FLUID FLOW
FOR CHEMICAL ENGINEERS
(EKC212)
Core Course
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by

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1 Principles and Basic Definitions of Fluids

1. Behaviour of fluids is essential to process engineering.
2. It consists of one of the foundations in unit operations.
3. Understanding of fluids is important in treating problems on the movement of fluids through
   - pipes
   - pumps
   - all kinds of process equipment
4. Fluids include
   - liquids
   - gases
   - vapour
5. The area of study of the above fluids is known as *fluid mechanics*.
6. It is in turn a part of a larger discipline called *continuum mechanics*
7. This has 2 main branches:
   - **fluid statics**: treats fluids in the equilibrium state of no shear stress
   - **fluid dynamics**: treats fluids when portions of the fluid are in motion relative to other parts.

1.1 Density

- Density is defined as the mass per unit volume of a particular fluid given by;

$$\begin{aligned}\text{Density}, \rho &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{m}{V} \qquad (1)\end{aligned}$$

- The higher an object's density, the higher its mass per unit volume
- The SI unit of density is the kilogram per cubic metre ($\mathrm{kg \cdot m^{-3}}$)
- Density is also measured in other units, such as;
  - grams per cubic centimetre ($\mathrm{g \cdot cm^{-3}}$)
  - megagrams per cubic metre ($\mathrm{Mg \cdot m^{-3}}$)
  - kilograms per litre ($\mathrm{kg \cdot l^{-1}}$)
  - pounds per cubic foot ($\mathrm{lb \cdot ft^{-3}}$)
  - pounds per cubic yard ($\mathrm{lb \cdot yd^{-3}}$)
  - pounds per cubic inch ($\mathrm{lb \cdot in^{-3}}$)
  - ounces per cubic inch ($\mathrm{oz \cdot in^{-3}}$)

- pounds per gallon (for U.S. or imperial gallons) ($\text{lb} \cdot \text{gal}^{-1}$)

- The maximum density of pure water at a pressure of one standard atmosphere is $999.861\ \text{kg} \cdot \text{m}^{-3}$; this occurs at a temperature of about 3.98°C (277.13 K).

1.2 Viscosity

- Viscosity is a measure of a fluid's resistance to flow.

- It describes the internal friction of a moving fluid.

- A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction.

- A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion.

- Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

- We define the viscosity of the fluid, denoted by η, as the ratio of the shear stress, F/A to the strain rate:

$$\begin{aligned} \text{Viscosity}, \eta &= \frac{\text{Shear stress}}{\text{Strain rate}} \\ &= \frac{F/A}{v/l} \end{aligned} \tag{2}$$

- Rearranging equation (2), we see that the force required for the motion of fluid is directly proportional to the speed:

$$F = \eta A \frac{v}{l} \tag{3}$$

- Viscosities of all fluids are strongly temperature dependent, increasing for gases and decreasing for liquids as the temperature increases.

- The unit of viscosity is that of force times distance, divided by area times speed. The SI unit is;
$$1\ \text{N} \cdot \text{m}/[\text{m}^2 \cdot (\text{m/s})] = 1\ \text{N} \cdot \text{s/m}^2 = 1\ \text{Pa} \cdot \text{s}$$

- The corresponding cgs unit, is the only viscosity unit in common use; it is called a poise, in honor of the French scientist Jean Louis Marie Poiseuille;

$$1\ \text{poise} = 1\ \text{dyn} \cdot \text{s/cm}^2 = 10^{-1}\ \text{N} \cdot \text{s/m}^2$$

- The centipoise and the micropoise are also used.

- The viscosity of water is 1.79 centipoise at 0°C and 0.28 centipoise at 100°C.

- For a *Newtonian fluid* the viscosity is independent of the speed v, and from equation (3) the force F is directly proportional to the speed.

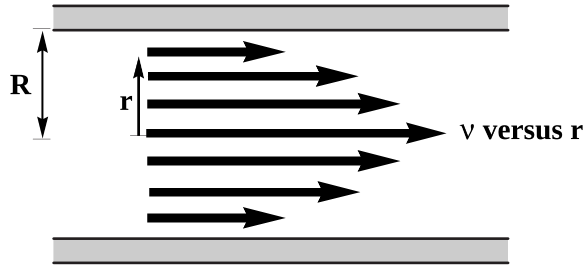

Figure 1: Velocity profile for a viscous fluid in a cylindrical pipe.

- Fluids that are suspensions or dispersions are often non-*Newtonian* in their viscous behavior.
- Figure 1 shows the flow speed profile for laminar flow of a viscous fluid in a long cylindrical pipe.
- The speed is greatest along the axis and zero at the pipe walls.
- The motion is like a lot of concentric tubes sliding relative to one another, with the central tube moving fastest and the outermost tube at rest.
- By applying equation (2) to a cylindrical fluid element, we could derive an equation describing the speed profile.
- The flow speed v at a distance r from the axis of a pipe with radius R is;

$$v = \frac{p_1 - p_2}{4\eta L}(R^2 - r^2) \tag{4}$$

1.3 Pressure

- Basic property of fluid (static).
- It is defined as a surface of force exerted by a fluid against the walls of its container. This can be denoted as;

$$\begin{aligned} p &= \frac{\text{force}}{\text{surface area}} \\ &= \frac{F}{A} \end{aligned} \tag{5}$$

- Pressure exists at every point within a fluid's volume.
- For a static fluid, pressure is independent of the orientation of any internal surface on which the pressure is assumed to act.
- Units of pressure is Pascal, (Pa).
- Other units include;
  - Newton per metre squared, (N/m^2)
  - atmosphere, (atm)
  - bars

- dynes per centimetre squared (dynes/cm^2)
- mm mercury, (mm Hg)
- pound force per square inch, (lb_f/in^2), (psi)
- feet water, (ft H_2O)

1.4 Unit Systems

- The official international system of units is the SI.
- Many physiochemical data are in cgs units and most calculations are in fgs units.
- Physical quantities:
  - they mostly consist of 2 main parts; a unit and a number.
  - a unit shows the quantity and gives a standard by which it is measured.
  - a number shows how many units are needed to make up the quantity.
  - no physical quantity is defined until both the number and the unit are given.
- SI Units:
  - it covers the entire field of science and engineering
  - units are derivable from;
    1. four proportionalities of chemistry and physics (chemistry, mechanics, gravity and thermodynamics).
    2. arbitrary standards for mass, length, time, temperature and mole.
    3. arbitrary choices for the numerical values of 2 proportionality constants.
- Basic Equations:
  - Newton's second law of motion is given by the the proportionality of force and momentum;
  $$F = k_1 \frac{d}{dt}(mu) \tag{6}$$
  - Newton's law of gravitation is given by the proportionality of force and the attraction of 2 particles of masses m_a and m_b a distance r apart;
  $$F = k_2 \frac{m_a m_b}{r^2} \tag{7}$$
  - The first law of thermodynamics if defined as the proportionality between the work performed by a closed system during a cycle and the heat absorbed by that system during the same cycle.
  $$Q_c = k_3 W_c \tag{8}$$
  - The proportionality between thermodynamic absolute temperature and the zero-pressure limit of the pressure-volume product of a definite mass of any gas.
  $$T = k_4 \lim_{p \to 0} \frac{pV}{m} \tag{9}$$

- equations (6) to (9) state that if there is a method available in order to measure the values of all variables and the numerical values of k is calculated, thus, the values of k is constant and it depends only on the units used for measuring the variables in the equation.

- Standard units;
  - standards are fixed arbitrarily for the quantities;
    1. **mass**: the standard of mass is kilogram (kg)and it is defined as the mass of the international kilogram, a platinum cylinder preserved at Sèrves, France.
    2. **length**: the standard length is metre, (m) and it is defined as the length if the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second.
    3. **time**: the standard time is second, (s) and it is defined as 9,192,631.770 frequency cycles of a certain quantum transition in an atom of ^{133}Ce
    4. **temperature**: the standard temperature is Kelvin, (K) and it is defined by assigning the value 273.16K to the temperature of pure water at its triple point, the unique temperature at which liquid water, ice and steam exist at equilibrium.
    5. **mole**: written as (mol), is defined as the amount of substance comprising as many elementary units as there are atoms in 12g of ^{12}C.

- Method of Determining Constants
  - using equations (6) and (7), m, m_a and m_b are in kilograms. Radius, r in metre and u in $\mathrm{m \cdot s^{-1}}$.
  - the constants k_1 and k_2 are not independent but related by eliminating F from the above equations:
$$\frac{k_1}{k_2} = \frac{\frac{d}{dt}(mu)}{\frac{m_a m_b}{r^2}} \tag{10}$$
  - either constants can be fixed arbitrarily, then the other one is found experimentally.
  - in SI system, k_1 is fixed at unity and k_2 is found experimentally, (6) then becomes;
$$F = \frac{d}{dt}(mu) \tag{11}$$
  - F is the force in newton, (N) used in both equations (6) and (7).
  - constant k_2 is given by G, known as the gravitational constant with value of $6.6726 \times 10^{-11}\mathrm{N \cdot m^2 \cdot kg^{-2}}$.
  - **work** and **energy** are measured in newton-metres, (N·m) which is also; $1\ \mathrm{J} = 1\ \mathrm{N \cdot m} = 1\ \mathrm{kg \cdot m^2 \cdot s^{-2}}$.
  - **power** is measured in joules per second, ($\mathrm{J \cdot s^{-1}}$) or watt, (W).
  - **heat** as can be seen from equation (8) is directly equal to work done;
$$Q_c = W_c$$
    where k_3 is fixed at unity with the units of joules.

- for **temperature**, the quantity, $\frac{pV}{m}$ could be measured in joules per unit kilogram. For a chosen type of gas, such quantity can be determined by measuring the pressure, p, volume, V and mass, m in a thermostat. Values of $\frac{pV}{m}$ at various p and at constant temperature can be extrapolated. For water at triple point, the limiting values is denoted as $\left(\frac{pV}{m}\right)_0$ which gives;

$$273.16 = k_4 \lim_{p\to 0}\left(\frac{pV}{m}\right)_0 \tag{12}$$

  for an experiment at temperature T, i.e. $\left(\frac{pV}{m}\right)_T$, the Kelvin temperature is defined as;

$$T = 273.16\frac{\lim_{p\to 0}\left(\frac{pV}{m}\right)_T}{\lim_{p\to 0}\left(\frac{pV}{m}\right)_0} \tag{13}$$

- CGS Units:
  - standard for mass is gram, (g)
  - standard for length is centimetre, (cm)
  - standard for time, temperature and mole are the same.
  - the units of force in CGS system is called *dyne*, (dyn). this is defined as 1 dyn = 1 g·cm·s^{-2}.
  - the units for **energy** and **work** is *erg* where 1 erg = 1 dyn·cm = 1×10^{-7}J.
  - gas constant denoted as R is given by;

$$\lim_{p\to 0}\left(\frac{pV}{nT}\right) = R \tag{14}$$

  - the constant R is given as 8.31447 J·K^{-1}·mol = 8.31447×10^7 erg·K^{-1}·mol

- FPS Engineering Units:
  - the conversion of the non-SI system is given as follows:
    * 1 lb = 0.45359237 kg
    * 1 ft = 2.54 × 12 × 10^{-2}m = 0.3048 m
    * the thermodynamic temperature is given in Rankine;

$$1°\mathrm{R} = \frac{1}{1.8}\mathrm{K}$$

    * relationship between Celsius and Fahrenheit scales is given by;

$$T°\mathrm{F} = 32 + 1.8°\mathrm{C}$$

  - Pound force, (lb$_f$)
    * it is defined as a standard gravitational field of force of one pound on a mass of one pound.
    * the standard acceleration of free fall in fps units is to five significant figures;

$$\begin{aligned} g_n &= \frac{9.80665}{0.3048} \\ &= 32.174\ \mathrm{ft\cdot s^{-1}} \\ &= 1\ \mathrm{lb_f} \end{aligned} \tag{15}$$

1.5 Units Conversion

- The 3 different unit systems are commonly used and they are related by the conversion factors.
- Only defined conversion factors for the base units are required since conversion factors for other units can be determined from them.
- Example: Calculate the conversion factors for;

  1. N to lb_f
  2. BTU to cal_{IT}
  3. atm to $lb_f \cdot in^{-2}$
  4. hp to kW

  Answer:

  1. Given that;
  $$1 \text{ N} = 1 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$$
  and
  $$1 \text{ lb} = 0.45359237 \text{ kg}$$
  also
  $$1 \text{ ft} = 2.45 \times 20^{-2} \text{ m} = 0.3048 \text{ m}$$
  $$1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1 \text{ lb}}{0.4535 \text{ kg}} \cdot \frac{1 \text{ ft}}{0.3048 \text{ m}} \tag{16}$$
  since
  $$1 \text{ lb} \cdot \text{ft} \cdot \text{s}^{-2} = \frac{0.3048}{9.80665} \text{lb}_\text{f}$$
  thus multiplied by equation (16)
  $$1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1 \text{ lb}}{0.4535 \text{ kg}} \cdot \frac{1 \text{ ft}}{0.3048 \text{ m}} \cdot \frac{0.3048}{9.80665} \text{lb}_\text{f} = 0.224809 \text{ lb}_\text{f} \tag{17}$$
  Therefore, $1 \text{ N} = 0.224809 \text{ lb}_\text{f}$
  2. Given that;
  $$1 \text{ BTU} \cdot \text{lb}^{-1} \cdot^\circ \text{F}^{-1} = 1 \text{ cal}_\text{IT} \cdot \text{g}^{-1} \cdot^\circ \text{C}^{-1}$$
  thus,
  $$1 \text{ BTU} = 1 \text{ cal}_\text{IT} \frac{\text{lb} \cdot^\circ \text{F}}{\text{g} \cdot^\circ \text{C}} \cdot \frac{0.4535 \text{ kg}}{1 \text{ lb}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \frac{1}{1.8} \frac{^\circ\text{C}}{^\circ\text{F}} = 252 \text{ cal}_\text{IT} \tag{18}$$
  3. Given that
  $$1 \text{ atm} = 1.01325 \times 10^5 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} \cdot \text{m}^{-2}$$
  convert the units on the RHS;

1.6 Dimensional Analysis

- Technique used to solve problems that cannot be tackled theoretically or by any mathematical means.
- Empirical method should be applied.
- One example is on correlating pressure loss from friction in a long cylindrical straight pipe.
- Variables which involve in measurement include;
  - length of the pipe
  - diameter of the pipe
  - flow rate of the liquid within the pipe
  - density of the liquid
  - viscosity of the liquid
- Any small change of the above variables will result in the change of the pressure drop in the pipe.
- The empirical method of correlating these variables to the pressure drop is by varying each variable separately while keeping the others constant.
- If a theoretical equation of a certain engineering/scientific phenomenon does not exist, there must be an equation which can relate all these variable and finally give one dimensionally homogeneous correlation.
- This method is known as dimensional analysis.
  - algebraic treatment of symbols which represent the units
  - no fitting of experimental data required
  - can be used to check consistency of units in equations as well as during units conversion and scale-up of data
  - this technique still cannot be applied if one does not have a complete information regarding the physics of the situation
  - such a problem can be easily resolved by referring to the basic differential equation of fluid flow which incorporate heat of conduction and diffusion
  - a *clean* numerical equation is impossible in dimensional analysis and one should devise certain experimental work in order to complete the solution of the problem
- There are 3 main rules in testing the dimensional consistency of an equation:
  1. sums of the exponents relating to any given dimension must be the same on both sides of the equation
  2. an exponent must itself be dimensionless
  3. all factors in an equation must be collectible into a set of dimensionless groups and these groups may carry exponents of any magnitude (they might not be whole numbers)

- Example: Check whether there is a consistency in any of these empirical equations;

1. $$h_i = 0.023G^{0.8}k^{0.67}c_p^{0.33}D^{-0.2}\mu^{-0.47}$$

2. $$\frac{h_j D_a}{k} = 4.9\left(\frac{D_a\bar{V}\rho}{\mu}\right)^{0.57}\left(\frac{c_p\mu}{k}\right)^{0.47}\left(\frac{D_a n}{\bar{V}}\right)^{0.17}\left(\frac{D_a}{L}\right)^{0.37}$$

3. $$D_v = 7.4\times 10^{-8}\frac{(\psi_B M_B)^{0.5}T}{\mu V_A^{0.6}}$$

- Answers:

1. Let the dimensions of the quantities involved in

$$h_i = 0.023G^{0.8}k^{0.67}c_p^{0.33}D^{-0.2}\mu^{-0.47}$$

are represented as follows:
$[h_i] = \mathrm{Ht^{-1}L^{-2}T^{-1}}$
$[D] = \mathrm{L}$
$[\bar{G}] = \mathrm{Mt^{-1}L^{-2}}$
$[k] = \mathrm{Ht^{-1}L^{-1}T^{-1}}$
$[c_p] = \mathrm{HM^{-1}T^{-1}}$
$[\mu] = \mathrm{ML^{-1}t^{-1}}$

Left hand side = Right hand side

$$\begin{aligned} h_i &= 0.023G^{0.8}k^{0.67}c_p^{0.33}D^{-0.2}\mu^{-0.47} \\ \mathrm{Ht^{-1}L^{-2}T^{-1}} &= \left[\mathrm{Mt^{-1}L^{-2}}\right]^{0.8}\left[\mathrm{Ht^{-1}L^{-1}T^{-1}}\right]^{0.67}\left[\mathrm{HM^{-1}T^{-1}}\right]^{0.33}[L]^{-0.2}\left[\mathrm{ML^{-1}t^{-1}}\right]^{-0.47} \\ &= \mathrm{M^{0.8}t^{-0.8}L^{-1.6}H^{0.67}t^{-0.67}L^{-0.67}T^{-0.67}H^{0.33}M^{-0.33}T^{-0.33}}L^{-0.2}\mathrm{M^{-0.47}L^{0.47}t^{0.47}} \\ &= \mathrm{M^{0.8-0.33-0.47}t^{-0.8-0.67+0.47}L^{-1.6-0.67-0.2+0.47}H^{0.67+0.33}T^{-0.67-0.33}} \end{aligned} \tag{19}$$

collect the indices of the same dimension gives;
M = 0
t = -1
L = -2
H = 1
T = -1
comparing these to the LHS of the equation confirms that the above correlation is consistent.

2. Let the dimensions of the quantities involved in

$$\frac{h_j D_a}{k} = 4.9\left(\frac{D_a\bar{V}\rho}{\mu}\right)^{0.57}\left(\frac{c_p\mu}{k}\right)^{0.47}\left(\frac{D_a n}{\bar{V}}\right)^{0.17}\left(\frac{D_a}{L}\right)^{0.37}$$

are represented as follows:
$[h_j] = \mathrm{Ht^{-1}L^{-2}T^{-1}}$

$[D_a] = \mathrm{L}$
$[k] = \mathrm{Ht^{-1}L^{-1}T^{-1}}$
$[\bar{V}] = \mathrm{Lt^{-1}}$
$[\rho] = \mathrm{ML^{-3}}$
$[\mu] = \mathrm{Mt^{-1}L^{-1}}$
$[c_p] = \mathrm{HM^{-1}T^{-1}}$
$[L] = \mathrm{L}$
$[n] = \mathrm{rev \cdot t^{-1}}$ (this is similar to rad·s^{-1}, which rad can be dropped in dimensional analysis)

$$\text{Left hand side} = \text{Right hand side}$$

$$\begin{aligned}
\frac{h_j D_a}{k} &= 4.9 \left(\frac{D_a \bar{V} \rho}{\mu}\right)^{0.57} \left(\frac{c_p \mu}{k}\right)^{0.47} \left(\frac{D_a n}{\bar{V}}\right)^{0.17} \left(\frac{D_a}{L}\right)^{0.37} \\
\frac{\mathrm{Ht^{-1}L^{-2}T^{-1} \cdot L}}{\mathrm{Ht^{-1}L^{-1}T^{-1}}} &= \left[\frac{\mathrm{L \cdot Lt^{-1} \cdot ML^{-3}}}{\mathrm{Mt^{-1}L^{-1}}}\right]^{0.57} \left[\frac{\mathrm{HM^{-1}T^{-1} \cdot Mt^{-1}L^{-1}}}{\mathrm{Ht^{-1}L^{-1}T^{-1}}}\right]^{0.47} \left[\frac{\mathrm{L \cdot t^{-1}}}{\mathrm{Lt^{-1}}}\right]^{0.17} \left[\frac{\mathrm{L}}{\mathrm{L}}\right]^{0.37} \\
1 &= 1 \cdot 1 \cdot 1 \cdot 1 \qquad (20)
\end{aligned}$$

This confirms that both sides of the equations are consistent.

1.7 Physical Characteristics of Fluids

- It is a substance that does not permanently resist distortion.
- Changing mass of a particular fluid will result in sliding of layers of fluid between each other until a new shape is formed.
- Development of shear stress which magnitudes depend on μ of the fluid and the rate of sliding.
- An equilibrium state is achieved when there is no shear stress (fluid at its final form).
- A fluid is at its definite density, ρ at a given pressure and temperature.
- Variation of ρ with changes of the above variables could be small or large.
- For *incompressible* fluid: slight change of ρ with moderate change of pressure and pressure.
- For *compressible* fluid: significant change of ρ.
- Liquids are mostly incompressible
- Gases are compressible fluids.
- Gases subjected to small % changes of pressure and temperature act as incompressible fluids (change of ρ can be ignored).

2 Fluid Statics and Its Applications

2.1 Basic equations

2.1.1 The concept of pressure in fluid statics

- It is an essential property of a static fluid.
- In a physical sense, pressure, p is defined as a surface force exerted by a fluid against the walls of its container.
- It exists at every point within a volume of fluid.
- It is independent of orientation of any internal surface which pressure is assumed to act.

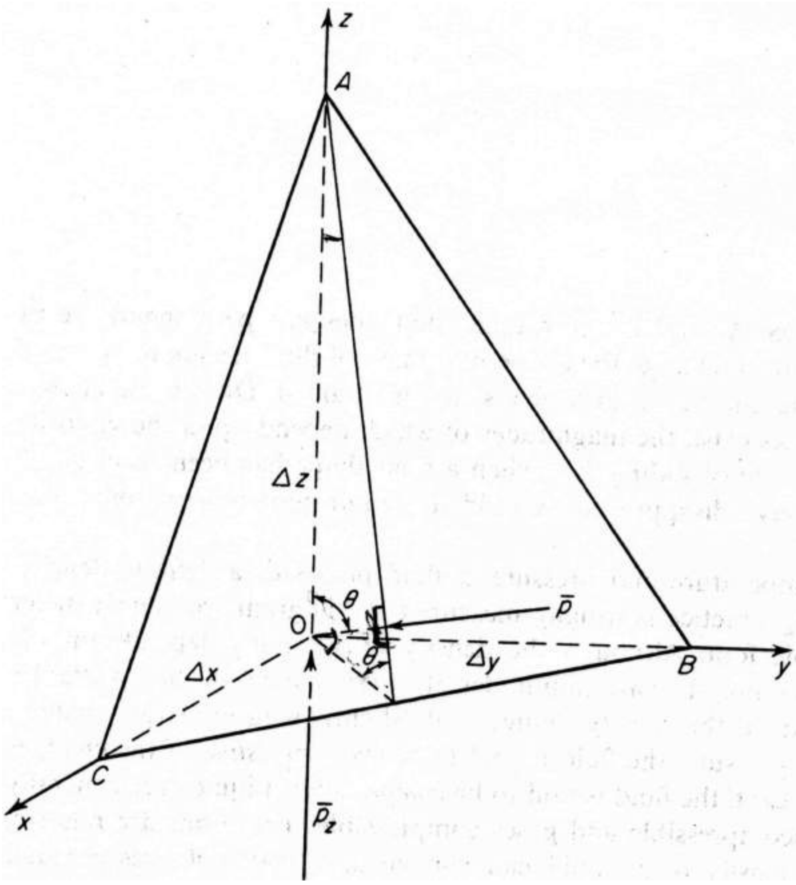

Figure 2: Forces acting on a tetrahedral statics shape.

- Refer to the tetrahedral shape (supplied separately), let all forces acting on the body are in the direction of z-axis (from outside or surrounding of fluid). The forces include:
  - force of gravity acting downward
  - pressure force acting upward on COB plane
  - vertical component of pressure force on place ABC acting downward.

- At equilibrium conditions, all resultant forces are zero and no shear stresses exist and therefore, these forces are normal to the surface on which they act.
- The surface area of COB form the diagram is given by'

$$\frac{1}{2}\Delta x \Delta y$$

and the average pressure acting of this surface is represented by $\bar{p}_z$. Since pressure is given by;

$$\bar{p}_z = \frac{F_z}{A_{\text{COB}}}$$

thus, the upward force can be determined by;

$$F_z = \frac{\bar{p}_z}{2}\Delta x \Delta y$$

Let $\bar{p}$ represents the average pressure acting of the surface of ABC with the area A_{ABC} given by;

$$A_{\text{ABC}} = \frac{1}{2}\frac{\Delta x \Delta y}{(\cos\theta)}$$

Therefore, the total force is determined by;

$$\bar{p}\Delta x \Delta y(2\cos\theta)$$

- The vertical component of this force acting downward is given by;

$$\text{Force}_{\text{COB}} = \frac{\bar{p}\Delta x \Delta y \cos\theta}{2\cos\theta} = \frac{1}{2}\bar{p}\Delta x \Delta y$$

- Volume of tetrahedron is given by;

$$\text{Volume, V}_{\text{tetra}} = \frac{1}{6}\Delta x \Delta y \Delta z$$

if the density of the fluid is, ρ, therefore the gravitational force acting on the fluid can be calculated by;

$$F_{\bar{p}} = \rho V \frac{g}{g_c}$$

thus;

$$\frac{\rho g}{6 g_c}\Delta x \Delta y \Delta z$$

Hence, resolving forces in z-direction;

$$\begin{aligned} \uparrow\ F_z - F_{\text{COB}} - F_{\bar{p}} &= 0 \\ \frac{1}{2}\bar{p}_z \Delta x \Delta y - \frac{1}{2}\bar{p}\Delta x \Delta y - \frac{\rho g}{6 g_c}\Delta x \Delta y \Delta z &= 0 \qquad (21) \end{aligned}$$

Simplifying (21) by dividing with $\Delta x \Delta y$ leads to;

$$\frac{1}{2}\bar{p}_z - \frac{1}{2}\bar{p} - \frac{\rho g}{6 g_c}\Delta z = 0 \qquad (22)$$

- Let the plane ABC move toward the origin, O and as the plane $\rightarrow 0$, $\Delta z \rightarrow 0$ and thus, $g \rightarrow 0$ and finally disappears. This will result in;

$$\bar{p} \rightarrow p$$

  and

$$\bar{p}_z \rightarrow p_z$$

  This is similar if one wants to write a force balance for x and y axes. Pressure terms in x, y and z directions will lead to a single term p;

$$p_x = p_y = p_z = p \tag{23}$$

2.1.2 Hydrostatic equilibrium

- For a stationary mass of a static fluid, p is constant in any cross section, parallel to the earth surface but varies with height.
- Refer to the given diagram.

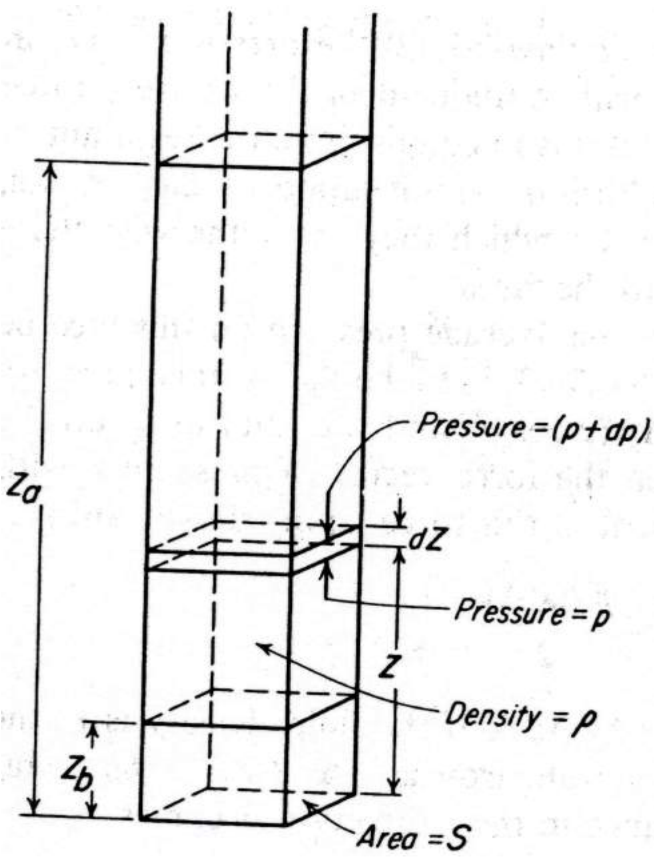

Figure 3: Hydrostatic equilibrium.

- The cross sectional area of the column is denoted as S.
- The pressure of the column is given by p
- The density of the fluid within the column is ρ

- Resolving all forces of a small volume of height dZ and cross sectional area S leads to 0.
- There are 3 types of forces acting on this volume which include;
  1. force from pressure p acting in an upward direction; $F_{up} = pS$
  2. force from pressure $(p + dp)$ acting in a downward direction given by $F_{down} = (p + dp)S$
  3. force of gravity acting downward given by $F_g = \left(\frac{g}{g_c}\right) \rho S dZ$
- Resolving all these 3 forces;

$$\begin{aligned} \uparrow \; F_{up} - F_{down} - F_g &= 0 \\ pS - (p + dp)S - \left(\frac{g}{g_c}\right) \rho S dZ &= 0 \\ pS - pS - dpS - \left(\frac{g}{g_c}\right) \rho S dZ &= 0 \\ dp + \left(\frac{g}{g_c}\right) \rho dZ &= 0 \qquad (24) \end{aligned}$$

  The above form of equation cannot be analytically solved for compressible fluids unless the variation of ρ with pressure is known throughout the column of fluid.
- It is normal in calculation to assumed ρ to be constant.
- Integrating (24) indefinitely at a constant density, gives;

$$\frac{p}{\rho} + \frac{g}{g_c} Z = C \qquad (25)$$

  and with definite integral from points a to b of the column height, gives;

$$\frac{p_b - p_a}{\rho} = \frac{g}{g_c}(Z_a - Z_b) \qquad (26)$$

2.1.3 Barometric equation

- Relationship between density and pressure of a particular gas is given by the *Ideal Gas Equation*;

$$\begin{aligned} pV &= nRT \\ pV &= \frac{m}{M} RT \\ \frac{m}{V} &= \frac{pM}{RT} \\ \rho &= \frac{pM}{RT} \qquad (27) \end{aligned}$$

- For a fluid in gaseous form, substitute (27) into (24) leads to;

$$dp + \left(\frac{g}{g_c}\right) \frac{pM}{RT} dZ = 0 \qquad (28)$$

Upon rearrangement gives;

$$\frac{1}{p}dp + \left(\frac{g}{g_c}\right)\left(\frac{M}{RT}\right)dZ = 0 \tag{29}$$

- Integrating differential equation (29);

$$\begin{aligned}
\int_b^b \frac{1}{p}dp &= -\int_a^b \left(\frac{g}{g_c}\right)\left(\frac{M}{RT}\right)dZ \\
\ln p_b - \ln p_b &= -\left(\frac{g}{g_c}\right)\left(\frac{M}{RT}\right)\int_a^b dZ \\
\ln\frac{p_b}{p_a} &= \left(\frac{g}{g_c}\right)\left(\frac{M}{RT}\right)(Z_a - Z_b) \\
\frac{p_b}{p_a} &= e^{\left[\frac{gM(Z_a - Z_b)}{g_c RT}\right]} \qquad (30)
\end{aligned}$$

- Equation (30) is commonly known as the *barometric equation.*

2.2 Hydrostatic Equilibrium in a Centrifugal Field

- In a centrifugal movement, a liquid is thrown in an outward direction from the axis of rotation.
- The free surface of the liquid takes the shape of a paraboloid of revolution.

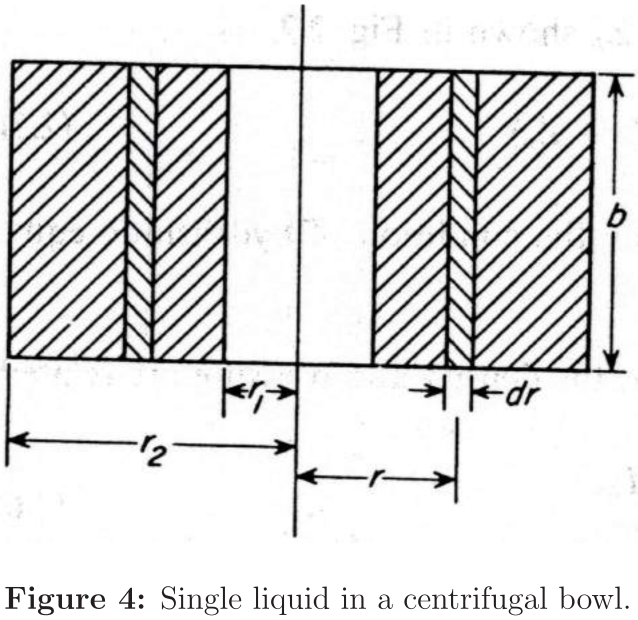

Figure 4: Single liquid in a centrifugal bowl.

- For an industrial centrifuge, the speed of rotation is very high that the force is much greater than the force of gravity that the liquid surface is virtually cylindrical and coaxial with the axis of rotation. (Refer to the given figure)
- r_1 is the radial distance from the axis of rotation to the free liquid surface.
- r_2 is the radius of the centrifugal bowl.

- The whole mass of the liquid rotates like a rigid body.
- Pressure drop of the rotating liquid with volume thickness, dr at a radius r;

$$\begin{aligned} F &= ma \\ dF &= adm \end{aligned} \tag{31}$$

and the acceleration of the element is given as;

$$a = \frac{\omega^2 r}{g_c}$$

substitute into (31) gives;

$$dF = \left(\frac{\omega^2 r}{g_c}\right) dm \tag{32}$$

- If ρ is the density of the given liquid, and b is the breadth of the ring, thus, the mass of the element can be written as;

$$m = \pi r^2 b \cdot \rho \tag{33}$$

which upon differentiating leads to;

$$\begin{aligned} dm &= d(\pi r^2 b \cdot \rho) \\ &= 2\pi r b \rho \cdot dr \end{aligned} \tag{34}$$

- Substitute (34) into (32);

$$\begin{aligned} dF &= \left(\frac{\omega^2 r}{g_c}\right) 2\pi r b \rho \cdot dr \\ &= \frac{1}{g_c} 2\pi \rho b \omega^2 r^2 dr \end{aligned} \tag{35}$$

- Since the small change of pressure is given by the force exerted by the element of liquid per unit area of the ring;

$$\begin{aligned} dp &= \frac{dF}{A} \\ &= \frac{1}{2\pi r b} dF \\ dp &= \frac{\omega^2 p r}{g_c} dr \end{aligned} \tag{36}$$

- Integrating equation (36) assuming that there is constant density, ρ;

$$\begin{aligned} \int_{p_1}^{p_2} dp &= \int_{r_1}^{r_2} \frac{\omega^2 \rho r}{g_c} dr \\ &= \frac{\omega^2 \rho}{g_c} \int_{r_1}^{r_2} r dr \\ p\Big]_{p_1}^{p_2} &= \frac{\omega^2 \rho}{2 g_c} (r_2^2 - r_1^2) \end{aligned} \tag{37}$$

2.3 Manometers

- It is a device used to measure pressure differences.
- Refer to the given figure below.

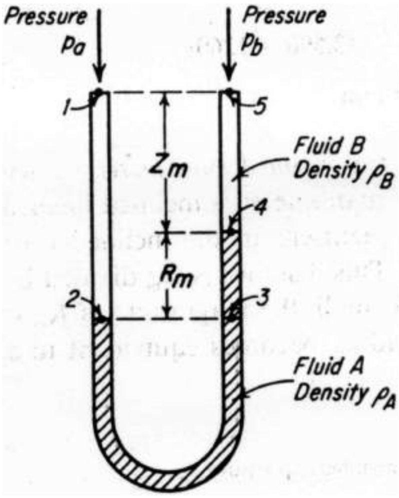

Figure 5: Simple manometer.

- The shaded area within the U tube is filled with liquid A with density ρ_A.
- In the other arms of the tube above the liquid are filled with fluid B with density, ρ_B.
- Fluid B is immiscible with liquid A and less dense than A.
- If a pressure p_A is exerted in the left hand side of the tube and p_B in the other, this gives the difference of pressure of $(p_A - p_B)$.
- The meniscus of a branch of the LHS tube is higher than the RHS and the vertical distance between the 2 meniscus R_m may be used to measure the pressure.
- The relationship between the pressure difference, $(p_A - p_B)$ and the meniscus, R_m can be derived as follows:
  1. start at pressure, p_a at point 1
  2. pressure at point 2 is $p_a + \frac{g}{g_c}(Z_m + R_m)\rho_A$
  3. the same is applied to point 3
  4. at point 4, pressure is less then that of point 3 by $\frac{g}{g_c}R_m\rho$
  5. pressure at point 5 is p_b is less than $\frac{g}{g_c}Z_m\rho_B$
- The overall statement can be summarised into a single form of equation given by;

$$\begin{aligned}
p_a - p_b + \frac{g}{g_c}\{(Z_m + R_m)\rho_B - R_m\rho_A - Z_m\rho_B\} &= 0 \\
p_a - p_b + \frac{g}{g_c}\{Z_m\rho_B + R_m\rho_B - R_m\rho_A - Z_m\rho_B\} &= 0 \\
p_a - p_b &= \frac{g}{g_c}R_m(\rho_A - \rho_B) \qquad (38)
\end{aligned}$$

- The above relationship is independent of Z_m as well as the dimension of the tube.
- However, this is only *true* if the pressure, p_a and p_b are measured in the same horizontal plane.
- If fluid B in the right arm of the tube is in gaseous form, the density, ρ_B is negligible compared to that of p_A and the term can be cancelled from equation (38).
- Example: A manometer shown in the diagram is used to measure the pressure drop across an orifice. Liquid A is a mercury with $\rho_{\text{Hg}} = 13590$ kg/m^3 and fluid B flows through the orifice and filling the manometer leads is brine with $\rho_{\text{NaCl}} = 1260$ kg/m^3. When pressure at the taps are equal, the level of Hg in the manometer is 0.9 m below the orifice taps. Under operating conditions, the gauge pressure at the upstream tap is 0.14 bar and the pressure at the downstream tap is 250 mm Hg below atmospheric. Calculate the reading of the manometer.

  Answer: Let the atmospheric pressure = 0

  Given that;

$$p_a = 0.14\text{bar} = 14000\text{Pa}$$

  While

$$\begin{aligned} p_b &= Z_b \frac{g}{g_c} \rho_{\text{Hg}} \\ &= -\frac{250}{1000}(9.81)(13590) \\ &= -33318\text{Pa} \end{aligned}$$

  Note that $g_c = 1$ when SI units is used.

  Then, using equation (38);

$$p_a - p_b = \frac{g}{g_c} R_m (\rho_{\text{Hg}} - \rho_{\text{NaCl}})$$

$$R_m = \frac{(p_a - p_b)}{g(\rho_{\text{Hg}} - \rho_{\text{NaCl}})}$$

  substitute all values into the above equation;

$$R_m = \frac{(14000 - (-33318))}{9.81(13590 - 1260)}$$

$$R_m = 0.391 \text{ m}$$

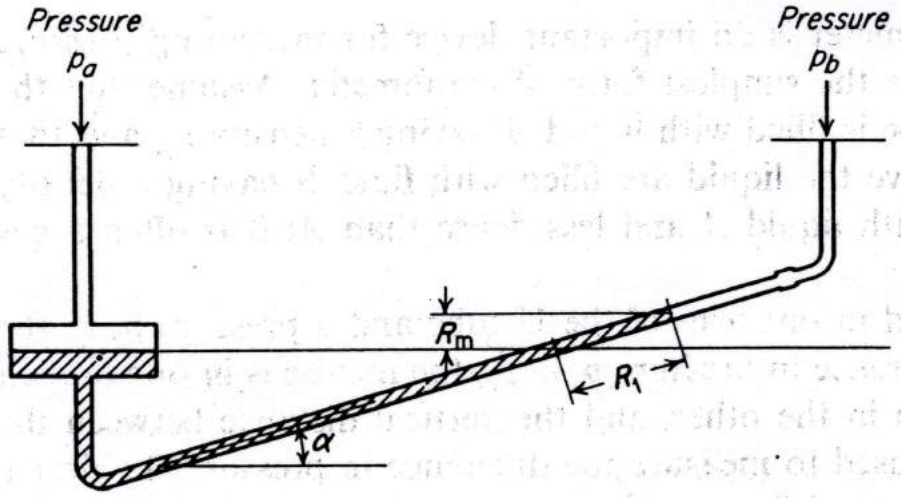

Figure 6: Simple inclined manometer.

- For an inclined manometer used to measure any small pressure diffferent, the meniscus in the inclined tube must move to a considerable distance along the tube which is given by;

$$R_m = R_1 \sin \alpha$$

The new relationship is now become;

$$p_a - p_b = \frac{g}{g_c}(\rho_A - \rho_B) R_1 \sin \alpha \tag{39}$$

- For this type of measuring instrument, it is necessary to provide an enlargement in the vertical leg such that the movement of the meniscus in the enlargement is negligible within the operating range of the instrument.

3 Flow of Compressible Fluids

- Other previous chapters only concerned with the *low speed* or incompressible flow where fluid velocity is much less than the speed of sound.
- When a fluid moves at speed comparable to the speed of sound, density changes become significant and fluid is now become compressible.
- It is much difficult to obtain in liquids, since high pressures of order 1000 atm are needed to generate sonic velocity.
- In gases however, a pressure of 2:1 ratio is likely to cause sonic flow. Compressible gas flow is quite common compared to that of incompressible type.
- 2 most important and distinctive effects of compressibility on flow are:
  - *choking*: the duct flow rate is limited by the sonic condition.
  - *shock wave*: discontinuous property changes in a supersonic flow.
- For nearly incompressible flow, the Mach number (Ma or N_{Ma}) is small;

$$\mathrm{Ma} = N_{\mathrm{Ma}} = \frac{v}{a} \ll 1$$

  where v is the velocity of flow and a is the velocity of sound.
- Under small Ma, changes in fluid density are everywhere in the flow field.
- For compressible flow, Mach numbers are greater than about 0.3 and density changes cannot be neglected.
- If density change is significant, it follows from equation of state that temperature and pressure changes are also substantial–these show that the energy equation can no longer be neglected and 4 types of equations should be considered and solved simultaneously for pressure, density, temperature and flow velocity;
  i. continuity equation
  ii. momentum equation
  iii. energy equation
  iv. equation of state
- This is rather complicated and reversible adiabatic (isentropic) flow assumption should be introduced in order to simplify the system. Other assumptions which will simplify the system further include:
  i. only one-dimensional flow is considered
  ii. the flow is at steady-state
  iii. no velocity gradient is considered within a cross section, thus, the average velocity is the same as that of the velocity of the flow

$$\overline{V} = v$$

  iv. friction is only restricted to the wall shear

v. no shaft work, $W_{sf} = 0$

vi. fluid is considered as ideal gas with constant specific heat.

vii. gravitational effect is negligible, therefore, no potential energy.

3.1 The Mach Number and the Speed of Sound

- It is a dominant parameter in compressible-flow analysis with different effects depending upon its magnitude.
- The following classifications are commonly used;

Mach number, (Ma)	**Types of flows**
Ma < 0.3	*incompressible flow*: density effects are negligible.
$0.3 <$ Ma < 0.8	*subsonic flow*: density effects are important but no shock waves appear.
$0.8 <$ Ma < 1.2	*transonic flow*: shock waves first appear, dividing subsonic and supersonic regions of the flow. Powered flight in the transonic region is difficult because of the mixed character of the flow field.
$1.2 <$ Ma < 3.0	*supersonic flow*: shock waves are present with no subsonic regions.
$3.0 <$ Ma	*hypersonic flow*: shock waves and other flow changes are especially strong.

Table 1: Classifications of Mach number.

- For an ideal gas, an isentropic path follows;

$$p\rho^{-\gamma} = \text{const.}$$

and since

$$\rho = \frac{pM}{RT}$$

substitute for ρ gives;

$$\begin{aligned}
p\left(\frac{pM}{RT}\right)^{-\gamma} &= \text{const.} \\
p_1\left(\frac{p_1 M}{RT_1}\right)^{-\gamma} &= p_2\left(\frac{p_2 M}{RT_2}\right)^{-\gamma} \\
p_1\left(\frac{p_1}{T_1}\right)^{-\gamma} &= p_2\left(\frac{p_2}{T_2}\right)^{-\gamma} \\
\frac{p_1}{p_2} &= \left(\frac{p_2}{p_1}\right)^{-\gamma}\left(\frac{T_2}{T_1}\right)^{\gamma} \\
\left(\frac{p_1}{p_2}\right)^{1-\gamma} &= \left(\frac{T_2}{T_1}\right)^{\gamma} \\
\frac{T_2}{T_1} &= \left(\frac{p_1}{p_2}\right)^{\left(\frac{1-\gamma}{\gamma}\right)} \\
T_2 p_2^{\frac{1-\gamma}{\gamma}} &= T_1 p_1^{\frac{1-\gamma}{\gamma}} \\
\Rightarrow \; Tp^{\frac{1-\gamma}{\gamma}} &= \text{const.}
\end{aligned} \tag{40}$$

- Also, using,

$$p\rho^{-\gamma} = \text{const.}$$

and upon differentiation gives;

$$\begin{aligned}
d(p\rho^{-\gamma}) &= d(\text{const.}) \\
\rho^{-\gamma}dp + pd\rho^{-\gamma} &= 0 \\
\frac{1}{\rho^{\gamma}}dp + p(-\gamma)\rho^{-\gamma-1}d\rho &= 0 \\
\frac{1}{\rho^{\gamma}}dp - \gamma p\frac{1}{\rho^{\gamma}}\frac{1}{\rho}d\rho &= 0 \\
\frac{1}{\rho^{\gamma}}\left(dp - \gamma p\frac{d\rho}{\rho}\right) &= 0 \\
dp - \gamma p\frac{d\rho}{\rho} &= 0 \\
\frac{dp}{p} - \gamma\frac{d\rho}{\rho} &= 0 \\
\frac{dp}{p} &= \gamma\frac{d\rho}{\rho}
\end{aligned} \tag{41}$$

Equation (41) can also be written in the form of;

$$\left(\frac{dp}{d\rho}\right) = \gamma\frac{p}{\rho}$$

- Since the speed of sound is given by;

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)}$$

substitute with $\left(\frac{dp}{d\rho}\right)$ gives;

$$a = \sqrt{\gamma \frac{p}{\rho}} \tag{42}$$

But according to the ideal gas law;

$$\frac{p}{\rho} = \frac{RT}{M}$$

thus,

$$a = \sqrt{\gamma \frac{RT}{M}}$$

- The above equation shows that the speed of sound or the *acoustical velocity* of an ideal gas is a function of temperature only.
- The speed of sound is actually the rate of propagation of a pressure pulse of infinitesimal strength through a still fluid (or thermodynamic property of fluid).
- Since Ma is given by $\frac{v}{a}$, therefore, substitute a into Ma and squaring both sides gives;

$$\begin{aligned}
\text{Ma} &= \frac{v}{\sqrt{\gamma \frac{RT}{M}}} \\
\text{Ma}^2 &= \left(\frac{v}{\sqrt{\gamma \frac{RT}{M}}}\right)^2 \\
&= \frac{v^2 M}{\gamma RT}
\end{aligned} \tag{43}$$

3.2 Specific Heat Ratio

- Calculations of compressible flow also depend upon a second dimensionless parameter which is the *specific heat ratio* of the gas, given by;

$$\gamma = \frac{c_p}{c_v}$$

  where c_p and c_v are the specific heat at constant pressure and volume respectively. The values of γ for eight common gases are given in the supplementary graph.
- As can be seen from the curves, k decreases slowly with temperature and lies between 1.0 and 1.7.
- Variation in γ have only a slight effect upon compressible flow computations and air with $\gamma = 1.40$ is the dominant fluid of interest.

3.3 Perfect Gas

- Most elementary treatments of compressible flow are confined to the perfect gas with constant specific heats;

$$p = \rho R T$$

and

$$R = c_p - c_v = \text{constant}$$

with

$$\gamma = \frac{c_p}{c_v} = \text{constant}$$

- For all real gases, c_p, c_v and k vary moderately with temperature.

- Therefore, since we only rarely deal with a very large temperature changes, it is quite reasonable to assume constant specific heats.

- The changes of internal energy $\hat{u}$ and enthalpy, h of a perfect gas can be computed for constant specific heats as;

$$\hat{u}_2 - \hat{u}_1 = c_v(T_2 - T_1) \tag{44}$$

and

$$h_2 - h_1 = c_p(T_2 - T_1) \tag{45}$$

- For variable specific heats, equations (44) and (45) must be integrated such that;

$$\hat{u} = \int c_v dT$$

and

$$h = \int c_p dT$$

3.4 Isentropic Process

- This approximation is common in compressible-flow theory

- For a pure substance, the entropy change can be computed using the first and the second laws of thermodynamics;

$$T ds = dh - \frac{dp}{\rho}$$

and from previous equation;

$$dh = c_p dT$$

for a perfect gas and solve for ds with;

$$\rho T = \frac{p}{R}$$

thus,

$$\int_1^2 ds = \int_1^2 c_p \frac{dT}{T} - R \int_1^2 \frac{dp}{p}$$

c_p in the above equation is a variable and its values have to be found from gas tables. However, for constant c_p, the analytical solution is given below;

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = c_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1} \tag{46}$$

- Equations (46) are used to compute the entropy change across a shock wave for an irreversible process.
- For isentropic flow, setting $s_2 = s_1$ and obtain the power-law relations for an isentropic perfect gas of the form;

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{47}$$

3.5 Equations Considered for Compressible Flow

- **Continuity Equation**:
  - it is given in the form of;

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0 \tag{48}$$

- **Steady Flow Total Energy Balance**:
  - the heat added to a fluid can be written as;

$$\frac{Q}{m} = H_2 - H_1 + \frac{v_2^2}{2} - \frac{v_1^2}{2} \tag{49}$$

where v_1 and v_2 are velocities of flow at points 1 and 2 respectively, H_1 and H_2 are enthalpies at points 1 and 2 respectively, m is the mass of material.

  - upon differentiating (49) leads to;

$$\begin{aligned} d\left(\frac{Q}{m}\right) &= d\left(H_2 - H_1 + \frac{v_2^2}{2} - \frac{v_1^2}{2}\right) \\ \Rightarrow \frac{1}{m} dQ &= dH + d\left(\frac{v^2}{2}\right) \end{aligned} \tag{50}$$

- **Mechanical Energy Balance with Wall Friction**:
  - according to the basic Bernoulli's equation with friction;

$$\frac{p_1}{\rho} + Z_1 g + \frac{\alpha_1 \overline{V}_1^2}{2} = \frac{p_2}{\rho} + Z_2 g + \frac{\alpha_2 \overline{V}_2^2}{2} + h_f$$

where the term h_f represents the friction generated per unit mass of fluid.

  - upon differentiating the equation gives;

$$\begin{aligned} d\left(\frac{p}{\rho} + Zg + \frac{\alpha \overline{V}^2}{2} + h_f\right) &= 0 \\ \frac{dp}{\rho} + g\, dZ + d\left(\frac{\alpha \overline{V}^2}{2}\right) + dh_f &= 0 \end{aligned} \tag{51}$$

from assumption (vii), since the gravitational effect is negligible and $\alpha = 1.0$ with $v = \overline{V}$, thus, the equation is reduced into;

$$\frac{dp}{\rho} + d\left(\frac{v^2}{2}\right) + dh_f = 0 \tag{52}$$

since

$$dh_f = f\frac{u^2}{2}\frac{dL}{r_H}$$

Substitute this into (52);

$$\frac{dp}{\rho} + d\left(\frac{v^2}{2}\right) + f\frac{u^2}{2}\frac{dL}{r_H} = 0 \quad (53)$$

which describes the mechanical energy balance incorporating the friction factor, f within the expression.

- **Equation for Velocity of Sound**:
  - It is termed as the *acoustical velocity*
  - The velocity of a very small compression-rarefaction wave moving adiabatically and frictionlessly through a medium at constant entropy/isentropic process.
  - This is given by;

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)_s} \quad (54)$$

with S represents the isentropic condition.

- **Equation of State of Ideal Gas**:
  - For the application of an ideal gas in the compressible gas flow, the density should be related to the temperature and pressure.

$$p = \frac{R}{M}\rho T \quad (55)$$

  - Putting in log form;

$$\begin{aligned} \ln p &= \ln\left(\frac{R}{M}\rho T\right) \\ \ln p &= \ln\frac{R}{M} + \ln\rho + \ln T \end{aligned} \quad (56)$$

  - Differentiating (56) gives;

$$\begin{aligned} d\ln p &= d\left(\ln\frac{R}{M} + \ln\rho + \ln T\right) \\ \frac{dp}{p} &= \frac{d\rho}{\rho} + \frac{dT}{T} \end{aligned} \quad (57)$$

  - The temperature term in the right hand side of the equation is independent of the specific heat, c_p. The enthalpy, H of gas at temperature T can be determined using;

$$dH = c_p dT \quad (58)$$

3.6 The Stagnation and Asterisk Conditions

3.6.1 Stagnation conditions

- Using the perfect gas equation from (49);

$$c_pT + \frac{1}{2}v^2 = c_pT_0$$

and upon rearrangement leads to;

$$1 + \frac{v^2}{2c_pT} = \frac{T_0}{T} \tag{59}$$

since

$$c_pT = \left(\frac{\gamma R}{\gamma - 1}\right)T = a^2\frac{1}{\gamma - 1}$$

ignoring the relative molecular mass, M and substitute into (59);

$$\begin{aligned} 1 + \frac{v^2}{2\left(a^2\frac{1}{\gamma-1}\right)} &= \frac{T_0}{T} \\ \Rightarrow\ 1 + \frac{(\gamma-1)}{2}\left(\frac{v}{a}\right)^2 &= \frac{T_0}{T} \end{aligned} \tag{60}$$

- Since

$$\mathrm{Ma} = \frac{v}{a}$$

thus,

$$\frac{T_0}{T} = 1 + \frac{(\gamma-1)}{2}\mathrm{Ma}^2 \tag{61}$$

- Similar goes to pressure, density and velocity of sound where the equations are given as follows;
  - for pressure;

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{1}{2}(\gamma-1)\mathrm{Ma}^2\right)^{\frac{\gamma}{\gamma-1}} \tag{62}$$

  - for density;

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{1}{2}(\gamma-1)\mathrm{Ma}^2\right)^{\frac{1}{\gamma-1}} \tag{63}$$

  - for velocity of sound;

$$\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \left(1 + \frac{1}{2}(\gamma-1)\mathrm{Ma}^2\right)^{\frac{1}{2}} \tag{64}$$

- In the above equations (59) to (64), the terms p_0, ρ_0, a_0 and T_0 are the stagnation values or the reference conditions of a flow.
- There can be only one stagnation condition for a given flow–this is only true for isentropic flows.

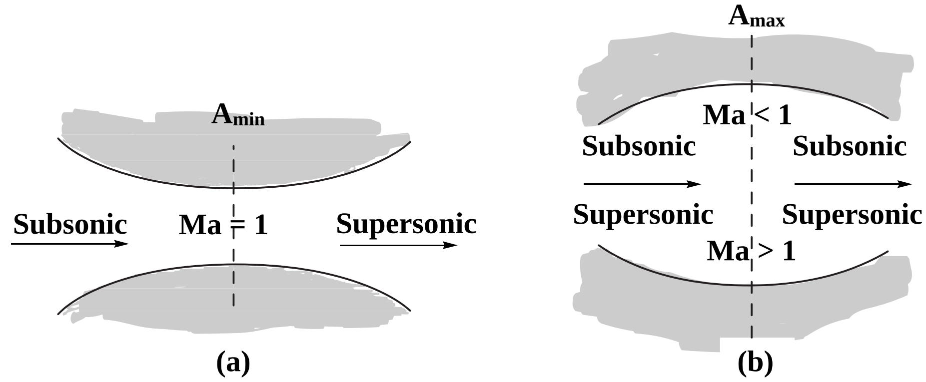

Figure 7: Flow through a throat: (a) smooth acceleration through sonic and supersonic flows. For flow through a bulge, (b) the flow at the bulge cannot be sonic on physical ground.

Duct geometry	**Subsonic,** Ma < 1	**Supersonic,** Ma > 1
$dA > 0$	$dv < 0$, $dp > 0$	$dv > 0, dp < 0$
$dA < 0$	$dv > 0$, $dp < 0$	$dv < 0$, $dp > 0$

Table 2: Effect of Mach number on the property change in duct flow.

- For non-isentropic flow, every point can have its own stagnation conditions, that is, if the flow is brought to rest locally at every point, there will be a series of stagnation points.
- For pressure and density, p_0 and ρ_0 are the values achieved when the system/flow is brought isentropically to rest.
- The quantities T_0 and a_0 are constant in an adiabatic non-isentropic flow.

3.6.2 Asterisk conditions

- The condition occurs when the velocity of a flow, v is relatively equal to the speed of sound, a;

$$v = a$$

  and

$$\text{Ma} = 1.0$$

- It is due to the increase of velocity when the area suddenly increases.
- Summary between subsonic and supersonic flows are given in the Table 3 above.
- The continuity equation for a sonic condition (when Ma > 1);

$$\rho v A = \rho^* v^* A^* \tag{65}$$

  or it can be written in the form of;

$$\frac{A}{A^*} = \frac{\rho^*}{\rho}\frac{v^*}{v}$$

- For isentropic flow of the function of Ma;

$$\frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho} = \left\{\frac{2}{\gamma+1}\left(1+\frac{1}{2}(\gamma+1)\mathrm{Ma}^2\right)\right\}^{\frac{1}{\gamma-1}} \tag{66}$$

- The velocity of fluid/gas can also be related to temperature using the continuity equation (65);

$$\begin{aligned}\frac{v^*}{v} &= \frac{1}{v}\sqrt{\frac{\gamma RT^*}{M}} \\ &= \frac{1}{v}\sqrt{\frac{\gamma RT^*}{M}}\left(\frac{T^*}{T_0}\right)^{\frac{1}{2}}\left(\frac{T_0}{T}\right)^{\frac{1}{2}} \end{aligned}\tag{67}$$

since $\frac{T_0}{T}$ is given by (61) with the asterisk condition for temperature of the form given by

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

when Ma = 1.0 thus;

$$\frac{v^*}{v} = \frac{1}{v}\sqrt{\frac{\gamma RT^*}{M}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}}\left(1+\frac{(\gamma-1)}{2}\mathrm{Ma}^2\right)^{\frac{1}{2}} \tag{68}$$

upon rearranging (68) gives;

$$\frac{v^*}{v} = \sqrt{\frac{\gamma RT^*}{v^2 M}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}}\left(1+\frac{(\gamma-1)}{2}\mathrm{Ma}^2\right)^{\frac{1}{2}} \tag{69}$$

since

$$\mathrm{Ma}^2 = \frac{v^2 M}{\gamma RT^*}$$

thus;

$$\begin{aligned}\frac{v^*}{v} &= \frac{1}{\mathrm{Ma}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}}\left(1+\frac{(\gamma-1)}{2}\mathrm{Ma}^2\right)^{\frac{1}{2}} \\ &= \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{(\gamma-1)}{2}\mathrm{Ma}^2\right)\right]^{\frac{1}{2}} \end{aligned}\tag{70}$$

- The cross-sectional area of the duct can also be incorporated into the asterisk condition following the expression given by;

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\frac{2+(\gamma-1)\mathrm{Ma}^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{71}$$

4 Flow Past Immersed Bodies

4.1 Drag and drag force

- It is defined as the force acting in the direction of flow exerted by the fluid on the solid.
- It is essentially a flow loss and must be overcome if the body is to move against the stream.
- Any body of any shape when immersed in a fluid stream will experience *forces* and moments from the flow. (At this point, we only consider *forces* acting on the body).
- This is shown in the given figure.
- According to Newton's Third Law of motion, and equal and opposite net force is exerted by the body on the fluid.
- When wall of the body is parallel with the direction of flow (see Figure 8 below) the

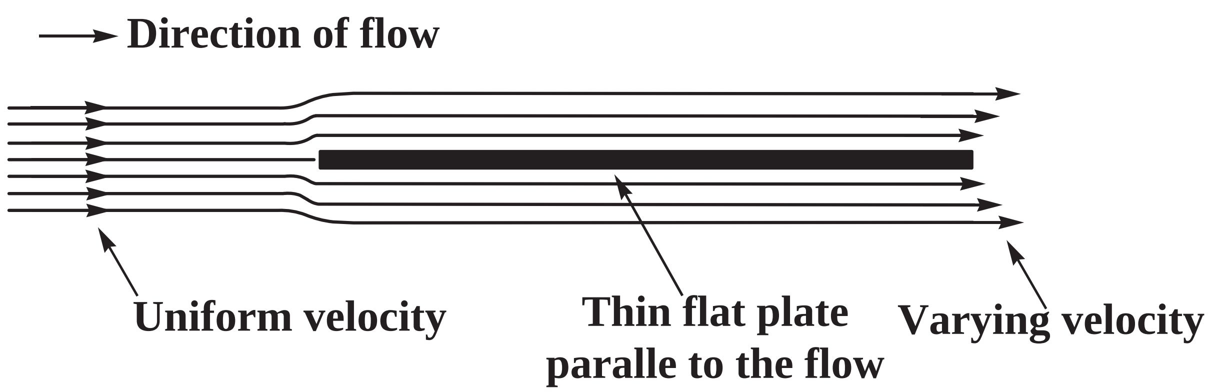

Figure 8: Flow past immersed flat plate.

only drag force is the wall shear, τ_w.

- If the wall of an immersed body makes an angle with the direction of flow, thus, the component of the wall shear in the direction of flow contributes to *drag*. (Refer to attached figure).
- The pressure from the fluid that acts normal to the wall (body) possesses a component in the direction of the flow which also contributes to the drag.
- Referring to Figure 9:
  - it shows the forces due to the pressure, p and shear forces acting on an element of an area dA inclined at an angle α to the direction of flow.
  - drag from the wall shear, τ_w, gives *wall drag* of the form;

$$F_w = \tau_w \sin\alpha \; dA$$

  - force due to pressure, p which forms *form drag* is given by;

$$F_f = p \cos\alpha \; dA$$

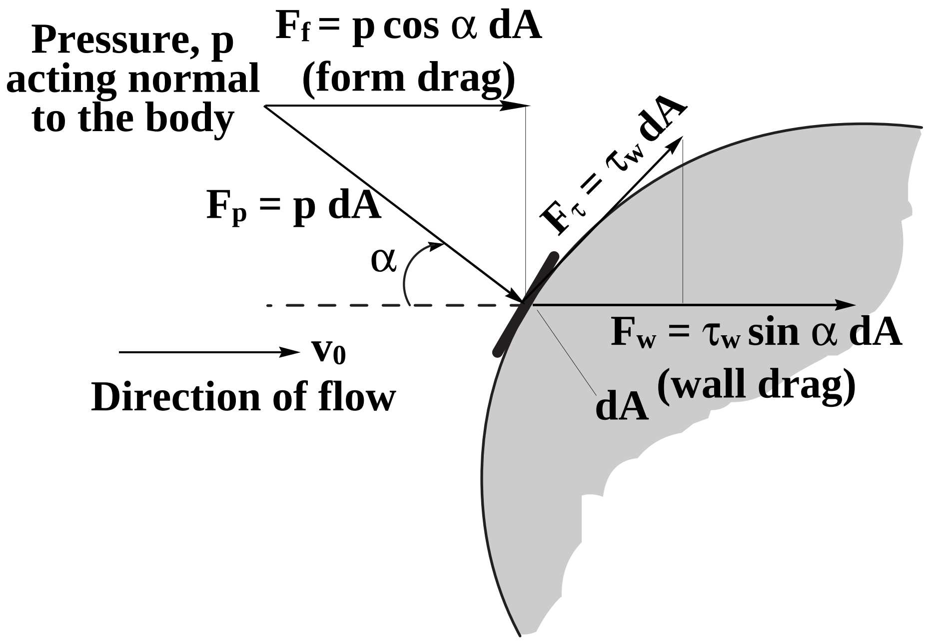

Figure 9: Formation of wall drag, F_w and form drag, F_f on an immersed body.

- and the total drag on the body is the sum of the integrals of these two forces.
- for a potential flow (flow of an ideal, incompressible fluid), there is no wall drag since $\tau_w = 0$.
- pressure drag in the direction of flow is also balanced by an equal force in the opposite direction and thus, the integral of of the form drag equals zero (no net drag for potential flow).

4.2 Drag coefficient

- It is defined by using the characteristic/projected area, A_p (depends on the body shape);

$$C_D = \frac{\mathrm{F_D}}{\frac{1}{2}\rho v_0^2 A_p}$$

where it is assumed that the velocity of the flow, v_0 is constant over the projected area.

- The coefficient is similar to that of the friction factor in pipes.
- The difference between the high pressure in the front stagnation region and the low pressure in the rear separated region causes a large drag contribution named the *pressure drag* coefficient given by;

$$C_{D,pressure} = \frac{p_s - p_r}{1/2 \rho v_0^2}$$

- This is added to the *friction drag* (drag due to shear) which gives;

$$C_D = C_{D,pressure} + C_{D,friction}$$

- For other shapes, it is necessary to specify the size and geometrical form of the body.
- The orientation w.r.t the flow of the fluid should also be taken into account (shape factors).
- For a cylindrical shape:
  1. diameter defines the dimension
  2. ratio of length to diameter gives the shape factor
  3. angle formed by the axis of cylinder and the direction of flow gives the orientation of the flow.
- For a smooth solid in an incompressible fluid, the drag coefficient for a particular shape depends on Re;

  $$C_D = \phi(Re_{particle})$$

  where $Re_{particle}$ is given by;

  $$\frac{G_0 D_p}{\mu} = \frac{\rho D_p v_0}{\mu}$$

- The relationship between Re and C_D can be plotted for different shapes of immersed bodies.
- The drag coefficients for compressible fluids increase with the increase in Mach number.
- From the complex nature of drag, it is not surprising that the variation of drag coefficient, C_D with Re is more complicated than that of the the friction factor, f with Re.
- This is due to the fact that drag coefficient is governed by both form drag and wall drag.
- Consider a sphere;
  - the theoretical equation follows *Stoke's Law* given previously by;

    $$F_D = C_D A_p \frac{\rho v_0^2}{2}$$

    and since

    $$C_D = \frac{24}{Re}$$

    thus substitute into above gives;

    $$F_D = \left(\frac{24}{Re}\right) A_p \frac{\rho v_0^2}{2}$$

    and the projected area, $A_p = \frac{\pi D^2}{4}$ thus;

    $$F_D = \left(\frac{24}{Re}\right) \left(\frac{\pi D^2}{4}\right) \frac{\rho v_0^2}{2}$$

    Upon simplification, leads to;

    $$F_D = 3\pi D v_0 \mu$$

    where D represents the diameter of the sphere.

- theoretically, *Stoke's Law* is only valid whenever Re is considerably less than unity (refer to the given C_D against Re correlation).
- the drag force equation above and the drag coefficient may be used with small error $\forall \; Re \leqslant 1$.
- at low velocities when the law is valid, the sphere moves through the fluid by deforming it.
- the wall shear results only from viscous forces.
- inertial forces are negligible.
- if solid wall exists within 20 to 30 diameters of the sphere, *Stoke's Law* must be corrected for the wall effect.
- such a flow is called *creeping flow*.
- the law, is particularly important for calculating
  * resistance of small particle (dust/fog moving through gases/liquid of low viscosity)
  * motion of larger particles through highly viscous liquids.
- as $Re \to 10$ (beyond the range of *Stoke's Law*), separation begins to occur at the point just forward the equatorial plane.
- wake covering the entire rear hemisphere is then formed.
- large form drag is developed as a result from the wakes.
- at a moderate Re, vortices disengaged from wake forming a series of moving vortices at the downstream fluid called *vortex street*.
- at high Re, vortices are no longer shed from the wake.
- stable boundary layer is formed, starting from the apex point at the projected area of the sphere.
- boundary layer grows and separates flowing freely around the wake after separation.
- according to the correlation chart of Re against C_D, the coefficient is nearly constant for spheres and cylinders and increases slightly with Re.
- an increase in Re results in transition of flow to turbulence;
  * initially in the free boundary layer
  * then boundary layer still attached to the front hemisphere of the sphere.
- when turbulence occurs;
  * separation point moves towards the rear of the body
  * the wake started to shrink
  * friction decreases
  * drag also decreases
  * result in the decrease in drag coefficient (from 0.45 to 0.10) for $Re \approx 3\times 10^5$
- Re where the attached boundary layer becomes turbulent $\to$ critical Re for drag (Re_{crit})
- the given curve (for sphere) only applies when the fluid approaching the sphere is not a turbulent type OR the sphere is moving through a static fluid.

  - if fluid is turbulent, Re_{crit} becomes smaller as the scale increases.

- Consider cylinders;
  - infinitely long cylinder:
    * at low Re, C_D does not inversely vary with Redue to the 2-D character of the flow around the cylinder.
  - short cylinder:
    * drag coefficient falls between values for spheres and long cylinders
    * varies inversely with Re (at very low values)

- Consider a disc:
  - does not show a decrease in C_D at Re_{crit}.
  - once separation occurs at the edge of the disc, the separated stream does not return to the back of the disc.
  - no shrinking of wake when boundary layer changes to turbulent.
  - such bodies are called *bluff bodies*
  - for $Re > 2000$, $C_D \approx 1$

- For irregularly shaped bodies/particles:
  - particles such as coal, sand etc.
  - similar to spheres of the same nominal size at $Re < 50$.
  - at $Re \approx 100$, the curve levels out and C_D values are 2 to 3 times those as spheres ($Re = 500$ to $Re = 3000$).
  - similar results also obtained for isometric particles.

4.3 Form drag and streamlining

- Can be minimised by forcing separation towards rear of body–streamlining.
- Streamlined body is given separately in the figure.
- Such a shape will give an increase in pressure in the boundary layer, that is the basic cause of separation.
- Perfect shape of streamlined body would give no wake and nearly no form drag.

4.4 Stagnation point

- It is the point where a streamline divides directly on the *nose* of an aerofoil-shape into 2 parts.
- The velocity at this point is 0.
- From the Bernoulli equation, the stagnation point can be described as;

$$\frac{p_{stag} - p_0}{\rho} = \frac{v_0^2}{2}$$

- The nominator at the left hand side of the equation shows the pressure increase for the streamline passing through a stagnation point.
- The value is larger compared to any other streamline–since all velocity head of the approaching stream is converted into pressure head.

4.5 Stagnation pressure

- The above equation is basically inaccurate for $Ma \gtrapprox 0.4$.
- Isentropic/adiabatic stagnation pressure should be used;

$$\left(\frac{p_{stag}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{stag}}{T_0}$$

since for a compressible fluids, the temperature ratio is given by;

$$\frac{T_a}{T_b} = \frac{1+\left(\frac{\gamma-1}{2}\right) Ma_b^2}{1+\left(\frac{\gamma-1}{2}\right) Ma_a^2}$$

substitute T_a with T_{stag} and T_b with T_0 also $Ma_a = 0$ and $Ma_b = Ma_0$ gives;

$$\frac{T_{stag}}{T_0} = 1 + \left(\frac{\gamma-1}{2}\right) Ma_0^2$$

- Since

$$\left(\frac{p_{stag}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{stag}}{T_0}$$

thus,

$$\left(\frac{p_{stag}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \left(\frac{\gamma-1}{2}\right) Ma_0^2$$

upon rearranging leads to;

$$\frac{p_{stag}}{p_0} = \left[1 + \left(\frac{\gamma-1}{2}\right) Ma_0^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

minus both sides by 1;

$$\frac{p_{stag}-p_0}{p_0} = \left[1 + \left(\frac{\gamma-1}{2}\right) Ma_0^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)} - 1$$

applying Binomial Theorem, $(a+b)^n = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array}\right) a^k n^{n-k}$ to the RHS of the above equation;

$$\begin{aligned} \left[1 + \left(\frac{\gamma-1}{2}\right) Ma_0^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)} - 1 &= \frac{Ma_0^2\gamma}{2} + \frac{Ma_0^4\gamma}{8} + \frac{2-\gamma}{48} Ma_0^8\gamma + \cdots \\ &= \frac{Ma_0^2\gamma}{2}\left(1 + \frac{Ma_0^2}{4} + \frac{2-\gamma}{24} Ma_0^4 + \cdots\right) \qquad (72) \end{aligned}$$

since the squared of Ma for an ideal gas is given by;

$$Ma_0^2 = \frac{\rho_0 v_0^2}{\gamma p_0}$$

therefore the RHS of equation (72) changes to;

$$\frac{\rho v_0^2}{2p}\left(1 + \frac{Ma_0^2}{4} + \frac{2-\gamma}{24}Ma_0^4 + \cdots\right)$$

thus;

$$\frac{p_{stag} - p_0}{p_0} = \frac{\rho_0 v_0^2}{2p_0}\left(1 + \frac{Ma_0^2}{4} + \frac{2-\gamma}{24}Ma_0^4 + \cdots\right)$$

multiply both sides by

$$\frac{p_0}{\rho_0}$$

leads to;

$$\frac{p_{stag} - p_0}{\rho_0} = \frac{v_0^2}{2}\left(1 + \frac{Ma_0^2}{4} + \frac{2-\gamma}{24}Ma_0^4 + \cdots\right)$$

- Comparing with the equation of the stagnation point previously, the quantity given in the bracket represents the correction factor for converting ideal fluids into a compressible one within the range of

$$0 \leqslant Ma < 1.0$$

Example:

A small particle of dust may be considered as a sphere. It has the density of 100 lb_m/ft and a diameter of 0.0001 in. It is settling in still air at 68°F and has been doing so for some time. How fast does the dust fall?

4.6 Friction in flow through beds of solids

- Examples of such a process include; filtration and countercurrent flow of liquid and gas through packed towers.
- Filtration:
  - bed of solids consists of small particles removed from the liquid by filter cloth/fine screen.
- Ion-exchange:
  - a single fluid either liquid or gas flows through a bed of granular solids.
- Resistance to flow of a fluid through the voids in a bed of solids is resultant of the total drag of all the particles in the bed.
- These phenomena occur according to Re: laminar flow, turbulent flow, form drag, separation and wake formation.
- Common methods of correlation are based on the estimates of the total drag of the fluid on the solid boundaries of the tortuous channels through bed of particles.
- Actual channels are:
  - irregular in shape
  - variable cross-section
  - variable orientation
  - highly interconnected
- For calculation, assume bed has uniform circular channels $\rightarrow$ total surface area (surface area per particle times the number of particles) and void volume matching that of the bed.
- Calculation is usually based on;
  - volume fraction particles in the bed
  - surface-volume ratio of particles
- For a spherical shape; the surface area of a particle is given by;

  $$s_p = \pi D_p^2$$

  and volume of a particle given by;

  $$v_p = \frac{1}{6}\pi D_p^3$$

  thus;

  $$\begin{aligned} \frac{s_p}{v_p} &= \frac{\pi D_p^2}{\frac{1}{6}\pi D_p^3} \\ &= \frac{6}{D_p} \end{aligned} \tag{73}$$

- For other irregular shapes of particles, surface-volume ratio should include sphericity Φ_s which is defined as the surface-volume ratio for a sphere of diameter D_p divided by the surface-volume ratio for the particle whose nominal size is D_p.

$$\begin{aligned}\Phi_s &= \frac{\text{surface–volume ratio for a sphere of diameter } D_p}{\text{surface–volume ratio for the particle whose nominal size is } D_p} \\ &= \frac{\frac{6}{D_p}}{\frac{s_p}{v_p}} \qquad (74)\end{aligned}$$

and upon rearranging leads to;

$$\frac{s_p}{v_p} = \frac{6}{\Phi_s D_p}$$

where values of Φ_s for granular solids are between 0.60 to 0.95.

- Volume fraction of a bed is given by;

$$(1 - \varepsilon)$$

where ε is the porosity/void fraction.

- For porous particle, pores are very small to permit significant flow through them, therefore, ε is used as an external void fraction of the bed (it doesn't represent the total porosity).

- The equivalent channel diameter, D_{eq} is determined using;

$$n\pi D_{\text{eq}} L = S_0 L(1-\varepsilon)\left(\frac{6}{\Phi_s D_p}\right) \qquad (75)$$

where n is the number of parallel channels, L is the length of the channels and S_0 is the cross-sectional area of the bed.

- Expanding equation (75) gives;

$$n\pi D_{\text{eq}} L = (S_0 L - S_0 L\varepsilon)\left(\frac{6}{\Phi_s D_p}\right) \qquad (76)$$

where the term $S_0 L\varepsilon$ defines the volume of the void space in the bed which is also given by the total volume of the n number of channel;

$$S_0 L\varepsilon = \frac{n\pi D_{\text{eq}}^2 L}{4} \qquad (77)$$

rearrange (77);

$$n\pi D_{\text{eq}} L = 4\frac{S_0 L\varepsilon}{D_{\text{eq}}}$$

and substitute into (76) and rearrange gives;

$$\begin{aligned}4\frac{S_0 L\varepsilon}{D_{\text{eq}}} &= (S_0 L - S_0 L\varepsilon)\left(\frac{6}{\Phi_s D_p}\right) \\ D_{\text{eq}} &= \frac{2}{3}\Phi_s D_p\left(\frac{\varepsilon}{1-\varepsilon}\right) \qquad (78)\end{aligned}$$

- Δp across channels depends on the average velocity, $\bar{V}$ where;

$$\overline{V} = \frac{\overline{V}_0}{\varepsilon}$$

where $\overline{V}_0$ is the superficial velocity and $\overline{V}$ is the average velocity in the channels.

- With these parameters, a correlation which predicts the pressure drop of a flow (at very **LOW** Re) through channels can be constructed.

- Using the Hagen-Poiseuille equation of the form;

$$\Delta p_s = 32\frac{\Delta L\overline{V}\mu}{D_{\mathrm{eq}}^2} \tag{79}$$

substitute D_{eq} and $\overline{V}$ into equation (79) gives;

$$\begin{aligned}
\Delta p_s &= 32\frac{\Delta L\left(\frac{\overline{V}_0}{\varepsilon}\right)\mu}{\frac{4}{9}\Phi_s^2 D_p^2\left(\frac{\varepsilon}{1-\varepsilon}\right)^2}\cdot\lambda_1 \\
&= 72\frac{\Delta L\overline{V}_0\mu\lambda_1}{\varepsilon\Phi_s^2 D_p^2}\left(\frac{1-\varepsilon}{\varepsilon}\right)^2 \\
\frac{\Delta p_s}{L} &= 72\frac{\overline{V}_0\mu\lambda_1}{\varepsilon\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^2} \qquad (80)
\end{aligned}$$

where λ_1 is the correction factor that accounts for the tortuosity and bent-type channels.

- An empirical correlation based on experimental values was found to be in the form of;

$$\frac{\Delta p_s}{L} = 150\frac{\overline{V}_0\mu}{\varepsilon\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^2} \quad \text{(Kozeny–Karman equation)}$$

which λ_1 has been included within the value of the constant where $\lambda_1 = 2.1$.

- As the flow rate increases, Re also increases (**HIGH** Re). Using the turbulent flow equation in pipes of the form;

$$\frac{\Delta p_s}{L} = 4f\frac{\rho\overline{V}^2}{2D_{\mathrm{eq}}} \tag{81}$$

similarly, substitute D_{eq} and $\overline{V}$ into equation (81) gives;

$$\begin{aligned}
\frac{\Delta p_s}{L} &= 2f\frac{\rho\left(\frac{\overline{V}_0}{\varepsilon}\right)^2}{\frac{2}{3}\Phi_s D_p\left(\frac{\varepsilon}{1-\varepsilon}\right)}\cdot\lambda_2 \\
&= 3f\frac{\rho\lambda_2\overline{V}_0^2(1-\varepsilon)}{\Phi_s D_p\varepsilon^3} \qquad (82)
\end{aligned}$$

where λ_2 is the correction factor that accounts for the tortuosity along the channels. The equation had also being studied experimentally and the empirical form is given below;

$$\frac{\Delta p_s}{L} = 1.75\frac{\rho\overline{V}_0^2(1-\varepsilon)}{\Phi_s D_p\varepsilon^3} \tag{83}$$

- The friction factor, f is assumed to be approximately 0.01 and the constant $\lambda_2 = 58$.
- Δp is largely affected by the kinetic energy (K.E) losses due to changes in channel cross section and flow direction.
- When fluid passes through particles:
  - channel becomes smaller and larger.
  - max. velocity is much greater than the average velocity.
  - rapid changes in channel area results in lost of K.E from expansion.
- To incorporate K.E losses, equation (12) is divided by;

$$\frac{\rho\overline{V}^2}{2}$$

with $\overline{V} = \frac{\overline{V}_0}{\varepsilon}$;

$$\begin{aligned}
\frac{\Delta p_s}{\frac{\rho L}{2}\left(\frac{\overline{V}_0}{\varepsilon}\right)^2} &= 1.75\rho\overline{V}_0^2\frac{(1-\varepsilon)}{\varepsilon^3}\frac{1}{\Phi_s D_p}\frac{2\varepsilon^2}{\rho\overline{V}_0^2} \\
&= 3.5\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{1}{\Phi_s D_p} \qquad (84)
\end{aligned}$$

- The term on the LHS is called the velocity heads.
- The previous two equations (80 and 84) are valid for only certain range of flow rates.
- A single equation than took into account the viscous losses and K.E losses over the entire range of flow rates is given by the *Ergun* equation;

$$\frac{\Delta p}{L} = 150\frac{\overline{V}_0\mu}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75\frac{\rho\overline{V}_0^2}{\Phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right) \qquad (85)$$

4.7 Motion of particles through fluids

- Example of such motions include:
  - removal of solids from liquid wastes–to allow discharge into public drainage systems
  - elimination of dust and fumes from flue gas/air
  - recovery of acid mists from waste gas
- There are 3 forces that act on a particle;
  - external force (gravitational or centrifugal)
  - buoyant force (acts parallel with the external force but in opposite direction)
  - drag force (appears whenever there is relative motion between particle and fluid, acts parallel with the direction of movement and in opposite direction)

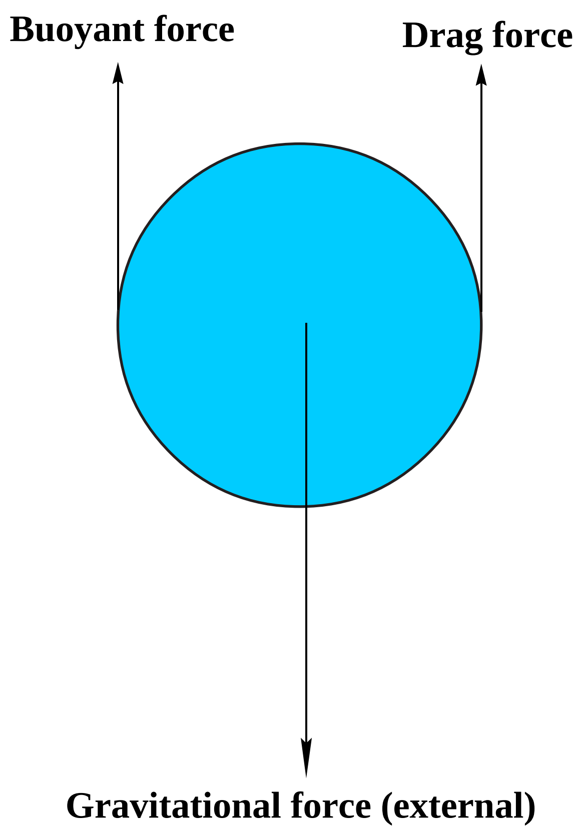

Figure 10: Three forces that act on a spherical bodies.

- For a particle moving through a fluid under the action of an external force, F_{ex}, the velocity of the particle relative to the fluid is v. The buoyant force is F_b and the drag force is given by F_D. Therefore the resultant force in the direction of the external force can be written as:

$$\begin{aligned} F_{ex} - F_b - F_D &= F \\ &= ma \\ &= m\frac{dv}{dt} \end{aligned} \tag{86}$$

- The external force, F_{ex} can be expressed as a product of the mass and acceleration, a_{ex} of the particle from this force, thus;

$$F_{ex} = ma_{ex} \tag{87}$$

- The buoyant force, F_b is the product of the mass of the fluid displaced by the particle and the acceleration from the external force. Volume of particle is given by $\frac{m}{\rho_p}$. Therefore, the mass of fluid displaced is $\frac{m}{\rho_p}\rho_f$. Hence, the buoyant force can be written as;

$$F_b = \frac{m\rho_f a_{ex}}{\rho_p} \tag{88}$$

- The drag force, F_D is previously given in page (3) of this notes;

$$F_D = C_D A_p \frac{\rho_f v_0^2}{2} \tag{89}$$

- Substitute equations (87), (88) and (89) into (86) leads to;

$$\begin{aligned} ma_{ex} - \frac{m\rho_f a_{ex}}{\rho_p} - C_D A_p \frac{\rho_f v_0^2}{2} &= m\frac{dv}{dt} \\ m\left(a_{ex} - \frac{\rho_f a_{ex}}{\rho_p} - C_D A_p \frac{\rho_f v_0^2}{2m}\right) &= m\frac{dv}{dt} \\ a_{ex}\left(1 - \frac{\rho_f}{\rho_p}\right) - C_D A_p \frac{\rho_f v_0^2}{2m} &= \frac{dv}{dt} \end{aligned} \tag{90}$$

- For a motion under the influence of gravity;

$$a_{ex} = g$$

thus, equation (90) changes to;

$$\frac{dv}{dt} = g\left(1 - \frac{\rho_f}{\rho_p}\right) - C_D A_p \frac{\rho_f v_0^2}{2m} \tag{91}$$

- For a motion under a centrifugal field;

$$a_{ex} = r\omega^2$$

thus;

$$\frac{dv}{dt} = r\omega^2\left(1 - \frac{\rho_f}{\rho_p}\right) - C_D A_p \frac{\rho_f v_0^2}{2m} \tag{92}$$

- **Terminal velocity** occurs when the acceleration decreases with time and approaches 0. At this point, the particle moves at a constant velocity i.e *terminal velocity*. Using equation (91);

$$\frac{dv}{dt} = g\left(1 - \frac{\rho_f}{\rho_p}\right) - C_D A_p \frac{\rho_f v_0^2}{2m}$$

putting $\frac{dv}{dt} = 0$ at constant velocity, leads to;

$$\begin{aligned} g\left(1 - \frac{\rho_f}{\rho_p}\right) - C_D A_p \frac{\rho_f v_0^2}{2m} &= 0 \\ v_t &= \sqrt{\frac{2gm(\rho_p - \rho_f)}{A_p C_D \rho_p \rho_f}} \end{aligned} \tag{93}$$

- For a motion in a centrifugal force, the velocity of the particle depends on the radius and acceleration is not constant if the particle is in motion with respect to the fluid. In a centrifugal force, $\frac{dv}{dt}$ is always small and if the term is neglected, a terminal velocity in a centrifugal field can be written as;

$$v_t = \omega\sqrt{\frac{2rm(\rho_p - \rho_f)}{A_p C_D \rho_p \rho_f}} \tag{94}$$

5 Transportation and Metering of Fluids

5.1 Pipe

- Pipe is normally circular in cross section with
  1. different sizes (diameters)
  2. materials
  3. thickness of wall
- Comparison between pipe and tubing:

Characteristics	**Pipe**	**Tubing**
Wall type	Heavy	Thin
Diameter	Large	Small
Length	Moderate	Several hundred feet (coils)
Threading	Possible (for metal pipe)	Not possible
Roughness	Slightly	Smooth
Join	Using screw, flanged or welded	Use fitting
Method of manufacturing	Welding/casting/ piercing	Extruded/ cold drawn
Material	Metals/alloys/ceramics	PVC/plastics

Table 3: Comparison between pipe and tubing.

- Size specification:
  - specified according to;
    - diameter
    - thickness
  - **for steel pipe**: the standard nominal diameters range from $\frac{1}{8}$ to 30 in (according to American practice).
    - standard sizes known as iron pipe size (IPS)/normal pipe size (NPS).
    - eg. 2-in nickel IPS = nickel pipe with same outside diameter as standard 2-in. steel pipe.
  - **for large pipe**: ($>$ 12 in diameter), nominal diameters = actual outside diameters.
  - **for small pipe**: nominal diameter $\neq$ any actual dimension.
  - **for tubing**: size is indicated by outside diameter.

    - normal value is actual outside diameter.
    - wall thickness = Birmingham Wire Gauge (BWG) number [24(light)–7(heavy)].

- Selection of pipe sizes:
  - optimum size of pipe should consider;
    - relative costs of investment
    - power
    - maintenance
    - pipe stocking
    - pipe fittings
  - for gravitational flow from overhead tank, low velocity should be used.

- Joints and fittings:
  - depend on the material of either pipes or tubing–based on the thickness of the wall.
  - for pipes made of brittle materials (glass/carbon/cast iron)
  - lengths of pipe (> 2 in.)–connected by flanges/welding.
  - **flange**: matching disks/rings of metal bolted together, compressing a gasket between their faces.
  - **welding**: normally used to joining large steel pipe for high pressure service–makes stronger joints than screw fitting.
  - **disadvantage of welding**: cannot be opened without destroying the pipes.

5.2 Valves

- **Purpose**: to slow down/stop the flow of a fluid.
- Other functions include:
  - can work on fully-open or fully-closed
  - reduce pressure and flow rate of a fluid
  - can allow only one directional flow
  - can allow only water and gas to pass while holding back the steam
  - can control temperature/pressure/liquid level
- Gate valves and globe valves:
  - most common type of valves
  - for gate valve:
    - diameter of opening where a fluid passes is nearly the same as the pipe.
    - direction of flow is the same.
    - large opening of valve leads to small pressure drop.
    - it is not recommended for controlling flow.

    * can only be used in fully-open or fully-closed position.
  - for globe valve:
    * normally used for controlling flow.
    * opening increases almost linearly with stem position.
    * wear is evenly distributed around the disk.
    * there is a change in direction whenever the fluid passes the disk.
    * this contributes to high pressure drop.

- Ball valves:
  - sealing element is spherical in shape
  - reduce the problems of alignment & *freezing* of element are less.
  - area of contact between moving element and seat is large–can be used as throttling service.
  - normally used to control flow.

- Check valves:
  - only permits one directional flow.
  - valve only opens when there is pressure form fluid in the direction of the flow.
  - it automatically stops when the flow stops **or** it tends to reverse its direction–this is done either by spring/gravity that pushes the disk.

5.3 Fluid-moving machinery

Most common technique of adding energy in moving fluid is by positive displacement/centrifugal action. This can be done by:

1. applying direct pressure to the fluid.
2. using torque to generate rotation.

5.3.1 Pumps

The 2 basic types of pumps include:

1. positive-displacement pumps (PDP).
   - force the fluid by changing volume.
   - when a cavity opens, fluid is admitted through an inlet, when it closes, fluid is squeezed through an outlet.
   - classification of PDP is given below:
     - reciprocating:
       (a) piston/plunger
       (b) diaphragm
     - rotary:
       (a) single rotor;

    i. sliding vane
    ii. flexible tube/lining
    iii. screw
    iv. peristaltic
(b) multiple rotors;
    i. gear
    ii. lobe
    iii. screw
    iv. circumferential piston

- it delivers a pulsating/periodic flow as the cavity volume opens, traps and squeezes the fluid.
- **advantage**: deliver any fluid regardless of viscosity.
- can operate up to very high pressure (300 atm) but only produces very low flow rate (100 gal/min).
- at constant rotation speed–it produces nearly constant flow rate & unlimited pressure rise with little effect on viscosity.
- flow rate cannot be varied except by changing the displacement/speed.

2. dynamic or momentum change pumps.
    - it adds momentum to the fluid by means of fast moving blades.
    - no closed volume–which means fluid increases momentum while moving through open passages and then converts its high velocity to increase in pressure by exiting into diffuser section.
    - this type of pump can be classified into 2:
        (a) rotary:
            i. centrifugal/radial exit flow
            ii. axial flow
            iii. mixed flow
        (b) special design:
            i. jet pump/ejector
            ii. electromagnetic pumps for liquid metal
            iii. fluid-actuated: gas-lift/hydraulic-ram
    - it can provide a higher flow rate compared to the PDP type
    - discharge is steadier but **not effective** in handling high viscosity liquids.
    - can provide very high flow rate (up to 300,000 gal/min)with moderate pressure rises (only a few atm).
    - it provides continuous constant-speed variation of performance–from near max. Δp at 0 flow (shutoff condition)to zero Δp at max. flow rate.
    - for high viscosity liquids: decrease the performance of this type of pump.

5.3.2 Developed head

- A typical use of pump is to provide energy to draw liquid from a reservoir and discharge a constant volumetric flow rate at exit of a pipeline, Z_b ft above the level of liquid.

- If the liquid enters the pump at point a, and it discharges at point b, thus the Bernoulli's equation can be written as;

$$\begin{aligned} H &= \left(\frac{p}{\rho g} + \frac{v^2}{2g} + z\right)_b - \left(\frac{p}{\rho g} + \frac{v^2}{2g} + z\right)_a \\ &= h_b - h_a \end{aligned} \tag{95}$$

  where H is the total pressure head and h_b and h_a are pump head supplied (suction) and losses (discharge) respectively with the pump friction $h_f = 0$.

- In pumps, the difference between the heights of suction and discharge is usually negligible, thus, the terms z_a and z_b can be dropped. The velocities of flow (v_a and v_b) are also approximately the same thus, leaving equation (95) as;

$$\begin{aligned} H &\approx \frac{p_2 - p_1}{\rho g} \\ &= \frac{\Delta p}{\rho g} \end{aligned} \tag{96}$$

- Since the total head is also given by;

$$H = \eta W_p$$

  thus,

$$\begin{aligned} W_p &= \frac{h_b - h_a}{\eta} \\ &= \frac{H}{\eta} \end{aligned} \tag{97}$$

  where W_p is the work done by the pump per unit mass of fluid and η is the pump efficiency.

5.3.3 Power requirement

- The power supplied to any particular pump from an external source can be determined using;

$$\begin{aligned} P_B &= \dot{m} W_p \\ &= \rho g Q W_p \\ &= \rho g Q \frac{H}{\eta} \end{aligned} \tag{98}$$

  with Q is the flow rate of fluid discharges from the pump.

- The power delivered to the fluid can be calculated using;

$$P_f = \dot{m}H \tag{99}$$

which is related to the power supplied by;

$$\begin{aligned} P_B &= \dot{m}\frac{H}{\eta} \\ &= \frac{P_f}{\eta} \end{aligned} \tag{100}$$

5.3.4 Suction lift and cavitation

- Equation (98) shows that power is closely related to the difference in pressure between discharge and suction.
- From the energy point of view, it doesn't matter whether the suction pressure is below or above the atmospheric pressure as long as the liquid remains liquid.
- But if the suction pressure is **slightly more** than the vapour pressure–some liquid may convert to vapour inside the pump–called *cavitation*.
- This **reduces** the pump capacity and severely erode the pump.
- If suction pressure is **less** than the vapour pressure–vaporisation in suction line occurs–no liquid is drawn into the pump.
- To avoid *cavitation*:
  - pressure at the inlet **must exceed** vapour pressure by a certain value–net positive suction head (NPSH).
  - for small centrifugal pumps: required values of NPSH (2-3 m).
  - for very large pumps: it can increase up to 15 m.
  - NPSH increases with increase in
    * pump capacity
    * impeller speed
    * discharge pressure
  - the NPSH can be calculated using:

$$\mathrm{NPSH} = \frac{1}{g}\left(\frac{p_i - p_v}{\rho}\right) - h_f - z_i \tag{101}$$

    where p_i and p_v are pressure at inlet of the reservoir and vapour pressure of the liquid respectively. h_f represents the friction head-loss between the reservoir and the pump inlet and z_i is the height of the pump inlet above the reservoir.

- In order to obtain a more accurate value of NPSH, the above equation can be subtracted with the term;

$$\frac{v^2}{2}$$

but it will give values between 30–60 cm and only specified by the min. required NPSH–important for positive-displacement pump.

- for non-volatile liquid:
  - $p_v = 0$.
  - friction head-loss is negligible, $h_f = 0$.
  - pressure at inlet is atmospheric.
  - max. possible suction is found by subtracting NPSH with barometric head.
  - for pumping cold water–max. suction is approximately 10.4 m

5.3.5 Positive-displacement pumps

This is a type of pump that alternately traps liquid in a chamber at the inlet and empties the liquid at a higher pressure through discharge. It can be categorised into 2:

- reciprocating pumps: contain a stationary cylindrical chamber with piston.
- rotary pumps: continuously moving chamber from inlet to discharge.

1. reciprocating pumps:
   - they include; piston pumps, plunger pumps and diaphragm pumps.
   - for piston pumps:
     - liquid is drawn through an inlet check valve into the cylinder by withdrawing of piston and forced out through discharge check valve on return stroke.
     - mostly double acting piston–one part is filled and the other part is emptied.
     - commonly 2 or more cylinders are used in parallel.
     - piston may be motor driven through reducing gears.
     - max. discharge for piston pumps–50 atm.
   - for plunger pumps:
     - use for higher pressure.
     - contain heavy-walled cylinder of small diameter with reciprocating plunger.
     - during its stroke–plunger fills all space of the cylinder.
     - normally single acting and motor driven.
     - can discharge at pressure of $\geqslant 1500$ atm.
   - for diaphragm pumps:
     - made of reciprocating flexible diaphragm metal/plastic/rubber.
     - eliminates the need for packing/seals being exposed to liquid.
     - can handle toxic/corrosive liquids.
     - handle only small-moderate amount of liquid $\backsimeq 100$ gal/min.
     - develop pressure up to 100 atm.
   - efficiency varies from
     - 40–50% for small pumps.
     - 70–90% for large pumps.
2. rotary pumps:

- they include; gear pumps, lobe pumps, screw pumps, cam pumps and vane pumps.
- it contains no check valve.
- minimise leakage due to close tolerance between moving and stationary parts.
- work well with clean and moderately viscous liquids.
- discharge pressure up to 200 atm or more.

5.3.6 Centrifugal pumps

- Liquid enters axially at the suction connection (1).
- In the rotating eye of the impeller, liquid spreads out radially and enters the channels between vanes at point (1').
- It then leaves the periphery of the impeller at point (2') and leaves the pump discharge at point (2).
- To construct an elementary theory of pump performance:

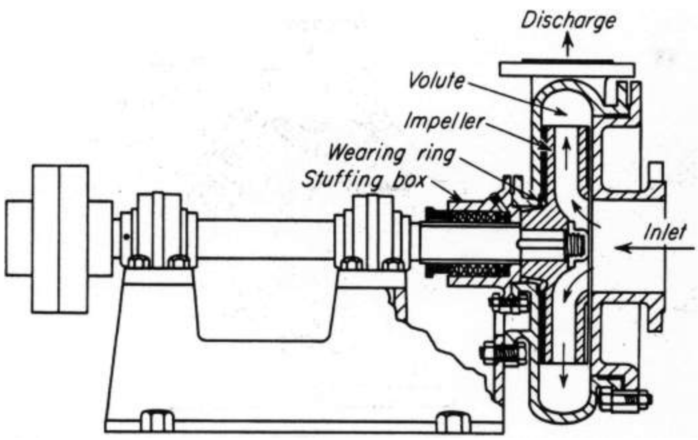

Figure 11: Centrifugal pump.

  - assume 1-dimensional flow
  - assume idealised fluid velocity vectors through impeller with angular momentum theorem.

- using a given diagram:
  - fluid is assumed to enter the impeller at

$$r = r_1$$

    with velocity component w_1 tangent to the blade angle β_1.
  - circumferential speed is given by;

$$u_1 = \omega r_1$$

    matching the tip speed of the impeller.

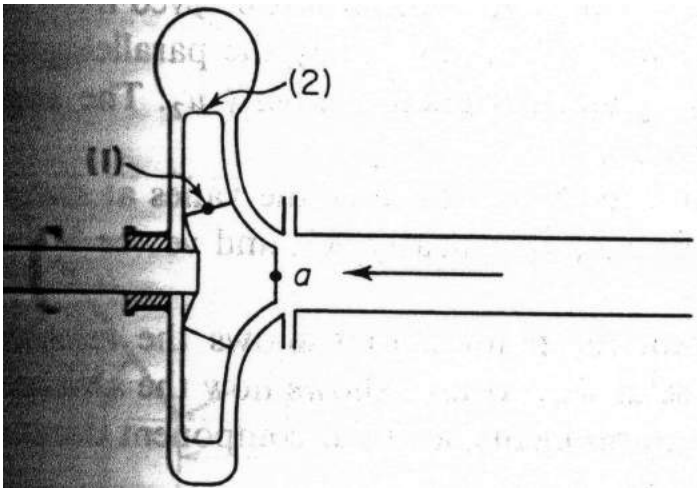

Figure 12: Schematic diagram of a centrifugal pump.

- absolute velocity is the vector sum of w_1 and u_1 given by V_1.
- the flow exits at $r = r_2$ with component w_2 parallel to the blade angle β_2 plus tip speed $u_2 = \omega r_2$ which gives the resultant velocity, V_2.
- the angular momentum theorem is given by;

$$T = \dot{m}(r_2 V_{t2} - r_1 V_{t1}) \tag{102}$$

which is the applied torque. In a radial flow, where $\alpha = 90°$ and $V_{t1} = 0$, therefore, $r_1 V_{t1} = 0$ and equation (102) reduces into

$$T = \dot{m} r_2 V_{t2}$$

- therefore, the power delivered to the fluid is thus;

$$\begin{aligned} P_B &= \omega T \\ &= \dot{m}(u_2 V_{t2} - u_1 V_{t1}) \\ &= \rho Q(u_2 V_{t2} - u_1 V_{t1}) \end{aligned} \tag{103}$$

OR

$$\begin{aligned} H &= \frac{P_B}{\dot{m}} \\ &= \frac{1}{g}(u_2 V_{t2} - u_1 V_{t1}) \end{aligned} \tag{104}$$

- for an ideal frictionless pump, the power equation can be written as;

$$P_{fr} = \dot{m}\omega r_2 V_{t2} \tag{105}$$

- head-flow relation for an ideal pump:
  - for an ideal pump, P_B reduces into P_{fr} and substitute into $H = \frac{P_{fr}}{\dot{m}}$ gives;

$$H = \omega r_2 V_{t2} \tag{106}$$

- since, $\omega r_2 = u_2$, then (106) reduces into;

$$H = u_2 V_{t2} \tag{107}$$

- from the given figure, it is known that

$$V_{t2} = u_2 - \frac{V_{t2}}{\tan\beta_2}$$

therefore;

$$H_r = u_2 \left(u_2 - \frac{V_{t2}}{\tan\beta_2} \right) \tag{108}$$

- if the pump volumetric flow rate is given by

$$Q_r = V_{t2} A_p$$

where A_p is the cross-sectional area of the channel around the circumference of the pump, then substitute into (108);

$$H_r = u_2 \left(u_2 - \frac{Q_r}{A_p \tan\beta_2} \right) \tag{109}$$

- it is obvious that the variables u_2, A_p and β_2 have constant values, thus, the head is relatively linear with the volumetric flow rate.
- the gradient of the head flow depends on the sign of $\tan\beta 2$ and therefore, β_2.
  * if $\beta_2 < 90°$, the line has (-ve) gradient.
  * if the line is having a (+ve) gradient (horizontal), the flow might be unstable.

- head-work relation in an ideal pump:
  - work done by a passing liquid through an ideal pump is given as;

$$\begin{aligned} W_p &= \frac{P_{fr}}{\dot{m}} \\ &= \omega r_2 V_{t2} \end{aligned} \tag{110}$$

  - this is equivalent to the head written using Bernoulli's equation in the form of;

$$W_p = \frac{p_2}{\rho} - \frac{p_1}{\rho} + \frac{V_2^2}{2} - \frac{V_1^2}{2} \tag{111}$$

- pump head-loss:
  - it is assumed that for an ideal pump, the angle between u_2 and w_2 equals the vane angle β_2.
  - for a real pump, the guidance is not perfect where the actual stream of fluid leaves at an angle less than β_2.
  - this is due to the fact that the velocity in the given cross-section is not uniform.

- characteristic curves:

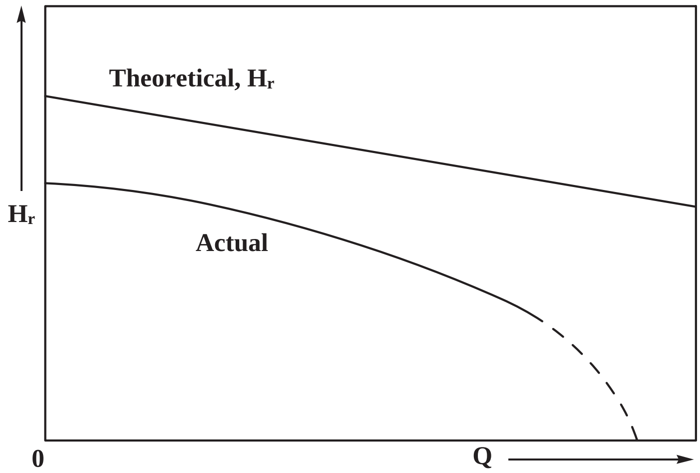

Figure 13: Head against flow rate characteristic of a centrifugal pump.

- for the head vs. flow rate curve:
  - the theoretical head is a straight line graph which follows equation (106) given previously.
  - the actual head is considerably less and drops rapidly to zero as the flow rate increases to a certain value in a given pump.
  - such a condition is known as the *zero-head flow rate*which is the max. flow the pump can deliver under any given conditions.
  - the operating floe rate is indeed below such a value.
- for the power vs. flow rate:

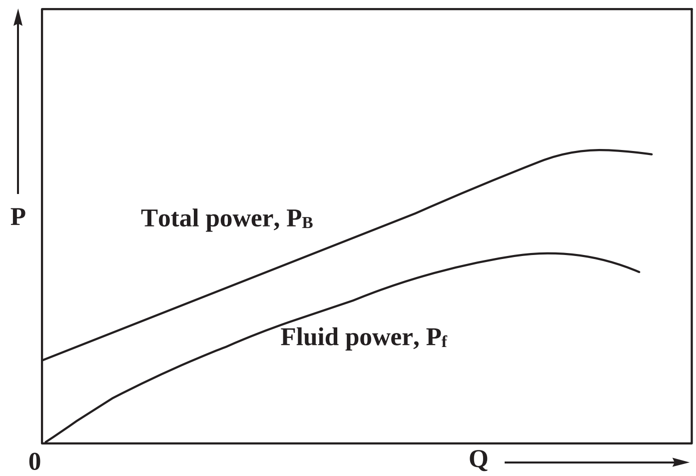

Figure 14: Power against flow rate characteristic of a centrifugal pump.

  - the difference between ideal and actual performance shows the power lost in the pump.
  - this is due to the fluid friction and shock losses which result from the mechanical energy conversion into the form of heat.
  - leakage is another form of losses which reduces the volume of the discharge.
  - disk friction is a type of friction occurs between the outer surface of the impeller and the liquid within the casing.

* bearing losses occur from the mechanical friction in the bearing and stuffing boxes of the pump.

– for the efficiency vs. flow rate:

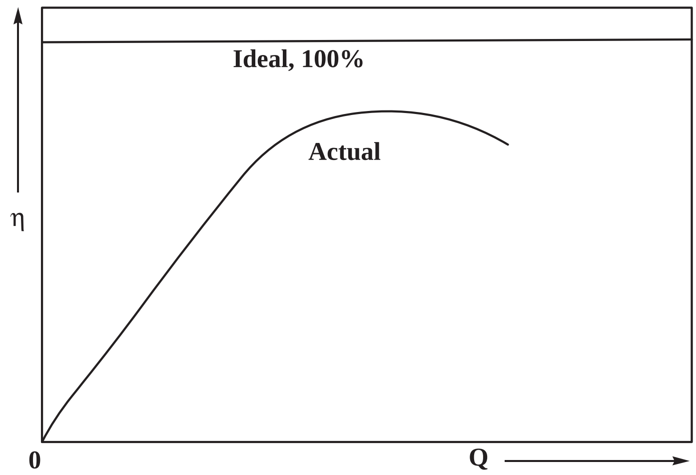

Figure 15: Efficiency against flow rate characteristic of a centrifugal pump.

* it is the ratio of the fluid power to the total power input to the pump.
* it is obtained from the power curve (Figure 14) which apparently shows a rapid increase with flow rate at low values.
* the efficiency than reaches a peak (max.)in the region of rated capacity.
* the values than fall as the flow rate increases and approaching a 0 head value.

5.3.7 Compressors

- A *compressor* is a pump which pumps a gas with

$$\frac{p_{out}}{p_{in}} \gg 1.0$$

- When pressure change of a gas while passing through a pump is small–the pump is a *blower/fan*.

- To compress a gas to a final (absolute) pressure of more than about 1.1 times its inlet pressure requires a *compressor*.

- For an isentropic, adiabatic and frictionless pressure change of an ideal gas, the relationship is given as;

$$\frac{T_{out}}{T_{in}} = \left(\frac{p_{out}}{p_{in}}\right)^{\frac{\gamma-1}{\gamma}} \tag{112}$$

- For a given gas, the temperature ratio increases with increase in the compression ratio $\frac{p_{out}}{p_{in}}$.
  – when compression ratio $\geqslant 10$ isentropic temperature become excessive.
  – for non-frictionless compressor (actual), heat due to friction is absorbed by the gas and and temperature more than isentropic temperature is obtained.

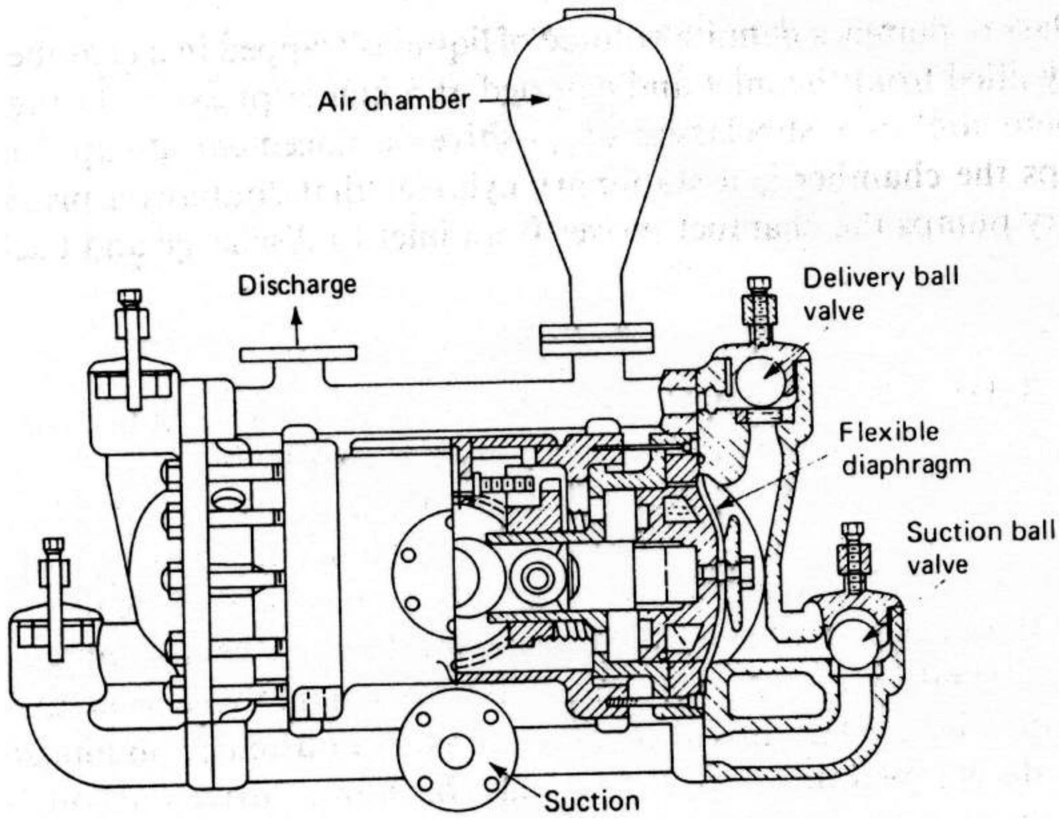

Figure 16: Compressor.

- Equations for compressors:
  - for adiabatic compression:
    - fluid follows an isentropic path.
    - the relationship is given by;
$$\frac{p}{\rho^{\gamma}} = \frac{p_a}{\gamma_a^{\gamma}} \tag{113}$$
    - since the work done by an ideal compressor is given by;
$$W_{ic} = \int_{p_a}^{p_b} \frac{dp}{\rho}$$
      substitute for p and solving the equation leads to;
$$W_{ic(ad)} = \frac{p_a \gamma}{(\gamma - 1)\rho_a} \left[\left(\frac{p_b}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \tag{114}$$
    - equation (114) shows the important of compression ratio $\frac{p_b}{p_a}$.
  - for isothermal compression:
    - normally occurs after the completion of cooling period within compressor.
    - there is no change in temperature (isothermal).
    - thus the relationship is given by;
$$\frac{p}{\rho} = \frac{p_a}{\rho_a} \tag{115}$$

* thus the work done is given by;

$$W_{ic(iso)} = \frac{RT_a}{M} \ln \frac{p_b}{p_a} \tag{116}$$

* for the given compression ratio, and the suction condition, the work requirenment in isothermal compression is less that that for adiabatic compression.
* thus, cooling is useful in compressors.

– compressor efficiency:

* efficiency is given by; the ratio of of theoretical work (fluid power) to the actual work (total power input).
* for isentropic compressor, this is denoted by:

$$\eta = \frac{W_{\text{isentropic}}}{W_{\text{actual}}} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \tag{117}$$

5.4 Measurement of flowing fluids

5.4.1 Venturi meter

The diagram of a venturi meter is given below:

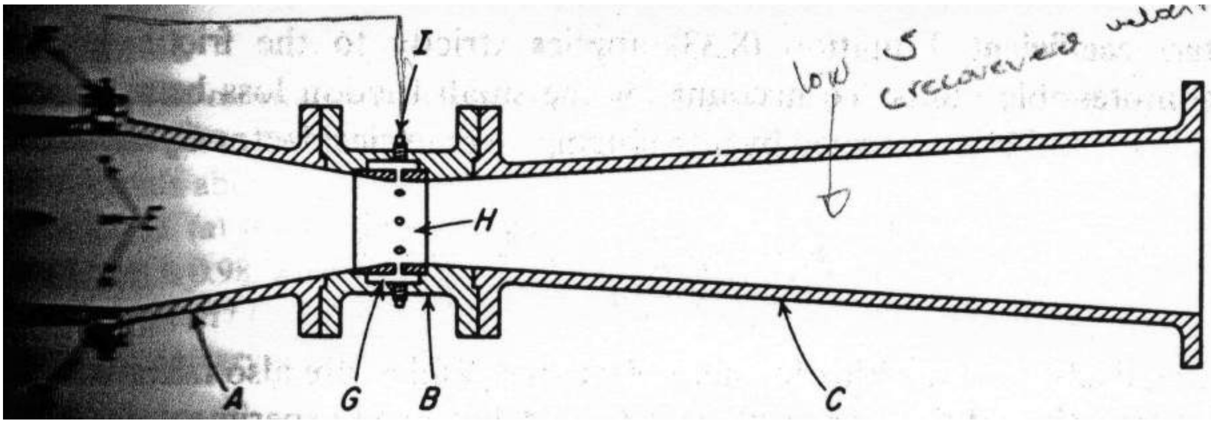

Figure 17: Venturi meter.

- It consists of a truncated cone with cross-sectional area perpendicular to flow decreases, short cylindrical section and a truncated cone with cross-sectional area increases to its original value.
- Pressure taps at the upstream and at the short cylindrical section.
- These are connected to a manometer.
- Equation that relates point (2) to point (1);

$$\frac{p_2 - p_1}{\rho} - \frac{v_2^2}{2} = 0 \tag{118}$$

- Applying the continuity equation leads to:

$$v_2 = \sqrt{\frac{\frac{2(p_1-p_2)}{\rho}}{1-\frac{A_2^2}{A_1^2}}} \tag{119}$$

- Example:

The venturi meter shown in the previous diagram has water flowing through it. The pressure difference $(p_1 - p_2)$ is 1 $\mathrm{lb}_f/\mathrm{in}^3$. The diameter at point (1) is 1 ft and that at point (2) is 0.5 ft. What is the volumetric flow rate through the meter?

Answer: Using equation (118) and substitute all values;

$$v_2 = \sqrt{\frac{\frac{2\times 1\mathrm{lb_f/in^2}}{62.3\mathrm{lb_m/ft^3}} \times \frac{144\mathrm{in^2}}{1\mathrm{ft^2}} \times \frac{32.2\mathrm{lb_m \cdot ft}}{1\mathrm{lb_f \cdot s^2}}}{1-\left(\frac{\frac{\pi}{4}\times 0.5^2}{\frac{\pi}{4}\times 1^2}\right)^2}}$$

$$v_2 = 12.7\mathrm{ft/s} = 3.9\mathrm{m/s}$$

Therefore, the flow rate;

$$Q = v_2 A_2 = 12.7 \cdot \frac{\pi}{4} \cdot 0.5^2 = 2.49\mathrm{ft^3/s} = 0.070\mathrm{m^3/s}$$

- It has been found that the value above is slightly higher than the actual observed value.
- This is due to the friction within the venturi meter and non-uniform flow across the pipe (assumed zero in the above calculation).
- Therefore to account this effect, the *coefficient of discharge*, C_v should be introduced and equation (118) changes into;

$$v_2 = C_v \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1-\frac{A_2^2}{A_1^2}\right)}} \tag{120}$$

- Example:

Rework the example above taking into account the experimental results given in the given graph. To solve this type of question:

1. need trial and error method
2. to get value of v, value of C_v is required, which is a function of v
3. assuming that
$$v = v_{\text{from previous example}} = 12.7\mathrm{ft/s}$$
4. calculate Re using this value;
$$Re = \frac{vd\rho}{\mu} = 2.9 \times 10^5$$

5. using the figure, at this Re, $C_v = 0.985$
6. therefore,

$$v_{\text{revised}} = 0.984 \times 12.7 = 12.5$$

7. steps (4) and (5) should be repeated using the revised values of v
8. since the shape of the curve above 2 $\times 10^5$ is constant, therefore, the values of C_v would be the same and 12.5 ft/s is a satisfactory estimate of the velocity.
9. the flow rate can then be calculated using;

$$Q = 12.5 \times \frac{\pi}{4} \times 0.5^2 = 0.068\text{m}^3/\text{s}$$

For an inclined-type venturi meter as shown below:

- The result is independent of the angle to the vertical of the venturi meter.
- Reason: the elevation change in the meter is already taken into account by the elevation change within the manometer legs.
- Applying Bernoulli's equation from point (1) to point (2) and solving the pressure difference at all points give;

$$v_2 = \sqrt{\frac{2g(z_3 - z_4)(p_2 - p_1)}{\rho_1 \left(1 - \frac{A_2^2}{A_1^2}\right)}} \tag{121}$$

- The above formula is *true* for any angle θ, and it can be concluded that if the venturi meter is connected such as shown in the figure, equation (119) can simply be applied.

5.4.2 Pitot tube

The diagram for pitot tube is given below:

- The tube is sometimes called an *impact tube* or *stagnation tube*.
- It consists of a bent transparent tube with one vertical leg projecting out of the flow (connected to a manometer) and another leg pointing directly upstream in the flow.
- At point (1), the flow is practically undisturbed by the presence of the tube.
- Hence, has the velocity that exists at location (2) if the tube were not presence.
- At point (2), the flow has been completely stopped by the tube which has been inserted and $v_2 = 0$.
- Using Bernoulli's equation between location (1) and (2);

$$\frac{p_2 - p_1}{\rho} - \frac{v_1^2}{2} = \text{constant} \tag{122}$$

- The static tube measures the static pressure p_1–since no velocity component perpendicular to its opening.
- The pressure p_2, measures the stagnation pressure of the fluid.
- The pressure difference between point (2) and the inlet of the manometer, is exactly balanced by the pressure difference due to gravity from point (1) to the other side of the meter.
- The pressure meter measures $p_2 - p_1$.
- For a well-designed pitot tube, the friction effect is negligible–the pressure difference reading can directly be used to calculate the velocity using;

$$v_1 = \sqrt{\frac{2\Delta p}{\rho}} \tag{123}$$

- Example:

Air at a density of 0.1 lb_m/ft^3 is flowing through a pitot tube shown in the previous diagram. The pressure difference gauge indicates a difference of 0.5 lb_f/ft^2. What is the air velocity?

Answer: Using equation (123);

$$v_2 = \frac{2 \times 0.5\mathrm{lb_m/in^2}}{0.1\mathrm{lb_m/ft^3}} \cdot \frac{144\mathrm{in^2}}{\mathrm{ft^2}} \cdot \frac{32.2\mathrm{lb_m \cdot ft}}{\mathrm{lb_f \cdot s^2}} = 215\mathrm{ft/s} = 65.6\mathrm{m/s}$$

5.4.3 Orifice meter

The diagram of the equipment is shown below:

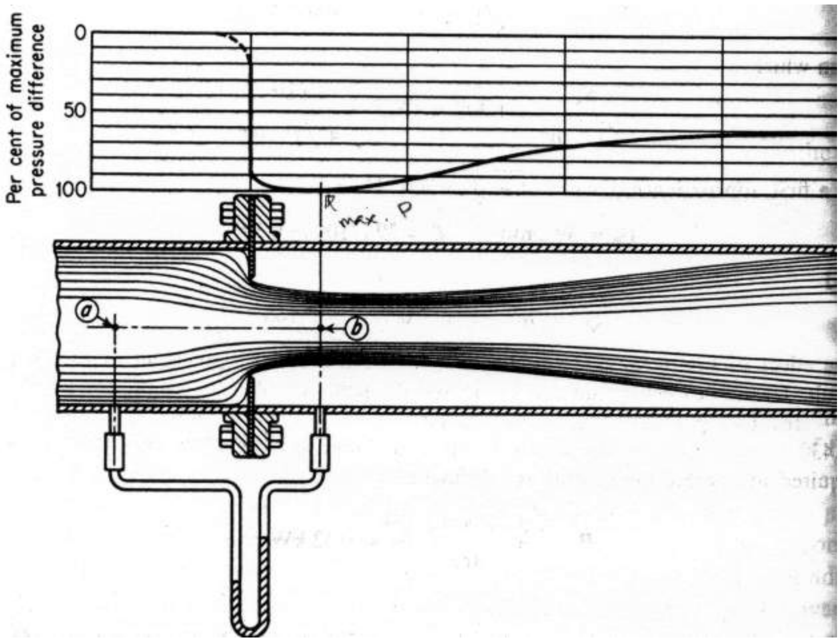

Figure 18: Orifice meter.

- Equipment such as venturi meter is relatively complex to construct and thus expensive.
- A simpler device such as orifice meter can substitute such an expensive equipment.
- It consists of a flat orifice plate with a circular hole in the middle.
- There is a pressure tap upstream from the orifice plate and another just downstream.
- If the flow is horizontal, Bernoulli's equation is applied ignoring the friction terms from point (1) and point (2).
- The final equation is similar to the one given by (119);

$$v_2 = C_o \sqrt{\frac{\frac{2(p_1 - p_2)}{\rho}}{1 - \frac{A_2^2}{A_1^2}}} \tag{124}$$

or by rearranging (124);

$$v_2 = \frac{C_o}{\sqrt{1-\beta^4}}\sqrt{\frac{2(p_1-p_2)}{\rho}} \tag{125}$$

- For process application, β should be between 0.20 and 0.75.
- If β is less than 0.25, the term $\sqrt{1-\beta^4} \rightarrow 1.0$ and if $C_o = 0.61$, thus equation (125) reduces to;

$$v_2 = 0.61\sqrt{\frac{2(p_1-p_2)}{\rho}} \tag{126}$$

- The mass flow rate of a flow passes through a orifice meter can be determined using the continuity equation;

$$\dot{m} = v_o A_o \rho$$

since

$$v_o = v_2$$

therefore,

$$\dot{m} = 0.61 A_o \sqrt{2\rho(p_1-p_2)}$$

- Example:

Water is flowing at a velocity of 1 m/s in a pipe of 0.4 m in diameter. In the pipe, there is an orifice with a hole diameter of 0.2 m. What is the measured pressure drop across the orifice?

Answer: Rearranging equation (125) leads to;

$$\Delta p = \frac{\rho v_2^2}{2C_o^2}\sqrt{1-\beta^4}$$

where

$$\beta = \frac{d_2}{d_1}$$

Calculating Re to determine the coefficient of orifice;

$$Re = \frac{v_2 d_2 \rho}{\mu} = 1.6 \times 10^6$$

using the correlation below; and the corresponding C_o is 0.62. Substitute in the equation gives;

$$\Delta p = 19.5\text{kPa}$$

5.4.4 Rotameter

The diagram of a rotameter is given below:

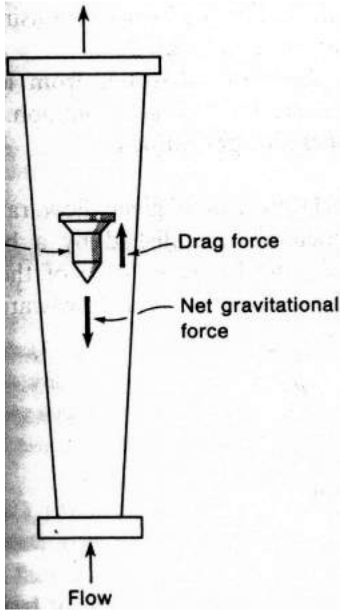

Figure 19: Rotameter.

- It uses a fixed pressure difference and a variable geometry, which is a simple function of a flow rate.
- It consists of a tapered transparent tube.
- The fluid to be measured flows upward.
- The float could be a spherical shape.
- Supposed that the flow is upwards and at a steady-state, resolving the forces around the ball;

$$\begin{aligned} F_D &= F_w - F_b \\ &= V_f \rho_f g - V_f \rho g \end{aligned} \tag{127}$$

since

$$V_f = \frac{m_f}{\rho_f}$$

thus,

$$F_D = m_f g \left(1 - \frac{\rho}{\rho_f}\right) \tag{128}$$

- For a rotameter operating an a certain fluid, the right hand side of equation (128) is constant and independent of flow rate.
- Thus F_D is also constant when flow rate increases–the position of the float must change to keep F_D constant.

- F_D can be expressed as;

$$F_D = A_f C_D \rho \frac{v_{max}^2}{2} \tag{129}$$

- If change in F_D is small–(for large rotameter)/moderate fluids μ–max. velocity stays the same when Q increases. This is based on the equation below:

$$Q = v_{max} \frac{\pi(d_{tube}^2 - d_{float}^2)}{4} \tag{130}$$

QUESTIONS

1. Air from a large vessel discharges into the atmosphere from a small orifice in its side. The pressure and temperature of the air in a vessel are 207 kN/m^2 absolute and 15°C respectively. The diameter of the orifice is 25 mm. Assuming R and γ to be 287 J/kg·K and 1.4 respectively, calculate the mass of air discharging per second. The atmospheric pressure is 103.5 kN/m^2 and C_o for the orifice is 0.64. (*Ans = 0.154 kg/s*)

2. The water in a tank is 1.8 m deep and over the surface is air at pressure 70 kN/m^2 above atmosphere. Find the rate of flow in m^3/s from an orifice of 50 mm diameter in the bottom of the tank. Given that $C_o = 0.6$. (*Ans = 0.0156 m^3/s*)

3. Starting from the Bernoulli's equation, and the continuity equation, derive an expression for the theoretical discharge through an orifice plate meter. Explain the reasons why expression must be modified with empirical factors to determine the actual discharge.

4. Derive an expression for the rate of flow through an inclined venturi meter and show that, if a U-tube type gauge is used to measure the pressure difference, the gauge reading will be the same for a given discharge irrespective of the inclination of the meter.

   A venturi meter measures the flow of oil of specific gravity 0.82 and has an entrance of 125 mm diameter and a throat of 50 mm diameter. There are pressure gauges at the entrance and at the throat, which is 300 mm above the entrance. If the coefficient for the meter, C_v is 0.97 and the pressure difference is 27.5 kN/m^2, what is the volumetric flow rate at the entrance? (*Ans = 0.01535 m^3/s*)

6 Agitation and Mixing of Liquids

6.1 Definition

Agitation:

- it is an induced motion of a material in a specified way.
- the pattern is normally circulatory.
- it is normally taken place inside a container.

Mixing:

- it is a random distribution of components with separate phases into and through one another.

6.2 Agitation of liquids

Purposes of agitation:

- suspending solid particles.
- blending miscible liquids.
- dispersing gas through liquid in the form of small bubbles.
- dispersing second liquid, immiscible with the first to form an emulsion/suspension of fine drops.
- promoting heat transfer between liquids and a coil/jacket.

6.2.1 Agitation equipment

- Liquids are normally agitated in a vessel/tank.
- Top of vessel can be either open or closed.
- Bottom of the tank/vessel should be rounded–avoid region where current cannot penetrate.
- The impeller creates a flow pattern–cause liquid to circulate through the vessel and return back to the impeller.

Impeller:

- Can be categorised into 2 classes:
  1. parallel currents with the impeller axis.
  2. tangential/radial currents with the impeller axis.
- Three types of impellers include:
  1. propellers
  2. paddles

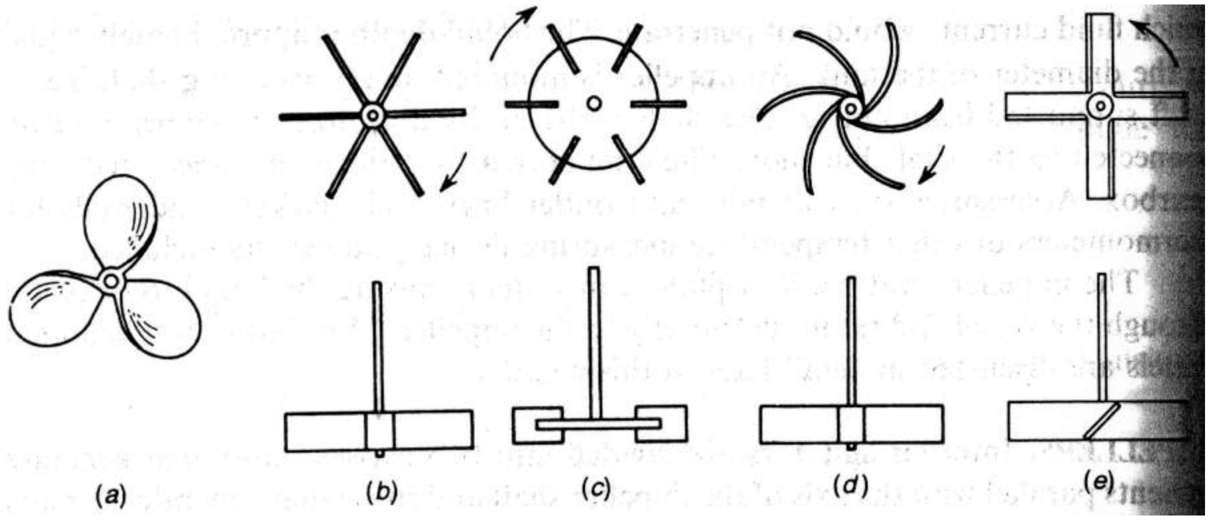

Figure 20: Different types of impellers.

3. turbines

- Propellers:
  - it creates an axial-flow.
  - apply for liquids of low viscosity.
  - flow of currents starts from the impeller and continue through the liquid at a given direction until it is deflected by the floor/wall.
  - propeller blades vigorously cut and shear the liquid.
  - it is effective in very large tanks.
  - ratio of the distance of the longitudinal revolution to the propeller diameter– *pitch* of propeller.
- Paddles:
  - normally use in stirring simple liquids.
  - can operate at low to moderate speeds.
  - paddles push the liquid radially and tangentially with no vertical motion at the impeller, unless blades are pitch.
  - currents generate from paddle-type impeller travel outward or downward.
  - *anchor* is one type of paddle agitator.
    - can prevent deposits on heat transfer surfaces.
    - only operates together with a higher speed paddle/other agitator (turning in opposite direction)
    - **disadvantage**: poor mixer.
  - in industry: paddles are used at speed between (20–150 rpm).
  - ratio of paddles diameter to the vessel diameter is typically (50–80%).
  - width of blade (1/6 to 1/10) of its length.
  - rate of agitation:

* **LOW** speeds–mild agitation for unbaffled vessel.
* **HIGH** speeds–require baffles.

- Turbines:
  - the shape of turbine impeller is similar to multi-bladed paddle agitator.
  - blades may be curved, pitch/vertical.
  - diameter (30–50%) of vessel diameter.
  - suitable for wide range of viscosity.
    * **LOW** μ: it generates strong currents which continue throughout vessel.
    * avoid stagnant pockets.
  - principal currents produced: radial and tangential.
  - tangential currents–produce *vortex* and *swirling*–method of preventing–baffles.

6.2.2 Flow patterns

- Patterns which appear as a result of using impeller depend on:
  1. type of impeller
  2. characteristic of fluid
  3. size
  4. proportionality of the tank, baffles and agitator
- Fluid velocity consists of 3 components:
  1. **radial flow**: acts in the direction perpendicular to the shaft of impeller.
  2. **longitudinal flow**: acts in the direction parallel with the shaft.
  3. **tangential flow**: (rotational), acts in the direction tangent to the circular path around the shaft.

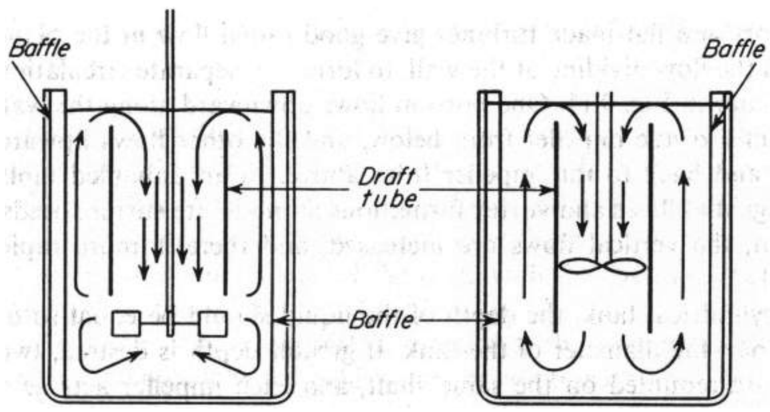

Figure 21: Different types of flow patterns resulted from different types of impellers; right hand side (turbine impeller) and left hand side (propeller).

- In all cases (for vertical shaft):
  - *radial* and *tangential* components are in horizontal plane.

  - for *longitudinal* component is vertical.

- To get a perfect mixing:

  - depends on *radial* and *longitudinal* components.
  - and *tangential* component is a disadvantageous (only if shaft is vertical and located in the middle of the tank);
    * create vortex in the liquid.
    * preserve stratification at various levels without having longitudinal flow between levels.
    * if solid particles present within tank–it tends to throw the particles to the outside of the centrifugal force region.
    * power absorbed by liquid is limited.
  - for vessel without baffles–circulatory flow always occur regardless of types of impellers and flow patterns.
  - for high impeller speed–gas from above the liquid will be drawn together during the mixing–**undesirable**.

- To avoid swirling:

  - mount the impeller away from the centre of the vessel and tilted in the direction perpendicular to the direction of flow.
  - use baffles.
  - for larger tank–mount the agitator at the side of the vessel (shaft in a horizontal plane at an angle with the radius).

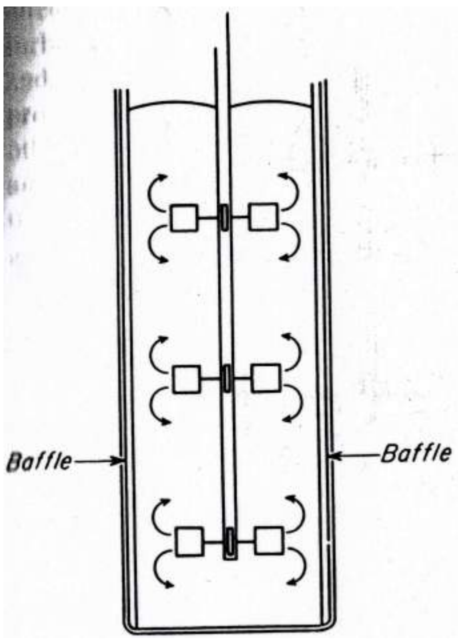

Figure 22: The use of baffle in an agitated tank.

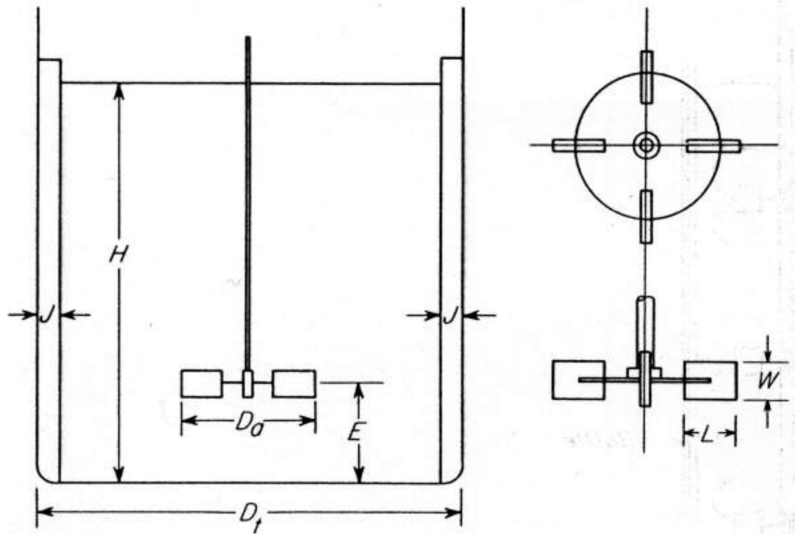

Figure 23: Measurement of turbine.

6.2.3 Designing turbine impeller

The dimension of a vessel and the turbine impeller is shown below:

- The design should consider:
    1. location of impeller
    2. proportion of vessel
    3. proportion of baffles
- These will definitely have effects on:
    1. circulation rate of liquid.
    2. velocity patterns.
    3. power consumed.
- According to the turbine impeller dimension, the proportions required are:

$$\frac{D_a}{D_t} = \frac{1}{3} \quad \frac{H}{D_t} = 1 \quad \frac{J}{D_t} = \frac{1}{12}$$

$$\frac{E}{D_t} = \frac{1}{3} \quad \frac{W}{D_a} = \frac{1}{5} \quad \frac{L}{D_a} = \frac{1}{4}$$

6.3 Velocities and power consumption in agitated vessels

For an effective mixing/agitation–the volume of fluid circulated in a vessel via an impeller must be sufficient to sweep out the entire vessel in a reasonable time.

Stream velocity leaving the impeller must be sufficient to carry currents to the remotest part of the vessel.

- Flow number:
  - the impeller of a turbine as basically the impeller of a pump–just without casing.
  - consider a flat-blade turbine impeller:
    * the velocity of the blade is v_2
    * actual tangential velocity, V'_{v2}

$$V'_{v2} = kv_2 \tag{131}$$

since

$$v_2 = \pi D_a n$$

thus,

$$V'_{v2} = k\pi D_a n \tag{132}$$

volumetric flow rate, Q;

$$Q = V'_{r2} A_p \tag{133}$$

where A_p is the area of the cylinder swept out by impeller blades given by;

$$A_p = \pi D_a W$$

with W is the blade width.

    * actual radial velocity, V'_{r2}

$$V'_{r2} = (v_2 - V'_{v2}) \tan \beta'_2 \tag{134}$$

substituting equation (132) gives;

$$V'_{r2} = \pi D_a n(1-k) \tan \beta'_2$$

    * total liquid velocity, V'_2

$$V'_2 = \alpha \pi n D_a \tag{135}$$

  - the volumetric flow rate based on the above velocity is given by

$$Q = K\pi^2 D_a^2 n W (1-k) \tan \beta'_2 \tag{136}$$

where K refers to a constant which allows for variable radial velocity over the width of the blade.

  - the ratio of the flow rate and the diameter of the impeller is given by the flow number, N_Q;

$$N_Q = \frac{Q}{nD_a^3} \tag{137}$$

- the flow number is a constant for each type of agitated vessel with specific type of turbine;
  - marine propeller: $N_Q = 0.5$
  - four-blade 45° turbine: $N_Q = 0.87$

- Power consumption:
  - This is an important information in order to design a stirred tank.
  - At the turbulent point of the flow, the power is estimated using;
    1. the flow rate produced by the impeller:

$$Q = nD_a^3 N_Q \tag{138}$$

    2. the kinetic energy per unit volume of the fluid:

$$E = \frac{1}{2}\rho V_2'^2 \tag{139}$$

  - with both equations above, the power requirement of agitated vessels can be calculated using;

$$P = \frac{1}{2}\rho n^3 D_a^5 (\alpha\pi)^2 N_Q \tag{140}$$

  - and upon rearrangement into a dimensionless form;

$$N_P = \frac{P}{n^3 D_q^5 \rho} \tag{141}$$

  - for example, for a six-blade turbine with the flow number, $N_Q = 0.3$, the power number, $N_P = 5.2$ if the ratio α of $\frac{V_2'}{v_2} = 0.9$.
  - at **LOW** Re: the will be a coincidence between lines of N_P and Re for baffled and an unbaffled tanks, thus;

$$N_P = \frac{K_L}{Re} \tag{142}$$

    where K_L is a constant at laminar flow regime and therefore;

$$P = K_L n^2 D_a^3 \mu \tag{143}$$

  - at **HIGH** Re: the power is independent of Re, thus μ does not play an important role and,

$$N_P = K_T \tag{144}$$

    where K_T is a constant at turbulent flow regime and;

$$P = K_T n^3 D_a^5 \rho \tag{145}$$

  - the magnitudes of K_L and K_T are normally specified.

- Power correlation:
  - required to estimate the the power of any agitated vessel.

– correlation can be found using dimensional analysis given information about:

1. tank measurement
2. impeller measurement
3. distance of impeller from tank floor
4. liquid depth
5. dimension of baffles (if applied)
6. fixed number of baffles
7. fix number of impeller blades

– other important variables which should be considered include;

1. viscosity of fluid, μ
2. density of fluid, ρ
3. speed of impeller, n

– for six-blade turbines, the correlation is given by the graph:

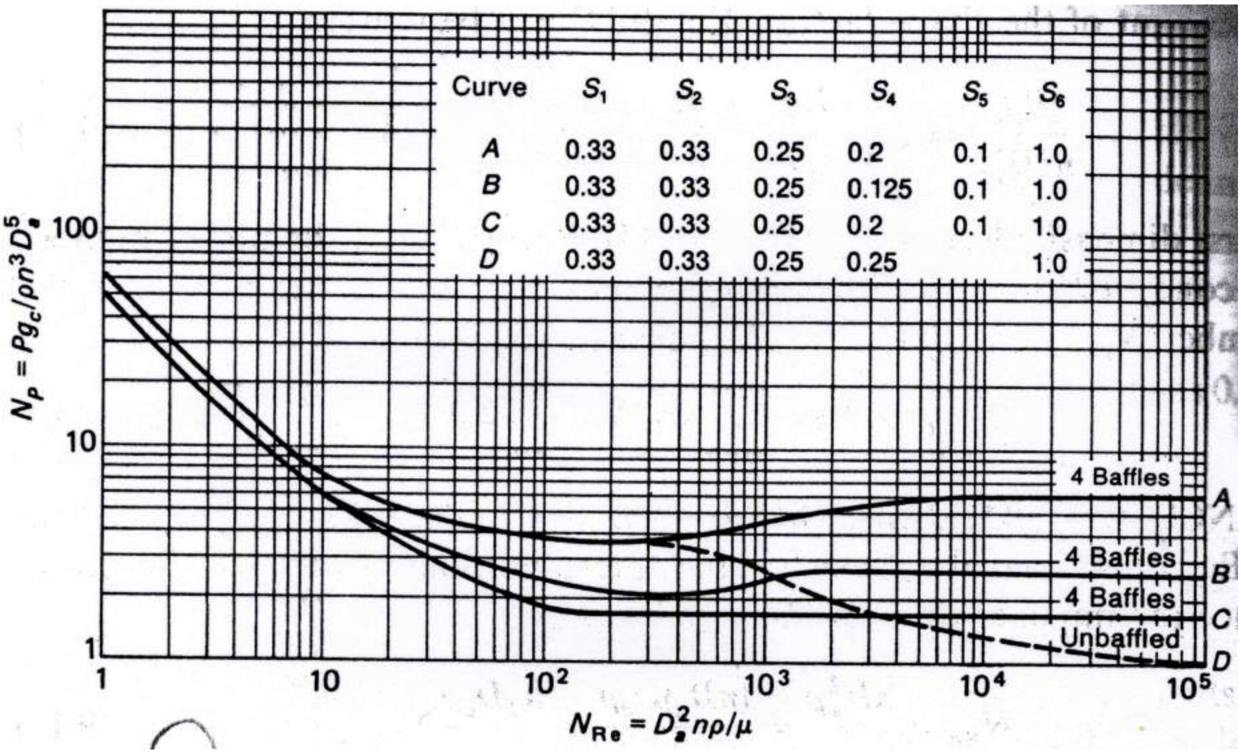

Figure 24: For six-blade turbines.

– for three-blade propellers, the graph is given below:

– mixing time of an agitated vessel can be found using the graph below:

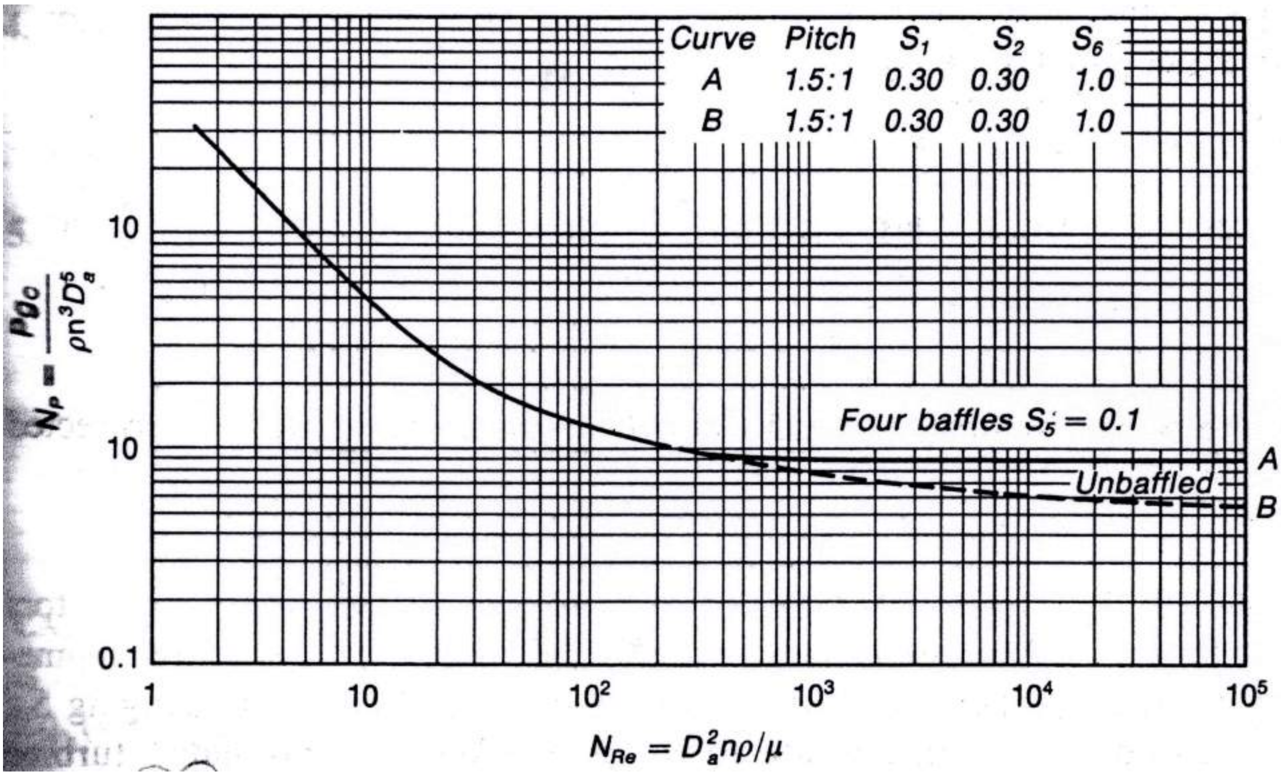

Figure 25: For three-blade propellers.

6.4 Scale-up of agitator design

Example:

A pilot-plant vessel with 305 mm diameter is agitated using a six-blade turbine impeller with 102 mm diameter. When the Re of the impeller is 10^4, the mixing time of the 2 liquids is 15 s. The power required is 2 hp per 1000 gal or 0.4 kW/m^3 of liquid.

a. What is the power input required to give the same mixing time in a vessel with 1830 mm diameter.

b. What is the mixing time for the same vessel as in (a) if the power input per unit volume was the same as in the pilot vessel.

Answer:

(a) It is given that Re of the pilot-plant type vessel is large, the correlation graph should be used. As shown in the graph, for $Re > 10^4$, the values of mixing times, nt_T is approximately constant. For a constant n_T, the speed of the liquid should remain the same in both vessels. The power input of an impeller is given by equation (140);

$$P = \frac{1}{2}\rho n^3 D_a^5 (\alpha\pi)^2 N_Q$$

and upon rearrangement gives;

$$\frac{P}{D_a^3} = K_T n^3 D_a^2 \rho$$

where for vessels of similar geometry;

$$P(\text{per unit volume}) \propto \frac{P}{D_a^3}$$

and from the above equation;

$$\frac{P}{D_a^3} = c_2 n^3 D_a^2$$

with constant c_2.
Let the notations as follows:

Power required for pilot-plant vessel $= P_{pp}$
Power required for large vessel $= P_l$
Diameter of the pilot-plant vessel $= D_{pp}$
Diameter of the large vessel $= D_l$
Speed of the pilot-plant vessel $= n_{pp}$
Speed of the large vessel $= n_l$
Taking the ratio of the two types of equipment;

$$\frac{\frac{P_l}{D_{a,l}^3}}{\frac{P_{pp}}{D_{a,pp}^3}} = \left(\frac{n_l}{n_{pp}}\right)^3 \left(\frac{D_{a,l}}{D_{a,pp}}\right)^2$$

since the speed should be the same for both vessel;

$$n_{T,pp} = n_{T,l}$$

therefore,

$$\frac{\frac{P_l}{D_{a,l}^3}}{\frac{P_{pp}}{D_{a,pp}^3}} = \left(\frac{1830}{305}\right)^2 = 6^2 = 36$$

Since the power input for the liquid in the pilot-plant scale is 2 hp, thus, for the large vessel;

$$P(\text{per unit volume}) = 2\text{hp} \times 36 = 72\text{hp}$$

(b) Should the power of the two types of vessels be the same;

$$\frac{n_l}{n_{pp}} = \left(\frac{D_{a,l}}{D_{a,pp}}\right)^{\frac{2}{3}}$$

for a constant mixing time;

$$\frac{n_l}{n_{pp}} = \frac{t_{T,pp}}{t_{T,l}}$$

therefore,

$$\frac{t_{T,l}}{t_{T,pp}} = \left(\frac{D_{a,l}}{D_{a,pp}}\right)^{\frac{2}{3}} = 6^{\frac{2}{3}} = 3.3$$

and the blending time for the large vessel, $t_{T,l}$ would be $3.3 \times 15 = 49.5$ s