Supply Chain Management: Inventory Management

Donglei Du

Faculty of Business Administration, University of New Brunswick, NB Canada Fredericton E3B 9Y2 (ddu@umbc.edu)

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Section 1

Introduction

Outline I

1. Introduce some basic concepts in inventory management
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   - Reorder point (ROP)
   - Lead time
   - Safety stock
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   - Service level
2. Introduce some basic inventory models, both deterministic and probabilistic.
   - deterministic models
     1. EOQ model
     2. EPQ model
   - probabilistic models
     1. Single-period model: Newsboy model
     2. Multiple-period model

Section 2

Inventory Management

What is Inventory? I

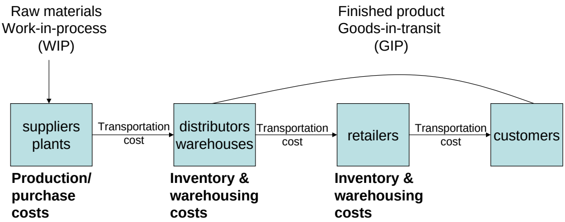
- Inventory is a stock or store of goods or services, kept for use or sale in the future. There are four types of inventory
  - Raw materials & purchased parts
  - Partially completed goods called work in progress (WIP)
  - Finished-goods inventories
  - Goods-in-transit to warehouses or customers (GIT)
- The motive for inventory: there are three motives for holding inventory, similar to cash.
  - Transaction motive: Economies of scale is achieved when the number of set-ups are reduced or the number of transactions are minimized.
  - Precautionary motive: hedge against uncertainty, including demand uncertainty, supply uncertainty
  - Speculative motive: hedge against price increases in materials or labor

What is inventory management?

- The objective of inventory is to achieve satisfactory levels of customer service while keeping inventory costs within reasonable bounds.
- Level of customer service: (1) in-stock (fill) rate (2) number of back orders (3) inventory turnover rate: the ratio of average cost of goods sold to average inventory investment
- Inventory cost: cost of ordering and carrying trade-off

An Example I

- Let us look at a typical supply chain consists of suppliers and manufacturers, who convert raw materials into finished materials, and distribution centers and warehouses, form which finished products are distributed to customers. This implies that inventory appears in the supply chain in several forms:



Section 3

Inventory models

Inventory Models I

- Inventory models come in all shapes. We will focus on models for only a single product at a single location. Essentially each inventory model is determined by three key variables:
  - demand:
    - deterministic, that is, known exactly and in advance.
    - random. Forecasting technique may be used in the latter case when historical data are available to estimate the average and standard deviation of the demand (will introduce forecasting techniques later).
  - costs:
    - order cost: the cost of product and the transportation cost
    - inventory holding cost: taxes and insurances, maintenance, etc.
  - physical aspects of the system: the physical structure of the inventory system, such as single or multiple-warehouse.
- Usually, there are two basic decisions in all inventory models:

Inventory Models II

1. How much to order?
2. When to order?

Section 4

Economic Order Quantity (EOQ)

Subsection 1

EOQ model

Economic Order Quantity (EOQ): Model description I

- The EOQ model is a simple deterministic model that illustrates the trade-offs between ordering and inventory costs.
- Consider a single warehouse facing constant demand for a single item. The warehouse orders from the supplier, who is assumed to have an unlimited quantity of the product.
- The EOQ model assume the following scenario:
  - Annual demand D is deterministic and occurs at a constant rate—constant demand rate: i.e., the same number of units is taken from inventory each period of time, such as 5 units per day, 25 units per week, 100 units per month and so on.
  - Order quantity are fixed at Q units per order.

Economic Order Quantity (EOQ): Model description II

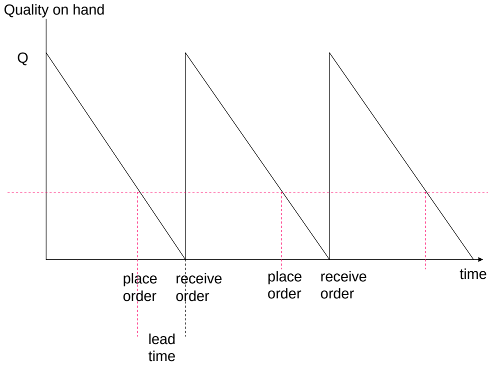
- A fixed ordering cost c_o per order, not depending on the quantity ordered.
- A holding cost c_h per unit per year, depending on the size of the inventory. Sometimes holding cost can usually be calculated as follows:

$$c_h = \text{annual holding cost rate} \times \text{annual cost of holding one unit in i}$$

- Shortages (such as stock-outs and backorder) are not permitted.
- There is no lead time or the lead time is constant.
- The planning horizon is long
- The objective is to decide the order quantity Q per order so as to minimize the total annual cost, including holding and setting-up costs.

Economic Order Quantity (EOQ): Analysis

- The following graph illustrates the inventory level as a function of time—a saw-toothed inventory pattern.



- *Inventory level (IL)* is the quantity on hand, which is different from *inventory position (IP)*, which is equal to inventory on-hand plus quantity on order minus backorder (if any).
- The maximum IL is Q, the minimum is 0, therefore the average IL is $\frac{Q}{2}$.
- Since

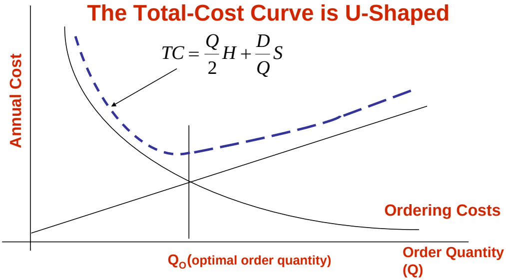
$$\begin{aligned}\text{Annual holding cost} &= \text{Average inventory} \times \text{Annual holding cost} \\ &= \frac{Q}{2} c_h\end{aligned}$$

$$\begin{aligned}\text{Annual ordering cost} &= \text{Number of orders per year} \times \text{ cost per orc} \\ &= \frac{D}{Q} c_o\end{aligned}$$

So

$$\begin{aligned}\text{Total annual cost} &= \text{annual holding cost} + \text{annual setup cost} \\ &= \frac{Q}{2} c_h + \frac{D}{Q} c_o\end{aligned}$$

- The trade-off of holding and ordering costs is illustrated in the following figure.



- Therefore, the optimal order quantity is achieved at the point where the two costs meet.
  1. The economic order quantity is
  $$Q^* = \sqrt{\frac{2Dc_o}{c_h}}$$
  2. The cycle time (the time between two consecutive orders) is
  $$\text{Cycle Time} = \frac{Q^*}{D}$$
  3. Number of orders per year
  $$\text{Number of orders per year} = \frac{1}{\text{cycle time}} = \frac{D}{Q^*}$$

An Example I

- A local distributor for a national tire company expects to sell approximately 9600 steel-belted radical tires of a certain size and tread design next year. Annual carrying cost is \$16 per tire, and ordering cost is \$75 per order. The distributor operates 288 days a year.
-
  1. What is the EOQ?

     $$Q^* = \sqrt{2Dc_o/c_h} = \sqrt{2(9600)75/16} = 300$$

  2. How many times per year would the store reorder if the ECQ is ordered?

     $$D/Q^* = 9600/300 = 32$$

An Example II

3. What is the length of an order cycle if the EOQ is ordered?

$$Q^*/D = 300/9600 = 1/32 \text{ year}$$

or

$$1/32 \times 288 = 9 \text{ working days}$$

4. What is the total cost.

$$TC = (Q^*/2)c_h + (D/Q^*)c_o = (300/2)16 + (9600/300)75 = 4800$$

Sensitivity analysis

- Estimating ordering and inventory costs is not an easy job, let alone accurate. Now let us see whether the EOQ model is robust to the costs, i.e., whether the order quantity is stable or not when the costs vary.
- Our claim is *EOQ model is <u>insensitive</u> to small variations or errors in the cost estimates.*

Subsection 2

When-to-order?

When-to-order: some concepts

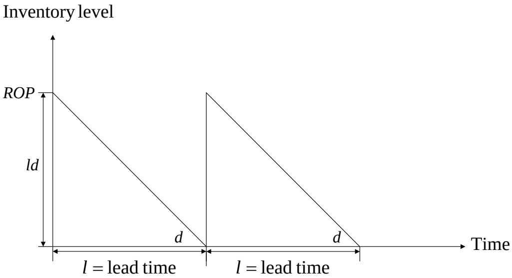
- The reorder point (ROP) is defined to be the inventory level at which a new order should be placed.
- The when-to-order decision is usually expressed in terms of a reorder point.
- We will consider deterministic and stochastic models separately.

When-to-order: deterministic demand and deterministic lead time

- Here we consider a simple deterministic demand and deterministic lead time model.
- Assume constant demand per period is d.
- Lead times is ℓ for a new order in terms of periods.
- Then

$$ROP = \ell d$$

- Note that in our previous EOQ model, we assume $\ell = 0$ so the reorder point is $ROP = 0$.


Inventory level
ROP
ld
d
d
Time
l = lead time
l = lead time

An Example I

- Mr. X takes two special tablets per day, which are delivered to his home seven days after an order is called in. At what point should Mr. X reorder?
- Solution: In this case, the daily demand and lead time are both constants (special random variable), and

$$\begin{aligned} d &= 2 \\ \ell &= 7 \end{aligned}$$

Therefore

$$\text{ROP} = \ell d = 2 \times 7 = 14$$

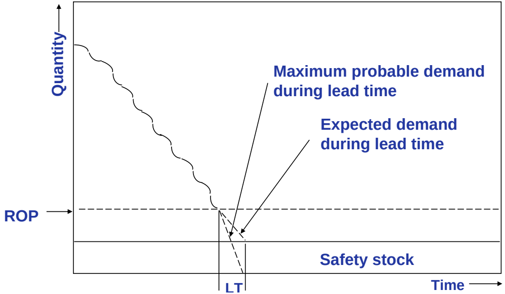
i.e., Mr. X should reorder when there are 14 tablets left.

When-to-order: stochastic demand and stochastic lead time

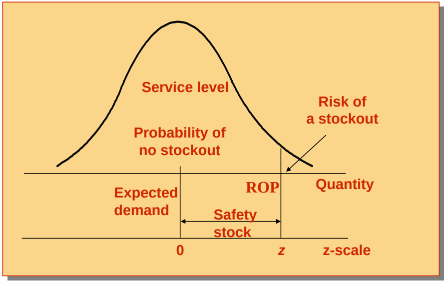
- The reorder point for stochastic demand and stochastic lead time is the expected demand during lead time and an extra safety stock, which serves to reduce the probability of experiencing a stockout during lead time. Formally

$$\text{ROP} = \text{Expected demand during lead time} + \text{Safety stock}$$

- Safety stock (SS): stock that is held in excess of expected demand due to variable demand rate and/or variable lead time. The following figure illustrates how safety stock can reduce the risk of stockout during lead time.



- Service level (SL): the probability that demand will not exceed supply during lead time (i.e., the probability of no stockout). So the risk of stockout is $1 - \text{SL}$.



Assumption

We assume

1. *Daily demands are i.i.d. random variables;*
2. *Daily demands are independent of the lead time.*

- The amount of safety stock depends on
  1. The desired service level $1-\alpha$.
  2. Daily demand random (rate) variable D_i for day i, and hence the unit of D_i is items/per day—suppose expectation and standard deviation of D_i are (μ_D, σ_D).
  3. Lead time random variable L, and hence the unit of L is number of days—suppose expectation and standard deviation are (μ_L, σ_L).
- So the *lead time demand*—demand during lead time, which is a new random variable with unit being number of items is equal to

$$D_L = \sum_{i=1}^{L} D_i = D_1 + \cdots + D_L$$

- Then, we use the *compound random variable* property to find the expectation and variance of D_L (note that here both D_i's and L are random variable and we cannot apply the rule of expectation and rule of variance directly):

$$\mathbb{E}[D_L] = \mathbb{E}\left[\sum_{i=1}^{L} D_i\right] = \mu_L \mu_D$$
$$\mathbb{V}[D_L] = \mathbb{V}\left[\sum_{i=1}^{L} D_i\right] = \mu_L \sigma_D^2 + \sigma_L^2 \mu_D^2$$

- Assume D_L follows a normal distribution, that is $D_L \sim N(\mathbb{E}[D_L], \sqrt{\mathbb{V}[D_L]})$.
- Then ROP should be chosen such that the probability of no stockout is at least $1-\alpha$, that is

$$\mathbb{P}(D_L \leq \text{ROP}) \geq 1 - \alpha$$

- A standard *given-p-value-find-x-value* calculation gives us the desired ROP:

$$\text{ROP} = \mathbb{E}[D_L] + z_\alpha \sqrt{\mathbb{V}[D_L]} = \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2}$$

An Example I

- A restaurant used an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manger is willing to accept no more than a 10 percent risk of stockout during lead time, which is two weeks. Assume the distribution of usage is Normal. What is the ROP?
- Solution: In this case, the weekly demand is random and the lead time is constant, and $\alpha = 0.10$, $\mu_D = 50$, $\mu_L = 2$, $\sigma_D = 3$, $\sigma_L = 0$. Therefore

$$\begin{aligned} \text{ROP} &= \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} \\ &= 2 \times 50 + 1.28\sqrt{2(3)^2} = 105.43 \end{aligned}$$

i.e., the ROP is approximately 106 jars.

Section 5

Economic Production Quantity (EPQ): model description

Subsection 1

EPQ model

Economic Production Quantity (EPQ): model description I

- Economic Production Quantity (EPQ), also called Economic Manufacturing Quantity (EMQ), is similar to EOQ model with one difference: instead of orders received in a single delivery, units are received incrementally during production, that is, constant production rate.
- The assumptions of the basic EPQ model are the following:
  - Annual demand D is deterministic at a fixed rate of d units/per day.
  - Annual production P is deterministic at a fixed rate of p units/per day, where $p > d$.
  - The setup cost c_o now has a different meaning—it is usually the cost to prepare for a production rum, which is independent of the production lot size.

Economic Production Quantity (EPQ): model description II

- A holding cost c_h per unit per year, depending on the size of the inventory.
- Shortages (such as stock-outs and backorder) are not permitted.
- There is no lead time or the lead time is constant.
- The planning horizon is long

- The following simple observation will be used shortly, which basically a unit transformation: year or day.

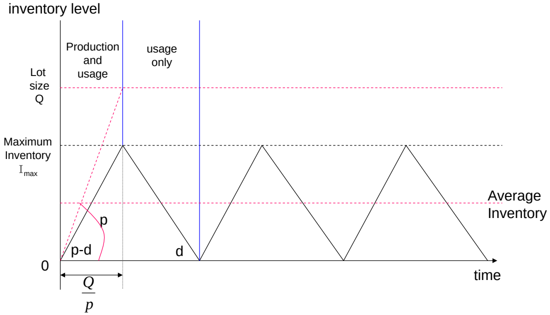
Observation

$$\frac{D}{P} = \frac{d}{p}$$

- The EPQ model is designed for production situations in which, once an order is placed, production begins and a constant number of units is produced to inventory each day until the production run has been completed.
- We introduce the concept of *lot size*, which is the number of units in an order. In general, if we let Q denote the production lot size, then the objective is to find the production lot size that minimizes the total holding and ordering cots.

Economic Production Quantity (EPQ): analysis

- During the production run, there are two activities (see the following figure):
  - Demand reduces the inventory
  - Production adds to the inventory



- Now we establish our total cost model analogously as before.
  - First, we consider the annual holding cost. Similarly as before we need to know the average inventory, which is equal to

$$\begin{aligned}\text{average inventory} &= \frac{1}{2}(\text{maximum inventory} + \text{minimum invento}\\ &= \frac{1}{2}\left((p-d)\frac{Q}{p} + 0\right)\\ &= \frac{1}{2}\left(1 - \frac{d}{p}\right)Q\end{aligned}$$

  - So the annual holding cost is

$$\begin{aligned}\text{Annual holding cost} &= (\text{Average inventory})(\text{Annual holding cost}\\ &= \frac{1}{2}\left(1 - \frac{d}{p}\right)Qc_h\end{aligned}$$

- Second, the annual setup cost has the same formula:

$$\begin{aligned}\text{Annual ordering cost} &= \text{(Number of orders per year)(cost per or} \\ &= \frac{D}{Q}c_o\end{aligned}$$

- So the total annual cost is

$$\begin{aligned}\text{Total annual cost} &= \text{annual holding cost+annual setu} \\ &= \frac{I_{\max}}{2}c_h + \frac{D}{Q}c_o \\ &= \frac{1}{2}\left(1-\frac{d}{p}\right)Qc_h + \frac{D}{Q}c_o \\ &\underbrace{=}_{\text{Observation 2}} \frac{1}{2}\left(1-\frac{D}{P}\right)Qc_h + \frac{D}{Q}c_o\end{aligned}$$

- The trade-off between the holding and setup costs are similar as the EOQ model. The optimal order quantity is achieved when these two costs meet:
  1. The economic production quantity is
  $$Q^* = \sqrt{\frac{2Dc_o}{c_h}}\sqrt{\frac{p}{p-d}}$$
  2. The run time (the production phase) is
  $$\text{Run Time} = \frac{Q^*}{p}$$
  3. The cycle time (the time between two consecutive runs) is
  $$\text{Cycle Time} = \frac{Q^*}{d}$$
  4. The maximum inventory is
  $$I_{\max} = \left(1 - \frac{d}{p}\right) Q$$

An Example I

- A toy manufacturer uses 48000 rubber wheels per year. The firm makes its own wheels at a rate of 800 per day. Carrying cost is \$1 per wheel per year. Setup cost for a production run is \$45. The firm operates 240 days per year. Determine the:
  1. The optimal run size
  2. Minimum total annual cost for carrying and setup.
  3. Cycle time for the optimal run size.
  4. Run time for the optimal run size.
- **Solution:** $D = 48000$ wheels per year, and hence $d = 48000/240 = 200$ wheels per day. $p = 800$ wheels per day, and hence $P = 800(240) = 192000$. $c_o = \$45$, $c_h = \$1$ per wheel per year,
  1. $Q^* = \sqrt{\frac{2(48000)45}{1}}\sqrt{\frac{800}{800-200}} = 2400$ wheels.

An Example II

2. $I_{\max} = \frac{2400}{800}(800 - 200) = 1800$
   $TC = \frac{I_{\max}}{2} c_h + \frac{D}{Q^*} c_o = \frac{1800}{2} \times 1 + \frac{48000}{2400} \times 45 = 1800$
3. Cycle Time $= \frac{2400}{200} = 12$ days.
4. Run Time $= \frac{2400}{800} = 3$ days.

Section 6

The Newsboy Problem-Unknown demand (probabilistic model)

Subsection 1

The newsvendor model

The Newsboy Problem-Unknown demand (probabilistic model): introduction

- The EOQ and EPQ models treat the world as if it were predictable by using forecast techniques. However this is not accurate in general.
- The newsboy model is a single-period inventory model, where one order is placed for the product; at the end of the period, the product is either sold out, or a surplus of unsold items will be sold for a salvage value. This happens in seasonal or perishable items, such as newspapers, seasonal clothing etc. Because we only order once for the period, the only inventory decision is how much to order. Of course, if the demand is deterministic, and known, then the solution is trivial. So the demand is assumed to be a random variable following some given probabilistic distribution.

The Newsboy Problem-Unknown demand (probabilistic model): model description

- The assumptions of the basic Newsboy model are the following:
  - Demand X is a random variable following a probabilistic distribution with cumulative distribution function F_X and density function f_X.
  - Order quantity Q units per order.
  - The purchasing wholesale price is w per unit.
  - The selling price is p per unit.
  - The salvage price is s.
  - The planning horizon is one-period.

- The objective is to decide the order quantity Q such that the expected total cost is minimized, including the long run total shortage and excess costs (see below). There are two opposite driving forces in this model.
  1. *shortage cost*—unrealized profit per item, which is the opportunity loss for underestimating the demand ($Q < X$):

     $$c_s = \text{selling price per unit} - \text{purchasing price per unit} = p - w$$

  2. *excess cost*—the difference between purchase and salvage cost, which is the loss of overestimating demand ($Q > X$):

     $$c_e = \text{purchasing price per unit} - \text{salvage price per unit} = w - s$$

  3. The expected total cost model is given by

     $$\begin{aligned}\mathbb{E}_X(TC) &= c_e\mathbb{E}_X(\max\{Q-X,0\}) + c_s\mathbb{E}_X(\max\{X-Q,0\}) \\ &= c_e\int_0^Q (Q-x)f(x)dx + c_s\int_Q^\infty (x-Q)f(x)dx\end{aligned}$$

4. The optimal solution Q^* depends on the underlying demand distribution.

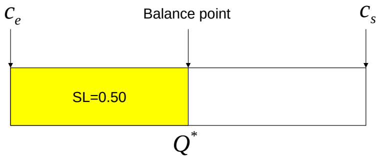
- we define the concept of *service level* (SL), usually denoted as α, as the probability that demand will not exceed the order quantity :

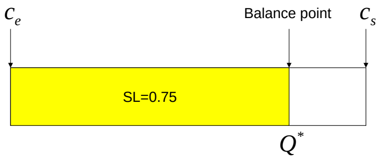
$$\text{SL} = \alpha = \frac{c_s}{c_s + c_e} = P(X \leq Q^*) = F_X(Q^*)$$

where F is the cumulative probability function. So

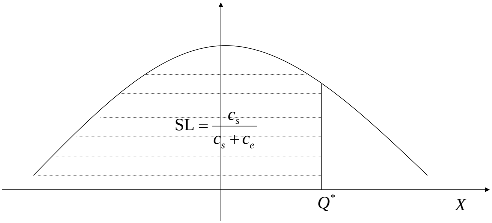
$$Q^* = F_X^{-1}(\alpha)$$

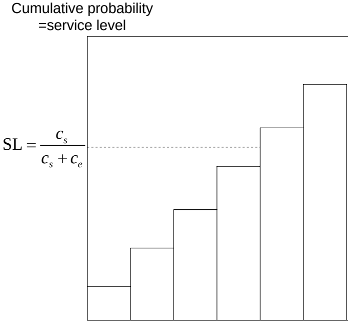
- The following figures illustration the service level for uniform demand distribution.





- A little note is necessary. When the order quantity is discrete rather than continuous (and hence X is a discrete random variable rather than a continuous one), the optimal Q^* satisfying the last equation usually does not coincide with a feasible order quantities—since it may be a fractional number. The solution is to order round up to the next highest order quantity (see the figures and example below for further illustration).





An Example I

- The owner of a newsstand wants to determine the number of Canada Post that must be stocked at the start of each day. It costs 30 cents to buy a copy, and the owner sells it for 75 cents. The sale of the newspaper typically occurs between 7:00 and 8:am. Newspapers left at the end of the day are recycled for an income of 5 cents a copy. How many copies should the owner stock every morning in order to minimize the total cost (or maximize the profit), assuming that the demand for the day can be approximated by
  1. A normal distribution with mean 300 copies and standard deviation 20 copies
  2. A discrete pdf defined as

D	200	220	300	320	340
f(D)	0.1	0.2	0.4	0.2	0.1
F(D)	0.1	0.3	0.7	0.9	1.0

An Example II

- **Solution**: $w = 30$, $p = 75$, $s = 5$. So $c_s = p - w = 75 - 30 = 45$, $c_e = w - s = 30 - 5 = 25$. Therefore the service level

$$\text{SL} = \frac{c_s}{c_s + c_e} = \frac{45}{70} = 0.6428$$

An Example I

-
  1. The probability of 0.6428 gives the z-value 0.36. Therefore optimal order $Q^* = 300 + 0.36(20) = 307.2$ or approximately 308 for normal distribution.
  2. Since the service level $0.3 < SL < 0.7$. The optimal order $Q^* = 300$ copies for discrete distribution.

Section 7

Multiple-period stochastic model: model description

Multiple-period stochastic model: model description

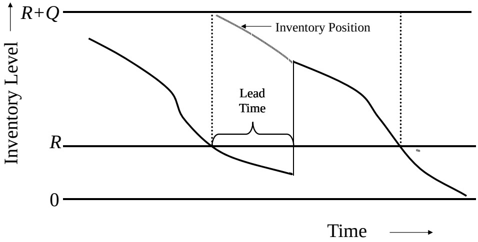
- Previously we consider the Newsvendor model, a single-period stochastic model. Now we consider a stochastic multiple period model.
  - The inventory system operates continuously with many repeating periods or cycles.
  - The lead time for a new order is L days and L is a random variable in general, reflecting the variation of delivery time.
  - The daily demand is D_i $(i = 1, \ldots, L)$ within the lead time, where the $D_i^\prime s$ are also random variables, , reflecting the variation of demand over time.
  - Inventory can be carried from one period to the next.

Multiple-period stochastic model: model description

- New orders can be placed based on one of the following two basic classes of reorder policies:
  - Event-driven: These are reorder policies that are driven by reorder point (ROP)—continuous review policy: in which inventory is reviewed every day and a decision is made about whether and how much to order.
  - Time-driven: These are reorder policies that are driven by time —periodic review policy: in which inventory is reviewed at regular intervals and an appropriate quantity is ordered after each review.

Continuous review policy: (Q, R) order-reorder policy

- (Q, R)-policy: Whenever the **inventory level** reaches the *reorder point* R, place an order of Q to bring the inventory position to the *order-up-to* level $R + Q$; for example Q can be chosen using the EOQ quantity.



- There are two decisions to be made in an (Q, R) order-reorder policy.
  - Decide the reorder level R—when-to-order.
  - Decide the reorder quantity Q and hence the order-up-to level—how much to-order.
- These two decisions should be made based on particular applications. Here we consider one situation where we can decide both quantities.

A situation where we can decide s and Q I

- The lead time L is a random variable.
- Daily demand D_i is random variable, $i = 1, \ldots, L$.
- The demand during lead time $D_L = \sum_{i=1}^{L} D_i$ follows a normal distribution, whose expectation and variance can be calculated similarly as before:

$$\begin{aligned}
\mathbb{E}[D_L] = \mathbb{E}\left[\sum_{i=1}^{L} D_i\right] &= \mu_L \mu_D \\
\mathbb{V}[D_L] = \mathbb{V}\left[\sum_{i=1}^{L} D_i\right] &= \mu_L \sigma_D^2 + \sigma_L^2 \mu_D^2
\end{aligned}$$

- There is an inventory holding cost of c_h per unit per day.
- There is a fixed order cost c_o per unit.
- no backorders: when there is a stockout, the order is lost.

A situation where we can decide s and Q II

- There is a required service level of $1-\alpha$. That is, the probability of no-stockout during lead time is $1-\alpha$

Solution

- Under the about conditions, we have

$$
\begin{aligned}
R &= \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} \\
Q &= \sqrt{\frac{2\mu_D c_o}{c_h}}
\end{aligned}
$$

An Example I

- Suppose the distributor of the TV sets is trying to set inventory policies at the warehouse for one the TV models. Assume that whenever the distributor places an order for TV sets, there is fixed ordering cost of $4500, which is independent of the order size. The cost of a TV set to the distributor is $250 and annual inventory holding cost is about 18 percent of the product cost. Lead time for a new order is about 2 weeks. The table below shows the number of TV sets sold in the last 12 months. Suppose the distributor would like to ensure 97% service level. What is the reorder level and the order-up-to level that the distributor should use?

months	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.
sales	200	152	100	221	287	176	151	198	246	309	98	156

Solution

1. The lead time $L = 2$ weeks. So $\mu_L = 2$ and $\sigma_L = 0$.
2. Based on the historical date, we can calculate the average monthly demand is 191.17 and the standard deviation of monthly demand is 66.53. To make the units consistent, we transform months to weeks, assuming 1 month $\approx$ 4.3 weeks:

$$\begin{aligned} \mu_D = \text{average weekly demand} &= \frac{\text{average monthly demand}}{4.3} \\ &= \frac{191.17}{4.3} = 44.58 \\ \sigma_D = \text{weekly standard deviation} &= \frac{\text{monthly standard deviation}}{\sqrt{4.3}} \\ &= \frac{191.17}{\sqrt{4.3}} = 32.05 \end{aligned}$$

3. The service level is $1 - \alpha = 0.97$ implying $z_\alpha = 1.88$ from the normal table.

1. The fixed ordering cost of $c_o = \$4500$.
2. Annual inventory holding cost per unit is $0.18(250)$ implying a weekly inventory holding cost per unit (assuming 1 year ≈ 52 weeks.

$$c_h = \frac{0.18(250)}{52} = 0.87$$

3. Now we are ready to decide both R and Q.

$$\begin{aligned} R &= &= \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} = 176 \\ Q &= &\sqrt{\frac{2\mu_D c_o}{c_h}} = 679 \end{aligned}$$

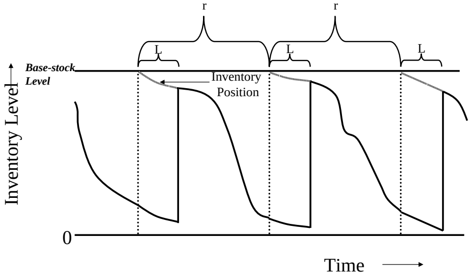
4. And hence the order-up-to level $R + Q = 176 + 679 = 855$.
5. To summarize, the distributor should place an order to raise the inventory position to 855 TV sets whenever the inventory level is below or at 176 units.

Periodic review policy: (s, S) policy I

- The periodic review policy is similar to the continuous review policy except that the former is triggered by time, which causes two major differences
  1. The fixed order cost plays no role here, as presumably the fixed cost is used to determine the cycle time. This implies there is only decision to make: how much to order. Usually we place an order that brings us to a desired *replenishment* level (the order-up-to-level point, or base-stock level).
  2. The periodic review policy must have stockout protection until the next order arrives, which is an order interval plus a lead time away, while the continuous review policy needs protection only during the lead time. Therefore the safety-stock is usually higher in periodic review policy.

Periodic review policy: (s, S) policy II

3. Therefore, at the time of the order, this order must raise the IP (inventory position) to the base-stock level. This level of IP should be enough to protect the warehouse against shortages until the next order arrives, which is after $r + L$ days, and hence the current order should be enough to cover the demand during a period of $r + L$ (or said differently, the IP should be enough to to cover the demand during the lead time and the order period).
4. The following figure explains the difference.



Periodic review policy: (s, S) policy III

Periodic review policy

- Periodic review policy: Let r be the review period (or order interval). For every period, place an order to bring the inventory to some desired replenishment position S.
- There is only one decision: the base-stock level to be raised: S.

A situation where we can decide S

- Review period is r, a deterministic number.
- The lead time L is a random variable.
- Daily demand D_i is random variable, $i = 1, \ldots, L$.
- The demand during lead time plus review period $D_L = \sum_{i=1}^{L+r} D_i$ follows a normal distribution, whose expectation and variance can be calculated similarly as before:

$$\begin{aligned}
\mathbb{E}[D_{L+r}] = \mathbb{E}\left[\sum_{i=1}^{L+r} D_i\right] &= (r + \mu_L)\mu_D \\
\mathbb{V}[D_{L+r}] = \mathbb{V}\left[\sum_{i=1}^{L+r} D_i\right] &= (r + \mu_L)\sigma_D^2 + \sigma_L^2\mu_D^2
\end{aligned}$$

- no backorders: when there is a stockout, the order is lost.
- There is a required service level of $1 - \alpha$. That is, the probability of no-stockout during lead time is $1 - \alpha$

Solution

- Under the about conditions, we have

$$S = \mu_D(\mu_L + r) + z_\alpha \sqrt{(\mu_L + r)\sigma_D^2 + \mu_D^2 \sigma_L^2}$$

An Example I

- Given the following information:
  - The distributor has historically observed weekly demand of: $\mu_D = 30$, $\sigma_D = 3$.
  - Replenishment lead time is $L = 2$ weeks.
  - Desired service level $1 - \alpha = 0.99$.
  - Review period is $r = 7$ weeks.

  Find the replenishment level.
- **Solution**:

$$
\begin{aligned}
S &= \mu_D(r + \mu_L) + z_\alpha \sqrt{(r + \mu_L)\sigma_D^2 + \mu_D^2 \sigma_L^2} \\
&= 30(2 + 7) + 2.33\sqrt{(2 + 7)3^2} = 291
\end{aligned}
$$

Section 8

Managing inventory in the supply chain

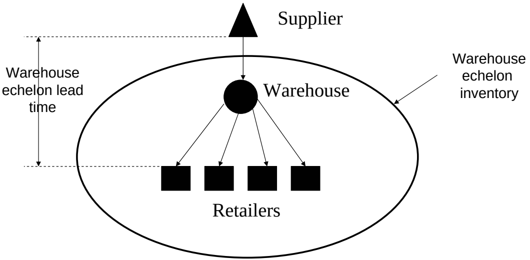
Managing inventory in the supply chain I

- Up to now, we only consider the single-facility managing its inventory in order to minimize its own cost as much as possible. However, in supply chain management, there are always multi-facility. We consider a multi-facility supply chain that satisfies the following requirements (for the purpose, consider a retail distribution system with a single warehouse serving a number of retailers):
  1. Inventory decisions are under centralized control by a decision-maker whose objective is to minimize systemwide cost.
  2. The decision-maker has access to inventory information across the supply chain.

Managing inventory in the supply chain II

- We introduce the concept of *echelon inventory*. In a distribution system, each stage or level (i.e., warehouses or retailers) is referred to as an echelon. The echelon inventory at each level of the system is equal to the inventory on hand at that echelon, plus all downstream inventory. For example, the echelon inventory at the warehouse is equal to the inventory a the warehouse, plus all inventory in transit to and in stock at the retailers.

The warehouse echelon inventory



- Based on the concept of echelon inventory, we suggest the following practical policy for managing the single warehouse multi-retailer system:

  Step 1. The individual retailers are managed using the the (Q, R)-policy based on its own inventory. Specifically, the ROP is calculated as follows for each retailer:

$$\begin{aligned}\text{ROP} &= \mathbb{E}\left(\sum_{i=1}^{L} D_i\right) + z_\alpha \sqrt{\mathbb{V}\left(\sum_{i=1}^{L} D_i\right)} \\ &= \mu_D \mu_L + z_\alpha \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2}\end{aligned}$$

Step 2. The warehouse also use the (Q, R) policy based on the echelon inventory at the warehouse. Specifically, the ROP is calculated as follows for the warehouse:

$$\begin{aligned} \text{ROP} &= \mathbb{E}\left(\sum_{i=1}^{L^e} D_i^e\right) + z_\alpha \sqrt{\mathbb{V}\left(\sum_{i=1}^{L^e} D_i^e\right)} \\ &= \mu_{D^e}\mu_{L^e} + z_\alpha \sqrt{\mu_{L^e}\sigma_{D^e}^2 + \mu_{D^e}^2\sigma_{L^e}^2} \end{aligned}$$

where L^e = echelon lead time = lead time between the retailers and the warehouse plus the lead time between the warehouse and its supplier; and D^e = demand across all retailers.

Practical issues

The top seven effective inventory reduction strategies by managers (from a recent survey) are

1. Periodic inventory review: this policy makes it possible to identify slow-moving and obsolete products and allow management to continuously reduce inventory levels.
2. Tight management of usage rates, lead times and safety stock. This allows the firm to make sure inventory is kept at the appropriate level.
3. Reduce safety stock levels: such as reducing lead time.
4. Introduce/enhance cycle counting practice: count more frequently than once every year—keep posted all the time!

1. ABC approach: Set up priorities for different items.
    - Class A: high-revenue products: 80% annual sales and 20% inventory SKU's: periodic review with shorter review time.
    - Class B: medium-revenue products: 15% annual sales and 75% inventory SKU's: periodic review with longer review time.
    - Class C: low-revenue products: 5% annual sales and 5% inventory SKU's: either no inventory or a high inventory of inexpensive products.
    - Shift more inventory or inventory ownership to suppliers
    - Quantitative approaches: such as the models we discussed earlier.