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Aerodynamic Flow Control

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1 Introduction	1
2 Scaling Parameters	2
3 Scope of AFC Applications	4
4 Computational Approaches to AFC	6
5 Conclusions and Outlook	9
List of Symbols	9
References	10

1 INTRODUCTION

Flow separation and its control dominated the thoughts of Prandtl when he formulated the boundary layer theory (see Tollmien, 1955). In developing the basic theory, he simultaneously demonstrated that flow separation is delayed by applying suction through a narrow slot cut through the surface of a cylinder. Although slot suction acted as a catalyst for future developments, research on slot suction for lift enhancement waned because the quest for high-speed flight necessitated thin wing sections. Nevertheless, suction through pores in the surface proved to be valuable in stabilizing the boundary layer and delaying transition to turbulence (e.g., Bussmann and Muenz, 1942). The potential for the practical application of boundary layer control (BLC) was demonstrated by blowing highly-pressurized air, conveyed

via relatively small diameter pipes (Baumann, 1921). The advent of jet propulsion and lift augmentation requirements of high-speed military jets catapulted blowing to the forefront of BLC in the mid-1950s. The massively produced Lockheed F-104 and the Mikoyan Mig-21 both used blowing over the surface of their flaps for landing. Although integration of propulsion with aerodynamics did not proceed beyond the laboratory stage, it gave rise to new concepts such as super-circulation and the “jet-flap”. The apogee of active BLC research was reached before the appearance of “Sputnik” whereupon a changeover of focus to space research had occurred. The vast scope of international research conducted during the first half-century is summarized in the classic volumes edited by Lachmann (1961).

After a partial hiatus, boundary layer control or, more currently, active flow control (AFC) re-emerged as a research area with wide application to aerodynamics. This interest was re-ignited as a result of two main factors: the one physical and the other technological. The physical driver was that separated flows could be manipulated and attached to surfaces, with attendant increases in lift, by means of periodic zero mass-flux (ZMF) perturbations whose input momentum was a small fraction of that required by steady blowing (e.g., Nishri and Wygnanski, 1998; Seifert, Darabi and Wygnanski, 1996). Perturbations could be introduced via a narrow slot or any other device, which is capable of producing an oscillatory momentum source. The efficacy of this method was explained by the fact that a boundary layer flow on the verge of separation contains an inflected velocity profile that is susceptible to periodic perturbations and amplifies them (see Flow Instabilities and Transition). This creates an array of spanwise vortices that periodically

sweep the surface transferring streamwise momentum from the outer flow to the surface, thus enabling the boundary layer to remain attached and overcome larger adverse pressure gradients (Darabi, 2002). This is in contrast to traditional BLC where suction removes the low-momentum fluid from the boundary layer while blowing re-energizes the same boundary layer. Spanwise-uniform periodic excitation exploits the instability of the boundary layer that amplifies the input, thereby leveraging its effects.

The technological driver and challenge behind AFC developments was, and still is, the ability to deploy autonomous actuators and sensors within, or attached to, aerodynamic surfaces. These are sometimes called micro-electro-mechanical systems (MEMS), but the most common length scales are $O(10^{-3})$ to $O(10^{-1})$ m. Two of the most common actuator types are fluidic actuators (e.g., ZMF, Nagib *et al.*, 2006) that are deployed behind the slot to produce oscillatory perturbations, and surface-mounted mechanical oscillators (e.g., Viets, Piatt and Ball, 1987; Neuburger and Wagnanski, 1987; Bar-Sever, 1989). Recent decades have witnessed significant advances in zero and nonzero mass-flux devices including piezoelectric (Chen *et al.*, 2000; Glezer and Amitay, 2002) valve-type (Seifert, Darabi and Wagnanski, 1996; Bachar, 2001; Seifert and Pack, 1999) and pulsed combustion or detonation-driven (Crittenden *et al.*, 2001) devices. The same is true for surface-mounted actuators that include piezoelectric (Seifert *et al.*, 1998), plasma-based (Sosa *et al.*, 2006; Post and Corke, 2004), arc filament (Samimy *et al.*, 2004), shape memory alloys (Wlezien *et al.*, 1998), and Lorentz force (Weier and Gerbeth, 2004) actuators. Perturbations produced by the actuators may be small relative to a characteristic velocity or vehicle dimension and thus exploit boundary layer instability; but they may also be much larger and hence “force” the flow, for example, by high-frequency alternating blowing and suction. Modern active flow control encompasses both steady and periodic approaches, as well as combinations of the two.

Our ability to understand and analyze these flows from first principles is severely limited for two reasons. First, the pre-existing turbulent flow complexity is exacerbated by a periodic unsteadiness that is often driven by instability mechanisms. Second, the wide variety of actuation devices listed above can have significantly different effects on the same basic flow field. Thus, theoretical methods (Gaster, Kit and Wagnanski, 1985; Reau and Tumin, 2002) can at best describe only qualitative trends and generally have a limited predictive capability. The main advances to date have been empirical, or semiempirical, and these form the basis of our rather superficial present understanding. Nevertheless, experiments that isolate controlling parameters remain an indispensable approach for advancing our knowledge. On

the other hand, with steady increase in computer power, computational fluid dynamics (CFD) is increasingly becoming a powerful tool for the prediction of the unsteady turbulent flows (Rumsey *et al.*, 2006).

This chapter provides a broad overview of modern active flow control. Passive flow control techniques such as vortex generators and turbulators (Chang, 1970), riblets (Walsh and Anders, 1989), and Gurney flaps (van Dam, Yen and Vijgen, 1999) are not reviewed here. Section 2 deals with the basic assumptions of actuation and includes empirically determined scaling laws. Some representative examples are described in Section 3. Section 4 describes CFD methodologies and their application to AFC, and Section 5 provides conclusions. This chapter does not explicitly address the control of free shear layers, jets, and wakes. A full treatise of these flows can be found in Joslin and Miller (2009).

2 SCALING PARAMETERS

Flow excitation, actuation, or forcing is a critical aspect in transitioning active control from the laboratory to real-world applications. On the one hand, actuators with sufficient authority must be developed that simultaneously provide a net system benefit. On the other hand, the correct location, frequency, orientation, type of actuator, etc., must be determined and here theoretical studies are only partially helpful.

2.1 Steady suction and blowing

Initially, mass-flux was used to assess the efficacy of separation control whenever suction or blowing was employed. This parameter was replaced by C_μ in the case of blowing, because the latter eliminated the dependence of lift on the width of the slot from which blowing emanated (Poisson-Quinton and Lepage, 1961). Large variations in C_μ were investigated ($0 < C_\mu < 1$), but the effects of slot width, or its location, were never systematically analyzed. Practical values of $C_\mu < 0.1$ required the inclusion of mass flow to improve the correlation of the data (Attinello, 1961) whenever the jet and stream velocities were comparable. Neglecting local pressure gradients and compressibility effects suggests the combined parameter $C_\mu[1-(U_\infty/U_j)] = C_\mu - 2C_Q$ where C_Q is a mass flow coefficient. The above-mentioned modification enables the subtraction of a scalar quantity based on mass flow (C_Q) from a vector quantity based on the jet momentum (C_μ), which is appropriate if the inclination of the jet to the oncoming flow is negligible. Since aeronautical applications focused on the reduction of landing speed, the effect of blowing on drag was not considered important and was left

unexplored. The focus on lift enabled many researchers to model the “jet flap” by assuming the flow to be inviscid and to predict a complete thrust recovery that was independent of the inclination of the jet leaving the trailing edge of the airfoil (e.g., Woods, 1958). Such independence was observed experimentally by Davidson (1956) and mostly verified later up to jet deflection angles that were almost orthogonal to the free-stream (Hynes, 1968).

Since many of the older investigations relied on force balance results, it was suggested that the lift increment attained by blowing encompassed two effects that sequentially depend on the level of C_μ . At low levels of C_μ , flow separation was prevented by energizing the boundary layer (providing BLC) and generating a lift increment $\Delta C_L \propto C_\mu$ (Attinello, 1961; Poisson-Quinton and Lepage, 1961). In this case, the jet momentum enabled the generation of a flowfield that could be approximated by a potential flow solution around an airfoil or wing (Williams, 1961). For C_μ exceeding a threshold level $C_{\mu,crit}$, the lift increment is smaller, $\Delta C_L \propto \sqrt{C_\mu}$ due to super-circulation (Spence, 1956). $C_{\mu,crit}$ separating the two flow regimes depends on the flap deflection and its size, but it is also sensitive to wing angle-of-attack (α), thickness, shape, sweep, and aspect ratio. On a typical 25% flap, $C_{\mu,crit} \approx 0.015 \tan(\delta_f)$, where δ_f is the flap deflection and provided $\alpha = 0^\circ$ (Poisson-Quinton and Lepage, 1961). Recent observations (Cerchie *et al.*, 2006) suggest that the assumptions giving rise to $C_{\mu,crit}$ might be erroneous because a wing may increase its lift due to the upstream entrainment of the fluidic actuation, be it blowing, suction, or ZMF forcing, without attaching the flow downstream of the slot.

2.2 Periodic perturbation and forcing

The introduction of frequency as a parameter, with or without a steady mass-flux component, correct amplitude scaling becomes an even more difficult task. Moreover, with the vast and ever growing range of actuation methods, a key task is to establish a common “output” parameter so that their relative effects on the flow or performance parameters of interest can be compared. Generalizing the steady blowing parameter to include a periodic component (see Seifert, Darabi and Wagnanski, 1996) is a common approach and produces the combined momentum coefficient:

$$C_{\mu,tot} = C_\mu + \langle C_\mu \rangle = \frac{J}{q_\infty L} + \frac{\langle J \rangle}{q_\infty L} \quad (1)$$

where J and $\langle J \rangle$ represent the steady and unsteady momentum addition, respectively (e.g., Seifert, Darabi and Wagnanski, 1996; Greenblatt and Wagnanski, 2000). In most cases, the

momentum components cannot be predicted from first principles – although lumped-element or reduced-order modeling is used (e.g., Gallas *et al.*, 2002) – and the actuator must be *calibrated* by directly measuring the velocity field and hence its momentum components. A notable exception is Lorentz force-type actuators (e.g., Weier and Gerbeth, 2004).

Although the above definition can readily be extended to include all devices that produce oscillatory momentum, the parameter $\langle C_\mu \rangle$ is not without its limitations. For example, the suction phase of the oscillatory cycle does not directly add momentum, and calibrations performed under quiescent conditions ($U_\infty = 0$) are not necessarily valid under test conditions ($U_\infty \neq 0$). To date, various experiments have been conducted, which indicate that this parameter is not universally valid, and alternate forms have been suggested (see Section 2.4).

The preferred method of pulsed control is to superimpose net positive or negative steady mass flux onto a nominally zero mass-flux device; certain pulsed valves have the disadvantage that the relative proportion of momentum cannot be varied. Some actuators operate in resonance at frequencies that are very much higher than those required for exciting a useful instability. In these instances, some form of low-frequency modulation is employed and in addition to $\langle C_\mu \rangle$, the duty cycle (% of time that the actuator operates) plays an important role.

2.3 Frequency scaling

When momentum is introduced in an oscillatory manner, the correct frequency scaling parameters are not known without empirical input. In general, different objectives – for example, attaching an otherwise separated flow, the avoidance of separation of an attached flow, or maximization of $C_{L,max}$ or L/D – will result in different numerical values of a selected parameter. The attachment of an initially separated shear layer to an inclined solid surface by means of small amplitude perturbations is directly related to the shear-layer receptivity to the perturbations and their amplitude. Forcefully reattached flow (Figure 1) indicates that the vortices amplified by the instability reinforce and regulate the eddies that would have been shed if the flow would have been separated creating a free mixing layer near the surface (Darabi, 2002).

Nishri and Wagnanski (1998) established that for a fully turbulent upstream boundary layer separating from a generic flap (i.e., an inclined surface) and in the absence of surface curvature, the optimum reduced frequency, that is, requiring the least momentum $\langle C_\mu \rangle_{min}$, was $F^+ \approx 1.3$, where $F^+ \equiv f_e X_{TE} / U_\infty$ (Figure 2). However, the optimum F^+ required to prevent the separation of an initially attached flow was somewhat higher, in the range $2 \leq F^+ \leq 4$.

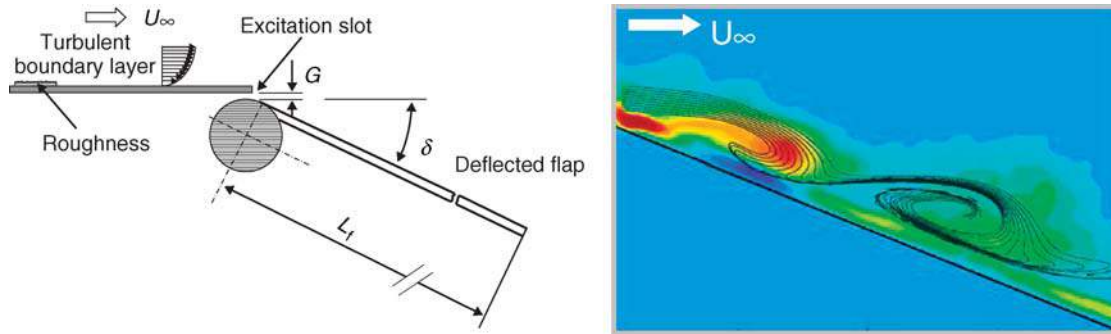


Figure 1. Forcefully reattached boundary layer by periodic excitation emanating from a slot showing a phase-averaged PIV “snapshot” (arbitrary phase). Shaded regions represent spanwise vorticity contours; black curves represent streak lines. Reproduced from Darabi (2002).

2.4 Extended parameter range and comparison of techniques

A number of different parameters have been proposed and assessed for the characterization of airfoil performance, usually lift coefficient. Nagib *et al.* (2006) conducted experiments on a highly deflected simple flap and concluded that the velocity ratio is a more appropriate parameter. They combined this with Strouhal number to form the new parameter $H = (U_j/U_\infty)/\sqrt{fL/U_\infty}$. Stalnov and Seifert (2008) considered five different scaling parameters for control using high-frequency and pulsed actuation and concluded that the Reynolds number-scaled momentum coefficient provided the best scaling for their data set.

A thick elliptical airfoil was tested by Cerchie *et al.* (2006), where slot thickness and location could be widely varied and AFC applied at both the leading and trailing edges

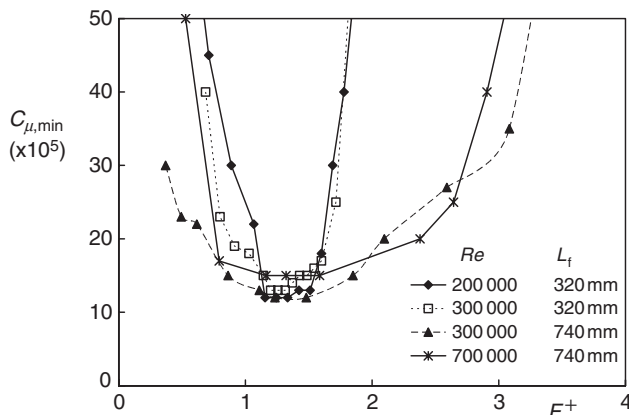


Figure 2. The minimum momentum coefficient required to attach an otherwise separated shear layer to a deflected surface as a function of reduced frequency. Reproduced with permission from Nishri and Wynanski (1998) © A. Darabi, 1998.

(Figure 3). The results of aft control (Figure 4) represent the dependence of ΔC_L on the product of the blowing velocity ratio and the slot width (h) for a given slot location and orientation at $\alpha = 0^\circ$. When the exponent $\beta = 1$, the abscissa $(U_j/U_\infty)^2(h/c)^\beta$ is equivalent to C_μ . The exponent β depends on h/c and is approximated by a polynomial function. For $h/c < 0.005$, the parameter is approximately equal to C_μ , and the significance of the slot width increases for $h/c > 0.005$. The flow is independent of Reynolds number, and the correlation renders a reasonable dependence of ΔC_L on the blowing parameter, although for $(U_j/U_\infty)^2(h/c)^\beta > 0.03$, there is considerable scatter. The deleterious effect of blowing on lift for $(U_j/U_\infty)^2(h/c)^\beta < 0.003$ should also be noted (see the upper inset). Good correlations were achieved for pitching moment (not shown), but drag data were seen to be highly dependent on the slot width. Steady suction generates lift at very low levels of mass flow without the deleterious effects observed in conjunction with steady blowing, and the lift coefficient data correlated well with the empirical parameter $C_Q(h/c)^\gamma Re^{0.4}$, where $\gamma = f(h/R)$. When zero mass-flux forcing was employed, a correlation was discerned between lift coefficient and the empirical parameter $\langle C_\mu \rangle (\theta F^+)^{0.5}$. Leading-edge steady blowing and suction were also considered. Blowing required large momentum coefficients ($C_\mu \approx 0.1\%$) to be effective while suction always produced a benefit; for example, at $C_\mu = 0.018\%$, stall was not observed up to $\alpha = 25^\circ$ with $C_{L,max}$ exceeding 2.

3 SCOPE OF AFC APPLICATIONS

An overwhelmingly large emphasis has been placed on two-dimensional airfoil studies, with the objective of producing “simplified high-lift” (i.e., use of simple flaps/slats along with AFC, rather than complex Fowler flaps/slats). When the

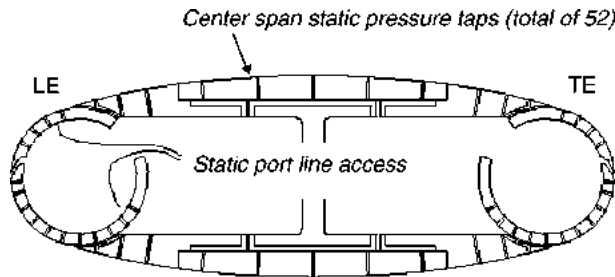


Figure 3. An elliptical airfoil designed to test the effects of various parameters used in active separation and circulation control. Reproduced with permission from Cerchie *et al.* (2006) © AIAA.

parameter F^+ is used, the vast majority of investigations citing effective control produce $0.3 \leq F^+ \leq 4$ at conventional low Reynolds numbers ($Re < 10^6$), provided an amplitude corresponding to $\langle C_\mu \rangle \sim 0.1\%$ is exceeded (Greenblatt and Wygnanski, 2000). It is generally accepted that the maximum lift attainable on airfoils and wings will depend on preventing separation from an initially attached boundary layer, and additional parameters such as boundary layer state and curvature are important. The majority of airfoil data indicate that maximum increases generally occur at lower reduced frequencies, typically at $0.5 < F^+ < 1$ (see Figure 5).

Streamwise curvature can have a profound effect on the efficacy of control, as well as the range of optimum frequencies and amplitudes. Perturbations introduced on a curved surface may be amplified by Kelvin–Helmholtz and Görtler

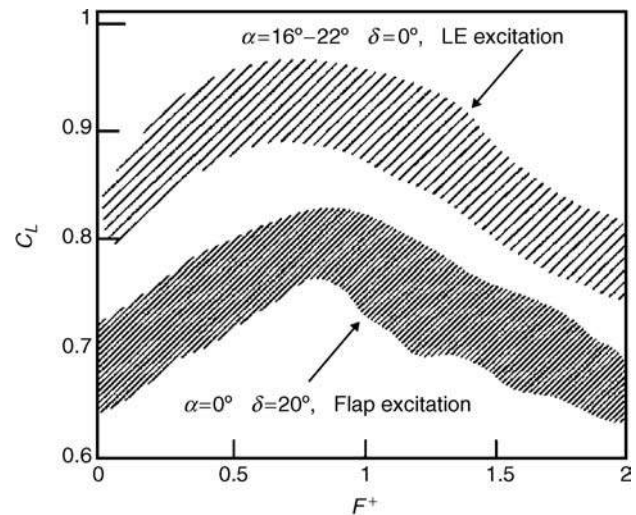


Figure 5. Lift coefficient as a function of reduced frequency on a NACA 0015 airfoil with ZMF slot and a flap at 75% chord deflected at angle δ ($300\,000 \leq Re \leq 600\,000$). Modified from Seifert, Darabi and Wygnanski (1996).

mechanisms simultaneously, resulting in a corrugated spanwise vortex structure (Neuendorf and Wygnanski, 1999). On airfoils, the leading-edge radius can place limitations on control strategies and significantly affect the leading parameters (Greenblatt and Wygnanski, 2003). The relatively “simple” classical problem of a cylinder placed in cross flow introduces additional parameters because the separation line is not

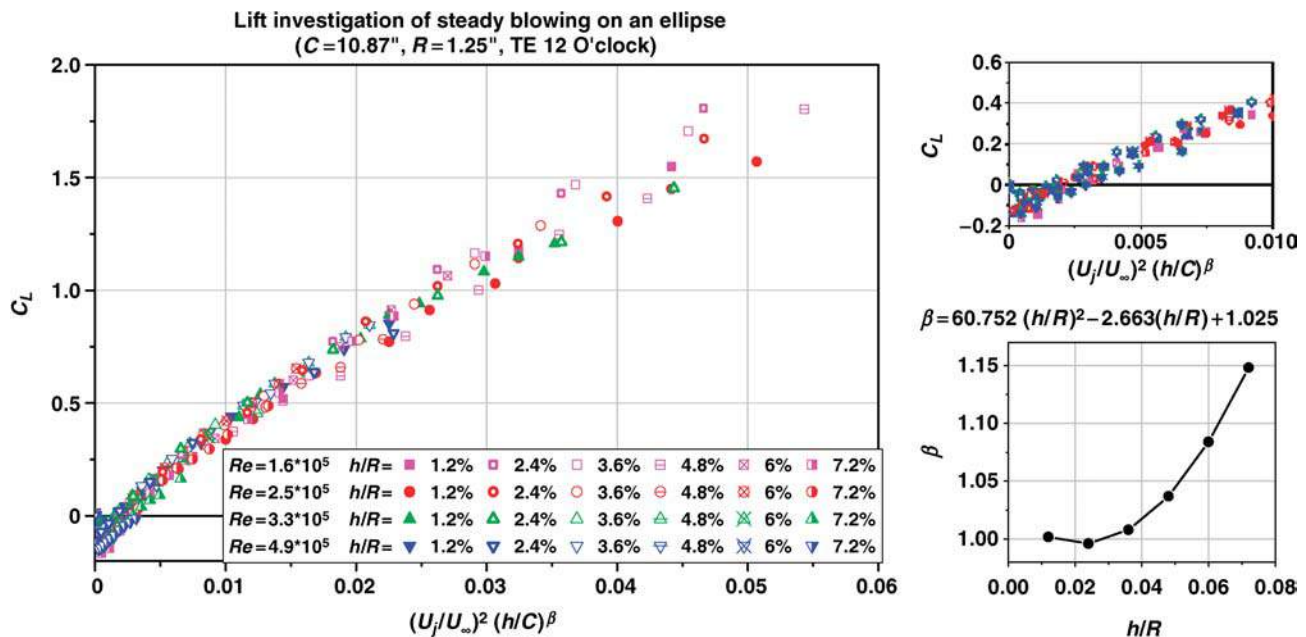


Figure 4. Empirical correlation of the lift generated by steady blowing from the trailing-edge region at $\alpha = 0^\circ$; Re and slot width were varied. Reproduced with permission from Cerchie *et al.* (2006) © AIAA.

precisely (geometrically) determined, it is flow-state dependent and multiple instabilities are present simultaneously (Naim *et al.*, 2007).

AFC has been demonstrated for a larger Reynolds number range, namely, $3 \times 10^3 \leq Re \leq 4 \times 10^7$, under significant compressibility effects $Ma \geq 0.55$, and in flight tests. At Reynolds numbers below 5×10^5 , where transition does not occur naturally and cannot be passively forced, active separation control may be the only effective method for delaying separation and generating useful lift. Dielectric barrier discharge plasma actuation has been demonstrated at Reynolds numbers as low as 3×10^3 (Greenblatt *et al.*, 2008). At the other end of the scale, periodic perturbations are effective up to Reynolds numbers of 4×10^7 and are essentially independent of Reynolds number for $Re > 8 \times 10^6$ (Figure 6; Seifert and Pack, 1999). The effects of compressibility in the absence of shocks are weak, and undesirable effects accompanying separation, such as vortex-shedding and buffet, can be significantly reduced or completely eliminated (Seifert and Pack, 2001). Separation resulting from shock wave/boundary-layer interaction can be ameliorated, providing that excitation is introduced upstream of separation.

Significant work has also been performed on three-dimensional flows, where standard sweep laws are appropriate for an infinitely swept cylinder (Naveh *et al.*, 1998). On finite unswept wings, the effect of control is

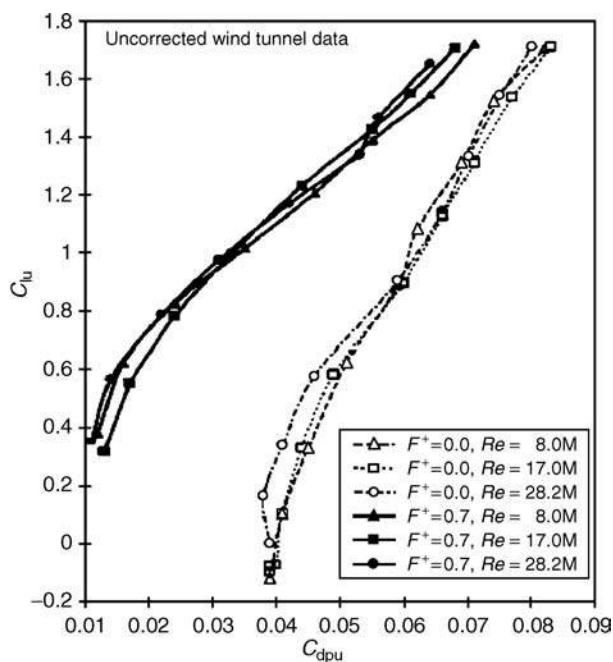


Figure 6. Effect of control on a NACA 0015 airfoil for $Ma = 0.2$ and $(C_{\mu}) = 0.05\%$. Uncorrected (subscript u) wind tunnel data. Reproduced with permission from Seifert and Pack (1999) © AIAA.

approximately uniform across the wingspan but remains effective to high angles of attack only near the tip. When sweep is introduced, a significant effect is noted inboard, but this effect degrades along the span and produces virtually no meaningful lift enhancement near the tip, irrespective of the tip configuration (Greenblatt and Washburn, 2008). In the former case, control strengthens the wing tip vortex; in the latter case, a simple semiempirical model, based on the trajectory or “streamline” of the evolving perturbation, served to explain the observations. In the absence of sweep, control on finite-span flaps is slightly more effective near the flap edges (Greenblatt and Washburn, 2008; Kiedaisch, Nagib and Demanett, 2006). Control over a highly deflected tip flap (40°) produced dramatically larger loads on the flap consistent with a strong vortex rolling up over the flap edge. This effect, combined with negligible changes to the loads upstream of the flap, has the potential to produce significant increases in yawing moments or controlled aerodynamic braking (Greenblatt, 2009).

Leading-edge vortices on delta wings at high angles of attack have been a significant research area because they are primarily responsible for the lift generation at low speeds (e.g., Mitchell and Délerly, 2001). As the incidence angle increases, the vortex “breaks down”, that is, it expands into a highly fluctuating structure with flow stagnation in the central part and is associated with loss of lift and unsteady loads. A number of studies have illustrated the profound effect of periodic perturbations on vortex breakdown and the resulting increase in delta wing loads (e.g., Margalit *et al.*, 2005). In general, the data are consistent with those on swept infinite cylinders, but the mechanism of control has still not been fully elucidated.

The above discussion relates to “time-invariant” active control, that is to say that the flow remains attached in a time-mean sense. The process by which the flow separates from, or attaches to, a surface is important when we want to control a process whose characteristic timescale is much larger than the typical period of eddy passage when periodic actuation is applied: $O(X_{sep}/U_\infty)$ (see Darabi and Wignanski, 2004a, 2004b). Typical examples include the response of vehicle control-surface flow (Amitay *et al.*, 2004), dynamic stall control (Greenblatt, Neuburger and Wignanski, 2001), and control of wake vortices (Greenblatt *et al.*, 2005).

4 COMPUTATIONAL APPROACHES TO AFC

A great deal of computational work has been performed during the past 20 years or so in the field of AFC, with much

of it accomplished using numerical solutions to forms of the Navier–Stokes equations. A brief overview of the main methodologies is given here, along with a few examples of applications in the literature.

4.1 CFD methodologies

In direct numerical simulation (DNS), the Navier–Stokes equations are solved directly: all scales of motion are resolved, and there is no modeling of turbulence. DNS requires that the grid should be fine enough to resolve features on the order of the Kolmogorov dissipation length scale $\eta = (v^3/\varepsilon)^{1/4}$, and the time step needs to be fine enough to resolve motions whose time scales are on the order of $\tau = (v/\varepsilon)^{1/2}$. The simulation also has to be run long enough so that the time and phase-averaged properties become temporally converged, and the numerics must be accurate enough (e.g., high-order, pseudo-spectral, or spectral) so that excessive dissipation does not corrupt the small-scale resolved features. Even with today's computers, it is impossible to achieve these resolutions at reasonably high Reynolds numbers, particularly for complex geometries. However, it is relatively common to perform “under-resolved” or “coarse-grid” DNS, with the justification that the larger resolved scales have the major influence in the flow control problem of interest. The disadvantage of this method is that it is difficult to assess the influence of under-resolving the finer scales.

To date, there have been only a limited number of applications of DNS or coarse-grid DNS to AFC applications (e.g., Barwoff, Wengle and Geggale, 1996; Wengle *et al.*, 2001; Postl and Fasel, 2006; Kotapati, Mittal and Cattafesta, 2007). When performing flow control computations with DNS, most researchers simplify the problem in the spanwise direction by solving over a finite slice, with periodic boundary conditions. The issue of inflow boundary conditions or initial conditions (ensuring turbulent flow) can be problematic; this is sometimes handled via an additional source term added to the equations or by inserting geometric features or blowing/suction to trip the flow.

In large eddy simulation (LES), a low-pass filter is applied to the Navier–Stokes equations. As a result, two unknown quantities emerge that must be modeled: the subgrid scale stress τ_{ik} and the subgrid scale heat flux Q_k . A commonly used explicit subgrid model is the Smagorinsky model (Smagorinsky, 1963), for which the turbulent viscosity that goes into the computation of τ_{ik} and Q_k is given by $\nu_t = (c_s \bar{\Delta})^2 |\bar{S}|$, where $|\bar{S}| = \sqrt{2\bar{S}_{ik}\bar{S}_{ik}}$, $\bar{S}_{ik} = (\partial u_i/\partial x_k + \partial u_k/\partial x_i)/2$, and $\bar{\Delta}$ is typically defined by some average measure of the local grid spacing. Because the

optimum filter width has been shown to be flow dependent, a so-called dynamic methodology is used, where c_s is a variable rather than a constant (Germano *et al.*, 1991). Although explicit modeling of the subgrid scale effects is very common, the method of implicit LES, or ILES, has emerged over the last several years as a popular alternative. With ILES, no explicit modeling is employed. Instead, the numerical method is selected such that the numerical error fulfills desired properties and effectively acts like a subgrid model (Grinstein and Fureby, 2007).

LES methodologies have been applied successfully to AFC applications (Dandois, Garnier and Sagaut, 2006a, 2006b; Dejoan and Leschziner, 2004; Chang, Collis and Ramakrishnan, 2002; Rizzetta and Visbal, 2003; You, Wang and Moin, 2006). Most used either some form of dynamic subgrid modeling or ILES. As it is with DNS, LES typically requires the use of low-dissipation numerical schemes so as not to excessively smear the flow features of the method it is attempting to resolve. The grid requirements are somewhat less restrictive for LES. As it is with DNS, ensuring turbulent conditions within the computational domain can be problematic.

Although DNS and LES are rapidly becoming usable tools in the CFD arsenal due to increase in computer memory and speed, Reynolds-averaged Navier–Stokes (RANS) methods are still by far the most commonly used to compute flow control problems. With RANS, the flow variables of the Navier–Stokes equations are decomposed into mean and fluctuating components and then time-averaged. It turns out that the RANS equations are identical in form to the spatially filtered equations used for LES. As it is for LES, the unknowns τ_{ik} and Q_k must be modeled. The turbulent stresses can be modeled in many ways, including using second-moment closure modeling, where a transport equation is solved for the turbulent dissipation rate (or related quantity) as well as for each of the six stress components. However, the most common method employed is with linear or nonlinear eddy viscosity models (see Gatski and Rumsey, 2002).

RANS and LES methodologies are derived differently and their variables have different meanings, but from a coding standpoint, they are identical except that the models used for obtaining the turbulent viscosity are very different. RANS turbulence models do not involve a filter width and always have an influence across the entire energy spectrum. Although gross unsteady motions can often be captured with RANS solved time accurately (sometimes referred to as unsteady RANS or URANS), small-scale turbulence flow features are never resolved as the grid is refined because the turbulent viscosity does not go away. This is by design: Reynolds averaging is attempting to represent the mean effects of the turbulence through the additional turbulent viscosity term.

A particular consideration for RANS applied to AFC is its ability to capture unsteady flows. The averaging process could be problematic, but it is generally held that if the timescale of any gross unsteady motion is much greater than the physical time step employed, which in turn is much greater than the time scales associated with the turbulence, then the use of RANS is fully justified (Anderson, Tanehill and Pletcher, 1984). Beyond this issue, given a reasonably accurate numerical scheme of at least second order and fine enough grid and time step, the issue of turbulence modeling always surfaces as a likely reason for discrepancies between CFD and experiment. As mentioned earlier, there are many RANS modeling choices, although arguably the Spalart and Allmaras (1994) one-equation SA model and the Menter (1994) two-equation $k - \omega$ shear-stress transport (SST) turbulence model have become the most widely used over the past 15 years for aerodynamic flow predictions. Another consideration is the difficulty associated with faithfully reproducing experimental boundary conditions and geometric features or irregularities.

Hybrid RANS/LES models take advantage of the fact that the LES and RANS implementations only differ in their treatment of τ_{ik} . It is a relatively simple matter in CFD codes to blend the two different types of models so that the RANS type is active near walls and the LES type is active in wakes and separated regions. For example, in the detached eddy simulation (DES) method (Spalart *et al.*, 1997), the baseline Spalart–Allmaras (SA) one-equation RANS model (Spalart and Allmaras, 1994) merely modifies the distance variable in its destruction term to be $\min(d, c_{des}\Delta)$, where $\Delta = \max(\Delta x, \Delta y, \Delta z)$. When $c_{des}\Delta$ is invoked, the SA equation behaves in a similar manner to the Smagorinsky model, in the sense that ν_t becomes proportional to $\Delta^2 |\bar{S}|$. In spite of its success in many applications, hybrid RANS/LES modeling still has many unresolved issues related to the interface region, gridding, and wall modeling that are being actively debated and researched (Piomelli *et al.*, 2002; Spalart, 2009).

For all of the methods – DNS, LES, RANS, and hybrid RANS/LES – oscillatory blowing and suction flow control is most often achieved via transpiration boundary conditions applied at a wall located either directly on the aerodynamic surface or within an internal duct or slot included in the computational geometry. Limited CFD work has been done with moving or distorting walls (e.g., Xia and Qin, 2006).

4.2 CFD validation for active flow control

A flow control validation workshop conducted in 2004, titled CFDVAL2004, involved three cases: nominally 2D synthetic jet into quiescent flow, 3D circular synthetic jet into turbulent

boundary-layer cross flow, and nominally 2D separation-flow-control over a wall-mounted aerodynamic hump shape. The workshop summary and initial results were published by Rumsey *et al.* (2006), and a follow-up survey on subsequent results for the workshop cases was given by Rumsey (2009). The lessons learned from the workshop and subsequent investigations, mainly regarding separation control, are briefly summarized here.

For both the nominally 2D and 3D circular synthetic jets, the boundary conditions at the slots were shown to play a key role. Correct modeling of the membrane and internal orifice was necessary for capturing the complex nature of the flow field. For the wall-mounted hump case, RANS results consistently predicted too little eddy viscosity in the separation region and consequently yielded too long a separation bubble. An example is shown in Figure 7 for the hump case with synthetic jet (oscillatory control in and out of the slot). Figure 7a shows time-averaged streamlines near the back of the hump model. Results are shown using three different RANS turbulence models, including a nonlinear explicit algebraic stress (EASM-ko) model (Rumsey and Gatski, 2001). All models yielded similar results, with too big a bubble compared to experiment. In Figure 7b, the models' underprediction of turbulent shear stress ($-u'v'$) magnitude in the separated region is evident throughout the blowing/suction cycle (four representative phases are shown). This problem with RANS models also shows up for other cases with separation and remains a key roadblock. If a physics-based correction could be found, it would dramatically improve RANS capabilities not only for AFC cases involving separation, but also for general aerodynamic configurations at off-design conditions.

As described in detail by Rumsey (2009), it turns out that by resolving the large-scale turbulence features in the separated region using hybrid RANS/LES, LES, or under-resolved DNS, much better results can be obtained for this case. (There is one caveat to this conclusion: hybrid RANS/LES models typically have trouble predicting cases with smaller bubbles, because it is difficult to generate enough eddy content quickly enough when transitioning from RANS mode to LES mode in the bubble.) The question now is whether these more expensive simulations can be used to help improve RANS turbulence models in the separated region.

Given that RANS could not predict separation extent correctly for the hump configuration, Rumsey and Greenblatt (2007) conducted an investigation to determine how well the trends could be captured for variations in control magnitude, control frequency, and Reynolds number. For the steady suction type of control, CFD appeared capable of predicting variations due to Reynolds number and suction strength to a reasonable degree of accuracy, but oscillatory control trends were not predicted well.

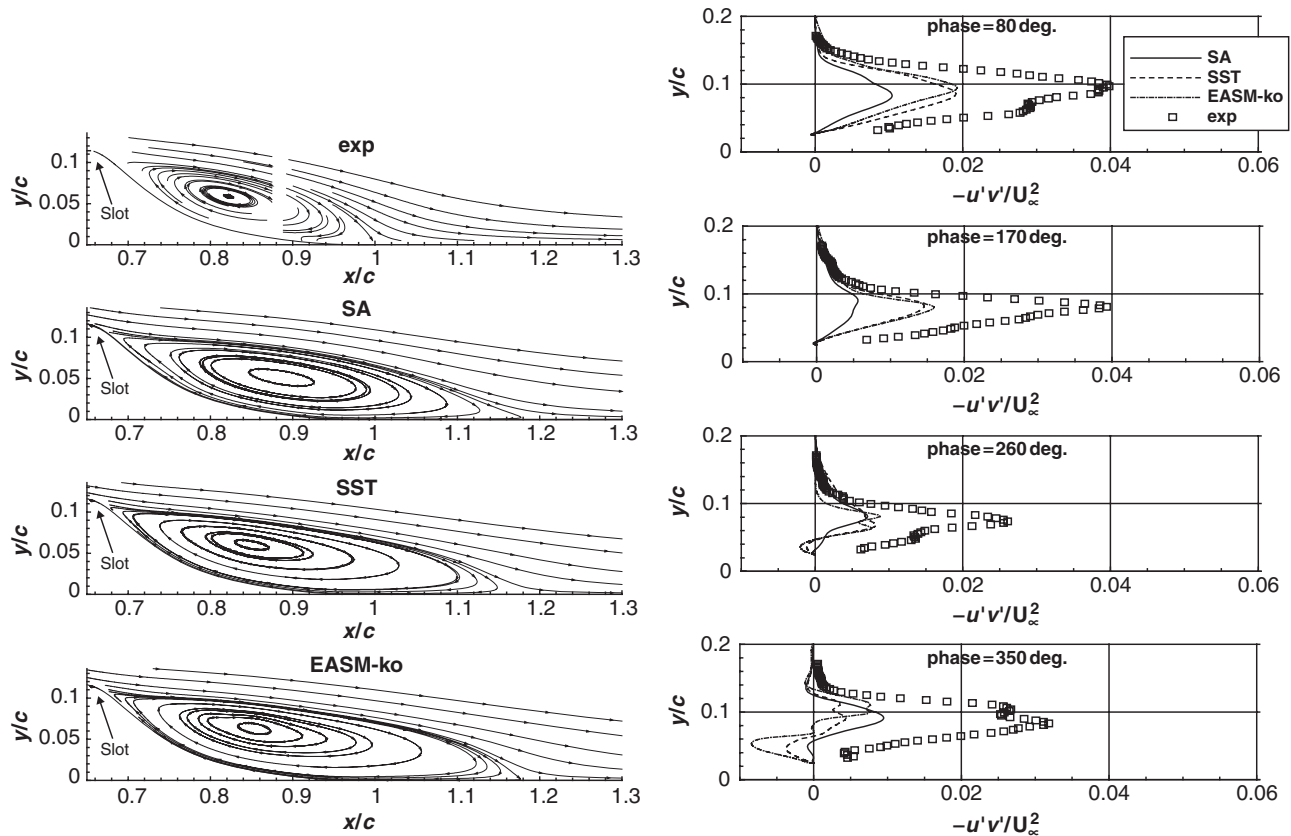


Figure 7. RANS results for the hump model with synthetic jet (a) time-averaged streamlines; (b) turbulent shear-stress profiles in separated region at $x/c = 0.8$. Reproduced with permission from Rumsey (2007).

5 CONCLUSIONS AND OUTLOOK

Active flow control has reemerged as an important research area in aerodynamics. The control of large coherent structures and the mechanical means developed to achieve this have been at the heart of these developments. As such, great potential exists for the effective control across wide Reynolds number and Mach number ranges. Research areas with potentially large payoffs include simplified high-lift systems, three-dimensional configurations, and vortex-dominated flows. The performance of experiments that elucidate new aspects or isolate controlling parameters should be considered a priority. Concurrently, the development of effective and system-efficient actuation methods remains a central challenge to the ultimate success of AFC.

CFD has emerged as the primary tool for prediction of these flows and much remains to be done because the commonly used RANS methods often fail to predict experimental trends. It is expected that the recent progress in the arena of simulation methods such as DNS, LES, and hybrid RANS/LES will continue. However, due to the expense of

these methods, effort should also be expended in improving RANS predictions. Carrying out dedicated experiments intended specifically for CFD validation will be invaluable for future progress.

LIST OF SYMBOLS

c	airfoil chord length
C_Q	slot suction coefficient, hU_s/cU_∞
C_μ	slot mean momentum coefficient, $J/c q_\infty$
$\langle C_\mu \rangle$	slot oscillatory momentum coefficient
f_e	separation control excitation frequency
F^+	reduced excitation frequency, $f_e L_f / U_\infty$
h	slot width
J	time mean slot momentum
$\langle J \rangle$	time mean slot momentum
L	flap length
M_a	Mach number
X_{TE}	distance from slot to trailing-edge
q_∞	freestream dynamic pressure

R	radius of curvature
Re	Reynolds number based on chord-length
U_j	peak jet slot blowing velocity
U_s	slot suction velocity
U_∞	freestream velocity
u_i	mean velocity component
x_i	position vector
β	empirically determined exponent
γ	empirically determined exponent
ν	kinematic molecular viscosity
ε	dissipation per unit mass
θ	momentum thickness
$\langle \rangle$	oscillatory component

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