

CHAPTER 2

Properties of Fluids

In this chapter we discuss a number of fundamental properties of fluids. An understanding of these properties is essential for us to apply basic principles of fluid mechanics to the solution of practical problems.

2.1 DISTINCTION BETWEEN A SOLID AND A FLUID

The molecules of a *solid* are usually closer together than those of a *fluid*. The attractive forces between the molecules of a solid are so large that a solid tends to retain its shape. This is not the case for a fluid, where the attractive forces between the molecules are smaller. An ideal elastic solid will deform under load and, once the load is removed, will return to its original state. Some solids are plastic. These deform under the action of a sufficient load and deformation continues as long as a load is applied, providing the material does not rupture. Deformation ceases when the load is removed, but the plastic solid does not return to its original state.

The intermolecular cohesive forces in a fluid are not great enough to hold the various elements of the fluid together. Hence a fluid will flow under the action of the slightest stress and flow will continue as long as the stress is present.

2.2 DISTINCTION BETWEEN A GAS AND A LIQUID

A fluid may be either a *gas* or a *liquid*. The molecules of a gas are much farther apart than those of a liquid. Hence a gas is very compressible, and when all external pressure is removed, it tends to expand indefinitely. A gas is therefore in equilibrium only when it is completely enclosed. A liquid is relatively incompressible, and if all pressure, except that of its own vapor pressure, is removed, the cohesion between molecules holds them together, so that the liquid does not expand indefinitely. Therefore a liquid may have a free surface, i.e., a surface from which all pressure is removed, except that of its own vapor.

A *vapor* is a gas whose temperature and pressure are such that it is very near the liquid phase. Thus steam is considered a vapor because its state is

normally not far from that of water. A gas may be defined as a highly superheated vapor; that is, its state is far removed from the liquid phase. Thus air is considered a gas because its state is normally very far from that of liquid air.

The volume of a gas or vapor is greatly affected by changes in pressure or temperature or both. It is usually necessary, therefore, to take account of changes in volume and temperature in dealing with gases or vapors. Whenever significant temperature or phase changes are involved in dealing with vapors and gases, the subject is largely dependent on heat phenomena (*thermodynamics*). Thus fluid mechanics and thermodynamics are interrelated.

2.3 DENSITY, SPECIFIC WEIGHT, SPECIFIC VOLUME, AND SPECIFIC GRAVITY

The *density* ρ (rho),¹ or more strictly, *mass density*, of a fluid is its *mass* per unit volume, while the *specific weight* γ (gamma) is its *weight* per unit volume. In the British Gravitational (BG) system (Sec. 1.5) density ρ will be in slugs per cubic foot (kg/m^3 in SI units), which can also be expressed as units of $\text{lb}\cdot\text{sec}^2/\text{ft}^4$ ($\text{N}\cdot\text{s}^2/\text{m}^4$ in SI units) (Sec. 1.5 and inside covers).

Specific weight γ represents the force exerted by gravity on a unit volume of fluid, and therefore must have the units of force per unit volume, such as pounds per cubic foot (N/m^3 in SI units).

Density and specific weight of a fluid are related as:

$$\rho = \frac{\gamma}{g} \quad \text{or} \quad \gamma = \rho g \quad (2.1)$$

Since the physical equations are dimensionally homogeneous, the dimensions of density are

$$\text{Dimensions of } \rho = \frac{\text{dimensions of } \gamma}{\text{dimensions of } g} = \frac{\text{lb}/\text{ft}^3}{\text{ft}/\text{sec}^2} = \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}^4} = \frac{\text{mass}}{\text{volume}} = \frac{\text{slugs}}{\text{ft}^3}$$

In SI units

$$\text{Dimensions of } \rho = \frac{\text{dimensions of } \gamma}{\text{dimensions of } g} = \frac{\text{N}/\text{m}^3}{\text{m}/\text{s}^2} = \frac{\text{N}\cdot\text{s}^2}{\text{m}^4} = \frac{\text{mass}}{\text{volume}} = \frac{\text{kg}}{\text{m}^3}$$

Note that density ρ is absolute, since it depends on mass, which is independent of location. Specific weight γ , on the other hand, is not absolute, since it depends on the value of the gravitational acceleration g , which varies with location, primarily latitude and elevation above mean sea level.

Densities and specific weights of fluids vary with temperature. Appendix A provides commonly needed temperature variations of these quantities for water

¹ The names of Greek letters are given in the List of Symbols on page xix.

and air. It also contains densities and specific weights of common gases at standard atmospheric pressure and temperature. We shall discuss the specific weight of liquids further in Sec. 2.6.

Specific volume v is the volume occupied by a unit mass of fluid.² We commonly apply it to gases, and usually express it in cubic feet per slug (m^3/kg in SI units). Specific volume is the reciprocal of density. Thus

$$v = \frac{1}{\rho} \quad (2.2)$$

Specific gravity s of a liquid is the dimensionless ratio

$$s_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water at standard temperature}}}$$

Physicists use 4°C (39.2°F) as the standard, but engineers often use 60°F (15.56°C). In the metric system the density of water at 4°C is 1.00 g/cm^3 (or 1.00 g/mL),³ equivalent to 1000 kg/m^3 , and hence the specific gravity (which is dimensionless) of a liquid has the same numerical value as its density expressed in g/mL or Mg/m^3 . Appendix A contains information on specific gravities and densities of various liquids at standard atmospheric pressure.

The specific gravity of a gas is the ratio of its density to that of either hydrogen or air at some specified temperature and pressure, but there is no general agreement on these standards, and so we must explicitly state them in any given case.

Since the density of a fluid varies with temperature, we must determine and specify specific gravities at particular temperatures.

SAMPLE PROBLEM 2.1 The specific weight of water at ordinary pressure and temperature is 62.4 lb/ft^3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution

$$\rho_{\text{water}} = \frac{\gamma_{\text{water}}}{g} = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/sec}^2} = 1.938 \text{ slugs/ft}^3 \quad \text{ANS}$$

$$\gamma_{\text{mercury}} = s_{\text{mercury}}\gamma_{\text{water}} = 13.56(62.4) = 846 \text{ lb/ft}^3 \quad \text{ANS}$$

$$\rho_{\text{mercury}} = s_{\text{mercury}}\rho_{\text{water}} = 13.56(1.938) = 26.3 \text{ slugs/ft}^3 \quad \text{ANS}$$

²Note that in this book we use a “rounded” lower case v (vee), to help distinguish it from a capital V and from the Greek ν (nu).

³One cubic centimeter (cm^3) is equivalent to one milliliter (mL).

SAMPLE PROBLEM 2.2 The specific weight of water at ordinary pressure and temperature is 9.81 kN/m^3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution

$$\begin{aligned}\rho_{\text{water}} &= \frac{9.81 \text{ kN/m}^3}{9.81 \text{ m/s}^2} = 1.00 \text{ Mg/m}^3 = 1.00 \text{ g/mL} && \text{ANS} \\ \gamma_{\text{mercury}} &= s_{\text{mercury}}\gamma_{\text{water}} = 13.56(9.81) = 133.0 \text{ kN/m}^3 && \text{ANS} \\ \rho_{\text{mercury}} &= s_{\text{mercury}}\rho_{\text{water}} = 13.56(1.00) = 13.56 \text{ Mg/m}^3 && \text{ANS}\end{aligned}$$

EXERCISES

- 2.3.1** If the specific weight of a liquid is 52 lb/ft^3 , what is its density?
- 2.3.2** If the specific weight of a liquid is 8.1 kN/m^3 , what is its density?
- 2.3.3** If the specific volume of a gas is $375 \text{ ft}^3/\text{slug}$, what is its specific weight in lb/ft^3 ?
- 2.3.4** If the specific volume of a gas is $0.70 \text{ m}^3/\text{kg}$, what is its specific weight in N/m^3 ?
- 2.3.5** A certain gas weighs 16.0 N/m^3 at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 12.0 N/m^3 ?
- 2.3.6** The specific weight of glycerin is 78.6 lb/ft^3 . Compute its density and specific gravity. What is its specific weight in kN/m^3 ?
- 2.3.7** If a certain gasoline weighs 43 lb/ft^3 , what are the values of its density, specific volume, and specific gravity relative to water at 60°F ? Use Appendix A.

2.4 COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

Fluid mechanics deals with both incompressible and compressible fluids, that is, with liquids and gases of either constant or variable density. Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. We may also consider gases to be incompressible when the pressure variation is small compared with the absolute pressure.

Ordinarily we consider liquids to be incompressible fluids, yet sound waves, which are really pressure waves, travel through them. This is evidence of the elasticity of liquids. In problems involving water hammer (Sec. 12.6) we must consider the compressibility of the liquid.

The flow of air in a ventilating system is a case where we may treat a gas as incompressible, for the pressure variation is so small that the change in density is of no importance. But for a gas or steam flowing at high velocity through a long pipeline, the drop in pressure may be so great that we cannot ignore the change in density. For an airplane flying at speeds below 250 mph (100 m/s), we

may consider the air to be of constant density. But as an object moving through the air approaches the velocity of sound, which is of the order of 760 mph (1200 km/h) depending on temperature, the pressure and density of the air adjacent to the body become materially different from those of the air at some distance away, and we must then treat the air as a compressible fluid (Chap. 13).

2.5 COMPRESSIBILITY OF LIQUIDS

The compressibility (change in volume due to change in pressure) of a liquid is inversely proportional to its **volume modulus of elasticity**, also known as the **bulk modulus**. This modulus is defined as

$$E_v = -v \frac{dp}{dv} = -\left(\frac{v}{dv}\right) dp$$

where v = specific volume and p = pressure. As v/dv is a dimensionless ratio, the units of E_v and p are identical. The bulk modulus is analogous to the modulus of elasticity for solids; however, for fluids it is defined on a volume basis rather than in terms of the familiar one-dimensional stress-strain relation for solid bodies.

In most engineering problems, the bulk modulus at or near atmospheric pressure is the one of interest. The bulk modulus is a property of the fluid and for liquids is a function of temperature and pressure. A few values of the bulk modulus for water are given in Table 2.1. At any temperature we see that the value of E_v increases continuously with pressure, but at any one pressure the value of E_v is a maximum at about 120°F (50°C). Thus water has a minimum compressibility at about 120°F (50°C).

Note that we often specify applied pressures, such as those in Table 2.1, in absolute terms, because atmospheric pressure varies. The units psia or kN/m² abs indicate absolute pressure, which is the actual pressure on the fluid, relative

TABLE 2.1 Bulk modulus of water E_v , psi^a

Pressure, psia	Temperature, °F				
	32°	68°	120°	200°	300°
15	293,000	320,000	333,000	308,000	
1,500	300,000	330,000	342,000	319,000	248,000
4,500	317,000	348,000	362,000	338,000	271,000
15,000	380,000	410,000	426,000	405,000	350,000

^a These values can be transformed to meganewtons per square meter by multiplying them by 0.006895. The values in the first line are for conditions close to normal atmospheric pressure; for a more complete set of values at normal atmospheric pressure, see Table A.1 in Appendix A. The five temperatures are equal to 0, 20, 48.9, 93.3, and 148.9°C, respectively.

to absolute zero. The standard atmospheric pressure at sea level is about 14.7 psia or 101.3 kN/m² abs (1013 mb abs) (see Sec. 2.9 and Table A.3). Bars and millibars were previously used in metric systems to express pressure; 1 mb = 100 N/m². We measure most pressures relative to the atmosphere, and call them gage pressures. This is explained more fully in Sec. 3.4.

The volume modulus of mild steel is about 26,000,000 psi (170 000 MN/m²). Taking a typical value for the volume modulus of cold water to be 320,000 psi (2200 MN/m²), we see that water is about 80 times as compressible as steel. The compressibility of liquids covers a wide range. Mercury, for example, is approximately 8% as compressible as water, while the compressibility of nitric acid is nearly six times greater than that of water.

In Table 2.1 we see that at any one temperature the bulk modulus of water does not vary a great deal for a moderate range in pressure. By rearranging the definition of E_v , as an approximation we may use for the case of a fixed mass of liquid at constant temperature

$$\frac{\Delta v}{v} \approx -\frac{\Delta p}{E_v} \quad (2.3a)$$

or

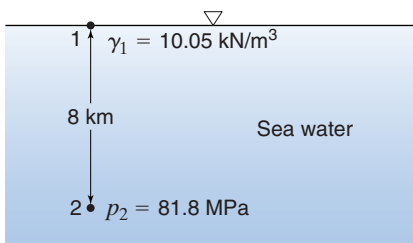
$$\frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v} \quad (2.3b)$$

where E_v is the mean value of the modulus for the pressure range and the subscripts 1 and 2 refer to the before and after conditions.

Assuming E_v to have a value of 320,000 psi, we see that increasing the pressure of water by 1000 psi will compress it only $\frac{1}{320}$, or 0.3%, of its original volume. Therefore we find that the usual assumption regarding water as being incompressible is justified.

SAMPLE PROBLEM 2.3 At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m³ and that the average volume modulus is 2.34×10^9 N/m² for that pressure range. (a) What will be the change in specific volume between that at the surface and at that depth? (b) What will be the specific volume at that depth? (c) What will be the specific weight at that depth?

Solution



- (a) Eq. (2.2): $v_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10050 = 0.000976 \text{ m}^3/\text{kg}$
 Eq. (2.3a): $\Delta v = -0.000976(81.8 \times 10^6 - 0)/(2.34 \times 10^9)$
 $= -34.1 \times 10^{-6} \text{ m}^3/\text{kg}$ **ANS**
- (b) Eq. (2.3b): $v_2 = v_1 + \Delta v = 0.000942 \text{ m}^3/\text{kg}$ **ANS**
- (c) $\gamma_2 = g/v_2 = 9.81/0.000942 = 10410 \text{ N/m}^3$ **ANS**

EXERCISES

- 2.5.1** To two significant figures what is the bulk modulus of water in MN/m^2 at 50°C under a pressure of 30 MN/m^2 ? Use Table 2.1.
- 2.5.2** At normal atmospheric conditions, approximately what pressure in psi must be applied to water to reduce its volume by 2%? Use Table 2.1.
- 2.5.3** Water in a hydraulic press is subjected to a pressure of 4500 psia at 68°F . If the initial pressure is 15 psia, approximately what will be the percentage decrease in specific volume? Use Table 2.1.
- 2.5.4** At normal atmospheric conditions, approximately what pressure in MPa must be applied to water to reduce its volume by 3%?
- 2.5.5** A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage.

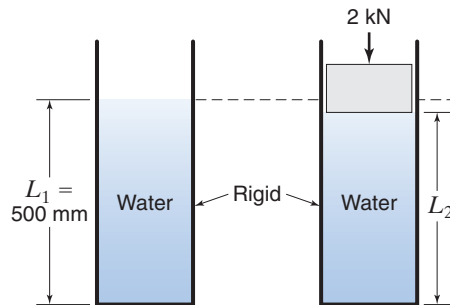


Figure X2.5.5

2.6 SPECIFIC WEIGHT OF LIQUIDS

The specific weights γ of some common liquids at 68°F (20°C) and standard sea-level atmospheric pressure⁴ with $g = 32.2 \text{ ft/sec}^2$ (9.81 m/s^2) are given in Table 2.2. The specific weight of a *liquid* varies only slightly with pressure, depending on the bulk modulus of the liquid (Sec. 2.5); it also depends on temperature, and the variation may be considerable. Since specific weight γ is equal to ρg , the

⁴ See Secs. 2.9 and 3.5.

TABLE 2.2 Specific weights γ of common liquids at 68°F (20°C), 14.7 psia (1013 mb abs) with $g = 32.2$ ft/sec² (9.81 m/s²)

	lb/ft ³	kN/m ³
Carbon tetrachloride	99.4	15.6
Ethyl alcohol	49.3	7.76
Gasoline	42	6.6
Glycerin	78.7	12.3
Kerosene	50	7.9
Motor oil	54	8.5
Seawater	63.9	10.03
Water	62.3	9.79

specific weight of a *fluid* depends on the local value of the acceleration of gravity in addition to the variations with temperature and pressure. The variation of the specific weight of water with temperature and pressure, where $g = 32.2$ ft/sec² (9.81 m/s²), is shown in Fig. 2.1. The presence of dissolved air, salts in solution, and suspended matter will increase these values a very slight amount. Ordinarily

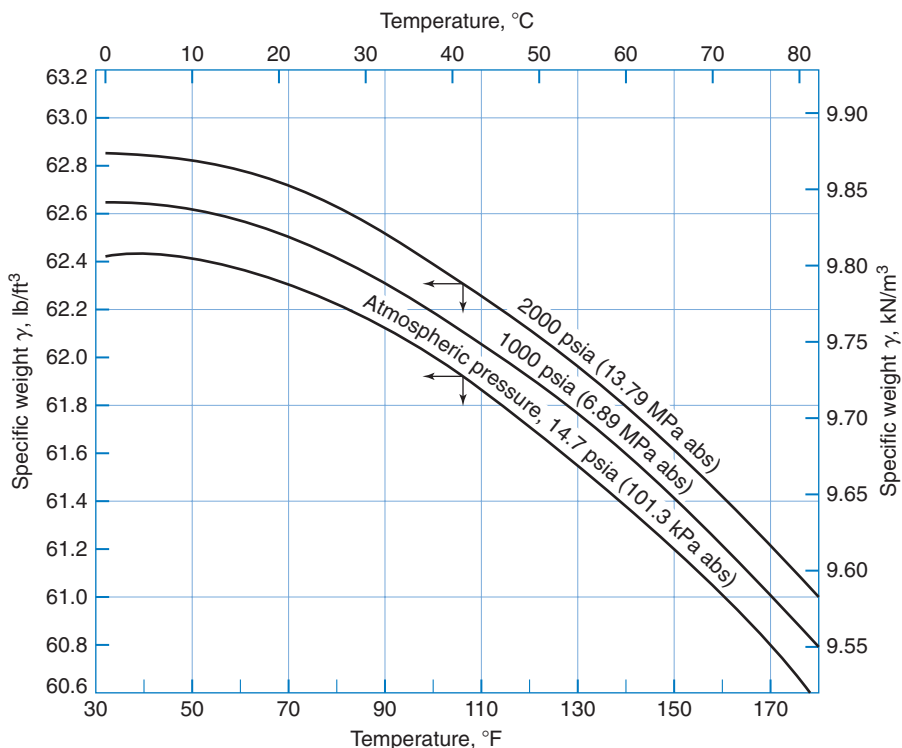
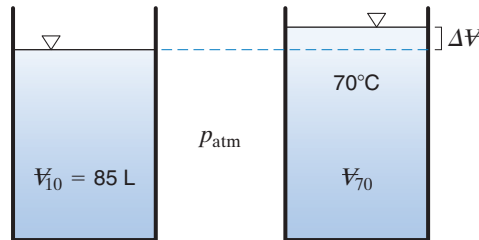


Figure 2.1 Specific weight γ of pure water as a function of temperature and pressure for the condition where $g = 32.2$ ft/sec² (9.81 m/s²).

we assume ocean water to weigh 64.0 lb/ft³ (10.1 kN/m³). Unless otherwise specified or implied by a given temperature, the value to use for water in the problems in this book is $\gamma = 62.4 \text{ lb/ft}^3$ (9.81 kN/m³). Under extreme conditions the specific weight of water is quite different. For example, at 500°F (260°C) and 6000 psi (42 MN/m²) the specific weight of water is 51 lb/ft³ (8.0 kN/m³).

SAMPLE PROBLEM 2.4 A vessel contains 85 L of water at 10°C and atmospheric pressure. If the water is heated to 70°C, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.

Solution



Volume, $V_{10} = 85 \text{ L} = 0.085 \text{ m}^3$

Table A.1: $\gamma_{10} = 9.804 \text{ kN/m}^3, \gamma_{70} = 9.589 \text{ kN/m}^3$

Weight of water, $W = \gamma V = \gamma_{10} V_{10} = \gamma_{70} V_{70}$

i.e., $9.804(0.085) \text{ kN} = 9.589 V_{70}; V_{70} = 0.08691 \text{ m}^3$

$\Delta V = V_{70} - V_{10} = 0.08691 - 0.08500 = 0.001906 \text{ m}^3$ at γ_{70}

$\Delta V/V_{10} = 0.001906/0.085 = 2.24\% \text{ increase} \quad \text{ANS}$

Must remove (at γ_{70}): $W\left(\frac{\Delta V}{V_{70}}\right) = \gamma_{70} \Delta V$

$= (9589 \text{ N/m}^3)(0.001906 \text{ m}^3) = 18.27 \text{ N} \quad \text{ANS}$

EXERCISES

- 2.6.1 Use Fig. 2.1 to find the approximate specific weight of water in lb/ft³ under the following conditions: (a) at a temperature of 60°C under 101.3 kPa abs pressure; (b) at 60°C under a pressure of 13.79 MPa abs.
- 2.6.2 Use Fig. 2.1 to find the approximate specific weight of water in kN/m³ under the following conditions: (a) at a temperature of 160°F under normal atmospheric pressure; (b) at 160°F under a pressure of 2000 psia.

- 2.6.3** A vessel contains 5.0 ft³ of water at 40°F and atmospheric pressure. If the water is heated to 80°F, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.
- 2.6.4** A cylindrical tank (diameter = 8.00 m and depth = 5.00 m) contains water at 15°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Assume the tank does not expand with the change in temperature. Use Appendix A.

2.7 PROPERTY RELATIONS FOR PERFECT GASES

The various properties of a gas, listed below, are related to one another (see, e.g., Appendix A, Tables A.2 and A.5). They differ for each gas. When the conditions of most real gases are far removed from the liquid phase, these relations closely approximate those of hypothetical *perfect gases*. Perfect gases, are here (and often) defined to have constant specific heats⁵ and to obey the *perfect-gas law*,

$$\frac{P}{\rho} = pv = RT \quad (2.4)$$

where p = absolute pressure (Sec. 3.4)
 ρ = density (mass per unit volume)
 v = specific volume (volume per unit mass, = $1/\rho$)
 R = a gas constant, the value of which depends upon the particular gas
 T = absolute temperature in degrees Rankine or Kelvin⁶

For air, the value of R is 1715 ft·lb/(slug·°R) or 287 N·m/(kg·K) (Appendix A, Table A.5); making use of the definitions of a slug and a newton (Sec. 1.5), these units are sometimes given as ft²/(sec²·°R) and m²/(s²·K), respectively. Since $\gamma = \rho g$, Eq. (2.4) can also be written

$$\gamma = \frac{gP}{RT} \quad (2.5)$$

from which the specific weight of any gas at any temperature and pressure can be computed if R and g are known. Because Eqs. (2.4) and (2.5) relate the various gas properties at a particular state, they are known as *equations of state* and as *property relations*.

In this book we shall assume that all gases are perfect. Perfect gases are sometimes also called ideal gases. Do not confuse a perfect (ideal) gas with an ideal fluid (Sec. 2.10).

⁵ Specific heat and other thermodynamic properties of gases are discussed in Sec. 13.1.

⁶ Absolute temperature is measured above absolute zero. This occurs on the Fahrenheit scale at -459.67°F (0° Rankine) and on the Celsius scale at -273.15°C (0 Kelvin). Except for low-temperature work, these values are usually taken as -460°F and -273°C . Remember that no degree symbol is used with Kelvin.

Avogadro's law states that all gases at the same temperature and pressure under the action of a given value of g have the same number of molecules per unit of volume, from which it follows that the specific weight of a gas⁷ is proportional to its molar mass. Thus, if M denotes **molar mass** (formerly called **molecular weight**), $\gamma_2/\gamma_1 = M_2/M_1$ and, from Eq. (2.5), $\gamma_2/\gamma_1 = R_1/R_2$ for the same temperature, pressure, and value of g . Hence for a perfect gas

$$M_1 R_1 = M_2 R_2 = \text{constant} = R_0$$

R_0 is known as the **universal gas constant**, and has a value of 49,709 ft·lb/(slug·mol·°R) or 8312 N·m/(kg·mol·K). Rewriting the preceding equation in the form

$$R = \frac{R_0}{M}$$

enables us to obtain any gas constant R required for Eq. (2.4) or (2.5).

For real (nonperfect) gases, the specific heats may vary over large temperature ranges, and the right-hand side of Eq. (2.4) is replaced by zRT , so that $R_0 = MzR$, where z is a compressibility factor that varies with pressure and temperature. Values of z and R are given in thermodynamics texts and in handbooks. However, for normally encountered monatomic and diatomic gases, z varies from unity by less than 3%, so the perfect-gas idealizations yield good approximations, and Eqs. (2.4) and (2.5) will give good results.

When various gases exist as a mixture, as in air, **Dalton's law of partial pressures** states that each gas exerts its own pressure as if the other(s) were not present. Hence it is the partial pressure of each that we must use in Eqs. (2.4) and (2.5) (see Sample Prob. 2.5). Water vapor as it naturally occurs in the atmosphere has a low partial pressure, so we may treat it as a perfect gas with $R = 49,709/18 = 2760$ ft·lb/(slug·°R) [462 N·m/(kg·K)]. But for steam at higher pressures this value is not applicable.

As we increase the pressure and simultaneously lower the temperature, a gas becomes a vapor, and as gases depart more and more from the gas phase and approach the liquid phase, the property relations become much more complicated than Eq (2.4), and we must then obtain specific weight and other properties from vapor tables or charts. Such tables and charts exist for steam, ammonia, sulfur dioxide, freon, and other vapors in common engineering use.

Another fundamental equation for a perfect gas is

$$pv^n = p_1 v_1^n = \text{constant} \quad (2.6a)$$

or

$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^n = \text{constant} \quad (2.6b)$$

where p is absolute pressure, v ($= 1/\rho$) is specific volume, ρ is density, and n may have any nonnegative value from zero to infinity, depending on the process to

⁷The specific weight of air (molar mass ≈ 29.0) at 68°F (20°C) and 14.7 psia (1013 mb abs) with $g = 32.2$ ft/sec² (9.81 m/s²) is 0.0752 lb/ft³ (11.82 N/m³).

which the gas is subjected. Since this equation describes the change of the gas properties from one state to another for a particular process, we call it a **process equation**. If the process of change is at a constant temperature (isothermal), $n = 1$. If there is no heat transfer to or from the gas, the process is **adiabatic**. A frictionless (and reversible) adiabatic process is an **isentropic** process, for which we denote n by k , where $k = c_p/c_v$, the ratio of specific heat at constant pressure to that at constant volume.⁸ This **specific heat ratio** k is also called the **adiabatic exponent**. For expansion with friction n is less than k , and for compression with friction n is greater than k . Values for k are given in Appendix A, Table A.5, and in thermodynamics texts and handbooks. For air and diatomic gases at usual temperatures, we can take k as 1.4.

By combining Eqs. (2.4) and (2.6), we can obtain other useful relations such as

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{\rho_2}{\rho_1}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \quad (2.7)$$

SAMPLE PROBLEM 2.5 If an artificial atmosphere consists of 20% oxygen and 80% nitrogen by volume, at 14.7 psia and 60°F, what are (a) the specific weight and partial pressure of the oxygen and (b) the specific weight of the mixture?

Solution

Table A.5: R (oxygen) = 1554 ft²/(sec²·°R),
 R (nitrogen) = 1773 ft²/(sec²·°R)

Eq. (2.5): 100% O₂: $\gamma = \frac{32.2(14.7 \times 144)}{1554(460 + 60)} = 0.0843$ lb/ft³

Eq. (2.5): 100% N₂: $\gamma = \frac{32.2(14.7 \times 144)}{1773(520)} = 0.0739$ lb/ft³

(a) Each ft³ of mixture contains 0.2 ft³ of O₂ and 0.8 ft³ of N₂.

So for 20% O₂, $\gamma = 0.20(0.0843) = 0.01687$ lb/ft³ **ANS**

From Eq. (2.5), for 20% O₂, $p = \frac{\gamma RT}{g} = \frac{0.01687(1554)520}{32.2}$
 $= 423$ lb/ft² abs = 2.94 psia **ANS**

Note that this = 20%(14.7 psia).

(b) For 80% N₂, $\gamma = 0.80(0.0739) = 0.0591$ lb/ft³.

Mixture: $\gamma = 0.01687 + 0.0591 = 0.0760$ lb/ft³ **ANS**

⁸Specific heat and other thermodynamic properties of gases are discussed in Sec. 13.1.

EXERCISES

- 2.7.1** A gas at 60°C under a pressure of 10000 mb abs has a specific weight of 99 N/m³. What is the value of R for the gas? What gas might this be? Refer to Appendix A, Table A.5.
- 2.7.2** A hydrogen-filled balloon of the type used in cosmic-ray studies is to be expanded to its full size, which is a 100-ft-diameter sphere, without stress in the wall at an altitude of 150,000 ft. If the pressure and temperature at this altitude are 0.14 psia and -67°F respectively, find the volume of hydrogen at 14.7 psia and 60°F that should be added on the ground. Neglect the balloon's weight.
- 2.7.3** Calculate the density, specific weight, and specific volume of air at 120°F and 50 psia.
- 2.7.4** Calculate the density, specific weight, and specific volume of air at 50°C and 3400 mb abs.
- 2.7.5** If natural gas has a specific gravity of 0.6 relative to air at 14.7 psia and 68°F, what are its specific weight and specific volume at that same pressure and temperature. What is the value of R for the gas? Solve without using Table A.2.
- 2.7.6** Given that a sample of dry air at 40°F and 14.7 psia contains 21% oxygen and 78% nitrogen by volume. What is the partial pressure (psia) and specific weight of each gas?
- 2.7.7** Prove that Eq. (2.7) follows from Eqs. (2.4) and (2.6).

2.8 COMPRESSIBILITY OF PERFECT GASES

Differentiating Eq. (2.6) gives $npv^{n-1}dv + v^n dp = 0$. Inserting the value of dp from this into $E_v = -(v/dv) dp$ from Sec. 2.5 yields

$$E_v = np \quad (2.8)$$

So for an isothermal process of a gas $E_v = p$, and for an isentropic process $E_v = kp$.

Thus, at a pressure of 15 psia, the isothermal modulus of elasticity for a gas is 15 psi, and for air in an isentropic process it is $1.4(15 \text{ psi}) = 21 \text{ psi}$. Assuming from Table 2.1 a typical value of the modulus of elasticity of cold water to be 320,000 psi, we see that air at 15 psia is $320,000/15 = 21,000$ times as compressible as cold water isothermally, or $320,000/21 = 15,000$ times as compressible isentropically. This emphasizes the great difference between the compressibility of normal atmospheric air and that of water.

SAMPLE PROBLEM 2.6 (a) Calculate the density, specific weight, and specific volume of oxygen at 100°F and 15 psia (pounds per square inch absolute; see Sec. 2.7). (b) What would be the temperature and pressure of this gas if we compressed it isentropically to 40% of its original volume? (c) If the process described in (b) had been isothermal, what would the temperature and pressure have been?

Solution

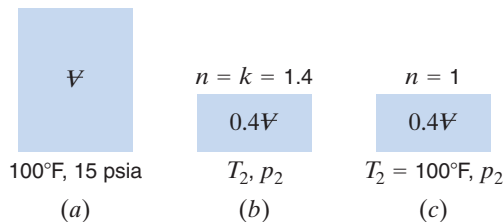


Table A.5 for oxygen (O_2): Molar mass $M = 32.0$, $k = 1.40$

$$(a) \text{ Sec. 2.7: } R \approx \frac{R_0}{M} = \frac{49,709}{32.0} = 1553 \text{ ft}\cdot\text{lb}/(\text{slug}\cdot^\circ\text{R}) \quad (\text{as in Table A.5})$$

$$\begin{aligned} \text{From Eq. (2.4): } \rho &= \frac{p}{RT} = \frac{15 \times 144 \text{ lb}/\text{ft}^2}{[1553 \text{ ft}\cdot\text{lb}/(\text{slug}\cdot^\circ\text{R})][(460 + 100)^\circ\text{R}]} \\ &= 0.00248 \text{ slug}/\text{ft}^3 \quad \mathbf{ANS} \end{aligned}$$

$$\text{With } g = 32.2 \text{ ft}/\text{sec}^2, \gamma = \rho g = 0.00248(32.2) = 0.0800 \text{ lb}/\text{ft}^3 \quad \mathbf{ANS}$$

$$\text{Eq. (2.2): } v = \frac{1}{\rho} = \frac{1}{0.00248} = 403 \text{ ft}^3/\text{slug} \quad \mathbf{ANS}$$

$$\begin{aligned} (b) \text{ Isentropic compression: } v_2 &= 40\% v_1 = 0.4(403) = 161.1 \text{ ft}^3/\text{slug} \\ \rho_2 &= 1/v_2 = 0.00621 \text{ slug}/\text{ft}^3 \end{aligned}$$

$$\begin{aligned} \text{Eq. (2.6) with } n = k: p v^k &= (15 \times 144)(403)^{1.4} = (p_2 \times 144)(161.1)^{1.4} \\ p_2 &= 54.1 \text{ psia} \quad \mathbf{ANS} \end{aligned}$$

$$\begin{aligned} \text{From Eq. (2.4): } p_2 &= 54.1 \times 144 \text{ psia} = \rho R T = 0.00621(1553)(460 + T_2) \\ T_2 &= 348^\circ\text{F} \quad \mathbf{ANS} \end{aligned}$$

$$(c) \text{ Isothermal compression: } T_2 = T_1 = 100^\circ\text{F} \quad \mathbf{ANS}$$

$$\begin{aligned} p v &= \text{constant: } (15 \times 144)(403) = (p_2 \times 144)(0.4 \times 403) \\ p_2 &= 37.5 \text{ psia} \quad \mathbf{ANS} \end{aligned}$$

SAMPLE PROBLEM 2.7 Calculate the density, specific weight, and specific volume of chlorine gas at 25°C and pressure of 600 kN/m² abs (kilonewtons per square meter absolute; see Sec. 2.7). Given the molar mass of chlorine (Cl₂) = 71.

Solution

$$\text{Sec. 2.7:} \quad R = \frac{R_0}{M} = \frac{8312}{71} = 117.1 \text{ N}\cdot\text{m}/(\text{kg}\cdot\text{K})$$

$$\begin{aligned} \text{From Eq. (2.4):} \quad \rho &= \frac{p}{RT} = \frac{600000 \text{ N/m}^2}{[117.1 \text{ N}\cdot\text{m}/(\text{kg}\cdot\text{K})][(273 + 25)\text{K}]} \\ &= 17.20 \text{ kg/m}^3 \quad \text{ANS} \end{aligned}$$

$$\text{With } g = 9.81 \text{ m/s}^2, \quad \gamma = \rho g = 17.20(9.81) = 168.7 \text{ N/m}^3 \quad \text{ANS}$$

$$\text{Eq. (2.2):} \quad v = \frac{1}{\rho} = \frac{1}{17.20} = 0.0581 \text{ m}^3/\text{kg} \quad \text{ANS}$$

EXERCISES

- 2.8.1** Methane at 22 psia is compressed isothermally, and nitrogen at 16 psia is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?
- 2.8.2** Methane at 140 kPa abs is compressed isothermally, and nitrogen at 100 kPa abs is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?
- 2.8.3** (a) If 10 m³ of nitrogen at 30°C and 125 kPa are expanded isothermally to 25 m³, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for nitrogen is 1.40.
- 2.8.4** Helium at 25 psia and 65°F is isentropically compressed to one-fifth of its original volume. What is its final pressure?

2.9 STANDARD ATMOSPHERE

Standard atmospheres were first adopted in the 1920s in the United States and in Europe to satisfy a need for standardization of aircraft instruments and aircraft performance. As knowledge of the atmosphere increased, and man's activities in it rose to ever greater altitudes, such standards have been frequently extended and improved.

The International Civil Aviation Organization adopted its latest *ICAO Standard Atmosphere* in 1964, which extends up to 32 km (105,000 ft). The International Standards Organization adopted an *ISO Standard Atmosphere* to

50 km (164,000 ft) in 1973, which incorporates the ICAO standard. The United States has adopted the *U.S. Standard Atmosphere*, last revised in 1976. This incorporates the ICAO and ISO standards, and extends to at least 86 km (282,000 ft or 53.4 mi); for some quantities it extends as far as 1000 km (621 mi).

Figure 2.2 graphically presents variations of temperature and pressure in the U.S. Standard Atmosphere. In the lowest 11.02 km (36,200 ft), called the *troposphere*, the temperature decreases rapidly and linearly at a *lapse rate* of $-6.489^{\circ}\text{C}/\text{km}$ ($-3.560^{\circ}\text{F}/1000\text{ ft}$). In the next layer, called the *stratosphere*, about 9 km (30,000 ft) thick, the temperature remains constant at -56.5°C (-69.7°F). Next, in the *mesosphere*, it increases, first slowly and then more rapidly, to a maximum of -2.5°C (27.5°F) at an altitude around 50 km (165,000 ft or 31 mi). Above this, in the upper part of the mesosphere known as the *ionosphere*, the temperature again decreases.

The standard absolute pressure⁹ behaves very differently from temperature (Fig. 2.2), decreasing quite rapidly and smoothly to almost zero at an altitude of

⁹ Absolute pressure is discussed in Secs. 2.7 and 3.4.

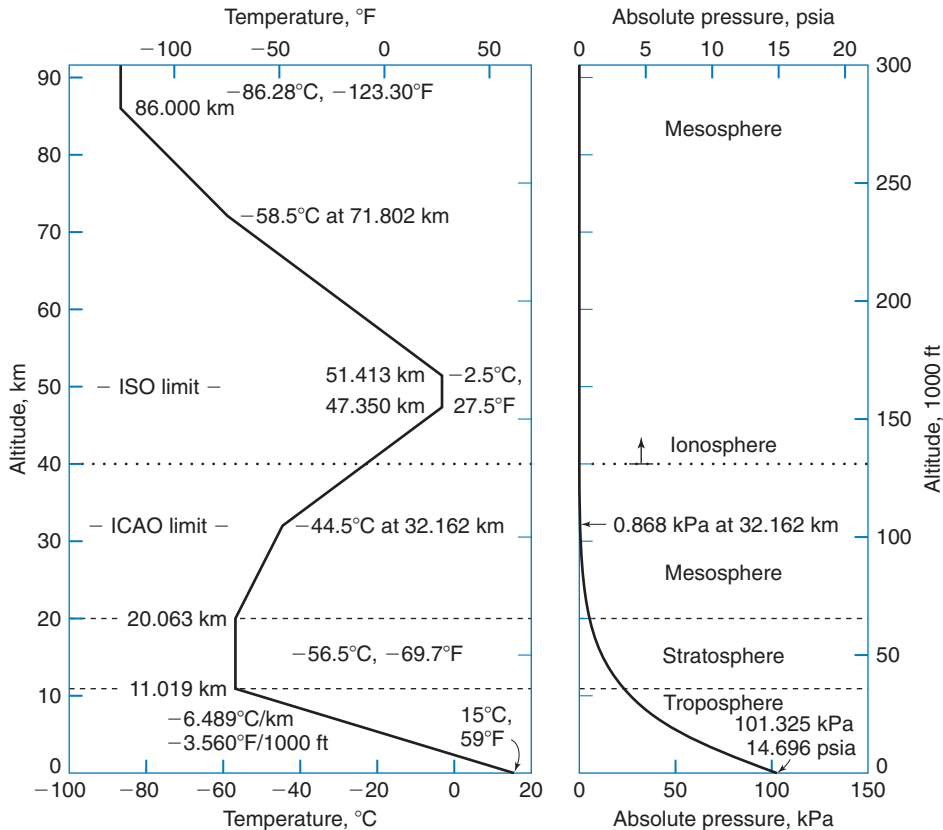


Figure 2.2 The U.S. Standard Atmosphere, temperature, and pressure distributions.

30 km (98,000 ft). The pressure profile was computed from the standard temperatures using methods of fluid statics (Sec. 3.2). The representation of the standard temperature profile by a number of linear functions of elevation (Fig. 2.2) greatly facilitates such computations (see Sample Prob. 3.1*d*).

Temperature, pressure, and other variables from the ICAO Standard Atmosphere, including density and viscosity, are tabulated together with gravitational acceleration out to 30 km and 100,000 ft in Appendix A, Table A.3. Engineers generally use such data in design calculations where the performance of high-altitude aircraft is of interest. The standard atmosphere serves as a good approximation of conditions in the atmosphere; of course the actual conditions vary somewhat with the weather, the seasons, and the latitude.

2.10 IDEAL FLUID

An *ideal* fluid is usually defined as a fluid in which there is no *friction*; it is *inviscid* (its viscosity is zero). Thus the internal forces at any section within it are always normal to the section, even during motion. So these forces are purely pressure forces. Although such a fluid does not exist in reality, many fluids approximate frictionless flow at sufficient distances from solid boundaries, and so we can often conveniently analyze their behaviors by assuming an ideal fluid. As noted in Sec. 2.7, take care to not confuse an ideal fluid with a perfect (ideal) gas.

In a *real* fluid, either liquid or gas, tangential or shearing forces always develop whenever there is motion relative to a body, thus creating fluid friction, because these forces oppose the motion of one particle past another. These friction forces give rise to a fluid property called viscosity.

2.11 VISCOSITY

The *viscosity* of a fluid is a measure of its resistance to shear or angular deformation. Motor oil, for example, has high viscosity and resistance to shear, is cohesive, and feels “sticky,” whereas gasoline has low viscosity. The friction forces in flowing fluid result from the cohesion and momentum interchange between molecules. Figure 2.3 indicates how the viscosities of typical fluids depend on temperature. As the temperature increases, the viscosities of all liquids *decrease*, while the viscosities of all gases *increase*. This is because the force of cohesion, which diminishes with temperature, predominates with liquids, while with gases the predominating factor is the interchange of molecules between the layers of different velocities. Thus a rapidly-moving gas molecule shifting into a slower-moving layer tends to speed up the latter. And a slow-moving molecule entering a faster-moving layer tends to slow down the faster-moving layer. This molecular interchange sets up a shear, or produces a friction force between adjacent layers. At higher temperatures molecular activity increases, so causing the viscosity of gases to increase with temperature.

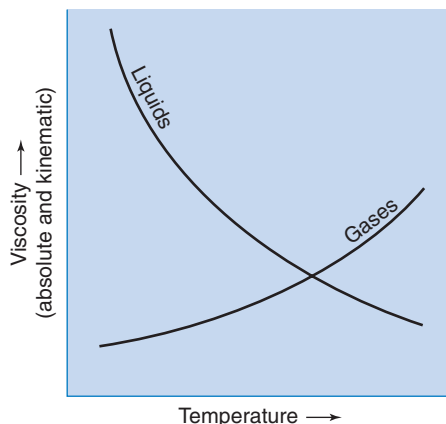


Figure 2.3
Trends in viscosity variation with temperature.

Figures A.1 and A.2 in Appendix A graphically present numerical values of absolute and kinematic viscosities for a variety of liquids and gases, and show how they vary with temperature.

Consider the classic case of two parallel plates (Fig. 2.4), sufficiently large that we can neglect edge conditions, a small distance Y apart, with fluid filling the space between. The lower plate is stationary, while the upper one moves parallel to it with a velocity U due to a force F corresponding to some area A of the moving plate.

At boundaries, particles of fluid adhere to the walls, and so their velocities are zero relative to the wall. This so-called ***no-slip condition*** occurs with all viscous fluids. Thus in Fig. 2.4 the fluid velocities must be U where in contact with the plate at the upper boundary and zero at the lower boundary. We call the form of the velocity variation with distance between these two extremes, as depicted in Fig. 2.4, a ***velocity profile***. If the separation distance Y is not too great, if the velocity U is not too high, and if there is no net flow of fluid through the space, the velocity profile will be linear, as in Fig. 2.4a. If, in addition, there is a small amount of bulk fluid transport between the plates, as could result from pressure-fed lubrication for example, the velocity profile becomes the sum of

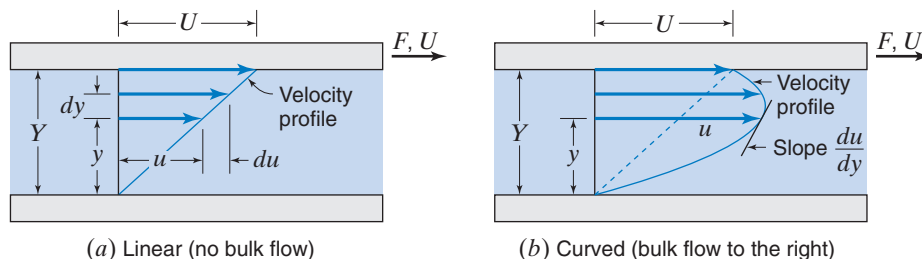


Figure 2.4
Velocity profiles.

the previous linear profile plus a parabolic profile (Fig. 2.4b); the parabolic additions to (or subtractions from) the linear profile are zero at the walls (plates) and maximum at the centerline. The behavior of the fluid is much as if it consisted of a series of thin layers, each of which slips a little relative to the next.

For a large class of fluids under the conditions of Fig. 2.4a, experiments have shown that

$$F \propto \frac{AU}{Y}$$

We see from similar triangles that we can replace U/Y by the velocity gradient du/dy . If we now introduce a constant of proportionality μ (mu), we can express the shearing stress τ (tau) between any two thin sheets of fluid by

$$\tau = \frac{F}{A} = \mu \frac{U}{Y} = \mu \frac{du}{dy} \quad (2.9)$$

We call Eq. (2.9) **Newton's equation of viscosity**, since Sir Isaac Newton (1642–1727) first suggested it. Although better known for his formulation of the fundamental laws of motion and gravity and for the development of differential calculus, Newton, an English mathematician and natural philosopher, also made many pioneering studies in fluid mechanics. In transposed form, Eq. (2.9) defines the proportionality constant

$$\mu = \frac{\tau}{du/dy} \quad (2.10)$$

known as the **coefficient of viscosity**, the **absolute viscosity**, the **dynamic viscosity** (since it involves force), or simply the **viscosity** of the fluid. We shall use “absolute viscosity” to help differentiate it from another viscosity that we will discuss shortly.

We noted in Sec. 2.1 that the distinction between a solid and a fluid lies in the manner in which each can resist shearing stresses. We will clarify a further distinction among various kinds of fluids and solids by referring to Fig. 2.5. In

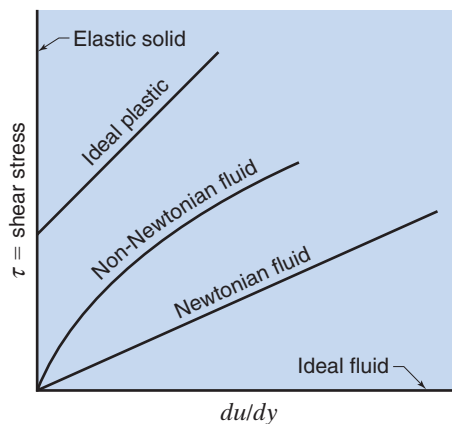


Figure 2.5

the case of a solid, shear stress depends on the *magnitude* of the deformation; but Eq. (2.9) shows that in many fluids the shear stress is proportional to the *time rate* of (angular) deformation.

A fluid for which the constant of proportionality (i.e., the absolute viscosity) does not change with rate of deformation is called a **Newtonian fluid**, and this plots as a straight line in Fig. 2.5. The slope of this line is the absolute viscosity, μ . The ideal fluid, with no viscosity (Sec. 2.10), falls on the horizontal axis, while the true elastic solid plots along the vertical axis. A plastic that sustains a certain amount of stress before suffering a plastic flow corresponds to a straight line intersecting the vertical axis at the yield stress. There are certain non-Newtonian fluids¹⁰ in which μ varies with the rate of deformation. These are relatively uncommon in engineering usage, so we will restrict the remainder of this text to the common fluids that under normal conditions obey Newton's equation of viscosity.

In a **journal bearing**, lubricating fluid fills the small annular space between a shaft and its surrounding support. This fluid layer is very similar to the layer between the two parallel plates, except it is curved. There is another more subtle difference, however. For coaxial cylinders (Fig. 2.6) with constant rotative speed ω (omega), the resisting and driving torques are equal. But because the radii at the inner and outer walls are different, it follows that the shear stresses

¹⁰ Typical non-Newtonian fluids include paints, printer's ink, gels and emulsions, sludges and slurries, and certain plastics. An excellent treatment of the subject is given by W. L. Wilkinson in *NonNewtonian Fluids*, Pergamon Press, New York, 1960.

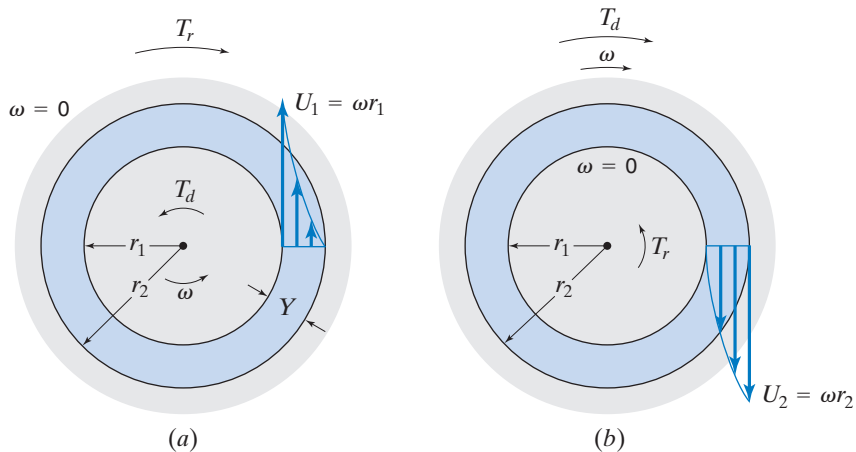


Figure 2.6 Velocity profile, rotating coaxial cylinders with gap completely filled with liquid. (a) Inner cylinder rotating. (b) Outer cylinder rotating. Z is the dimension at right angles to the plane of the sketch. Resisting torque = driving torque and $\tau \propto (du/dy)$.

$$\tau_1(2\pi r_1 Z)r_1 = \tau_2(2\pi r_2 Z)r_2, \quad \left(\frac{du}{dy}\right)_1 = \left(\frac{du}{dy}\right)_2 \frac{r_2^2}{r_1^2}$$

and velocity gradients there must also be different (see Fig. 2.6 and equations that accompany it). The shear stress and velocity gradient must vary continuously across the gap, and so the velocity profile must curve. However, as the gap distance $Y \rightarrow 0$, $du/dy \rightarrow U/Y = \text{constant}$. So, when the gap is very small, we can assume the velocity profile to be a straight line, and we can solve problems in a similar manner as for flat plates.

The dimensions of absolute viscosity are force per unit area divided by velocity gradient. In the British Gravitational (BG) system the dimensions of absolute viscosity are as follows:

$$\text{Dimensions of } \mu = \frac{\text{dimensions of } \tau}{\text{dimensions of } du/dy} = \frac{\text{lb/ft}^2}{\text{fps/ft}} = \text{lb}\cdot\text{sec/ft}^2$$

In SI units

$$\text{Dimensions of } \mu = \frac{\text{N/m}^2}{\text{s}^{-1}} = \text{N}\cdot\text{s/m}^2$$

A widely used unit for viscosity in the metric system is the **poise** (P), named after Jean Louis Poiseuille (1799–1869). A French anatomist, Poiseuille was one of the first investigators of viscosity. The poise = $0.10 \text{ N}\cdot\text{s/m}^2$. The **centipoise** (cP) ($= 0.01 \text{ P} = 1 \text{ mN}\cdot\text{s/m}^2$) is frequently a more convenient unit. It has a further advantage in that the viscosity of water at 68.4°F is 1 cP. Thus the value of the viscosity in centipoises is an indication of the viscosity of the fluid relative to that of water at 68.4°F .

In many problems involving viscosity the absolute viscosity is divided by density. This ratio defines the **kinematic viscosity** ν (nu), so called because force is not involved, the only dimensions being length and time, as in kinematics (Sec. 1.1). Thus

$$\nu = \frac{\mu}{\rho} \quad (2.11)$$

We usually measure kinematic viscosity ν in ft^2/sec in the BG system, and in m^2/s in the SI. Previously, in the metric system the common units were cm^2/s , also called the **stoke** (St), after Sir George Stokes (1819–1903), an English physicist and pioneering investigator of viscosity. Many found the **centistoke** (cSt) ($0.01 \text{ St} = 10^{-6} \text{ m}^2/\text{s}$) a more convenient unit to work with.

An important practical distinction between the two viscosities is the following. The absolute viscosity μ of most fluids is virtually independent of pressure for the range that is ordinarily encountered in engineering work; for extremely high pressures, the values are a little higher than those shown in Fig. A.1. The kinematic viscosity ν of gases, however, varies strongly with pressure because of changes in density. Therefore, if we need to determine the kinematic viscosity ν at a nonstandard pressure, we can look up the (pressure-independent) value of μ and calculate ν from Eq. (2.11). This will require knowing the gas density, ρ , which, if necessary, we can calculate using Eq. (2.4).

The *measurement* of viscosity is described in Sec. 11.1.

SAMPLE PROBLEM 2.8 A 1-in-wide space between two horizontal plane surfaces is filled with SAE 30 Western lubricating oil at 80°F. What force is required to drag a very thin plate of 4-ft² area through the oil at a velocity of 20 ft/min if the plate is 0.33 in from one surface?

Solution

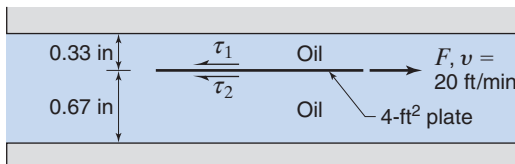


Fig. A.1:

$$\mu = 0.0063 \text{ lb}\cdot\text{sec}/\text{ft}^2$$

Eq. (2.9): $\tau_1 = 0.0063 \times (20/60)/(0.33/12) = 0.0764 \text{ lb}/\text{ft}^2$

Eq. (2.9): $\tau_2 = 0.0063 \times (20/60)/(0.67/12) = 0.0394 \text{ lb}/\text{ft}^2$

From Eq. (2.9): $F_1 = \tau_1 A = 0.0764 \times 4 = 0.305 \text{ lb}$

From Eq. (2.9): $F_2 = \tau_2 A = 0.0394 \times 4 = 0.158 \text{ lb}$

$$\text{Force} = F_1 + F_2 = 0.463 \text{ lb} \quad \text{ANS}$$

SAMPLE PROBLEM 2.9 In Fig. S2.9 oil of absolute viscosity μ fills the small gap of thickness Y . (a) Neglecting fluid stress exerted on the circular underside, obtain an expression for the torque T required to rotate the truncated cone at constant speed ω . (b) What is the rate of heat generation, in joules per second, if the oil's absolute viscosity is $0.20 \text{ N}\cdot\text{s}/\text{m}^2$, $\alpha = 45^\circ$, $a = 45 \text{ mm}$, $b = 60 \text{ mm}$, $Y = 0.2 \text{ mm}$, and the speed of rotation is 90 rpm?

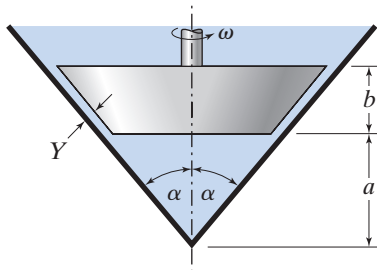


Figure S2.9

Solution

(a) $U = \omega r$; for small gap Y , $\frac{du}{dy} = \frac{U}{Y} = \frac{\omega r}{Y}$

Eq. (2.9): $\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{Y}$; $dA = 2\pi r ds = \frac{2\pi r dy}{\cos \alpha}$

$$\text{From Eq. (2.9): } dF = \tau dA = \frac{\mu \omega r}{Y} \left(\frac{2\pi r dy}{\cos \alpha} \right)$$

$$dT = r dF = \frac{2\pi \mu \omega}{Y \cos \alpha} r^3 dy; \quad r = y \tan \alpha$$

$$dT = \frac{2\pi \mu \omega \tan^3 \alpha}{Y \cos \alpha} y^3 dy$$

$$T = \frac{2\pi \mu \omega \tan^3 \alpha}{Y \cos \alpha} \int_a^{a+b} y^3 dy; \quad \frac{y^4}{4} \Big|_a^{a+b} = \left[\frac{(a+b)^4}{4} - \frac{a^4}{4} \right]$$

$$T = \frac{2\pi \mu \omega \tan^3 \alpha}{4Y \cos \alpha} [(a+b)^4 - a^4] \quad \text{ANS}$$

$$(b) [(a+b)^4 - a^4] = (0.105 \text{ m})^4 - (0.045 \text{ m})^4 = 0.0001175 \text{ m}^4$$

$$\omega = \left(90 \frac{\text{rev}}{\text{min}} \right) \left(2\pi \frac{\text{radians}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 3\pi \text{ rad/s} = 3\pi \text{ s}^{-1}$$

$$\begin{aligned} \text{Heat generation rate} = \text{power} = T\omega &= \frac{2\pi \mu \omega^2 \tan^3 \alpha}{4Y \cos \alpha} [(a+b)^4 - a^4] \\ &= \frac{2\pi (0.20 \text{ N}\cdot\text{s/m}^2) (3\pi \text{ s}^{-1})^2 (1)^3 [0.0001175 \text{ m}^4]}{4(2 \times 10^{-4} \text{ m}) \cos 45^\circ} \\ &= 23.2 \text{ N}\cdot\text{m/s} = 23.2 \text{ J/s} \quad \text{ANS} \end{aligned}$$

EXERCISES

- 2.11.1** At 60°F what is the kinematic viscosity of the gasoline in Fig. A.2, the specific gravity of which is 0.680? Give the answer in both BG and SI units.
- 2.11.2** To what temperature must the fuel oil with the higher specific gravity in Fig. A.2 be heated in order that its kinematic viscosity may be reduced to three times that of water at 40°F?
- 2.11.3** Compare the ratio of the absolute viscosities of air and water at 70°F with the ratio of their kinematic viscosities at the same temperature and at 14.7 psia.
- 2.11.4** A flat plate 200 mm × 750 mm slides on oil ($\mu = 0.85 \text{ N}\cdot\text{s/m}^2$) over a large plane surface (Fig. X2.11.4). What force F is required to drag the plate at a velocity v of 1.2 m/s, if the thickness t of the separating oil film is 0.6 mm?

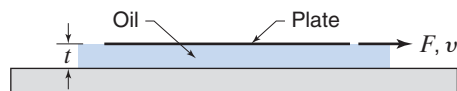


Figure X2.11.4

- 2.11.5** Refer to Fig. X2.11.4. A flat plate 2 ft × 3 ft slides on oil ($\mu = 0.024 \text{ lb}\cdot\text{sec/ft}^2$) over a large plane surface. What force F is required to drag the plate at a velocity v of 4 ft/sec, if the thickness t of the separating oil film is 0.025 in?

- 2.11.6** A liquid has an absolute viscosity of 3.2×10^{-4} lb·sec/ft². It weighs 56 lb/ft³. What are its absolute and kinematic viscosities in SI units?
- 2.11.7** (a) What is the ratio of the absolute viscosity of water at a temperature of 70°F to that of water at 200°F? (b) What is the ratio of the absolute viscosity of the crude oil in Fig. A.1 ($s = 0.925$) to that of the gasoline ($s = 0.680$), both being at a temperature of 60°F? (c) In cooling from 300 to 80°F, what is the ratio of the change of the absolute viscosity of the SAE 30 Western oil to that of the SAE 30 Eastern oil? Refer to Appendix A.
- 2.11.8** A space 16 mm wide between two large plane surfaces is filled with SAE 30 Western lubricating oil at 35°C (Fig. X2.11.8). What force F is required to drag a very thin plate of 0.4 m² area between the surfaces at a speed $v = 0.25$ m/s (a) if the plate is equally spaced between the two surfaces, and (b) if $t = 5$ mm? Refer to Appendix A.

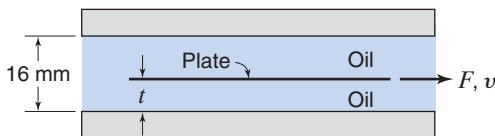


Figure X2.11.8

- 2.11.9** A journal bearing consists of an 80-mm shaft in an 80.4-mm sleeve 120 mm long, the clearance space (assumed to be uniform) being filled with SAE 30 Western lubricating oil at 40°C (Fig. X2.11.9). Calculate the rate at which heat is generated at the bearing when the shaft turns at 150 rpm. Express the answer in kN·m/s, J/s, Btu/hr, ft·lb/sec, and hp. Refer to Appendix A.

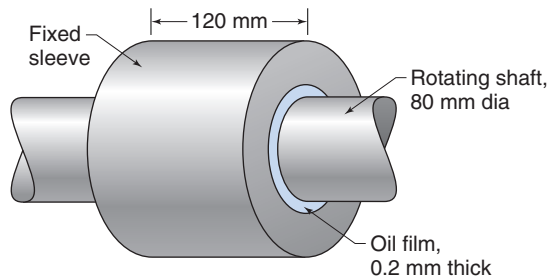


Figure X2.11.9

- 2.11.10** In using a rotating-cylinder viscometer, a bottom correction must be applied to account for the drag on the flat bottom of the inner cylinder. Calculate the theoretical amount of this torque correction, neglecting centrifugal effects, for a cylinder of diameter d , rotated at a constant angular velocity ω , in a liquid of absolute viscosity μ , with a clearance Δh between the bottom of the inner cylinder and the floor of the outer one.
- 2.11.11** Assuming a velocity distribution as shown in Fig. X2.11.11, which is a parabola having its vertex 12 in from the boundary, calculate the velocity gradients for $y = 0, 3, 6, 9,$ and 12 in. Also calculate the shear stresses in lb/ft² at these points if the fluid's absolute viscosity is 600 cP.

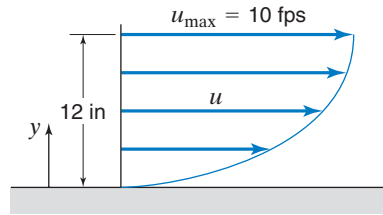


Figure X2.11.11

- 2.11.12** Air at 50 psia and 60°F is flowing through a pipe. Table A.2 indicates that its kinematic viscosity ν is 0.158×10^{-3} ft²/sec. (a) Why is this ν value incorrect? (b) What is the correct value?

2.12 SURFACE TENSION

Liquids have cohesion and adhesion, both of which are forms of molecular attraction. **Cohesion** enables a liquid to resist tensile stress, while **adhesion** enables it to adhere to another body.¹¹ At the interface between a liquid and a gas, i.e., at the liquid surface, and at the interface between two **immiscible** (not mixable) liquids, the out-of-balance attraction force between molecules forms an imaginary surface film which exerts a tension force in the surface. This liquid property is known as **surface tension**. Because this tension acts in a surface, we compare such forces by measuring the tension force per unit length of surface. When a second fluid is not specified at the interface, it is understood that the liquid surface is in contact with air. The surface tensions of various liquids cover a wide range, and they decrease slightly with increasing temperature. Values of the surface tension for water between the freezing and boiling points vary from 0.00518 to 0.00404 lb/ft (0.0756 to 0.0589 N/m); Table A.1 of Appendix A contains more typical values. Table A.4 includes values for other liquids. **Capillarity** is the property of exerting forces on fluids by fine tubes or porous media; it is due to both cohesion and adhesion. When the cohesion is of less effect than the adhesion, the liquid will wet a solid surface it touches and rise at the point of contact; if cohesion predominates, the liquid surface will depress at the point of contact. For example, capillarity makes water rise in a glass tube, while mercury depresses below the true level, as shown in the insert in Fig. 2.7, which is drawn to scale and reproduced actual size. We call the curved liquid surface that develops in a tube a **meniscus**.

A cross section through capillary rise in a tube looks like Fig. 2.8. From free-body considerations, equating the lifting force created by surface tension to

¹¹ In 1877 Osborne Reynolds demonstrated that a $\frac{1}{4}$ -in.-diameter column of mercury could withstand a tensile stress (negative pressure, below atmospheric) of 3 atm (44 psi or 304 kPa) for a time, but that it would separate upon external jarring of the tube. Liquid tensile stress (said to be as high as 400 atm) accounts for the rise of water in the very small channels of xylem tissue in tall trees. For practical engineering purposes, however, we assume liquids are incapable of resisting any direct tensile stress.

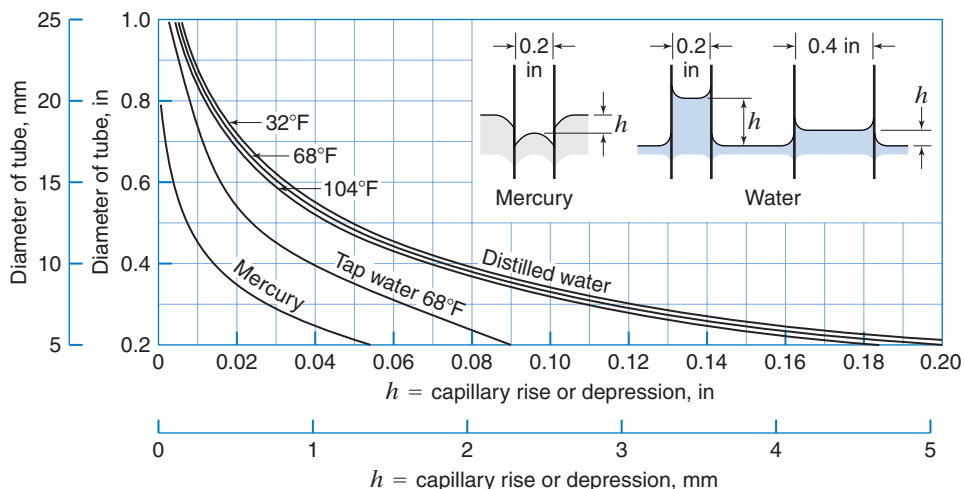


Figure 2.7
Capillarity in clean circular glass tubes, for liquid in contact with air.

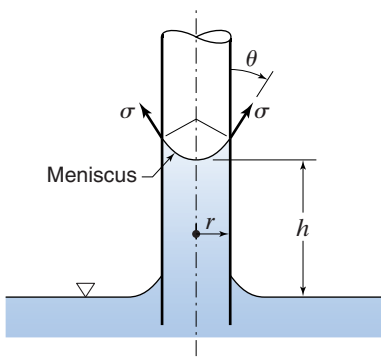


Figure 2.8
Capillary rise.

the gravity force,

$$2\pi r\sigma \cos \theta = \pi r^2 h \gamma$$

so

$$h = \frac{2\sigma \cos \theta}{\gamma r} \tag{2.12}$$

where σ = surface tension (sigma) in units of force per unit length

θ = wetting angle (theta)

γ = specific weight of liquid

r = radius of tube

h = capillary rise¹²

¹² Measurements to a meniscus are usually taken to the point on the centerline.

We can use this expression to compute the *approximate* capillary rise or depression in a tube. If the tube is clean, $\theta = 0^\circ$ for water and about 140° for mercury. Note that the meniscus (Figs. 2.7 and 2.8) lifts a small volume of liquid, near the tube walls, in addition to the volume $\pi r^2 h$ used in Eq. (2.12). For larger tube diameters, with smaller capillary rise heights, this small additional volume can become a large fraction of $\pi r^2 h$. So Eq. (2.12) overestimates the amount of capillary rise or depression, particularly for larger diameter tubes. The curves of Fig. 2.7 are for water or mercury in contact with air; if mercury is in contact with water, the surface tension effect is slightly less than when in contact with air. For tube diameters larger than $\frac{1}{2}$ in (12 mm), capillary effects are negligible.

Surface tension effects are generally negligible in most engineering situations. However, they can be important in problems involving capillary rise, such as in the soil water zone; without capillarity most forms of vegetable life would perish. When we use small tubes to measure fluid properties, such as pressures, we must take the readings while aware of the surface tension effects; a true reading would occur if surface tension effects were zero. These effects are also important in hydraulic model studies when the model is small, in the breakup of liquid jets, and in the formation of drops and bubbles. The formation of drops is extremely complex to analyze, but is, for example, of critical concern in the design of inkjet printers, a multi-billion-dollar business.

SAMPLE PROBLEM 2.10 Water at 10°C stands in a clean glass tube of 2-mm diameter at a height of 35 mm. What is the true static height?

Solution

Table A.1 at 10°C : $\gamma = 9804 \text{ N/m}^3$, $\sigma = 0.0742 \text{ N/m}$.

Sec. 2.12 for clean glass tube: $\theta = 0^\circ$.

$$\begin{aligned} \text{Eq. (2.12): } \quad h &= \frac{2\sigma}{\gamma r} = \frac{2(0.0742 \text{ N/m})}{(9804 \text{ N/m}^3)0.001 \text{ m}} \\ &= 0.01514 \text{ m} = 15.14 \text{ mm} \end{aligned}$$

Sec. 2.12: True static height = $35.00 - 15.14 = 19.86 \text{ mm}$ **ANS**

EXERCISES

- 2.12.1** Tap water at 68°F stands in a glass tube of 0.32-in diameter at a height of 4.50 in. What is the true static height?
- 2.12.2** Distilled water at 20°C stands in a glass tube of 6.0-mm diameter at a height of 18.0 mm. What is the true static height?
- 2.12.3** Use Eq. (2.12) to compute the capillary depression of mercury at 68°F ($\theta = 140^\circ$) to be expected in a 0.05-in-diameter tube.

- 2.12.4** Compute the capillary rise in mm of pure water at 10°C expected in an 0.8-mm-diameter tube.
- 2.12.5** Use Eq. (2.12) to compute the capillary rise of water to be expected in a 0.28-in.-diameter tube. Assume pure water at 68°F. Compare the result with Fig. 2.7.

2.13 VAPOR PRESSURE OF LIQUIDS

All liquids tend to evaporate or vaporize, which they do by projecting molecules into the space above their surfaces. If this is a confined space, the partial pressure exerted by the molecules increases until the rate at which molecules reenter the liquid is equal to the rate at which they leave. For this equilibrium condition, we call the vapor pressure the **saturation pressure**.

Molecular activity increases with increasing temperature and decreasing pressure, and so the saturation pressure does the same. At any given temperature, if the pressure on the liquid surface falls below the saturation pressure, a rapid rate of evaporation results, known as **boiling**. Thus we can refer to the saturation pressure as the **boiling pressure** for a given temperature, and it is of practical importance for liquids.¹³

We call the rapid vaporization and recondensation of liquid as it briefly passes through a region of low absolute pressure **cavitation**. This phenomenon is often very damaging, and so we must avoid it; we shall discuss it in more detail in Sec. 5.10.

Table 2.3 calls attention to the wide variation in saturation vapor pressure of various liquids; Appendix A, Table A.4 contains more values. The very low vapor pressure of mercury makes it particularly suitable for use in barometers. Values for the vapor pressure of water at different temperatures are in Appendix A, Table A.1.

TABLE 2.3 Saturation vapor pressure of selected liquids at 68°F (20°C)

	psia	N/m ² abs	mb abs
Mercury	0.000025	0.17	0.0017
Water	0.34	2340	23.4
Carbon tetrachloride	1.90	13100	131
Gasoline	8.0	55200	552

¹³ Values of the saturation pressure for water for temperatures from 32 to 705.4°F can be found in J. H. Keenan, *Thermodynamic Properties of Water including Vapor, Liquid and Solid States*, John Wiley & Sons, Inc., New York, 1969, and in other steam tables. There are similar vapor tables published for ammonia, carbon dioxide, sulfur dioxide, and other vapors of engineering interest.

SAMPLE PROBLEM 2.11 At approximately what temperature will water boil if the elevation is 10,000 ft?

Solution

From Appendix A, Table A.3, the pressure of the standard atmosphere at 10,000-ft elevation is 10.11 psia. From Appendix A, Table A.1, the saturation vapor pressure p_v of water is 10.11 psia at about 193°F (by interpolation). Hence the water at 10,000 ft will boil at about 193°F. **ANS**

Compared with the boiling temperature of 212°F at sea level, this explains why it takes longer to cook at high elevations.

EXERCISES

2.13.1 At what pressure in millibars absolute will 70°C water boil?

2.13.2 At approximately what temperature will water boil in Mexico City (elevation 7400 ft)? Refer to Appendix A.

PROBLEMS

- 2.1** If the specific weight of a gas is 12.40 N/m³, what is its specific volume in m³/kg?
- 2.2** A gas sample weighs 0.108 lb/ft³ at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 0.075 lb/ft³?
- 2.3** If a certain liquid weighs 8600 N/m³, what are the values of its density, specific volume, and specific gravity relative to water at 15°C? Use Appendix A.
- 2.4** Find the change in volume of 15.00 lb of water at ordinary atmospheric pressure for the following conditions: (a) reducing the temperature by 50°F from 200°F to 150°F; (b) reducing the temperature by 50°F from 150°F to 100°F; (c) reducing the temperature by 50°F from 100°F to 50°F. Calculate each and note the trend in the changes in volume.
- 2.5** Initially when 1000.00 mL of water at 10°C are poured into a glass cylinder, the height of the water column is 1000.0 mm. The water and its container are heated to 70°C.

Assuming no evaporation, what then will be the depth of the water column if the coefficient of thermal expansion for the glass is 3.8×10^{-6} mm/mm per °C?

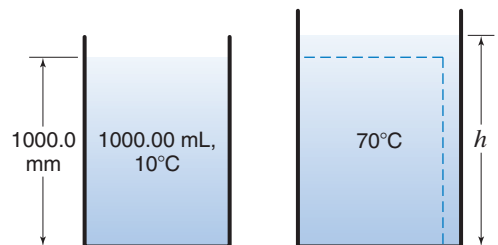


Figure P2.5

- 2.6** At a depth of 4 miles in the ocean the pressure is 9520 psi. Assume that the specific weight at the surface is 64.00 lb/ft³ and that the average volume modulus is 320,000 psi for that pressure range. (a) What will be the change in specific volume between that at the surface and at that depth? (b) What will be the specific volume at that depth? (c) What will be the specific weight at that depth? (d) What is the

percentage change in the specific volume?
 (e) What is the percentage change in the specific weight?

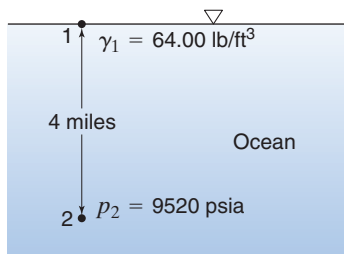


Figure P2.6

- 2.7** Water at 68°F is in a long, rigid cylinder of inside diameter 0.600 in. A plunger applies pressure to the water. If, with zero force, the initial length of the water column is 25.00 in, what will its length be if a force of 420 lb is applied to the plunger. Assume no leakage and no friction.

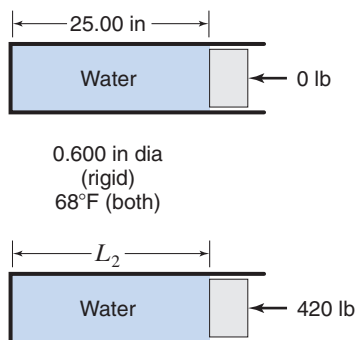


Figure P2.7

- 2.8** Find the change in volume of 10 m^3 of water for the following situations: (a) a temperature increase from 60°C to 70°C with constant atmospheric pressure, (b) a pressure increase from zero to 10 MN/m^2 with temperature remaining constant at 60°C, (c) a temperature decrease from 60°C to 50°C combined with a pressure increase of 10 MN/m^2 .
- 2.9** A heavy closed steel chamber is filled with water at 40°F and atmospheric pressure. If the temperature of the water and the chamber is raised to 80°F, what will be the

new pressure of the water? The coefficient of thermal expansion of the steel is $6.6 \times 10^{-6} \text{ in/in per } ^\circ\text{F}$; assume the chamber is unaffected by the water pressure. Use Table A.1 and Fig. 2.1.

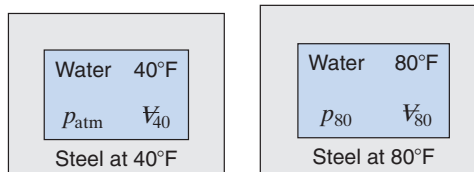


Figure P2.9

- 2.10** Repeat Exer. 2.6.4 for the case where the tank is made of a material that has a coefficient of thermal expansion of $4.6 \times 10^{-6} \text{ mm/mm per } ^\circ\text{C}$.
- 2.11** (a) Calculate the density, specific weight, and specific volume of oxygen at 20°C and $50 \text{ kN/m}^2 \text{ abs}$. (b) If the oxygen is enclosed in a rigid container of constant volume, what will be the pressure if the temperature is reduced to -100°C ?
- 2.12** (a) If water vapor in the atmosphere has a partial pressure of 0.50 psia and the temperature is 90°F, what is its specific weight? (b) If the barometer reads 14.50 psia, what is the partial pressure of the (dry) air, and what is its specific weight? (c) What is the specific weight of the atmosphere (air plus the water vapor present)?
- 2.13** (a) If water vapor in the atmosphere has a partial pressure of 3500 Pa and the temperature is 30°C, what is its specific weight? (b) If the barometer reads 102 kPa abs, what is the partial pressure of the (dry) air, and what is its specific weight? (c) What is the specific weight of the atmosphere (air plus the water vapor present)?
- 2.14** If the specific weight of water vapor in the atmosphere is 0.00065 lb/ft^3 and that of the (dry) air is 0.074 lb/ft^3 when the temperature is 70°F, (a) what are the partial pressures of the water vapor and the dry air in psia, (b) what is the specific weight of the atmosphere (air and water vapor), and (c) what is the barometric pressure in psia?

- 2.15** If an artificial atmosphere consists of 20% oxygen and 80% nitrogen by volume, at 101.32 kN/m² abs and 20°C, what are (a) the specific weight and partial pressure of the oxygen, (b) the specific weight and partial pressure of the nitrogen, and (c) the specific weight of the mixture?
- 2.16** When the ambient air is at 70°F, 14.7 psia, and contains 21% oxygen by volume, 4.5 lb of air are pumped into a scuba tank, capacity 0.75 ft³. (a) What volume of ambient air was compressed? (b) When the filled tank has cooled to ambient conditions, what is the (gage) pressure of the air in the tank? (c) What is the partial pressure (psia) and specific weight of the ambient oxygen? (d) What weight of oxygen was put in the tank? (e) What is the partial pressure (psia) and specific weight of the oxygen in the tank?
- 2.17** (a) If 10 ft³ of carbon dioxide at 50°F and 15 psia is compressed isothermally to 2 ft³, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for carbon dioxide is 1.28.
- 2.18** (a) If 350 L of carbon dioxide at 20°C and 120 kN/m² abs is compressed isothermally to 50 L, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The isentropic exponent k for carbon dioxide is 1.28.
- 2.19** Helium at 180 kN/m² abs and 20°C is isentropically compressed to one-fifth of its original volume. What is its final pressure?
- 2.20** The absolute viscosity of a certain gas is 0.0234 cP while its kinematic viscosity is 181 cSt, both measured at 1013 mb abs and 100°C. Calculate its approximate molar mass, and suggest what gas it may be.
- 2.21** A hydraulic lift of the type commonly used for greasing automobiles consists of a 10.000-in-diameter ram that slides in a 10.006-in-diameter cylinder (Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.0038 ft²/sec and

specific gravity of 0.83. If the rate of travel of the ram v is 0.5 fps, find the frictional resistance, F when 6 ft of the ram is engaged in the cylinder.

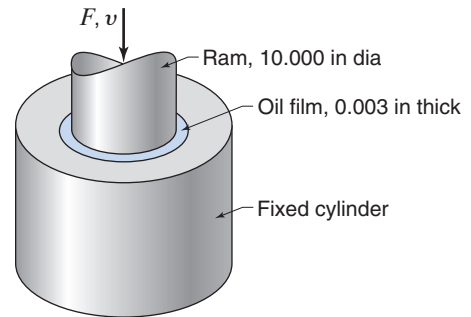


Figure P2.21

- 2.22** A hydraulic lift of the type commonly used for greasing automobiles consists of a 280.00-mm-diameter ram that slides in a 280.18-mm-diameter cylinder (similar to Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.00042 m²/s and specific gravity of 0.86. If the rate of travel of the ram is 0.22 m/s, find the frictional resistance when 2 m of the ram is engaged in the cylinder.
- 2.23** A journal bearing consists of an 8.00-in shaft in an 8.01-in sleeve 10 in long, the clearance space (assumed to be uniform) being filled with SAE 30 Eastern lubricating oil at 100°F. Calculate the rate at which heat is generated at the bearing when the shaft turns at 100 rpm. Refer to Appendix A. Express the answer in Btu/hr.

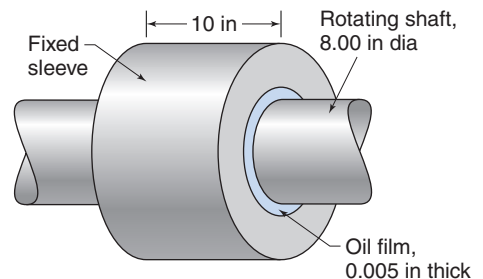


Figure P2.23

- 2.24** Repeat Prob. 2.23 for the case where the sleeve has a diameter of 8.50 in. Compute as accurately as possible the velocity gradient in the fluid at the shaft and sleeve.
- 2.25** A disk spins within an oil-filled enclosure, having 2.4-mm clearance from flat surfaces each side of the disk. The disk surface extends from radius 12 to 86 mm. What torque is required to drive the disk at 660 rpm if the oil's absolute viscosity is $0.12 \text{ N}\cdot\text{s}/\text{m}^2$?
- 2.26** It is desired to apply the general case of Sample Prob. 2.9 to the extreme cases of a journal bearing ($\alpha = 0$) and an end bearing ($\alpha = 90^\circ$). But when $\alpha = 0$, $r = \tan \alpha = 0$, so $T = 0$; when $\alpha = 90^\circ$, contact area = ∞ due to b , so $T = \infty$. Therefore devise an alternative general derivation that will also provide solutions to these two extreme cases.
- 2.27** Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed. Its pressure is now 200 kPa abs and its temperature is 20°C . Table A.2 indicates that its kinematic viscosity ν is $15 \times 10^{-6} \text{ m}^2/\text{s}$. (a) Why is this ν incorrect? (b) What is the correct value?
- 2.28** Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed isentropically. Its pressure is now 194.5 kPa abs and its temperature is 80°C . Table A.2 indicates that its kinematic viscosity ν is $20.9 \times 10^{-6} \text{ m}^2/\text{s}$. (a) Why is this ν incorrect? (b) What is the correct value? (c) What would the correct value be if the compression were isothermal instead?
- 2.29** Pure water at 50°F stands in a glass tube of 0.04-in diameter at a height of 6.78 in. Compute the true static height.
- 2.30** (a) Derive an expression for capillary rise (or depression) between two vertical parallel plates. (b) How much would you expect 10°C water to rise (in mm) if the clean glass plates are separated by 1.2 mm?
- 2.31** By how much does the pressure inside a 2-mm-diameter air bubble in 15°C water exceed the pressure in the surrounding water?
- 2.32** Determine the excess pressure inside an 0.5-in-diameter soap bubble floating in air, given the surface tension of the soap solution is $0.0035 \text{ lb}/\text{ft}$.
- 2.33** Water at 170°F in a beaker is placed within an airtight container. Air is gradually pumped out of the container. What reduction below standard atmospheric pressure of 14.7 psia must be achieved before the water boils?
- 2.34** At approximately what temperature will water boil on top of Mount Kilimanjaro (elevation 5895 m)? Refer to Appendix A.