Chapter 12

Static Equilibrium & Elasticity

Static Equilibrium & Elasticity

Conditions for Equilibrium

The Center of Gravity

Examples of Static Equilibrium

Stability of Rotational Equilibrium

Stress and Strain

The Conditions For Equilibrium

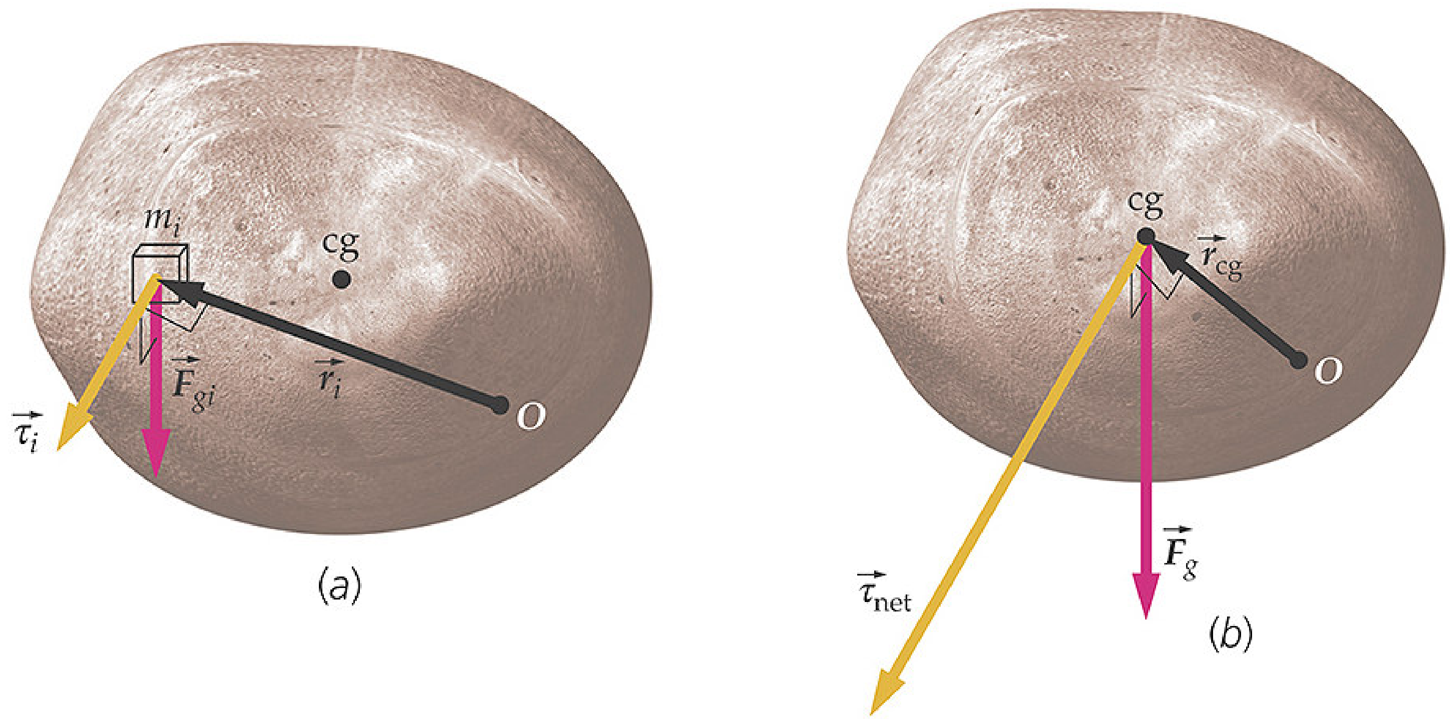
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum \tau = 0$$

CM and CG are the same in the absence of a field gradient



Examples of Field Gradients

Gravity

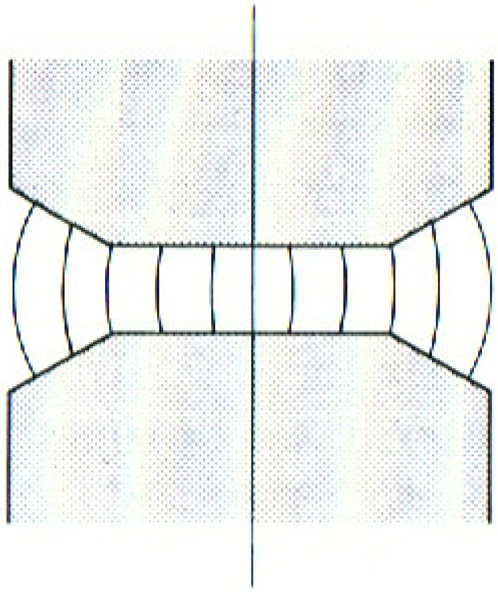
Magnetic Field

Dielectric in a Capacitor

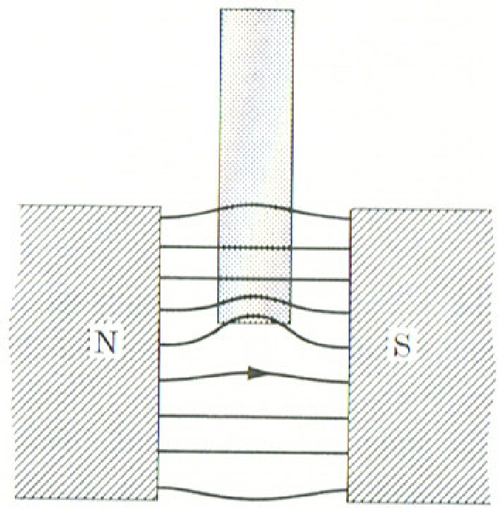
Electrostatic Hold down

Laser Beam Cleaner

Fringing Magnetic Field

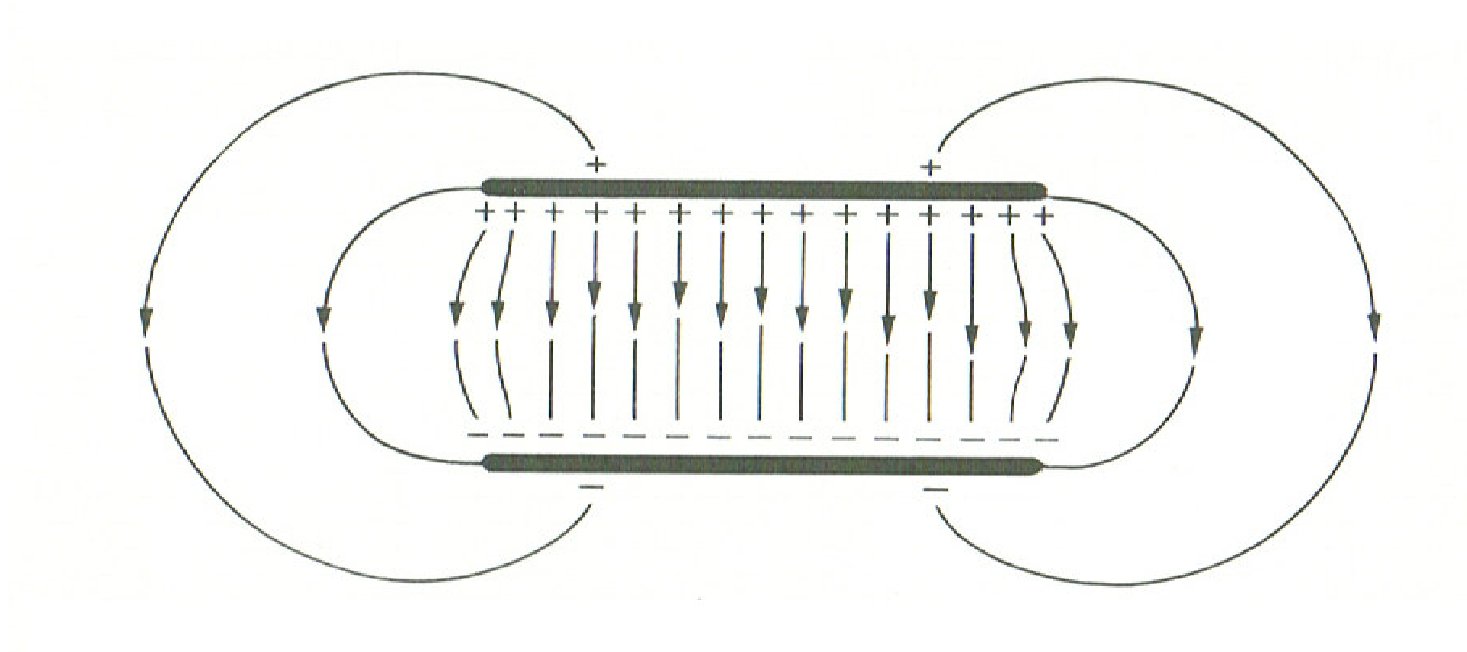


$$\mathbf{F} = (\mathbf{m} \cdot \nabla)\mathbf{B}$$

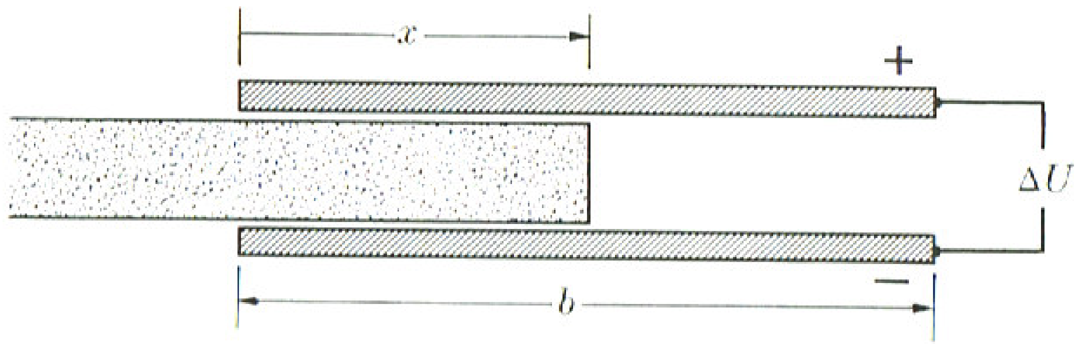


A paramagnetic rod inserted between the pole faces of a magnet.

Fringing Electric Field



Dielectric in an E-Field Gradient



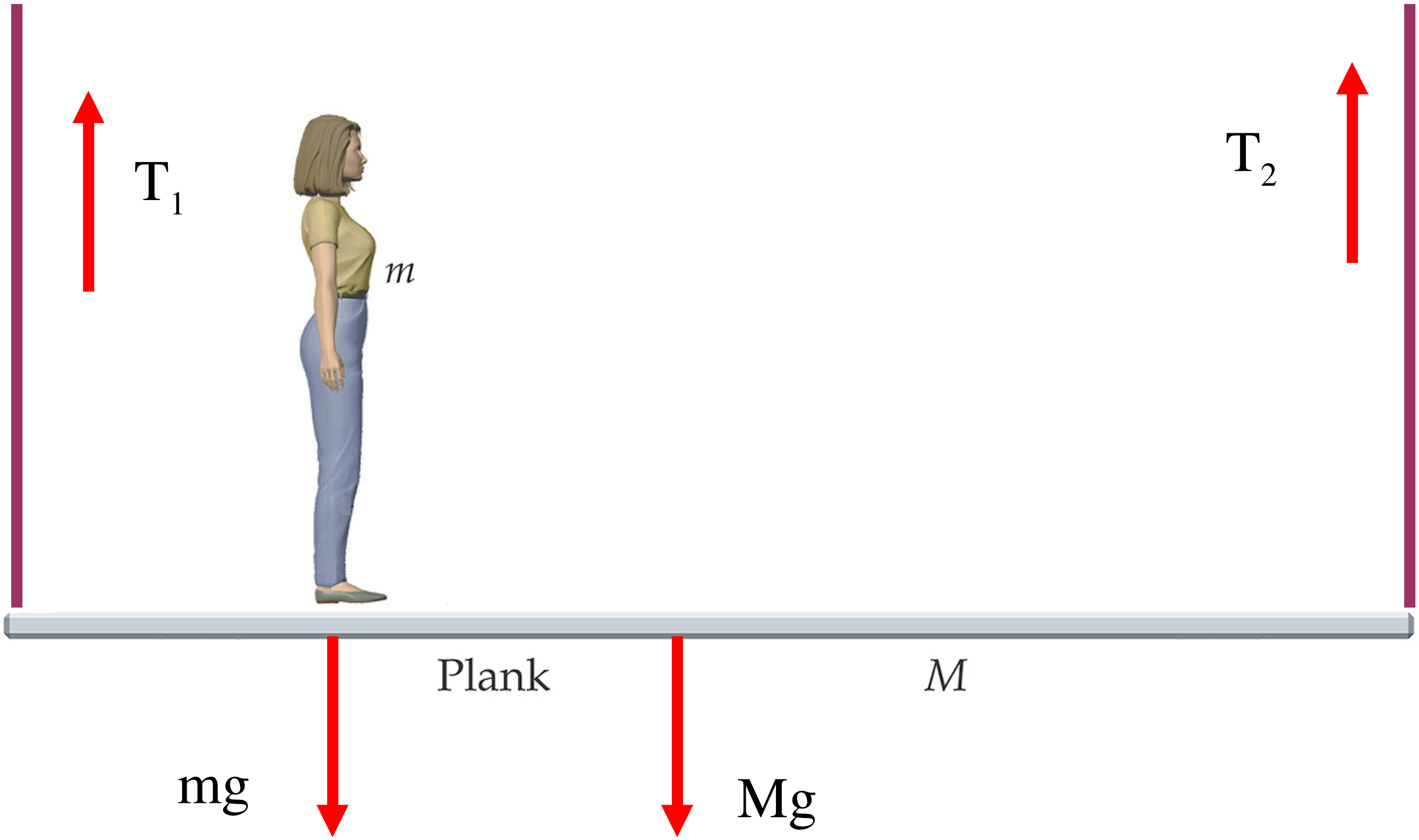
Dielectric slab partially withdrawn from between two charged plates.

Examples of Field Gradients

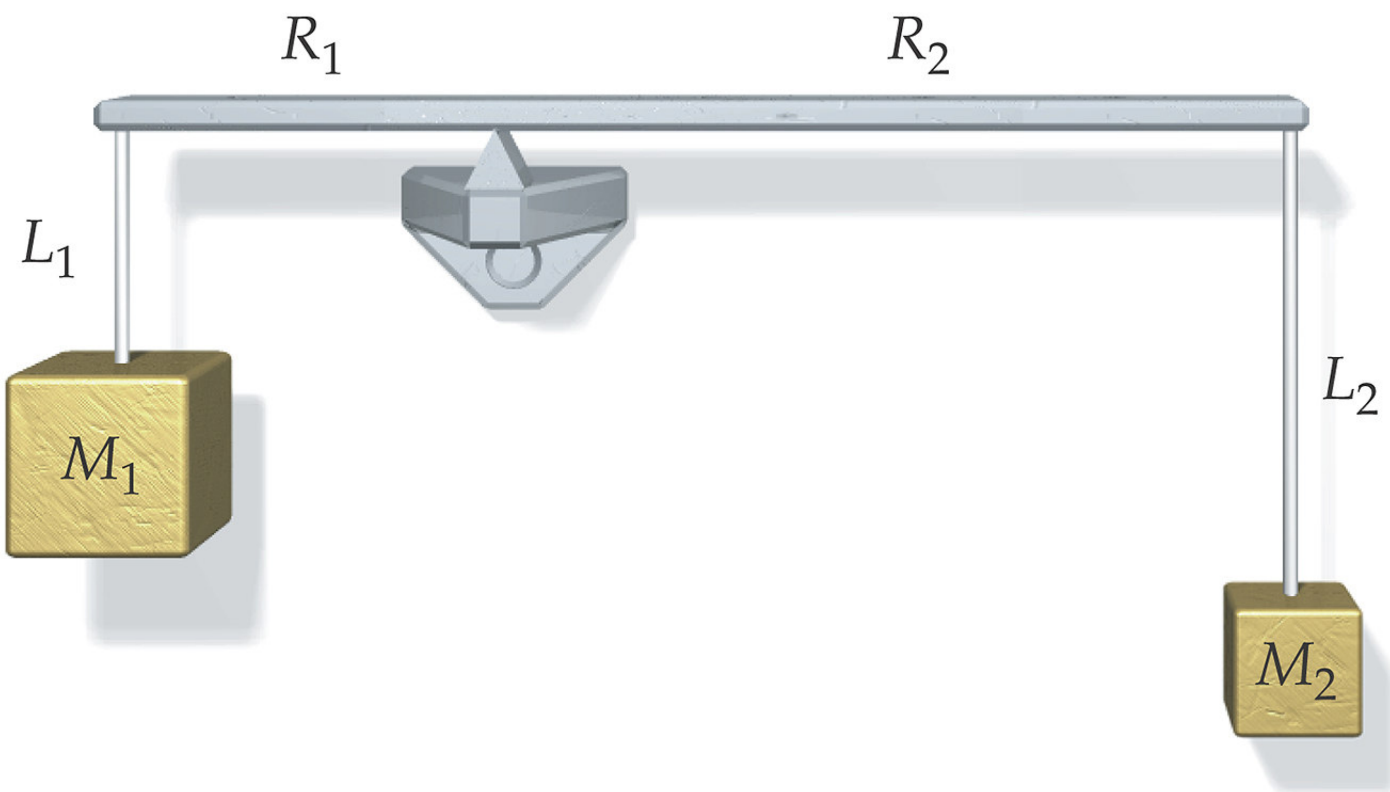
- Electrostatic Hold down
- Laser Beam Cleaner

Applications

Walking the Plank

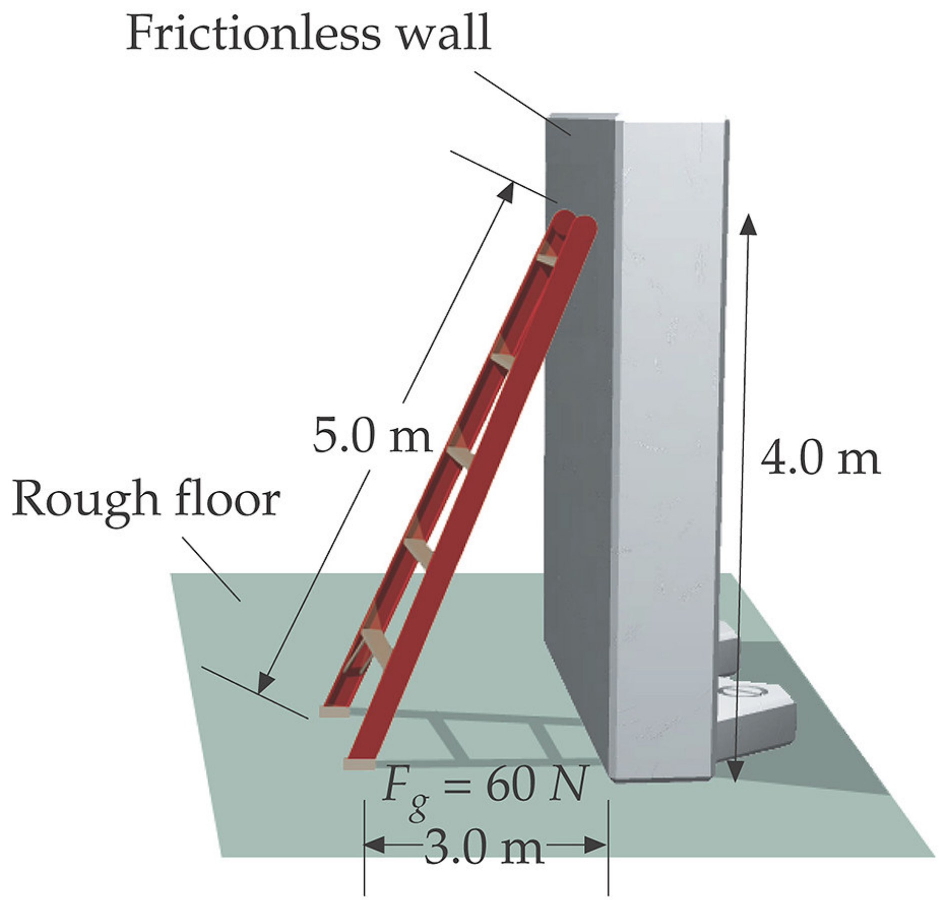


Use as an example

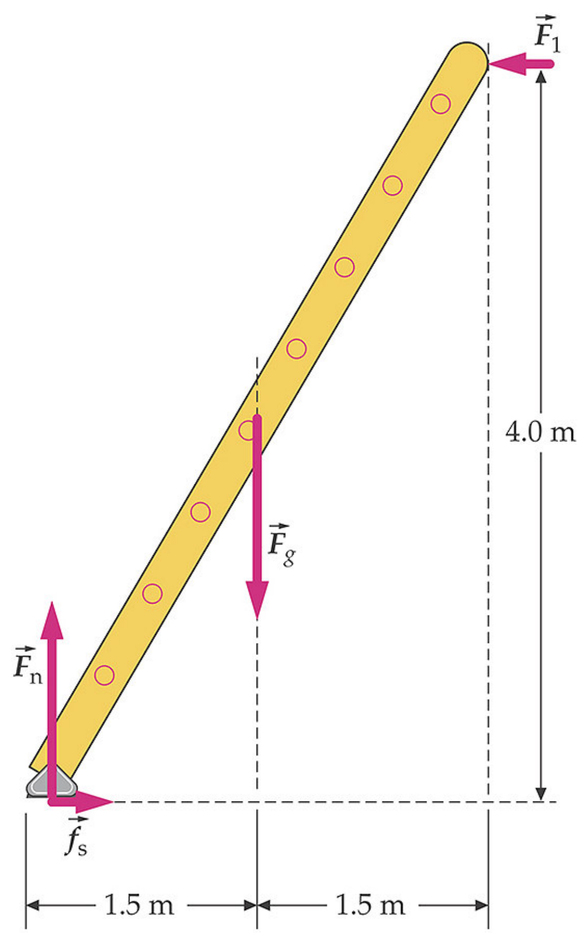


Meter Stick Balance

The Classic Ladder Problem



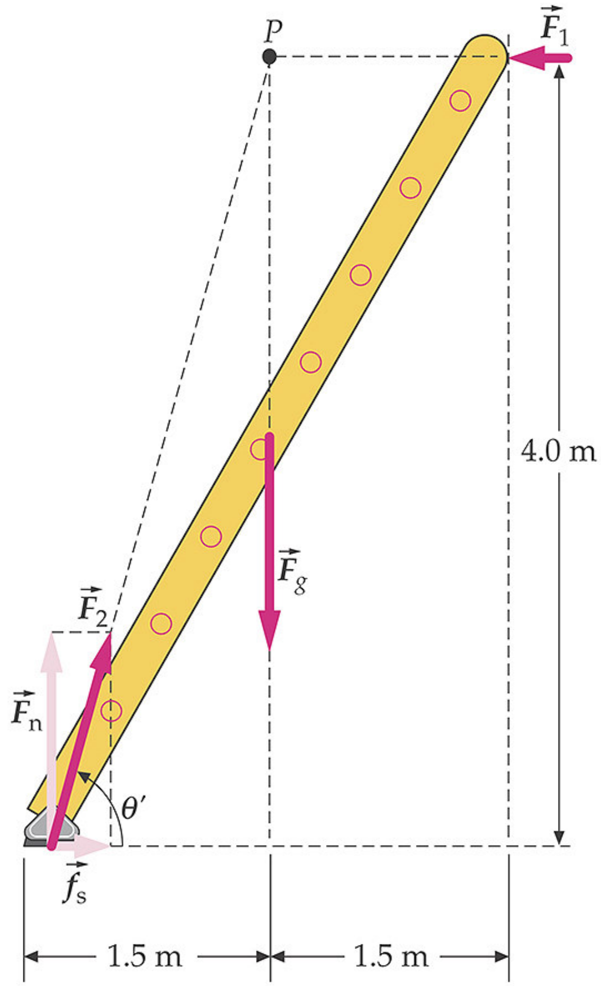
Ladder Free Body Diagram



Why?

Three forces acting on an object in equilibrium.

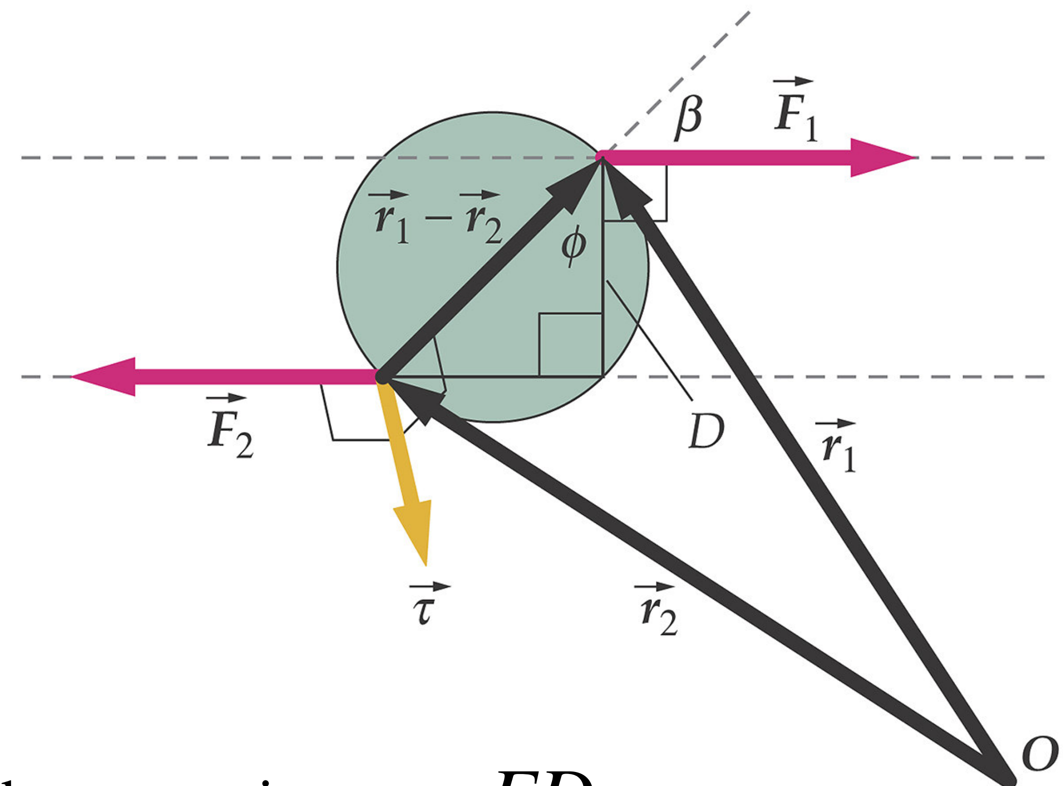
If two of the force's lines of action intersect in a point then the third force's line of action will also intersect in the same point.



Force Couple

The purpose of a force couple is to cause a rotation with any translation.

$$\vec{\tau} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1$$



The magnitude of the torque is $\tau = FD$

Your 1st Thought: Where's the Hidden Camera?

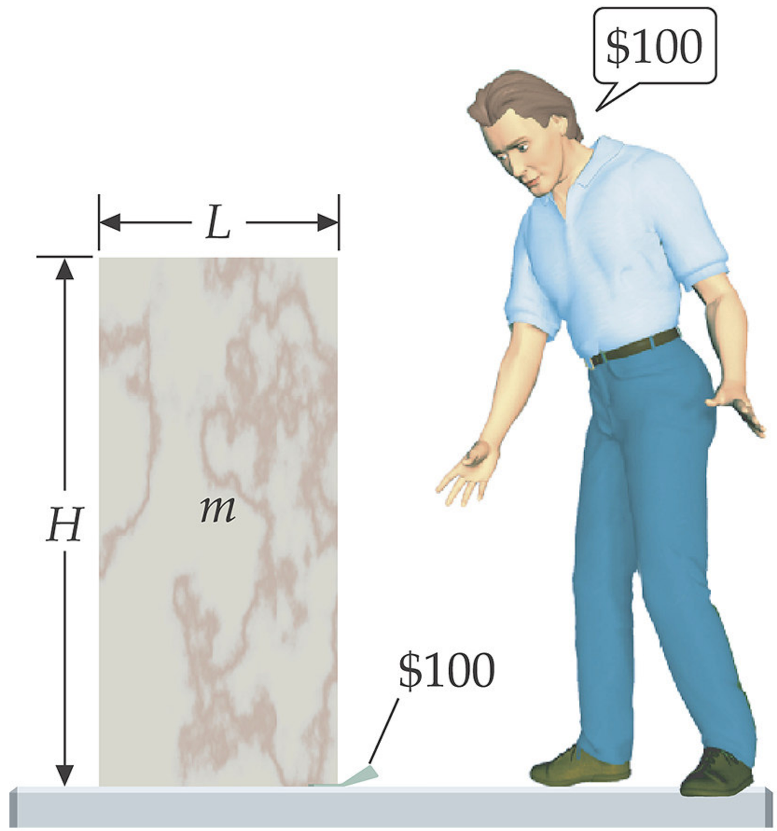
Your 2nd thought:

Where's some scrap paper so that I can draw a free body diagram to analyze this situation?

Your 3rd thought:

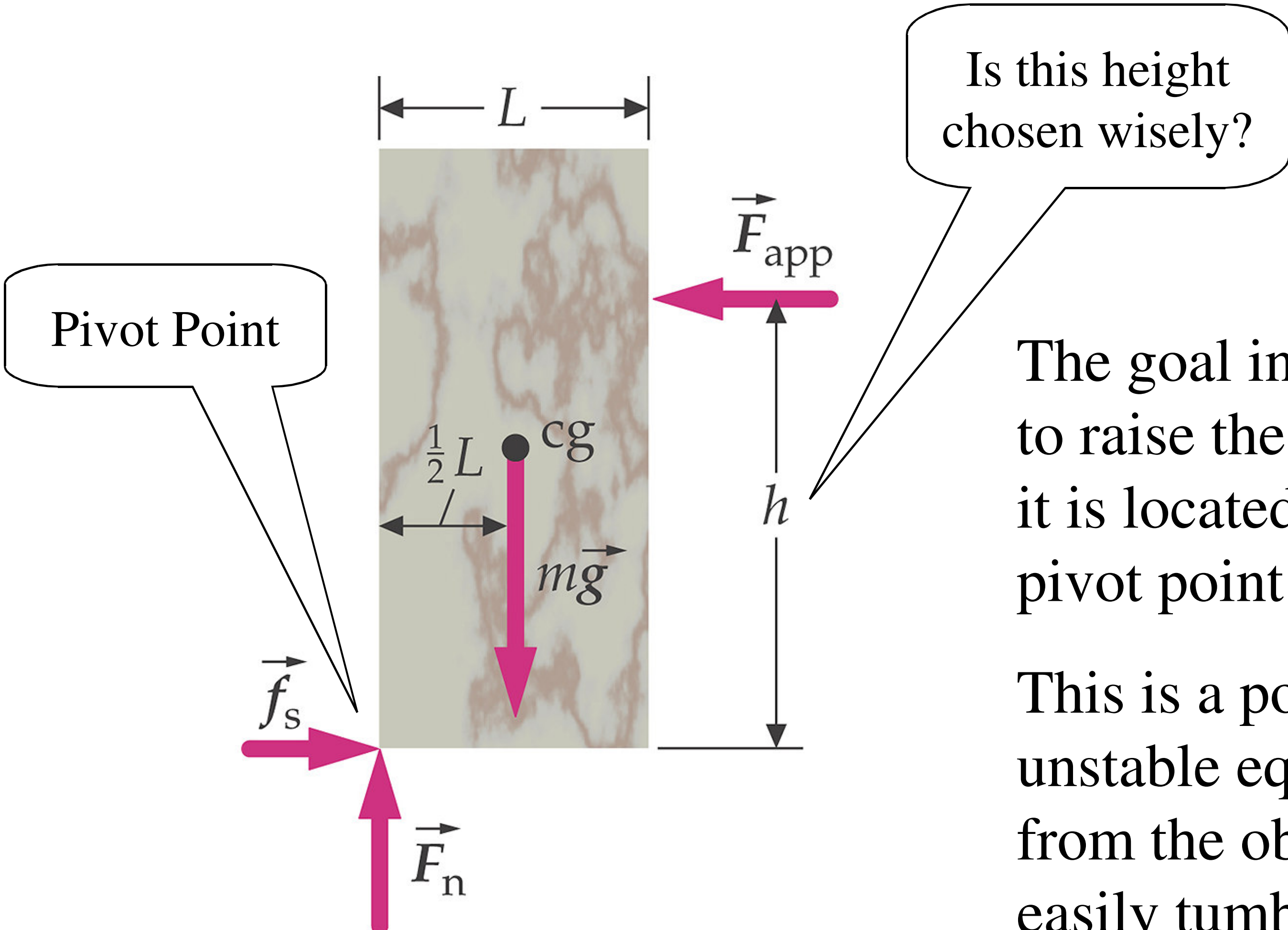
Where's my formula sheet?

By then someone else got the $100 bill.



If he had memorized his formulas he'd be $100 richer now.

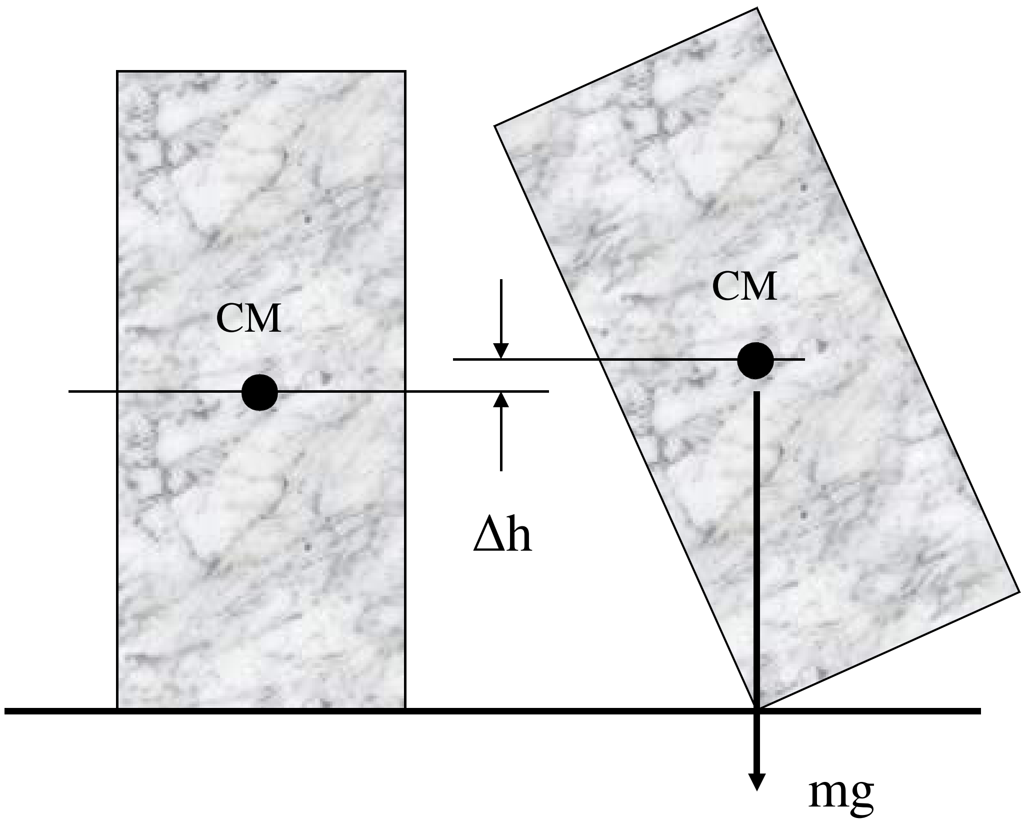
Tipping Analysis



The goal in tipping is to raise the CM so that it is located over the pivot point.

This is a point of unstable equilibrium from the object will easily tumble over.

Tipping Analysis

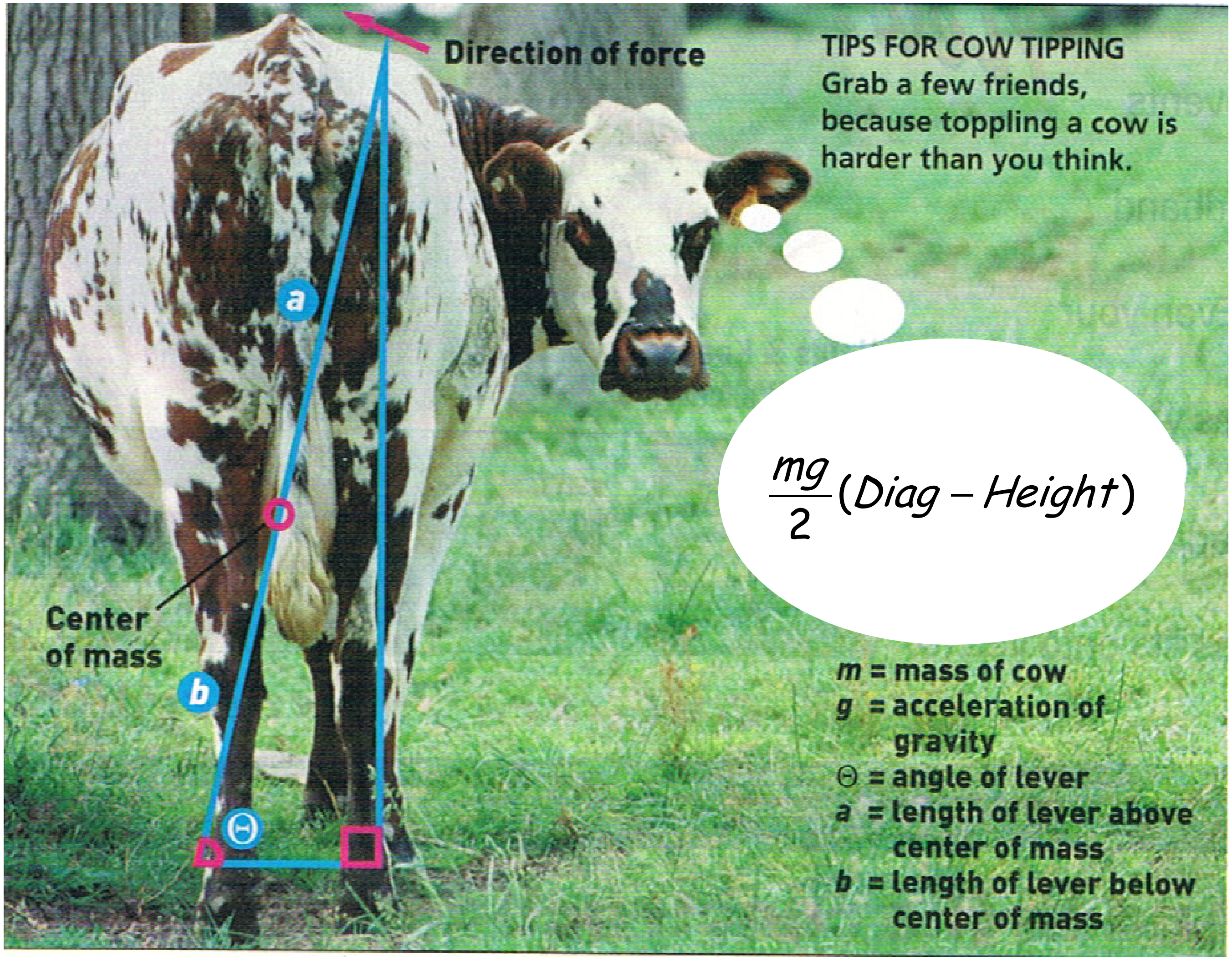


The energy needed to tip the block is the energy needed to raise the CM a height Δh.

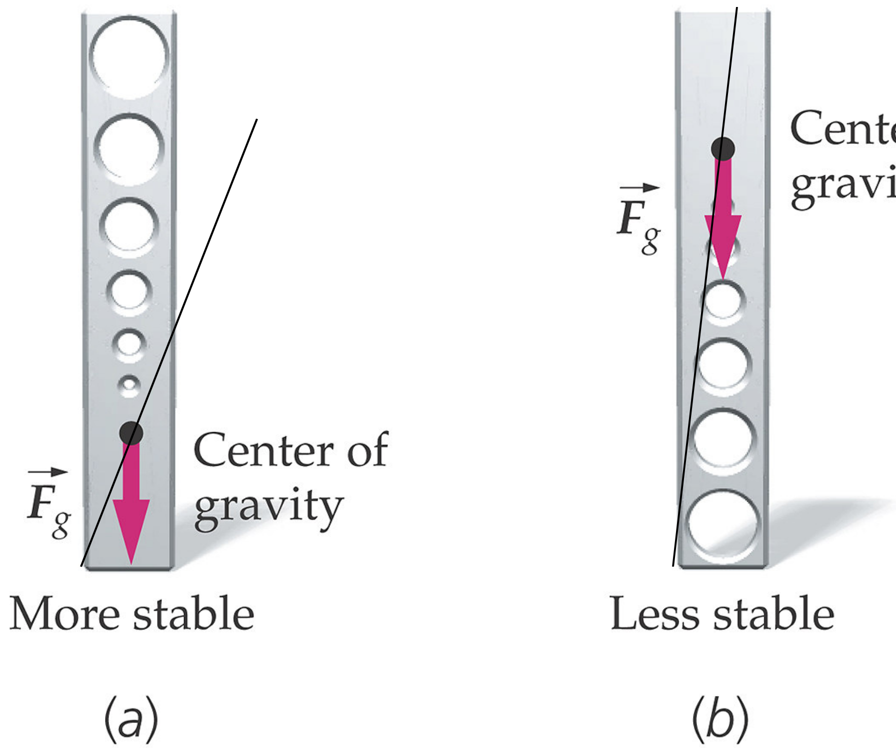
Let H be the height and L be the width of the base.

Δh is one half of the difference between the length of the diagonal and the height

Cow Tipping Technique

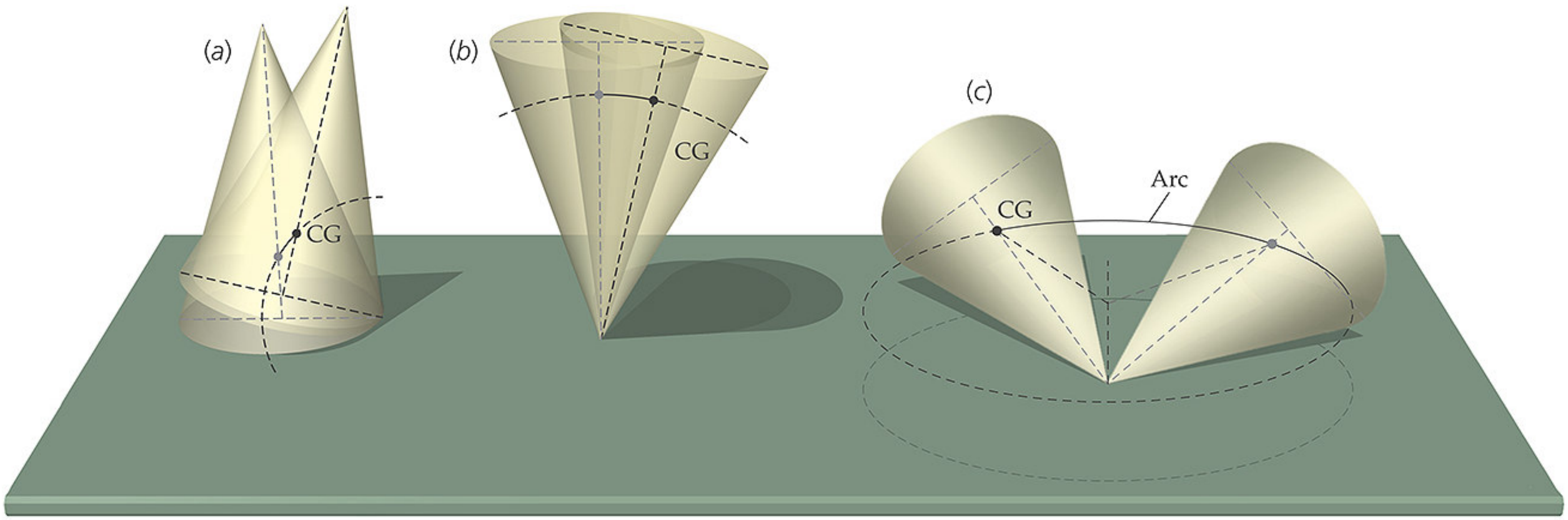


Lower the Center of Mass for More Stability



Leverage and Angle

Stability of Rotational Equilibrium



Stable Unstable Neutral

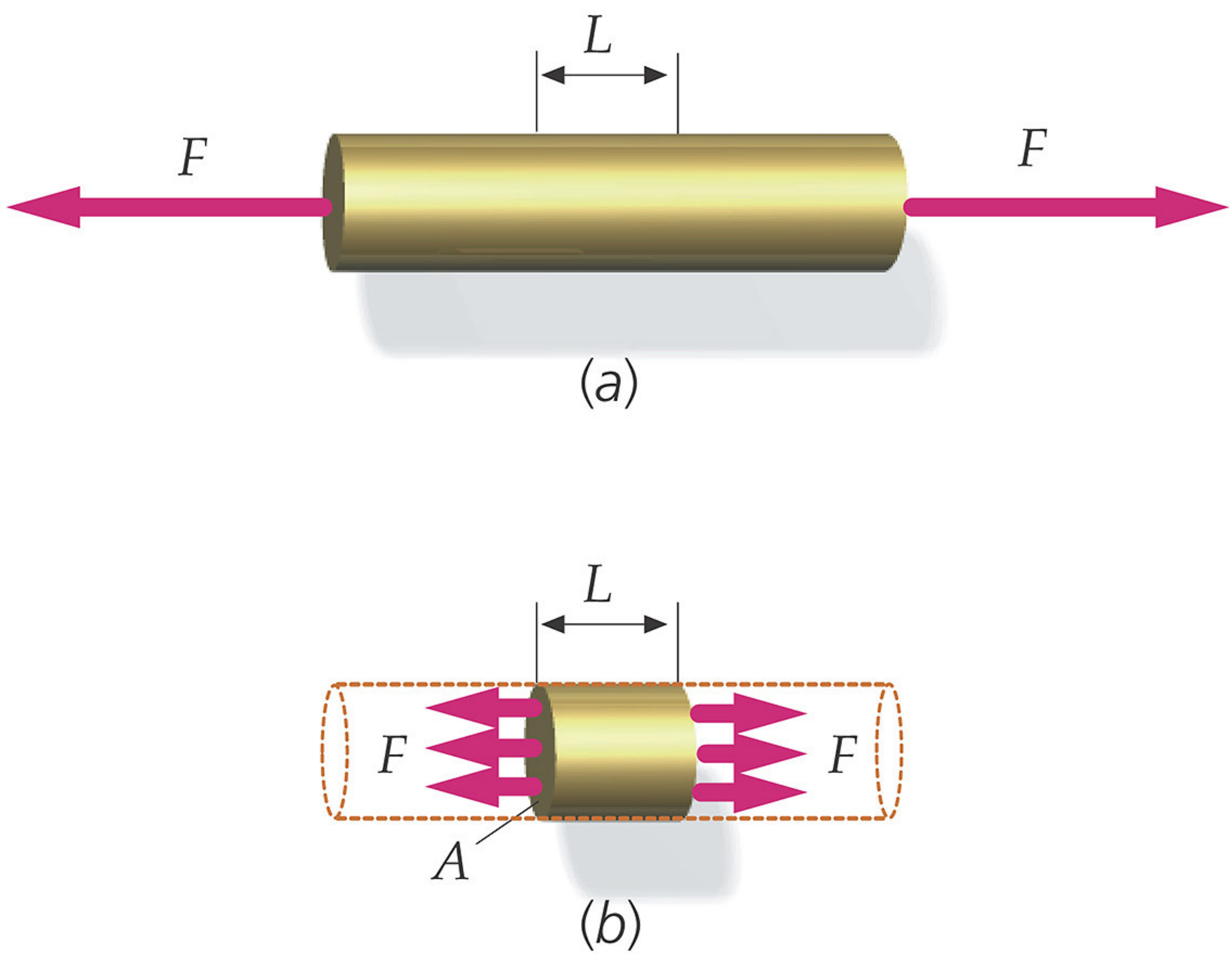
Follow the motion of the center of mass

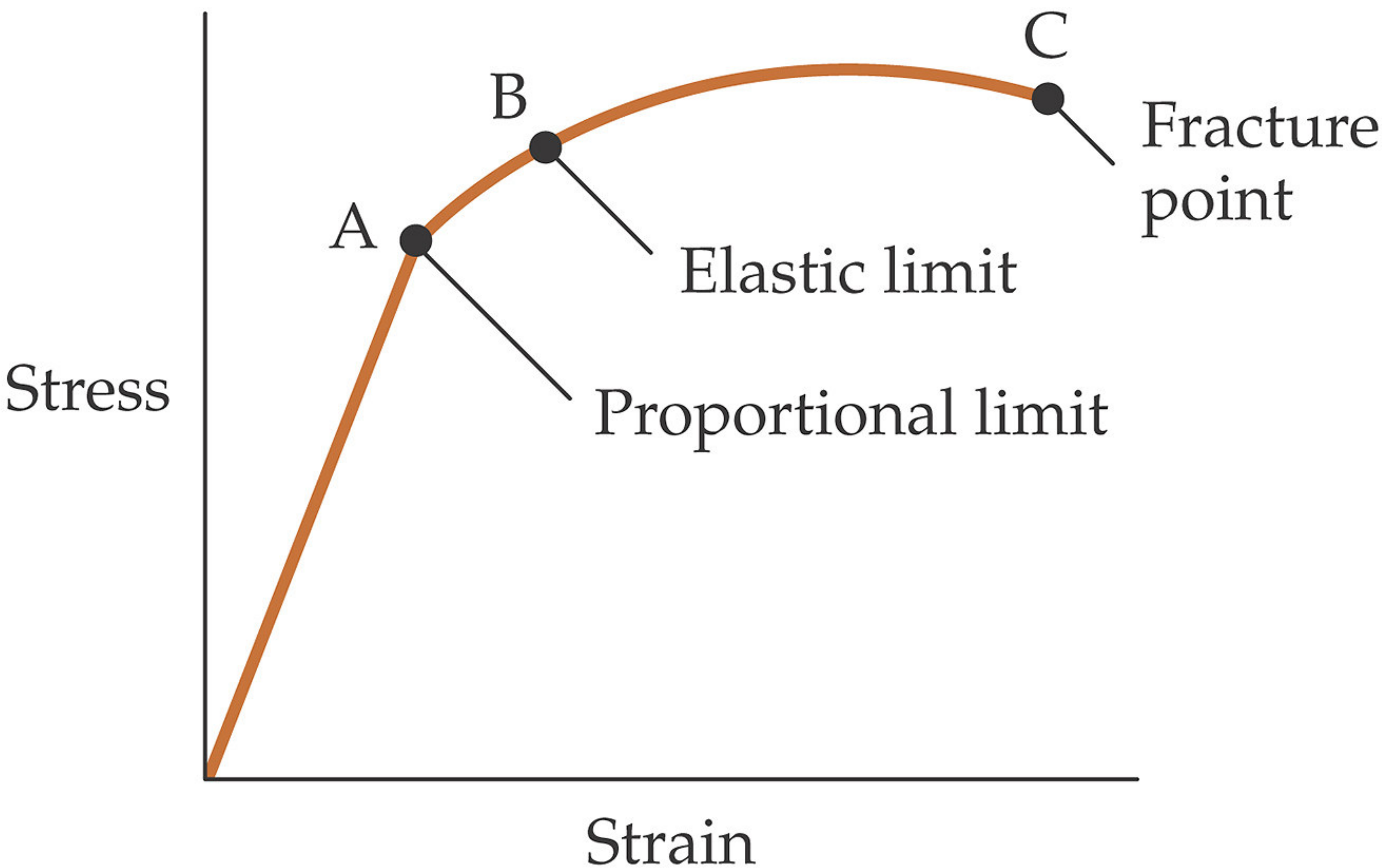
Elasticity

Elastic Deformation of Solids

A **deformation** is the change in size or shape of an object.

An elastic object is one that returns to its original size and shape after contact forces have been removed. If the forces acting on the object are too large, the object can be permanently distorted.


L
F
F
(a)
L
F
F
A
(b)


C
B
Fracture
point
A
Elastic limit
Stress
Proportional limit
Strain

Hooke's Law

Apply a force to both ends of a long wire. These forces will stretch the wire from length L to L+ΔL.

Stress and Strain

Define:

$$\text{strain} = \frac{\Delta L}{L}$$

The fractional change in length

$$\text{stress} = \frac{F}{A}$$

Force per unit cross-sectional area

Stretching ==> Tensile Stress

Squeezing ==> Compressive Stress

Hooke's Law

Hooke's Law ($F \propto x$) can be written in terms of stress and strain (stress $\propto$ strain).

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

The spring constant *k* is now $$k = \frac{YA}{L}$$

Y is called Young's modulus and is a measure of an object's stiffness. Hooke's Law holds for an object to a point called the **proportional limit**.

Table 12-1 Young's Modulus Y and Strengths of Various Materials[†]

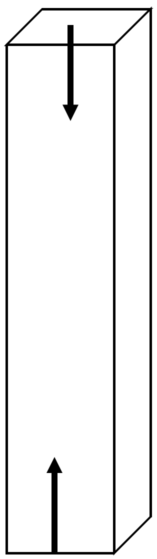
Material	Y, GN/m^2[‡]	Tensile strength, MN/m^2	Compressive strength, MN/m^2
Aluminum	70	90	
Bone			
Tensile	16	200	
Compressive	9		270
Brass	90	370	
Concrete	23	2	17
Copper	110	230	
Iron (wrought)	190	390	
Lead	16	12	
Steel	200	520	520

[†] These values are representative. Actual values for particular samples may differ.

[‡] $1\ \text{GN} = 10^3\ \text{MN} = 1 \times 10^9\ \text{N}$.

A steel beam is placed vertically in the basement of a building to keep the floor above from sagging. The load on the beam is 5.8×10^4 N and the length of the beam is 2.5 m, and the cross-sectional area of the beam is 7.5×10^{-3} m^2.

Find the vertical compression of the beam.



$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

For steel Y = 200×10^9 Pa.

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right) = \left(\frac{5.8\times10^4\ \text{N}}{7.5\times10^{-3}\ \text{m}^2}\right)\left(\frac{2.5\ \text{m}}{200\times10^9\ \text{N/m}^2}\right) = 1.0\times10^{-4}\ \text{m}$$

Example : A 0.50 m long guitar string, of cross-sectional area $1.0\times10^{-6}\ \text{m}^2$, has a Young's modulus of 2.0×10^{9} Pa.

By how much must you stretch a guitar string to obtain a tension of 20 N?

$$\frac{F}{A} = Y\,\frac{\Delta L}{L}$$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right) = \left(\frac{20.0\ \text{N}}{1.0\times10^{-6}\ \text{m}^2}\right)\left(\frac{0.5\ \text{m}}{2.0\times10^{9}\ \text{N/m}^2}\right)$$

$$= 5.0\times10^{-3}\ \text{m} = 5.0\ \text{mm}$$

Beyond Hooke's Law

Elastic Limit

If the stress on an object exceeds the **elastic limit**, then the object will not return to its original length.

Breaking Point

An object will fracture if the stress exceeds the **breaking point**. The ratio of maximum load to the original cross-sectional area is called tensile strength.

Ultimate Strength

The **ultimate strength** of a material is the maximum stress that it can withstand before breaking.

An acrobat of mass 55 kg is going to hang by her teeth from a steel wire and she does not want the wire to stretch beyond its elastic limit. The elastic limit for the wire is 2.5×10^{8} Pa.

What is the minimum diameter the wire should have to support her?

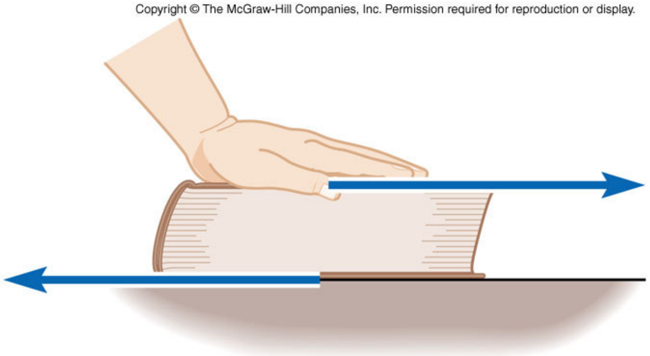
Want $$\text{stress} = \frac{F}{A} < \text{elastic limit}$$

$$A > \frac{F}{\text{elastic limit}} = \frac{mg}{\text{elastic limit}}$$

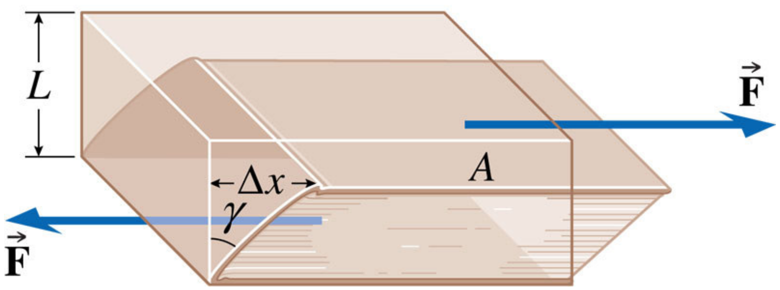
$$\pi\left(\frac{D}{2}\right)^2 > \frac{mg}{\text{elastic limit}}$$

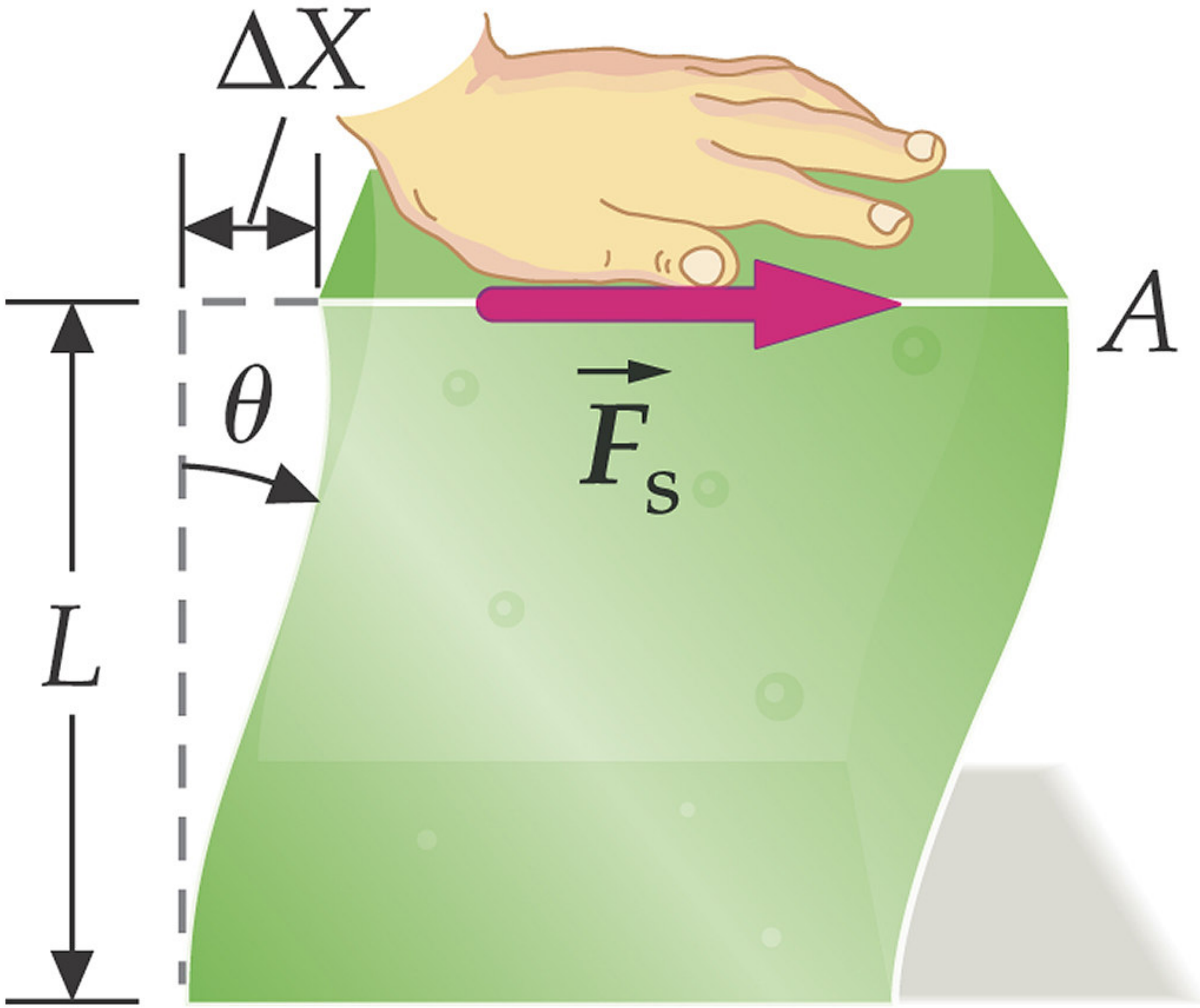
$$D > \sqrt{\frac{4mg}{\pi * \text{elastic limit}}} = 1.7\times10^{-3}\ \text{m} = 1.7\ \text{mm}$$

Shear Deformations



A **shear deformation** occurs when two forces are applied on opposite surfaces of an object.




ΔX
A
θ
$\vec{F}_s$
L

Stress and Strain

Define:

$$\text{Shear Stress} = \frac{\text{Shear Force}}{\text{Surface Area}} = \frac{F}{A}$$

$$\text{Shear Strain} = \frac{\text{displacement of surfaces}}{\text{separation of surfaces}} = \frac{\Delta x}{L}$$

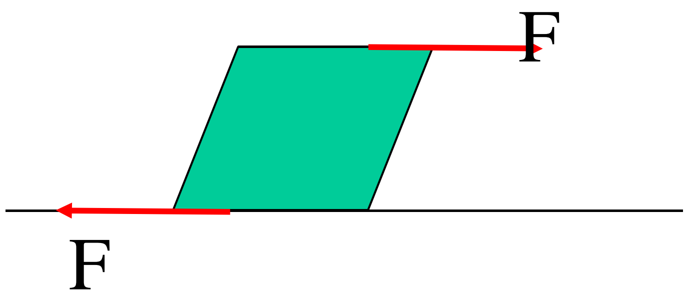
Hooke's law (stress$\propto$strain) for shear deformations is

$$\frac{F}{A} = S\,\frac{\Delta x}{L}$$

where S is the shear modulus

Example : The upper surface of a cube of gelatin, 5.0 cm on a side, is displaced by 0.64 cm by a tangential force. The shear modulus of the gelatin is 940 Pa.

What is the magnitude of the tangential force?



$$\frac{F}{A} = S\,\frac{\Delta x}{L}$$

From Hooke's Law:

$$F = SA\frac{\Delta x}{L}$$

$$= \left(940\ \mathrm{N/m^2}\right)\left(0.0025\ \mathrm{m^2}\right)\left(\frac{0.64\ \mathrm{cm}}{5.0\ \mathrm{cm}}\right) = 0.30\ \mathrm{N}$$

Table 12-2 Approximate Values of the Shear Modulus M_s of Various Materials

Material	M_s, GN/m^2
Aluminum	30
Brass	36
Copper	42
Iron	70
Lead	5.6
Steel	84
Tungsten	150

Volume Deformations

An object completely submerged in a fluid will be squeezed on all sides.

$$\text{volume stress} = \text{pressure} = \frac{F}{A}$$

The result is a volume strain;

$$\text{volume strain} = \frac{\Delta V}{V}$$

For a **volume deformation,** Hooke's Law is (stress$\propto$strain):

$$\Delta P = -B\,\frac{\Delta V}{V}$$

where B is called the bulk modulus. The bulk modulus is a measure of how easy a material is to compress.

An anchor, made of cast iron of bulk modulus 60.0×10^{9} Pa and a volume of $0.230\ \text{m}^3$, is lowered over the side of a ship to the bottom of the harbor where the pressure is greater than sea level pressure by 1.75×10^{6} Pa.

Find the change in the volume of the anchor.

$$\Delta P = -B\frac{\Delta V}{V}$$

$$\Delta V = -\frac{V\Delta P}{B} = -\frac{\left(0.230\ \text{m}^3\right)\left(1.75\times10^{6}\ \text{Pa}\right)}{60.0\times10^{9}\ \text{Pa}}$$

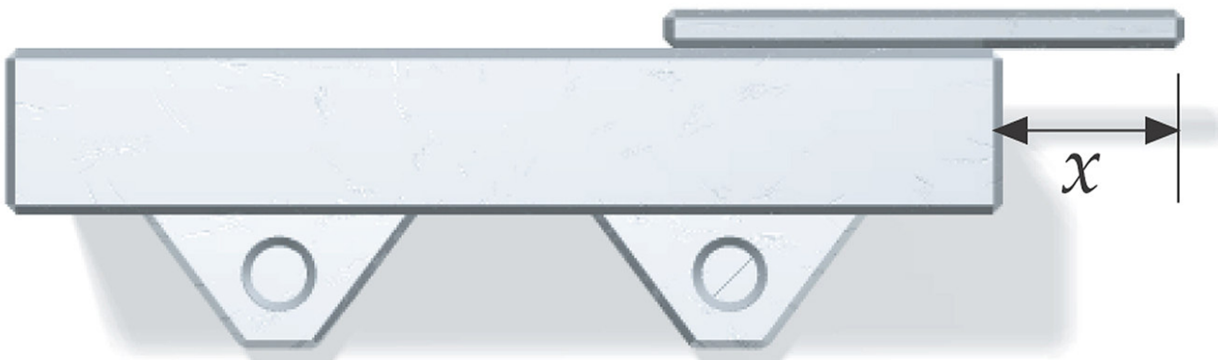
$$= -6.71\times10^{-6}\ \text{m}^3$$

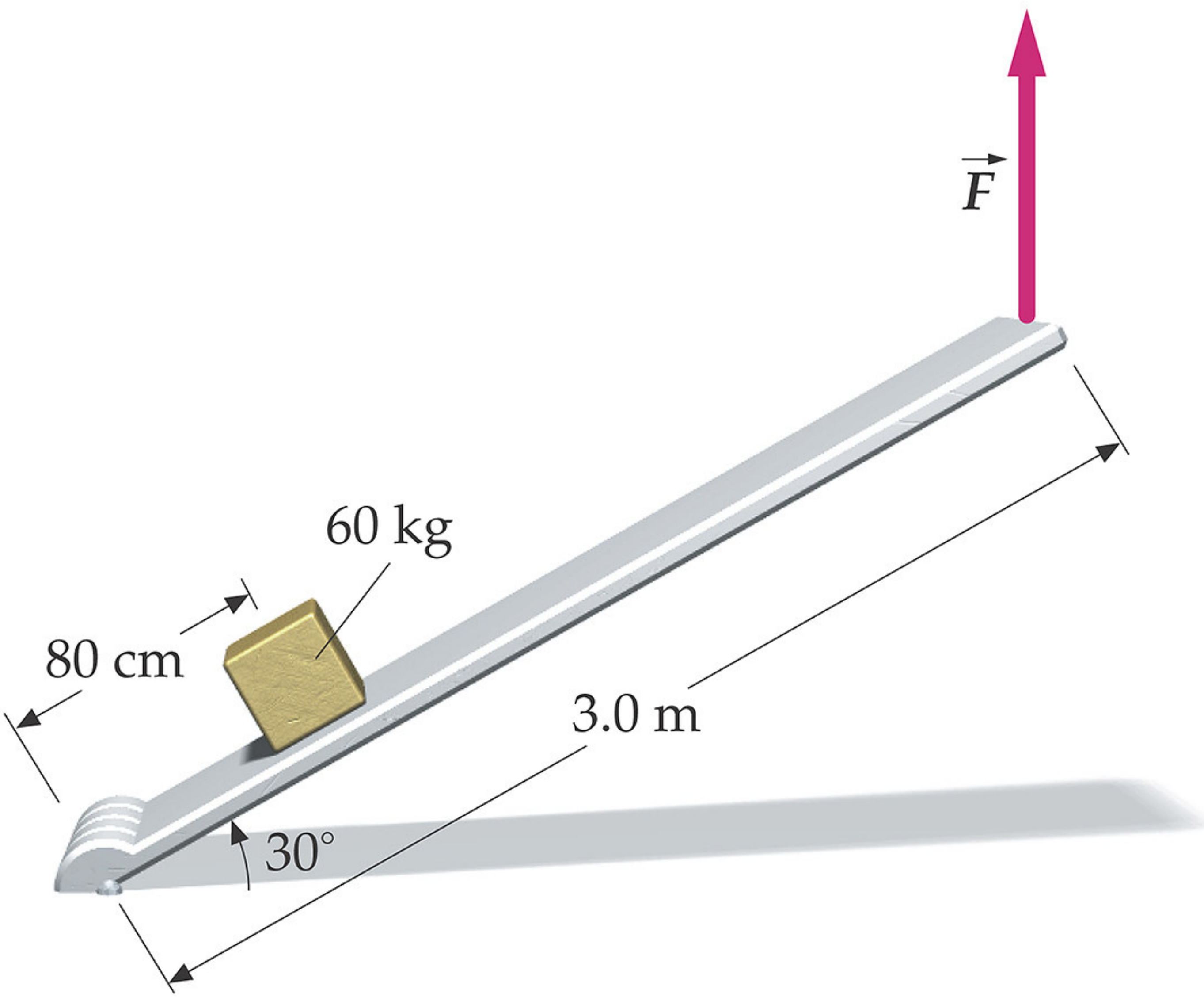
Deformations Summary Table

	Tensile or compressive	Shear	Volume
Stress	Force per unit cross-sectional area	Shear force divided by the area of the surface on which it acts	Pressure
Strain	Fractional change in length	Ratio of the relative displacement to the separation of the two parallel surfaces	Fractional change in volume
Constant of proportionality	Young's modulus (Y)	Shear modulus (S)	Bulk Modulus (B)

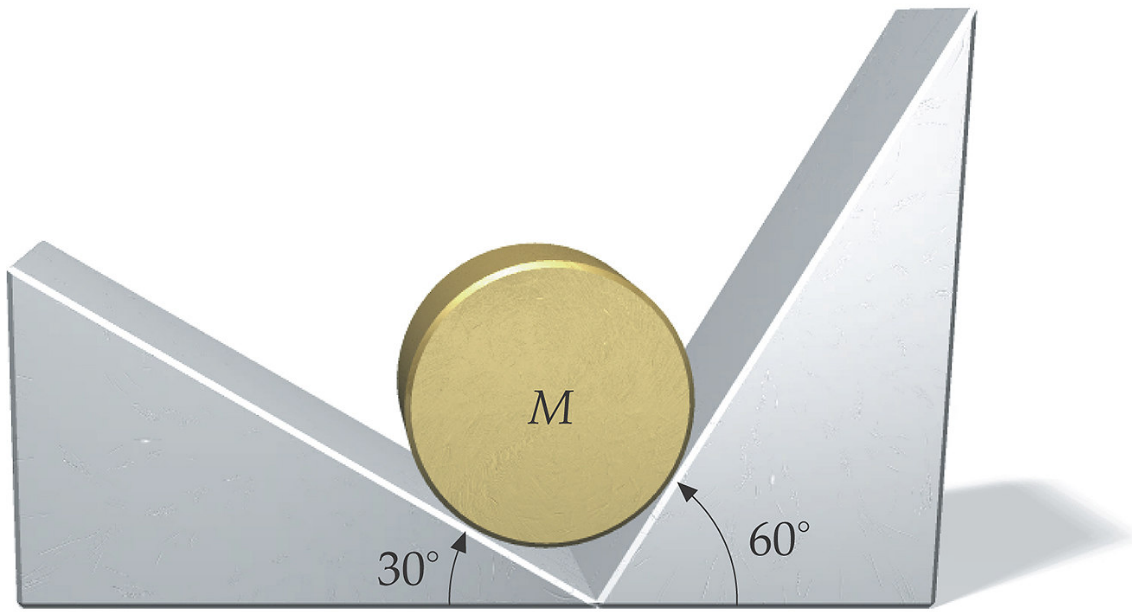
Examples

A 60kg student wants to walk out on the 300kg beam. What value of x will allow this?




$\vec{F}$
60 kg
80 cm
3.0 m
30°

Find the force exerted by each plane on the cylinder



The bar is 85kg, What is the tension in the cable and the force exerted by the wall on the steel bar?