Engineering Mechanics [Static]-Lecture Notes

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Scope:

This course is provided to the engineering students with the basic skills in static and strength of materials. It provides a clear and thorough demonstration of the theory and application of engineering static and strength of materials.

Among the main concepts that are covered in this course are vectors, equilibrium of a particle, equilibrium of a rigid body, trusses and frames, internal forces, centroids, and moment of inertia, concept of stress, stress and strain-axial loading and pure bending

Purposes:

- To understand and use the general ideas of force vectors and equilibrium of particle and rigid body. كيفية تحليل القوى والاتزان للاجسام والجسيمات الصلبة
- To understand and use the general ideas of structural analysis and internal force and friction.التحليل الانشائي والقوى الداخلية والاحتكاك
- To understand and use the general ideas of centre of gravity, centroids and moments of inertia. كيفية حساب المركز الهندسي ومركز الثقل وعزم القصور الذاتي

Part One: Static

Contents: [General principals, Force vectors, Equilibrium of a particle, Force system resultants, Equilibrium of a Rigid Body, Structural Analysis, Internal Forces, Friction, Center of Gravity and Centroid, Moments of Inertia]

Ref: Engineering Mechanics -Statics, Twelfth Edition, R. C. Hibbeler, 2009.

General Principals

1.1. Introduction

Statics is the study of bodies that are at rest or move with constant velocity. We can consider statics as a special case of dynamics, in which the acceleration is zero. علم الاستاتيكا او السكون يهتم بحالة الاجسام الثابتة اي الساكنة

1.2. Fundamental Concepts

Before we begin our study, it is important to understand the meaning of certain fundamental concepts and principles.

Length: Length is used to locate the position of a point in space and thereby describe the size of a physical system. كمية الطول يستخدم لتحديد موقع النقطة في فراغ ووصف حجم الانظمة الفيزيائية

Time: Although the principles of statics are time independent. This quantity plays an important role in the study of dynamics. الاستاتيكا لاتعتمد على الزمن لكن الزمن مهم في الديناميكيا وحركة الاجسام

Mass: Mass is a measure of a quantity of matter. خاصية فيزيائية للاجسام وقياس مايحويه الجسم من مادة

Force: Force is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall. A force is completely characterized by its magnitude, direction, and point of application. القوة تستخدم لدفع او سحب جسم ما من جسم لاخر نتيجة الاتصال المباشر بينهم. القوة توصف بمقدارها-اتجاهها-ونقطة التسليط

Particle has a mass, but it size can be neglected. الجسيم له كتلة لكن ممكن اهمال حجمه

Rigid Body A rigid body can be considered as a combination of a large number of Particles الجسم الصلب او الجاسئ هو الحالة المثالية لجسم صلب المسافة بين الجسيمات تبقى فيه ثابتة عبر الزمن بغض النظر عن القوى الخارجية المسلطة عليه. عادة يعتبر الجسم الجاسئ على انه توزيع مستمر للكتلة

Newton's first law: A particle originally at rest or moving in a straight line with constant velocity, tends to remain in this State provided the particle is not subjected to an unbalanced force (Fig.1-1). يظل الجسم في حالته الساكنة (اما السكون التام او التحريك في خط مستقيم بسرعة ثابتة) مالم تؤثر عليه قوة تغير من هذه الحالة. يشير القانون انه اذا كان مجموع الكميات الموجهة من القوى التي تؤثر على جسم ما صفرا, فسوف يظل هذا الجسم ساكنا. ولمثل فان اي جسمة متحرك سيظل على حركته بسرعة ثابتة في حالة عدم وجود اي قوى تؤثر عليه مثل قوى الاحتكاك

$$\sum_{i=1}^{N} F_1 = 0$$

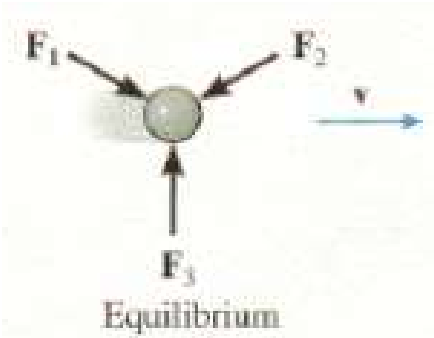


Fig. 1.1

Newton's second law: A particle acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force (Fig. 1-2). If "F" is applied to a particle or mass "m", this law may be expressed mathematically as:

اذا اثرت قوة او مجموعة قوى على جسم ما فانها تكسبه تسارعا او عجلة يتناسب مع محصلة القوى المؤثرة ومعامل التناسب هو الكتلة للجسم

F=m.a

Fig. 1.2

Newton's third Law: The mutual forces of action between two particles are equal, opposite, and collinear (Fig. 1-3).

لكل قوة فعل قوة رد تساويها بلمقدار ومعاكسة لها بلاتجاه

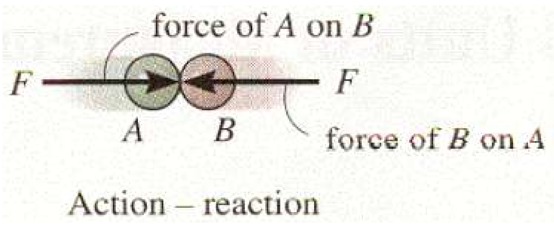


Fig. 1.3

Newton's Law of Gravitational Attraction: Shortly after formulating his three laws of motion. Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically:

قانون الجذب العام هو قانون فيزيائي استنباطي ينص على انه توجد قوة تجاذب بين اي جسمين في الكون تتناسب طرديا مع حاصل ضرب كتلتيهما وعكسيا مع مربع المسافة بينهما,

$$F = G\frac{m_1 m_2}{r^2}$$

Where

F: Force of gravitational between the two particles.

G: Universal constant of gravitation, according to experimental evidence:

$$G=66.73\ 10^{-12}\ m^3/kg.s^2$$

m_1, m_2: mass of each of the two particles.

r: distance between the two particles.

Weight: Weight refers to the gravitational attraction of the earth on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

الوزن هو قوة جذب الارض للجسم

W=mg

Measurements give **$g=9.8006\ m/s^2$**

Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2 kg body weights 19.62 N, and so on (Fig. 1-4).

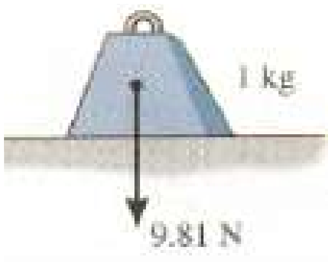


Fig. 1.4

1.3. Units of Measurement:

SI units: The International System of units. Abbreviated SI is a modern version, which has received worldwide recognition. As shown in Table 1.1. The SI system defines the length in meters (m), time in seconds (s), and mass in kilograms (kg). In the SI system the unit of force, the Newton is a derived unit. Thus, 1 Newton (N) is equal to a force required to give 1 kilogram of mass and acceleration of 1 m/s^2. النظام العالمي للوحدات هو النظام الاوسع انتشارا ويستخدم في مختلف دول العالم ويسمى النظام المتري

US customary: In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in Pounds (lb). The unit of mass, called a slug, 1 slug is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1 lb (1 slug = 1 lbs^2/ft). وهو نظام وحدات يستخدم بكثرة في بريطانيا والـــولايات المتحدة الامريــكيـــة

Table 1.1 Systems of Units				
Name	**Length**	**Time**	**Mass**	**Force**
International Systems of Units	*meter*	*seconds*	*kilogram*	*Newton**
SI	*m*	*s*	*kg*	*N* $\frac{kg.m}{s^2}$
US Customary	*foot*	*second*	*Slug**	*pound*
FPS	*ft*	*s*	$\frac{lb.s^2}{ft}$	*lb*
***Derived unit**				

Conversion of Units:

Table 1.2 provides a set of direct conversion factors between FPS and SI units for the basic quantities. Also in the FPS system, recall that:

1ft=12 in (inches), 1 mile=5280 ft, 1 kp (kilo pound) =1000 Ib, 1 ton=2000 Ib

Table 1.2 Conversion factors			
Quantities	**Unit of Measurement (FPS)**	**equals**	**Unit of Measurement (SI)**
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

البادئات **Prefixes**: When a numerical quantity is either very Large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table 1.3. Each represents a multiple or submultiples of a unit, which, if applied successively, moves the decimal point of a numerical quantity to every third place. For example, 4000000N=4000kN (kilo-Newton)=4MN (mega-Newton), or 0.005m=5mm (milli·meter).

Multiple المضاعفات الاعلى

Submultiple المضاعفات الاوطاء

البادئات Prefixes كلمة او رقم او حرف تضاف واحد قبل الاخر

Add (something) at the beginning as a prefix or introduction.

Table 1.3 Prefixes			
	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	K
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Exercises 1.1: Convert 2 km/h to m/s. How many ft/s is this?

Exercises 1.2: Convert the quantities 300 lb.s and 52 slug/ft^3 to appropriate SI units.

Exercises 1.3: Evaluate each of the following and express with SI units having an appropriate prefix: (a) (50 mN) (6 GN) (b) (400 mm) (0.6 MN) 2 (c) 45 MN3/900 Gg.

Exercise 1.4*:* Round off the following numbers to three significant figures: (a) 4.65735 m (b) 55.578 s (c) 4555 N (d) 2768 kg

Exercise 1.5: Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) μMN (b) N/μm (c) MN/ks2 (d) kN/ms.

Exercise 1.6: Represent each of the following combinations of units in the correct SI form: (a) Mg/ms (b) N/mm (c) mN/(kg. μs).

Exercise 1.7: A rocket has a mass of 250 103 slugs on earth. Specify (a) its mass in SI units and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is gm=5.30 ft/s2, determine to 3 significant figures (c) its weight in units, and (d) its mass in SI units.

Exercise 1.8: If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

Exercise 1.9: The Pascal (Pa) is actually a very small units of pressure. To show this, convert 1 Pa=1 N/m2 to lb/ft2. Atmospheric pressure at sea level is 14.7 lb/in2. How many Pascals is this?

Exercise 1.10: Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

Exercise 1.11: Determines the mass in kilograms of an object that has a weight of: (a) 20 mN (b) 150 kN (c) 60 MN

Chapter Two

Force Vectors

2.1. Scalar and vectors

A **scalar** is any **positive** or **negative physical quantity** that can be completely specified by its **magnitude**.الكمية القياسية هي اي كمية موجبة او سالبة والتي يمكن تعيينها بمعرفة مقدارها ووحدة قياسها

A **vector** is any physical quantity that requires both a **magnitude** and **direction** for its complete description. A vector is shown **graphically** by an **arrow**. The **length** of the arrow represents the **magnitude** of the vector, and a fixed axis defines the **direction** of its line of action. The **head** of the arrow indicates the **sense** of **direction of the vector** (Fig 2-1). الكميات المتجهة هي كميات فيزيائية يمكن تعيينها بمعرفة مقدارها العددي واتجاهها و نقطة تاثيرها ومحور عملها

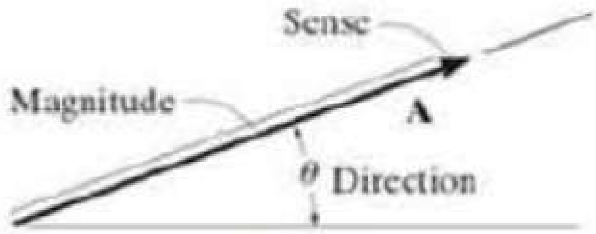
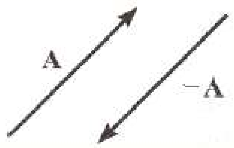


Fig. 2–1

For **handwritten** work, it is often **convenient** to denote a vector quantity by simply drawing an arrow on top it $\vec{A}$.
In **print**, vector quantities are represented by **bold** face letters such as **A**, and its **magnitude** of the vector is **italicized**, *A*.

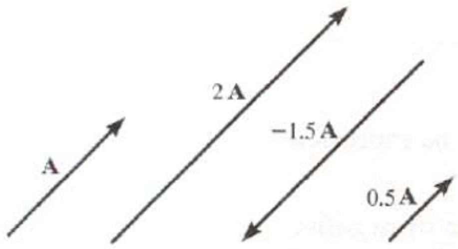
2.2. Vector operations

Multiplication and division of vector by a scalar: ضرب المتجهات
If a vector is **multiplied** by a **positive scalar**, its **magnitude is increased** by that amount. When **multiplied by a negative scalar** it will also **change the directional** sense of the vector (Fig 2-2).



Vector **A** and its negative counterpart

Fig. 2–2



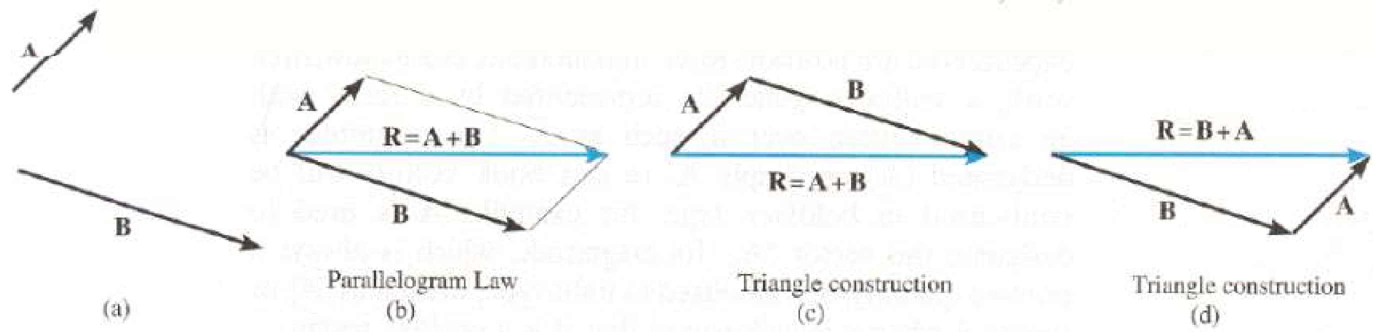
Scalar Multiplication and Division

Fig. 2–3

2.3. Vector addition:

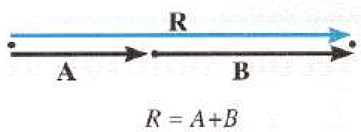
All vector quantities **obey** the **parallelogram law** of addition. Fig 2-3 and Fig 2-4 and Fig 2-5 illustrates the addition of vectors $\vec{A}$ and $\vec{B}$. to obtain a resultant **R**. جمع المتجهات بطريقة الرسم البياني قاعدة متوزازي الاضلاع

قاعدة المثلث
متجهات على نفس الخط



Vector Addition

Fig. 2–4



Addition of collinear vectors

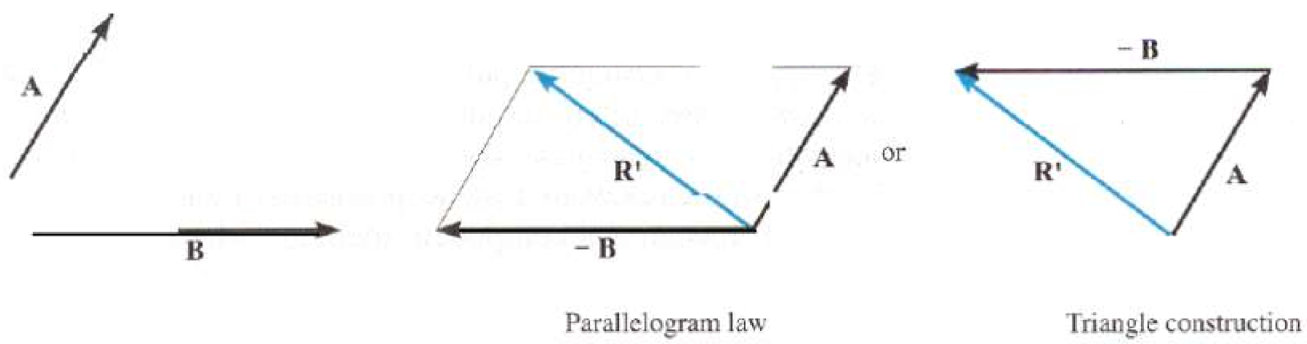
Fig. 2–5

2.4. Vector subtraction:

The resultant of the difference between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ of the same type may be expressed as:

$$\mathbf{R'=A-B=A+(-B)}$$

Fig 2-6 illustrates subtraction of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.



Vector Subtraction

Fig. 2–6

2.5. Vector addition of forces:

Experimental evidence has shown that a **force** is a **vector quantity** since it has a specified **magnitude**, **direction**, and **sense** and it **adds according to the parallelogram law.**
القوى عبارة عن متجهات

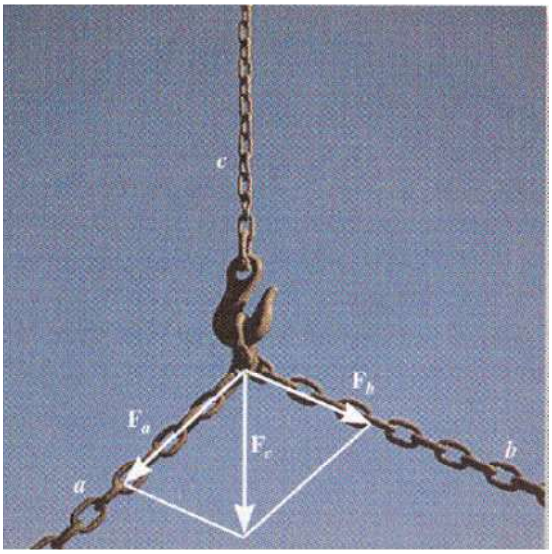


Fig. 2.7 تاثير القوى على دبوس

2.6. Finding a resultant force:

The two component forces $\vec{F}_1$ and $\vec{F}_2$ acting on the pin in Fig 2-7 can be added together to form the **resultant** force

$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2}$$

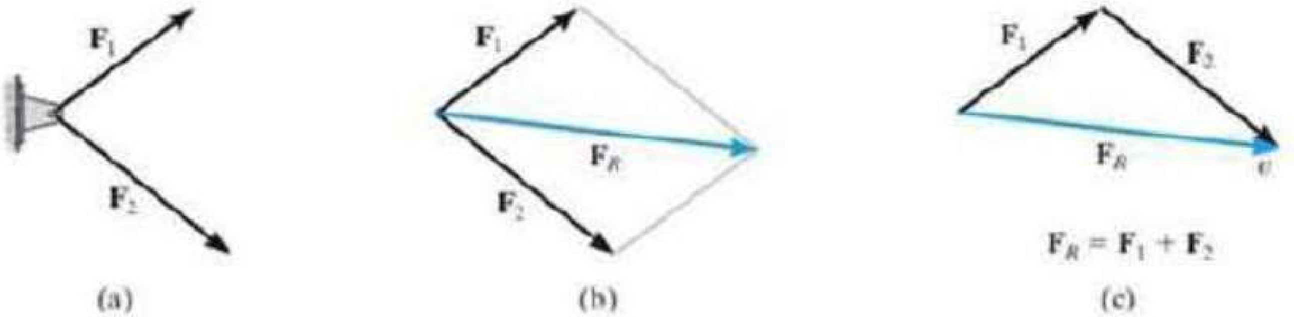


Fig. 2-7

2.7. Finding the components of a force:

Sometimes it is necessary to **resolve a force** into two components in order to **study its pulling and pushing effect** in two specific directions.

احيانا يتطلب تحليل القوى الى مركبتين لدراسة السحب او الدفع في اتجاهيين معينين

For example, in Fig 2.8, F is to be resolved into two components along two members, defined by *u* and *v* (Fig 2.8)

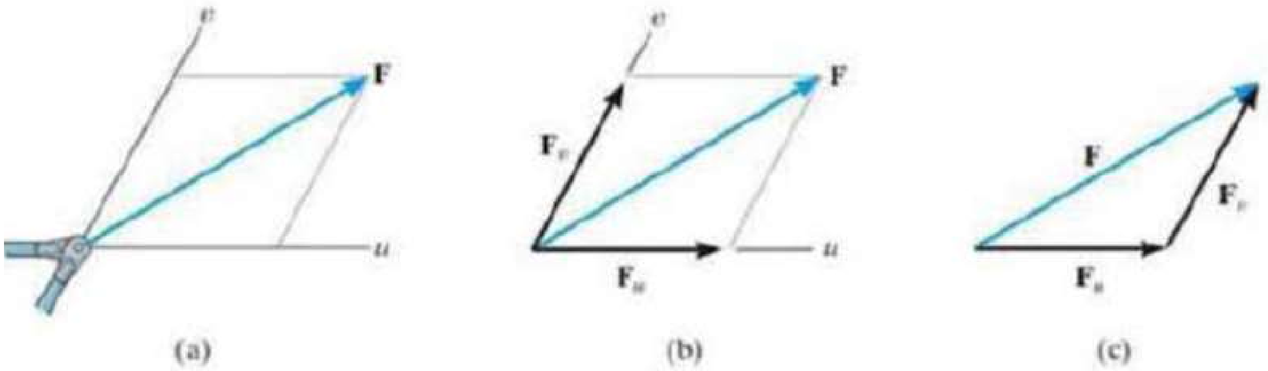


Fig. 2-8

2.8. Addition of several forces:

If **more than, two forces** are to be **added successive applications of the parallelogram law** can be carried out in order to obtain the resultant force. For example, if the three forces $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, $\vec{\mathbf{F}}_3$ **,** act at a point o, the **resultant** of any two of the forces is found ($\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$) and then this resultant is **added** to the **third force, yielding** the **resultant of all three forces** ($\vec{\mathbf{F}}_R = (\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2) + \vec{\mathbf{F}}_3$) (Fig 2-9).

استعمال طريقة الرسم البياني بشكل متتالي
هنا نجمع المحصلة الكلية $\vec{F}_1$ و $\vec{F}_2$ للحصول على المحصلة الكلية $\vec{F}_R$

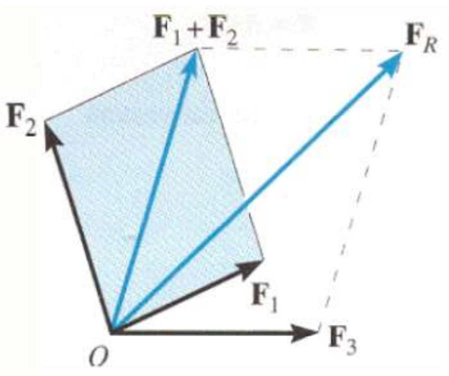


Fig. **2–8**

2.9. Trigonometry analysis:

Redraw a half portion of the parallelogram to illustrate the **triangular** head to tail addition of the components. From this triangle, the **magnitude of the resultant force** can be determined using **the law of cosines,** and its direction is determined from the **law of sines.** The magnitudes of two force components are determined from the law of Sines. The formulas are given in Fig 2-10

Cosine law:

الطريقة التحليلية تستخدم قاعدة الجيب والجيب تمام للمثلثات

$$C = \sqrt{A^2 + B^2 - 2ABCosc}$$

Sine law

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{sinc}$$

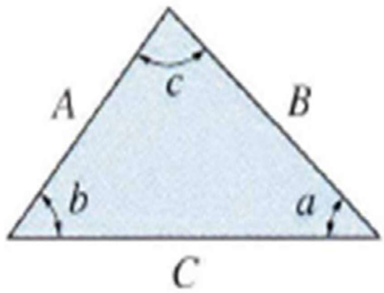
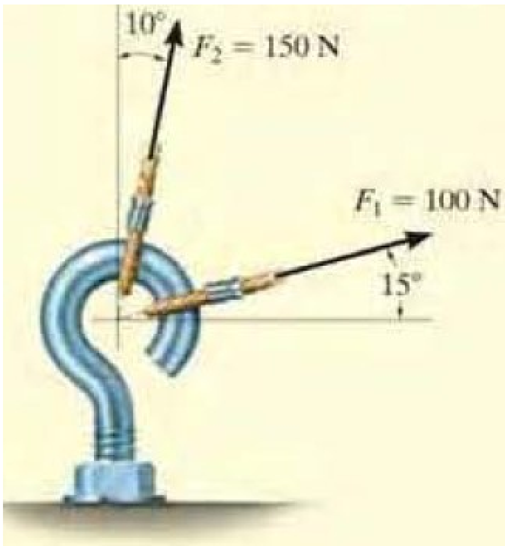


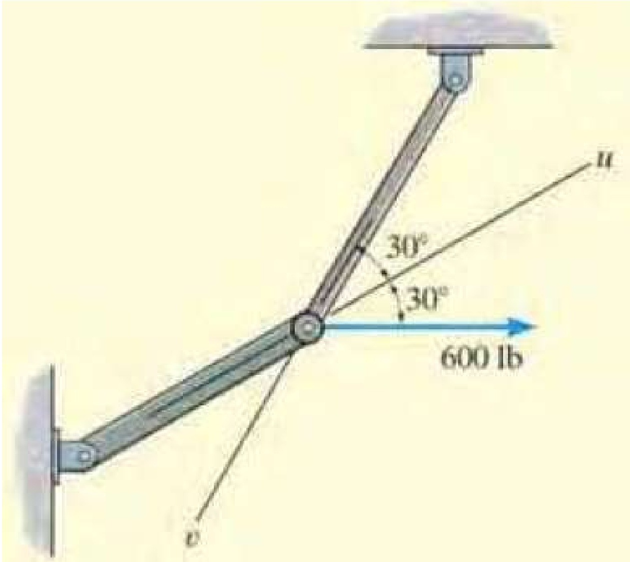
Fig. 2-10

Exercises.

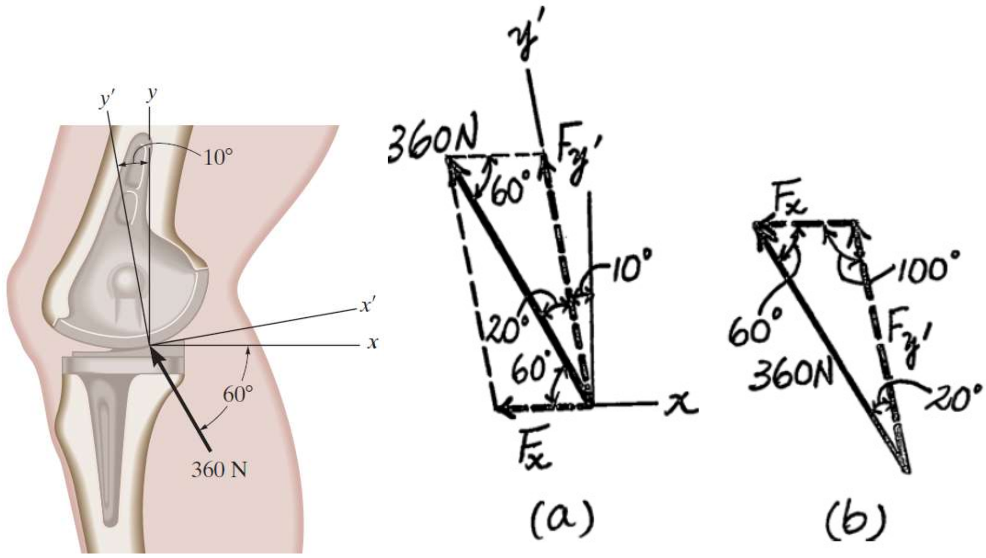
Ex. 2.1: The screw eye in Fig 2-11 is subjected to two forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. Determine the magnitude and direction of the resultant force.



Ex. 2.2: Resolve the horizontal 600lb force in fig 2.12 into components action along the *u* and *v* axes and determine the magnitudes of these components.



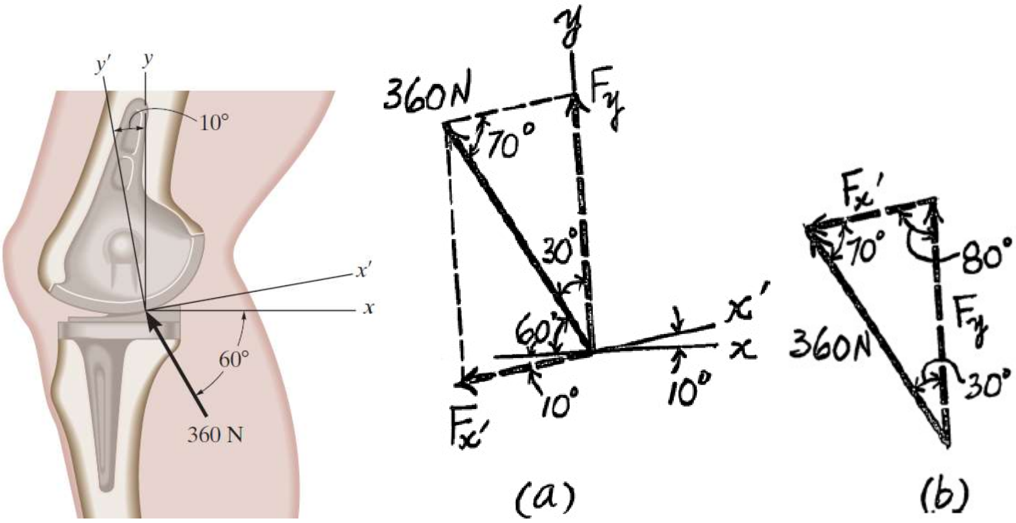
Ex. 2.12. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the *x* and *y*` axes.



$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ}; \qquad F_x = -125 \text{ N} \qquad \text{Ans}$$

$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ}; \qquad F_{y'} = 317 \text{ N} \qquad \text{Ans}$$

2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the *x*` and *y* axes.



$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183\ \text{N} \quad \textbf{Ans}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344\ \text{N} \quad \textbf{Ans}$$

2.10. Addition of a system of coplanar forces

When a force in resolved into two components along the *x* and *y* axes the components are then called **rectangular components**. The rectangular components of force F shown in Fig 2.23 are found using the parallelogram law, so that:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_x + \vec{\mathbf{F}}_y$$
$$F_x = F\cos\Theta$$
$$F_y = F\sin\theta$$

التمثيل هنا يكون بتحويلها الى مركبات سينية وصادية على المحاور الديكارتية

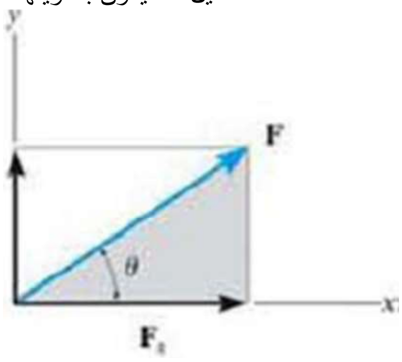
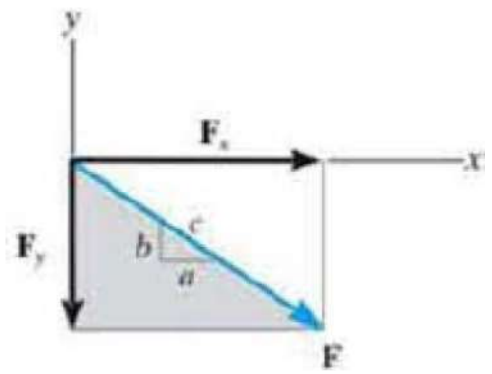


Fig 2-23

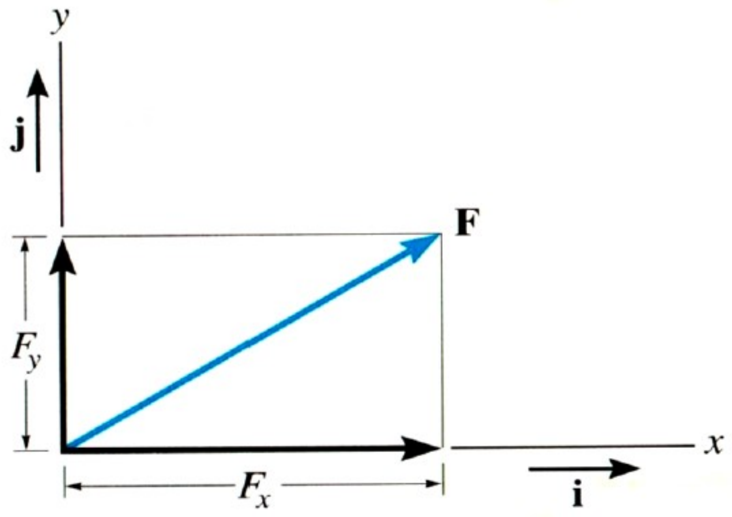
Instead of using the angle $\boldsymbol{\theta}$, the direction of $\vec{\mathbf{F}}$ can also be defined **using a small "slope" triangle**, such as shown in fig 2.24

هنا يمكن استغلال المثلث الصغير المقترن بلقوة لمعرفة مركبات القوة



$\frac{F_x}{F} = \frac{a}{c} \rightarrow F_x = F\left(\frac{a}{c}\right) F$ And $\frac{F_y}{F} = \frac{b}{c} \rightarrow F_y = -F\left(\frac{b}{c}\right) F$

It is also possible to represent the *x* and *y* components of a force in terms of **Cartesian unit vectors i** and **j** (Fig 2.25).



(a)
Fig. 2.25

We can express $\vec{\mathrm{F}}$ as a **Cartesian vector**.

$$\vec{F} = F_x\vec{\imath} + F_y\vec{\jmath}$$

In **coplanar force** resultant case, **each force is resolved** into its *x* and *y* components, and then the respective components are added using **scalar algebra** since they are collinear. For example, consider the three concurrent forces in Fig 2.26.

في حالة محصلة القوى المستوية, كل قوة ممكن تحليلها الى الاحداثيات x and y وبعدها تضاف الى القيم الاتجاهية المرتبطة بلقيمة لانها على خط واحد Collinear كما في الشكل

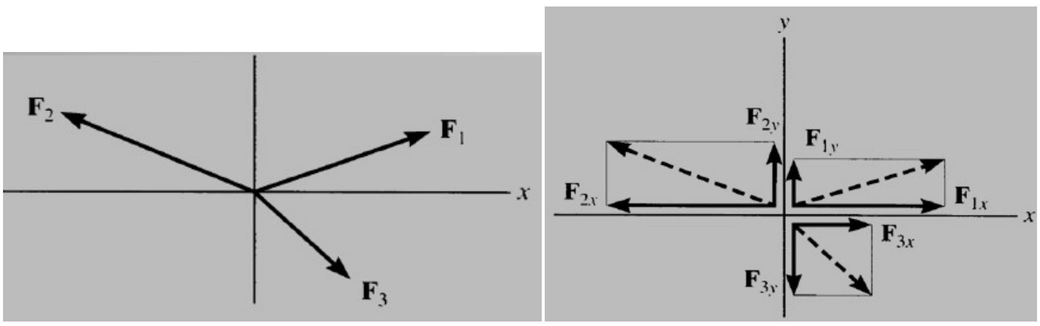


Fig. 2.26

Each force is represented as a Cartesian vector.

$$\vec{F}_1 = F_{1x}\,\vec{\imath} + F_{1y}\,\vec{\jmath}$$
$$\vec{F}_2 = -F_{2x}\,\vec{\imath} + F_{2y}\,\vec{\jmath}$$
$$\vec{F}_3 = F_{3x}\,\vec{\imath} - F_{3y}\,\vec{\jmath}$$

The vector resultant is therefore.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

We can **represent** the components of the resultant force of any number of coplanar forces symbolically by the **algebraic sum the x and y** components of all the forces.

$$\vec{F}_{Rx} = \sum F_x = F_{1x} - F_{2x} + F_{3x}$$
$$\vec{F}_{Rx} = \sum F_x = F_{1y} + F_{2y} + F_{3y}$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition as shown in Fig 2-27. **The magnitude of $\mathbf{F_R}$** is then found from the by **Pythagorean Theorem**: that is:

كمية وزاوية المحصلة يكون باستعمال نظرية بيثاغورث للمثلثات القائمة:

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$
$$\theta = \tan^{-1}\left|\frac{F_{Rx}}{F_{Rx}}\right|$$

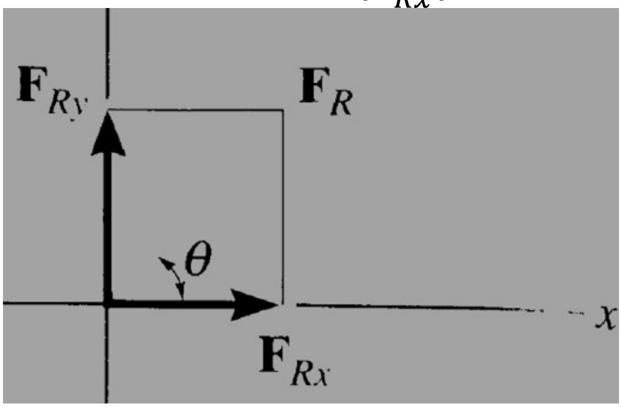
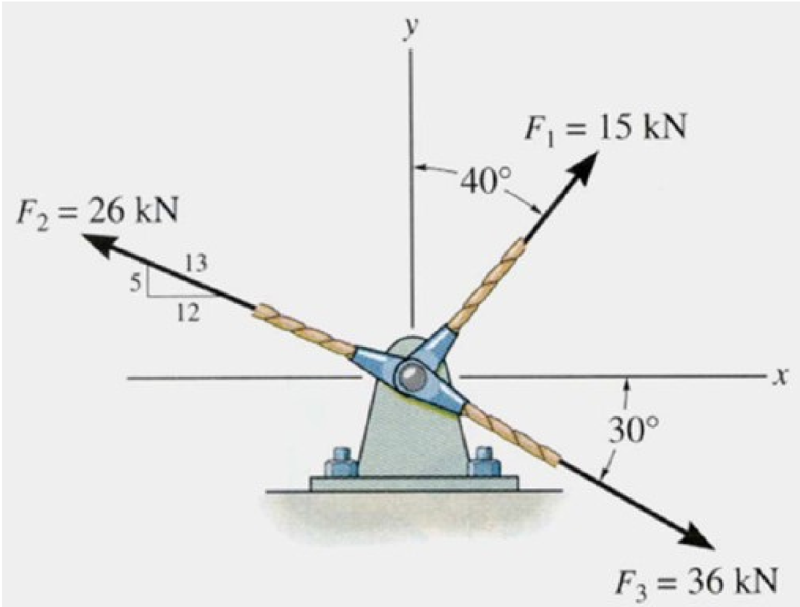


Fig. 2-27

Given: Three concurrent forces acting on a bracket.
Find: The magnitude and angle of the resultant force.



Plan:

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

$\mathbf{F_1}$ = { 15 sin 40° **i** + 15 cos 40° **j** } kN

= { 9.642 **i** + 11.49 **j** } kN

$\mathbf{F_2}$ = { -(12/13)26 **i** + (5/13)26 **j** } kN

= { -24 **i** + 10 **j** } kN

$\mathbf{F_3}$ = { 36 cos 30° **i** – 36 sin 30° **j** } kN

= { 31.18 **i** – 18 **j** } kN

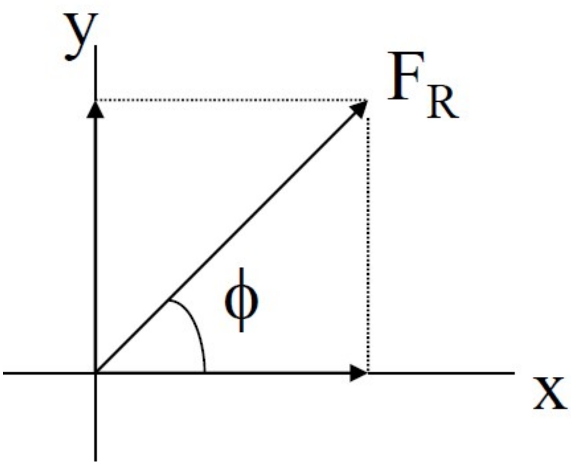
Summing up all the **i** and **j** components respectively, we get,

$\mathbf{F_R}$ = { (9.642 – 24 + 31.18) **i** + (11.49 + 10 – 18) **j** } kN

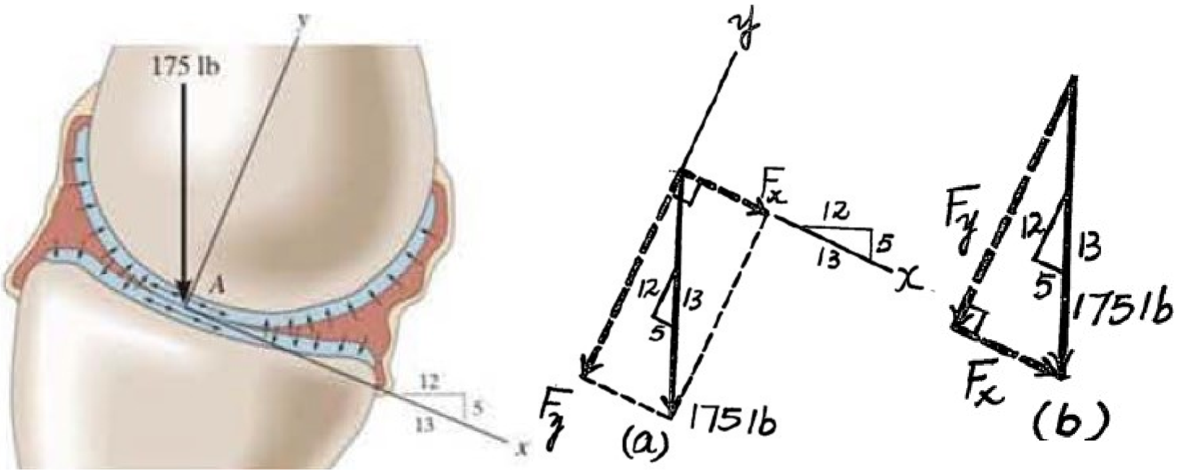
= { 16.82 **i** + 3.49 **j** } kN

$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2$ kN

$\phi = \tan^{-1}(3.49/16.82) = 11.7°$



Ex. The contact point between the femur and tibia bones of the leg is at *A*. If a vertical force of 175 lb is applied at this point, determine the components along the *x* and *y* axes. Note that the *y* component represents the normal force on the load-bearing region of the bones. Both the *x* and *y* components of this force cause synovial fluid to be squeezed out of the bearing space.



F_x=175(5/13)=67.3 Ib
F_y=-175(12/13)=-162 Ib

2.5. Cartesian vectors

A vector $\vec{\mathbf{A}}$ may have three rectangular components along the *x, y, z* coordinate axes and is represented by the vector sum of its **three rectangular components** (Fig 2-38).

القوى المؤثرة في البعد الثالث لها مركبات حسب المحاور x,y,z

$$\vec{\mathbf{A}}=\vec{\mathbf{A}}_x+\vec{\mathbf{A}}_y+\vec{\mathbf{A}}_z$$

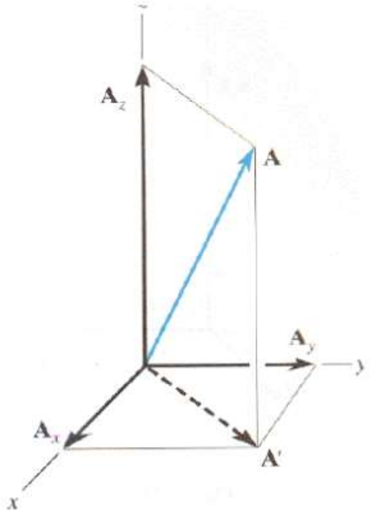


Fig. 2.38

In three dimensions, the set of Cartesian unit $\vec{\imath}, \vec{\jmath}$, $\vec{k}$ is used to designate the directions of the x, y, z axes, respectively. The positive Cartesian unit vectors are shown in Fig 2-39.

وحدات المحاور I,j,k تستخدم لتعيين الاتجاهات في المحاور x,y,z على التوالي. المتجهات الموجبة موضحة في الشكل الاتي:

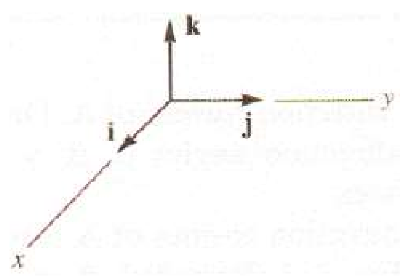


Fig. 2.39

We can write $\vec{\mathbf{A}}$ in **Cartesian vector** form as:

تستخدم متجهات الوحدة الديكارتية $\vec{\imath}, \vec{\jmath}, \vec{k}$ لتعريف المتجه $\vec{\mathbf{A}}$

$$\vec{\mathbf{A}}=A_x\vec{\imath}+A_y\vec{\jmath}+A_z\vec{k}$$

The **magnitude** of $\vec{\mathbf{A}}$ is expressed in Cartesian vector from as:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The **direction** of $\vec{\mathbf{A}}$ is defined by the **coordinate direction** angles α, β, and γ (Fig 2.40).

$$\text{Cos}\,\alpha = \frac{A_x}{A} \qquad \text{Cos}\,\beta = \frac{A_y}{A} \qquad \text{Cos}\,\gamma = \frac{A_z}{A}$$

With

$$cos^2\alpha + cos^2\beta + cos^2\gamma=1$$

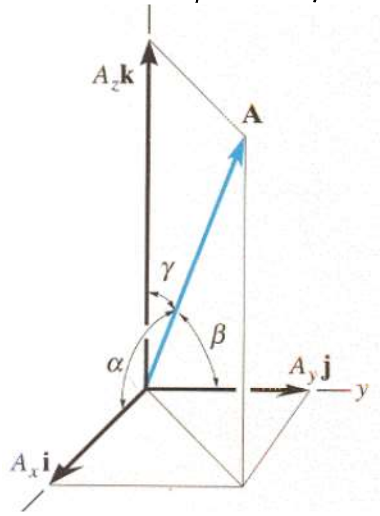


Fig. 2.40

The addition (or subtraction) of two or more vectors is greatly simplified in terms of their Cartesian components. For example, the resultant $\vec{\mathbf{R}}$ in Fig 2.41 is written as:

حالة جمع متجهين في الفضاء:

$$\vec{\mathbf{R}} = (A_x + B_x)\vec{\imath} + (A_y + B_y)\vec{\jmath} + (A_z + B_z)\vec{k}$$

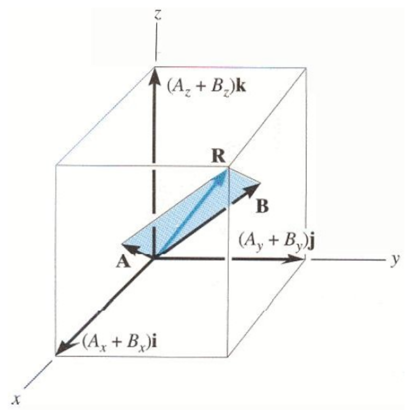
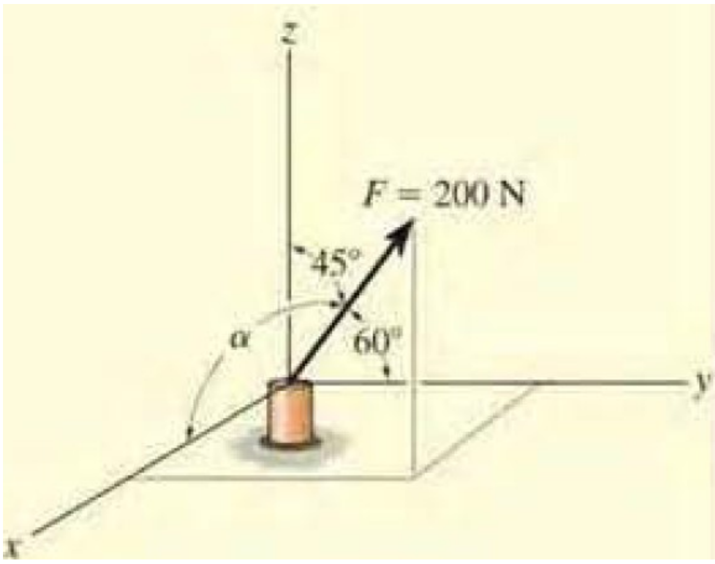


Fig. 2.41

If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as:

$$\overrightarrow{F_R} = \sum \vec{F} = \sum F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

Ex. Express the force F in Fig as a Cartesian Vector



Since two coordinates direction angle are specified, α third angle must be determined

$\cos^2 \alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 \alpha + \cos^2 60^o + \cos^2 45^o = 1$

$\cos^2 \alpha = \sqrt{1-(0.5)^2-(0.707)^2} = \pm 0.5$

Hence, two positions exist, namely:

$$\alpha = \cos^{-1}(0.5) = 60^{\circ}$$

Or

$$\alpha = \cos^{-1}(-0.5) = 120^{\circ}$$

By inspection it is necessary that α=60°, since $\mathbf{F}_x$ must be in the +x direction. Using eq 2-9 with F=200 N, we have:

F=*F*cosα**i**+Fcosβ**j**+*F*cosγ**k**

=(200 cos60° N)**i**+(200 cos 60° N)**j**+(200 cos 45° N)**k**

={100.0**i**+100.0**j**+141.4**k}N**

Show that indeed the magnitude of F=200 N.

2.6. Position Vectors

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2-48. This vector is also designated by the **symbol *r***. As a smaller convention اتفاقية, we will sometimes refer to this vector with two subscripts to indicate from and to the point where it is directed. Thus, ***r*** can also be designated as $\boldsymbol{r}_{AB}$. Also, note that $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$ in Fig. 2-48, are referenced with only one subscript since they extend from the origin of coordinates.

From Fig. 2-48, by the **head-to-tail vector** addition, using the **triangle** We require:

$\overrightarrow{r_A} + \vec{r} = \overrightarrow{r_B}$

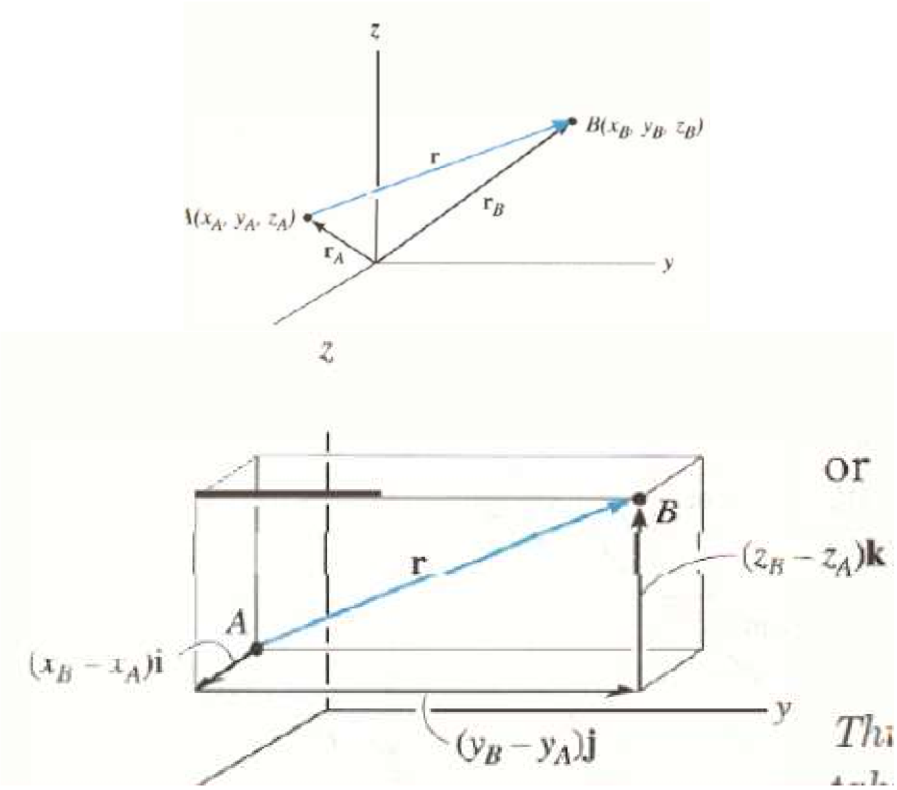


Fig. 2-48

Solving for $\vec{r}$ and expressing $\overrightarrow{r_A}$ and $\overrightarrow{r_B}$ in Cartesian vector form yields:

$$\vec{r} = \overrightarrow{r_B} - \overrightarrow{r_A} = (x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} + (z_B - z_A)\vec{k}$$

Ex.

2.7. Dot Product (scalar product)

the dot Product of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ written $\vec{\mathbf{A}}.\vec{\mathbf{B}}$ and read $\vec{\mathbf{A}}$ dot $\vec{\mathbf{B}}$ is defined as the product of the magnitudes of A and B and the cosine of the angle θ between their tails (Fig 2.50).

$\vec{\mathbf{A}}.\vec{\mathbf{B}} = ABcos\theta$ (2.1)

Where

$$0° \leq \theta \leq 180°$$

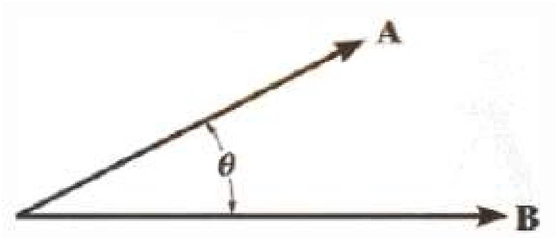


Fig. 2.50

Equation 2.1 must be used to find the dot product for any two Cartesian unit vectors.

For example:

$\vec{\imath}.\vec{\imath} = (1)(1)cos0°$ $\vec{\jmath}.\vec{\jmath} = 1$ $\vec{k}.\vec{k} = 1$

$\vec{\imath}.\vec{\jmath} = (1)(1)cos90°$ $\vec{\jmath}.k = 0$ $\vec{\jmath}.\vec{k} = 0$

If we want to find the dot product of two general vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ that are expressed in Cartesian vector form, then we have:

$$\vec{\mathbf{A}}.\vec{\mathbf{B}} = (A_x\vec{\imath} + A_y\vec{\jmath} + A_k\vec{k})(B_x\vec{\imath} + B_y\vec{\jmath} + B_k\vec{k})$$

$$\vec{\mathbf{A}}.\vec{\mathbf{B}} = A_xB_x(\vec{\imath}.\vec{\imath}) + A_xB_y(\vec{\imath}.\vec{\jmath}) + A_xB_z(\vec{\imath}.\vec{k}) + A_yB_x(\vec{\jmath}.\vec{\imath}) + A_yB_y(\vec{\jmath}.\vec{\jmath}) + A_yB_y(\vec{\jmath}.\vec{k}) + A_zB_x(\vec{k}.\vec{\imath}) + A_zB_y(\vec{k}.\vec{\jmath}) + A_zB_z(\vec{k}.\vec{k}) +$$

$$\vec{\mathbf{A}}.\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z \qquad (2.2)$$

We deduce that the angle forces between two vectors can be written as:

$$\theta = cos^{-1}(\frac{\vec{\mathbf{A}}.\vec{\mathbf{B}}}{AB})$$

Where

$$0° \leq \theta \leq 180°$$

We note that if:

$\vec{\mathbf{A}}.\vec{\mathbf{B}} = 0 \Longrightarrow\ \theta = cos^{-1}0 = 90$

So that $\vec{\mathbf{A}}$ will be perpendicular to $\vec{\mathbf{B}}$.

In the case of line *a* as shown in figure 2-51, and if the direction of the line is specified by the unit $\overrightarrow{\mathbf{u_a}}$ **a** then since $\overrightarrow{u_a} = 1$, we can determine the magnitude of $\mathbf{A_a}$ directly from the dot product

$$A_a = Acos\theta$$

$$\overrightarrow{A_a}\overrightarrow{u_a} = A.1.cos\theta = Acos\theta \Longrightarrow \quad A_a = \overrightarrow{A_a}\overrightarrow{u_a}$$

Notice that if this result is positive, then $\overrightarrow{\mathbf{A_a}}$ has a directional sense which is the same as , whereas if A_a is a negative scalar, then $\overrightarrow{\mathbf{u_a}}$ has the opposite sense of direction $\overrightarrow{u_a}$. The component $\overrightarrow{\mathbf{A_a}}$ represented as a vector is therefore:

$$\overrightarrow{A_a} = A_a\overrightarrow{u_a}$$

The component of $\vec{\mathbf{A}}$ that is perpendicular to line a ($\vec{\mathbf{A}}\ \perp$) can also be obtained from Figure 2-51. Therefore

$\vec{\mathbf{A}} \perp = \mathrm{A}\sin\theta$ with $\theta = cos^{-1}(\frac{\vec{\mathbf{A}}.\overrightarrow{u_a}}{AB})$

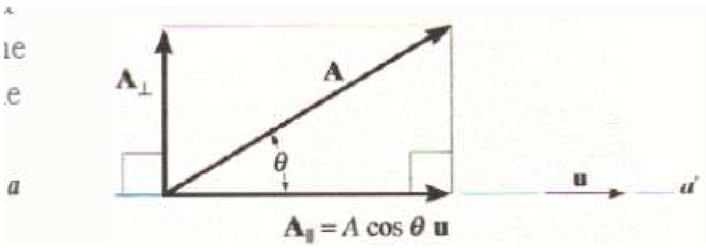


Fig. 2.51

Alternatively as if A_a is known then by Pythagorean's theorem we can also write:

$$\mathbf{A}_{\perp} = \sqrt{\mathbf{A}^2 - \mathbf{A}_a^2}$$

Ex.

Equilibrium of a Particle

3.1. Condition for the equilibrium of a particle

A particle is said to be in equilibrium if it **remains at rest** if **originally at rest, or has a constant velocity** if **originally in motion**. To maintain equilibrium, it is necessary to satisfy Newton's first law of motion which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as:

$$\sum \vec{F} = 0 \qquad (3.1)$$

شرط توازن الجسم هو ان تكون محصلة القوى الخارجية المؤثرة عليه تساوي صفرا

Where $\sum \vec{F}$ is the vector sum of all the forces acting on the particle.

3.2. The free body diagram

A **drawing** that **shows** the particle with **all the forces** that act on it is called a **free body diagram** (**FBD**).

مخطط الجسم الحر هو تمثيل تصويري يستخدم لتحليل القوى المؤثرة على الجسم الحر.

We will consider **a springs connections** often encountered in particle equilibrium problems.

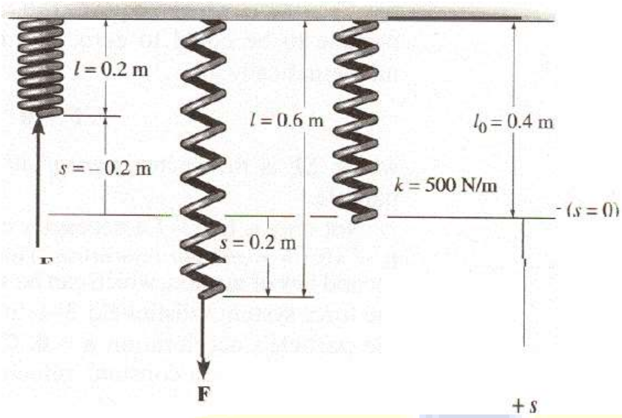
Springs: If a linearly elastic spring of undeformed length l_0 is used to support a particle, **the length of the spring will change in direct proportion to the force F acting on it**, Fig 3.1. A **characteristic** that defines the **elasticity of a spring** is the **spring constant** or **stiffness** *k*. The magnitude of force exerted on a linearly elastic spring is stated as:

النوابض ذات الملف الحلزوني هي عناصر مرنة تقاوم القوى الضاغطة المطبقة باتجاه محورها، وهي تخضع لقانون هوك للمرونة الذي يعتبر بان الامتداد الناتج يتناسب مباشرة مع الحمل.

$$F = K \times S$$

Where:

$$s = l - l_o$$



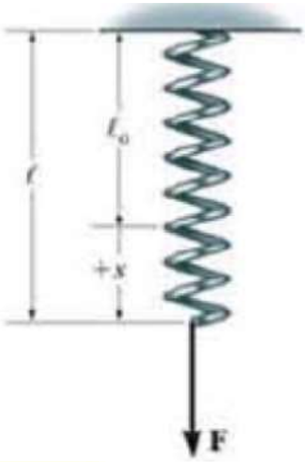


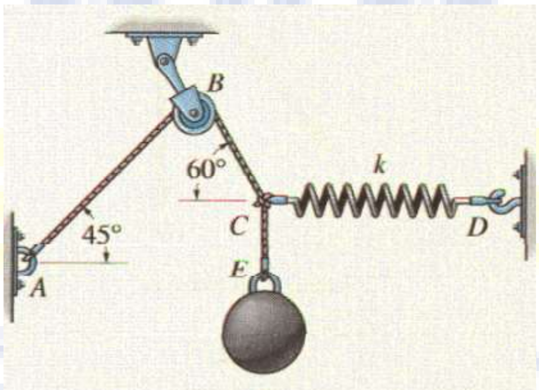
Fig. 3.1

S هو الفرق بين موضع الجسم الجديد وموقعه الاصلي. K هو ثابت المرونة.

Ex. P 84

The following example shows **a drawing of the free body diagram** of a sphere; The sphere in Figure below has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord CE, and the knot at c.

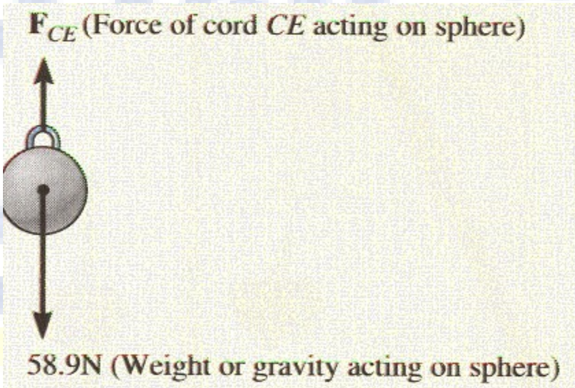
يظهر مخطط الجسم الحر قوى التلامس المؤثرة على الجسم الكروي والحيل و العقدة C .



(a)

Solution

Sphere By inspection, there are only two forces acting on the sphere. Namely, its weight and the force of cord CE. The sphere has a weight **d 6** kg (9.81 m/s²) = 58.9 N. The force-body diagram is shown in ***Fig. 3-36.***



(b)

Cord CE. When the cord CE is isolated from its surroundings, its free body diagram shows only two forces acting on it. Namely, the force of the sphere and the force of the knot, Fig. 3-3. Notice that FCE shown here is equal but opposite to that shown in b, and a, a consequence of Newton's third law. In addition, FCE and FEC pull on the cord and keep it in tension so that it collapse. For equilibrium, FCE = FEC.

Knot. The knot at C is subjected to three forces, Fig. 3.3d. They are caused by the cords CBA and CE and the spring CD. As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord subjects the knot to this force.

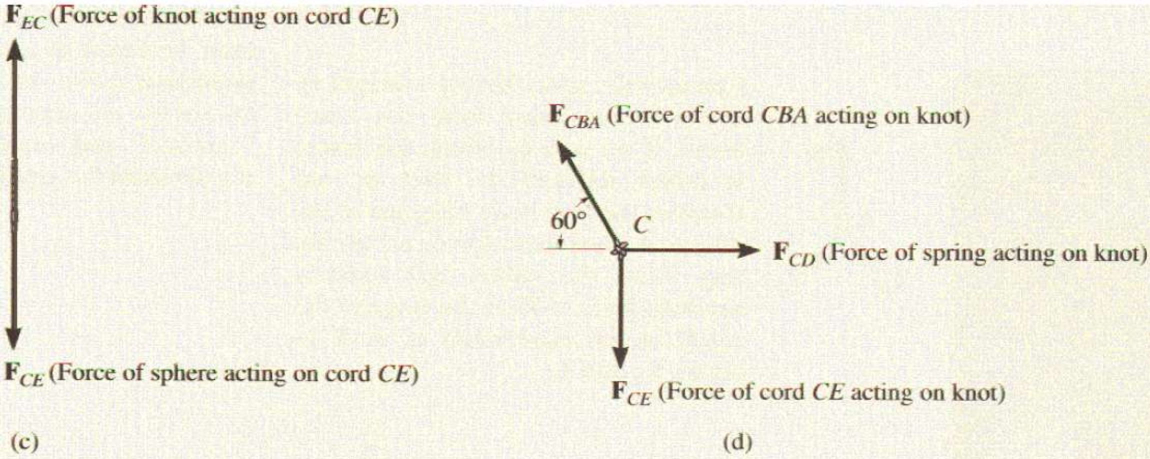


Fig. 3.3

3.3. Coplanar force systems

If a particle is subjected to a system of coplanar forces as in Fig 3-4, then each force can be resolved into it is $\vec{\imath}$ an $\vec{\jmath}$ components. **For equilibrium, these forces must sum to produce a zero free resultant**.

$$\sum \vec{F}=0$$

$$\sum F_x \vec{\imath} + \sum F_y \vec{\jmath}=0$$

بعد تحليل القوى الى مركباتها الافقية والعمودية يمكن كتابة معادلات الاتزان الجسم في اتجاه كل محور.

Hence,

$$\sum F_x=0$$

مجموع القوى في اتجاه المحور x تساوي 0.

$$\sum F_y=0$$

مجموع القوى في اتجاه المحور y تساوي 0.

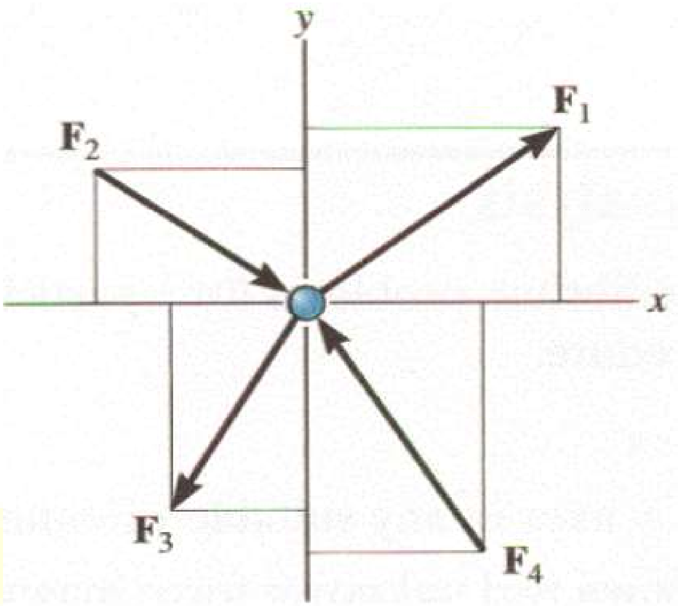
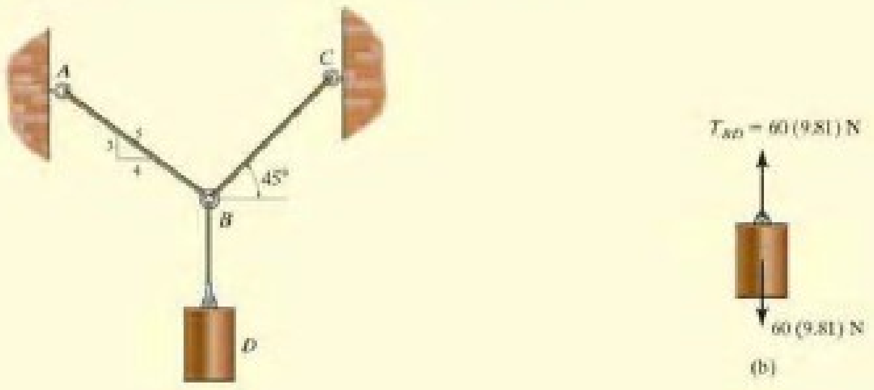


Fig. 3-4

Ex. 3.2

EXAMPLE 3.2

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3-6a.



(a)

(b)

SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be $T_{BD} = 60(9.81)$ N, Fig. 3-6b. The forces in cables BA and BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig. 3-6c. The magnitudes of $\mathbf{T}_A$ and $\mathbf{T}_C$ are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45° - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad T_C \sin 45° + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45° + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

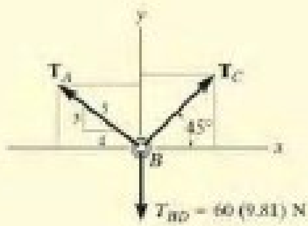
So that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.



(c)

Fig. 3-6

3.4. Three dimensional force systems

In the case of three dimensional force system, as in fig 3.10, we can resolve the forces into their respective, $\vec{\imath}, \vec{\jmath}, \vec{k}$ components **For equilibrium**, so that:

$$\sum F_x\,\vec{\imath} + \sum F_y\,\vec{\jmath} + \sum F_k\,\vec{k} = 0$$

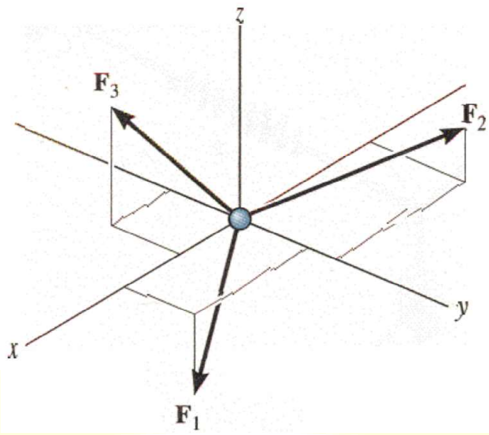
بعد تحليل القوى الى مركباتها الثلاثية الابعاد يمكن كتابة معادلات الاتزان الجسم في اتجاه كل محور.

To satisfy this equation we require:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

مجموع القوى في اتجاه x تساوي 0

مجموع القوى في اتجاه y تساوي 0
مجموع القوى في اتجاه z تساوي 0



Ex. 3.5

EXAMPLE 3.5

A 90-lb load is suspended from the hook shown in Fig. 3–10a. If the load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3–10b.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be used. Considering components directed along each positive axis as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30° - \left(\tfrac{4}{5}\right)F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30° + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \left(\tfrac{3}{5}\right)F_C - 90 \text{ lb} = 0 \qquad (3)$$

Solving Eq. (3) for F_C, then Eq. (1) for F_D, and finally Eq. (2) for F_B, yields

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 207.8 \text{ lb} \qquad \textit{Ans.}$$

The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

NOTE: Since the results for all the cable forces are positive, each cable is in tension; that is, it pulls on point A as expected, Fig. 3–10b.

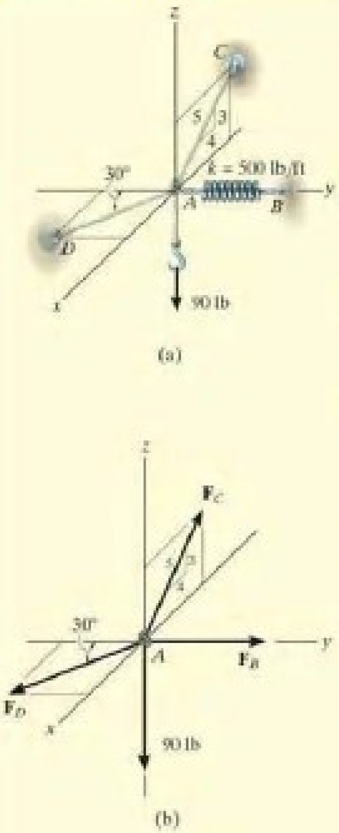
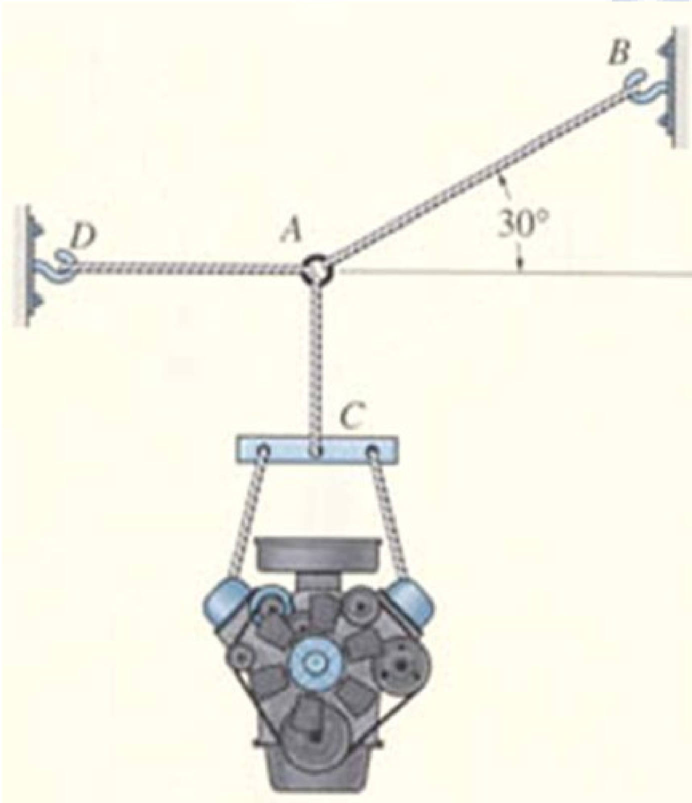
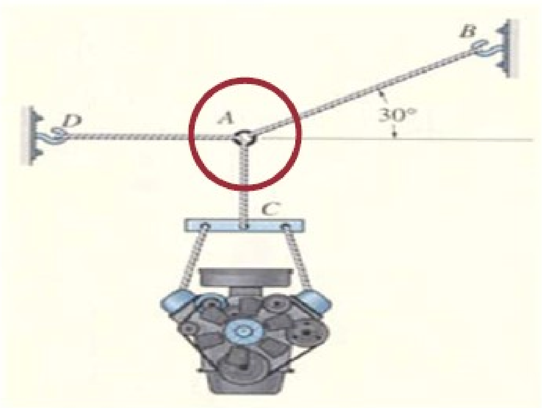


Fig. 3–10

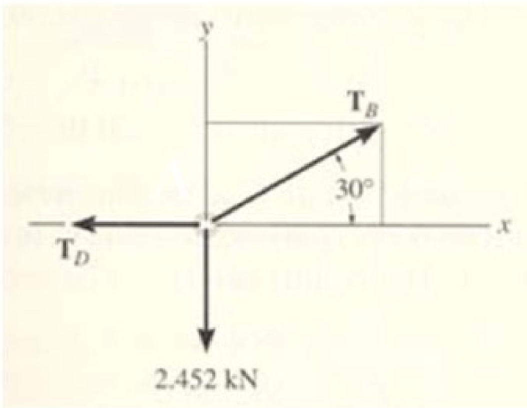
Example: This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium.
To determine the tensions in the cables for a given weight of the engine, we need to learn how to draw a free body diagram and apply equations of equilibrium.



How?
1. Imagine the particle to be isolated or cut free from its surroundings.
2. Show all the forces that act on the particle.
Active forces: They want to move the particle.
Reactive forces: They tend to resist the motion.
3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and/or directions as variables.



Note : Engine mass = 250 Kg



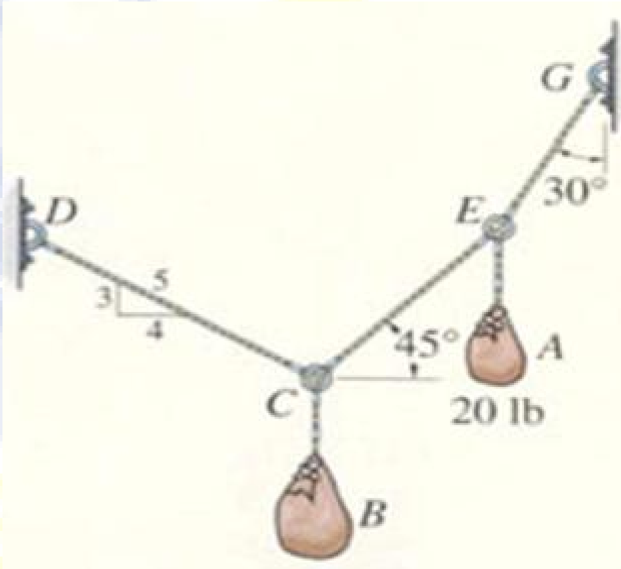
FBD at A

EQUATIONS OF 2-D EQUILIBRIUM
Since particle A is in equilibrium, the net force at A is zero.
So $\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} = 0$
or $\Sigma \mathbf{F} = 0$

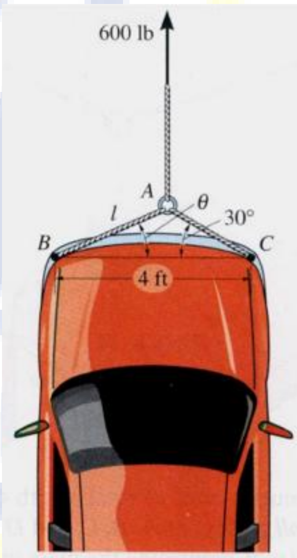
In general, for a particle in equilibrium, $\Sigma \mathbf{F} = 0$ or
$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = 0 = 0\mathbf{i} + 0\mathbf{j}$ (A vector equation)

Or, written in a scalar form,
$\Sigma F_x = 0$ and $\Sigma F_y = 0$
These are two scalar equations of equilibrium (EofE). They can be used to solve for up to two unknowns.

Write the scalar EofE:
$+ \rightarrow \Sigma F_x = T_B \cos 30^\circ - T_D = 0$
$+ \uparrow \Sigma F_y = T_B \sin 30^\circ - 2.452 \text{ kN} = 0$
Solving the second equation gives: $T_B = 4.90$ kN
From the first equation, we get: $T_D = 4.25$ kN

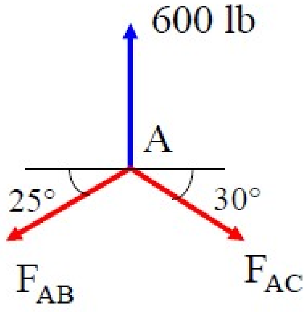


Example: Given: Sack A weighs 20 lb. and geometry is as shown.
Find: Forces in the cables and weight of sack B.
Plan:
1. **Draw** a FBD for Point E.
2. **Apply** EofE at Point E to solve for the unknowns (TEG & TEC).
3. **Repeat** this process at C.



Example: Given: The car is towed at constant speed by the 600

lb force and the angle θ is 25°.
Find: The forces in the ropes AB and AC.
Plan:
1. Draw a FBD for point A.
2. Apply the E-of-E to solve for the forces in ropes AB and AC.



FBD at point A

Applying the scalar E-of-E at A, we get;

$+ \rightarrow \sum F_x = F_{AC} \cos 30° - F_{AB} \cos 25° = 0$

$+ \rightarrow \sum F_y = -F_{AC} \sin 30° - F_{AB} \sin 25° + 600 = 0$

Solving the above equations, we get;

$F_{AB} = 634$ lb

$F_{AC} = 664$ lb

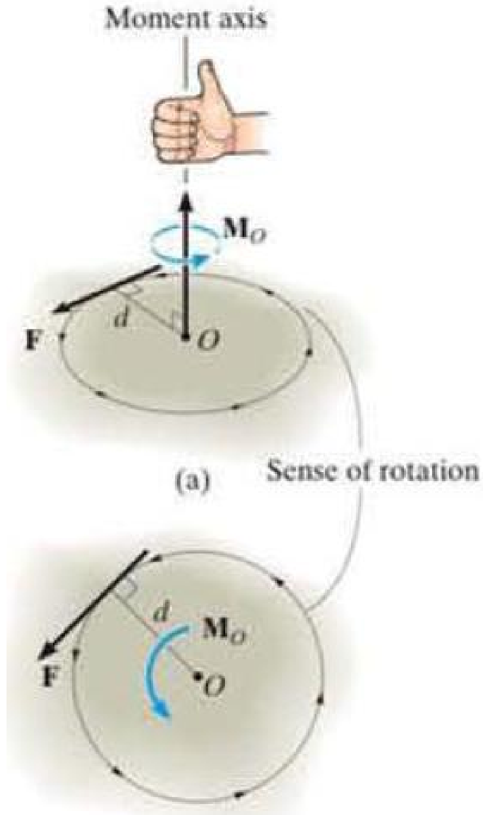
1. Moment of a force scalar formulation

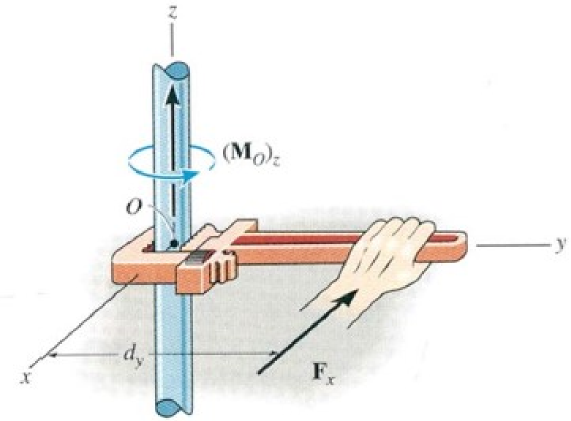
When a force is applied to a body, it will produce tendency for the body to rotate. Called sometime Torque, but most often it is called $\overrightarrow{M}_o$the moment of a force or simply the moment. The handle of the wrench it will tend to turn the bolt about point 0. The magnitude of the moment proportional to the force F, and the perpendicular distance or moment arm *d*.

The $\overrightarrow{M}_o$moment about **point O**, or about an axis passing through O and perpendicular to the plane, **is a vector quantity** since it has a specified **magnitude and direction** (fig 4-1).

The magnitude of $\overrightarrow{M}_o$ is:

$M_o = F.d$





Where *d* **is the moment arm** or **perpendicular distance from the axis at point O** to the line of action of the force. **Units** of moment are **N.m** or **lb.ft**.

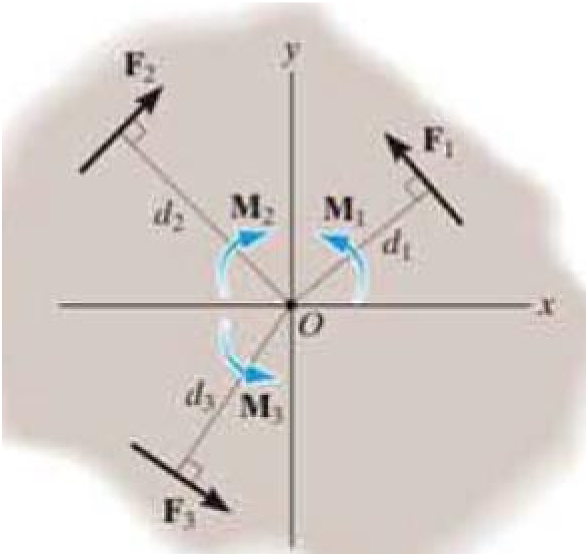
The **direction** of $\vec{\boldsymbol{M}}_o$ is defined by its moment axis which is **perpendicular** to the **plane** that contains the force **F** and its moment arm *d*. The **right-hand rule** is used establish the sense of the direction of $\vec{M}_o$.

For two-dimensional problems, where all the forces lie within the *x-y* plane, fig 4-2, the resultant moment $\left(\vec{M}_R\right)_o$ **about point O** (the z-axis) can be determined by finding the **algebraic sum of the moments** caused by all the forces in the system. As a **convention,** we will generally consider **positive moments** as a **counterclockwise** since they are directed along the positive z-axis (out of page).

Clockwise moments will be **negative**.

Using the sign convention, the resultant moment in fig 4-3 is therefore:

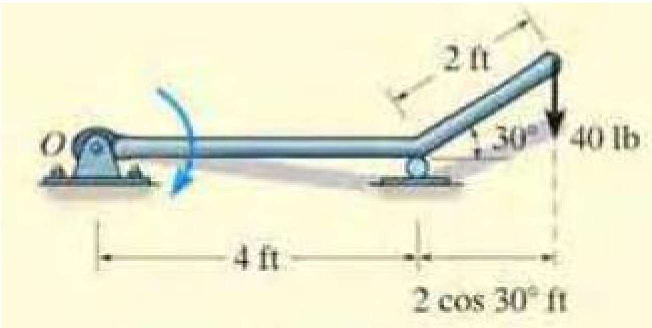
$(M_R)_0 = \Sigma Fd$

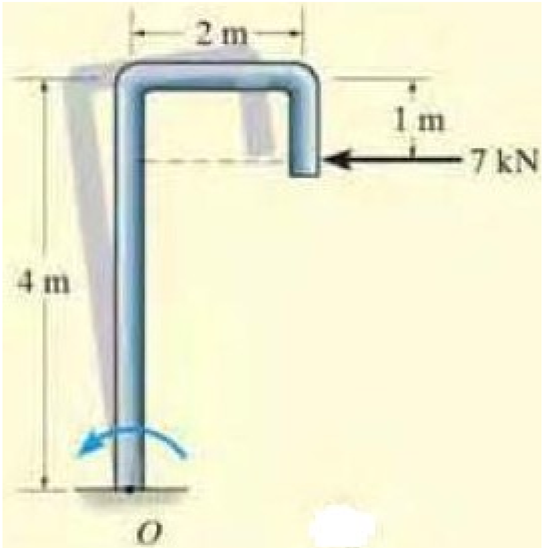


$(M_R)_0 = F_1 d_1 - F_2 d_2 + F_3 d_3$

Examples

For each case illustrated in Fig. 4-4. Determine the moment of the force about point O.





2. Cross product

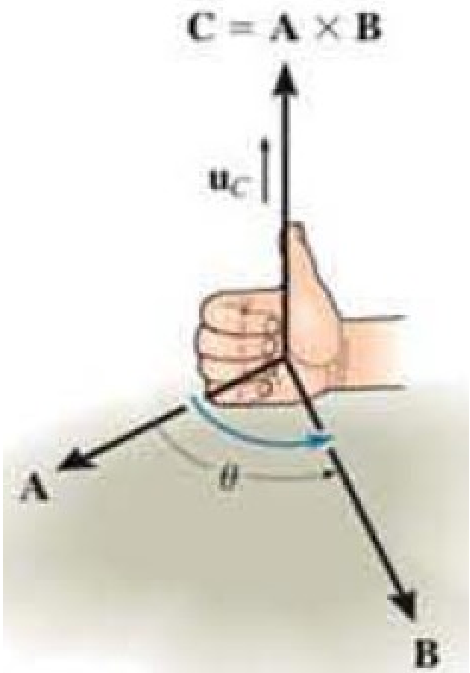
The cross product of two vectors $\vec{A}$ and $\vec{B}$ yields the vector C that is written:

$$\vec{C} = \vec{A} x \vec{B}$$

and read $\vec{C}$ equals $\vec{A}$ cross $\vec{B}$

The magnitude of $\vec{C}$ is defined as the product of the magnitude $\vec{A}$, and $\vec{B}$, and the sine of angle ɵ between their tails (0≤ɵ≤180), thus:

C=AB Sine ɵ



$\vec{\mathbf{C}}$ has a direction that is perpendicular to the plane containing $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ such that $\vec{\mathbf{C}}$ is specified by the right-hand rule.

Knowing both the magnitude and direction of $\vec{\mathbf{C}}$, we can write

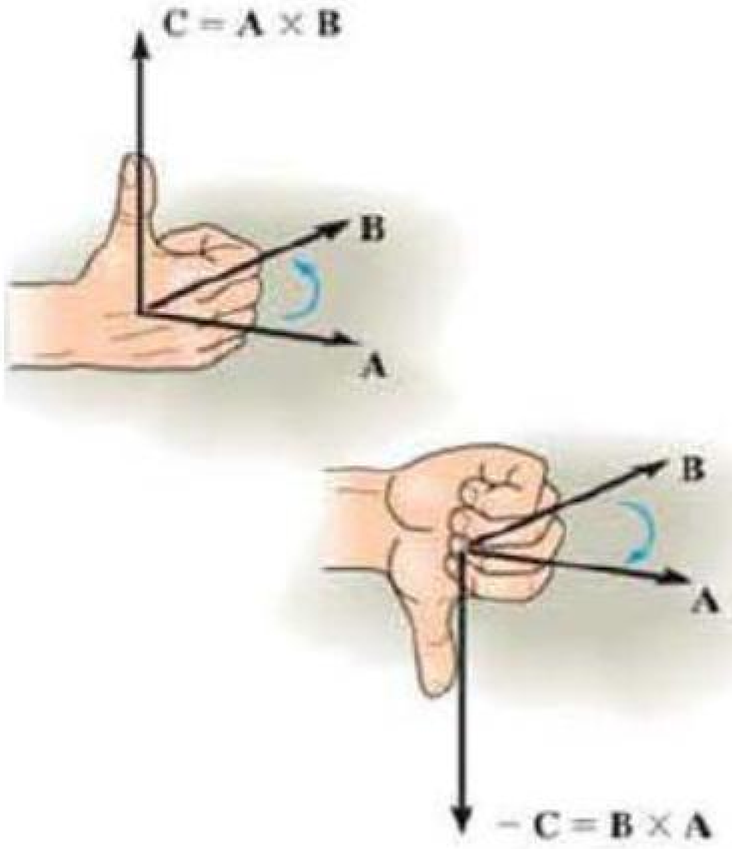
$$\vec{\mathbf{C}} = \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} = (A\,B\,\sin\theta)\overrightarrow{\mathbf{u_c}}$$

Where the scalar *(A B* sin θ) defines the magnitude of $\vec{\mathbf{C}}$ and the unit vector $\overrightarrow{\mathbf{u_c}}$ defines the direction of $\vec{\mathbf{C}}$ (fig 4-4).

Laws of operation:

$\vec{A}x\vec{B} \neq \vec{B}x\vec{A}$

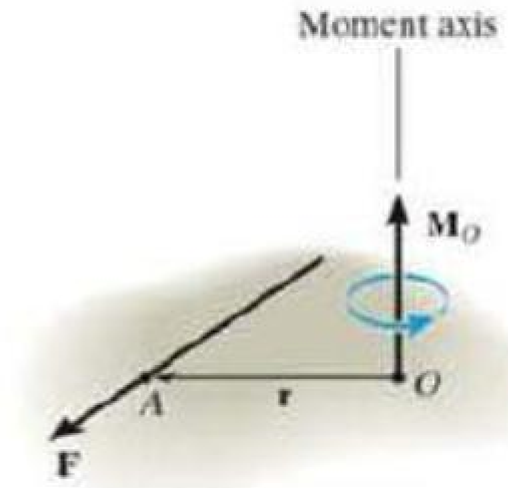
$\vec{A}x\vec{B} = -\vec{B}x\vec{A}$ (**Commutative law is not valid**)



3. Moment of a force – vector formulation

The moment of a force F about a point O (fig 4-8) can be expressed using the vector cross product namely:

$$\vec{M}_o = \vec{r} \times \vec{F}$$



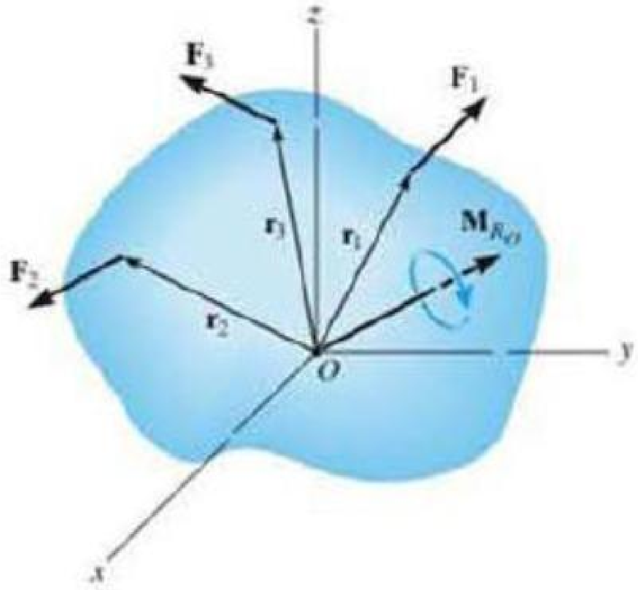
Here $\vec{r}$ represents a position vector direct from O to any point on the line of action of $\vec{\boldsymbol{F}}$. The magnitude of the cross product is defined from Eq. 4-3 as:

$$M_o = rF \sin \theta$$

4. Resultant Moment of a system of forces

If a body is acted upon by a system of forces (fig 4-11), the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as:

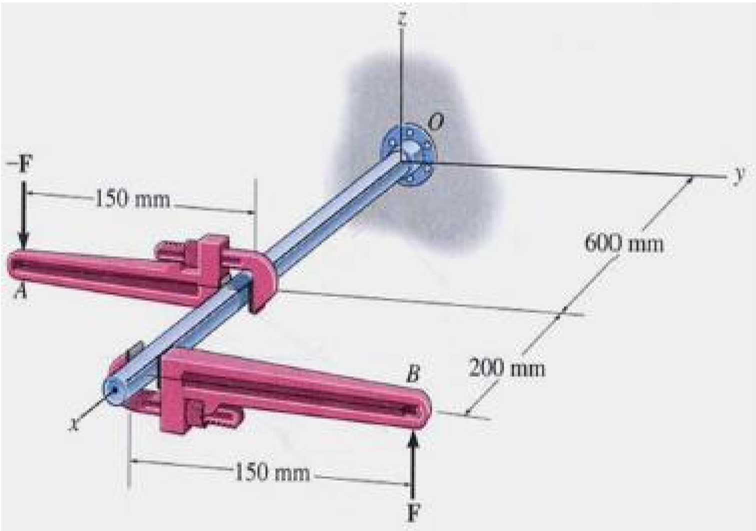
$$\overrightarrow{M_{Ro}} = \sum (\vec{r} x \, \vec{F} \,)$$

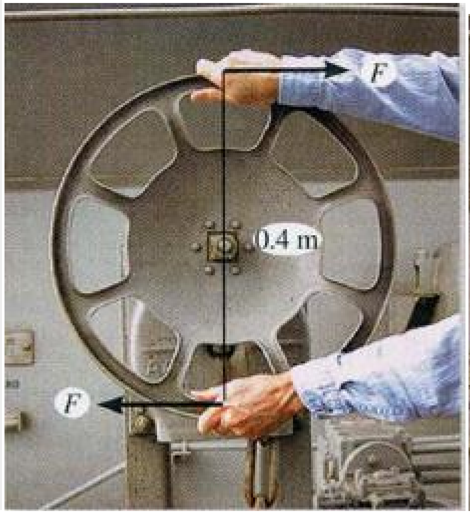


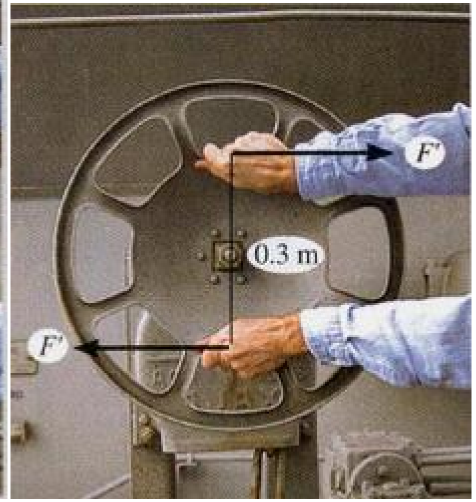
5. Moment of a Couple

A torque or moment of 12 N · m is required to rotate the wheel.

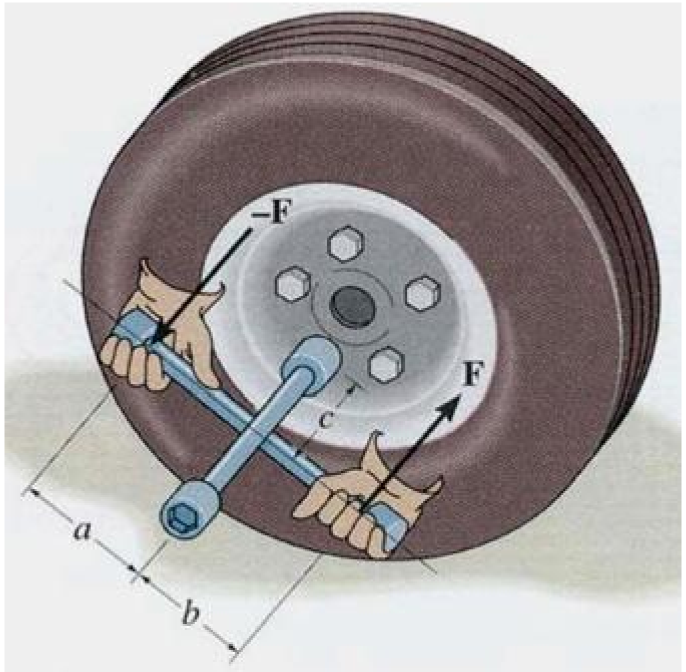
Which one of the two grips of the wheel above will require less force to rotate the wheel?







The crossbar lug wrench is being used to loosen a lug net. What is the effect of changing dimensions a, b, or c on the force that must be applied?

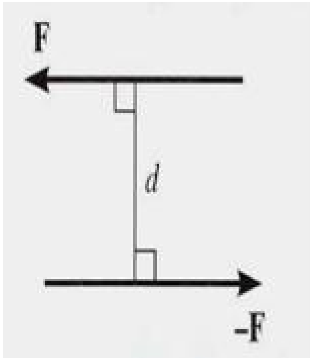


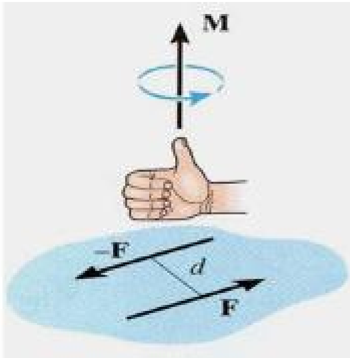
A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a **perpendicular distance d**. The moment of a couple is defined as:

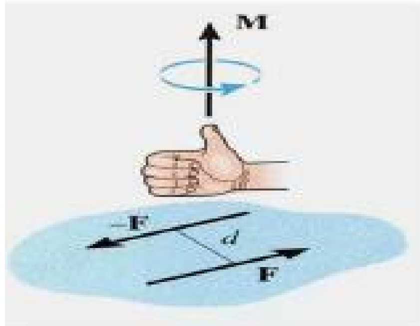
$\mathbf{M_O} = \mathbf{F} \times \mathbf{d}$ (using a scalar analysis) or as

$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$ (using a vector analysis).

Here **r** is any position vector from the line of action of **–F** to the line of action of **F**.





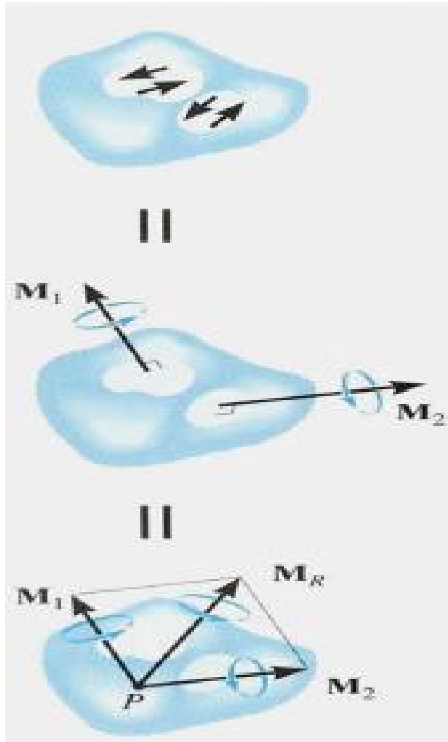


The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals **F d.**

Moments due to couples can be added using the same rules as adding any vectors.

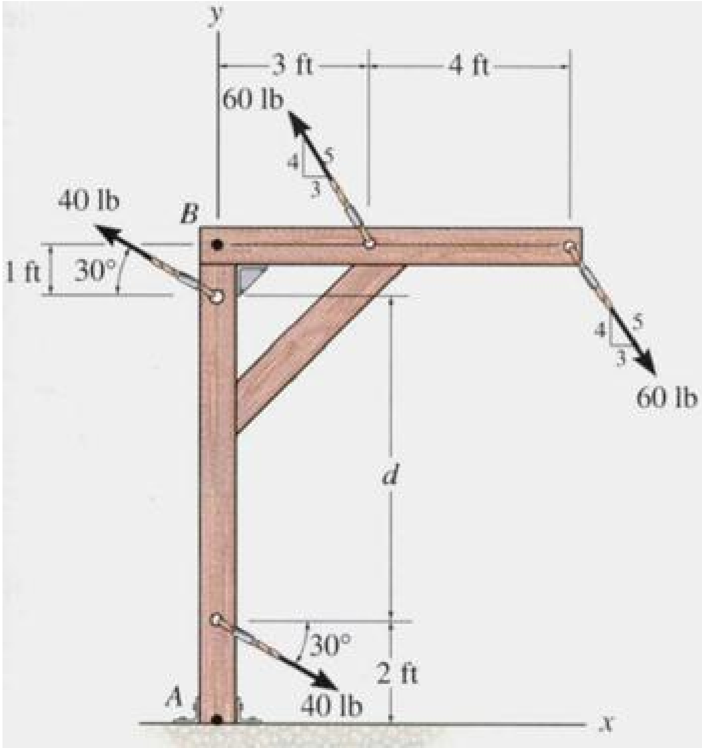
Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added using the same rules as adding any vectors.

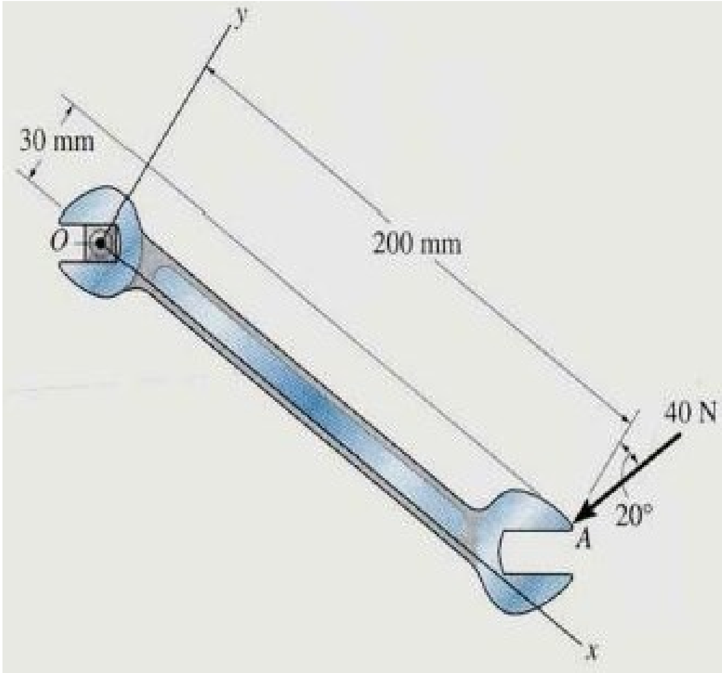


6. Example - Scalar Approach

Given: Two couples act on the beam and d equals 8 ft.
Find: The resultant couple
Plan:
1) Resolve the forces in x and y directions so they can be treated as couples.
2) Determine the net moment due to the two couples.



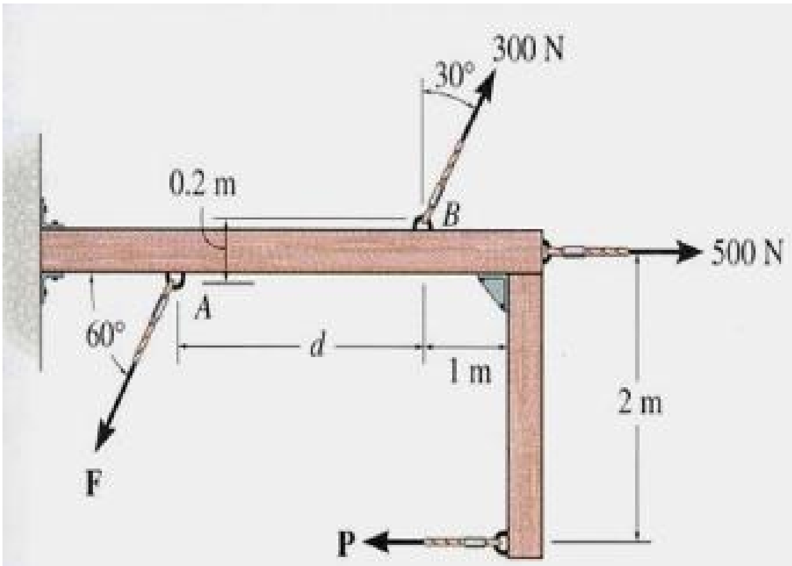
7. Example - Scalar Approach



Given: A 40 N force is applied to the wrench.
Find: The moment of the force at O.
Plan: 1) Resolve the force along x and y-axes.
2) Determine M_O using scalar analysis.

8. Example - Scalar Approach

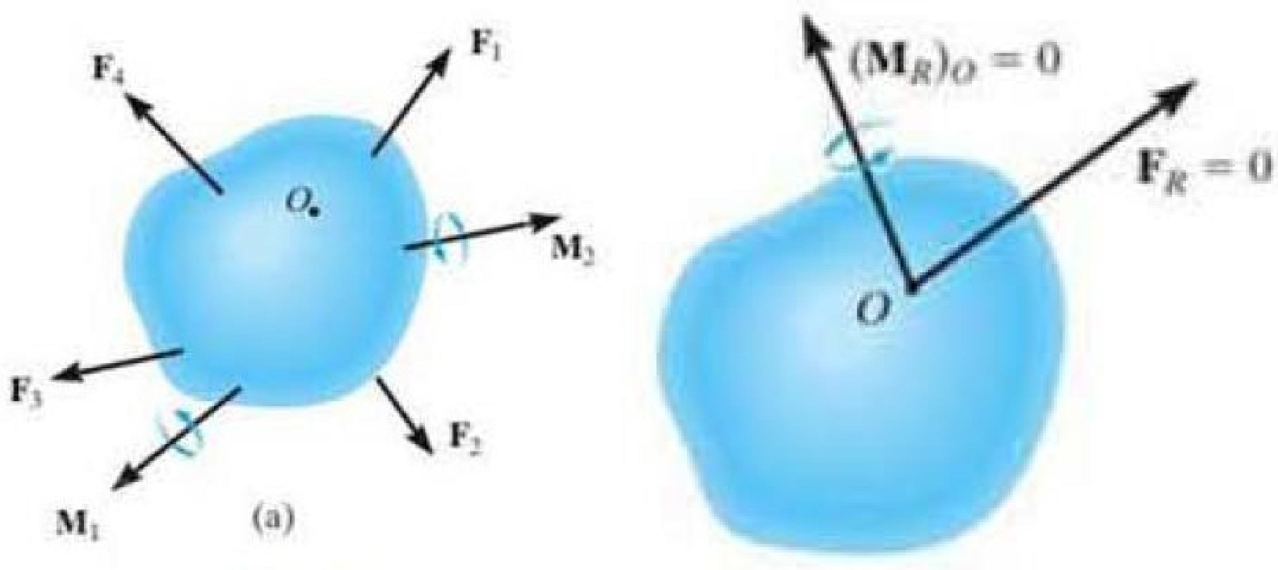
Given: Two couples act on the beam. The resultant couple is zero.
Find: The magnitudes of the forces P and F and the distance d.
PLAN:
1) Use definition of a couple to find P and F.
2) Resolve the 300 N force in x and y directions.
3) Determine the net moment.
4) Equate the net moment to zero to find d.



Equilibrium of a Rigid Body

Dr. A. Al-Mukhtar

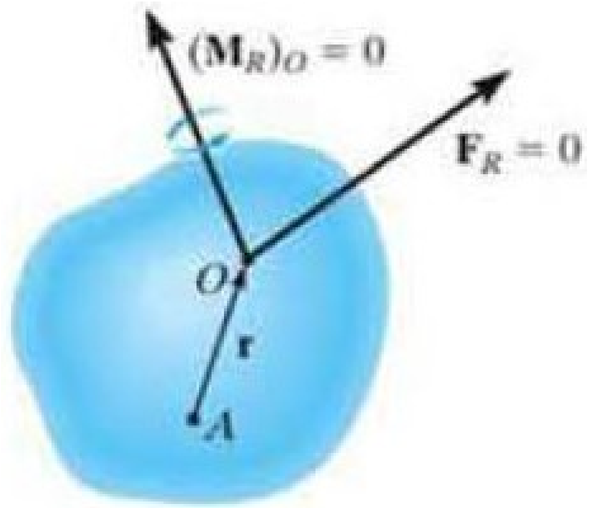
Conditions for Rigid Body Equilibrium



$F_R = \Sigma F = 0$

$(M_R)_0 = \Sigma M_0 = 0$

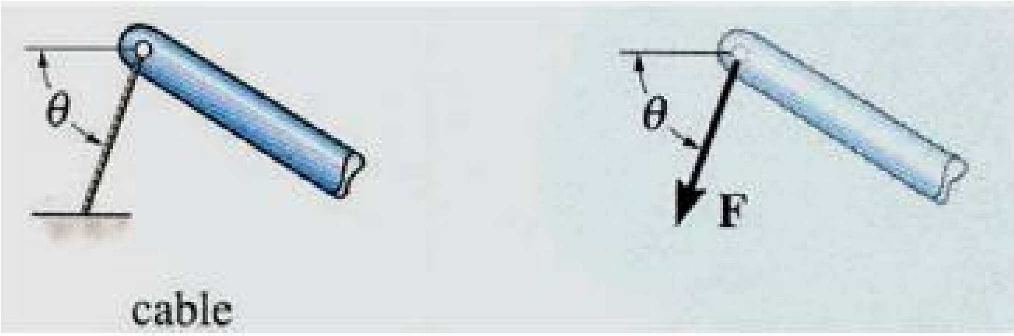
The conditions of Equilibrium for Rigid Body

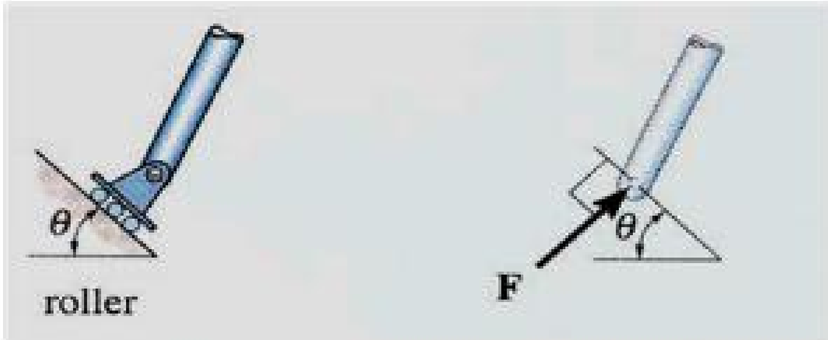


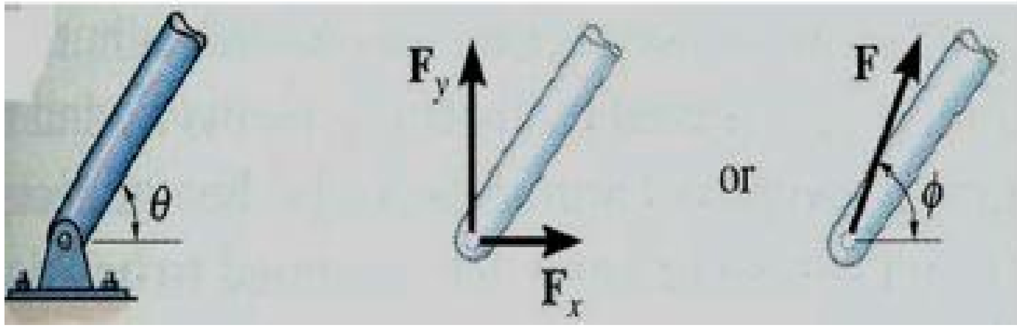
$$\Sigma \mathbf{M}_A = \mathbf{r} \times \mathbf{F}_R + (\mathbf{M}_R)_O = \mathbf{0}$$

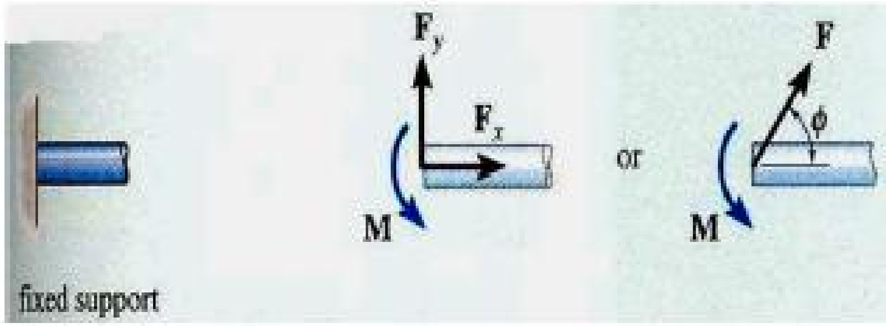
Support Reactions In 2-D

A few examples are shown above. Other support reactions are given in your textbook (see **Table 5-1**).







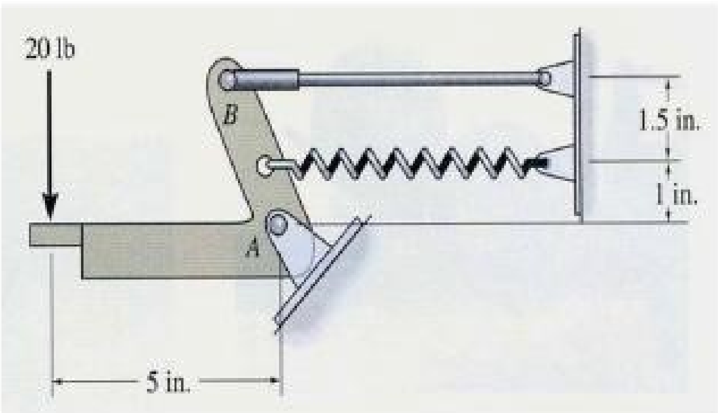


Generally, if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction. Similarly, if rotation is prevented, a couple moments are exerted on the body.

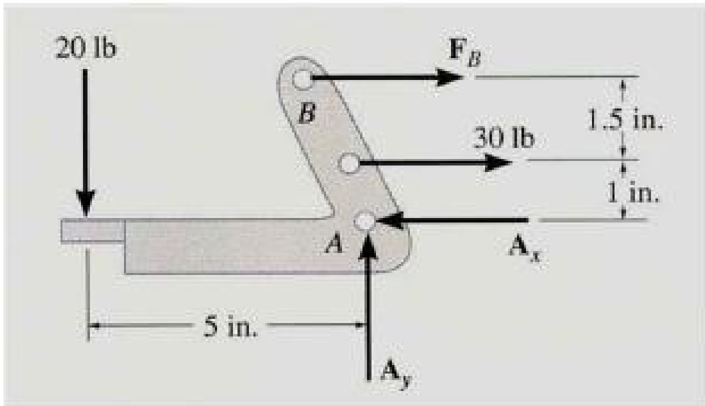
Free-Body Diagrams-Example:

Given: An operator applies 20 lb to the foot pedal. A spring with k = 20 lb/in is stretched 1.5 in.

Draw: A free-body diagram of the foot pedal.



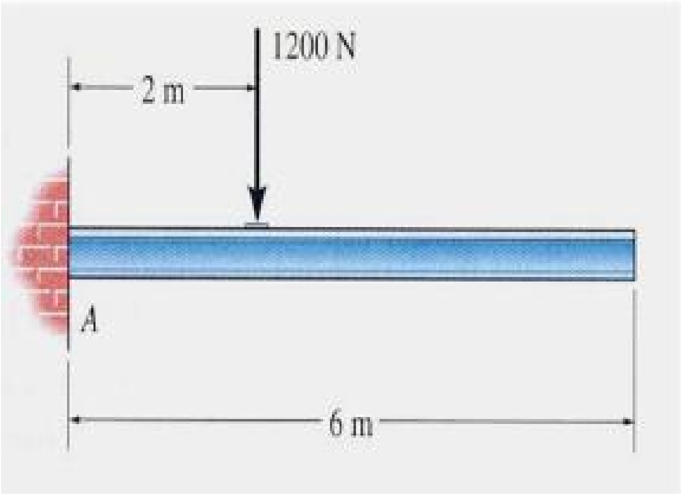
The idealized model



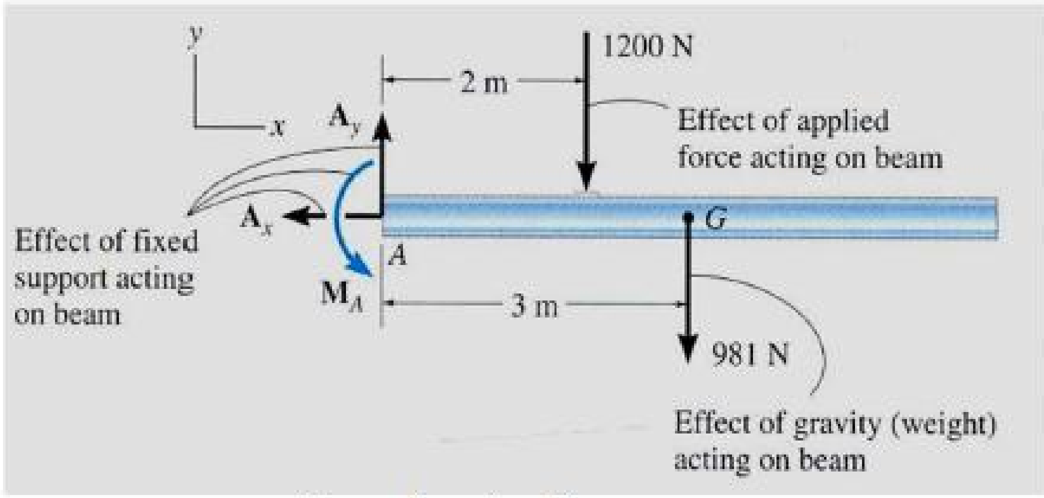
The free-body diagram

Ex:

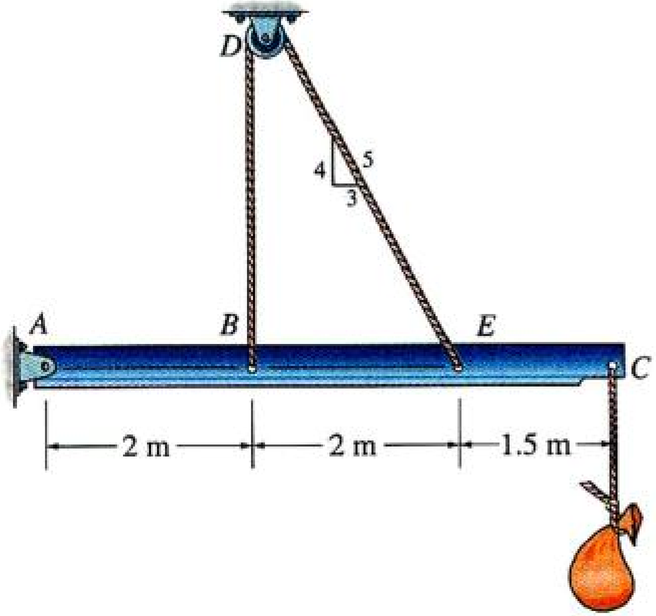
100 Kg beam

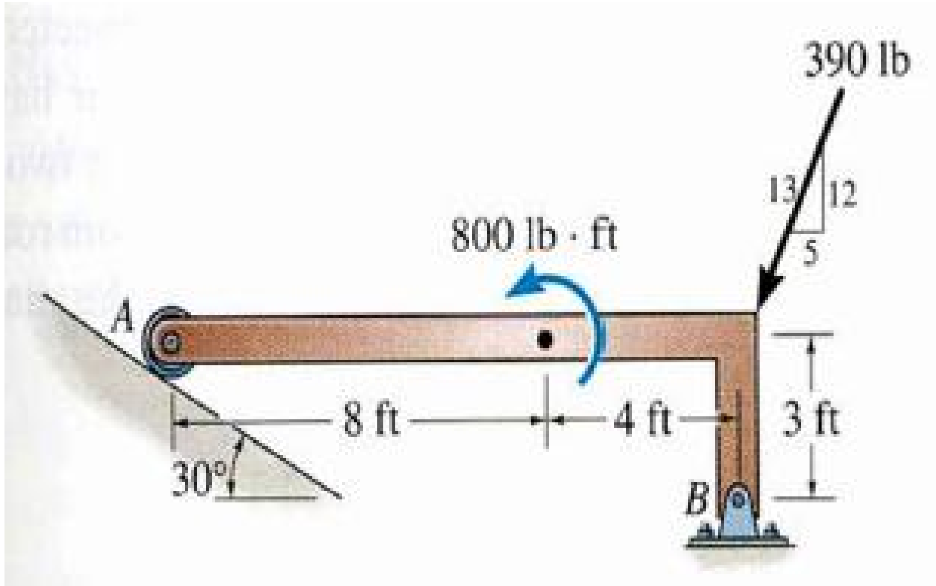


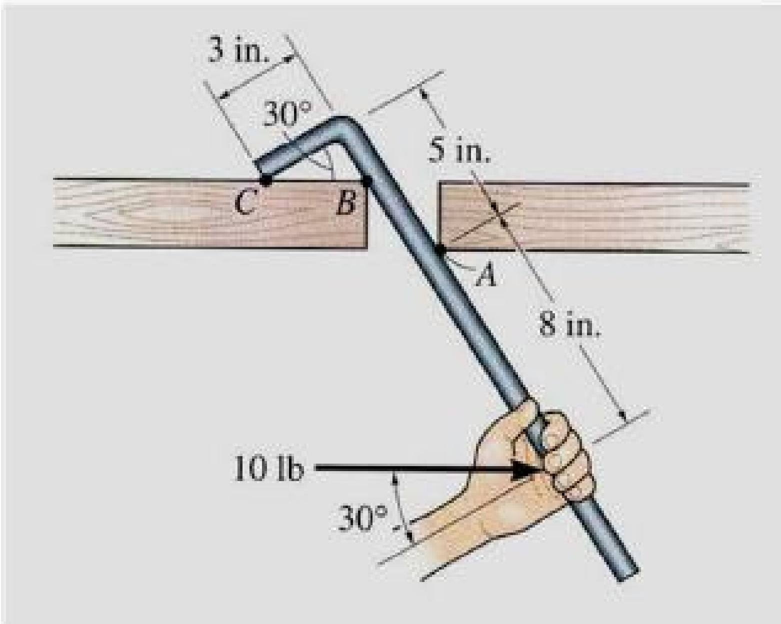
Idealized model

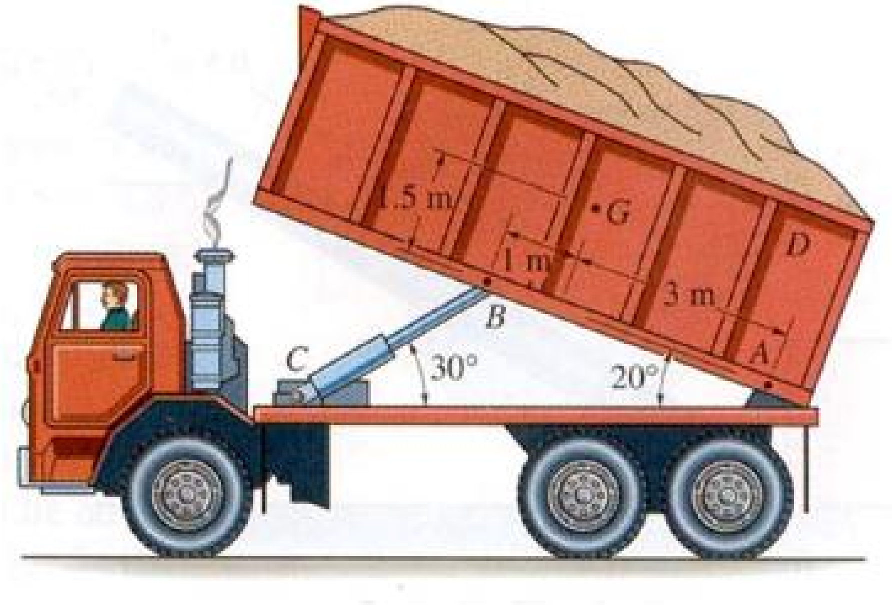


Free-body diagram









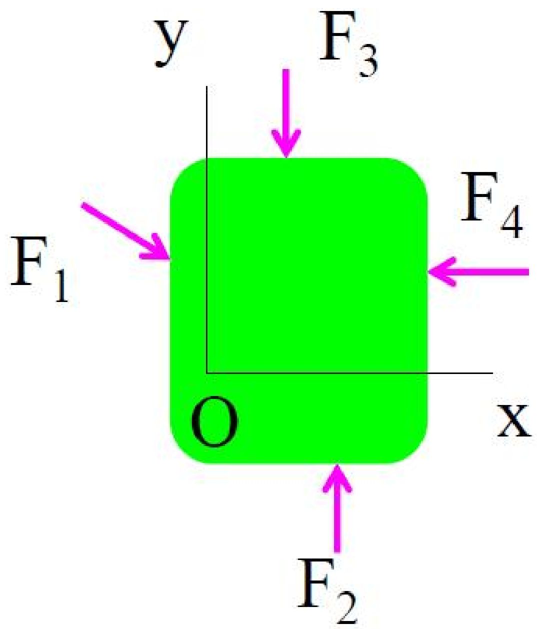
Draw a FBD of the bar, which has smooth points of contact at A, B, and C.

Draw a FBD of the 5000 lb dumpster (D). It is supported by a pin at A and the hydraulic cylinder BC (treat as a short link).

Equation of Equilibrium

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier in Section 5.1). This 2-D condition can be represented by the three scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$$

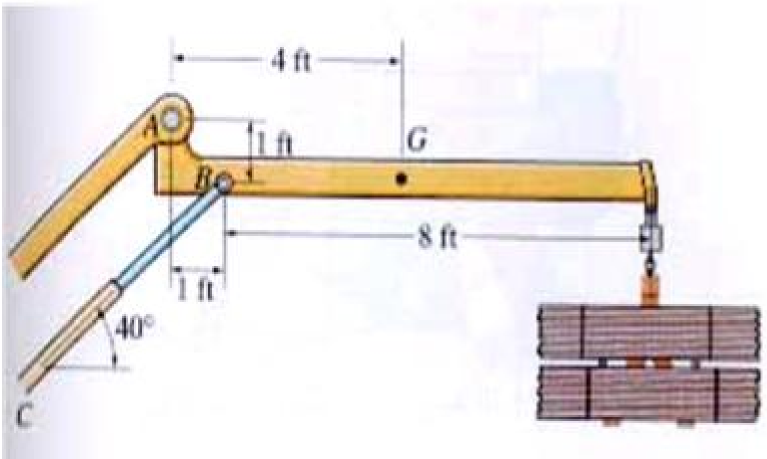


Where point O is any arbitrary point.

Example:

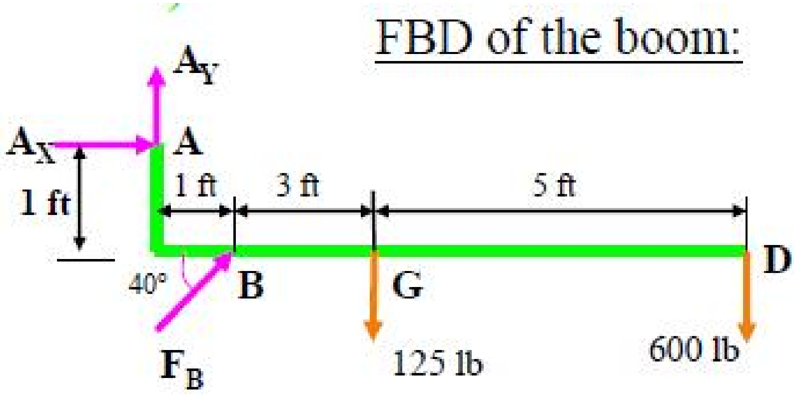
Given: Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

Find: Support reactions at A and B.



Plan:

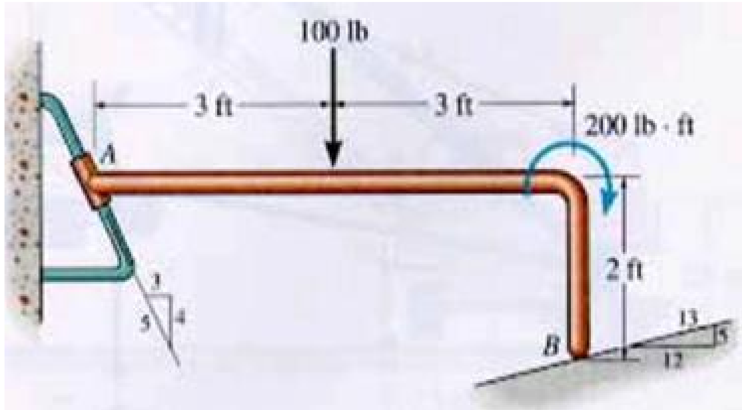
1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.



Ex:

Given: The load on the bent rod is supported by a smooth inclined surface at B and a collar at A. The collar is free to slide over the fixed inclined rod.

Find: Support reactions at A and B.



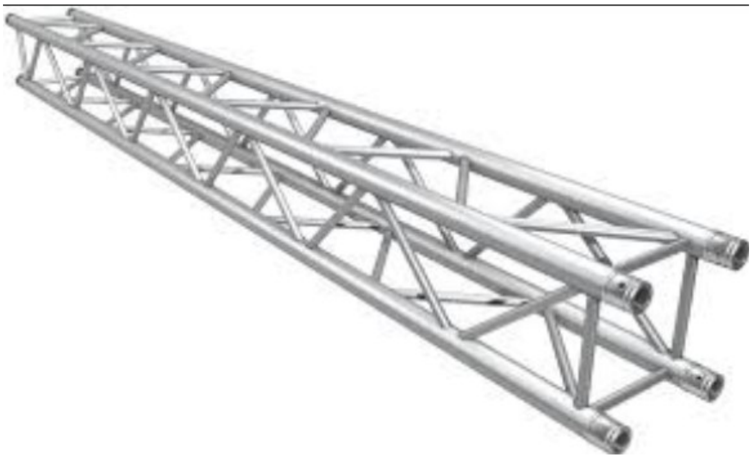
Plan:

1. Establish the x – y-axes.
2. Draw a complete FBD of the bent rod.
3. Apply the E-of-E to solve for the unknowns.

Structural Analysis

What is the structure?

Trusses are used for roof. The analysis is how we can determine the forces in truss member, hence select their size !



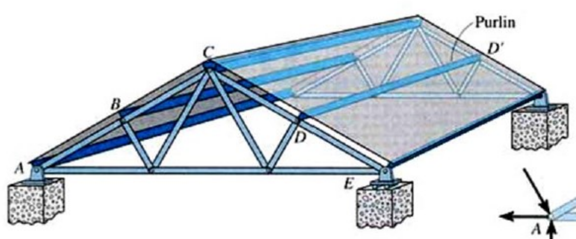
How we can design the truss geometry to minimize the cost?

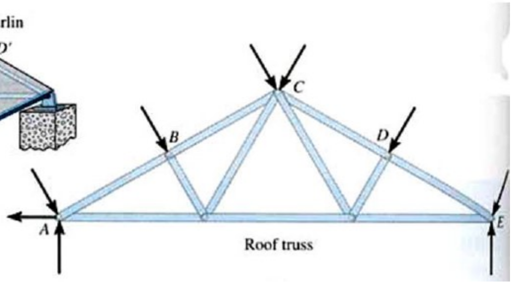
Applications

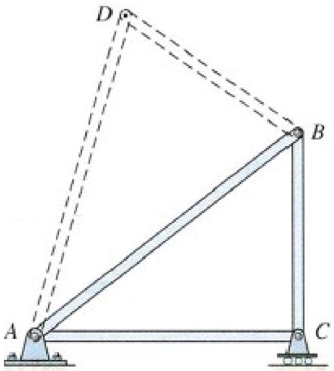
Simple trusses

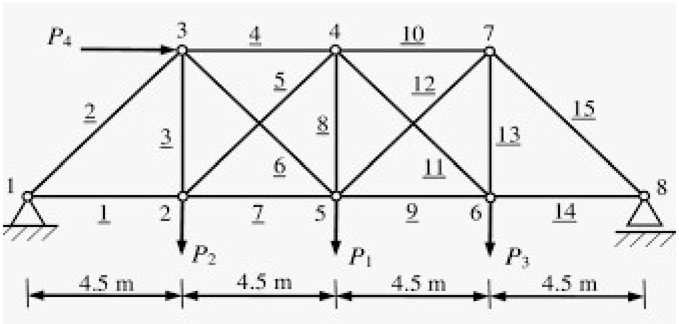
A truss is a structure composed of slender members joined together at their end points. If a truss, along with the imposed load, lies in a single plane (as shown at the top right), then it is called a planar truss.

A simple truss is a planar truss that begins with a triangular element and can be expanded by adding two members and a joint. For these trusses, the number of members (M) and the number of joints (J) are related by the equation M = 2 J – 3.







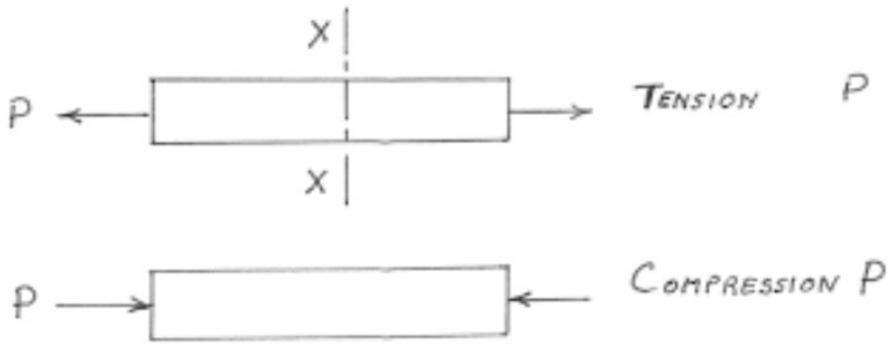


Simple truss subjected to point load

Analysis & Design Assumptions

When designing both the member and the joints of a truss, first it is necessary to determine the forces in each truss member. This is called the force analysis of a truss. When doing this, two assumptions are made:
1. All loads are applied at the joints. The weight of the truss members is often neglected, as the weight is usually small as compared to the forces supported by the members.

2. The members are joined together by smooth pins. This assumption is satisfied in most practical cases where the joints are formed by bolting or welding.
With these two assumptions, the members act as two-force members. They are loaded in either tension or compression. Often compressive members are made thicker to prevent buckling?



The methods of joints

In this method of solving for the forces in truss members, the equilibrium of a joint (pin) is considered. All forces acting at the joint are shown in a FBD. This includes all external forces (including support reactions) as well as the forces acting in the members. Equations of equilibrium ($\Sigma FX= 0$ and $\Sigma FY = 0$) are used to solve for the unknown forces acting at the joints.

STEPS FOR ANALYSIS

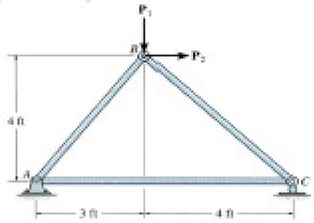
1. If the support reactions are not given, draw a FBD of the entire truss and determine all the support reactions using the equations of equilibrium.

2. Draw the free-body diagram of a joint with one or two unknowns. Assume that all unknown member forces act in tension (pulling the pin) unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, $\Sigma F_X = 0$ and $\Sigma F_Y = 0$, to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.

Examples

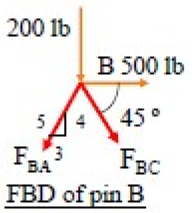


EXAMPLE

Given: $P_1 = 200$ lb, $P_2 = 500$ lb

Find: The forces in each member of the truss.

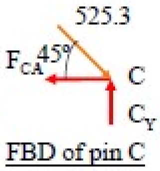
Plan: First analyze pin B and then pin C



$$+\rightarrow \Sigma F_X = 500 + F_{BC} \cos 45° - (3/5) F_{BA} = 0$$

$$+\uparrow \Sigma F_Y = -200 - F_{BC} \sin 45° - (4/5) F_{BA} = 0$$

$F_{BA} = 214$ lb (T) and $F_{BC} = -525.3$ lb (C)

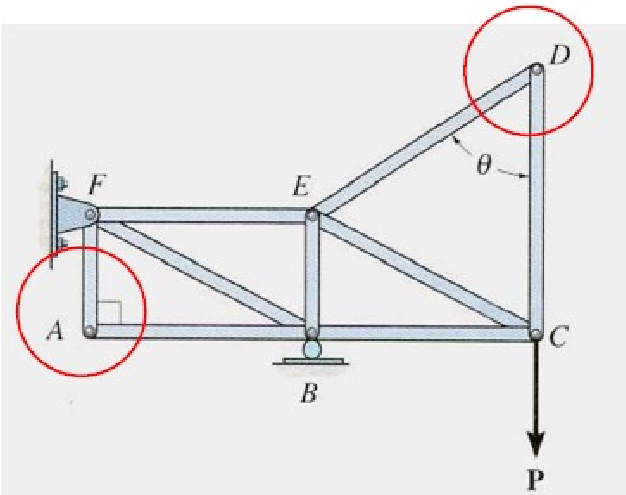


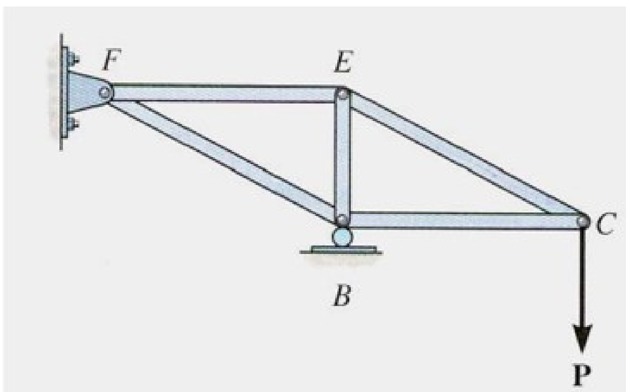
$$+\rightarrow \Sigma F_X = -F_{CA} + 525.3 \cos 45° = 0$$

$F_{CA} = 371$ (T)

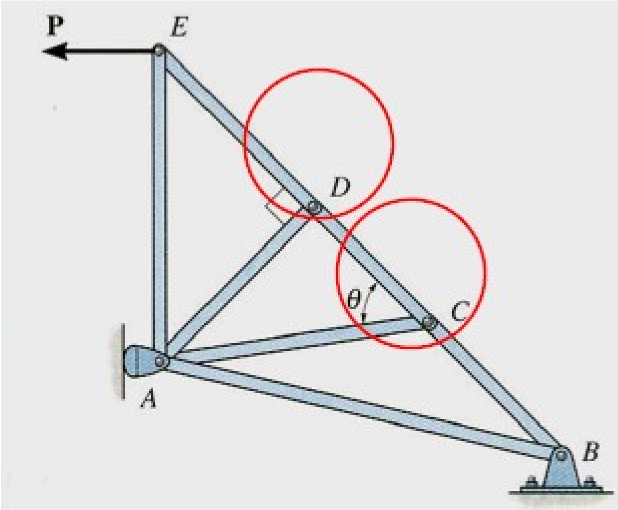
ZERO-FORCE MEMBERS

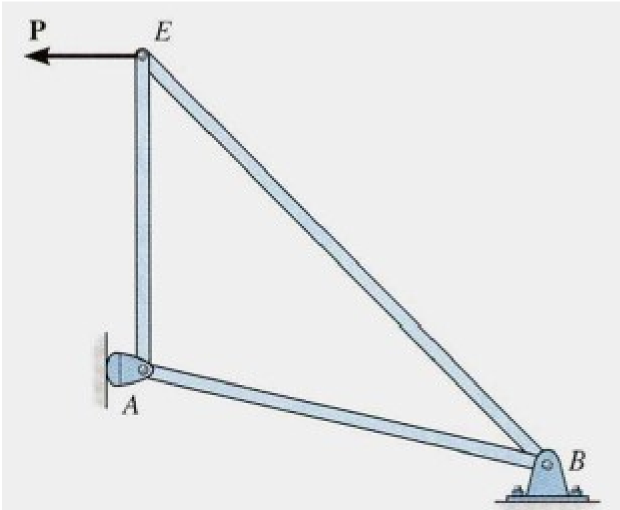
If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero force members. In this example members DE, CD, AF, and AB are zero force members. You can easily prove these results by applying the equations of equilibrium to joints D and A. Zero-force members can be removed (as shown in the figure) when analyzing the truss.





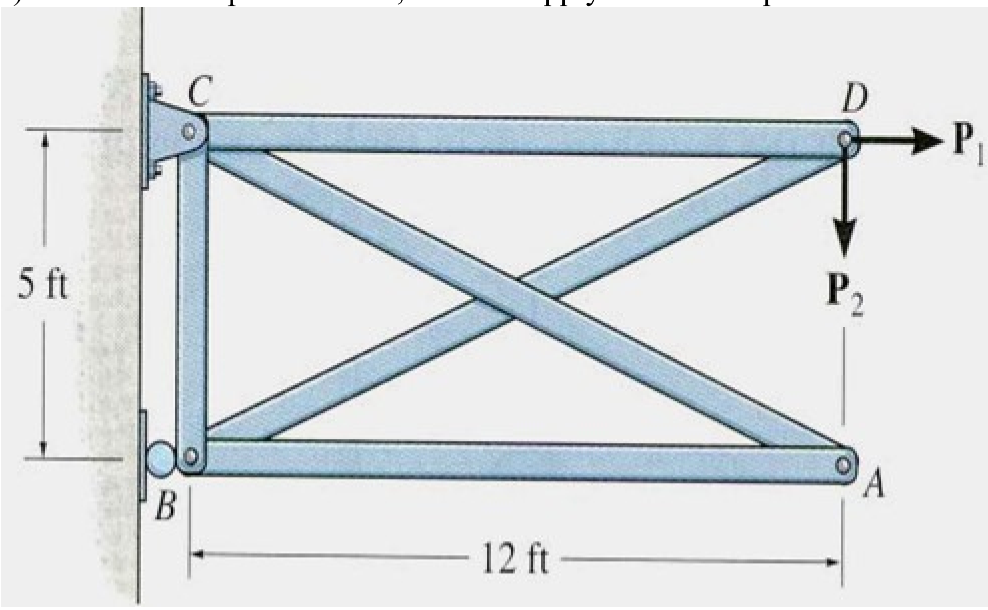
If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member. Again, this can easily be proven. One can also remove the zero-force member, as shown, on the left, for analyzing the truss further. Please note that zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.





GROUP PROBLEM SOLVING

Given: P1 = 240 lb and
P2 = 100 lb
Find: Determine the force in all the truss members (do not forget to mention whether they are in T or C).
Plan:
a) Check if there are any zero-force members.
b) Draw FBDs of pins D and B, and then apply EE at those pins to solve for the unknowns.



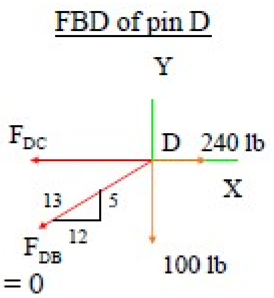
Solution:
Members AB and AC are zero-force members.

(continued)

Analyzing pin D:

$$- + \sum F_Y = -100 - (5/13)\,F_{DB} = 0$$

$$F_{DB} = -260 \text{ lb (C)}$$



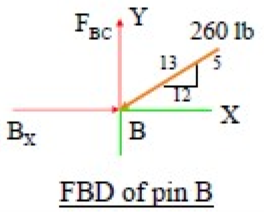
$$\rightarrow + \sum F_X = 240 - F_{DC} - (12/13)(-260) = 0$$

$$F_{DC} = 480 \text{ lb (T)}$$

Analyzing pin B:

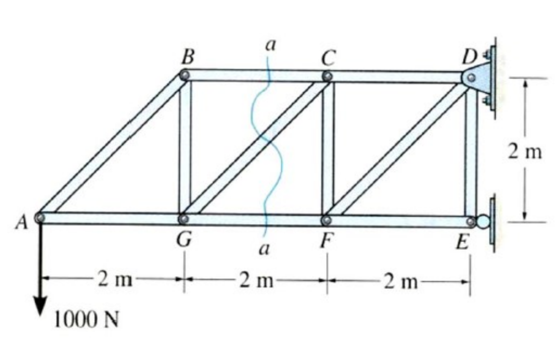
$$-+ \sum F_Y = F_{BC} - (5/13)\,260 = 0$$

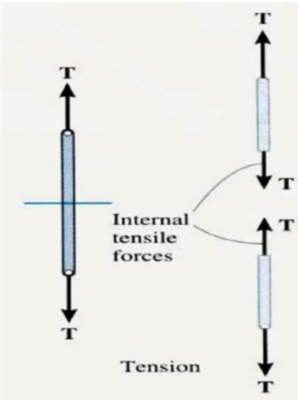
$$F_{BC} = 100 \text{ lb (T)}$$

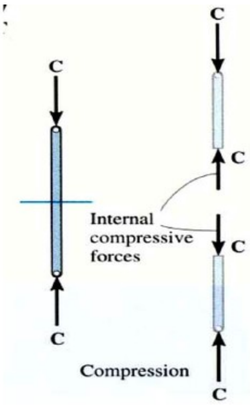


The methods of section

Using for long truss that used for construct the bridges. The method of joints requires that many joints be analyzed before we can determine the forces in the middle part of the truss. Is there another method to determine these forces directly?

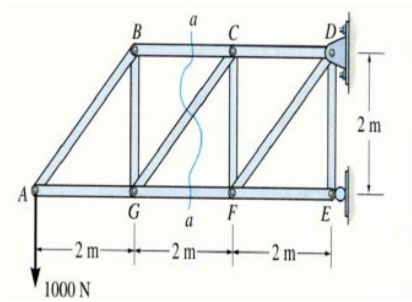


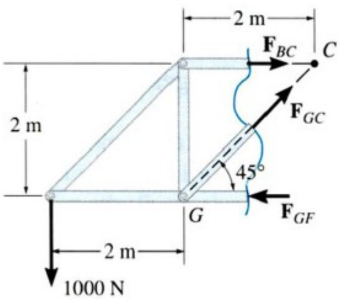


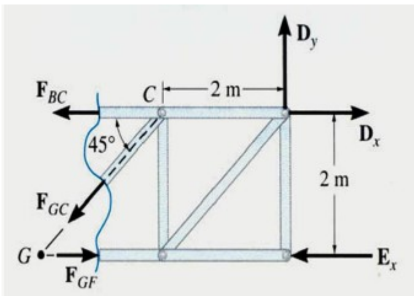


In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss. Since truss members are subjected to only tensile or compressive

forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude. This result is based on the equilibrium principle and Newton's third law.



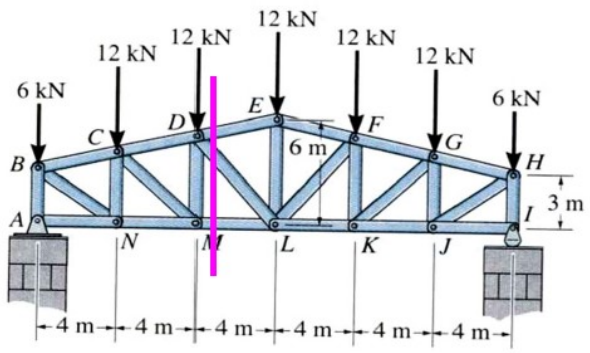




1. Decide how you need to "cut" the truss. This is based on:
a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).

2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).
3. If required, determine the necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.
4. Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. (Please note that you can also assume forces to be either tension or compression by inspection as was done in the figures above.)
5. Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.

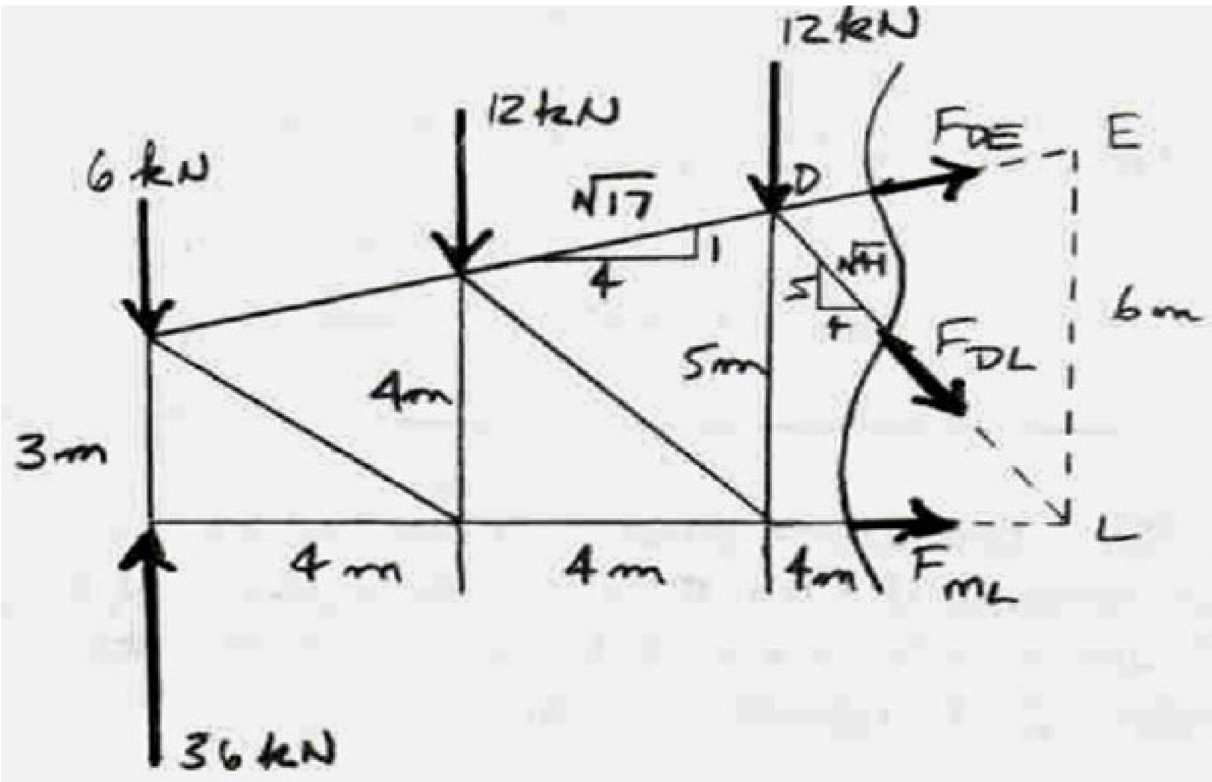
EXAMPLE



Given: Loads as shown on the roof truss.

Find: The force in members DE, DL, and ML.

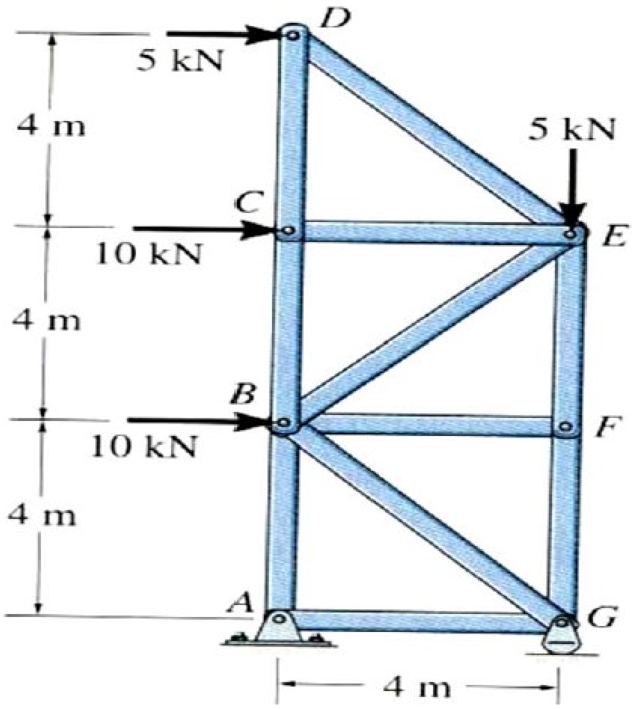
Plan:
a) Take a cut through the members DE, DL, and ML.
b) Work with the left part of the cut section. Why?
c) Determine the support reaction at A. What are they?
d) Apply the EofE to find the forces in DE, DL, and ML.



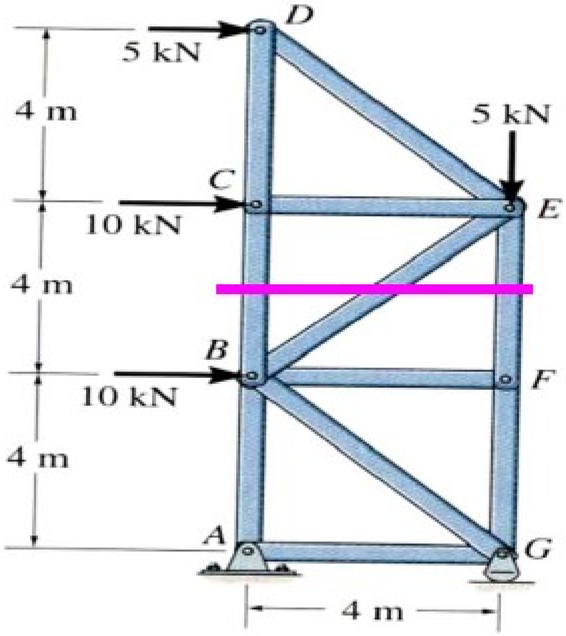
Analyzing the entire truss, we get $\Sigma F_X = A_X = 0$. By symmetry, the vertical support reactions are
$A_Y = I_Y = 36$ kN
(ccw) $+ M_D = -36\ (8) + 6\ (8) + 12\ (4) + F_{ML}\ (5) = 0$
$F_{ML} = 38.4$ kN (T)

$+\Sigma M_L = -36\ (12) + 6\ (12) + 12\ (8) + 12\ (4) - F_{DE}\ (4/\sqrt{17})(6) = 0$
$F_{DE} = -37.11$ kN or 37.1 kN (C)
$\rightarrow + \Sigma F_X = 38.4 + (4/\sqrt{17})\ (-37.11) + (4/\sqrt{41})\ F_{DL} = 0$
$F_{DL} = -3.84$ kN or 3.84 kN (C)

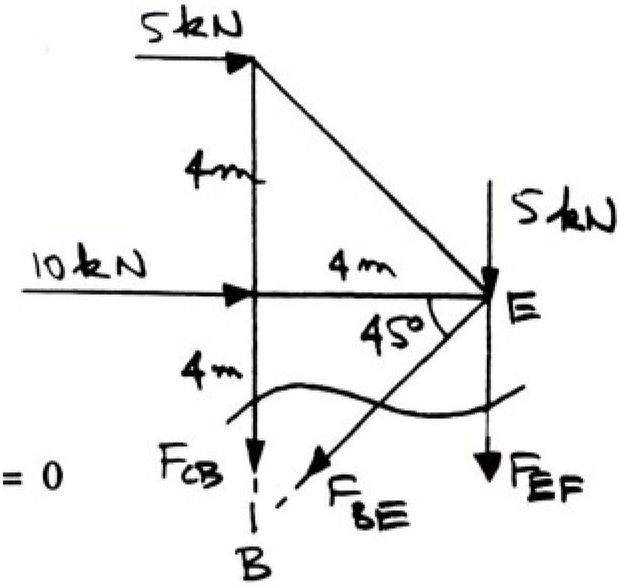
Examples



Given: Loading on the truss as shown.
Find: The force in members BC, BE, and EF.
Plan:
a) Take a cut through the members BC, BE, and EF.
b) Analyze the top section (no support reactions!).
c) Draw the FBD of the top section.
d) Apply the equations of equilibrium such that every equation yields answer to one unknown.



= 0



$+\rightarrow \Sigma F_X = 5 + 10 - F_{BE} \cos 45º = 0$

FBE = 21.2 kN (T)

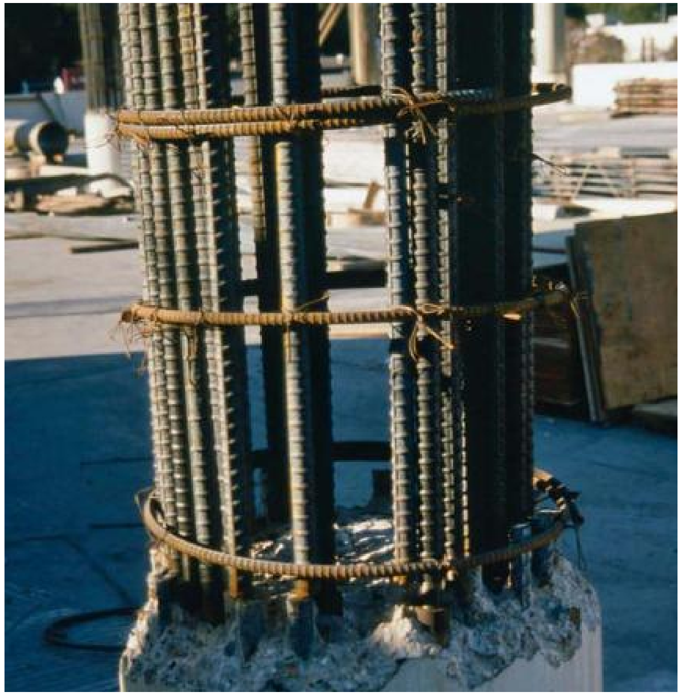
(ccw)+ Σ ME = – 5(4) + FCB (4) = 0

FCB = 5 kN (T)

(ccw)+ $\Sigma M_B = -5\,(8) - 10\,(4) - 5\,(4) - FEF\,(4) = 0$

FEF = – 25 kN or 25 kN (C)

Engineering Mechanics



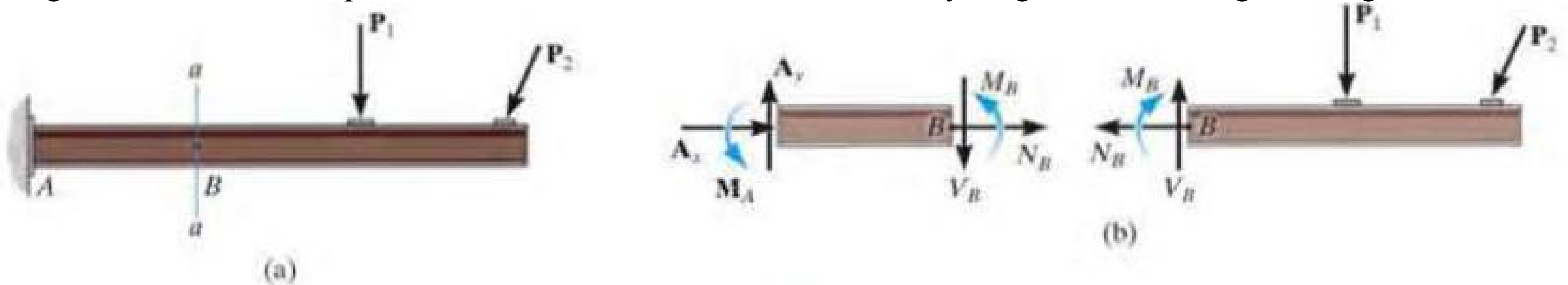
Chapter 7

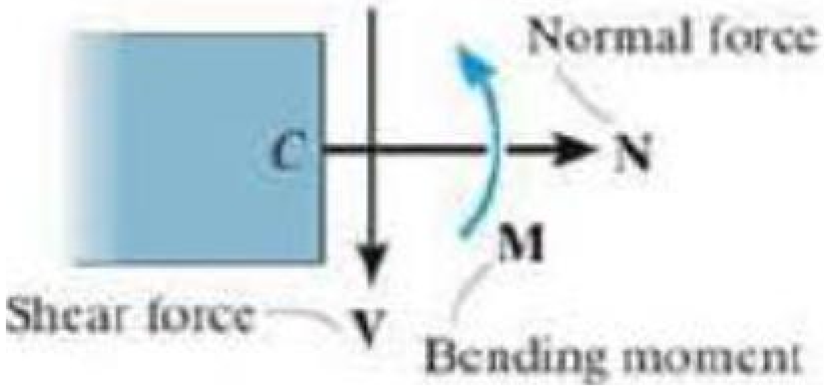
Internal Forces

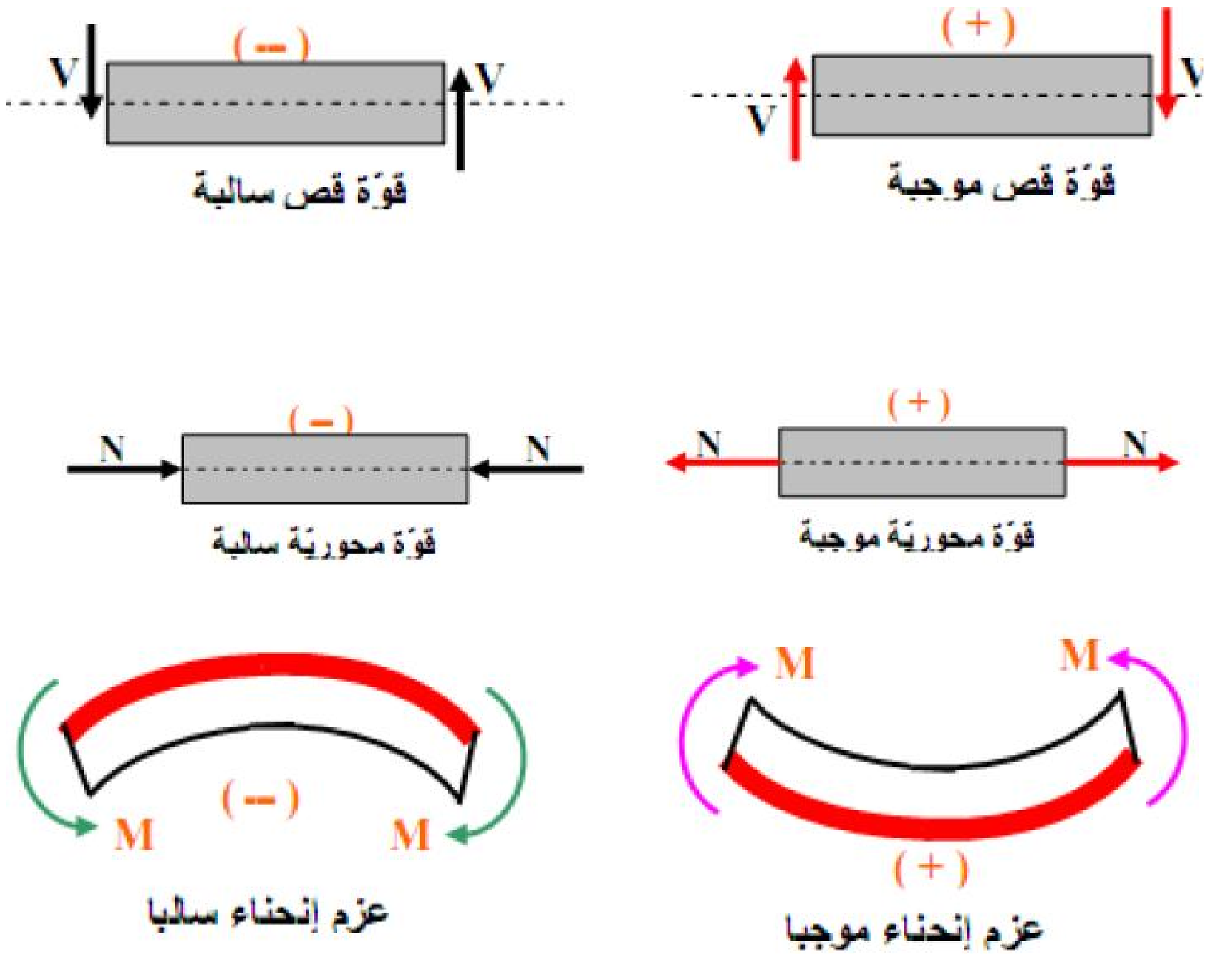
Dr. A. Al-Mukhtar

7.1. Internal Forces

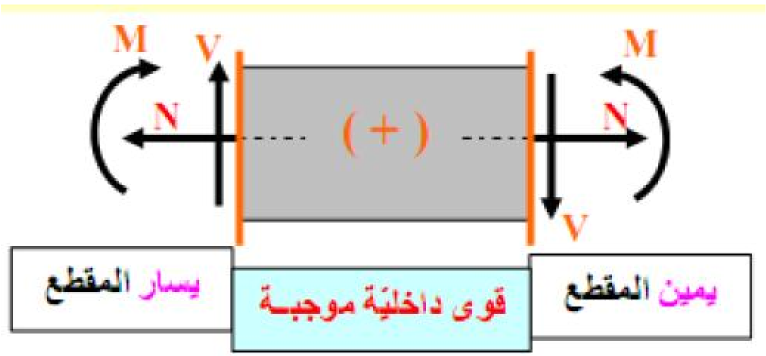
To design a structural or mechanical member it is necessary to know the loading acting within the member in order to be sure the material can resist this loading. Internal loadings can be determined by using the **method of sections**. To illustrate this method, consider the cantilever beam in Fig. 7- 1a. If the internal loadings acting on the cross section at point B are to be determined, we must pass an imaginary section a-a perpendicular to the axis of the beam through point B and then separate the beam into two segments. The internal loadings acting at B will then be exposed and become external on the free-body diagram of each segment, fig. 7- 1b.



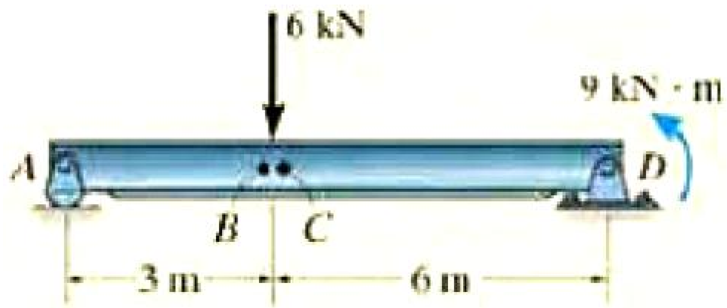




7.2. Sign Convention



Ex: Determine the normal force; shear force, and bending moment acting just to the left point B, and just to the right, point C, of the 6-kN force on the beam in Fig. 7-4.



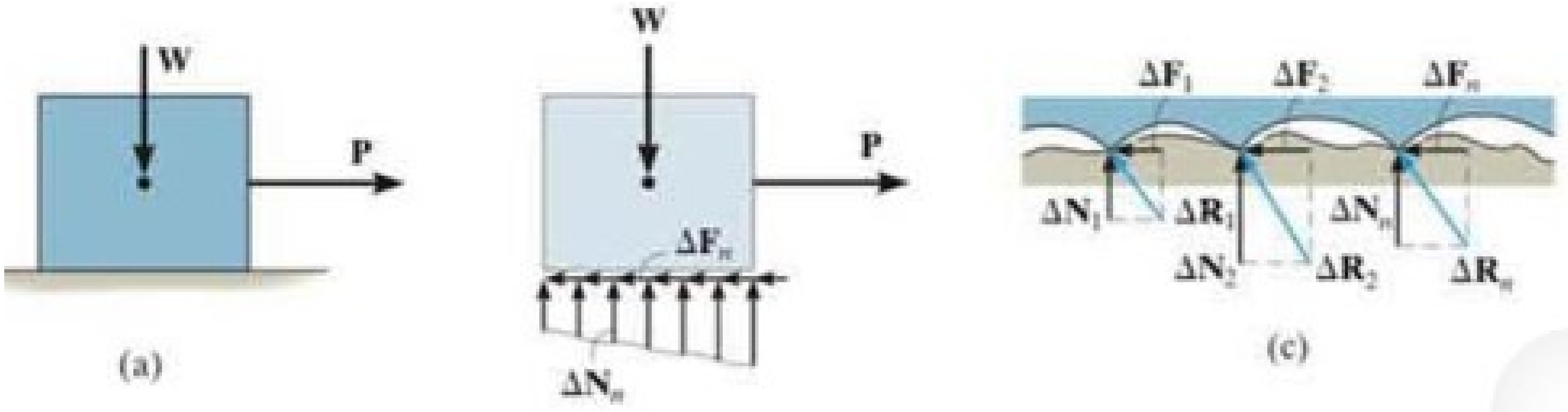
7.3. Shear and Moment Equations and Diagrams

Chapter 8

Friction

Dr.-Ing. A. Al-Mukhtar

- Friction is the **resistant force against movement** between the two surfaces.
- The force always acts **tangent** to the surface and **opposite** direction



- **Theory of Dry friction**
- **No lubrication**, and with the presence of **projections**.
- As shown on the FBD: Normal force: **ΔN_a and frictional force ΔF_a along the contacting surface, and the weight of the body $\vec{W}$**.
- For equilibrium, the normal force must act upward to balance the weight, and the frictional force acts to the left to prevent the applied force $\vec{P}$ from moving the block to the right.
- Between the two surfaces many microscopic **irregularities** between two surfaces, as a results $\overrightarrow{\Delta R_n}$ are developed at each point of contact, which contribute between the frictional and normal components, respectively $\overrightarrow{\Delta F_n}$, $\overrightarrow{\Delta N_n}$
- **Impending Motion**
- The frictional force $\vec{F}$ may not greater than the applied force $\vec{P}$.

- P is slowly increased; F correspondingly increases until it attains a certain maximum value. The max. Value F_x called the **limiting static frictional force**. If this value is reached, the block will move (unstable equilibrium). Any increasing in P will cause the block to move.
- The limiting static frictional Force F is directly proportional to the resultant normal force N.
- $F_s = \mu_s N$ where μ is the coefficient of static friction.
- The angle of static friction $\theta = \tan^{-1}(\frac{F_s}{N}) = \tan^{-1}(\frac{\mu_s N}{N}) = \tan^{-1}\mu_s$
- μ_s Depends on the variable conditions of roughness and cleanliness of contacting surfaces. F_x can be determined directly by an experiment that involves the two materials to be used.

Table 8-1 Typical Values for μ_s	
Contact Materials	*coefficient of static Friction μ_s*
Metal on ice	0.03 - 0.05
Wood on wood	0.30 - 0.70
Leather on wood	0.20 - 0.50
Leather on metal	0.30 - 0.60
Aluminum on aluminum	1.10 - 1.7

Ex: The uniform crate shown in Fig below has a mass 20 Kg. if a force P=80 N is applied to the crate, determine if its remains in equilibrium. The coefficient of static friction is μ_s =0.3.