Open Channel Hydraulics Lectures

Environmental Engineering Department Collage of Engineering University of Tikrit – 3rd Stage 2019

Ahmed Saadi Mahmood

Open channel hydraulics

References:

- Open Channel Hydraulics , by V.T Chow.

- Civil Engineering Hydraulics, by Hinderson

- Open Channel Hydraulics, by R. E. Featherstone and C. Nalluri.

- Open Channel Hydraulics ,by Richard, H. French.

- Open Channel Hydraulics ,by Terry W. Sturm.

Open channel flow and its classifications:

Introduction :

A passage through which water flows with its free surface in contact with the atmosphere is known as "Open Channel". The water therefore runs under the atmospheric pressure throughout the open channel open at the top.

Classification of Channels:

Open channel may be classified as described below:

a- Channel of regular section or irregular section.

The rectangular, trapezoidal, circular or semicircular section are examples of channels of regular section. The example of channels of irregular section are stream or rivers.

b- Natural or artificial channel

Open channel may be either natural or artificial. Streams or rivers are example of natural channels. Artificial channel are man made.

c- Prismatic or non- prismatic channels

In prismatic channels, the cross-section and slope remain uniform throughout its length. Artificial channels are prismatic channels. The bed slope and cross section of non-prismatic channels do not remain uniform throughout the length. Natural channels are non-prismatic channels.

Classification of Flow:

The flow through the channel may be classified in different types as in the case of the pipe flow:

a- Uniform Flow

The flow is to be uniform when the velocity of the flow dose note change both in magnitude and direction from one section to another in part of channel of uniform cross-section. The Depth of flow must also remain constant throughout its length for uniform flow.

b- Non-Uniform Flow

A flow is said to be non-uniform when the velocity and depth of flow varies from one section to another.

c- Steady flow

A flow with constant rate of discharge is known as steady flow. Also, steady flow defined as the flow where the characteristics at a point are not change with time.

d- Unsteady Flow

A flow in which the rate of discharge dose not remain constant.

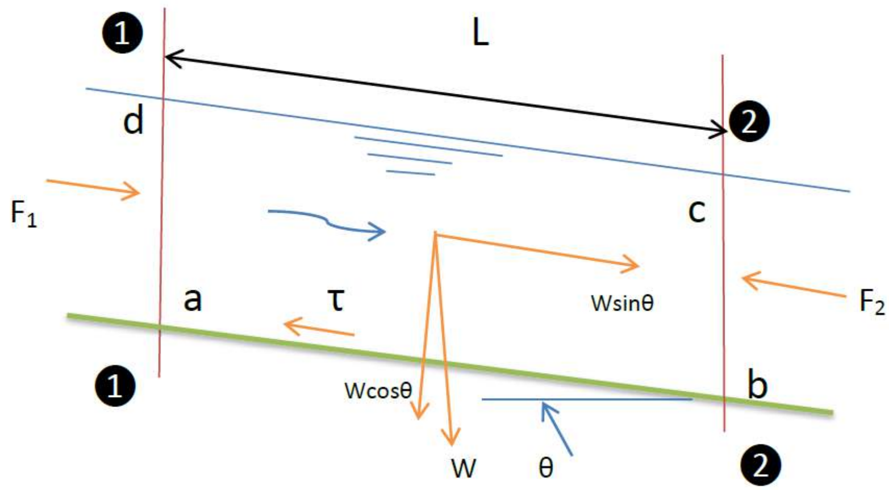
e- Gradually Varied Flow

When the change in depth of flow is gradual the flow is said gradually varied flow.

f- Rapidly Varied Flow

if the change in the depth of flow is abrupt and the transition is conferred to short length only. It is said to be rabidly varied flow.

Chezy's Formula for Uniform Flow



Assuming a section of length L between section (1-1) and section (2-2) moving along the channel as shown in the figure. The water body (abcd) is subjected to the following forces:

- Hydrostatic force (F1) upstream end of the body.
- Hydrostatic force (F2) downstream end of the body.
- Weight of water body (W)
- Frictional resistance force (τ*PL)

Since the water through the channel flows with uniform velocity, therefore the net accelerating or retarding force is equal to zero.

Hence:-

$$F_1 - F_2 + W\sin\theta - \tau \times PL = 0 \quad \text{..(1)}$$

$F_1 = F_2$ (Because the depth of the flow is constant)

$$W = A \times L \times \gamma = AL \times \rho g$$

$$\tau_0 = C_f \times \rho \times \frac{v^2}{2}$$

$$Sin\,\theta = \frac{h_L}{L} = S_0$$

Substituting values of W and τ in the equation (1)

$$AL\rho g \sin\theta - \frac{1}{2} C_f \, \rho v^2 PL = 0$$

$$v^2 = \frac{2Ag \sin\theta}{C_f \, P} = \frac{2g}{C_f} \times \frac{A}{P} \sin\theta$$

$$\frac{A}{P} = R_h$$

$$\frac{2g}{C_f} = C$$

$$v = C\sqrt{R \times S_0} \text{ .... Chezy Equation}$$

$$Q = A \times v$$

$$Q = CA\sqrt{R_h \, S_0}$$

Where:

Q=Discharge

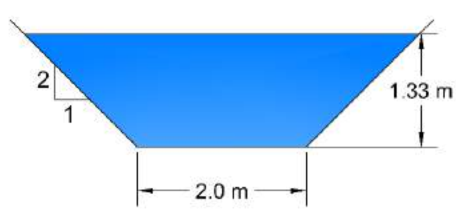
A=Cross section area

C= Chezy Coefficient

R_h= Hydraulic radius

S_0= Longitudinal Slope

Example :A trapezoidal channel has base width of 2m and side slope of 1 horizontal to 2 vertical. The depth of flow in the channel is 1.33m. Find Chezy constant if the discharge through this channel is 7200 liter/sec and the longitudinal slope of this channel is 1 to 530.



Sol. :

$$A = By + zy^2$$

$$A = 2(1.33) + 0.5(1.33)^2 = 3.544 \, m2$$

$$P = B + 2y\sqrt{1+z^2}$$

$$P = 2 + 2(1.33)\sqrt{1+(0.5)^2} = 4.98\ m$$

$$R_h = \frac{A}{P} = \frac{3.544}{4.98} = 0.713m$$

$$Q = \frac{7200}{1000} = 7.2\ \frac{m3}{sec}$$

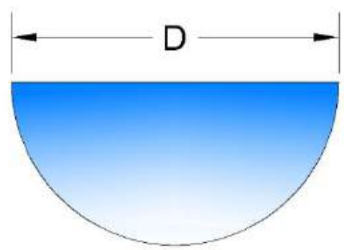
$$Q = CA\sqrt{R_h\, S_0}$$

$$7.2 = C(3.544)\sqrt{(0.713)\ (\frac{1}{530})}$$

$$C = 55.3$$

Example: The discharge through a semi-circular open channel is 10m3/sec. The bed slope is (1 to 1650) and the channel is running full. Find the diameter of this channel if the Chezy coefficient (C) is 70.

Sol.:



$$Q = CA\sqrt{R_h\, S_0}$$

$$A = \frac{\pi R^2}{2} \quad , \quad P = \pi R$$

$$R_h = \frac{0.5\pi R^2}{\pi R} = \frac{R}{2}$$

$$10 = 70(\frac{\pi R^2}{2})\sqrt{(\frac{R}{2})(\frac{1}{1650})}$$

$$R^{\frac{5}{2}} = \frac{10}{35\pi(0.017)}$$

$$R = 1.95m$$

$$D = 2 \times 1.95 = 3.9m$$

The Manning's Formula:

One of the best as well as one of the most widely used formula for uniform flow in open channels is that published by the Irish engineer (Robert Manning). Manning found from many tests, that the value of (C) in the Chezy formula varied approximately as ($R^{1/6}$), so he proposed the following relation:

$$C = \frac{1}{n} R_h^{\frac{1}{6}}$$

Where n is constant and depends on the channel material. Now, Substituting the value of C in the following equation:-

$$V = C\sqrt{R_h S_0}$$

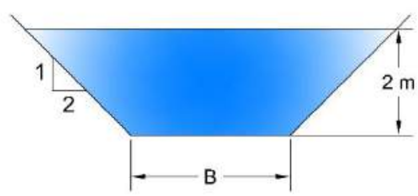
$$V = \frac{1}{n} R_h^{\frac{1}{6}} \sqrt{R_h S_0}$$

$Q = \frac{1}{n} A\, R_h^{\frac{2}{3}}\ S_0^{\frac{1}{2}}$ ------------------ In SI Units System (m , kg,…).

$Q = \frac{1.49}{n} A\, R_h^{\frac{2}{3}}\, S_0^{\frac{1}{2}}$ ------------------- In UK Units System (ft, Ib,….).

Example: An open channel of trapezoidal section has side slope of 2 horizontal to 1 vertical and carries water of rate equal to 15 m^3/sec. The bed slope of the channel is 0.5 per one kilometer and depth of flow is 2m .

a- Find the bed width of channel assuming n =0.018.
b- If the channel at the above cross section is used to discharge 6 m^3/sec of water at the velocity of 0.5 m/sec and the depth of flow being 2m. determine the bed width and bed slope of the channel.



<u>Sol.:</u>

a- Q=15 m^3/sec, $S = \frac{0.5}{1000} = 0.0005$

$$Q = \frac{1}{n} A\, R^{\frac{2}{3}}\, S^{\frac{1}{2}}$$

$$A = By + Zy^2$$

$A = 2B + (2)(2)^2 = 2B + 8$

P= $B + 2y\sqrt{1 + Z^2}$

P= $B + 2(2)\sqrt{1 + (2)^2} = B + 4\sqrt{5}$

$R = \frac{A}{P} = \frac{2B + 8}{B + 4\sqrt{5}}$

$15 = \frac{1}{0.018}\ (2B + 8)(\frac{2B + 8}{B + 4\sqrt{5}})^{\frac{2}{3}}\ (0.0005)^{\frac{1}{2}}$

$\frac{15 \times 0.018}{(0.0005)^{\frac{1}{2}}} = \frac{(2B + 8\ )^{\frac{5}{3}}}{(B + 4\sqrt{5}\ )^{\frac{2}{3}}}$

$B = 1.75\ m$

b- Q= 6 m^3/sec, V=0.5 m/sec , y=2m , n=0.018 , z=2m , S=?

$A = \frac{Q}{v} = \frac{6}{0.5} = 12\ m^2$
$A = By + Zy^2$
$12 = 2B + 2(2)^2 = 2B + 8$
$B = 2\ m$

$Q = \frac{1}{n}\ A\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}$

P= $B + 2y\sqrt{1 + Z^2}$

P= $2 + 2(2)\sqrt{1 + (2)^2} = 10.94\ m$

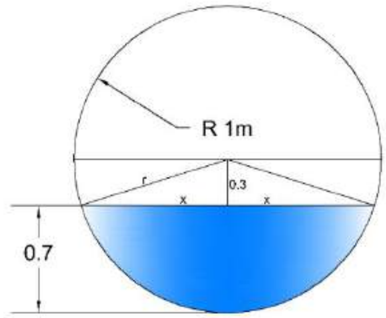
$R = \frac{A}{P} = \frac{12}{10.94} = 1.096\ m$

$6 = \frac{1}{0.018}\ 12 \times\ (1.096)^{\frac{2}{3}}\ S^{\frac{1}{2}}$

$S = 0.0000715$

Example: A circular Sewage of 1 m radius has longitudinal slope of (1 to 250) .find the discharge though the sewage if depth of flow is 700 mm assuming (n) equal to 0.015

Sol.:



$$Q = \frac{1}{n}\ A\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}$$

$$A = Area\ of\ sector - Area\ of\ triangle$$

$$Area\ of\ sector = Area\ of\ circle\ \times \frac{2\theta}{360}$$

$$\cos\theta = \frac{h}{r}$$

$$r = d + h$$

$$h = r - d = 1 - 0.7 = 0.3\ m$$

$$\cos\theta = \frac{0.3}{1}\ \ ,\ \ \theta = 72.54°$$

$$\sin\theta = \frac{x}{r}$$

$$\text{Sin}(72.54) = \frac{x}{1}\ ,\qquad x = 0.95\ m$$

$$Area\ of\ sector = \pi r^2 \times \frac{2\theta}{360}$$

$$Area\ of\ sector = \pi(1)^2 \times \frac{2(72.54)}{360} = 1.266\ m^2$$

$$Area\ of\ triangle\ = \frac{1}{2} \times 2x \times h$$

$$Area\ of\ triangle\ = \frac{1}{2} \times 2(0.95) \times (0.3) = 0.285\ m^2$$

$$A = 1.266 - 0.285 = 0.981\ m^2$$

$$P\ of\ sector = 2\pi r \times \frac{2\theta}{360}$$

$$P = 2\pi(1) \times \frac{(72.54)}{180} = 2.53\ m$$

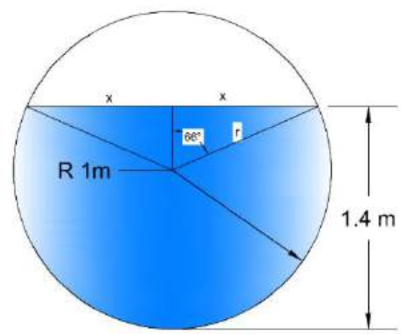
$$R = \frac{A}{P} = \frac{0.981}{2.53} = 0.387m$$

$$Q = \frac{1}{0.015}\ (0.981)\ (0.387)^{\frac{2}{3}}\ (\frac{1}{250})^{\frac{1}{2}}$$

$$Q = 2.196\ m^3/sec$$

Example: Use the Same data of the previous example with depth of flow (y) equal to 1.4 m. find the discharge ?

Sol.:



$$Q = \frac{1}{n}\ A\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}$$

$$\cos\theta = \frac{h}{r}$$

$$h = d - r = 1.4\ - 1 = 0.4\ m$$

$$\cos\theta = \frac{0.4}{1}\quad ,\ \ \theta = 66°$$

$$\sin\theta = \frac{x}{r}$$

$$\text{Sin}(66) = \frac{x}{1}\,, \qquad x = 0.91\ m$$

$$2\theta = 2 \times 66 = 132$$

$$2\alpha = 360 - 2\theta = 360 - 132 = 228°$$

Method 1

$$Area\ of\ flow = Area\ of\ circle - Area\ sector(2\theta) + Area\ of\ triangle$$

$$Area\ of\ flow = \pi r^2 - \pi r^2 \times \frac{2\theta}{360} + \frac{1}{2} \times 2x \times h$$

$$Area\ of\ flow = \pi(1)^2 - \pi(1)^2 \times \frac{132}{360} + \frac{1}{2} \times 2(0.91) \times (0.4) = 2.35\ m^2$$

Method 2

$$Area\ of\ flow = Area\ sector(2\alpha) + Area\ of\ triangle$$

$$Area\ of\ flow = \pi r^2 \times \frac{2\alpha}{360} + \frac{1}{2} \times 2x \times h$$

$$Area\ of\ flow = \pi(1)^2 \times \frac{228}{360} + \frac{1}{2} \times 2(0.91) \times 0.4 = 2.35\ m^2$$

Method 1

$$P = watted\ perimeter\ of\ circle - watted\ perimeter\ of\ sector(2\theta)$$

$$P = 2\pi r - 2\pi r \frac{2\theta}{360} = 2\pi(1) - 2\pi(1)\frac{132}{360} = 3.97\ m$$

Method 2

$$P = watted\ perimeter\ of\ sector(2\alpha)$$

$$P = 2\pi r \frac{2\alpha}{360} = 2\pi(1)\frac{228}{360} = 3.97\ m$$

$$R = \frac{A}{P} = \frac{2.35}{3.97} = 0.59\ m$$

$$Q = \frac{1}{0.015}\ (2.35)\ (0.59)^{\frac{2}{3}}\ (\frac{1}{250})^{\frac{1}{2}}$$

$$Q = 6.97\ \ m^3/sec$$

Condition of the Most Economical Cross Section :

The main purpose of channel is to transport water therefore the cross section of any geometrical shape channel which gives maximum discharge is known as most economical cross section.

In other words the channel of most efficient cross section needs minimum of excavation work for the given discharge through the channel is given by:

$Q = A \times v$ and $v = \frac{1}{n} R_h^{\frac{2}{3}} S^{\frac{1}{2}}$

For a given value of roughness factor (n); area of flow (A) and the hydraulic slope (S) the discharge is maximum if the hydraulic radius (R_h) is maximum but since:-

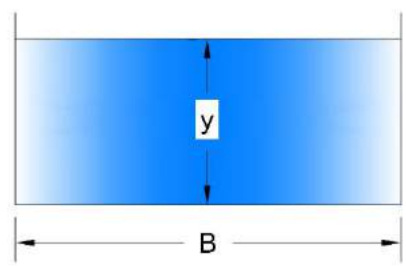
$$R_h = \frac{A}{P}$$

Hence , R_h is maximum if wetted parameter (P) is minimum.

Condition of the Most Economical Rectangular Section :

Consider a channel of rectangular section as shown the figure:

B= bed width
y= Depth of flow



$A = B \times y$

$B = \frac{A}{y}$

$P = B + 2y$

$P = \frac{A}{y} + 2y$

$\frac{dP}{dy} = \frac{-A}{y^2} + 2 = 0$

$A = 2y^2$

$B \times y = 2y^2$

$B = 2y$

This is the width of channel should be twice water depth for maximum discharge.

That is mean the most economical rectangular section is one half of square.

$$R_h = \frac{A}{P} = \frac{B.y}{B+2y}$$

$$R_h = \frac{2y.y}{2y+2y} = \frac{2y^2}{4y}$$

$$R_h = \frac{y}{2}$$

Example: find the discharge and best properties for a rectangular channel having cross section area 4.5 m2 the bed slope is 0.001 and n is 0.013.

Sol.:

$$Q = \frac{1}{n} A\, R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$A = B.y$$

$$R_h = \frac{y}{2}$$

$$4.5 = 2y.y$$

$$y^2 = \frac{4.5}{2} = 2.25\, m$$

$$y = 1.5\, m$$

$$B = 2y = 2(1.5) = 3\, m$$

$$Q = \frac{1}{0.013}(4.5)\left(\frac{1.5}{2}\right)^{\frac{2}{3}}(0.01)^{\frac{1}{2}} = 9.03\, \frac{m^2}{sec}$$

Example: find the best properties for rectangular channel to carry 1.5 m3/sec of water when the bed slope is (1to 3000) taken n equal to 0.015?

Sol.:

$$Q = \frac{1}{n} A\, R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

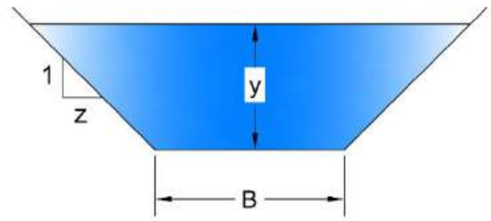
$$A = B.y = 2y^2$$

$$1.5 = \frac{1}{0.015}(2y^2)\left(\frac{y}{2}\right)^{\frac{2}{3}}\left(\frac{1}{3000}\right)^{\frac{1}{2}}$$

$$y = 0.92\, m \qquad ,B = 2(0.92) = 1.84\, m$$

Condition of the Most Economical Trapezoidal Section :

Consider a channel of trapezoidal section as shown the figure:



$$A = By + Zy^2$$

$$B = \frac{A}{y} - Zy$$

$$P = B + 2y\sqrt{1+Z^2}$$

$$P = \frac{A}{y} - Zy + 2y\sqrt{1+Z^2}$$

$$\frac{dP}{dy} = \frac{-A}{y^2} - Z + 2\sqrt{1+Z^2}$$

$$\therefore \frac{A}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{By + Zy^2}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{y(B+Zy)}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{(B+Zy+Zy)}{y} = 2\sqrt{1+Z^2}$$

$$\frac{(B+2Zy)}{y} = 2\sqrt{1+Z^2}$$

$$R_h = \frac{A}{P} = \frac{By + Zy^2}{B + 2y\sqrt{1+Z^2}} = \frac{y(B+Zy)}{B + (B+2Zy)}$$

$$= \frac{y(B+Zy)}{2B + 2Zy}$$

$$R_h = \frac{y(B+Zy)}{2(B+Zy)}$$

$$\therefore R_h = \frac{y}{2}$$

The most economical trapezoidal section is one half of hexagon $, \theta = 60°$

Example: find the width of the best efficient channel if it has side slope and bed slope (1:1) and (1 to 1000) respectively the discharge is 15 m3/sec and Chezy coefficient C=60?

Sol.:

$$Q = CA\sqrt{R_h\,S_0}$$

$$\frac{B + 2Zy}{y} = 2\sqrt{1 + Z^2}$$

$$B = 2\sqrt{2}y - 2y \quad , B = 0.828y$$

$$15 = 60(By + Zy^2)\sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$\frac{15}{60} = (0.828y^2 + y^2)\sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$\frac{0.25}{\sqrt{0.0005}} = 1.828y^{\frac{5}{2}}$$

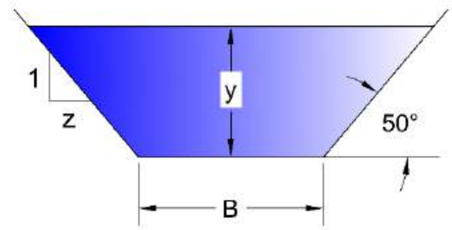
$$11.18 = 1.828y^{\frac{5}{2}}$$

$$y = 2.064\ m$$

$$B = 0.828(2.064) = 1.71\ m$$

Example: A trapezoidal channel having side slope equal to 50° with the horizontal as shown in the figure and laid on a slope of (1to 1000) the cross section area of the channel is 2 m^2 .find the discharge of this channel for the most economical cross section? Use C=66

Sol:



$$\tan 50° = \frac{1}{Z}$$

$$Z = 0.839$$

$$\frac{B + 2Zy}{y} = 2\sqrt{1 + Z^2}$$

$$B = 2y\sqrt{1 + Z^2} - 2Zy$$

$$B = 2y\sqrt{1 + (0.839)^2} - 2(0.839)y$$

$$B = 0.932y \qquad \text{-----------------------------------}(1)$$

$$A = By + Zy^2$$

$$2 = (0.932y)y + (0.839)y^2$$

$$y = 1.0627\ m$$

$$B = 0.932(1.0627) = 0.99\ m$$

$$R_h = \frac{y}{2} = \frac{1.0627}{2} = 0.531m$$

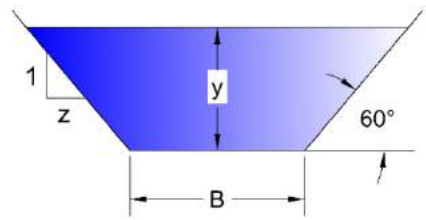
$$Q = CA\sqrt{R_h\ S_o}$$

$$Q = 66 \times 2\sqrt{(0531)(\frac{1}{1000})}$$

$$Q = 3.043\ m^3/sec$$

Example: A trapezoidal channel having side slope equal to 60° with the horizontal as shown in the figure and laid on a slope of (1to 750) carries discharge of 10 m^3/sec. find the width of the base and depth of flow for the most economical section, Take C=66.

Sol.:



$$\tan 60° = \frac{1}{Z}$$

$$Z = 0.5774$$

$$\frac{B + 2Zy}{y} = 2\sqrt{1 + Z^2}$$

$$B = 2y\sqrt{1 + Z^2} - 2Zy$$

$$B = 2y\sqrt{1 + (0.5774)^2} - 2(0.5774)y$$

$$B = 1.155y \qquad \text{------------------------------------(1)}$$

$$A = By + Zy^2$$

$$A = (1.155y)y + (0.577)y^2$$

$$A = 1.7325y^2 \qquad \text{------------------------------------(2)}$$

$$R_h = \frac{y}{2}$$

$$Q = CA\sqrt{R_h\ S_o}$$

$$10 = 66 \times (1.7325y^2)\ \sqrt{(\frac{y}{2})(\frac{1}{750})}$$

$$y = 1.625\ m$$

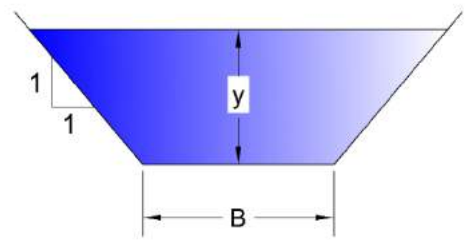
$$B = 1.155y$$

$$B = 1.155(1.625) = 1.876\ m$$

Example: A trapezoidal channel having side slope (1:1) it is required to discharge 13.75 m3/sec of water with a bed slope (1 to 1000). If this channel is unlined and the value of C=44 and when this channel is lined with concrete the value C=60. The cost per cubic meter of excavation is four times the cost per square meter of lining. The channel is to be the most efficient one find whether the lined channel or the unlined channel will be cheaper what will be the dimensions of the economical channel?

Sol.:

1- When the channel is unlined for most economical section.



$$Q = CA\sqrt{R_h \ S_o}$$
$$B + 2Zy = 2y\sqrt{1 + Z^2}$$
$$B + 2y = 2y\sqrt{2}$$
$$B = 0.828y$$
$$A = By + Zy^2$$
$$A = (0.828y)y + y^2 = 1.828y2$$
$$R_h = \frac{y}{2}$$
$$13.75 = 44 \times (1.828y^2)\sqrt{(\frac{y}{2})(\frac{1}{1000})}$$
$$y = 2.256\,m$$
$$B = 0.828(2.256) = 1.876 \ m$$
$$A = 1.828(2.256)^2 = 9.303 \ m^2$$

Let the cost of lining of square meters of concrete =x
Cost of excavation per m^3= 4 x
The cost of excavation per (1m) length of channel = volume of excavation =4x.

$$Area \ \times 1 \times 4x = 9.303 \times 1 \times 4x = 37.212x$$

2-When the channel is lined

$$Q = CA\sqrt{R_h \ S_o}$$
$$13.75 = 60 \times (1.828y^2)\sqrt{(\frac{y}{2})(\frac{1}{1000})}$$
$$y = 1.993\,m$$
$$B = 0.828(1.993\,) = 1.65 \ m$$

$A = 1.828(1.993)^2 = 7.26\ m^2$

The cost of lined channel = cost of excavation + cost of lining

$(A \times 1 \times 4x) + (P * 1 * x)$

$P = B + 2y\sqrt{1 + Z^2}$

$P = 1.65 + 2(1.993)\sqrt{1 + 1}$

$P = 7.287\ m$

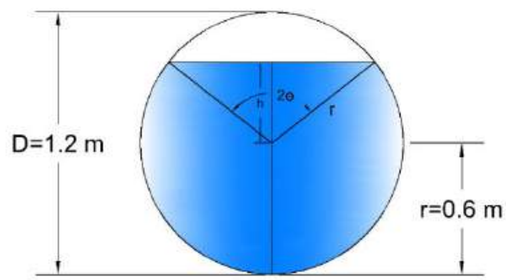
$(7.26 * 1 * 4x) + (7.287\ * 1 * x) = 36.327\ x$

The lined channel is cheaper

Example: A Circular channel of 1.2 m diameter is laid on a slope 1to 1500 . find the discharge through the channel when velocity of flow is maximum . Take n=0.015

Sol.:

For maximum velocity y=0.81 D



$$y = 0.81(1.2) = 0.972\,m$$

$$h = y - r$$

$$h = 0.972 - 0.6 = 0.372\,m$$

$$\cos\theta = \frac{h}{r}$$

$$\cos\theta = \frac{0.372}{0.6} = 0.62 \quad ,\theta = 51.68°$$

$$\sin\theta = \frac{x}{r}$$

$$\sin 51.68 = \frac{x}{0.6} \quad , \quad x = 0.47$$

$$2\theta = 2 \times 51.68 = 103.36°$$

$$2\alpha = 360 - 103.36 = 256.64°$$

Area= Area of sector + Area of triangle

$$= \pi r^2 \frac{2\alpha}{360} + \frac{1}{2} 2x \times h$$

$$= \pi(0.6)^2 \frac{256.64}{360} + 0.47 \times 0.372$$

$$= 0.806 + 0.175 = 0.98\,m^2$$

$$P = watted\ perimeter\ of\ sector$$

$$P = 2\pi r \frac{2\alpha}{360}$$

$$P = 2\pi(0.6)\frac{256.64}{360}$$

$$P = 2.69\,m$$

$$R = \frac{A}{P}$$

$$R = \frac{0.98}{2.69} = 0.36\,m$$

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

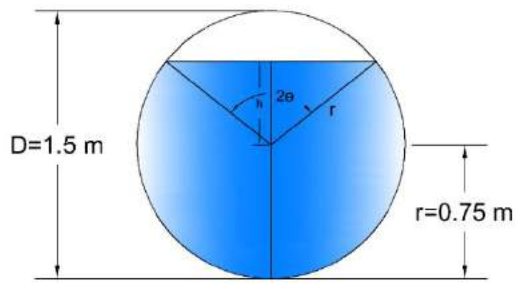
$$Q = \frac{1}{0.015}(0.98)(0.36)^{\frac{2}{3}}(\frac{1}{1500})^{\frac{1}{2}}$$

$$Q = 0.853\ m^3/sec$$

Example: A Circular channel of 1.5 m diameter is laid on a slope 1to 1000 . find the maximum discharge through this channel. Assume C=55

Sol.:

For maximum discharge y=0.95 D



$$y = 0.95(1.5) = 1.425\ m$$

$$h = y - r$$

$$h = 1.425 - 0.75 = 0.675\ m$$

$$\cos\theta = \frac{h}{r}$$

$$\cos\theta = \frac{0.675}{0.75} = 0.62 \quad ,\theta = 25.84°$$

$$\sin\theta = \frac{x}{r}$$

$$\sin 25.84 = \frac{x}{0.75} \qquad , \quad x = 0.33\ m$$

$$2\theta = 2 \times 25.84 = 51.68°$$

$$2\alpha = 360 - 51.68 = 308.32°$$

Area= Area of sector + Area of triangle

$$= \pi r^2 \frac{2\alpha}{360} + \frac{1}{2} 2x \times h$$

$$= \pi (0.75)^2 \frac{308.32}{360} + 0.33 \times 0.675$$

$$= 1.513 + 0.223 = 1.736\ m^2$$

$$P = watted\ perimeter\ of\ sector$$

$$P = 2\pi r \frac{2\alpha}{360}$$

$$P = 2\pi(0.75)\frac{308.32}{360}$$

$$P = 4.036\ m$$

$$R = \frac{A}{P}$$

$$R = \frac{1.736}{4.036} = 0.43\ m$$

$$Q = CA\sqrt{R\,S}$$

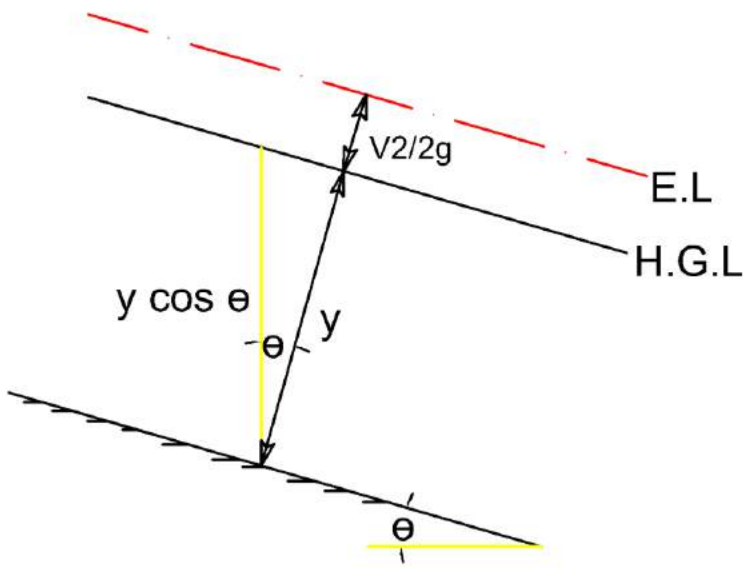
$$Q = 55 \times 1.736 \sqrt{(0.43)\left(\frac{1}{1000}\right)}$$

$$Q = 1.975\ m^3/sec$$

Non Uniform Flow in Open Channel:

If change in channel cross section or channel discharge or depth of flow in any section , the flow is said non uniform flow .

Energy of flowing liquid in open channel.



The Bernoulli's Equation

$$\frac{P}{\gamma}+\frac{v^2}{2g}+Z=Constant$$

The equation is also applicable in open channel flow:-

$$\frac{P}{\gamma}=\mathrm{y}\cos\theta \quad ,\theta\approx 0\ , \quad \cos\theta\approx 1$$

$$\frac{P}{\gamma}=y$$

$$\therefore The\ total\ energy=\ y+\frac{v^2}{2g}+Z$$

Specific Energy of channels :

The fact that the bed of the channel may not be exactly horizontal . but $\theta = 0$

Then taking the bed channel as a datum, therefore

$Z = 0$

$$The\ total\ energy(E) = y + \frac{v^2}{2g}$$

For rectangular channel , $v = \frac{Q}{A} = \frac{q*B}{y*B} = \frac{q}{y}$

$$\therefore E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

Example: A rectangular channel 4 m wide is discharging water at rate of 12 m^3/sec. Find specific energy of water if the depth of flow is 2 m.

Sol.:

Method 1

$$E = y + \frac{v^2}{2g} + Z$$

$$v = \frac{Q}{A} = \frac{12}{4 \times 2} = 1.5\ m/sec$$

$$\therefore E = 2 + \frac{(1.5)^2}{2 \times 9.81} = 2.114\ m$$

Method 2

$$E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

$$Q = q \times B\ \ ,q = \frac{Q}{B} = \frac{12}{4} = 3\ m^3/sec$$

$$E = 2 + \frac{1}{2(9.81)}\left(\frac{3}{2}\right)^2 = 2.114\ m$$

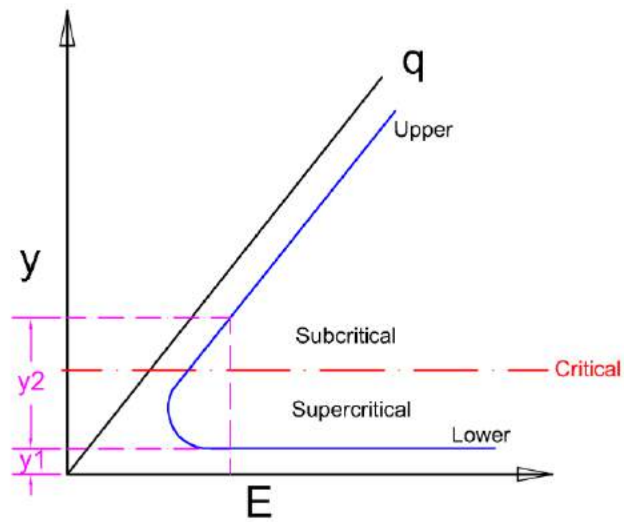
Specific Energy and Alternate Depths of Flow in Rectangular Channel:

$$E = y + \frac{v^2}{2g} = y + \frac{1}{2g}\,(\frac{q}{y})^2$$

$$E - y = \frac{1}{2g} * \frac{q^2}{y^2}$$

$$(E - y)y^2 = \frac{q^2}{2g}$$

1-A Plot of **E** with respect to **y** for constant **q** gives a specific energy discharge as shown in the figure:



2-Each different value of **q** will gives a different curve.
3-Each value of **E** gives two values possible different values of **y** these two values known alternate depths.
4-The two alternate depths represent two locally different flow ,slow and deep in the upper limb of the curve and fast and low in the lower limb of the curve.
5-The upper limb represent the subcritical flow and the lower limb represent the supercritical flow.

$$Fr > 1 \quad Supercritical\ flow$$
$$Fr = 1 \quad Critical\ flow$$
$$Fr < 1 \quad Subcritical\ flow$$

$$Fr = \frac{v}{\sqrt{gy}}$$

6-At point **C** for a given **q** , the value of **E** is minimum and the flow at this point referred to as critical flow . the depth of flow at that point is critical flow (**y_c**) and the velocity is critical velocity (**v_c**)

7-A relation for critical in wide rectangular channel can be found by deafferenting **E** of equation with respect to **y**:-

$$E = y + \frac{1}{2g} * \frac{q^2}{y^2}$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} \;,\quad 0 = 1 - \frac{q^2}{gy^3} \;,\quad 1 = \frac{q^2}{gy^3}$$

$$q^2 = gy^3$$

$$Q = vA = q * B = v_c(y_c * B)$$

$$v_c = \frac{q}{y_c} \;,\quad q = v_c * y_c$$

$$q^2 = {v_c}^2 * {y_c}^2$$

$$g{y_c}^3 = {v_c}^2 * {y_c}^2$$

$${v_c}^2 = gy_c$$

$$v_c = \sqrt{gy_c}$$

$$\frac{q}{y_c} = \sqrt{gy_c}$$

$$\frac{q^2}{{y_c}^2} = gy_c \rightarrow {y_c}^3 = \frac{q^3}{g}$$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} \qquad \text{for rectangular channel}$$

$${v_c}^2 = gy_c \quad \rightarrow y_c = \frac{{v_c}^2}{g}$$

$$\frac{y_c}{2} = \frac{{v_c}^2}{2g}$$

$$E = y + \frac{v^2}{2g}$$

$$E_{min} = y_c + \frac{{v_c}^2}{2g}$$

$$E_{min} = y_c + \frac{y_c}{2} = \frac{3y_c}{2}$$

$$\therefore E_{min} = 1.5y_c$$

$$Or \;\; y_c = \frac{2}{3} E_{min}$$

Example: A rectangular channel of bed width 4m is discharging water at rate 10 m3/sec . plot the specific energy curve for this flow and from curve determine the following:-

1- The critical depth (y_c).
2- The minimum specific energy (E_{min}).
3- Alternate depths of flow for specific energy equal to 2m.
4- Check the result with these obtained by calculation.
5- What will be the type of flow if the depth is ;
   a- 0.6 m
   b- 2 m.

Sol.:

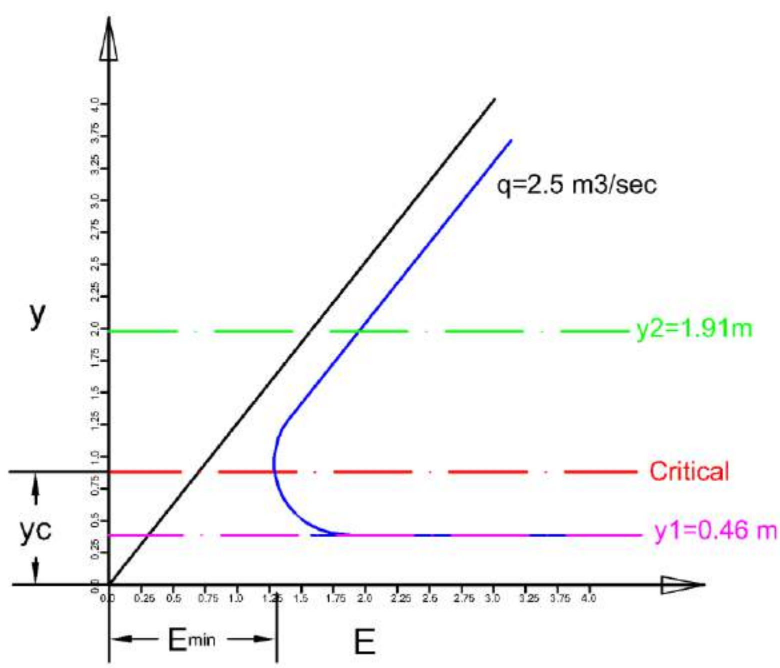
$$Q = 10\,m^3/sec$$

$$E = y + \frac{1}{2g} * \frac{q^2}{y^2}$$

$$q = \frac{Q}{B} = \frac{10}{4} = 2.5\ \frac{m^3}{sec}/m$$

$$E = y + \frac{1}{2(9.81)} * \frac{(2.5)^2}{y^2} = y + \frac{6.25}{19.62}(\frac{1}{y^2})$$

y	0.125	0.25	0.375	0.5	0.625	0.75	0.8	0.9	1.0	1.125	1.375	1.5	1.75	2	2.5
E	2.051	5.37	2.65	1.78	1.44	1.32	1.3	1.295	1.32	1.38	1.58	1.64	1.85	2.08	2.55



4- $y_c = (\frac{q^2}{g})^{\frac{1}{3}} = (\frac{(2.5)^2}{9.81})^{\frac{1}{3}} = 0.8605\ m$

$E_{min} = 1.5y_c = 1.5(0.8605) = 1.29m$

When E=2m

$$2 = y + \frac{1}{19.62} * \frac{(2.5)^2}{y^2}$$

$y_1 = 0.46\ m$

$y_2 = 1.91\ m$

5- $y = 0.6\ m < y = 0.86\ m$

$\therefore The\ flow\ is\ supercritical$

$$Or \quad Fr = \frac{v}{\sqrt{gy}}$$

$$v = \frac{q}{y_1} = \frac{2.5}{0.6} = 4.16\ m/sec$$

$$Fr = \frac{4.16}{\sqrt{9.81 \times 0.6}} = 1.7 > 1$$

$\therefore The\ flow\ supercritical$

$y > 2m$

$yc = 0.86\ m$

$\therefore The\ flow\ is\ subcritical$

$$Or \quad Fr = \frac{v}{\sqrt{gy}}$$

$$v = \frac{q}{y_2} = \frac{2.5}{2} = 1.25\ m/sec$$

$$Fr = \frac{1.25}{\sqrt{9.81 \times 2}} = 0.29 < 1$$

$\therefore The\ flow\ is\ subcritical$

Example: A rectangular channel of most efficient cross section is laid on of (1to 1500) and discharging water at a rate of 40 m3/sec . determine the type of flow in this channel . Take C=60

Sol.:

$Q = CA\sqrt{RS}$

For most efficient section $B = 2y$

$\therefore A = 2y^2$

$$40 = 60(2y^2)\sqrt{\frac{y}{2}(\frac{1}{1500})}$$

$y = 3.2\ m$

$B = 2(3.2) = 6.4\ m$

$y_c = (\frac{q^2}{g})^{\frac{1}{3}}$

$$q = \frac{Q}{B} = \frac{40}{6.4} = 6.25\ \frac{m^3}{sec}/m$$

$y_c = (\frac{(6.25)^2}{9.81})^{\frac{1}{3}} = 1.58\ m$

$\therefore y = 3.2\ m > y_c = 1.58\ m$

$\therefore the\ flow\ is\ subcritical$

$$Or \quad Fr = \frac{v}{\sqrt{gy}}$$

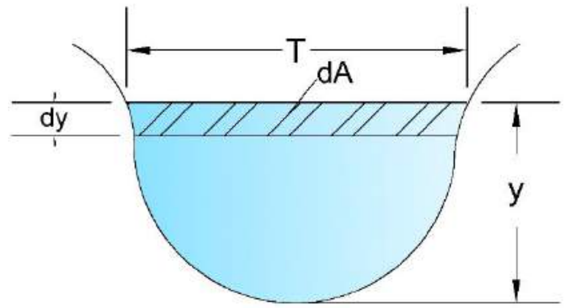
$$v = \frac{Q}{A} = \frac{40}{6.4(3.2)} = 1.95\ m/sec$$

$$Fr = \frac{1.95}{\sqrt{9.81 \times 3.2}} = 0.34 < 1$$

$\therefore The\ flow\ is\ subcritical$

Specific Energy and Critical Depths for Non-Rectangular Channel:

At section of any type of channel the specific energy of flowing equation:-



$$E = y + \frac{v^2}{2g}$$

$$E = y + \frac{1}{2g} * \left(\frac{Q}{A}\right)^2$$

$$E = y + \frac{Q^2}{2gA^2}$$

$$\frac{dE}{dy} = 1 + \frac{Q^2}{2g}\left(-\frac{2}{A^3}\right)\frac{dA}{dy} = 0$$

$$\frac{Q^2}{gA^3} * \frac{dA}{dy} = 1$$

$$dA = T * dy\,, \qquad \frac{dA}{dy} = T$$

T= Width of water surface at depth of water equal y.

$$\frac{Q^2}{gA^3} * T = 1 \qquad , \frac{Q^2}{A^3} = \frac{g}{T}$$

$$\frac{Q^2}{{A_c}^3} = \frac{g}{T_c}$$ At critical condition

$$\frac{Q^2}{{A_c}^2} = \frac{gA_c}{T_c}$$

$${v_c}^2 = \frac{gA_c}{T_c}$$

$$v_c = \sqrt{\frac{gA_c}{T_c}}$$

$$(y_c)_{ave} = \frac{A_c}{T_c}$$

$$v_c = \sqrt{g y_{c\,(ave)}}$$

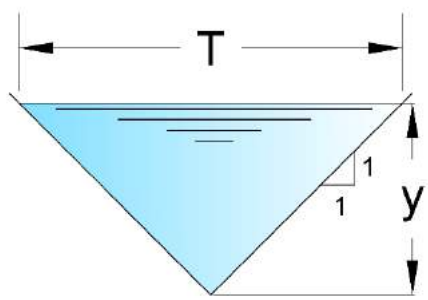
$y_{c\,(ave)}$= The average depth for critical condition.

$$\frac{A}{T} = y_{(ave)}$$
$$v_c{}^2 = g y_{c\,(ave)}$$
Now writing the equation for minimum specific energy equation
$$E_{min} = y_c + \frac{v_c{}^2}{2g}$$
$$E_{min} = y_c + \frac{g y_{c\,(ave)}}{2g}$$
$$E_{min} = y_c + \frac{y_{c\,(ave)}}{2}$$

$$\therefore Fr = \frac{v_c}{\sqrt{g y_{c\,(ave)}}}$$

Example: As shown in figure, water flow uniformly at a steady rate of 14 Cfs in very long triangle flume that has side slope 1:1 . the flume is laid on a slope 0.001 and n= 0.015 . Is the flow in this flume subcritical or supercritical? and find minimum specific energy?

Sol.:



$$A = \frac{1}{2}(2y)(y) = y^2$$
$$P = 2\sqrt{y^2 + y^2} = 2y\sqrt{2}$$
$$R = \frac{A}{P} = \frac{y^2}{2y\sqrt{2}} = \frac{y}{2\sqrt{2}}$$
$$Q = \frac{1.49}{n} A\, R^{\frac{2}{3}} S^{\frac{1}{2}}$$
$$14 = \frac{1.49}{0.012} * (y^2) * \left(\frac{y}{2\sqrt{2}}\right)^{\frac{2}{3}} * (0.006)^{\frac{1}{2}}$$
$$y = 1.494\, ft$$
$$\frac{Q^2}{A_c{}^3} = \frac{g}{T_c} \qquad T_c = 2Zy_c = 2y_c$$
$$\frac{(14)^2}{(y_c{}^2)^3} = \frac{32.2}{2y_c}$$

$y = 1.6848 \;\; > y_c = 1.494$

$\therefore the\ flow\ is\ supercritical$

$$(y_c)_{ave} = \frac{A_c}{T_c} = \frac{{y_c}^2}{2y_c} = \frac{y_c}{2} = \frac{1.648}{2} = 0.824$$

$$E_{min} = y_c + \frac{y_{c(ave)}}{2}$$

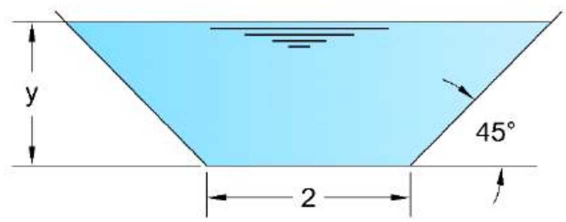
$$E_{min} = 1.648 + \frac{0.824}{2}$$

$$E_{min} = 2.06\ ft$$

Example: A channel of trapezoidal section , 2m wide at the base with side sloping 45° with the horizontal . This channel carries water at rate of 6 m^3/sec. Find the following :-

1- The critical depth (y_c).
2- What is the type of flow if the depth of water is 1m .
3- Compute Fr for normal depth of flow =1.2m .

Sol.:



$$\tan 45° = \frac{1}{Z}$$

$$Z = 1$$

$$A = B * y_c + Z * {y_c}^2$$
$$= 2y_c + {y_c}^2$$

$$T = B + 2Zy_c$$
$$= 2 + 2y_c$$

$$\frac{Q^2}{{A_c}^3} = \frac{g}{T_c}$$

$$\frac{6^2}{(2y_c + y_c)^3} = \frac{9.81}{(2 + 2y_c)}$$

$$\frac{6^2}{9.81} = \frac{(2y_c + y_c)^3}{(2 + 2y_c)}$$

$$yc = 0.838\, m$$
$$y = 1m > \; yc = 0.838\, m$$
$$\therefore The\ flow\ is\ subcritical$$

$$Or$$
$$A = B * y + Z * y^2$$
$$= 2 * (1) + (1) * (1)^2 = 3m$$

$$v = \frac{Q}{A} = \frac{6}{3} = 2\ \ m/sec$$

$$Fr = \frac{v}{\sqrt{g y_{av.}}}$$

$$y_{av.} = \frac{A}{T} = \frac{3}{2 + 2(1)(1)} = 0.75$$

$$Fr = \frac{2}{\sqrt{9.81 \times 0.75}} = 0.737 < 1$$

$\therefore The\ flow\ is\ subcritical$

$y = 1.2m$

$A = B * y + Z * y^2$

$= 2 * (1.2) + (1) * (1.2)^2 = 3.84m$

$$v = \frac{Q}{A} = \frac{6}{3.84} = 1.56\ \ m/sec$$

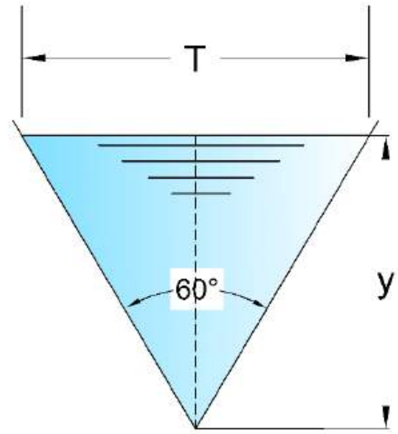
$$Fr = \frac{v}{\sqrt{g y_{av.}}}$$

$$y_{av.} = \frac{A}{T} = \frac{3.84}{2 + 2(1)(1.2)} = 0.87$$

$$Fr = \frac{1.56}{\sqrt{9.81 \times 0.87}} = 0.737 < 1$$

$\therefore The\ flow\ is\ subcritical$

Example: Compute the critical depth for flow at 0.4 m3/sec through the cross section of figure **.** And what is the type of flow if the depth of water is 0.5 m?.



Sol.:

$$A = \frac{1}{2}2(y_c \tan 30) \times y_c$$
$$= 0.5774{y_c}^2$$

$$T = 2(y_c \tan 30)$$
$$= 1.155\, y_c$$

$$\frac{Q^2}{{A_c}^3} = \frac{g}{T_c}$$

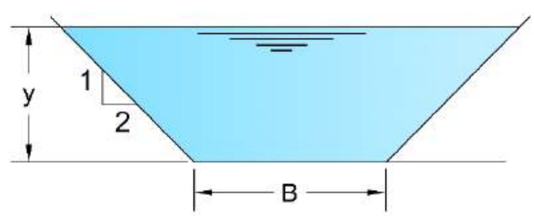
$$\frac{0.4^2}{(0.5774{y_c}^2)^3} = \frac{9.81}{1.155\, y_c}$$

$$y_c = 0.628\, m$$

$$y = 0.5m < \ yc = 0.628\, m$$

$$\therefore The\ flow\ is\ supercritical$$

H.W: A channel of trapezoidal carry water at rate equal to of 10 m^3/sec. Manning n=0.02 and bed slope is 0.0025 the side slope of this channel (Z) is 2:1.
Compute the bed width (B) of this channel if the allowable velocity of flow is 1.5 m/sec



Classification of Slopes:

As already described, the type of flow depends upon the depth of flow. There are three types of following depth:

1- Normal Depth (y_n)

When a channel of given slope carries uniform flow, then the depth of flow in the channel is called (Normal Depth). It is called normal because the area of flow must be taken normal to the direction of flow which in case of uniform is parallel to the bed.

2- Critical Depth (y_c)

Already the definition of the critical depth have been given the flow is critical when it is equal to normal depth.

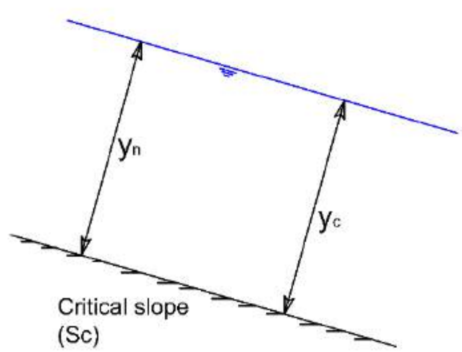
3- Actual Depth (y)

If the flow in a channel is of gradually varied type, then the depth of flow actually occurring at a section is called (Actual Depth).

Now, based upon the depth and type of flow, the bed slope may be classified in to following five types:

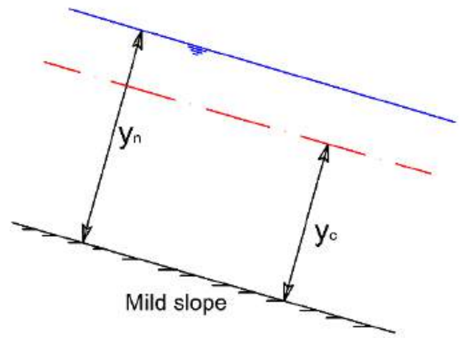
1- Critical Slope (S_c).

The slope of channel bed is said to be critical, when the normal depth (y_n) is equal to the critical depth (y_c). The critical slope can be calculated by Manning's formula if the depth (y_c) is known. (y_c=y_n)



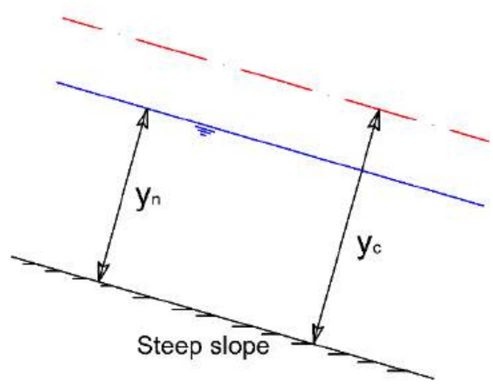
2- Mild Slope

The mild slope of a channel may be defined as a slope less than the critical slope. In this case the normal depth of flow will be greater than the critical depth ($y_n > y_c$).



3- Steep Slope

When the slope of a channel is more than critical slope, it is said to be steep slope. Hence, the normal depth for uniform flow will be less than critical depth ($y_n < y_c$) and the flow being super critical flow.



4- Horizontal Slope

A channel with zero slope is said to be of horizontal slope.

5- Adverse Slope

This is negative slope, where the bed is rises in the direction of flow.

Example: Find the alternate depths for a rectangular channel with bed with equal to 3.6 m and carrying a discharge of 8.64 m3 /sec . The specific energy (E) being 1.7 m . determine the necessary slope to maintain uniform flow at the above depths and name these slopes .Also , find the critical slope . what are the Froud's number at the flow depths assume n=0.015.

Sol.:

$$E = y + \frac{v^2}{2g}$$

$$E = y + \frac{1}{2g} * \frac{q^2}{y^2}$$

$$q = \frac{Q}{B} = \frac{8.64}{3.6} = 2.4\ \frac{m^3}{sec}/m$$

$$1.7 = y + \frac{1}{2g} * \frac{2.4^2}{y^2}$$

$$y_1 = 0.48\ m$$

$$y_2 = 1.59\ m$$

$$For\ y_1 = 0.48\ m$$

$$A_1 = B * y_1 = 3.6 * 0.48 = 1.76\ m^2$$

$$P_1 = B + 2y_1 = 3.6 + 2 * 0.48 = 4.58\ m$$

$$R_1 = \frac{A_1}{P_1} = \frac{1.76}{4.58} = 0.38\ m$$

$$Q = \frac{1}{n} A_1\ R_1^{\frac{2}{3}}\ S_1^{\frac{1}{2}}$$

$$8.64 = \frac{1}{0.015}(1.76)\ (0.38)^{\frac{2}{3}}\ S_1^{\frac{1}{2}}$$

$$S_1 = 0.019$$

$$For\ y_2 = 1.59\ m$$

$$A_2 = B * y_2 = 3.6 * 1.59 = 5.75\ m^2$$

$$P_2 = B + 2y_2 = 3.6 + 2 * 1.59 = 6.78\ m$$

$$R_2 = \frac{A_2}{P_2} = \frac{5.75}{6.78} = 0.84\ m$$

$$Q = \frac{1}{n} A_2\, R_2^{\frac{2}{3}}\, S_2^{\frac{1}{2}}$$

$$8.64 = \frac{1}{0.015}(5.75)\,(0.84)^{\frac{2}{3}}\, S_2^{\frac{1}{2}}$$

$$S_2 = 0.00064$$

In order to name the above slope it must be Sc therefore y_c must be given :-

$$y_c = (\frac{q^2}{g})^{\frac{1}{3}}$$

$$y_c = (\frac{2.4^2}{9.81})^{\frac{1}{3}} = 0.83\, m$$

$$A_c = B * y_c = 3.6 * 0.83 = 2.98\, m^2$$
$$P_c = B + 2y_c = 3.6 + 2 * 0.83 = 5.26\, m$$

$$R_c = \frac{A_c}{P_c} = \frac{2.98}{5.26} = 0.56\, m$$
$$Q = \frac{1}{n} A_c\, R_c^{\frac{2}{3}}\, S_c^{\frac{1}{2}}$$

$$8.64 = \frac{1}{0.015}(2.98)\,(0.56)^{\frac{2}{3}}\, S_c^{\frac{1}{2}}$$

$$S_c = 0.00409$$

$$\therefore\ S_1 = 0.019 > S_c = 0.00409 \qquad \therefore S_1\ \ Steep\ Slope\ (Super)$$

$$\therefore\ S_2 = 0.00064 < S_c = 0.00409 \qquad \therefore S_2\ \ Mild\ Slope\ (Sub)$$

$$Fr_1 = \frac{v_1}{\sqrt{g y_1}}$$
$$v_1 = \frac{Q}{A_1} = \frac{8.64}{1.76} = 4.9\, m/sec$$

$$Fr_1 = \frac{4.9}{\sqrt{9.81 * 0.48}} = 2.23 > 1 \quad Supercritical$$

$$Fr_2 = \frac{v_2}{\sqrt{gy_2}}$$

$$v_1 = \frac{Q}{A_2} = \frac{8.64}{5.75} = 1.51\ m/sec$$

$$Fr_2 = \frac{1.51}{\sqrt{9.81 * 1.59}} = 0.38 < 1 \quad Subcritical$$

$$Fr_c = \frac{v_c}{\sqrt{gy_c}}$$

$$v_c = \frac{Q}{A_c} = \frac{8.64}{2.98} = 2.891\ m/sec$$

$$Fr_c = \frac{2.891}{\sqrt{9.81 * 0.83}} = 1.01 \approx 1 \quad Critical\ \ flow.$$