Aerodynamics Formula Overview

Equation of state

$$p = \rho RT \tag{1}$$

Relationship between gravity with geometric height

$$g = g_0 \left(\frac{r}{r + h_G}\right)^2 \tag{2}$$

Hydrostatic equation
Geometric altitude:

$$dp = -\rho g dh_G \tag{3}$$

Geopotential altitude:

$$dp = -\rho g_0 dh \tag{4}$$

Relationship geopotential and geometric altitudes

$$dh = \frac{r^2}{(r + h_G)^2} dh_G \quad \Rightarrow \quad h = \frac{r}{r + h_G} h_G \tag{5}$$

Standard atmosphere
Isothermal layer

$$\frac{p}{p_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0(h-h_1)}{RT}} \tag{6}$$

Gradient layer

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}} \tag{7}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)} \tag{8}$$

$$T = T_1 + a(h - h_1) \tag{9}$$

Euler equation

$$dp = -\rho V dV \tag{10}$$

Steady frictionless incompressible flow
Simplified continuity equation:

$$A_1 V_1 = A_2 V_2 \tag{11}$$

Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{12}$$

Curved flow

$$\frac{dp}{dr} = \rho \frac{V^2}{r} \tag{13}$$

Thermodynamics
Enthalpy:

$$h = e + pv = e + RT \tag{14}$$

First law of thermodynamics:

$$\delta q + \delta w = de \tag{15}$$

At constant pressure:

$$\delta q = de + pdv \tag{16}$$

At constant volume:

$$\delta q = dh - vdp \tag{17}$$

Specific heat at constant volume:

$$c_v = \frac{\delta q}{dT} \tag{18}$$

Specific heat at constant pressure:

$$c_p = \frac{\delta q}{dT} \tag{19}$$

$$de = c_v dT \quad \Rightarrow \quad e = c_v T \tag{20}$$

$$dh = c_p dT \quad \Rightarrow \quad h = c_p T \tag{21}$$

$$R = c_p - c_v \tag{22}$$

$$\gamma = \frac{c_p}{c_v} \tag{23}$$

Mach number

$$M = \frac{V}{a} \tag{24}$$

$$a = \sqrt{\gamma RT} \tag{25}$$

Isentropic flow
Continuity equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{26}$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{27}$$

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \tag{28}$$

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \tag{29}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma-1}} \tag{30}$$

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2 \tag{31}$$

Measurement of airspeed
Incompresible flow ($M < 0.3$):

$$V_{true} = \sqrt{\frac{2\,(p_0 - p)}{\rho}} \tag{32}$$

$$V_e = \sqrt{\frac{2\,(p_0 - p)}{\rho_s}} \tag{33}$$

Subsonic compressible flow ($0.3 < M < 1$):

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_1} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{34}$$

$$V_{cal}^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_s} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{35}$$

Relationship true airspeed and equivalent airspeed:

$$V_e = V\sqrt{\frac{\rho}{\rho_s}} \tag{36}$$

Area velocity relation

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{dV}{V} \tag{37}$$

Conclusions:
$M < 1$: For the velocity to increase, the area must decrease
$M > 1$: For the velocity to increase, the area must increase
$M = 1$: The velocity will be sonic always at the throat

Reynold's number

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} \tag{38}$$

Viscous flow

$$\tau_w = \mu\left(\frac{dV}{dy}\right)_{y=0} \tag{39}$$

$$c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} \tag{40}$$

Laminar boundary layer:

$$\delta = \frac{5.2x}{\sqrt{Re_x}} \tag{41}$$

$$c_{f_x} = \frac{0.664}{\sqrt{Re_L}} \tag{42}$$

$$C_f = \frac{D_f}{q_\infty S} = \frac{1.328}{\sqrt{Re_L}} \tag{43}$$

Turbulent boundary layer

$$\delta = \frac{0.37x}{Re_x^{0.2}} \tag{44}$$

$$c_{f_x} = \frac{0.0592}{Re_L^{0.2}} \tag{45}$$

$$C_f = \frac{D_f}{q_\infty S} = \frac{0.074}{Re_L^{0.2}} \tag{46}$$

Drag due to viscous effects:

$$D_{profile} = D_{friction} + D_{pressure} \tag{47}$$

Airfoil nomenclature
Relation lift and drag force with normal and axial force:

$$L = N\cos\alpha - A\sin\alpha \tag{48}$$

$$D = N\sin\alpha + A\cos\alpha \tag{49}$$

AC, CP and the pitching moment

$$C_{m,CP} = 0 \tag{50}$$

$$\frac{dC_{m,AC}}{d\alpha} = 0 \tag{51}$$

$$C_{mQ_2} = C_{mQ_1} + C_n\left(\frac{x_{Q_2} - x_{Q_1}}{c}\right) \tag{52}$$

Lift, drag and moment for 2-dimensional wings

$$L = q_\infty\, S\, c_l \tag{53}$$

$$D = q_\infty\, S\, c_d \tag{54}$$

$$M = q_\infty\, S\, c\, c_m \tag{55}$$

Pressure coefficient:

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \tag{56}$$

Prandt-glauert rule:

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}} \tag{57}$$

$$c_n = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right) dx \tag{58}$$

Induced drag for 3-dimensional wings:

$$D_i = L\alpha_i = L\frac{C_L}{\pi A} \tag{59}$$

$$C_{D,i} = \frac{C_L^2}{\pi e A} \tag{60}$$

($e = 1$ for elliptical shapes)

$$A = \frac{b}{c} = \frac{b^2}{S} \tag{61}$$

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + \frac{57.3a_0}{\pi e_1 A}} \tag{62}$$

$$D_{total} = D_{profile} + D_{induced} \tag{63}$$

Mach waves

$$\mu = \arcsin\frac{1}{M} \tag{64}$$

$$M_{cr,swept} = \frac{M_{cr,airfoil}}{\cos\Omega} \tag{65}$$