Lecture I: Basic Physics

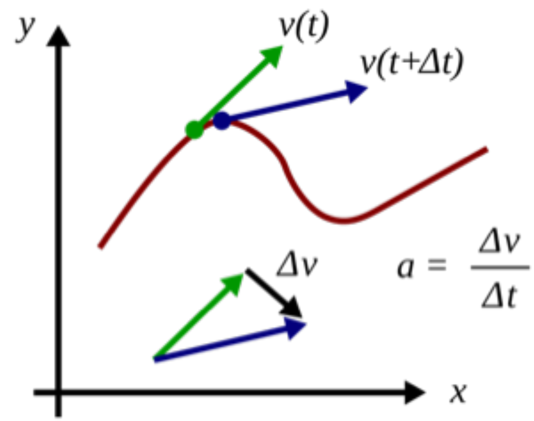
Velocity

- Velocity: Instantaneous change in position $\vec{v} = {}^{d\vec{x}}/_{dt}$
- Suppose object position $\overrightarrow{x_o}$ and **constant** velocity $\vec{v}$. After time step Δt:
  - $\overrightarrow{x_o}(t + \Delta t) = \overrightarrow{x_o}(t) + \vec{v}\Delta t$
  - $\Delta\overrightarrow{x_o} = \overrightarrow{x_o}(t + \Delta t) - \overrightarrow{x_o}(t) = \vec{v}\Delta t.$

- …$\vec{v}$ is never constant in practice
  - A function of time $\vec{v}(t)$.
  - Position is integrated in time: $\overrightarrow{x_o}(t) = \overrightarrow{x_o} + \int_0^t \vec{v}(s)\,ds.$
  - Velocity SI units: ${}^{m}/_{sec}$

Acceleration

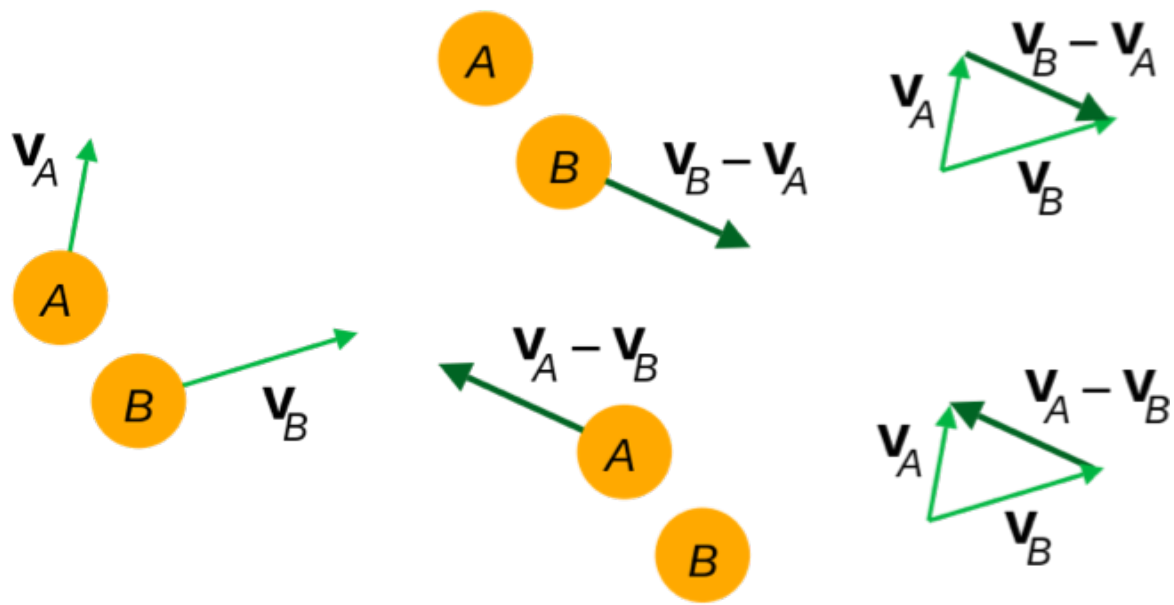
- Instantaneous change in velocity: $\vec{a} = {d\vec{v}}/{dt}$.
  - Constant acceleration: $\Delta\vec{v} = \vec{a}\Delta t$.
  - Otherwise, integrate: $v(t) = \mathrm{V} + \int_0^t a(s)\, ds$.



- Note vector quantities!
  - Position: trajectory of a point.
  - Velocity: tangent to trajectory curve.
    - Speed: absolute value of velocity.
  - Acceleration: the change in the tangent.

Relative Quantities

- Using coordinates, our vector quantities are relative to the chosen axis system (origin + xyz direction)
- They are viewpoint dependent.
- The derivation/integration relations are invariant!

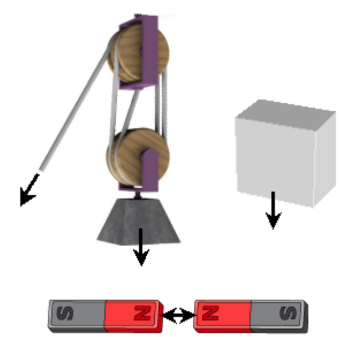


Forces

- Acceleration is induced by a force.

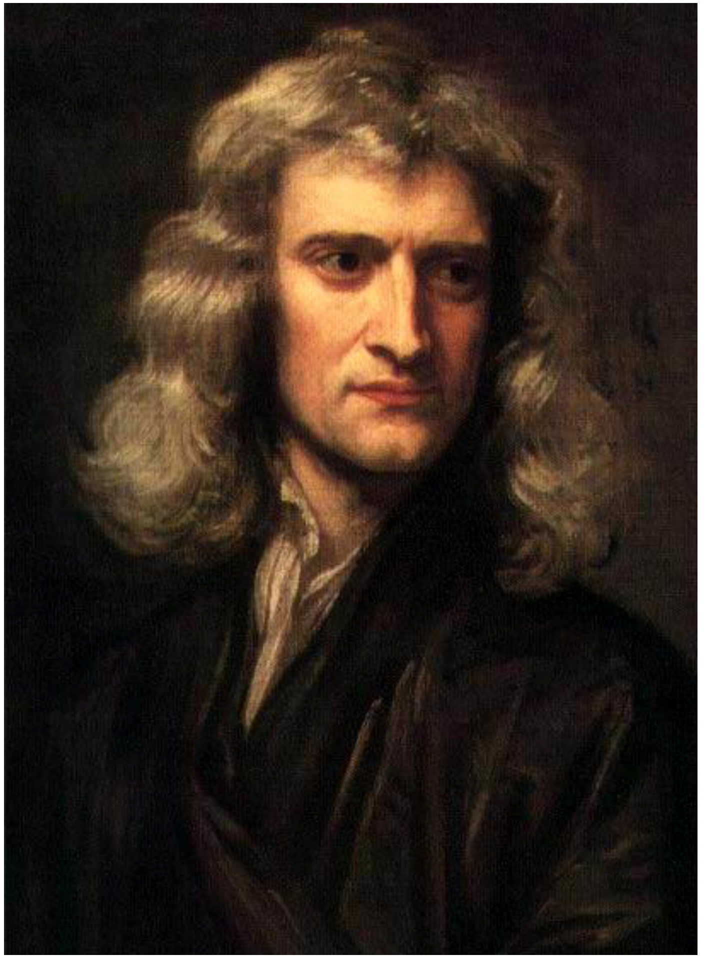
- Direction of force = direction of associated acceleration.

- Net force (and net acceleration): the sum of all acting forces.



Newton's laws of motion

- In the late 17th century, Sir Isaac Newton described three laws that govern all motion on Earth.
- ...ultimately, an approximation
  - Small scale: quantum mechanics.
  - Big scale: theories of relativity.



1^{st} Law of Motion

- Sum of forces on an object is null ⇔ there is no change in the motion

If $F_{net} = 0$, *there is no change in motion*

- With zero force sum:
  - An object at rest stays at rest.
  - A moving object perpetuates in the same velocity.
- Behavior of objects in the outer space.

2^{nd} Law of Motion

- Each force induces a co-directional acceleration in linear to the mass of the object:

$$\vec{F}_{net} = m \cdot \vec{a}$$

m is the mass and $\vec{a}$ the acceleration

- Consequently:
  - More force ⇔ faster speed-up.
  - Same force ⇔ lighter objects accelerate faster than heavy objects.

3rd Law of Motion

- Forces have consequences:

When two objects come into contact, they exert equal and opposite forces upon each other.

- All forces are actually interactions between bodies!

What happens here?

Gravity

- Newton's Law of Gravitation: the gravitation force between two masses A and B is:

$$\vec{F}_g = \vec{F}_{A\to B} = -\vec{F}_{B\to A} = G\,\frac{m_A m_B}{r^2}\,\overrightarrow{u_{AB}}$$

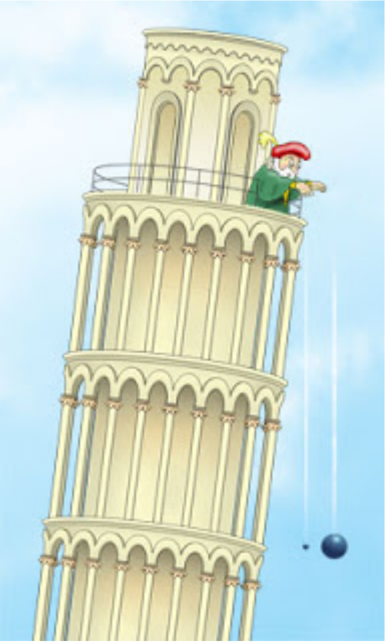
G: gravitational constant $6.673\times 10^{-11}\ [m^3 kg^{-1} s^{-2}]$.

$r = |\vec{p}_A - \vec{p}_B|$: the distance between the objects.

$\vec{u}_{AB} = {\vec{p}_A - \vec{p}_B}/{|\vec{p}_A - \vec{p}_B|}$: the unit direction between them.

Gravity on Earth

- By applying Newton's 2nd law to an object with mass m on the surface of the Earth, we obtain:



http://lannyland.blogspot.co.at/2012/12/10-famous-thought-experiments-that-just.html

$$\vec{F}_{net} = \vec{F}_g = m \cdot \vec{a}$$

$$G\frac{m \cdot m_{Earth}}{{r_{Earth}}^2} = m \cdot a$$

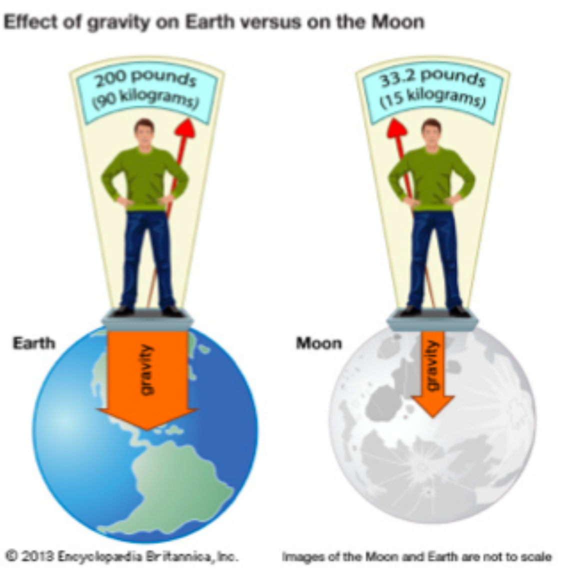
$$G\frac{m_{Earth}}{{r_{Earth}}^2} = a$$

Mass of object is canceled out!

$$a = g_{Earth} = 6.673 \times 10^{-11} \frac{5.98 \times 10^{24}}{(6.377 \times 10^{6})^2} \approx 9.81\, m/s^2$$

Gravity on Other Planets

- On Earth at altitude h: $a = G\frac{m_{Earth}}{(r_{Earth}+h)^2}$
- On the Moon
  - $m_{moon} = 7.35\times10^{22}\ kg$
  - $r_{moon} = 1738\ km$
  - $g_{moon} = 1.62\ m/s^2$
- On Mars
  - $m_{mars} = 6.42\times10^{23}\ kg$
  - $r_{mars} = 3403\ km$
  - $g_{mars} = 3.69\ m/s^2$



Weight

- Weight ⇔ gravitational force

$$\overrightarrow{W} = m \cdot \vec{g}$$

- We weigh different on the moon (but have the same mass…)

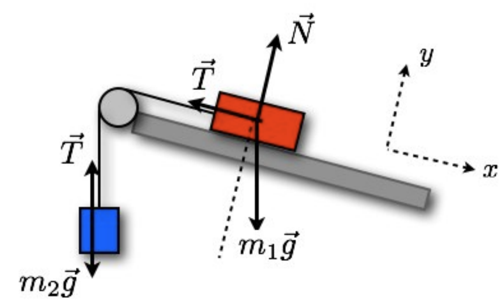
- Force units: $[kg \cdot \frac{m}{sec^2}]$.
  - Denoted as Newtons $[N]$.

Free-Body Diagram

- To get acceleration: sum forces and divide by mass (D'Alembert's principle):

$$\vec{F}_{net} = \sum \vec{F}_i = m \cdot \vec{a}$$

- Forces add up linearly as vectors.
  - Important: when all are represented in the same axis system!
- The Free-Body Diagram includes:
  - Object shape: center of mass, contact points.
  - Applied forces: direction, magnitude, and point of application.



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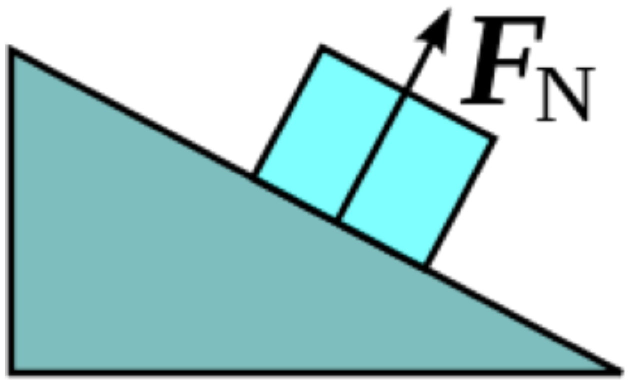
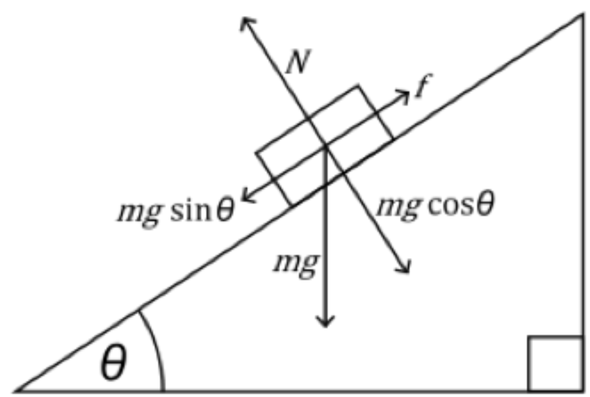
Normal force

- Force acting as a reaction to contact.
  - Direction is normal to the surface of contact.
  - Magnitude enough to cancel the weight so object doesn't go through the plane.

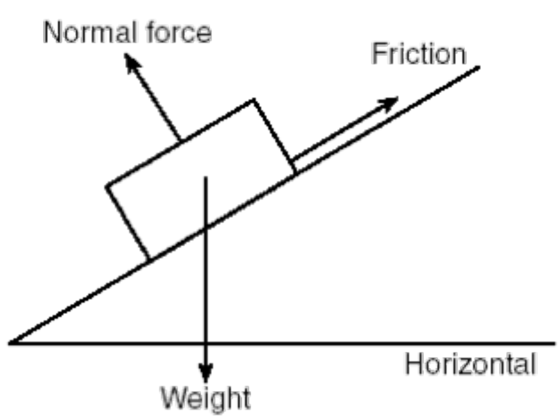


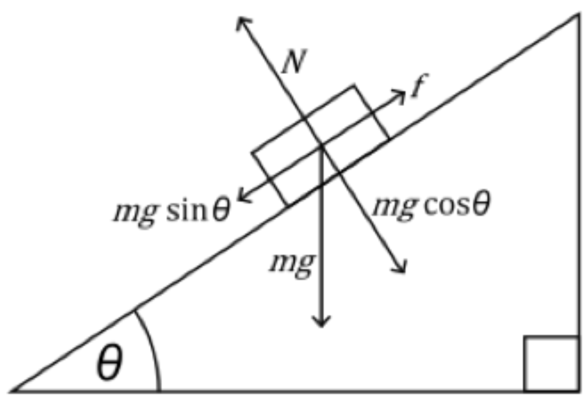


- Here, $\vec{F}_N = \vec{W}\cos(\alpha) = m\vec{g}\cos(\theta)$
- Related to collision handling (more later).
- Object slides down plane with remaining force: $\vec{W}\sin(\theta)$.

Friction

- Can the object stay in total equilibrium?
  - An extra tangential friction force must cancel $\overrightarrow{W}\sin(\theta)$.
- Ability to resist movement.
  - Static friction keeps an object on a surface from moving.
  - Kinetic friction slows down an object in contact.





Friction

- Static friction: a threshold force.
  - object will not move unless tangential force is stronger.

- Kinetic friction: when the object is moving.

- Depends on the materials in contact.
  - smoother ⇔ less friction.

- Coefficient of friction μ determines friction forces:
  - Static friction: $F_s = \mu_s F_N$
  - Kinetic friction: $F_k = \mu_k F_N$

Friction

- The kinetic coefficient of friction is always smaller than the static friction.

- If the tangential force is larger than the static friction, the object moves.

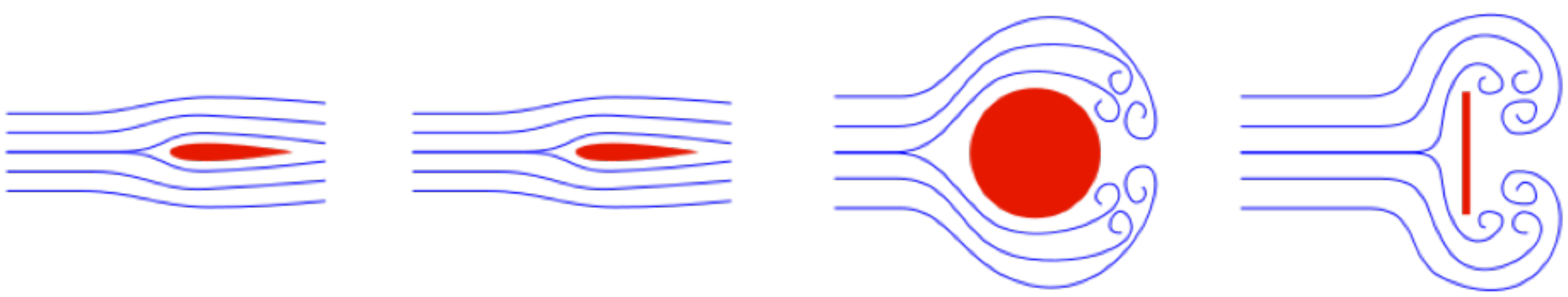
- If the object moves while in contact, the kinetic friction is applied to the object.

Friction

Surface Friction	**Static (μ_s)**	**Kinetic (μ_k)**
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.041	0.04
Brake lining on cast iron	0.4	0.3
Rubber on concrete (dry)	1.0	0.9
Rubber on concrete (wet)	0.30	0.25
Metal on ice	0.022	0.02
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Nickel on nickel	1.1	0.53
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53

Fluid resistance

- An object moving in a fluid (air is a fluid) is slowed down by this fluid.
- This is called fluid resistance, or drag, and depends on several parameters, e.g.:
  - High velocity ⇔ larger resistance.
  - More surface area ⇔ larger resistance (“bad aerodynamics”).



Fluid resistance

- At high velocity, the drag force $F_{D_{high}}$ is quadratic to the relative speed v of the object:

$$F_{D_{high}} = -\frac{1}{2} \cdot \rho \cdot v^2 \cdot C_d \cdot A$$

- ρ is the density of the fluid (1.204 for air at $20℃$)
- C_d is the drag coefficient (depends on the shape of the object).
- A is the reference area (area of the projection of the exposed shape).

Fluid resistance

- At low velocity, the drag force is approximately linearly proportional to the velocity

$$\overrightarrow{F_{D_{low}}} \approx -b \cdot \vec{v}$$

where b depends on the properties of the fluid and the shape of the object.

- High/low velocity threshold is defined by Reynolds Number (Re).

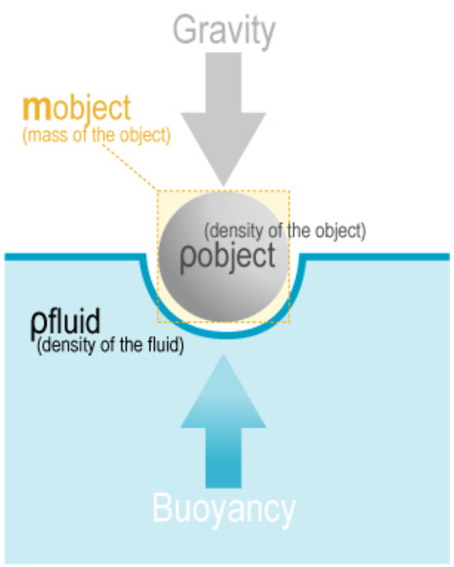
Buoyancy

1.14

- Develops when an object is immersed in a fluid.
- A function of the volume of the object V and the density of the fluid ρ:

$$F_B = \rho \cdot g \cdot V$$

- Considers the difference of pressure above and below the immersed object.
- Directed straight up, counteracting the weight.

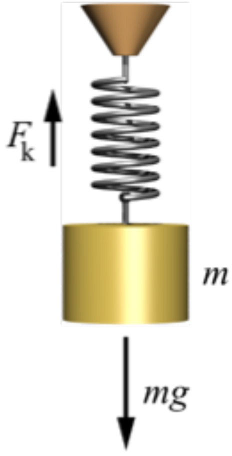


Springs

- React according to *Hook's Law* on extension and compression, *i.e.* on the relative displacement.

- The relative length l to the rest length l_0 determines the applied force:

$$F_k = -K(l - l_0)$$



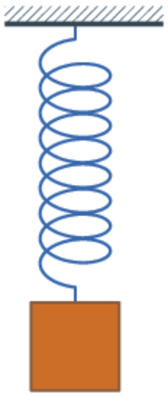
- K is the *spring constant* (in N/m).
- Scalar spring: two directions.

Dampers

- Without interference, objects may oscillate infinitely.
- Dampers slow down the oscillation between objects A and B connected by a spring.
- Opposite to the relative speed between the two objects:

$$\vec{F}_C = -C(\vec{v}_A - \vec{v}_B)$$

  - C is the damping coefficient.
  - Resulting force applied on A (opposite on B).
- Similar to friction or drag at low velocity!



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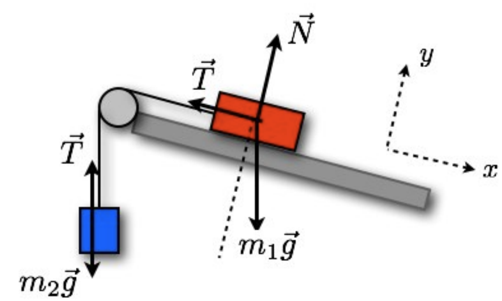

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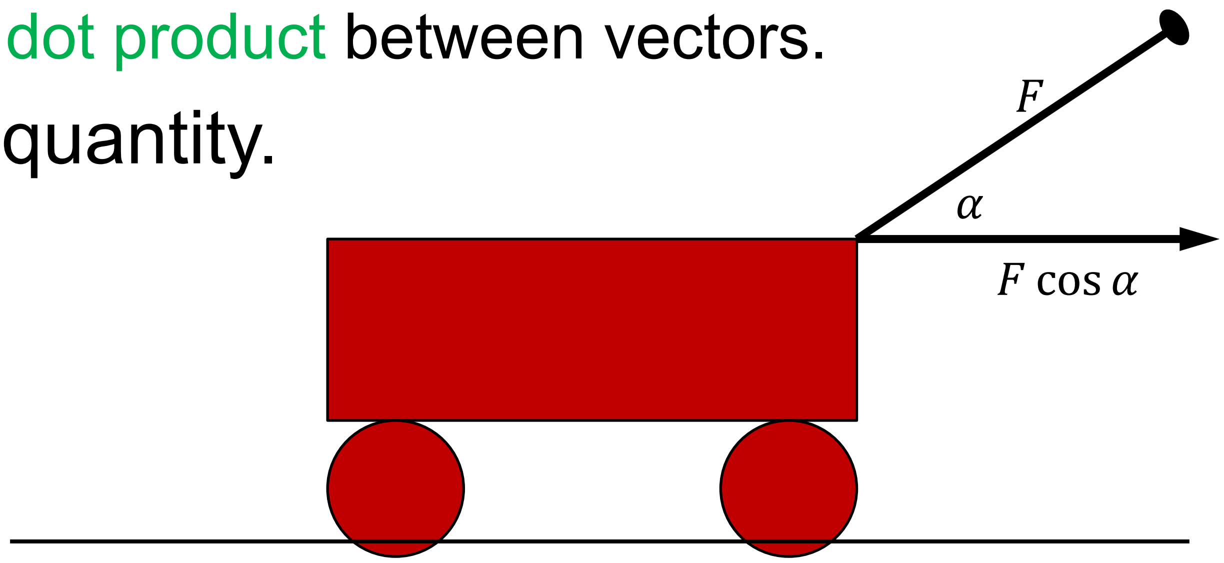
Work

- A force $\vec{F}$ does work W (in $Joule = N \cdot m$), if it achieves a displacement $\Delta\vec{x}$ in the direction of the displacement:

$$W = \vec{F} \cdot \Delta\vec{x}$$

  - Note dot product between vectors.
- Scalar quantity.



Kinetic energy

- The kinetic energy E_K is the energy of an object in velocity:

$$E_K = \frac{1}{2}\, m\, |\vec{v}|^2$$

  - The faster the object is moving, the more energy it has.
  - The energy is a scalar (relative to speed $v = |\vec{v}|$, regardless of direction).
  - Unit is also Joule:

$$kg(m/sec)^2 = \left(kg * \frac{m}{sec^2}\right) m = N * m = J$$

Work-Energy theorem

- The Work-Energy theorem: net work ⇔ change in kinetic energy:

$$W = \Delta E_K = E_K(t + \Delta t) - E_K(t)$$

i.e.

$$\vec{F} \cdot \Delta\vec{x} = \frac{1}{2} m(v(t + \Delta t)^2 - v(t)^2)$$

  - Very similar to Newton's second law...

Potential energy

- (Gravitational) Potential energy is the energy 'stored' in an object due to relative height difference.
  - The amount of work that would be done if we were to set it free.

$$E_P = m \cdot g \cdot h$$

  - Simple product of the weight $W = m \cdot g$ and height h.
  - Also measured in Joules (as here $kg \cdot \frac{m}{sec^2} \cdot m$).

- Other potential energies exist (like a compressed spring).

Conservation of mechanical energy

- Law of conservation: in a closed system, energy cannot be created or destroyed.
  - Energy may switch form.
  - May transfer between objects.
  - Classical example: falling trades potential and kinetic energies.

$$E_K(t+\Delta t) + E_P(t+\Delta t) = E_K(t) + E_P(t)$$

i.e.

$$\frac{1}{2}mv(t+\Delta t)^2 + mgh(t+\Delta t) = \frac{1}{2}mv(t)^2 + mgh(t)$$

Conservation: Example

- A roller-coaster cart at the top of the first hill
  - Much potential energy, but only a little kinetic energy.
  - Going down the drop: losing height, picking up speed.
  - At the bottom: almost all potential energy switched to kinetic, cart is at its maximum speed.

Conservation of Mechanical Energy

- External forces are usually applied:
  - Friction and air resistance.
  - Where does the “reduced” energy go?
  - Converted into heat and air displacements (sound waves, wind).
- We compensate by adding an extra term E_O to the conservation equation:

$$E_K(t+\Delta t) + E_P(t+\Delta t) + E_O = E_K(t) + E_P(t)$$

  - if $E_O > 0$, some energy is ‘lost’.

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Momentum

- The linear momentum $\vec{p}$: the mass of an object multiplied by its velocity:

$$\vec{p} = m * \overrightarrow{v}$$

- Heavier object/higher velocity ⬄ more momentum (more difficult to stop).
- unit is $[kg \cdot {}^{m}/_{sec}]$.
- Vector quantity (velocity).

Impulse

- A change of momentum:

$$\vec{J} = \Delta\vec{p}$$

- Compare:
  - Impulse is change in momentum.
  - Work is change in energy.

- Unit is also $[kg \cdot \frac{m}{sec}]$ (like momentum).

- Impulse ⇔ force integrated over time:

$$\vec{J} = \int_0^t \vec{F} dt = m \int_0^t \vec{a} dt = m\Delta\vec{v}.$$

Conservation of Momentum

- Law of conservation: in a closed system (no external forces\impulses), momentum cannot be created or destroyed.
- Compare: conservation of energy.
- Implied from 3rd law.
  - Objects react with the same force exerted on them.
- Special case of Noether's theorem: every physical system (With a symmetric action) has a conservation law.

Emmy Noether