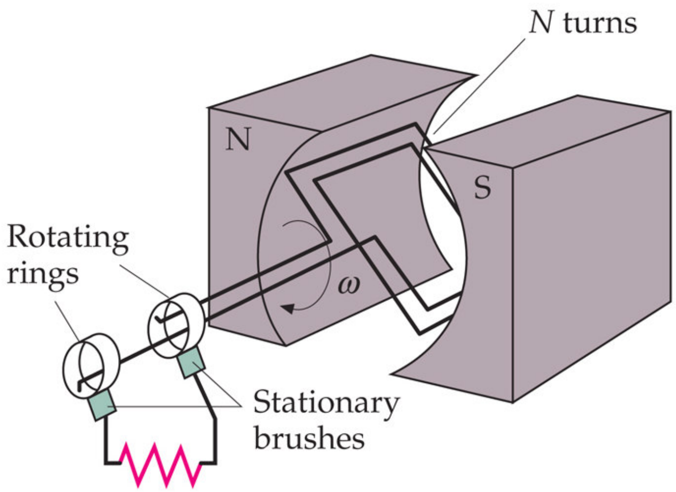
- Alternating Current - Generator
- Wave Nomenclature & RMS
- AC Circuits: Resistor; Inductor; Capacitor
- Transformers - not the movie
- LC and RLC Circuits - No generator
- Driven RLC Circuits - Series
    - Impedance and Power
- RC and RL Circuits - Low & High Frequency
- RLC Circuit - Solution via Complex Numbers
- RLC Circuit - Example
- Resonance

Generators



By turning the coils in the magnetic field an emf is generated in the coils thus turning mechanical energy into alternating (AC) power.

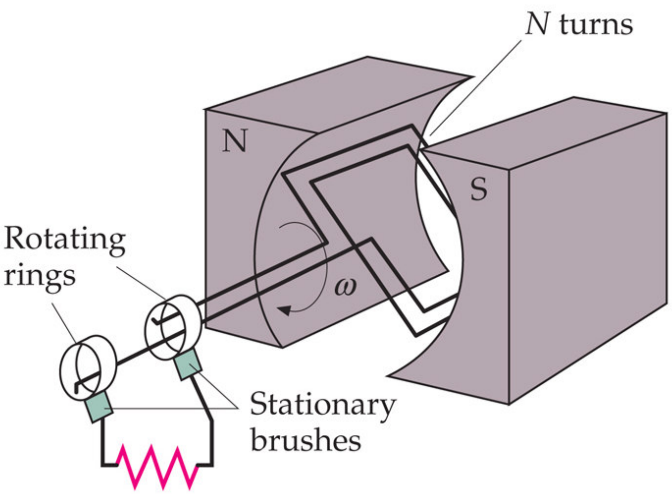
Generators



Rotating the Coil in a Magnetic Field Generates an Emf

- Examples: Gasoline generator
- Manually turning the crank
- Hydroelectric power

Generators



$$\varphi_m = NBAcos\theta \quad \theta = \omega t$$

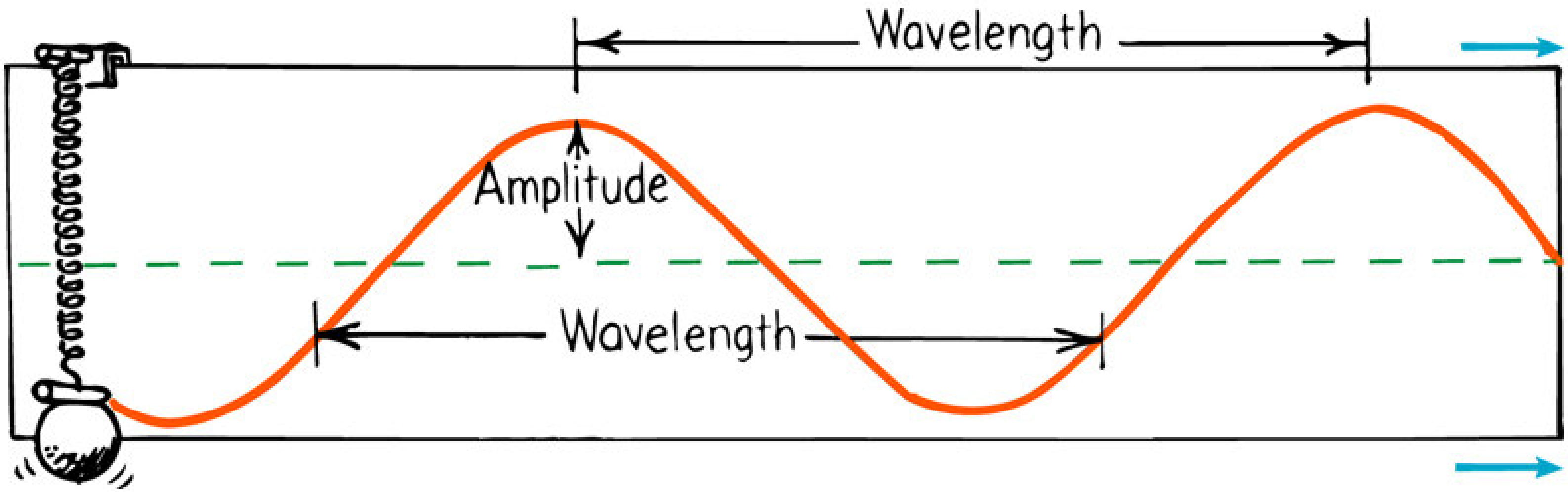
$$\varphi_m = NBAcos\omega t$$

$$\mathcal{E} = -\frac{d}{dt}\varphi_m = NBA\omega sin\omega t$$

$$\mathcal{E} = \mathcal{E}_{peak} sin\omega t; \quad \mathcal{E}_{peak} = NBA\omega$$

Wave Nomenclature and RMS Values

Wave Nomenclature



$$A_{peak\text{-}peak} = A_{p\text{-}p} = 2A_{peak} = 2A_p; \quad A_p = A_{p\text{-}p}/2$$

Shifting Trig Functions

$$x = A \begin{Bmatrix} sin \\ cos \end{Bmatrix} \left[\omega t - \varphi \right]$$

$$x = A \begin{Bmatrix} sin \\ cos \end{Bmatrix} \left[2\pi \frac{t}{T} - \varphi \right]$$

The minus sign means that the phase is shifted to the right.

A plus sign indicated the phase is shifted to the left

$$x = A\, sin\left[\omega t - \tfrac{\pi}{2} \right]$$

$$x = A\left(sin\omega t\, cos\tfrac{\pi}{2} - sin\tfrac{\pi}{2}\, cos\omega t \right)$$

$$x = A\left(sin\omega t\,(0) - (1)cos\omega t \right)$$

$$x = -Acos\omega t$$

Shifting Trig Functions

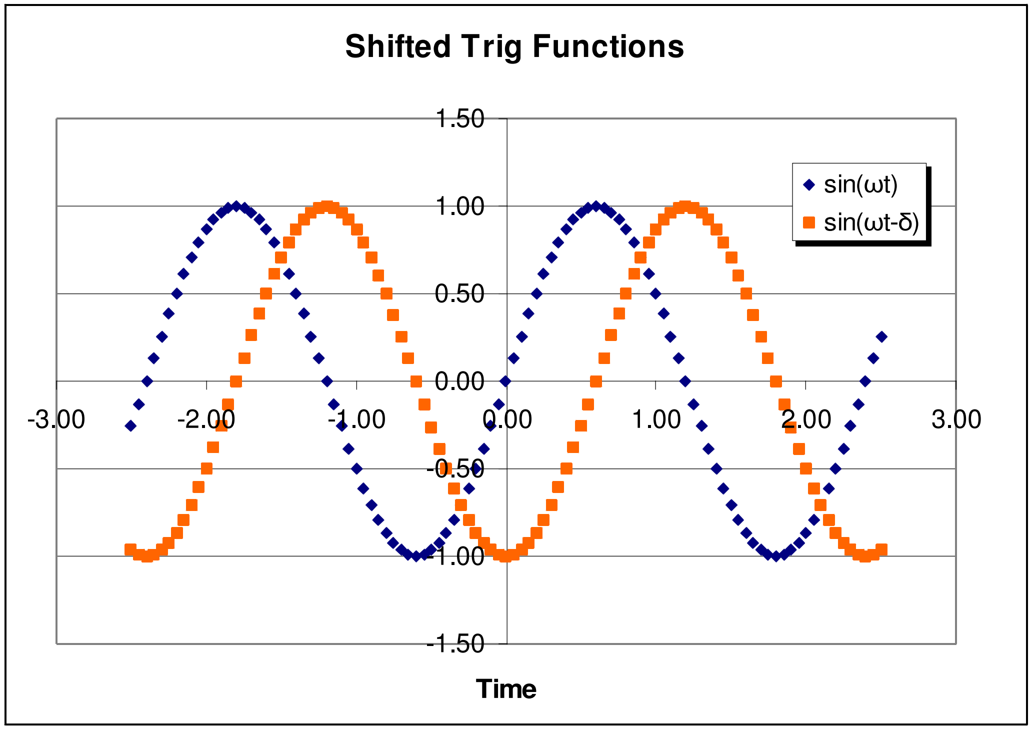
$$\sin\left[\omega t - \frac{\pi}{2}\right] = 0$$

$$\omega t - \frac{\pi}{2} = 0$$

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega}\frac{1}{} ; \qquad \frac{1}{\omega} = \frac{T}{2\pi}$$

$$t = \frac{\pi}{2}\frac{T}{2\pi} = \frac{T}{4}$$



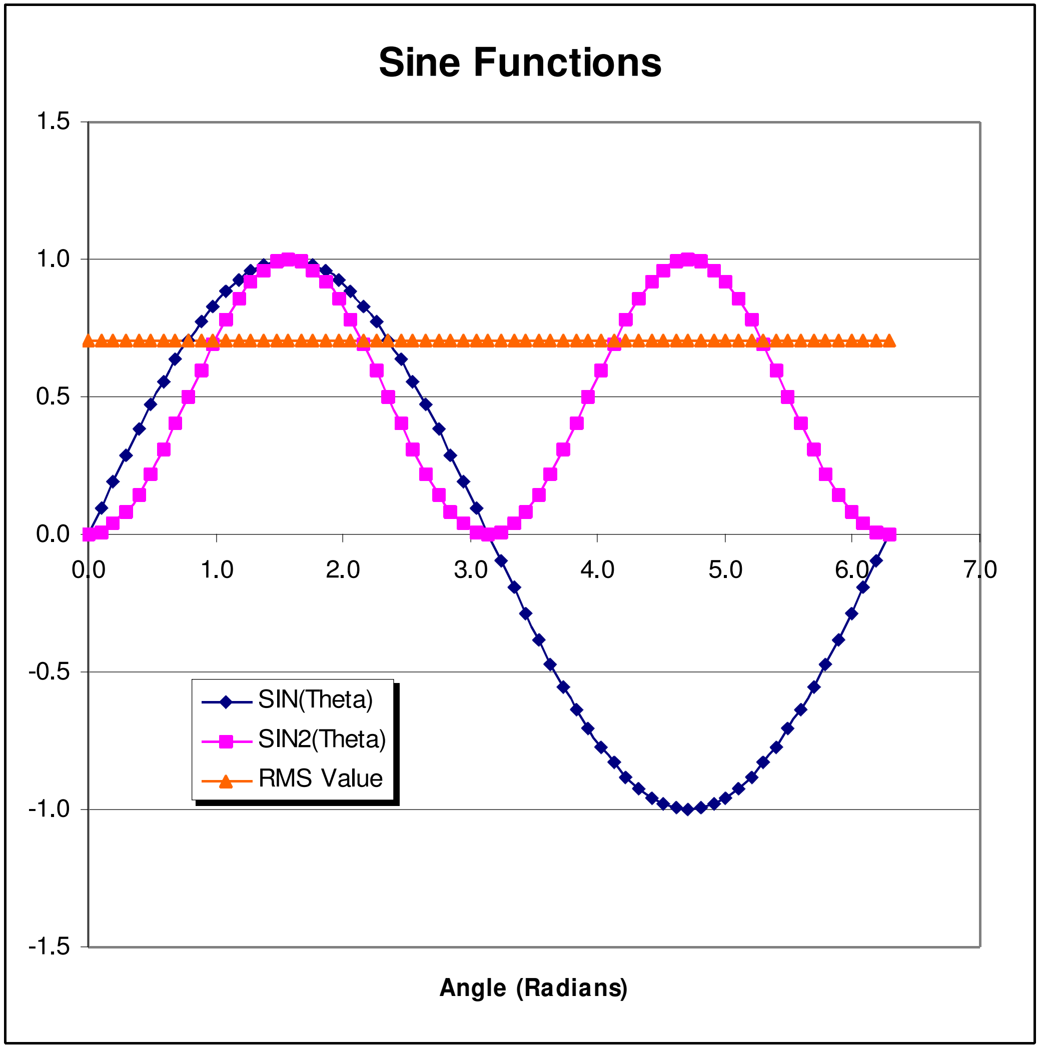
Root Mean Squared

The root mean squared (rms) method of averaging is used when a variable will average to zero but its effect will not average to zero.

Procedure

- Square it (make the negative values positive)
- Take the average (mean)
- Take the square root (undo the squaring operation)

Root Mean Squared Average



Average of a Periodic Function

$$\langle V \rangle = V_{avg} = \frac{1}{T}\int_{o}^{T} V(t)dt; \quad V(t) = V_p sin\omega t$$

$$V_{avg} = \frac{1}{T}\int_{o}^{T} V_p sin\omega tdt = \frac{V_p}{\omega T}\int_{0}^{\omega T} sinxdx = -\frac{V_p}{\omega T}\int_{cos(0)}^{cos(\omega T)} d(cosx)$$

$$V_{avg} = -\frac{V_p}{\omega T}(1 - 1) = 0$$

Root Mean Squared

$$\left\langle V^2 \right\rangle = \left(V^2\right)_{avg} = \frac{1}{T}\int_o^T V^2(t)dt; \quad V(t) = V_p sin\omega t$$

$$\left(V^2\right)_{avg} = \frac{V_p^2}{T}\int_o^T sin^2\,\omega t dt = \frac{V_p^2}{\omega T}\ \pi = \frac{V_p^2}{2}$$

$$\left(V^2\right)_{avg} = \frac{V_p^2}{2}$$

$$V_{RMS} \equiv \sqrt{\left(V^2\right)_{avg}} = \frac{1}{\sqrt{2}}V_p = 0.707V_p$$

Root Mean Squared

Root — Square — Mean

$$V_{RMS} \equiv \sqrt{\left(V^2\right)_{avg}} = \frac{1}{\sqrt{2}} V_p = 0.707 V_p$$

The RMS voltage (V_{RMS})is the DC voltage that has the same effect as the actual AC voltage.

RMS Power

$$P_{avg} = \frac{1}{2} V_p I_p$$

$$since \quad V_{RMS} = \frac{V_p}{\sqrt{2}} \; and \; I_{RMS} = \frac{I_p}{\sqrt{2}}$$

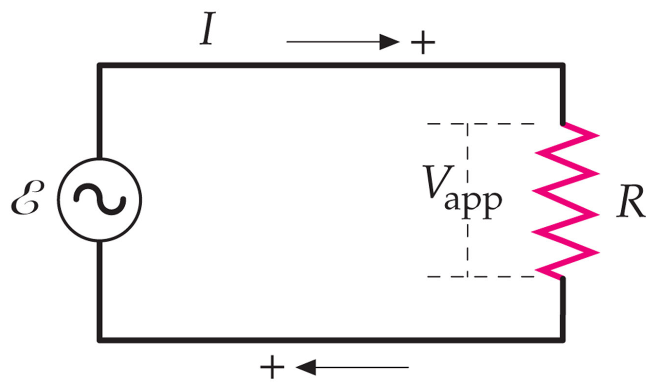
$$P_{avg} = \frac{1}{2}\left(\sqrt{2}\, V_{RMS}\right)\left(\sqrt{2}\, I_{RMS}\right)$$

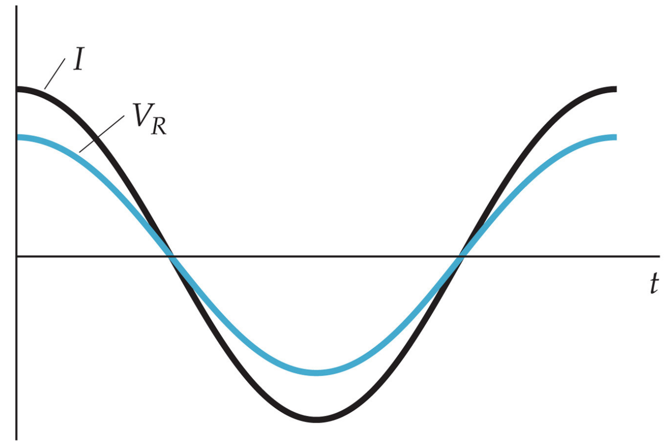
$$P_{avg} = V_{RMS} I_{RMS}$$

The average AC power is the product of the DC equivalent voltage and current.

Resistor in an AC Circuit

Resistor in an AC Circuit





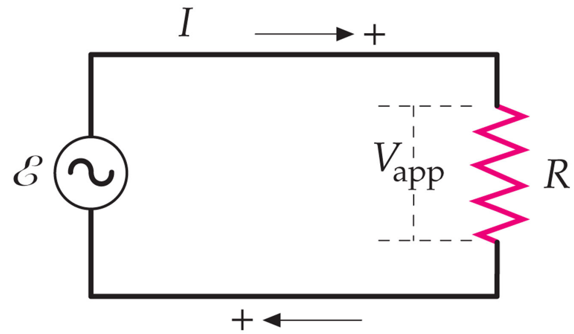
For the case of a resistor in an AC circuit the V_R across the resistor is in phase with the current I through the resistor.

In phase means that both waveforms peak at the same time.

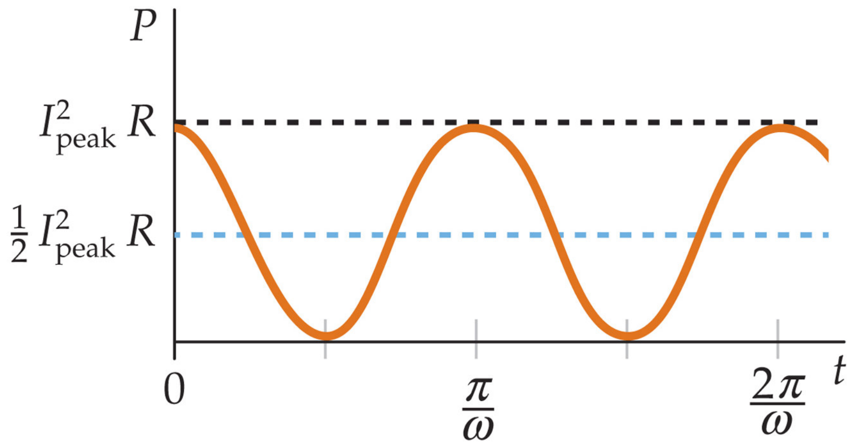
Resistor in an AC Circuit

$$P(t) = I^2(t)R = \left(I_p cos\omega t\right)^2 R$$

$$P(t) = I_p^2 R cos^2\omega t$$



The instantaneous power is a function of time. However, the average power per cycle is of more interest.



$$P_{avg} = \frac{1}{T}\int_0^T P(t)dt$$

$$P_{avg} = \frac{1}{T}\int_0^T I_p^2 R cos^2\omega t dt$$

$$P_{avg} = \frac{I_p^2 R}{T}\int_0^T cos^2\omega t dt = \frac{I_p^2 R}{\omega T}\pi = \frac{1}{2} I_p^2 R = \left(\frac{I_p}{\sqrt{2}}\right)^2 R = I_{RMS}^2 R$$

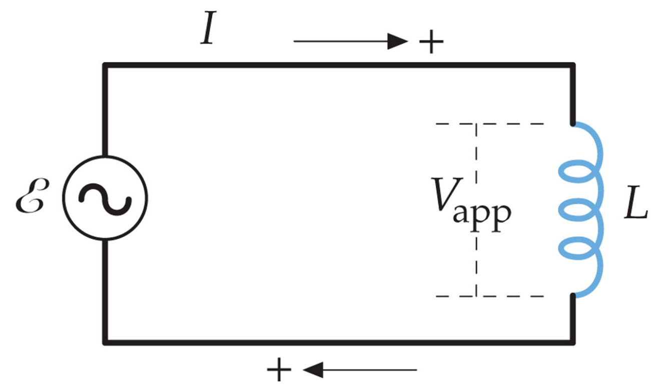
Inductors in an AC Circuit

Coils & Caps in an AC Circuit

	Low Frequency	High Frequency
Capacitor	Open	Short
Inductor	Short	Open

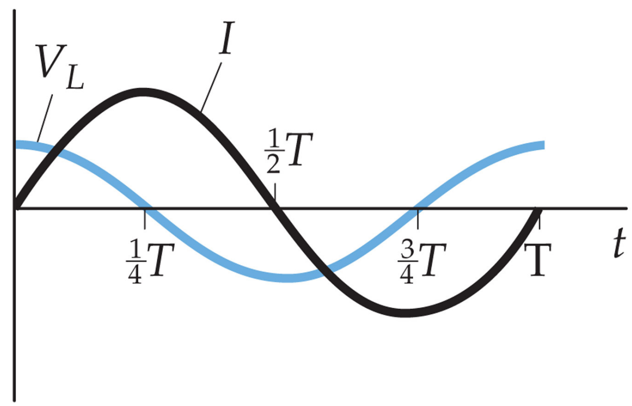
Inductors in an AC Circuit

$$\varepsilon_{peak} \cos\omega t = V_{L\,peak} \cos\omega t = L\frac{dI}{dt}$$



$$I = \frac{V_{L\,peak}}{L}\int \cos\omega t\,dt = \frac{V_{L\,peak}}{\omega L}\sin\omega t$$

$$I = I_p \sin\omega t = I_p \cos\left(\omega t - \pi/2\right)$$



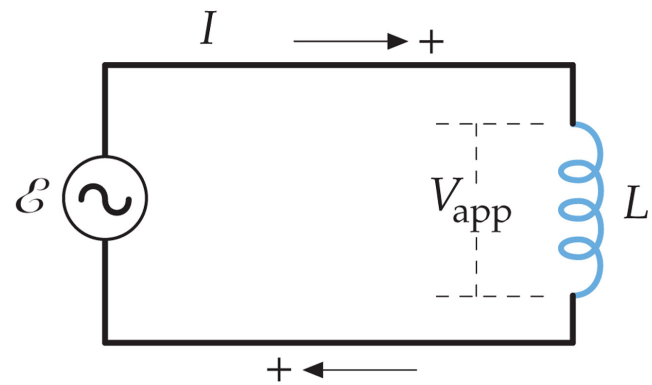
For the case of an inductor in an AC circuit the V_L across the inductor is 90^0 ahead of the current I through the inductor.

Inductors in an AC Circuit

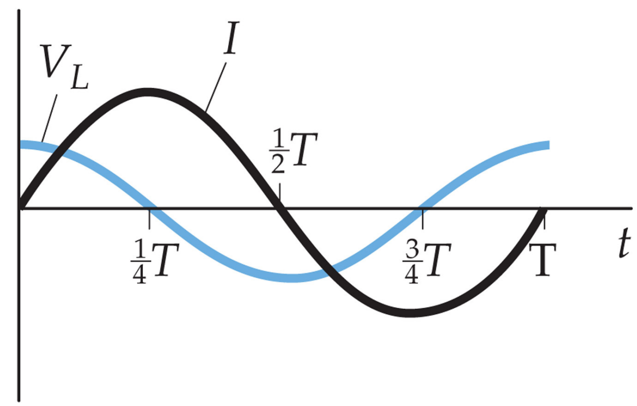
$$I = I_p sin\omega t = \frac{V_{L\ peak}}{\omega L} cos\left(\omega t - \pi/2\right)$$

$$I_p = \frac{V_{L\ peak}}{\omega L} = \frac{V_{L\ peak}}{X_L}$$

$$X_L = \omega L$$



X_L is the inductive reactance



Average Power - Inductors

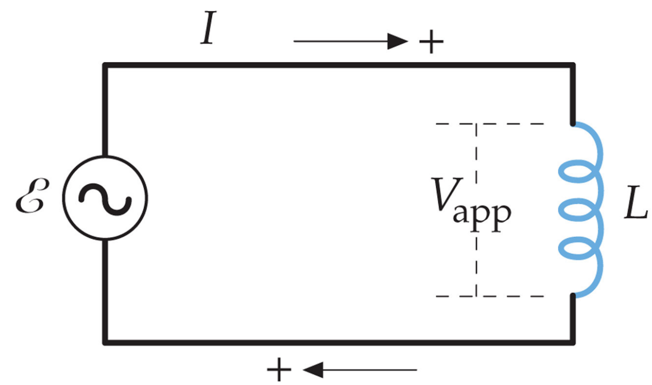
$$P(t) = V_L I = \left(V_{L\,peak} cos\omega t\right)\left(I_p sin\omega t\right)$$

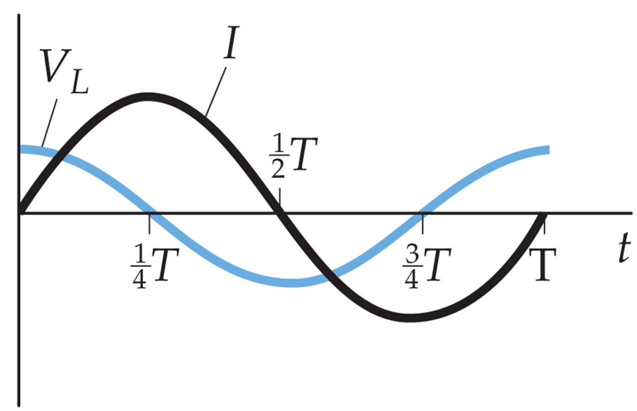
$$P(t) = V_{L\,peak} I_p cos\omega t\ sin\omega t$$

$$P_{avg} = \frac{1}{T}\int_0^T V_{L\,peak} I_p cos\omega t\ sin\omega t dt$$

$$P_{avg} = \frac{V_{L\,peak} I_p}{T}\int_0^T cos\omega t\ sin\omega t dt$$

$$P_{avg} = \frac{V_{L\,peak} I_p}{2T}\int_0^T sin2\omega t dt = 0$$





Inductors don’t dissipate energy, they store energy.

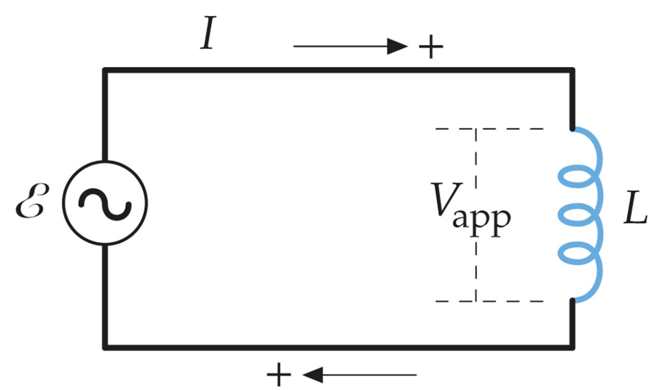
Average Power - Inductors

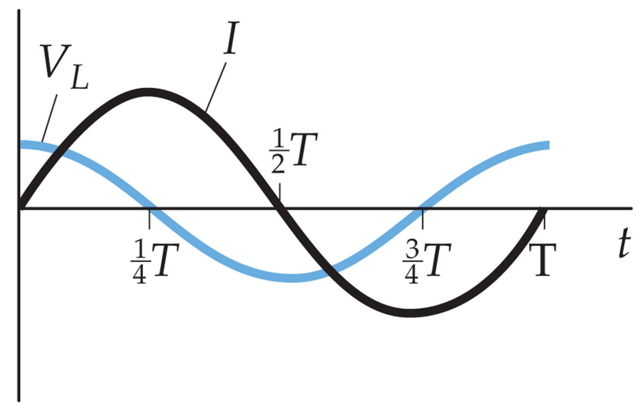
Inductors don't dissipate energy, they store energy.

The voltage and the current are out of phase by 90^o.

As we saw with Work, energy changed only when a portion of the force was in the direction of the displacement.

In electrical circuits energy is dissipated only if a portion of the voltage is in phase with the current.





Capacitors in an AC Circuit

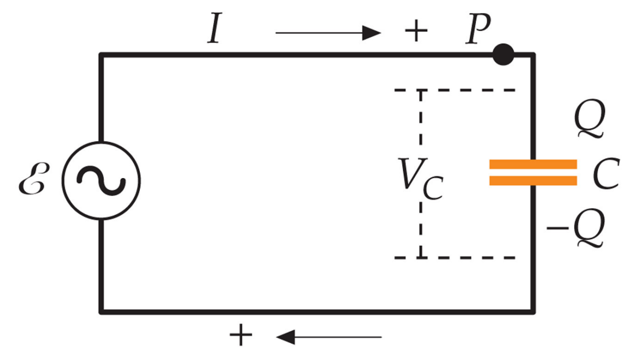
Capacitors in an AC Circuit

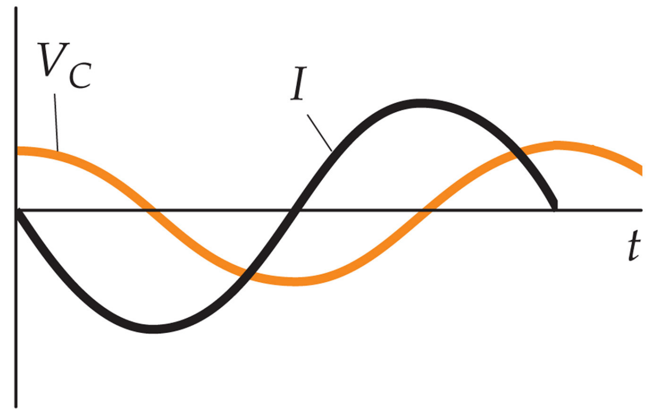
$$V_C = \mathcal{E}_p cos\omega t = V_{C\,p} cos\omega t$$

$$Q = V_C C = V_{C\,p} C cos\omega t = Q_p cos\omega t$$

$$I = \frac{dQ}{dt} = -\omega Q_p sin\omega t = -I_p sin\omega t$$

$$I = -\omega Q_p sin\omega t = I_p cos\left(\omega t + \pi/2\right)$$



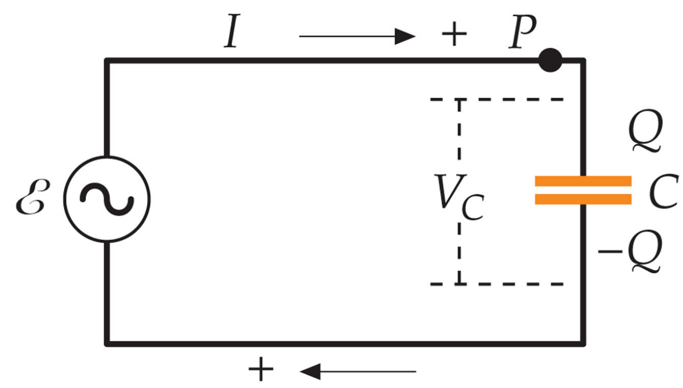


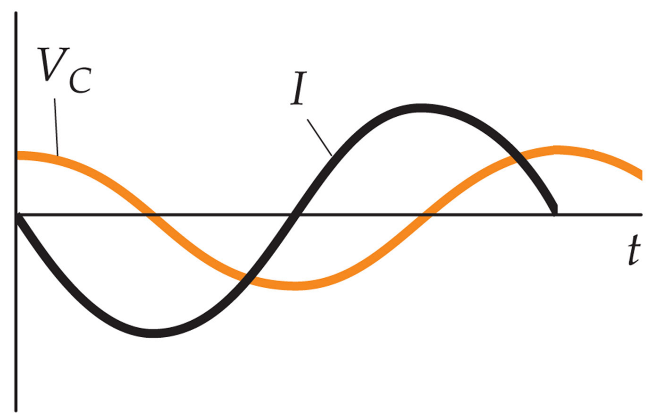
For the case of a capacitor in an AC circuit the V_C across the capacitor is 90^0 behind the current I on the capacitor.

Capacitors in an AC Circuit

$$I_p = \omega Q_p = \omega C V_{Cp} = \frac{V_{Cp}}{1/\omega C} = \frac{V_{Cp}}{X_C}$$

$$X_C = \frac{1}{\omega C}$$

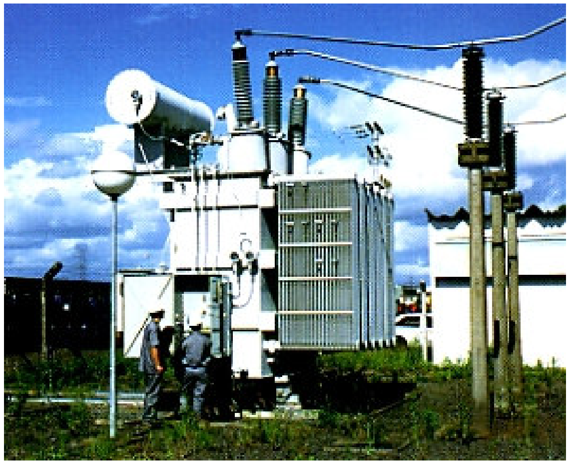




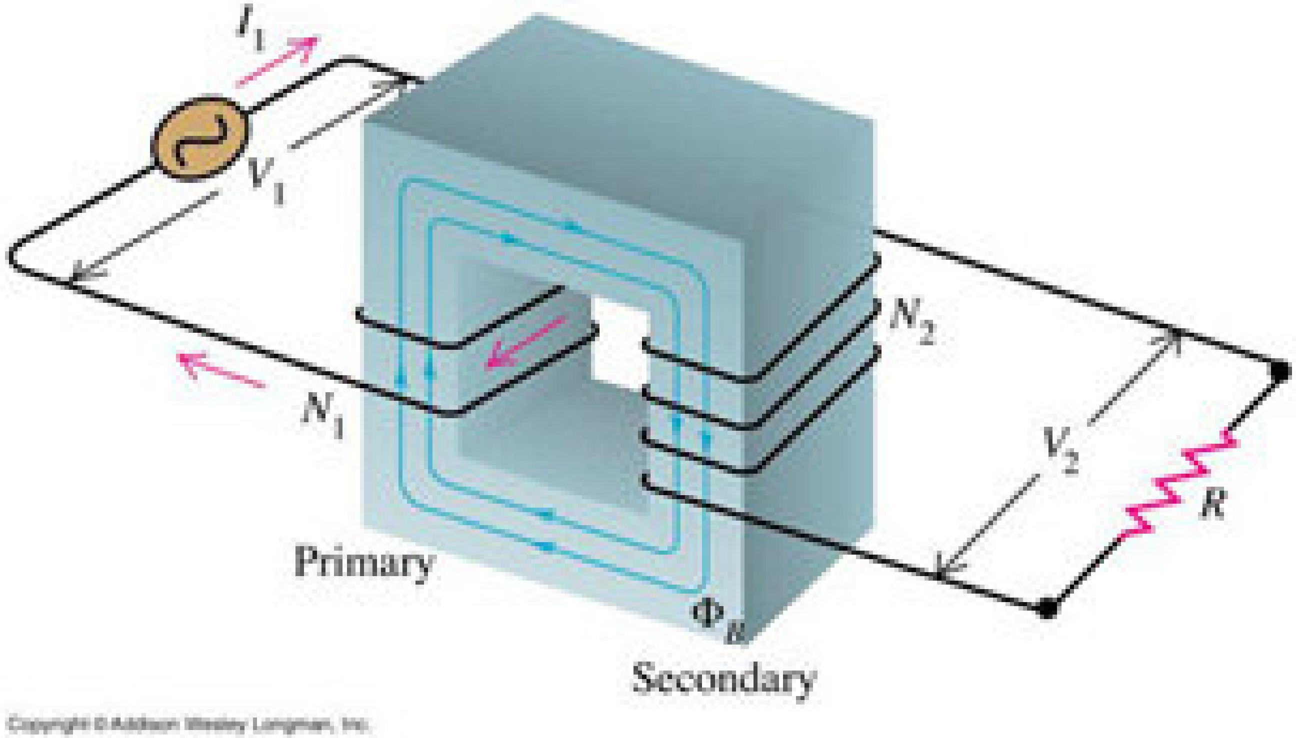
X_C is the capacitive reactance.

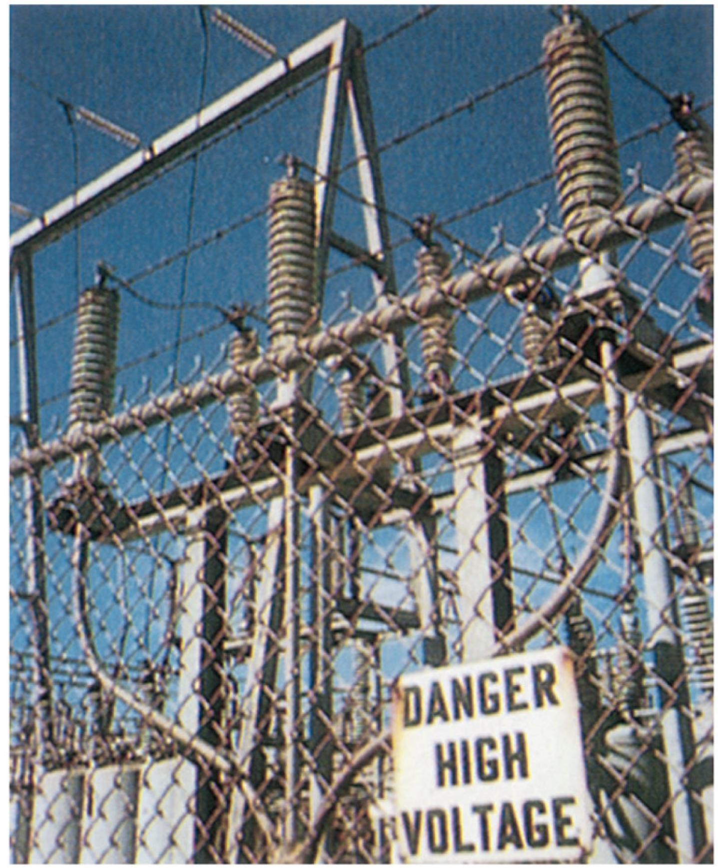
Electrical Transformers

Electrical Transformers

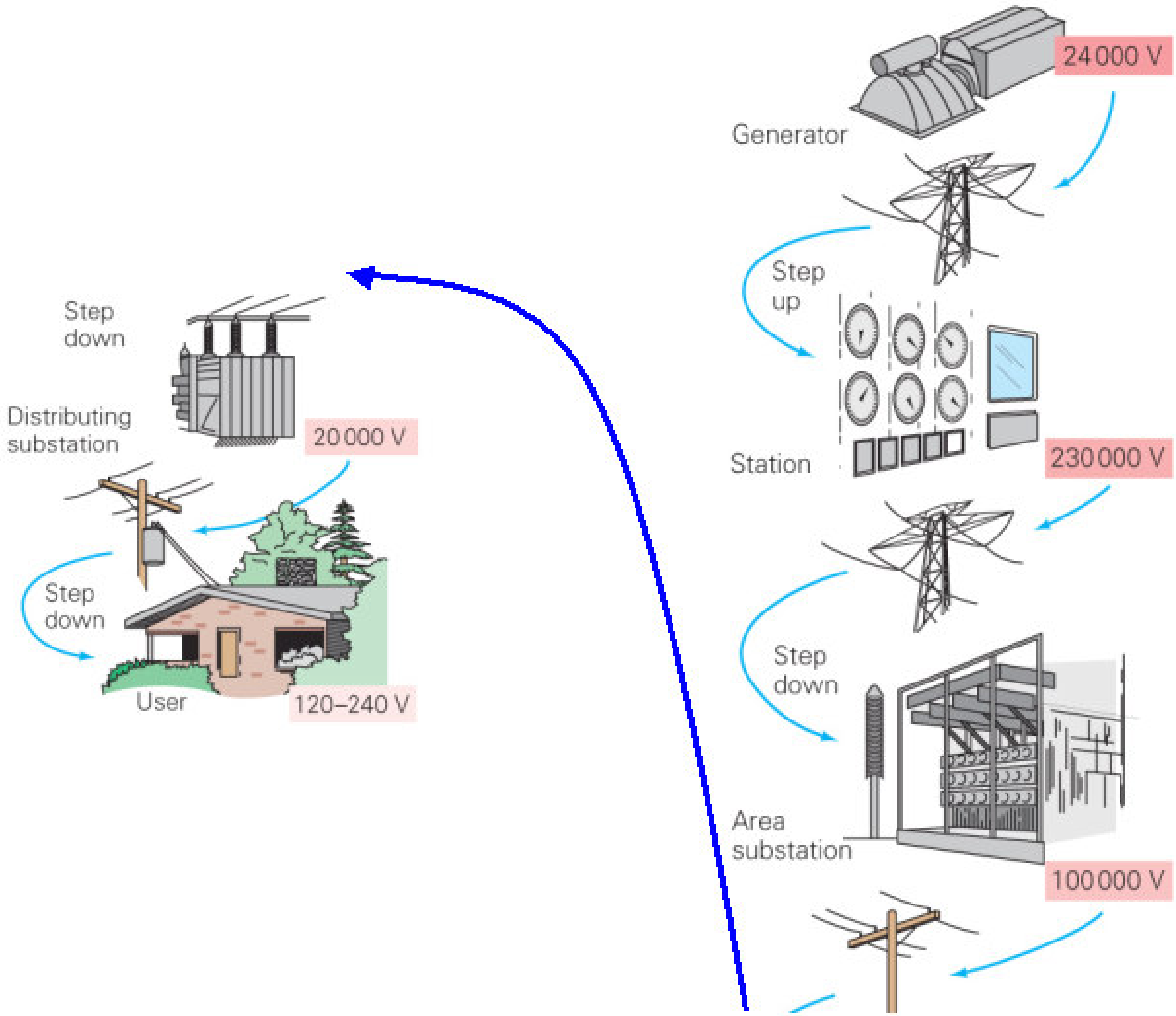


Electrical Transformers




DANGER
HIGH
VOLTAGE

Electrical Transformers



Electrical Transformers

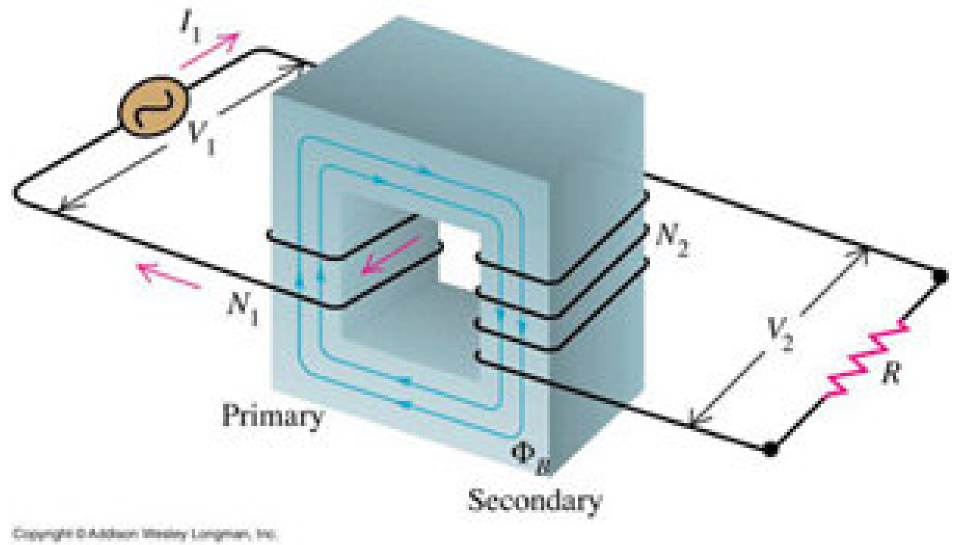
Both coils see the same magnetic flux and the cross sectional areas are the same

$$B = \mu_0 n I$$

$$\mu_0 n_1 I_1 = \mu_0 n_2 I_2$$

$$n_1 I_1 = n_2 I_2$$

$$I_2 = \frac{n_1}{n_2} I_1$$



$$\frac{I_1}{I_2} = \frac{n_2}{n_1} = \frac{N_2/L}{N_1/L} = \frac{N_2}{N_1}$$

Electrical Transformers

Conservation of Energy

Primary Power = Secondary Power

$$V_{in} I_1 = V_{out} I_2$$

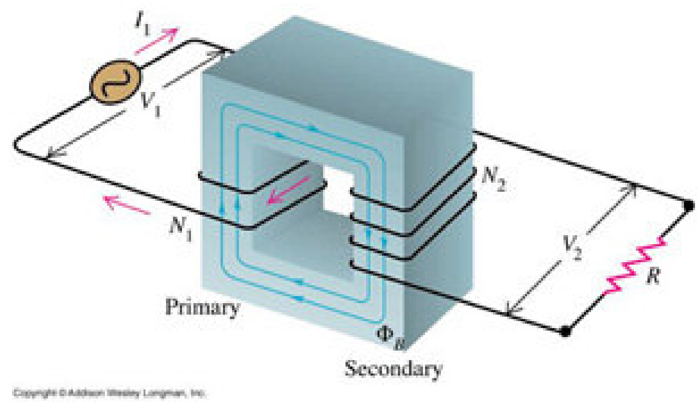
$$\frac{V_{out}}{V_{in}} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$V_{out} = \frac{N_2}{N_1} V_{in}$$

Induced voltage/loop

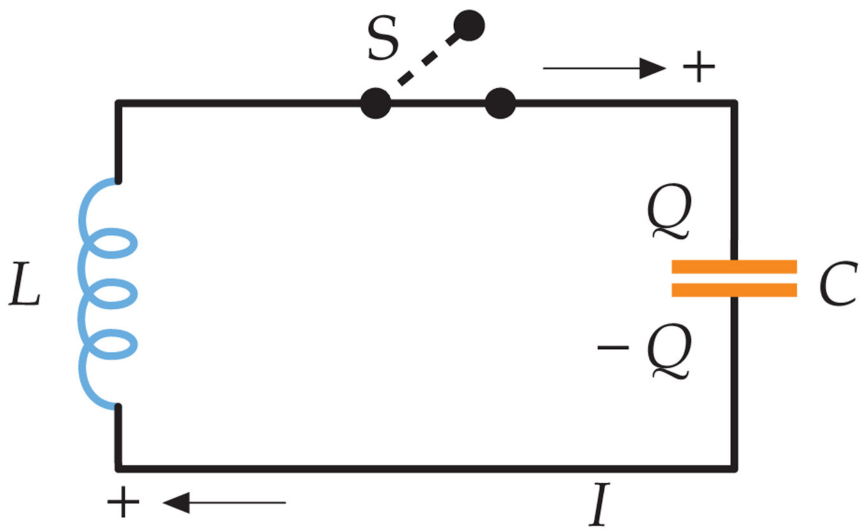
More loops => more voltage

Voltage steps up but the current steps down.



LC and RLC Circuits Without a Generator

LC Circuit - No Generator

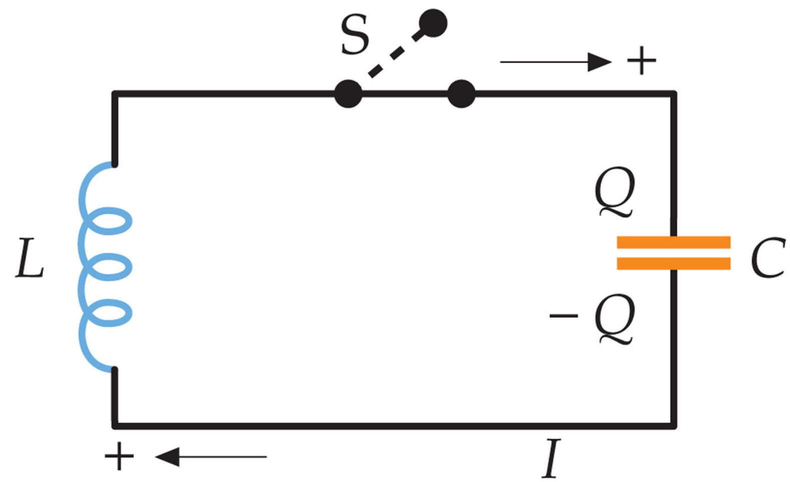


To start this circuit some energy must be placed in it since there is no battery to drive the circuit. We will do that by placing a charge on the capacitor

Since there is no resistor in the circuit and the resistance of the coil is assumed to be zero there will not be any losses.

LC Circuit - No Generator

Apply Kirchhoff's rule



$$L\frac{dI}{dt}+\frac{Q}{C}=0$$

$$Since\ \ I=\frac{dQ}{dt}$$

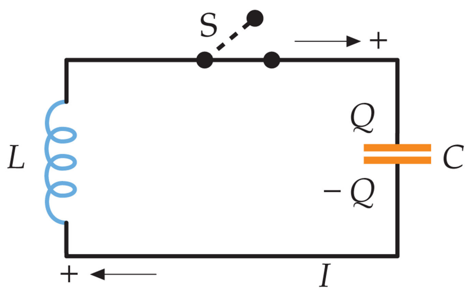
$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0$$

$$\frac{d^2Q}{dt^2}=-\frac{1}{LC}Q$$

← This is the harmonic oscillator equation

$$\omega_R=\frac{1}{\sqrt{LC}}$$

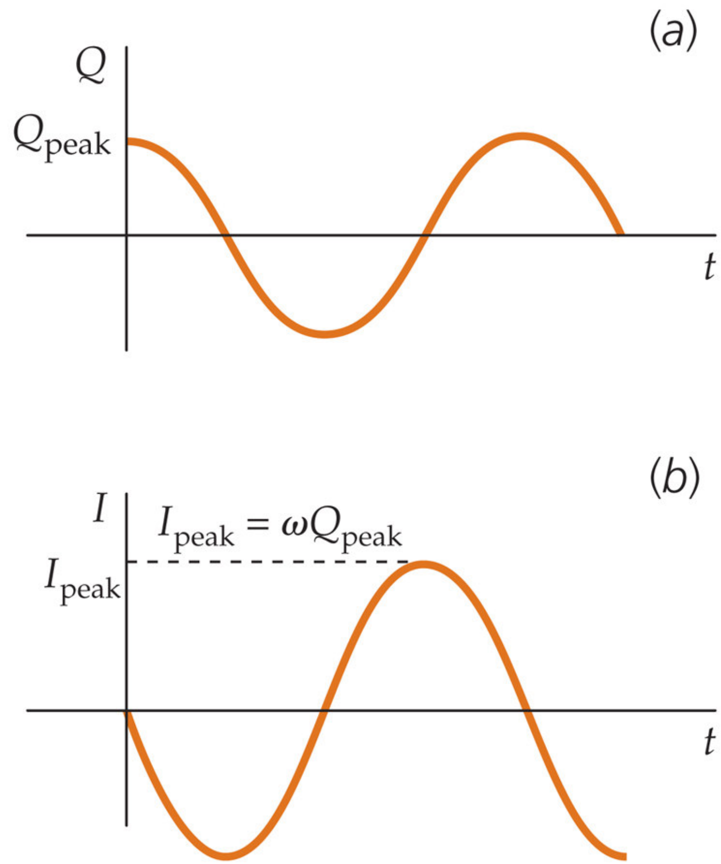
LC Circuit - No Generator



$$Q(t) = Q_p cos\omega t$$

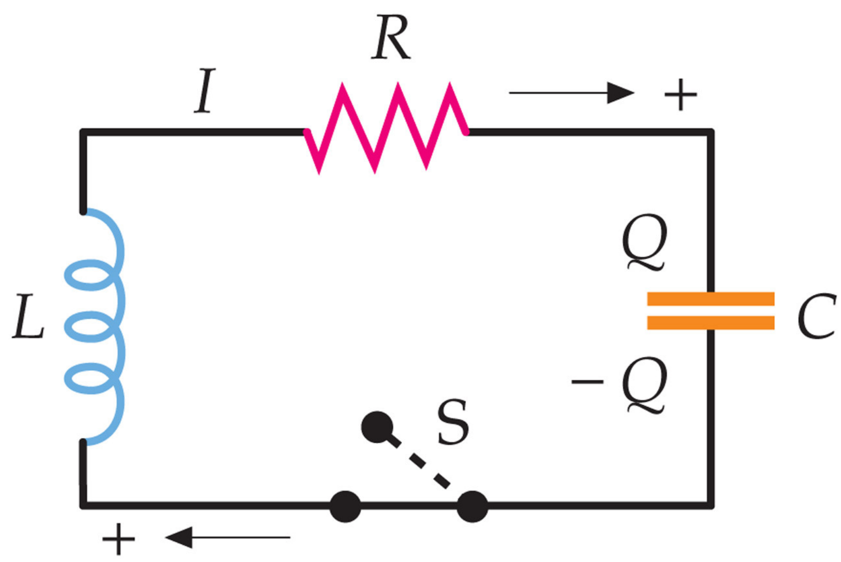
$$I(t) = \frac{dQ}{dt} = -\omega Q_p sin\omega t$$

$$I(t) = -\omega Q_p cos\left(\omega t + \pi/2\right)$$



The circuit will oscillate at the frequency ω_R. Energy will flow back and forth from the capacitor (electric energy) to the inductor (magnetic energy).

RLC Circuit - No Generator

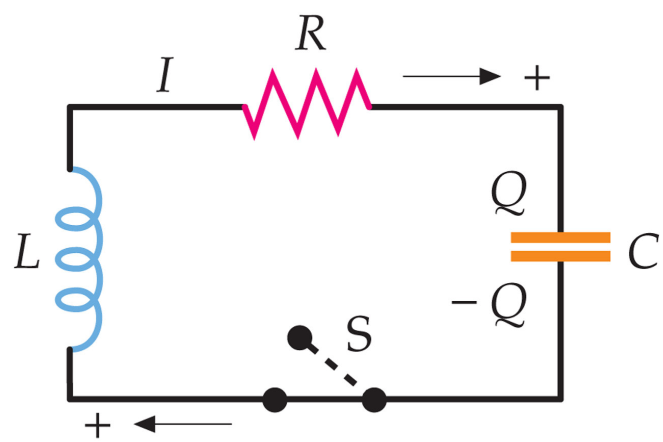


Like the LC circuit some energy must initially be placed in this circuit since there is no battery to drive the circuit. Again we will do this by placing a charge on the capacitor

Since there is a resistor in the circuit now there will be losses as the energy passes through the resistor.

RLC Circuit - No Generator

Apply Kirchhoff's rule



$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0\,; \quad I = \frac{dQ}{dt}$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

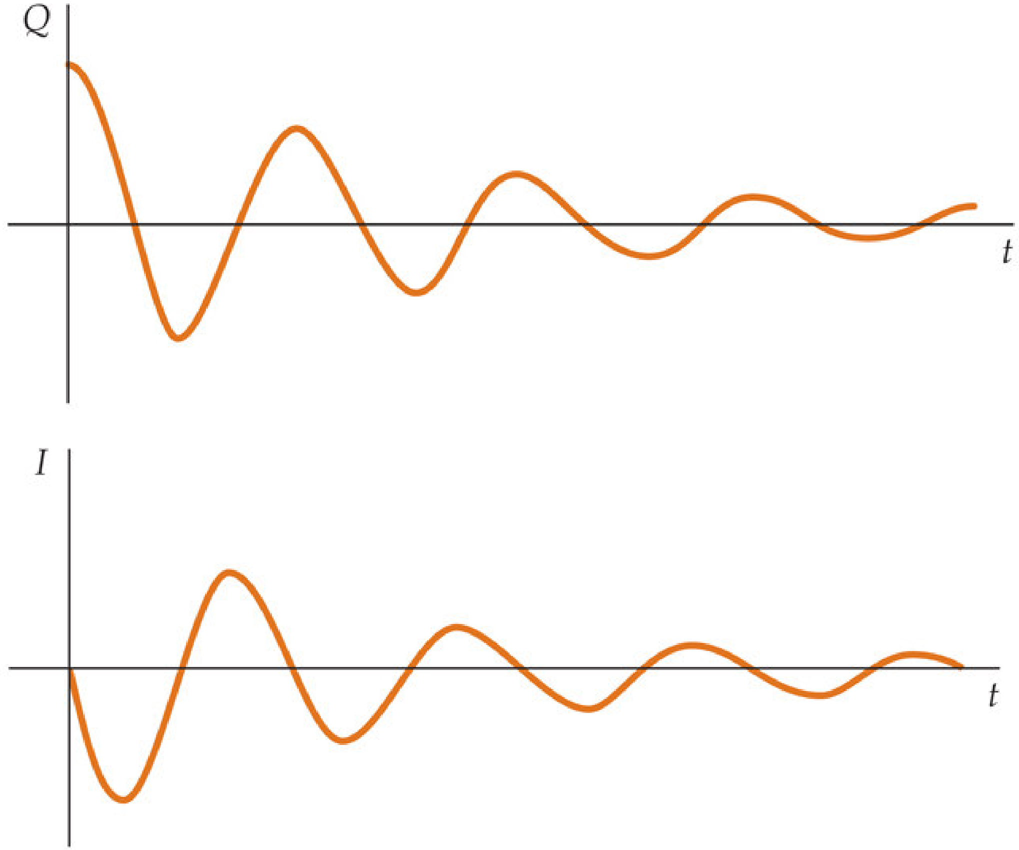
Restoring force "kx"

Damping term - friction

"ma" term

The damping term causes a damping of the natural oscillations of the circuit.

RLC Circuit - No Generator



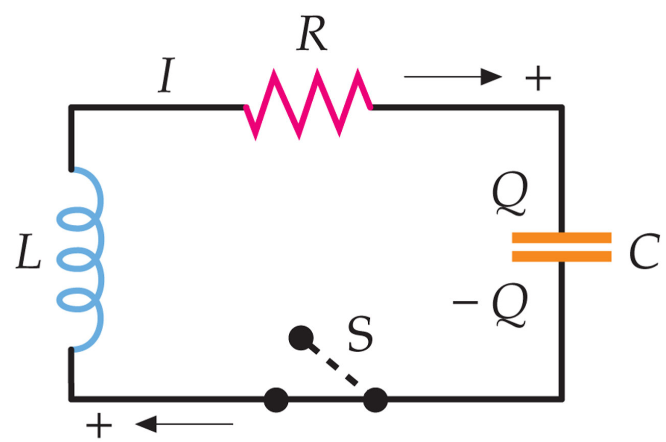
RLC Circuit - No Generator

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = 0$$ Multiply by I

$$LI\frac{dI}{dt} + I^2R + \frac{QI}{C} = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}LI^2\right] + I^2R + \frac{d}{dt}\left[\frac{1}{2}\frac{Q^2}{C}\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}\right] = -I^2R$$

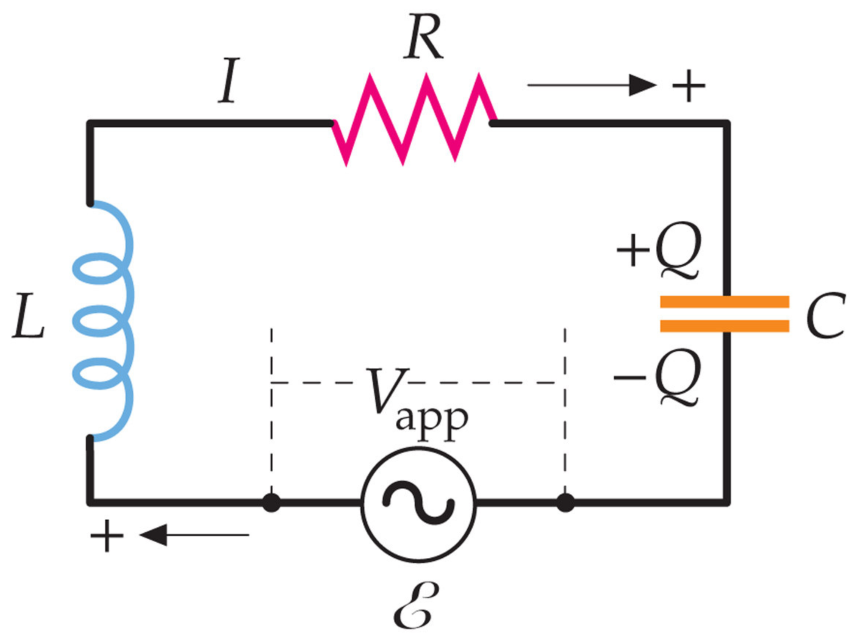


The rate of change of the stored energy = - Power dissipated in the resistor

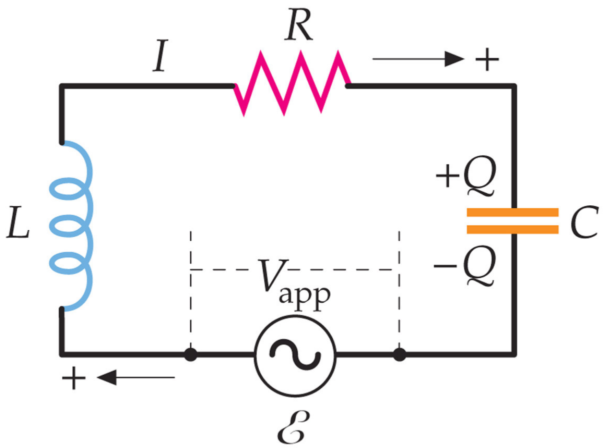
Series RLC Circuit with Generator

We have already examined the components in this circuit to understand the phase relations of the voltage and current of each component

Now we will examine the power relationships



Series RLC Circuit with Generator



$$\mathcal{E}(t) = \mathcal{E}_{peak} sin\omega t = \mathcal{E}_p sin\omega t$$

Apply Kirchhoff's Loop rule to the circuit

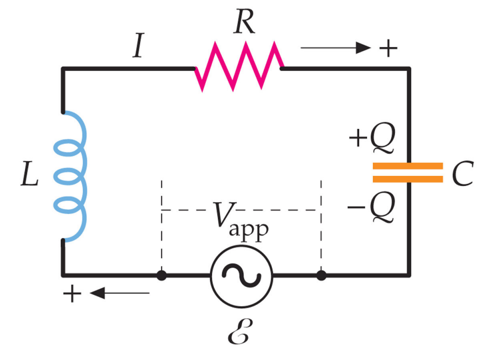
$$RI(t) + L\frac{dI(t)}{dt} + \frac{Q(t)}{C} = \mathcal{E}(t)$$

$$\frac{dQ}{dt} = I \quad \Rightarrow \quad Q(t) = Q_o + \int_0^t I(t')dt'$$

$$RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(t')dt' = \mathcal{E}(t); \;\; with\, Q_0 = 0$$

Series RLC Circuit with Generator

$$RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(t')dt' = \mathcal{E}(t)$$



$$Steady\ state \Rightarrow I(t) = I_p sin\omega t$$

$$\frac{dI(t)}{dt} = \omega I_p cos\omega t; \quad \int_0^t I(t')dt' = \int_0^t I_p sin\omega t' dt' = -\frac{I_p}{\omega}cos\omega t$$

$$RI_p sin\omega t + \omega L I_p cos\omega t - \frac{1}{\omega C} I_p cos\omega t = \mathcal{E}_p sin\omega t$$

Series RLC Circuit with Generator

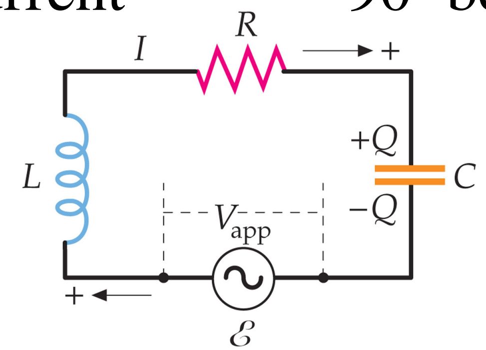
$$RI_p sin\omega t + \omega LI_p cos\omega t - \frac{1}{\omega C} I_p cos\omega t = \mathcal{E}_p sin\omega t$$

Change all "cos" to "sin" by shifting the angle

$$RI_p sin\omega t + \omega LI_p sin\left(\omega t + \pi/_2\right) + \frac{1}{\omega C} I_p sin\left(\omega t - \pi/_2\right) = \mathcal{E}_p sin\omega t$$

The inductive voltage is 90° ahead of the current

The capacitive voltage is 90° behind of the current



Impedance in a Series RLC Circuit

$$RI_p \sin\omega t + \omega L I_p \sin\left(\omega t + \pi/2\right) + \frac{1}{\omega C} I_p \sin\left(\omega t - \pi/2\right) = \mathcal{E}_p \sin\omega t$$

The coefficients are voltages

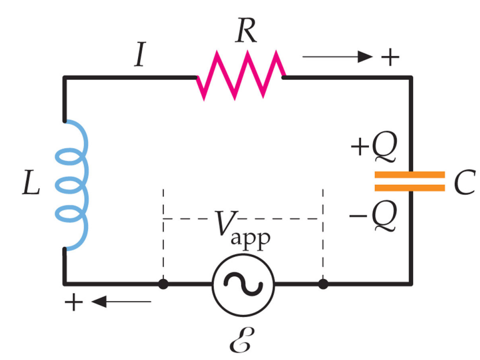
R	L	C
RI_p	$\omega L I_p$	$I_p/\omega C$
	$X_L I_p$	$X_C I_p$

$$X_L = \omega L \quad X_C = 1/\omega C$$

X_L is the inductive reactance.

X_C is the capacitive reactance.

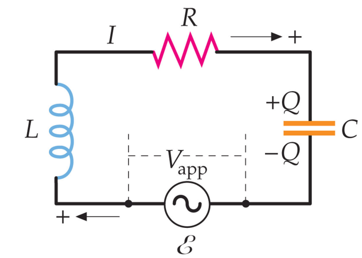
The R and X_L and X_C values are called impedances. That is a generlized term for resistance since they all have units of ohms.



Power in a Series RLC Circuit

Now we go back to the original equation and multiply by

$I(t) = I_p sin\omega t$ and integrate over one cycle: 0 => T



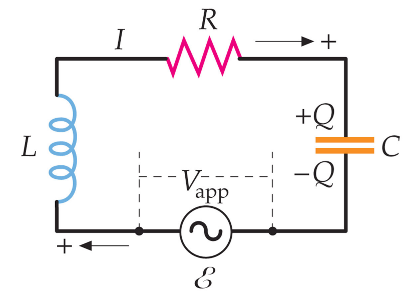
$$RI_p sin\omega t + \omega LI_p cos\omega t - \frac{1}{\omega C} I_p cos\omega t = \mathcal{E}_p sin\omega t$$

$$RI_p^2 \int_o^T sin^2\omega tdt + \omega LI_p^2 \int_o^T sin\omega tcos\omega tdt - \frac{I_p^2}{\omega C} \int_o^T sin\omega tcos\omega t\ dt = \mathcal{E}_p I_p \int_o^T sin^2\omega tdt$$

$$\int_0^T sin^2 \omega tdt = \pi \qquad \int_0^T sin\omega t\ cos\omega tdt = 0$$

Power in a Series RLC Circuit

$$RI_p^2(\pi) + \omega L I_p^2(0) - \frac{I_p^2}{\omega C}(0) = E_p I_p(\pi)$$



$$RI_p^2 = E_p I_p$$

Power in resistor

Power out of battery

- Power is only dissipated in the resistor.
- The inductor stores energy in its magnetic field.
- The capacitor stores its energy in its electric field.

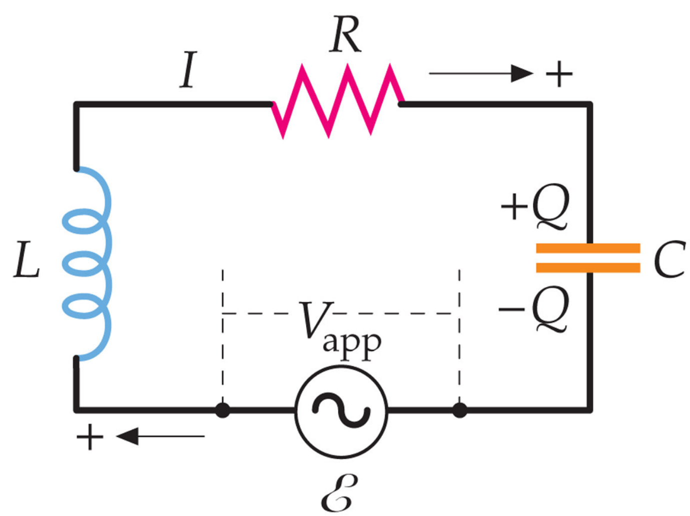
Series RLC Circuit with Generator

We have used this equation to demonstrate the behavior of the three types of components: R, L and C, but-

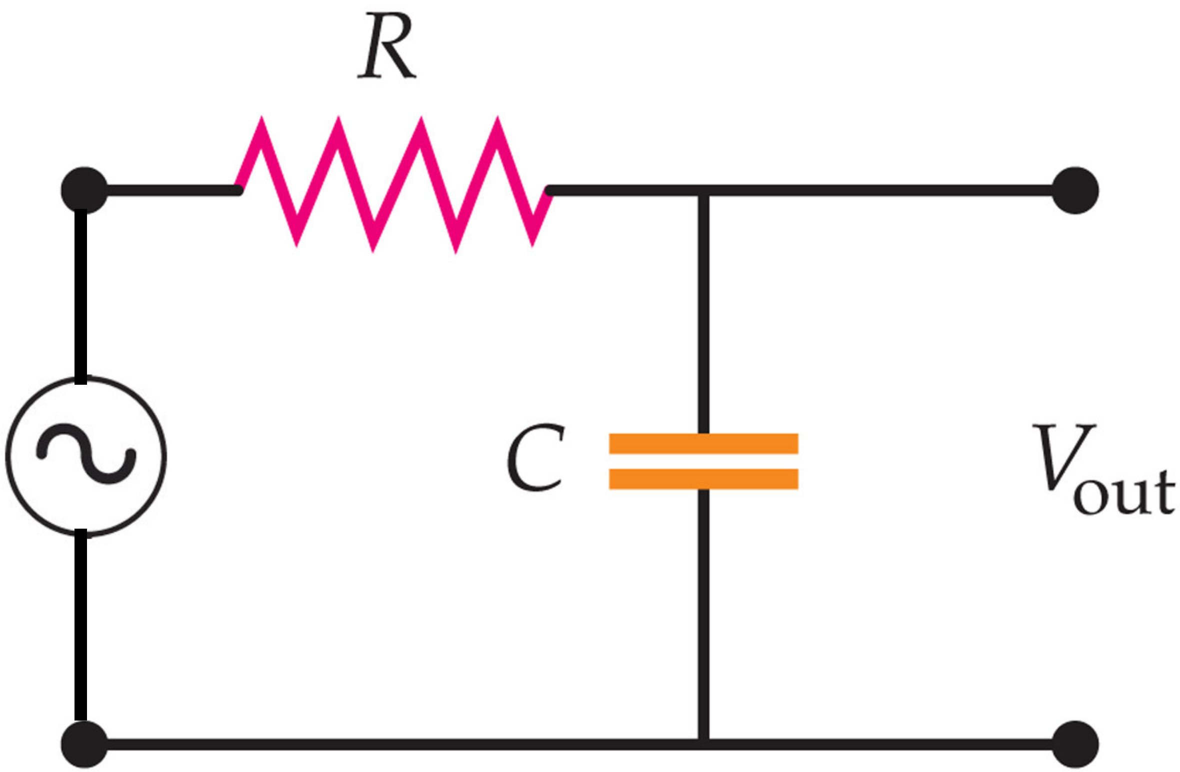
We still haven't solved the equation

$$RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(t')dt' = \mathcal{E}(t);\ \ with\ Q_0 = 0$$

Before we actually solve it we need to introduce complex variables that will be used in the solution.



The RC Circuit

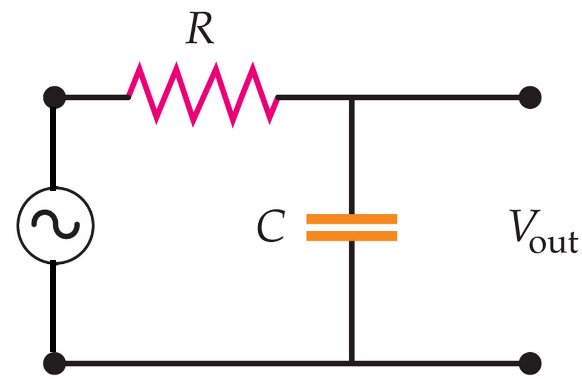


The RC Circuit - Low Freq

$$RI_p \sin\omega t + \omega L I_p \cos\omega t - \frac{1}{\omega C} I_p \cos\omega t = \mathcal{E}_p \sin\omega t$$

Let L => 0

$$RI_p \sin\omega t - \frac{1}{\omega C} I_p \cos\omega t = \mathcal{E}_p \sin\omega t$$

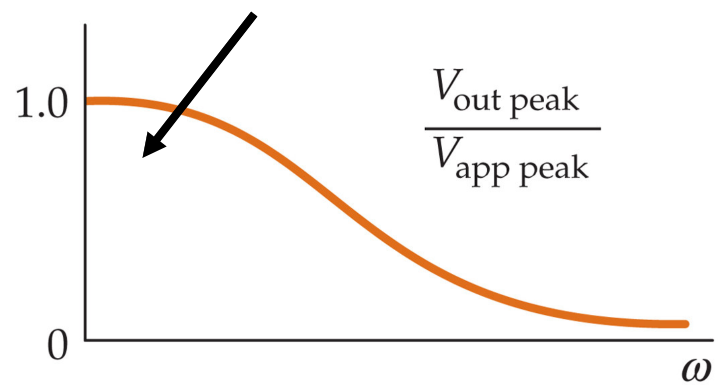


$$For\ \omega t \simeq 0;\ \cos\omega t \simeq 1;\ \sin\omega t \simeq 0$$

Low ω

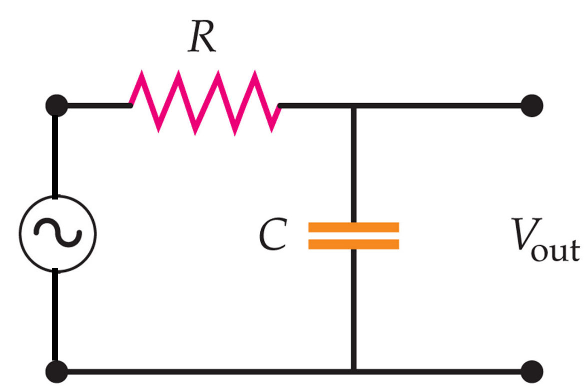
$$-\frac{1}{\omega C} I_p = \mathcal{E}_p(0)$$

$$I_p = 0 \Rightarrow Open\ circuit$$



The RC Circuit - High Freq

$$RI_p \sin\omega t - \frac{1}{\omega C} I_p \cos\omega t = \mathcal{E}_p \sin\omega t$$



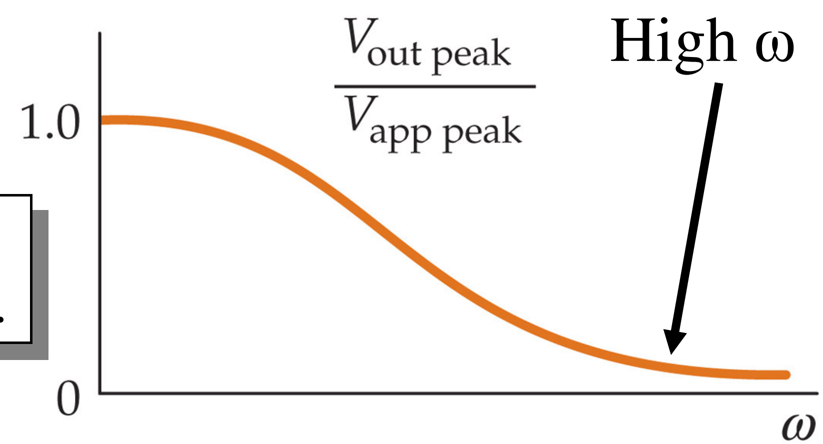
$$\text{For } \omega \gg {}^{1}\!/_{CR}$$

$$I_p \sin\omega t - \frac{1}{\omega CR} I_p \cos\omega t = \frac{\mathcal{E}_p}{R} \sin\omega t$$

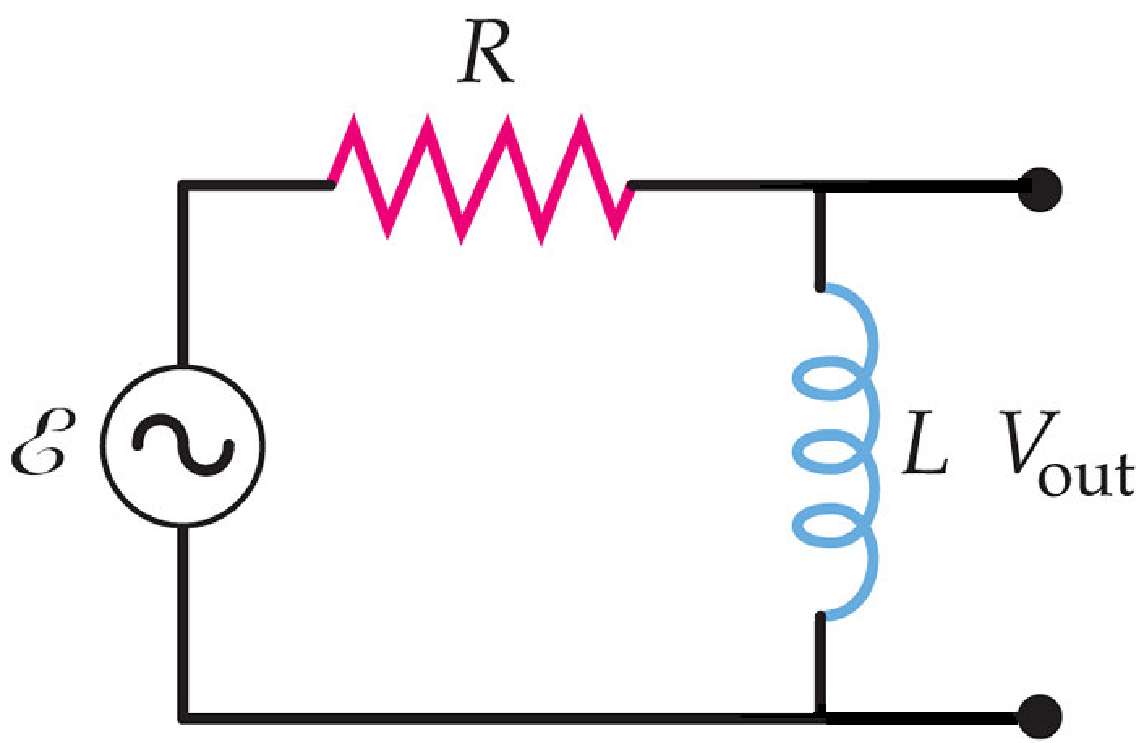
$$I_p \sin\omega t = \frac{\mathcal{E}_p}{R} \sin\omega t$$

$$I_p = \frac{\mathcal{E}_p}{R}$$

At high frequency the cap acts as a short circuit.



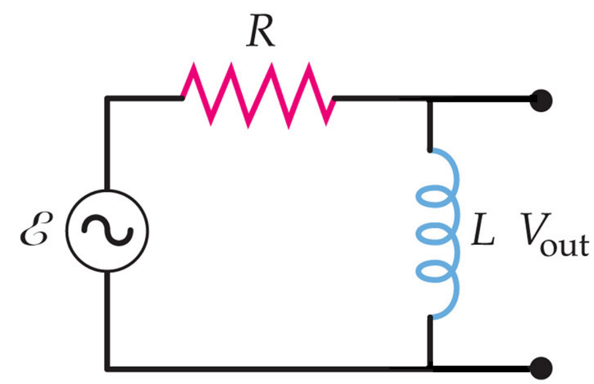
The RL Circuit



The RL Circuit - Low Freq

$$RI_p sin\omega t + \omega LI_p cos\omega t - \frac{1}{\omega C} I_p cos\omega t = \mathcal{E}_p sin\omega t$$

Let C => 0



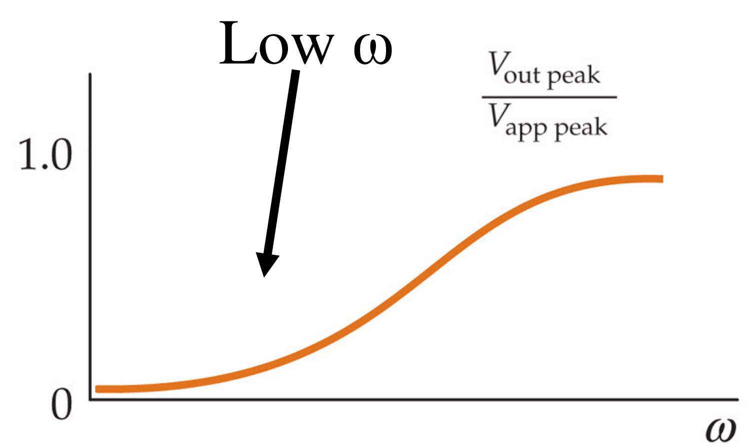
$$RI_p sin\omega t + \omega LI_p cos\omega t = \mathcal{E}_p sin\omega t$$

$$For\ \omega L \ll 1$$

$$RI_p sin\omega t = \mathcal{E}_p sin\omega t$$

$$RI_p = \mathcal{E}_p$$

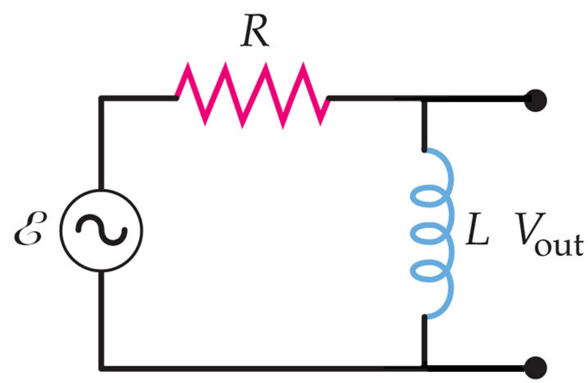
At low frequency L acts as a short circuit.



The RL Circuit - High Freq

$$RI_p \sin\omega t + \omega L I_p \cos\omega t = \mathcal{E}_p \sin\omega t$$

$$For\ \omega L \gg 1$$



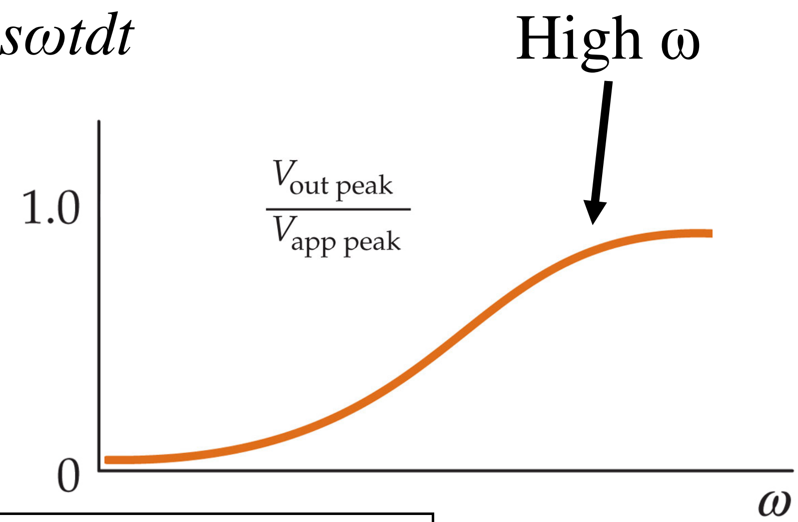
$$\omega L I_p \cos\omega t = \mathcal{E}_p \sin\omega t$$

multiply by cosωt and average

$$\omega L I_p \frac{1}{T}\int_0^T \cos^2\omega t\,dt = \mathcal{E}_p \frac{1}{T}\int_0^T \sin\omega t \cos\omega t\,dt$$

$$\omega L I_p \pi = 0$$

$$I_p = 0 \Rightarrow Open\ circuit$$



At high frequency L acts as an open circuit.

Coils & Caps in an AC Circuit

	Low Frequency	High Frequency
Capacitor	Open	Short
Inductor	Short	Open

Complex Numbers for AC Circuits

Complex Numbers for AC Circuits

The basic complex (imaginary) number is "i."

To avoid confusion we replace "i" with "j"

$$j = \sqrt{-1}$$

$$j^2 = jj = \sqrt{-1}\sqrt{-1} = -1$$

$$j^3 = jj^2 = j(-1) = -j$$

$$j^4 = j^2 j^2 = (-1)(-1) = +1$$

$$j^5 = jj^4 = j(+1) = j$$

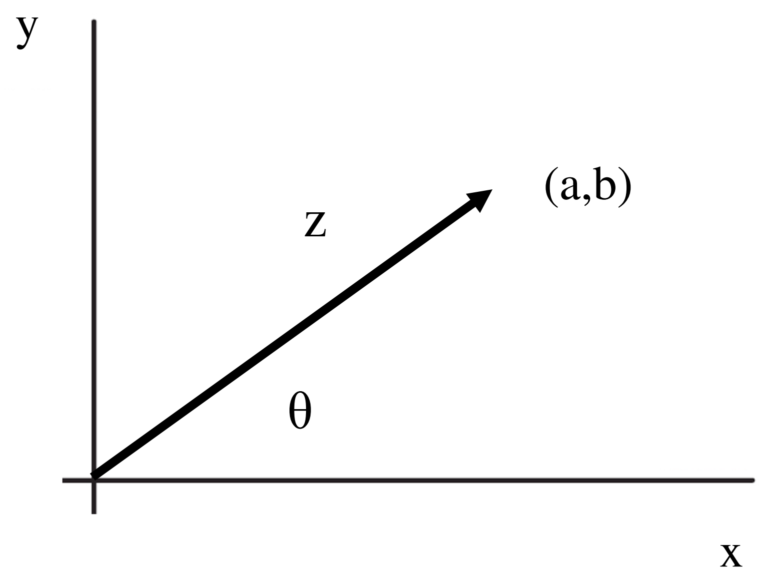
Complex Numbers for AC Circuits

Let a and b be real numbers

Then z is a complex number and z* is the complex conjugate

$$z = a + bj$$

$$z^* = a - bj$$



Complex Numbers for AC Circuits

The magnitude of z

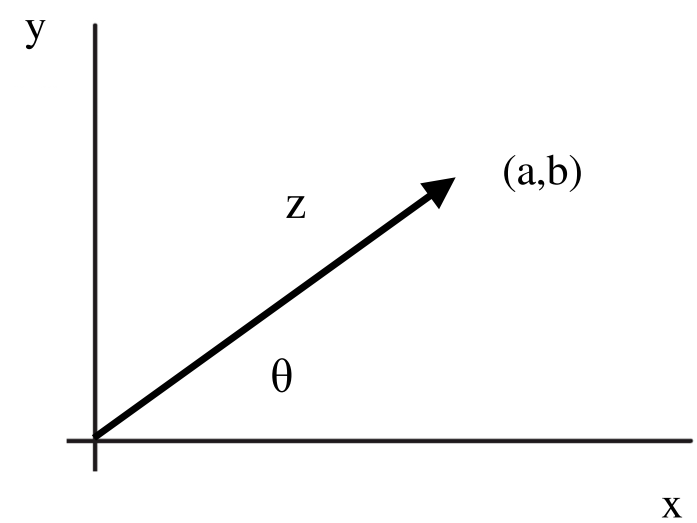
$$Magn(z) = |z| = \sqrt{(z^*)z} = \sqrt{(a - bj)(a + bj)}$$

$$|z| = \sqrt{a^2 + abj - abj - j^2b^2}$$

$$|z| = \sqrt{a^2 + b^2}$$

$$tan\theta = b/a; \quad \theta = tan^{-1}(b/a)$$

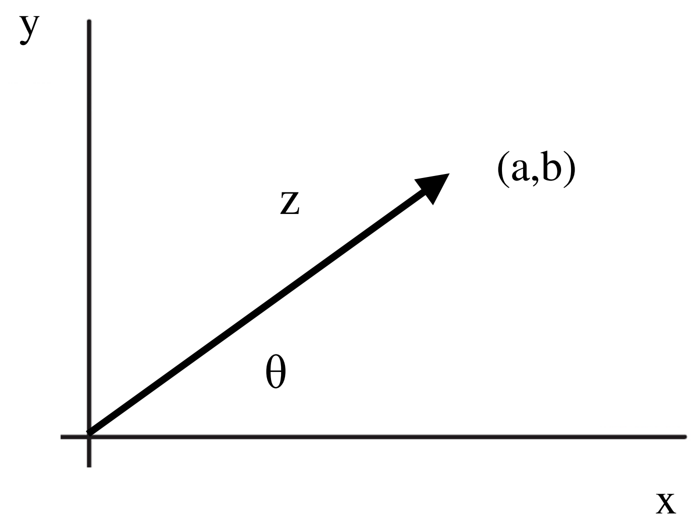
$$z = |z|(cos\theta + jsin\theta) = |z|e^{j\theta}$$



The exponential representation of a complex number will prove useful in solving the RLC differenial eqn.

Complex Numbers for AC Circuits

$$z = |z|(cos\theta + jsin\theta) = |z|e^{j\theta}$$



$e^{j\theta}$ can be viewed as a rotation operator in a complex space

$$e^{j\pi/2} = j$$

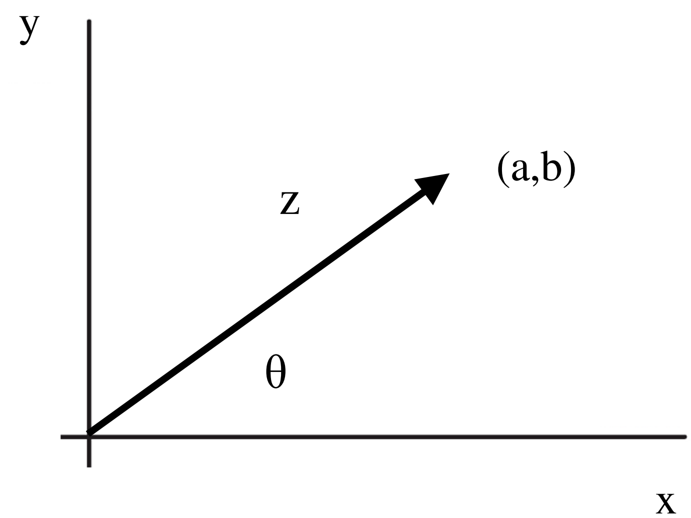
$$e^{j\pi} = -1$$

$$e^{j3\pi/2} = -j$$

$$e^{j2\pi} = e^{0} = +1$$

Why Complex Numbers ?

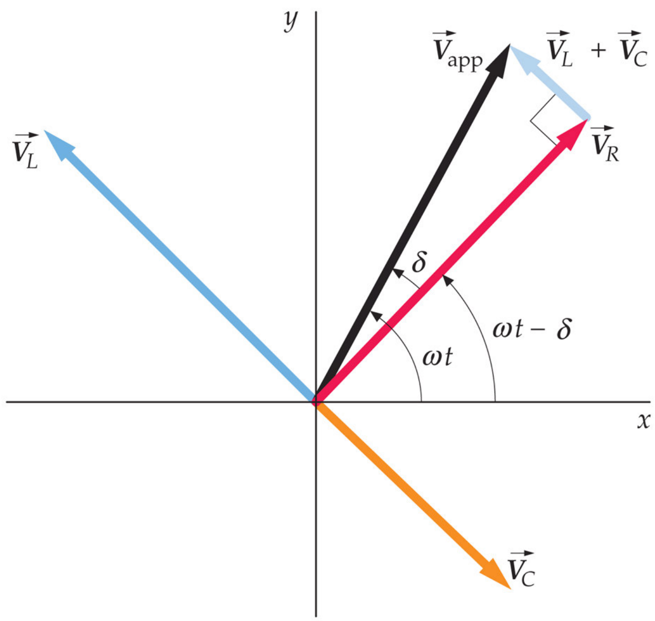
$$z = |z|(cos\theta + jsin\theta) = |z|e^{j\theta}$$



Complex numbers simplify the solution of the integral-differential equations encountered in series RLC AC circuits.

The use of complex numbers simplifies the lead-lag nature of the voltage and current in AC circuits.

Phasor Notation



This diagram depicts a series RLC circuit driven at a frequency that causes the inductive voltage to be greater than the capacitive voltage.

This gives the circuit an overall inductive nature - the current (in phase with V_R) is lagging the applied voltage V_{app}.

All of these voltage vectors (phasors) have a common time component ($e^{j\omega t}$) and so they all rotate at this common frequency. By suppressing this common rotation the concepts are easier to understand.

RLC Circuit Solution

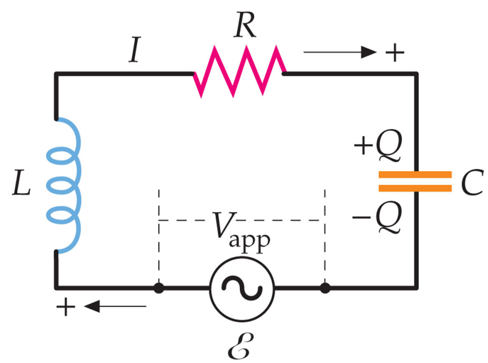
$$RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(t')dt' = \mathcal{E}(t)$$

$$I(t) = I_p e^{j\omega t}$$

The solution of a differential equation begins with the selection of a trial solution

$$\frac{dI}{dt} = j\omega I_p e^{j\omega t} = j\omega I \ ;$$

$$\int I(t)dt = \frac{I}{j\omega}$$



$$RI + j\omega LI + \frac{I}{j\omega C} = E$$

$$\left[R + j\omega L - \frac{j}{\omega C}\right]I = E$$

$$R + j\left[\omega L - \frac{1}{\omega C}\right] = \frac{E}{I}$$

RLC Circuit Solution

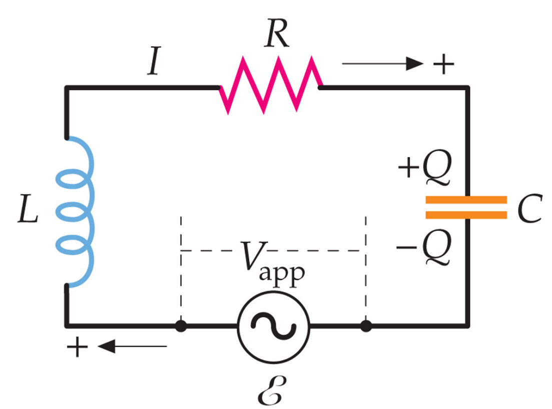
$$R + j\left[\omega L - \frac{1}{\omega C}\right] = \frac{E}{I}$$

These are complex variables

$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right] = \frac{E}{I}$$

$$Z = \frac{E}{I}$$

The quantity Z is called the impedance and it is a complex variable



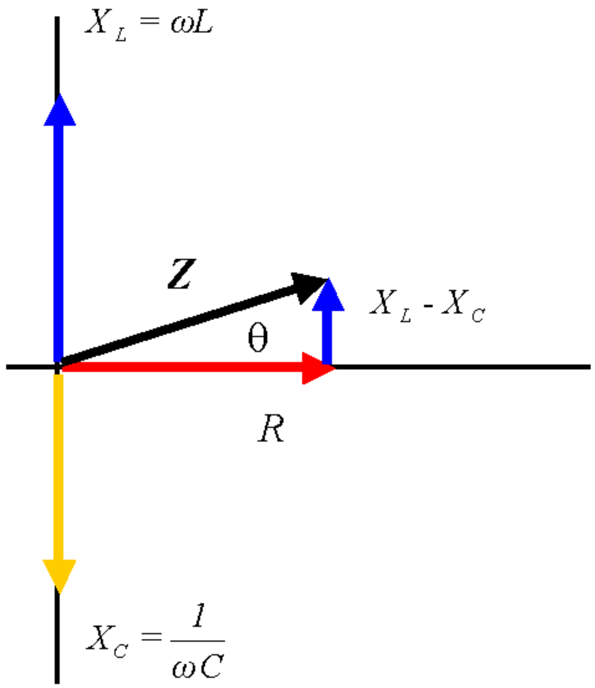
E = I Z is a complex version of Ohm's Law

Complex Impedance

$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

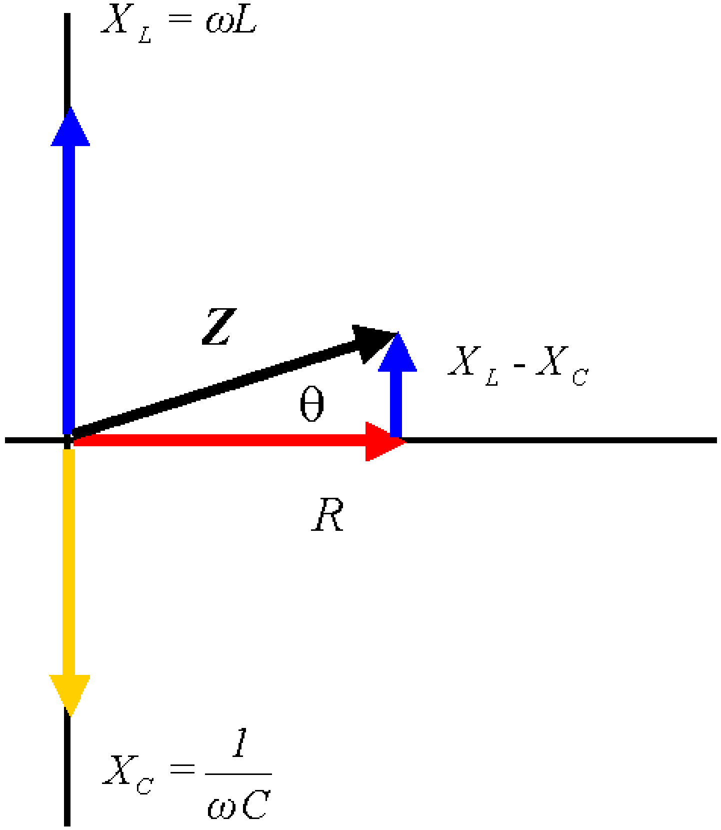
$$|z| = \sqrt{(z^*)z} = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

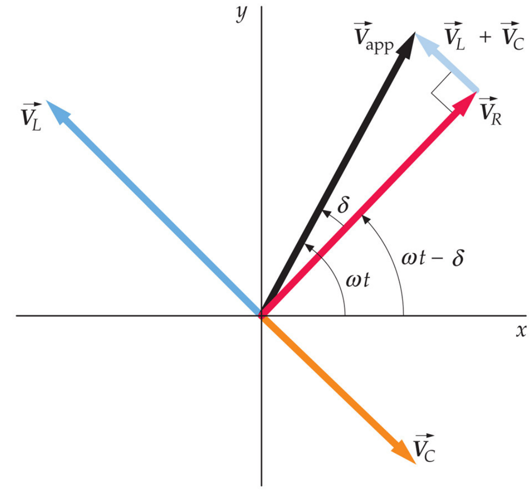
$$tan\theta = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

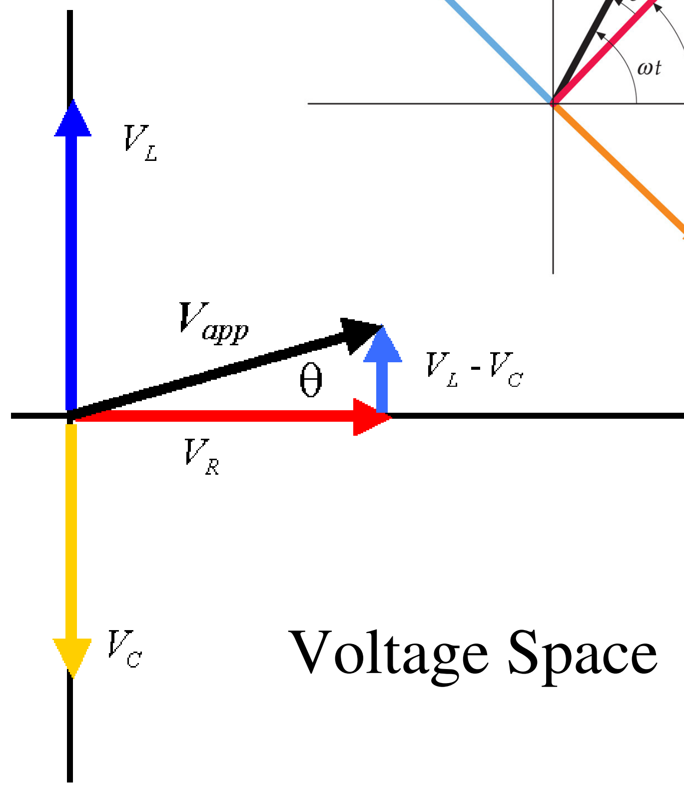


Relative Voltage Phases - Inductive

Impedance Space



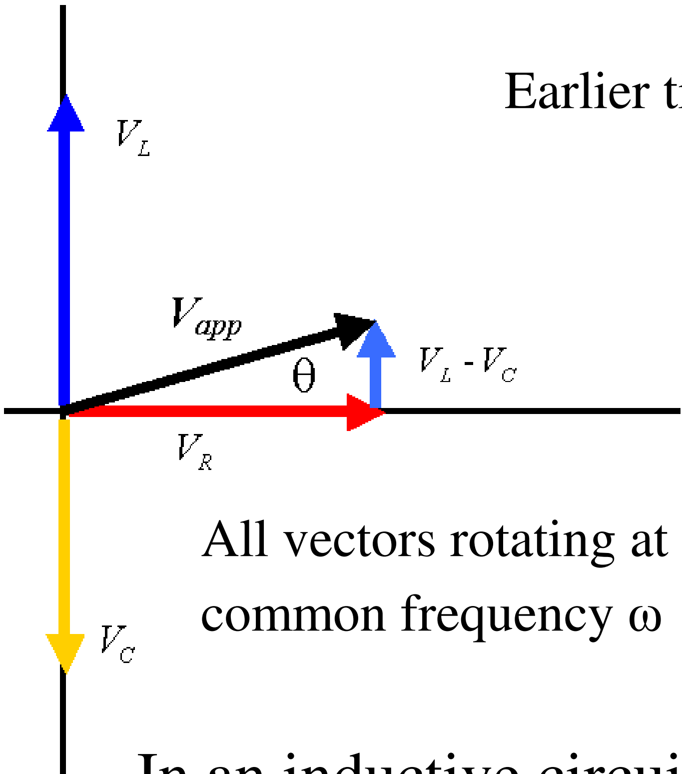




Voltage Space

Phases in an Inductive AC Circuit

RLC series

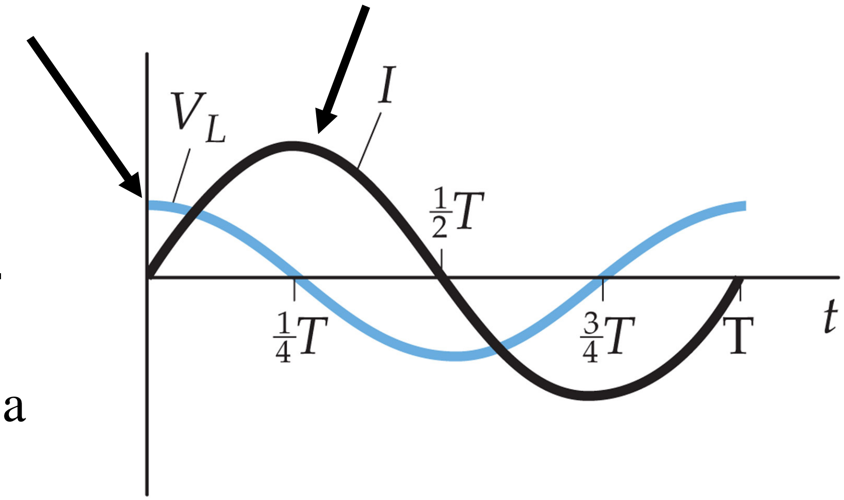


All vectors rotating at a common frequency ω

L only series

Earlier time

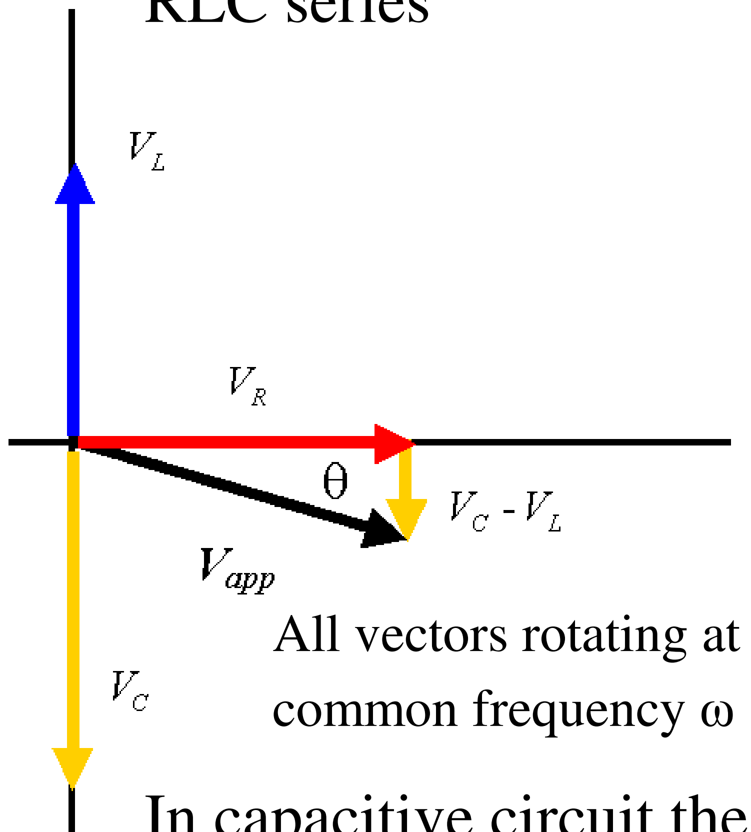
Later time



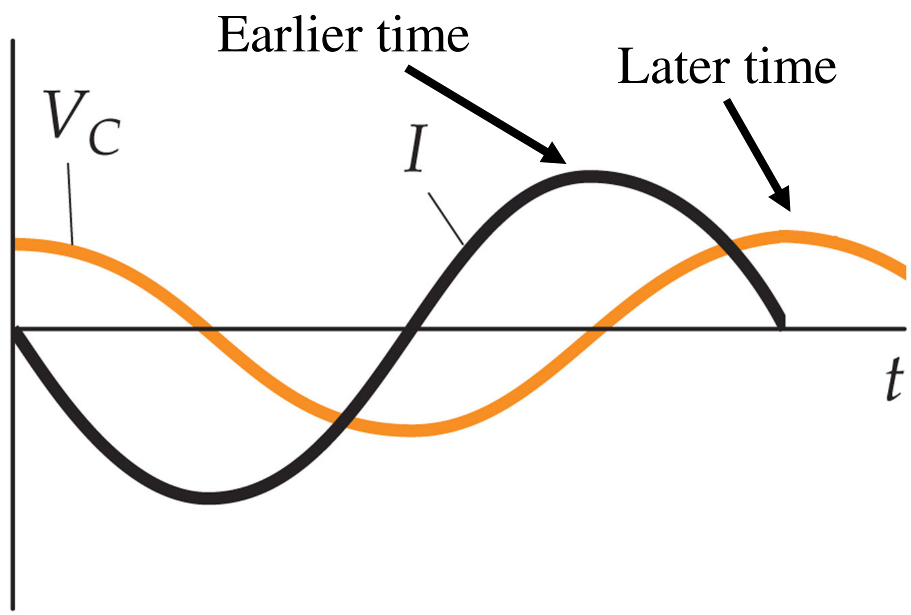
In an inductive circuit the voltage peaks first and the current peaks later.

Phases in a Capacitive AC Circuit

RLC series



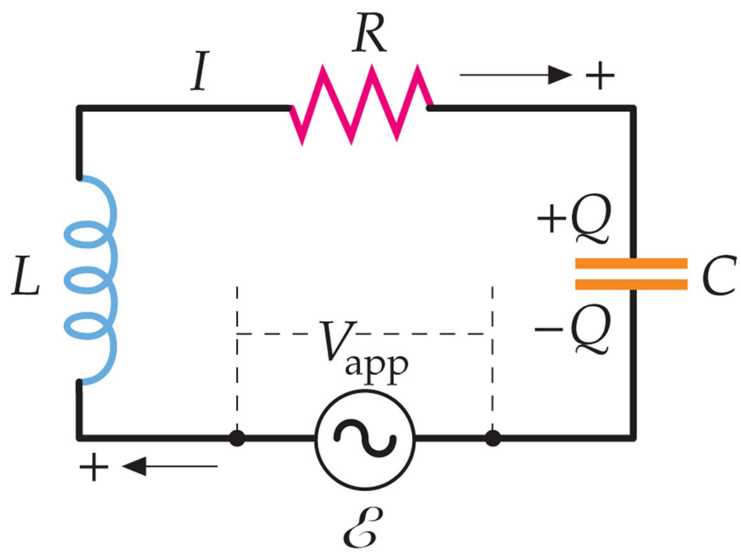
C only series



All vectors rotating at a common frequency ω

In capacitive circuit the current peaks first and the voltage peaks later.

RLC Series AC Circuit Example



R= 250Ω, L = 1.20mH,

C = 1.80μF, V_p = 120v, f = 60Hz

Determine the following:

(a.) X_L - Inductive reactance

(b.) X_C - Capacitive reactance

(c.) Z - Impedance

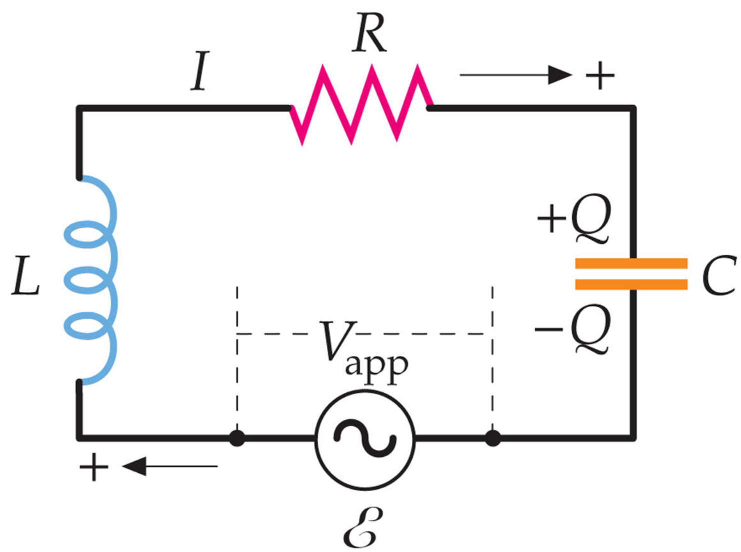
(d.) θ - Phase angle

(e.) I_p - Peak current

(f.) I_{RMS} - RMS current

(g.) ω_R - Resonance frequency

(h.) P_{avg} - Average Power



R= 250Ω, L = 1.20mH,

C = 1.80μF, V_p = 120v, f = 60Hz

Determine the following:

(a.) X_L - Inductive reactance

(b.) X_C - Capacitive reactance

(c.) Z - Impedance

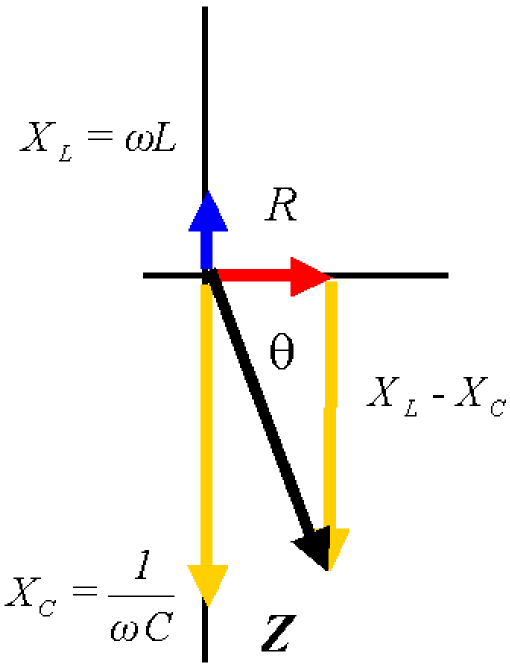
First calc: ω = 2πf = 2(3.14)60 = 377 rad/s

$X_L = \omega L = 377(1.20\text{x}10^{-3}) = 0.452\Omega$

$X_C = 1/\omega C = 1/((377)(1.80\text{x}10^{-6})) = 1474\Omega$

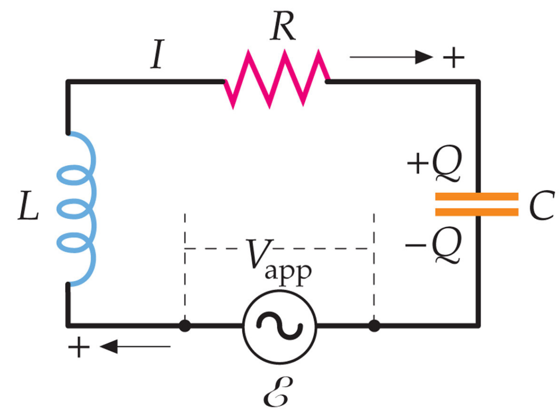
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{250^2 + (0.452 - 1474)^2}$$

$$Z = 1495\ \Omega$$



R= 250Ω, L = 1.20mH,

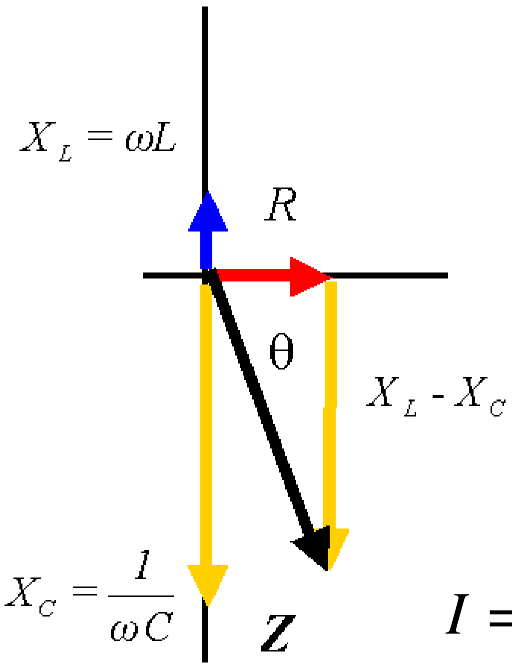
C = 1.80μF, V_p = 120v, f = 60Hz



Determine the following:

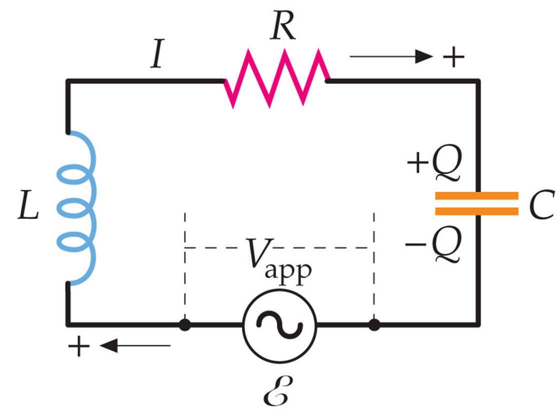
(d.) θ - Phase angle

$$\theta = tan^{-1}\left[\frac{X_L - X_C}{R}\right] = tan^{-1}\left[\frac{0.452 - 1474}{250}\right] = tan^{-1}\left[\frac{-1474}{250}\right] = -80.4^0$$



R= 250Ω, L = 1.20mH,

C = 1.80μF, V_p = 120v, f = 60Hz



Determine the following:

(e.) I_p - Peak current

(f.) I_{RMS} - RMS current

$$I = \frac{V}{Z} = \frac{V_p e^{+j\omega t}}{Z_p e^{+j\theta}} = \frac{V_p}{Z_p} e^{+j(\omega t - \theta)} = I_p e^{+j(\omega t - \theta)}$$

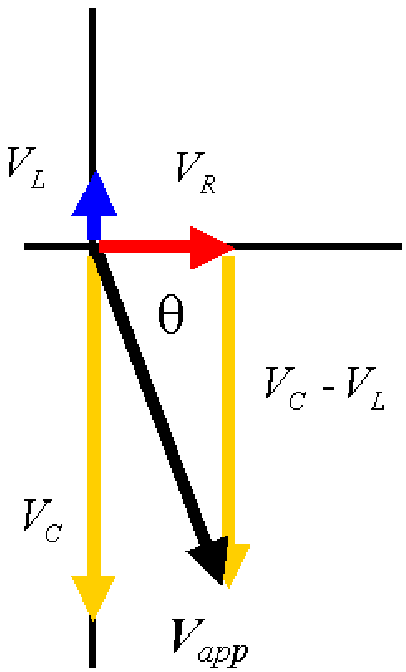
1st minus from the division.

$$I_p = \frac{V_p}{|Z|} = \frac{120}{1495} = 80.3mA$$

$$I = I_p e^{+j(\omega t - \theta)} = 80.3mAe^{+j(\omega t + 80.4)}$$

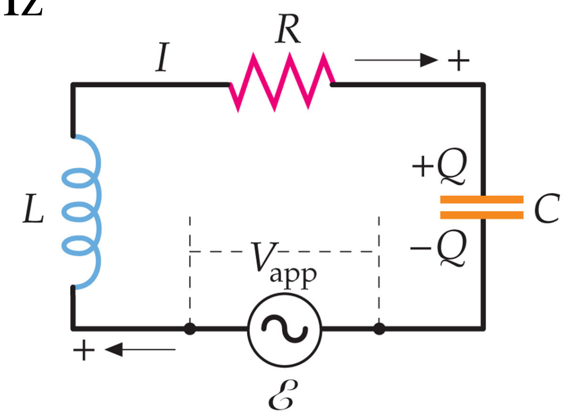
2nd minus from the angle.

$$I_{RMS} = \frac{I_p}{\sqrt{2}} = 0.707 I_p = 56.7mA$$



R= 250Ω, L = 1.20mH,

C = 1.80μF, V_p = 120v, f = 60Hz

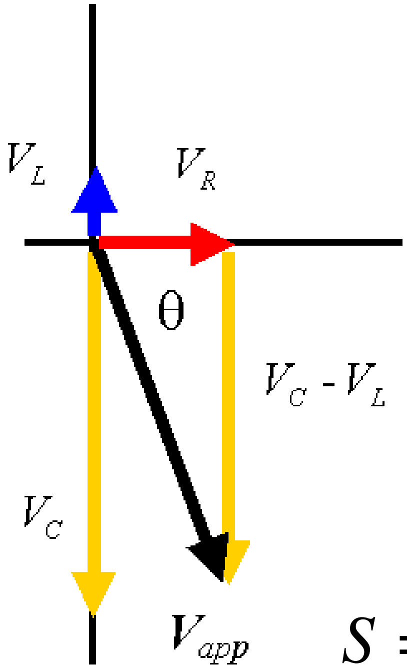


Determine the following:

(g.) ω_R - Resonance frequency

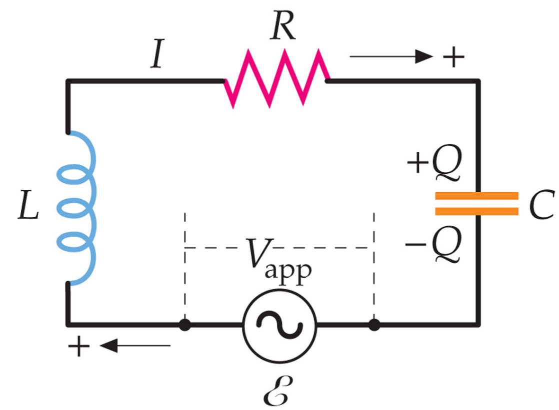
$$\omega_R = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.20x10^{-3})(1.80x10^{-6})}} = 21.5krad/s$$

$$f = \frac{\omega}{2\pi} = \frac{21.5x10^3}{6.28} = 3.42kHz$$



R= 250Ω, L = 1.20mH,

C = 1.80μF, V_p = 120v, f = 60Hz



Determine the following:

(h.) P_{avg} - Average Power

$$S = \frac{1}{2} V I^* \quad V = V_p e^{+j\omega t} \quad I = I_p e^{+j(\omega t + 80.4)}$$

$$S = \frac{1}{2}\left(V_p e^{+j\omega t}\right)\left(I_p e^{-j(\omega t + 80.4)}\right) = \frac{1}{2} V_p I_p e^{+j(\omega t - \omega t - 80.4)}$$

$$S = \frac{1}{2} V_p I_p e^{-j(80.4)} = \frac{V_p}{\sqrt{2}} \frac{I_p}{\sqrt{2}} e^{-j(80.4)} = V_{RMS} I_{RMS} e^{-j(80.4)}$$

Determine the following: (h.) P_{avg} - Average Power

$$S = \frac{1}{2} V_p I_p e^{-j(80.4)} = \frac{V_p}{\sqrt{2}} \frac{I_p}{\sqrt{2}} e^{-j(80.4)} = V_{RMS} I_{RMS} e^{-j(80.4)}$$

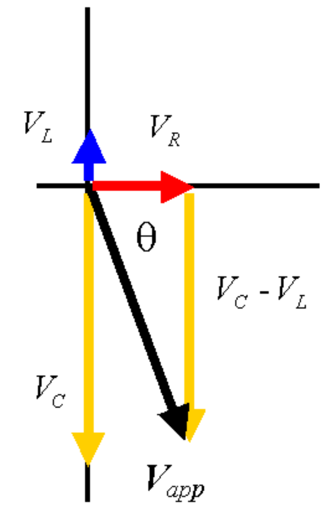
$$S = V_{RMS} I_{RMS} \left(cos(80.4) - jsin(80.4) \right)$$

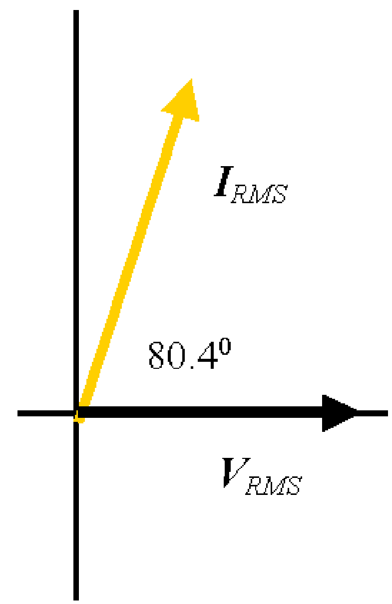
$$S = V_{RMS} I_{RMS} cos(80.4) - jV_{RMS} I_{RMS} sin(80.4)$$

$$P_{avg} = V_{RMS} I_{RMS} cos(80.4)$$

Power Factor

$$P_{avg} = 84.3\left(56.7x10^{-3}\right)(0.167) = 0.803W$$





Voltages

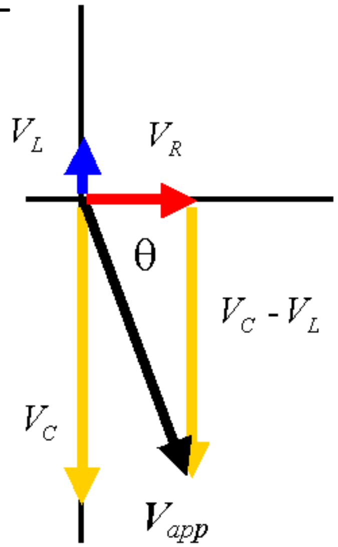
$$V_R = I_{RMS}R = \left(56.7x10^{-3}\right)\left(250\right) = 14.2V$$

$$V_L = I_{RMS}X_L = \left(56.7x10^{-3}\right)\left(377\right)\left(1.20x10^{-3}\right) = 0.0265V$$

$$V_C = I_{RMS}X_C = \left(56.7x10^{-3}\right)/\left(377\left(1.80x10^{-6}\right)\right) = 83.6V$$

$$V = \sqrt{V_R^{\,2} + (V_L - V_C)^2} = \sqrt{(14.2)^2 + (0.0256 - 83.6)^2}$$

$$V = 84.8 = V_{RMS}$$



Resonance in a Series RLC Circuit

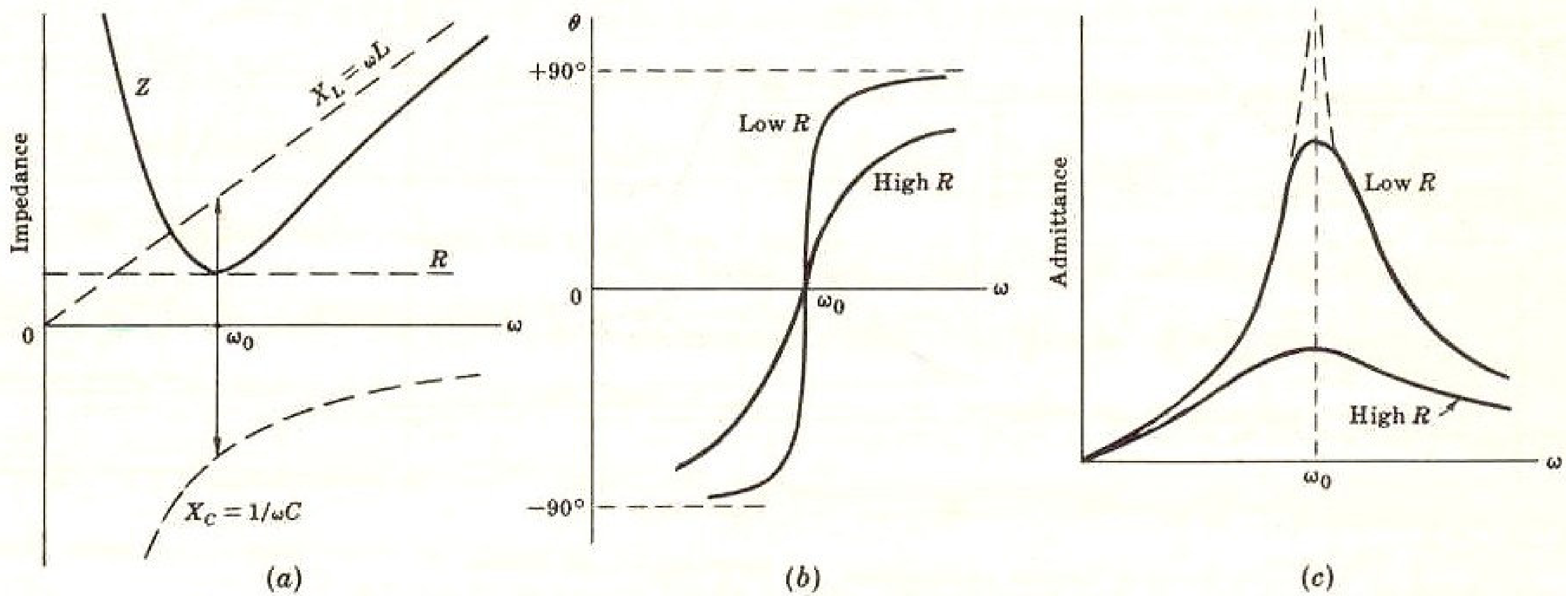


Fig. 8-2. Series Circuit Z, θ and Y as Functions of ω.

Resonance in a Series RLC Circuit

Power Transfer and Resonance

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + + \frac{Q}{C} = V_p cos\omega t$$

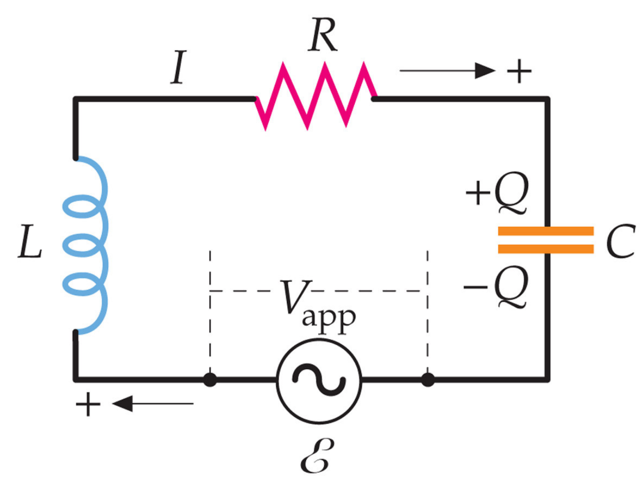
$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$P_{avg} = I_{RMS}^2 R = \frac{V_p^2}{Z^2} R$$

$$I = I_p cos\left(\omega t - \delta\right) = \frac{V_p}{|Z|} cos\left(\omega t - \delta\right)$$

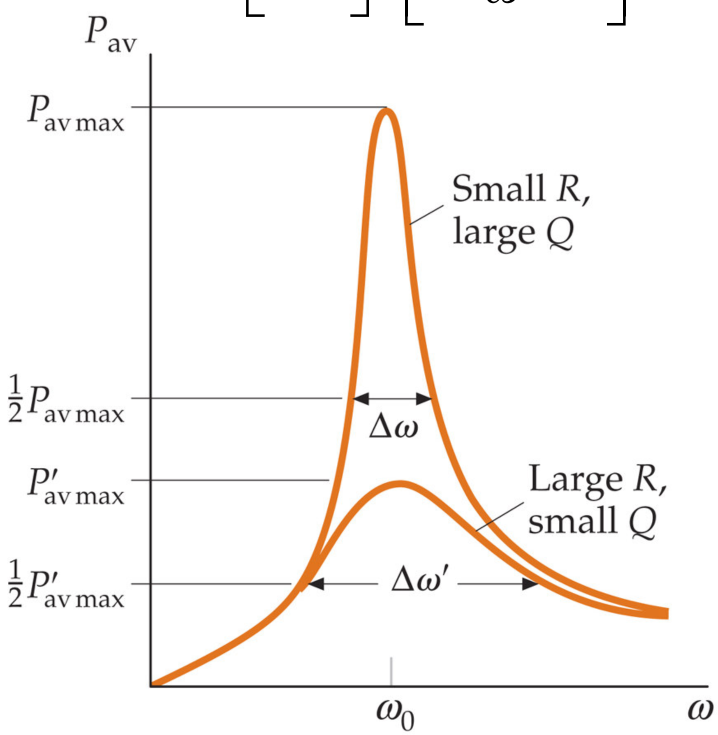
$$Z^2 = R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2$$

$$P_{avg} = \frac{\dfrac{V_p^2}{R}}{1 + \left[\dfrac{\omega L}{R}\right]^2 \left[\dfrac{\omega^2 - \omega_0^2}{\omega^2}\right]^2}$$



Q Factor = Measure of Stored Energy

$$P_{avg} = \frac{V_p^2 / R}{1 + \left[\frac{\omega L}{R}\right]^2 \left[\frac{\omega^2 - \omega_0^2}{\omega^2}\right]^2}$$



Q-Factor $Q = 2\pi \frac{E}{\Delta E} = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$

E = Total Energy and

ΔE is the dissipated energy

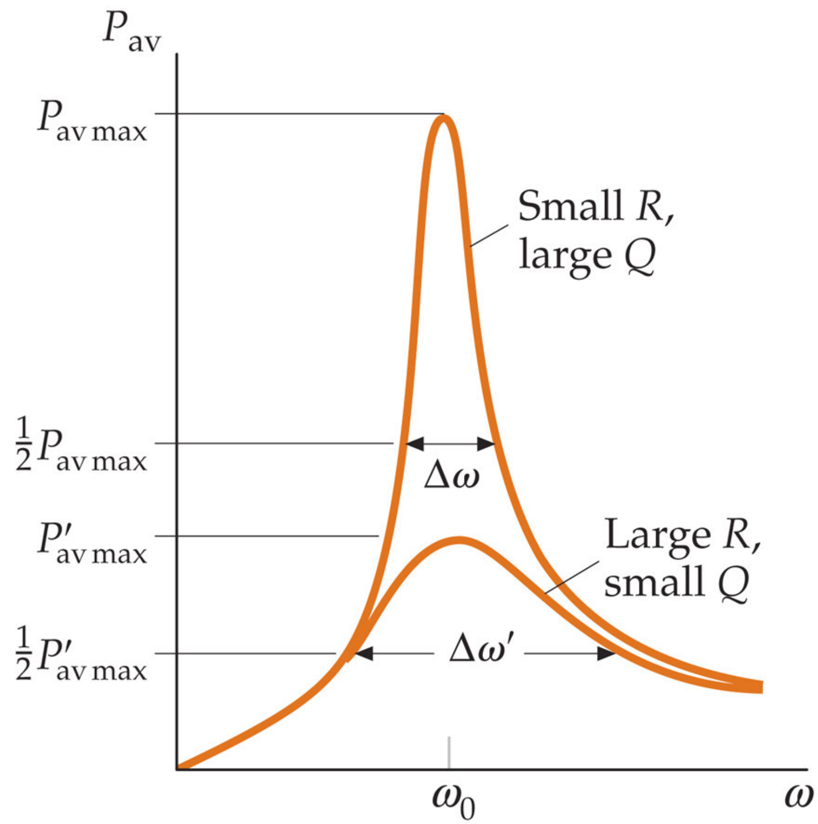
$$\Delta\omega = FWHM$$

FWHM = Full Width at Half Maximum

As an approximation

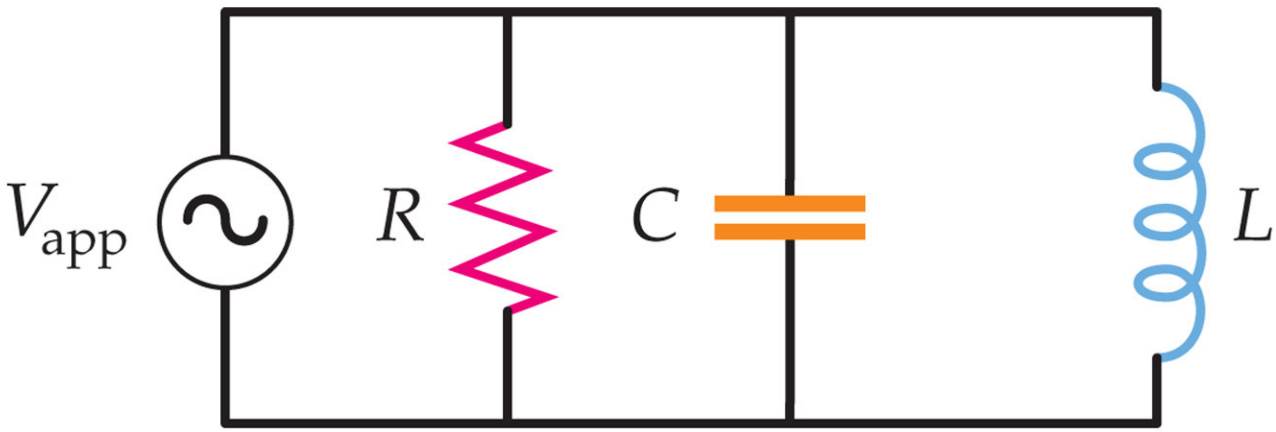
$$Q \simeq \frac{\omega_0 L}{R}$$

Resonance in a Series RLC Circuit



RLC Parallel Circuit

We're not covering this type of circuit



Extra Slides

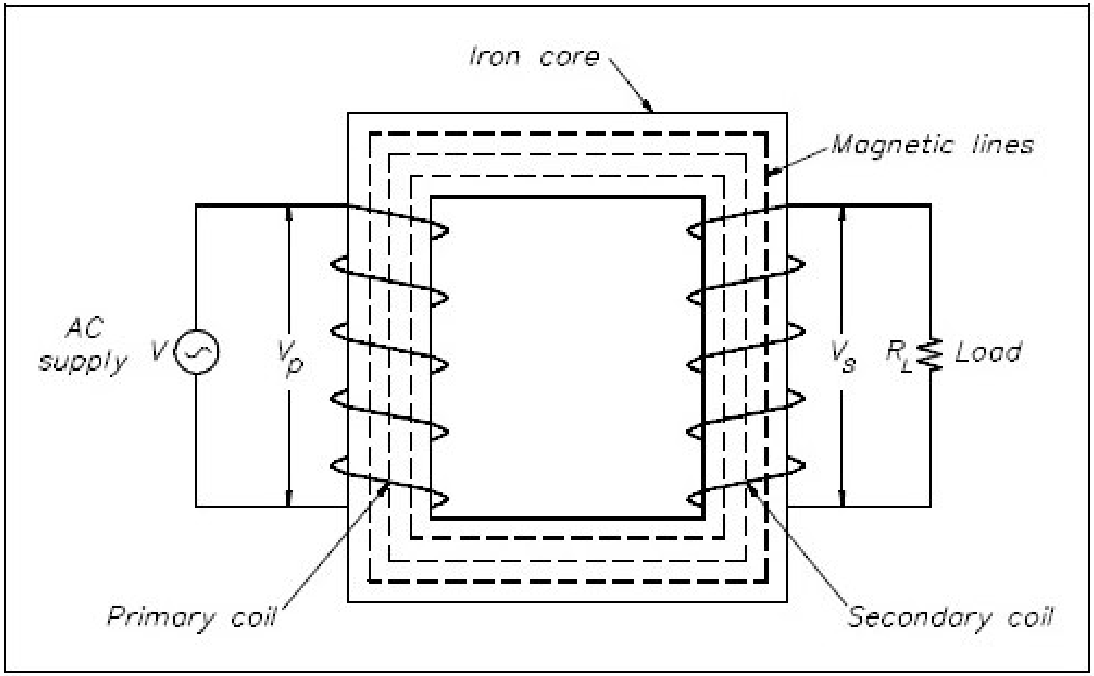


Figure 1 Core-Type Transformer

When alternating voltage is applied to the primary winding, an alternating current will flow that will magnetize the magnetic core, first in one direction and then in the other direction. This alternating flux flowing around the entire length of the magnetic circuit induces a voltage in both the primary and secondary windings. Since both windings are linked by the same flux, the voltage induced per turn of the primary and secondary windings must be the same value and same direction. This voltage opposes the voltage applied to the primary winding and is called counter-electromotive force (CEMF).