Engineering Mechanics (Statics)

Stage: First Year

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Department of Petroleum and Gas Engineering

Course Number: PGE102: Engineering Mechanics (Statics)

Instructor: Haider K. Sakban

Credit hours: 3

Textbook: Hibbeler R. C., Engineering Mechanics, Statics , 14th ed, 2015

References:

1. M. E. Plesha, Engineering Mechanics Statics, 1st ed, 2010.
2. A. Bedford, Engineering Mechanics Statics, 5th ed, 2008.

Course Contents:

This course covers: principles of statics, Resultant of a force system, Equilibrium of a force system, Moment of a force, Friction, centroid and center of gravity, Moment of inertia, analysis of internal forces, Strain, Stress-Strain diagram, Hook's law, Shearing deformation, Poisson's Ratio, Volumetric strain, Thin walled cylinders, Thermal stress, Shear and bending moment in beam.

Grading Policy:

The final letter grade will be computed using the following criteria:

- Homework/Quizzes 5%
- Midterm Exam I 17.5%
- Midterm Exam II 17.5%
- Final Exam 60%

Contents

1. Chapter One: Review and Fundamental concepts:

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: rigid-body mechanics, deformable-body mechanics, and fluid mechanics. In this semester we will study rigid-body mechanics since it is a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids. Furthermore, rigid-body mechanics is essential for the design and analysis of many types of structural members, mechanical components, or electrical devices encountered in engineering. Rigid-body mechanics is divided into two areas: **statics** and **dynamics**.

Statics deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas ***dynamics*** is concerned with the accelerated motion of bodies. We can consider statics as a special case of dynamics, in which the acceleration is zero; however, statics deserves separate treatment in engineering education since many objects are designed with the intention that they remain in equilibrium.

Scientific method:

- Recognize a question (unexplained fact)
- Make an educated guess (hypothesis)
- Make prediction about the consequences of the hypothesis
- Perform an experiment or make calculations
- Formulate a general rule

1.1. Fundamental concepts and units of measurement

The following four quantities are used throughout mechanics:

1. Length
2. Time
3. Mass
4. Force

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.

TABLE 1–2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

1.2. General procedure for analysis

- ✓ Read the problem carefully and try to correlate the actual physical situation with the theory studied.
- ✓ Tabulate the problem data and draw to a large scale any necessary diagrams.
- ✓ Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous.
- ✓ Solve the necessary equations, and report the answer with no more than three significant figures.
- ✓ Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

Example:

Convert 2 km/h to m/s. How many ft /s is this?

Solution:

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$\frac{2\,km}{h} = \frac{2\,\cancel{km}}{\cancel{h}}\left(\frac{1000m}{\cancel{km}}\right)\left(\frac{1\,\cancel{h}}{3600\,s}\right) = \frac{2000\,m}{3600\,s} = 0.556\,m/s$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\,\frac{m}{s} = 0.556\,\frac{\cancel{m}}{s}\left(\frac{1\,ft}{0.3048\,\cancel{m}}\right) = 1.82\,ft/\,s$$

2. Chapter Two: Force vectors:

A ***scalar*** is any positive or negative physical quantity that can be completely specified by its *magnitude*. Examples of scalar quantities include length, mass, and time.

A ***vector*** is any physical quantity that requires both a *magnitude* and a *direction* for its complete description. Examples of vectors encountered in statics are force, position, and moment.

2.1. Vector Operations

Procedure for Analysis:

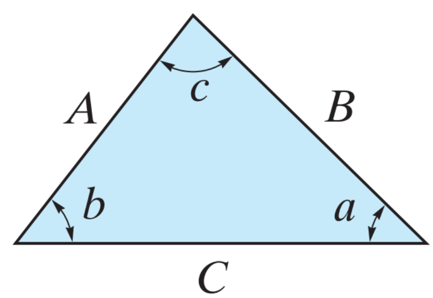
1. Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.
2. From this triangle, the ***magnitude of the resultant force*** can be determined using ***the law of cosines***, and its ***direction*** is determined from ***the law of sines***. The magnitudes of two force components are determined from the law of sines. The formulas are:

Cosine law:

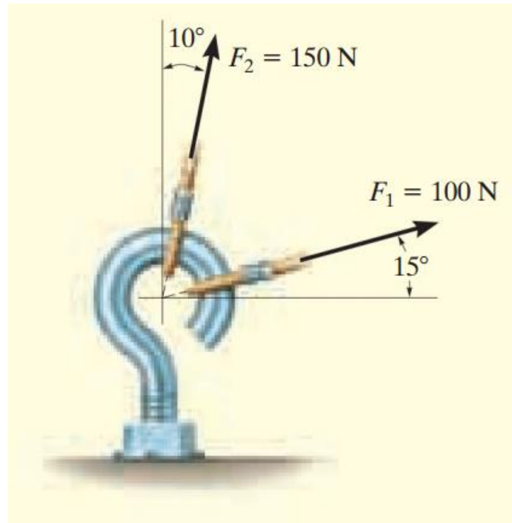
$$C = \sqrt{A^2 + B^2 - 2AB\cos(c)}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



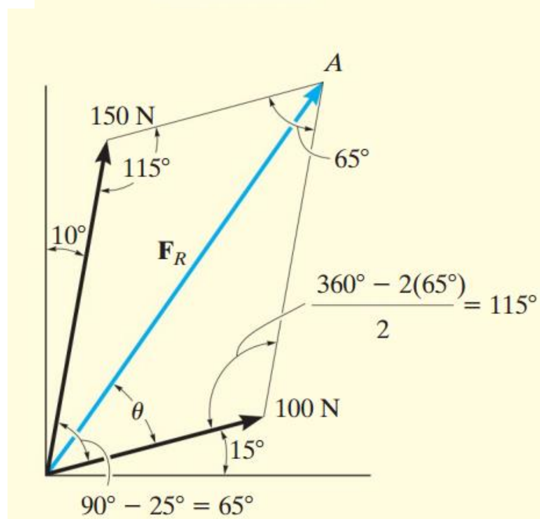
Example: The screw eye in Figure below is subjected to two forces, F1 and F2. Determine the ***magnitude*** and ***direction*** of the resultant force.



Solution:

The two unknowns are

The magnitude of F_R and the angle θ (theta).



Using the law of cosines:

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115^0}$$

$$F_R = \sqrt{10000 + 22500 - 30000\,(-0.4226)}$$

$$F_R = 212.6$$

Applying the law of sines to determine θ,

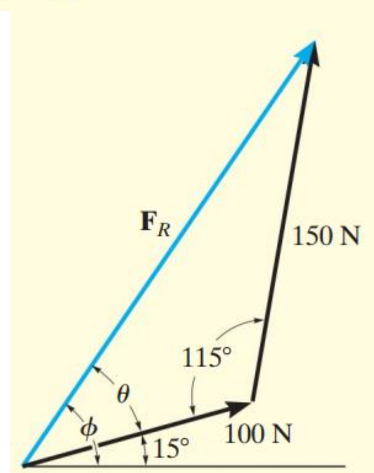
$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^0}$$

$$\sin\theta = \frac{150\,(\sin 115^0)}{212.6}$$

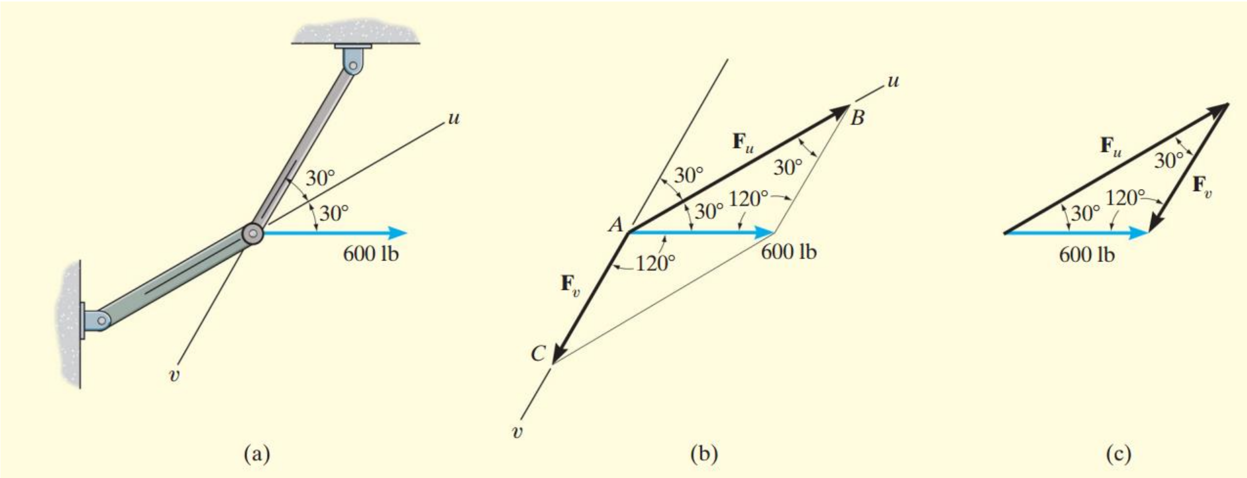
$$\theta = 39.8^0$$

Thus, the direction Φ (phi) of F_R, measured from the horizontal, is:

$$\Phi = 39.8^0 + 15^{\,0} = 54.8^0$$



Example: Resolve the horizontal 600-lb force in Figure (a) below into components acting along the *u* and *v* axes and determine the **magnitudes** of these components.



Solution:

The two unknowns are the magnitudes of **F_u** and **F_v**. Applying the law of sines,

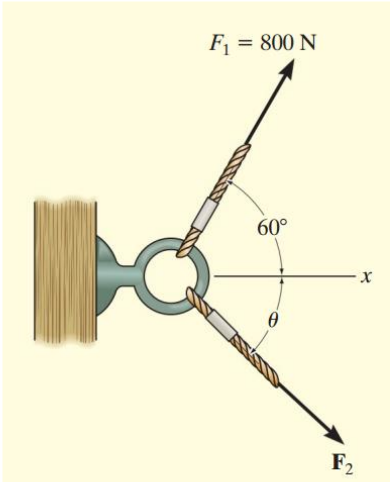
$$\frac{F_u}{\sin 120^0} = \frac{600}{\sin 30^0}$$

$$F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30^0} = \frac{600}{\sin 30^0}$$

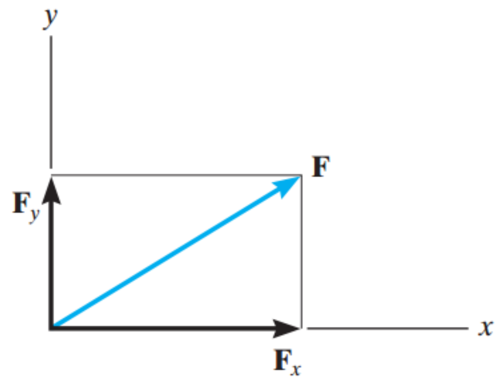
$$F_v = 600 \text{ lb}$$

HW: It is required that the resultant force acting on the eyebolt in Figure below be directed along the positive *x* axis and that **F**2 have a ***minimum*** magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

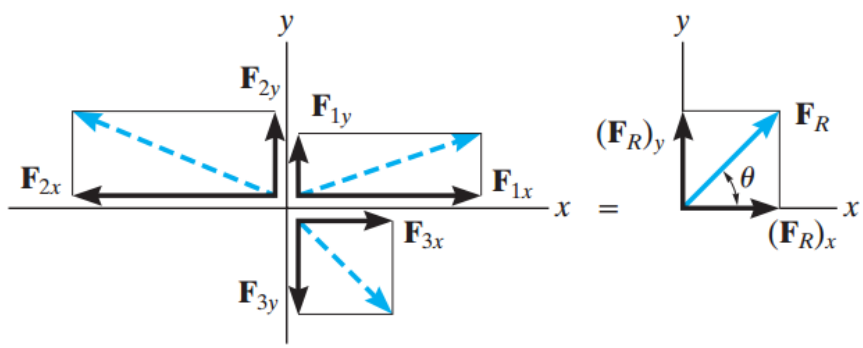


2.2.Rectangular Components :Two Dimensions

Vectors F_x and F_y are rectangular components of F.



The resultant force is determined from the algebraic sum of its components.



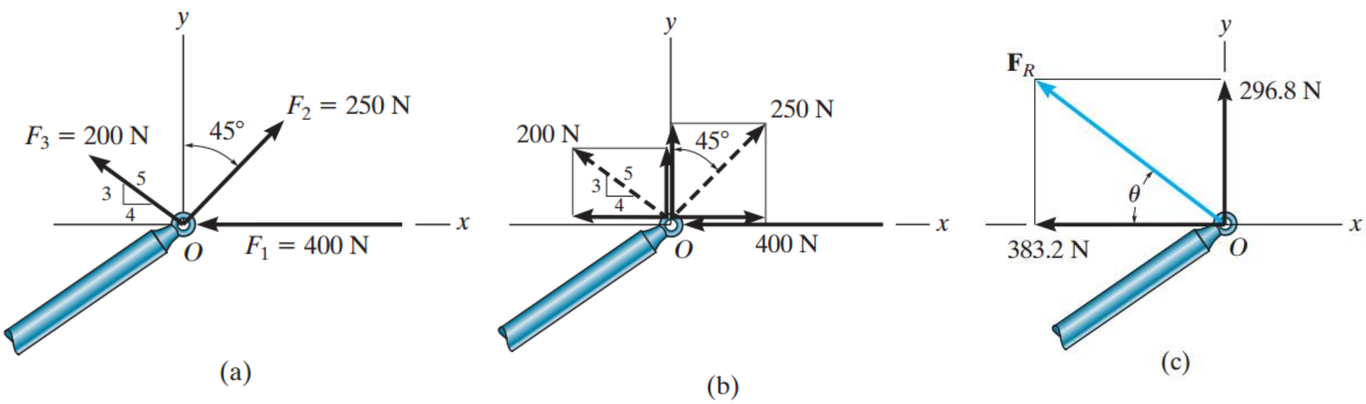
$$(F_R)_x = \Sigma F_x$$

$$(F_R)_y = \Sigma F_y$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Example: The end of the boom O in Figure (a) below is subjected to three concurrent and coplanar forces. Determine the ***magnitude*** and ***direction*** of the resultant force.



Solution:

Each force is resolved into its x and y components, Figure (b), Summing the x-components and y-components:

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400 \text{ N} + 250 \sin 45° \text{ N} - 200\left(\tfrac{4}{5}\right) \text{ N}$$
$$= -383.2 \text{ N} = 383.2 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45° \text{ N} + 200\left(\tfrac{3}{5}\right) \text{ N}$$
$$= 296.8 \text{ N} \uparrow$$

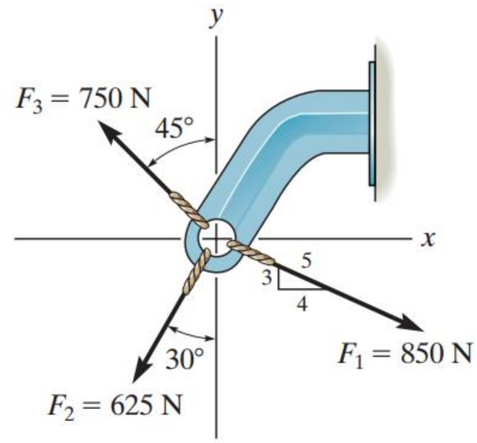
The resultant force, shown in Figure c, has a magnitude of:

$$F_R = \sqrt{(-383.2 \text{ N})^2 + (296.8 \text{ N})^2}$$
$$= 485 \text{ N}$$

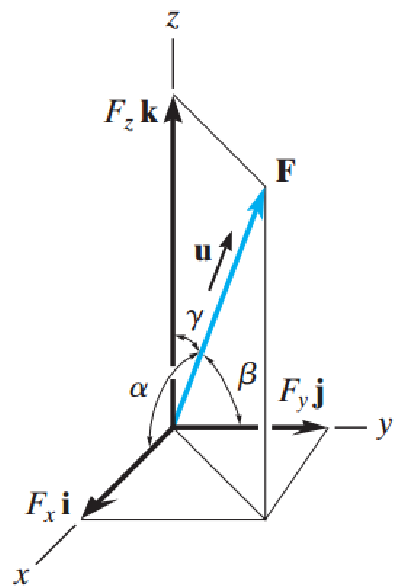
The direction angle θ is:

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8°$$

HW: Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



2.3. Rectangular Components: Three Dimensions



The magnitude of F is determined from the positive square root of the sum of the squares of its components.

$$F = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

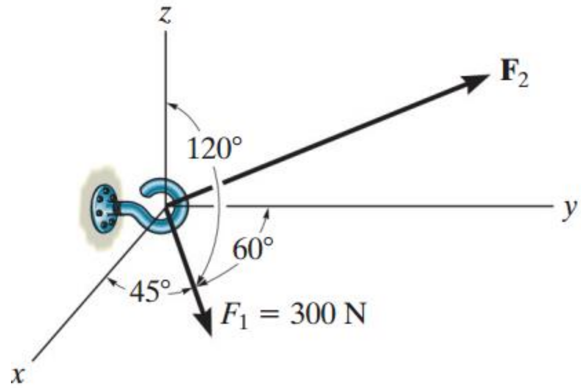
To determine α, β, and γ:

$$\cos\alpha = \frac{F_x}{F}, \qquad \cos\beta = \frac{F_y}{F}, \qquad \cos\gamma = \frac{F_z}{F}$$

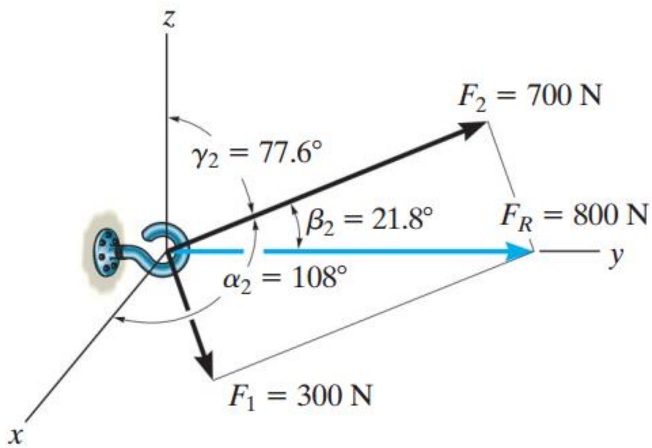
If only *two* of the coordinate angles are known, the third angle can be found using this equation:

$$\cos\alpha + \cos\beta + \cos\gamma = 1$$

Example: Two forces act on the hook shown in Fig. below. Specify the magnitude of F_2 and its coordinate direction angles so that the resultant force F_R acts along the positive y-axis and has a magnitude of 800 N.



(a)



(b)

Solution:

$\sum Fx = 0$ (Because F_R acts along the y-axis)

$0 = 300\cos 45^0 + F_{2x}$

$F_{2x} = -212.1$ N

$\sum Fy = 800\ N$ (Because F_R acts along the y-axis)

$800 = 300\cos 60^0 + F_{2y}$

$F_{2y} = 650$ N

$\sum Fz = 0$ (Because F_R acts along the y-axis)

$0 = 300\cos 120^0 + F_{2z}$

$0 = -150 + F_{2z}$

$F_{2z} = -150$ N

$$F_2 = \sqrt{{F_{2x}}^2 + {F_{2y}}^2 + {F_{2z}}^2}$$

$$F_2 = \sqrt{(-212.1)^2 + (650)^2 + (150)^2}$$

$F_2 = 700\ N$

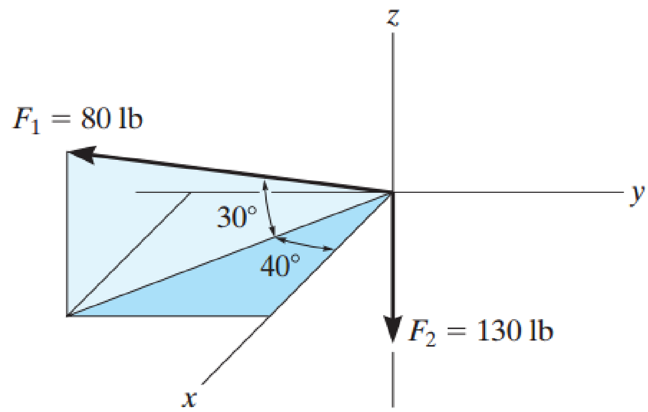
$\cos\alpha_2 = \frac{F_{2x}}{F} = \frac{-212.1}{700};\qquad \alpha_2 = 108^0$

$\cos\beta_2 = \frac{F_{2y}}{F} = \frac{650}{700};\qquad \beta_2 = 21.8^0$

$\cos\gamma_2 = \frac{F_{2z}}{F} = \frac{150}{700};\qquad \gamma_2 = 77.6^0$

HW: Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.

Solution:



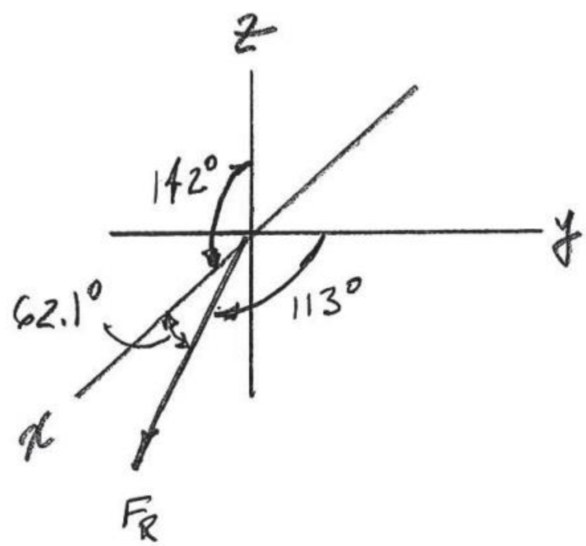
$$F_{1x} = 80\ \cos 30\ \cos 40 = 53.1\ ^{Ib} = F_x$$

$$F_{1y} = 80\ \cos 30\ \sin 40 = \ (-)\ 44.5\ ^{Ib} = \ F_y$$

$$F_{1z} = 80\ \sin 30 = \ 40\ ^{Ib}$$

$$F_z = 40 - 130 = -90\ ^{Ib}$$

$$F_R = \sqrt{53.1\ ^2 + 44.5^2 + (-90)^2} = \ 113.6^{Ib}$$



$$\cos \alpha = \frac{F_x}{F_R} = \frac{53.1}{113.6}; \qquad \alpha = 62.1^0$$

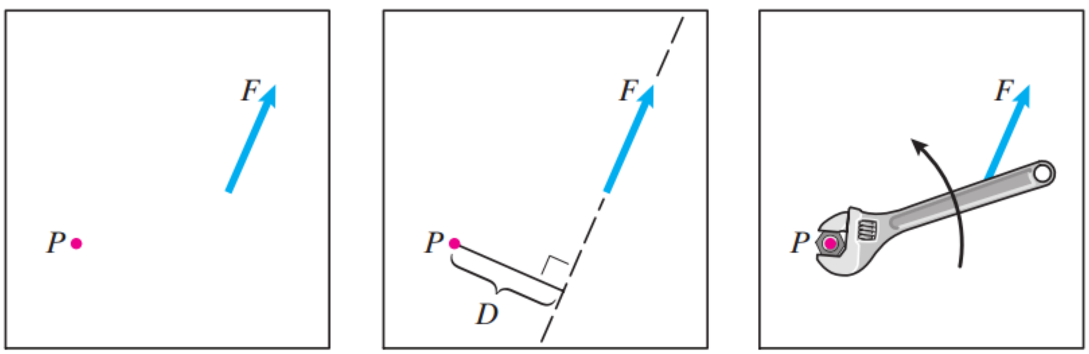
$$\cos \beta = \frac{F_y}{F_R} = \frac{-44.5}{113.6}; \qquad \beta = 113^0$$

$$\cos \gamma = \frac{F_z}{F_R} = \frac{-90}{113.6}; \qquad \gamma = 142^0$$

2.4. Moment of a force

Consider a force of magnitude F and a point P, and let's view them in the direction perpendicular to the plane containing the force vector and the point (Fig. a). The *magnitude of the moment* of the force about P is the product DF, where D is the perpendicular distance from P to the line of action of the force (Fig. b). In this example, the force would tend to cause counterclockwise rotation about point P. That is, if we imagine that the force acts on an object that can rotate about point P, the force would cause counterclockwise rotation (Fig. c). We say that the *direction of the moment* is counterclockwise. *We define counterclockwise moments to be positive and clockwise moments to be negative.* (This is the usual convention, although we occasionally encounter situations in which it is more convenient to define clockwise moments to be positive.) Thus, the moment of the force about P is

$$M_P = F * D$$



(a) (b) (c)

Example: what is the moment of the 40KN about point A?

Solution:

The perpendicular distance from *A* to the line of action of the force is

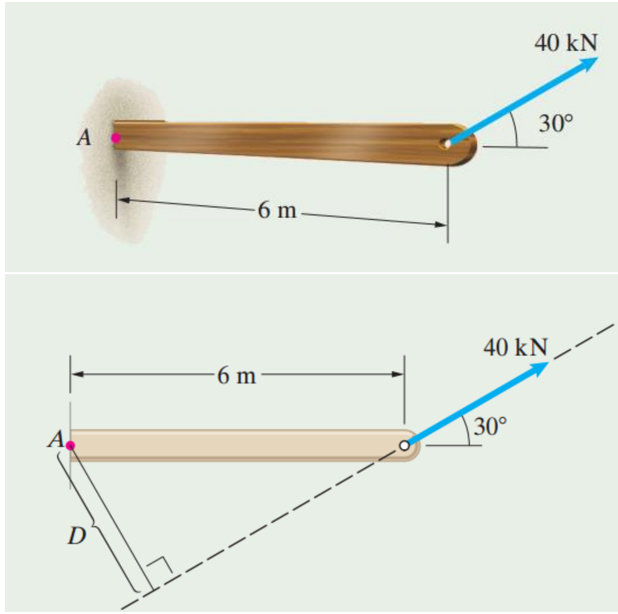
$D = (6\text{ m})\sin 30° = 3\text{ m}.$

Therefore the magnitude of the moment is

$(3\text{ m})(40\text{ kN}) = 120\text{ kN-m}.$

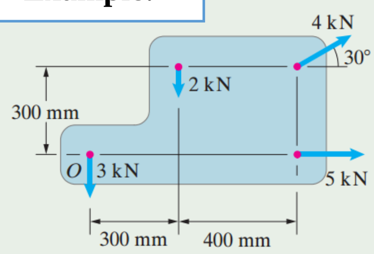
The direction of the moment is counterclockwise, so

$M_A = 120\text{ kN-m}.$



Practice Problem Resolve the 40-kN force into horizontal and vertical components and calculate the sum of the moments of the components about *A*.

Example:



Four forces act on the machine part. What is the sum of the moments of the forces about the origin *O*?

Strategy

We can determine the moments of the forces about point *O* directly from the given information except for the 4-kN force. We will determine its moment by expressing it in terms of components and summing the moments of the components.

Solution

Moment of the 3-kN Force The line of action of the 3-kN force passes through *O*. It exerts no moment about *O*.

Moment of the 5-kN Force The line of action of the 5-kN force also passes through *O*. It too exerts no moment about *O*.

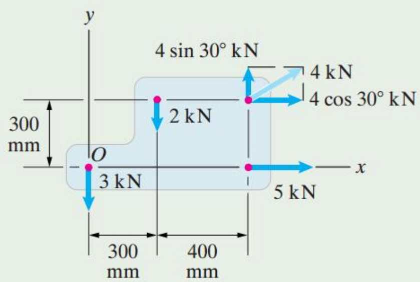
Moment of the 2-kN Force The perpendicular distance from *O* to the line of action of the 2-kN force is 0.3 m, and the direction of the moment about *O* is clockwise. The moment of the 2-kN force about *O* is

$$-(0.3 \text{ m})(2 \text{ kN}) = -0.600 \text{ kN-m}.$$

(Notice that we converted the perpendicular distance from millimeters into meters, obtaining the result in terms of kilonewton-meters.)

Moment of the 4-kN Force In Fig. a, we introduce a coordinate system and express the 4-kN force in terms of *x* and *y* components. The perpendicular distance from *O* to the line of action of the *x* component is 0.3 m, and the direction of the moment about *O* is clockwise. The moment of the *x* component about *O* is

$$-(0.3 \text{ m})(4 \cos 30^\circ \text{ kN}) = -1.039 \text{ kN-m}.$$



(a) Resolving the 4-kN force into components.

The perpendicular distance from point *O* to the line of action of the *y* component is 0.7 m, and the direction of the moment about *O* is counterclockwise. The moment of the *y* component about *O* is

$$(0.7 \text{ m})(4 \sin 30^\circ \text{ kN}) = 1.400 \text{ kN-m}.$$

The sum of the moments of the four forces about point *O* is

$$\Sigma M_0 = -0.600 - 1.039 + 1.400 = -0.239 \text{ kN-m}.$$

The four forces exert a 0.239 kN-m clockwise moment about point *O*.

3. Chapter Three: Equilibrium for a Rigid Body

The objectives of this chapter are:

- ✓ To develop the equations of equilibrium for a rigid body.
- ✓ To introduce the concept of the free-body diagram for a rigid body.
- ✓ To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

3.1. Conditions for Rigid-Body Equilibrium

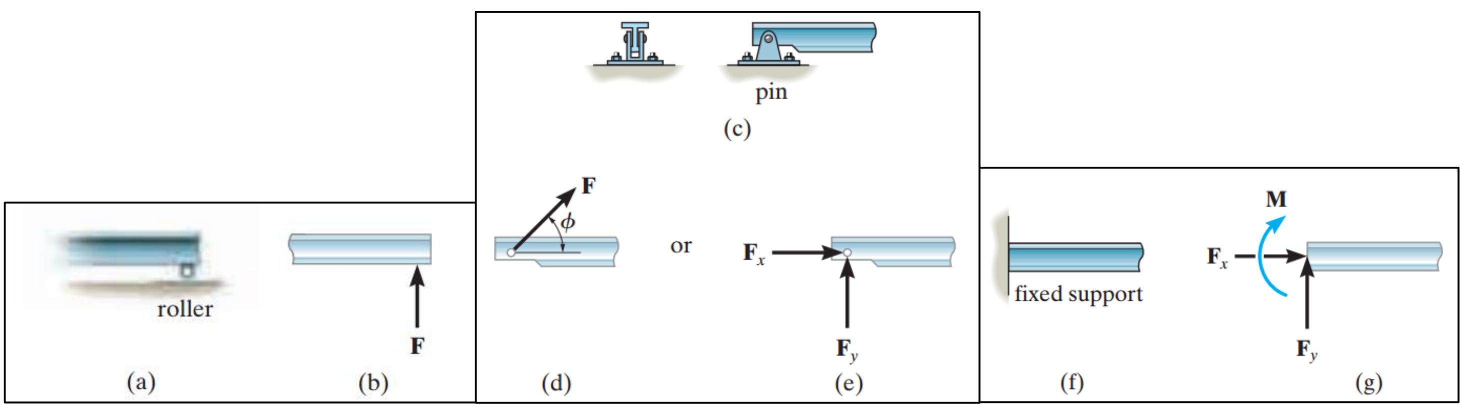
The body is said to be in ***equilibrium*** when resultant force and couple moment are both equal to zero. Mathematically, the equilibrium of a body is expressed as:

$$\sum Fx = 0$$

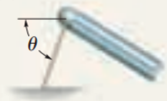
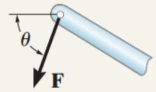
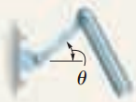
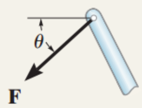
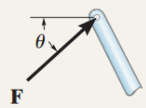
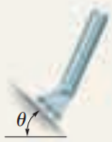
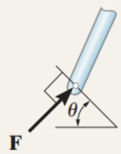
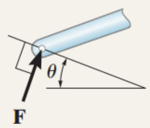
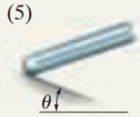
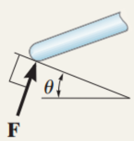
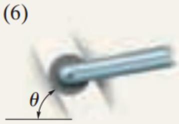
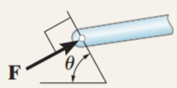
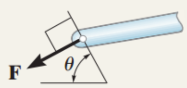
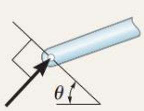
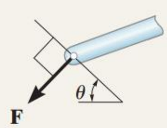
$$\sum Fy = 0$$

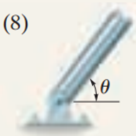
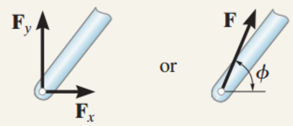
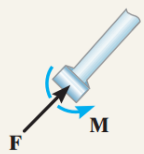
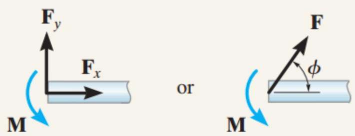
$$\sum M = 0$$

Main Support Reactions



Roller Pin (hinge) Fixed

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

3.2. Free-Body Diagrams

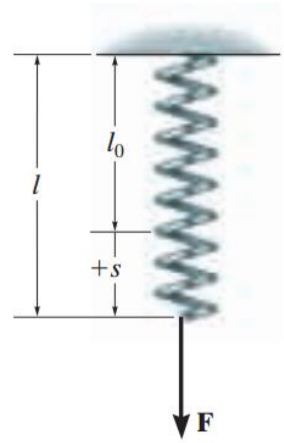
A ***free-body diagram*** is a sketch of the outlined shape of the body, which represents it as being isolated or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show all the forces and couple moments that the surroundings exert on the body so that these effects can be accounted for when the equations of equilibrium are applied.

To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

- ✓ **Draw Outlined Shape.**
- ✓ **Show All Forces and Couple Moments.**
- ✓ **Identify Each Loading and Give Dimensions.**

Note: the weight **W** of the body locates at the **center of gravity**.

Springs:



(a)

If a linearly elastic spring (or cord) of undeformed length l_0 is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it, Fig. a.

A characteristic that defines the "elasticity" of a spring is the spring constant or stiffness **k**. The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_0$, measured from its unloaded position, is:

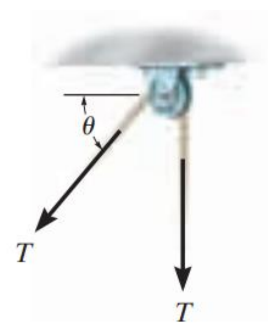
$$F = ks$$

For example, if the spring in **Fig. a** has an unstretched length of 0.8 m and a stiffness **k = 500 N/m** and it is stretched to a length of 1 m, so that $s = l - l_0 = 1\ m - 0.8\ m = 0.2\ m$, then a force:

F = ks =500 N /m (0.2 m) = 100 N.

Cables and Pulleys

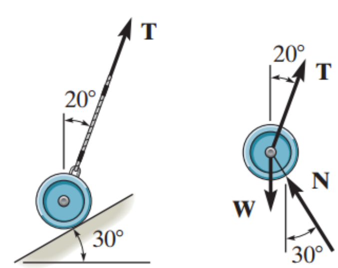
A cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable. Hence, for any angle θ, shown in Fig. b, the cable is subjected to a constant tension **T** throughout its length.



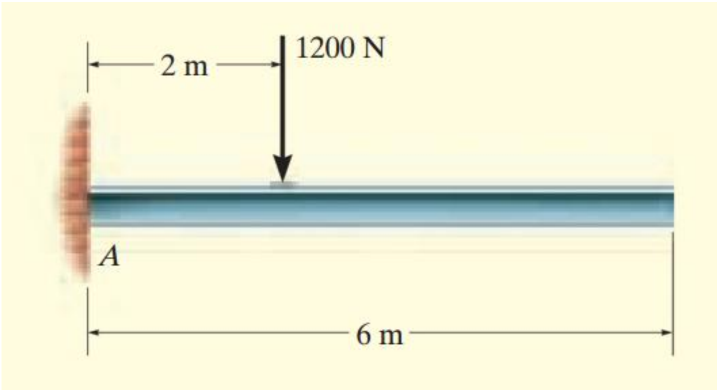
Cable is in tension
(b)

Smooth Contact:

If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.



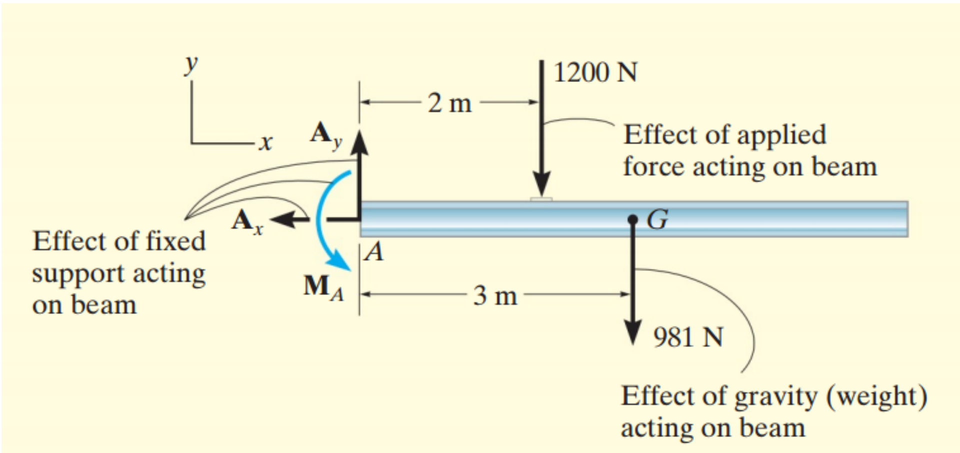
Example: Draw the free-body diagram of the uniform beam shown in Fig. below. The beam has a mass of 100 kg.



(a)

Solution:

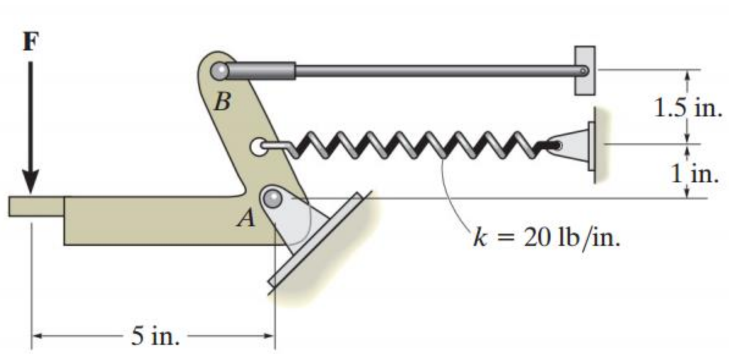
Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as Ax, Ay, and MA. The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, W = 100(9.81) N = 981 N, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.



(b)

Example: Draw the free-body diagram of the foot lever shown in Fig. a and b below. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force on the link at B is 20 lb.

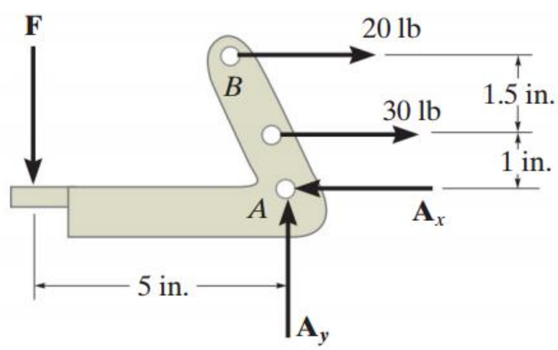
(a)



(b)

Solution:

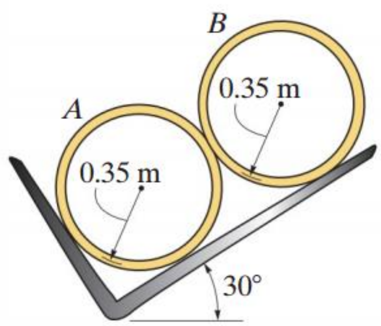
Since the pin at A is removed, it exerts force components A_x and A_y on the lever. The link exerts a force of **20 lb**, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be **k = 20 lb /in.**, then since the stretch **s = 1.5 in.**, using Eq. , **F_s = ks = 20 lb/in. (1.5 in.) = 30 lb**. Finally, the operator's shoe applies a vertical force of **F** on the pedal.



(c)

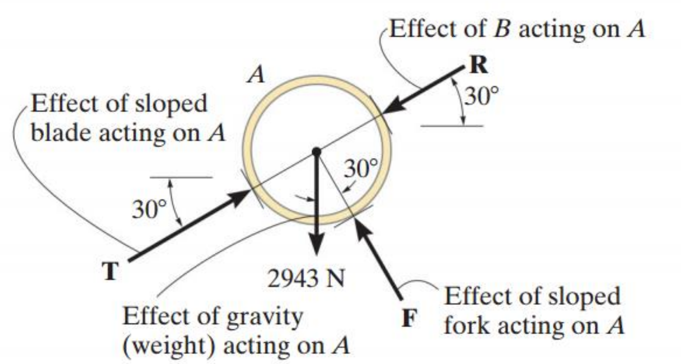
Example: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. a and b below. Draw the free-body diagrams for each pipe and both pipes together.

(a)

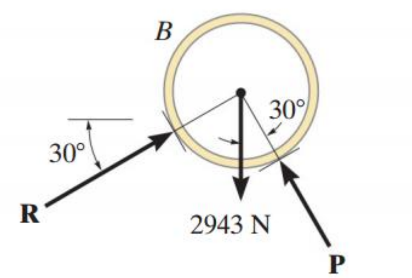


(b)

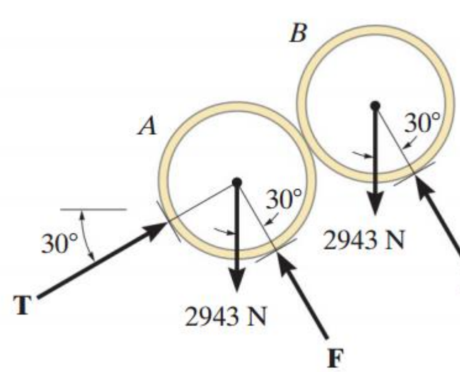
Solution: W = 300(9.81) N = 2943 N



(c)



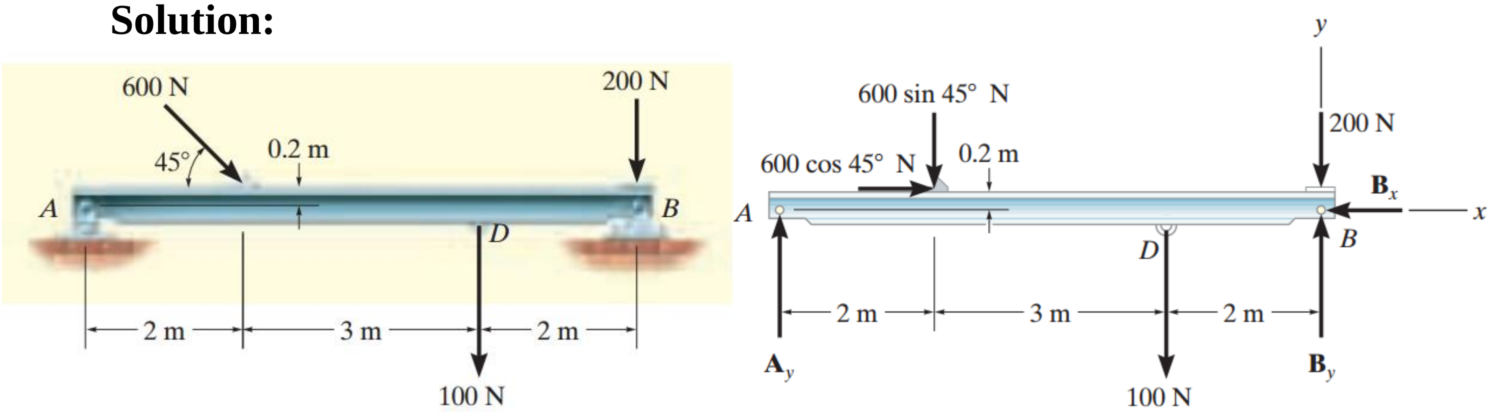
(d)



(e)

Example: Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the roller at A as shown in Fig. a. below. Neglect the weight of the beam.

Solution:



(a) (b)

Equations of Equilibrium. Summing forces in the x direction yields

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 600\cos 45^\circ\text{ N} - B_x = 0$$

$$B_x = 424\text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft +\Sigma M_B = 0; \qquad 100\text{ N}(2\text{ m}) + (600\sin 45^\circ\text{ N})(5\text{ m})$$

$$- (600\cos 45^\circ\text{ N})(0.2\text{ m}) - A_y(7\text{ m}) = 0$$

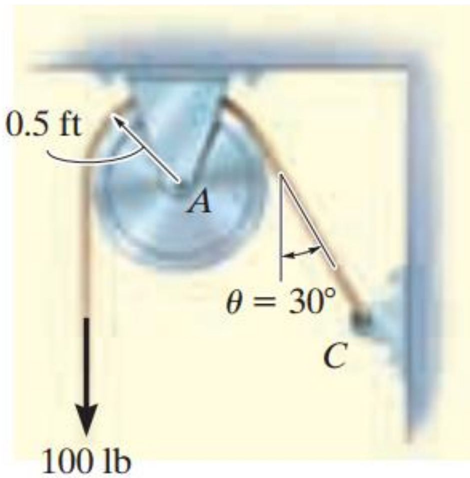
$$A_y = 319\text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

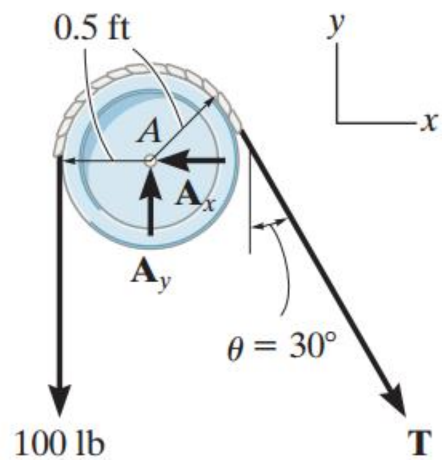
$$+\uparrow\Sigma F_y = 0; \qquad 319\text{ N} - 600\sin 45^\circ\text{ N} - 100\text{ N} - 200\text{ N} + B_y = 0$$

$$B_y = 405\text{ N} \qquad \textit{Ans.}$$

Example: The cord shown in Fig. a. below supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



(a)



Solution:

$$\circlearrowleft + \Sigma M_A = 0; \qquad 100 \text{ lb } (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$$

$$T = 100 \text{ lb} \qquad \textit{Ans.}$$

Using this result,

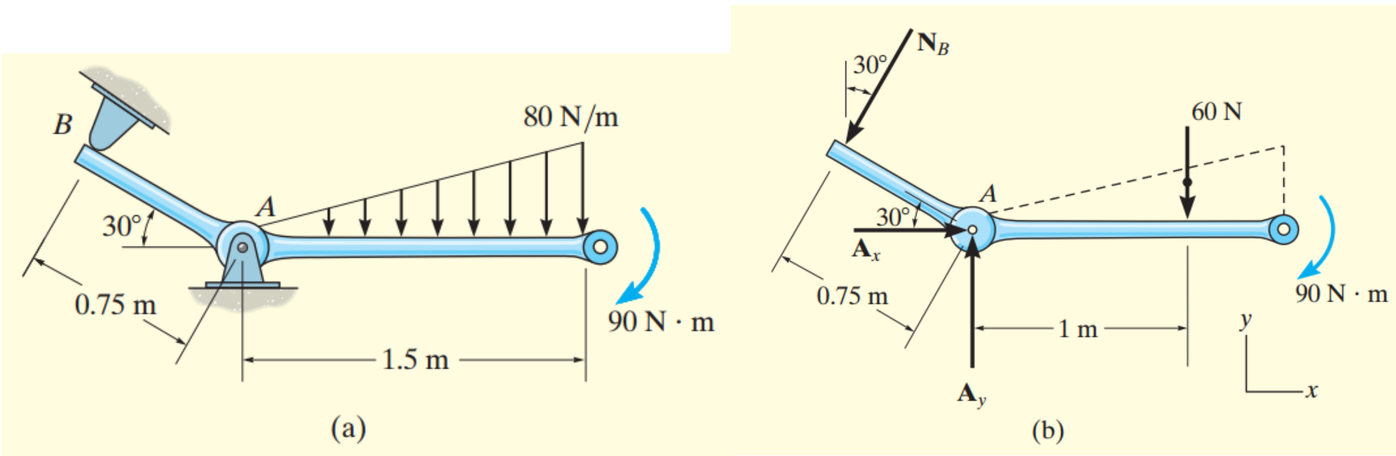
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 100 \sin 30° \text{ lb} = 0$$

$$A_x = 50.0 \text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 100 \text{ lb} - 100 \cos 30° \text{ lb} = 0$$

$$A_y = 187 \text{ lb} \qquad \textit{Ans.}$$

Example: The member shown in Fig. a. below is pin connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.



(a) (b)

Solution:

Free-Body Diagram. As shown in Fig. **(b)**, the supports are removed and the reaction $\mathbf{N}_B$ is perpendicular to the member at B. Also, horizontal and vertical components of reaction are represented at A. The resultant of the distributed loading is $\frac{1}{2}(1.5\text{ m})(80\text{ N/m}) = 60\text{ N}$. It acts through the centroid of the triangle, 1 m from A as shown.

Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft +\Sigma M_A = 0; \quad -90\text{ N}\cdot\text{m} - 60\text{ N}(1\text{ m}) + N_B(0.75\text{ m}) = 0$$

$$N_B = 200\text{ N}$$

Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 200\sin 30^\circ\text{ N} = 0$$

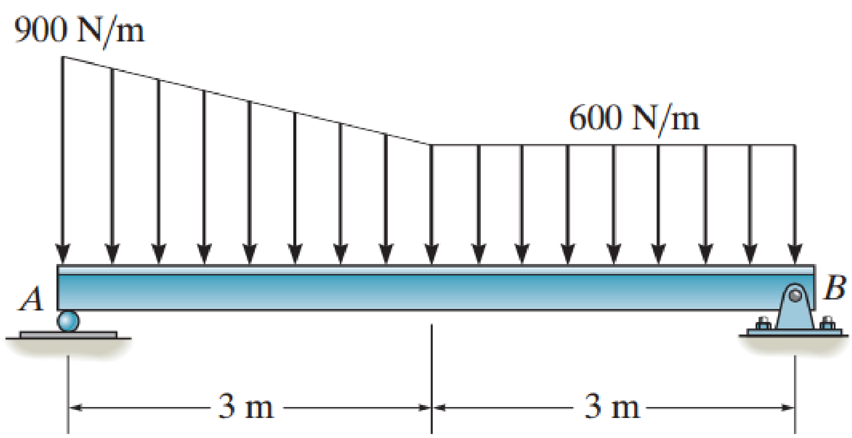
$$A_x = 100\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 200\cos 30^\circ\text{ N} - 60\text{ N} = 0$$

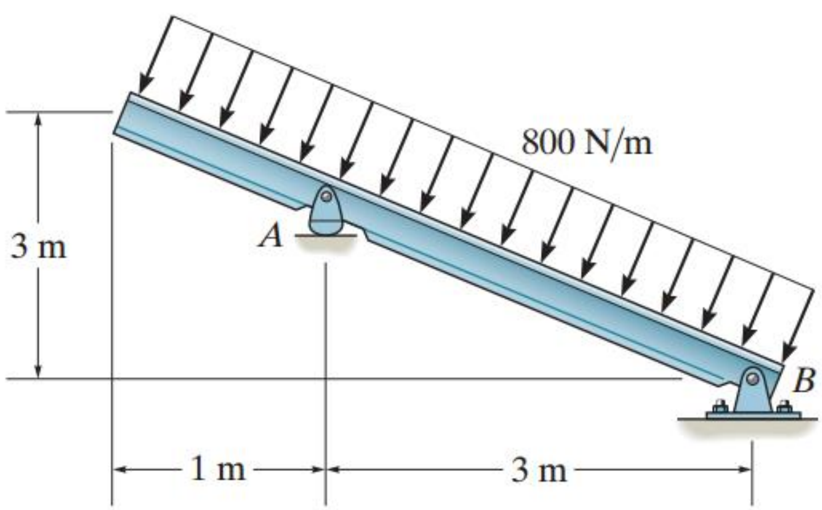
$$A_y = 233\text{ N} \qquad \textit{Ans.}$$

HW:

1. Determine the reactions at the supports:

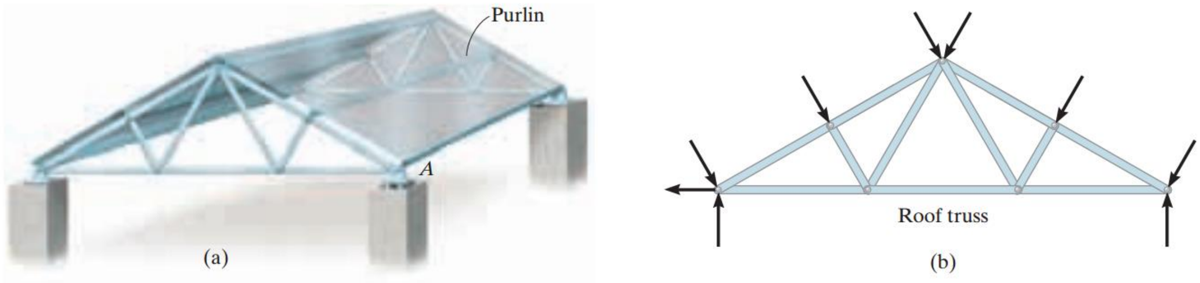


2. Determine the reactions at the supports:



4. Chapter four: Structural Analysis

A ***truss*** is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars.



(a) (b)

Simple trusses are composed of **triangular** elements. The members are assumed to be **pin connected** at their ends and loads applied at the joints.

Assumptions for Design

- ✓ All loadings are applied at the joints.
- ✓ The members are joined together by smooth pins.

4.1. Analysis of trusses:

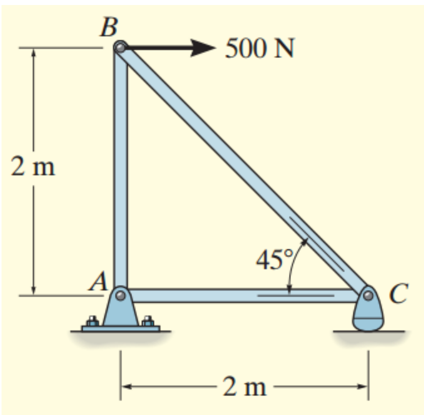
In order to analyze or design a truss, it is necessary to determine the force in each of its members. There are two main methods:

1. Method of joints.
2. Method of sections.

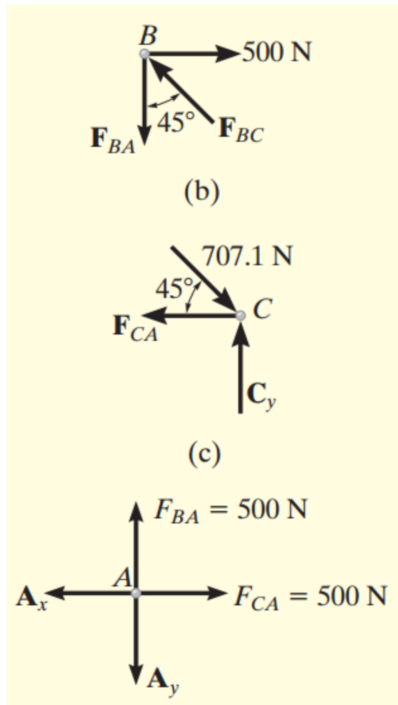
4.1.1. The Method of Joints

This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint.

Example: Determine the force in each member of the truss shown in Fig. *a* and indicate whether the members are in tension or compression.



(a)



(b) (c) (d)

Solution:

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint B.

Joint B. The free-body diagram of the joint at B is shown in Fig. b. applying the equations of equilibrium, we have:

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 500\text{ N} - F_{BC}\sin 45^\circ = 0 \qquad F_{BC} = 707.1\text{ N (C)} \qquad \textit{Ans.}$$
$$+\uparrow\Sigma F_y = 0; \qquad F_{BC}\cos 45^\circ - F_{BA} = 0 \qquad F_{BA} = 500\text{ N (T)} \qquad \textit{Ans.}$$

Since the force in member BC has been calculated, we can proceed to analyze joint C to determine the force in member CA and the support reaction at the rocker.

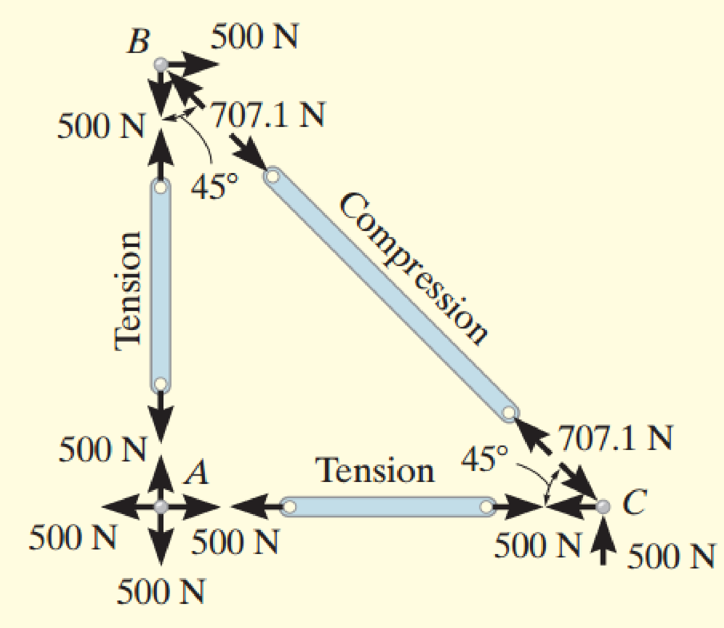
Joint C. From the free-body diagram of joint C, Fig. c, we have:

$$\xrightarrow{+} \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45^\circ \text{ N} = 0 \qquad F_{CA} = 500 \text{ N (T)} \qquad \textit{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45^\circ \text{ N} = 0 \qquad C_y = 500 \text{ N} \qquad \textit{Ans.}$$

Joint A. Although it is not necessary, we can determine the components of the support reactions at joint A using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. d, we have:

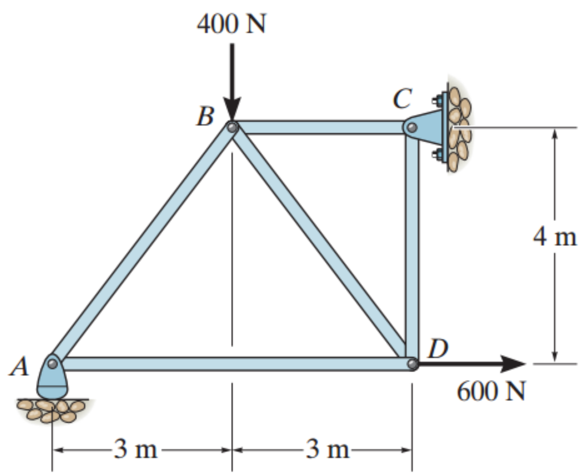
$$\xrightarrow{+} \Sigma F_x = 0; \quad 500 \text{ N} - A_x = 0 \qquad A_x = 500 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \quad 500 \text{ N} - A_y = 0 \qquad A_y = 500 \text{ N}$$

The results of the analysis are summarized in Fig. e below:



(e)

HW: Determine the force in each member of the truss shown in Fig. below. Indicate whether the members are in tension or compression.

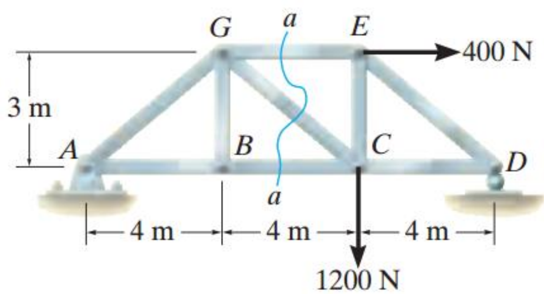


(a)

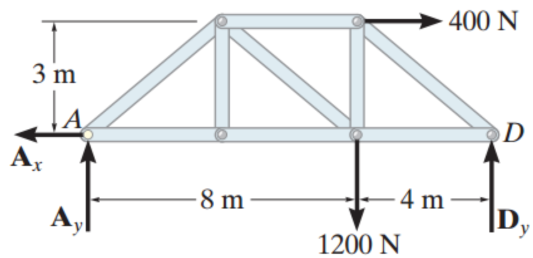
4.1.2. The Method of Sections

When we need to find the force in only a few members of a truss, we can analyze the truss using ***the method of sections***. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium.

Example: Determine the force in members GE, GC, and BC of the truss shown in Fig. a. Indicate whether the members are in tension or compression.



(a)



(b)

Solution:

Section a a in Fig. a has been chosen since it cuts through the three members whose forces are to be determined. In order to use the method of sections, however, it is first necessary to determine the external reactions at A or D. Why? A free-body diagram of the entire truss is shown in Fig. b. applying the equations of equilibrium, we have:

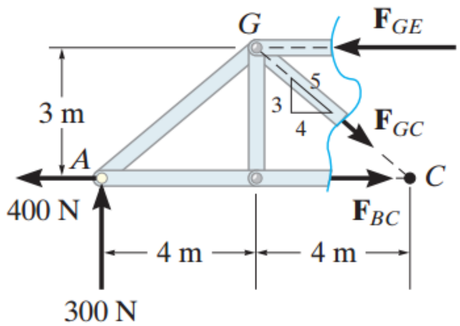
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 400\text{ N} - A_x = 0 \qquad A_x = 400\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -1200\text{ N}(8\text{ m}) - 400\text{ N}(3\text{ m}) + D_y(12\text{ m}) = 0$$

$$D_y = 900\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200\text{ N} + 900\text{ N} = 0 \qquad A_y = 300\text{ N}$$

Free-Body Diagram. For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, Fig. c.



(c)

Equations of Equilibrium. Summing moments about point G eliminates F_{GE} and F_{GC} and yields a direct solution for F_{BC}.

$$\circlearrowleft + \Sigma M_G = 0; \quad -300\text{ N}(4\text{ m}) - 400\text{ N}(3\text{ m}) + F_{BC}(3\text{ m}) = 0$$

$$F_{BC} = 800\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

In the same manner, by summing moments about point *C* we obtain a direct solution for F_{GE}.

$$\circlearrowleft + \Sigma M_C = 0; \quad -300\text{ N}(8\text{ m}) + F_{GE}(3\text{ m}) = 0$$

$$F_{GE} = 800\text{ N} \quad (\text{C}) \qquad \textit{Ans.}$$

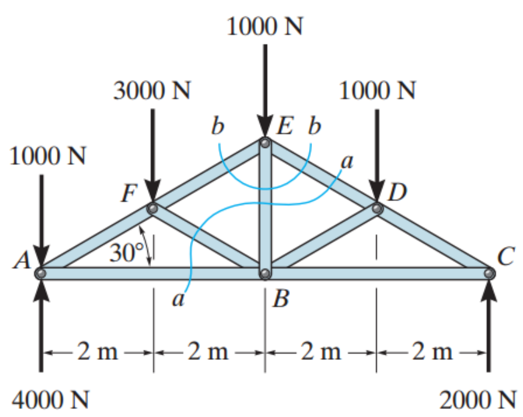
Since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GE}$ have no vertical components, summing forces in the *y* direction directly yields F_{GC}, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad 300\text{ N} - \tfrac{3}{5}F_{GC} = 0$$

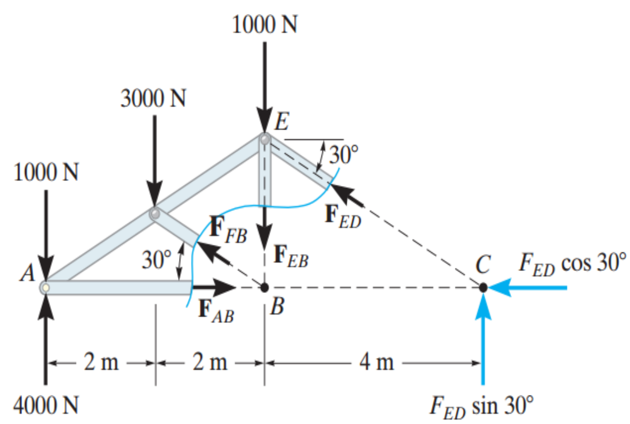
$$F_{GC} = 500\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

NOTE: Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, $\Sigma M_C = 0$ requires $\mathbf{F}_{GE}$ to be *compressive* because it must balance the moment of the 300-N force about *C*.

Example: Determine the force in member EB of the roof truss shown in Fig. a. Indicate whether the member is in tension or compression.



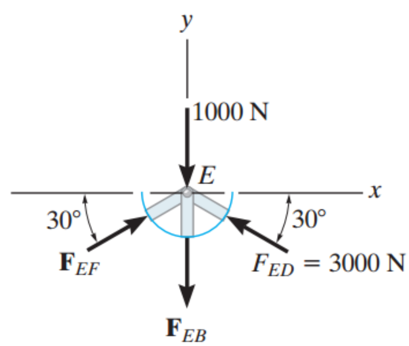
(a)



(b)

Solution:

Free-Body Diagrams. By the method of sections, any imaginary section that cuts through EB, Fig.a, will also have to cut through three other members for which the forces are unknown. For example, section aa cuts through ED, EB, FB, and AB. If a free-body diagram of the left side of this section is considered, Fig. b, it is possible to obtain F_{ED} by summing moments about B to eliminate the other three unknowns; however, F_{EB} cannot be determined from the remaining two equilibrium equations. One possible way of obtaining F_{EB} is first to determine F_{ED} from section aa, then use this result on section bb, Fig. 6–18a, which is shown in Fig. c. ***Here the force system is concurrent and our sectioned free-body diagram is the same as the free-body diagram for the joint at E.***



(c)

Equations of Equilibrium. In order to determine the moment of F_{ED} about point B, Fig. b, we will use the principle of transmissibility and slide the force to point C and then resolve it into its rectangular components as shown. Therefore,

$$\circlearrowleft +\Sigma M_B = 0; \qquad 1000\text{ N}(4\text{ m}) + 3000\text{ N}(2\text{ m}) - 4000\text{ N}(4\text{ m}) + F_{ED}\sin 30^\circ(4\text{ m}) = 0$$

$$F_{ED} = 3000\text{ N} \quad (\text{C})$$

Considering now the free-body diagram of section bb, Fig. c, we have:

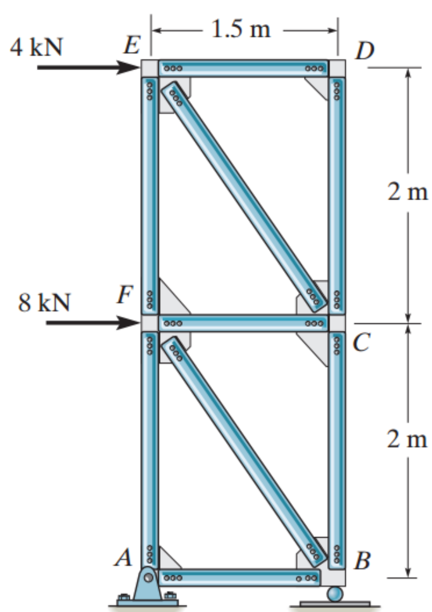
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{EF}\cos 30^\circ - 3000\cos 30^\circ\text{ N} = 0$$

$$F_{EF} = 3000\text{ N} \quad (\text{C})$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(3000\sin 30^\circ\text{ N}) - 1000\text{ N} - F_{EB} = 0$$

$$F_{EB} = 2000\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

HW: Determine the force in members AF, BF, and BC, and state if the members are in tension or compression.



5. Chapter Five: Friction

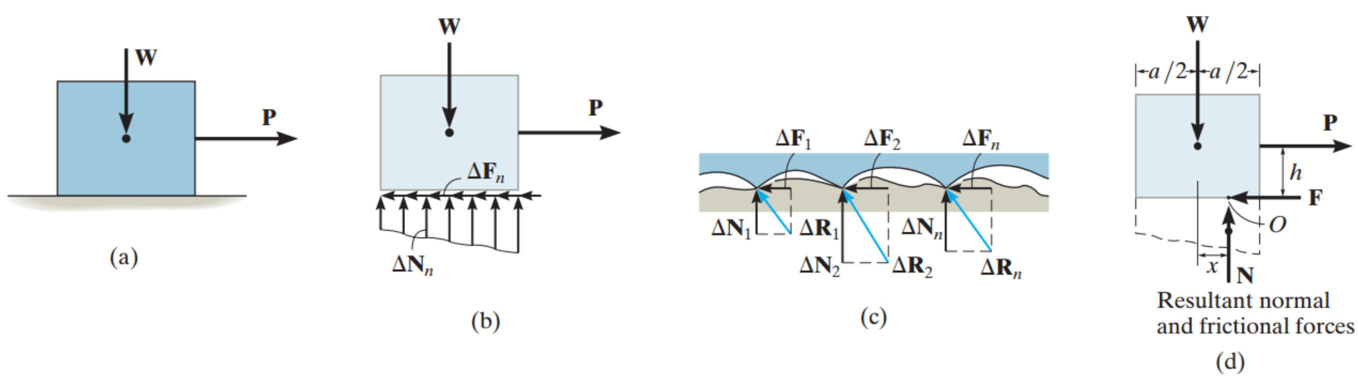
Friction is a force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts tangent to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

Dry friction occurs between the contacting surfaces of bodies when there is no lubricating fluid. Another type of friction, called ***fluid friction***, is studied in fluid mechanics.

In this chapter, we will study the effects of dry friction, which is sometimes called ***Coulomb friction*** since its characteristics were studied extensively by the French physicist Charles-Augustin de Coulomb in 1781.

5.1. Theory of Dry Friction

✓ **Equilibrium:**



The effect of the distributed normal and frictional loadings is indicated by their resultants **N** and **F** on the free-body diagram, Fig. d. Notice that **N** acts a distance **x** to the right of the line of action of **W**, Fig. d. This location, which coincides with the centroid or geometric center of the normal force distribution in Fig. b, is necessary in order to balance the "tipping effect" caused by **P**. For example, if **P** is applied at a height h from the surface, Fig. d, then moment equilibrium about point ***O*** is satisfied if $\mathbf{Wx = Ph}$ **or** $\mathbf{x = Ph/W}$.

✓ **Impending Motion:** In cases where the surfaces of contact are rather "slippery," the frictional force F may not be great enough to balance P, and consequently the block will tend to slip.

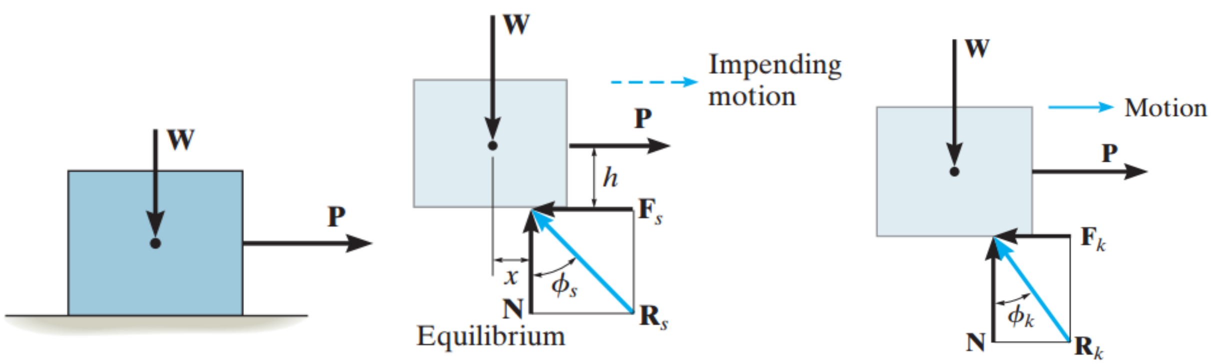
$$F_s = \mu_s N$$

Where:

F$_s$, called the limiting static frictional force, μ_s (mu "sub" s), is called the coefficient of static friction, and **N** the normal force.

The angle ϕ_s (phi "sub" *s*) that **R***s* makes with **N** is called the ***angle of static friction***.
From the figure below:

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$



(a) (b) (c)

✓ **Motion:** If the magnitude of **P** acting on the block is increased so that it becomes slightly greater than **F**$_s$, the frictional force at the contacting surface will drop to a smaller value **F**$_k$, called the ***kinetic frictional force***.

$$F_k = \mu_k N$$

Here the constant of proportionality, μ_k, is called the ***coefficient of kinetic friction***.

In this case, the resultant force at the surface of contact, $\mathbf{R_k}$, has a line of action defined by ϕ_k. This angle is referred to as the ***angle of kinetic friction***, where:

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$$

Characteristics of Dry Friction:

- The frictional force acts *tangent* to the contacting surfaces in a direction *opposed* to the *motion* or tendency for motion of one surface relative to another.

- The maximum static frictional force F_s that can be developed is independent of the area of contact, provided the normal pressure is not very low nor great enough to severely deform or crush the contacting surfaces of the bodies.

- The maximum static frictional force is generally greater than the kinetic frictional force for any two surfaces of contact. However, if one of the bodies is moving with a *very low velocity* over the surface of another, F_k becomes approximately equal to F_s, i.e., $\mu_s \approx \mu_k$.

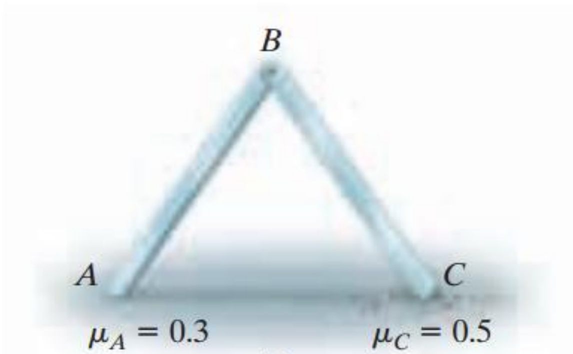
- When *slipping* at the surface of contact is *about to occur*, the maximum static frictional force is proportional to the normal force, such that $F_s = \mu_s N$.

- When *slipping* at the surface of contact is *occurring*, the kinetic frictional force is proportional to the normal force, such that $F_k = \mu_k N$.

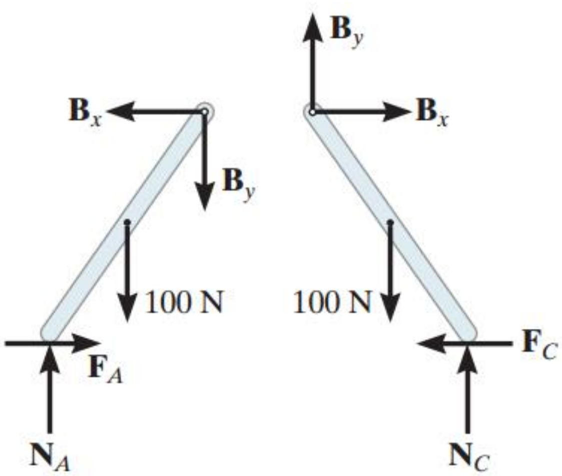
5.2.Types of Friction Problems

In general, there are three types of static problems involving dry friction. They can easily be classified once free-body diagrams are drawn and the total number of unknowns are identified and compared with the total number of available equilibrium equations.

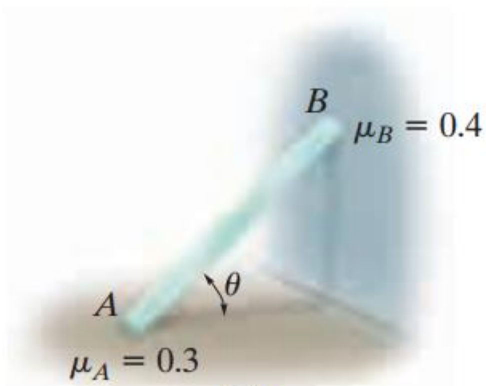
<u>No Apparent Impending Motion</u>. Problems in this category are strictly equilibrium problems, which require the number of unknowns to be ***equal*** to the number of available equilibrium equations. Once the frictional forces are determined from the solution, however, their numerical values must be checked to be sure they satisfy the inequality $F \leq \mu_s N$; otherwise, slipping will occur and the body will not remain in equilibrium.



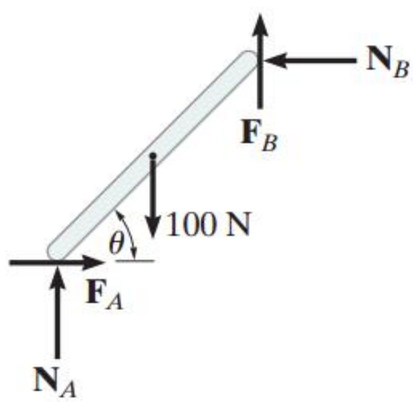
(a)



Impending Motion at All Points of Contact. In this case the total number of unknowns will ***equal*** the total number of available equilibrium equations ***plus*** the total number of available frictional equations, $F = \mu N$. When motion is impending at the points of contact, then $F_s = \mu_s N$; whereas if the body is slipping, then $F_k = \mu_k N$.

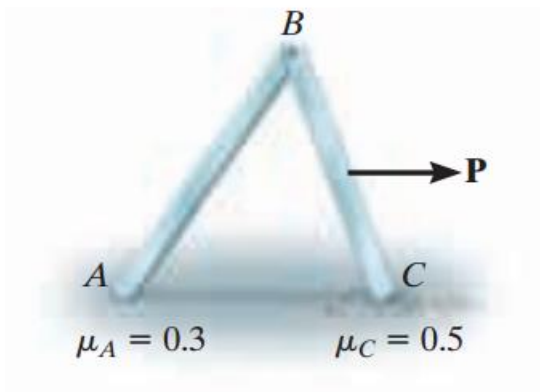


(a)

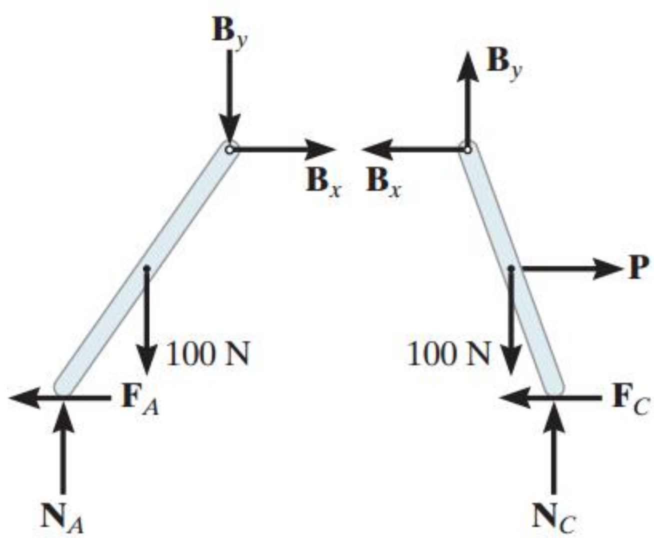


(b)

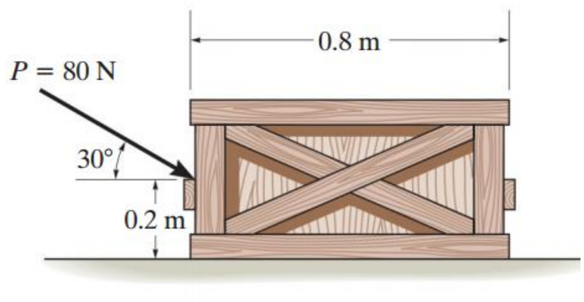
Impending Motion at Some Points of Contact. Here the number of unknowns will be ***less*** than the number of available equilibrium equations plus the number of available frictional equations or conditional equations for tipping. As a result, several possibilities for motion or impending motion will exist and the problem will involve a determination of the kind of motion which actually occurs.

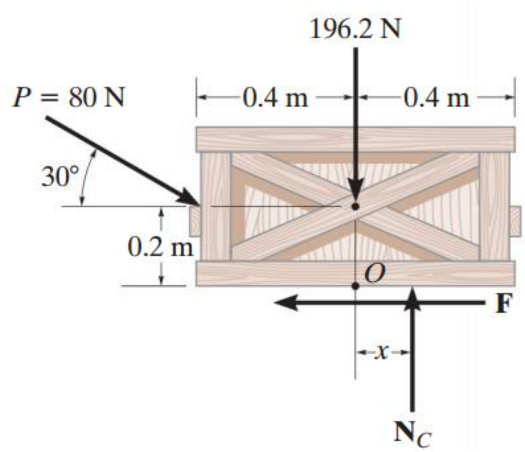


(a)



Example: The uniform crate shown in Fig. a has a mass of 20 kg. If a force P = 80 N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.





Solution:

The resultant normal force N_C must act a distance x from the crate's center line in order to counteract the tipping effect caused by P. There are three unknowns, F, N_C, and x, which can be determined strictly from the three equations of equilibrium.

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 80 \cos 30^\circ \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \quad 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

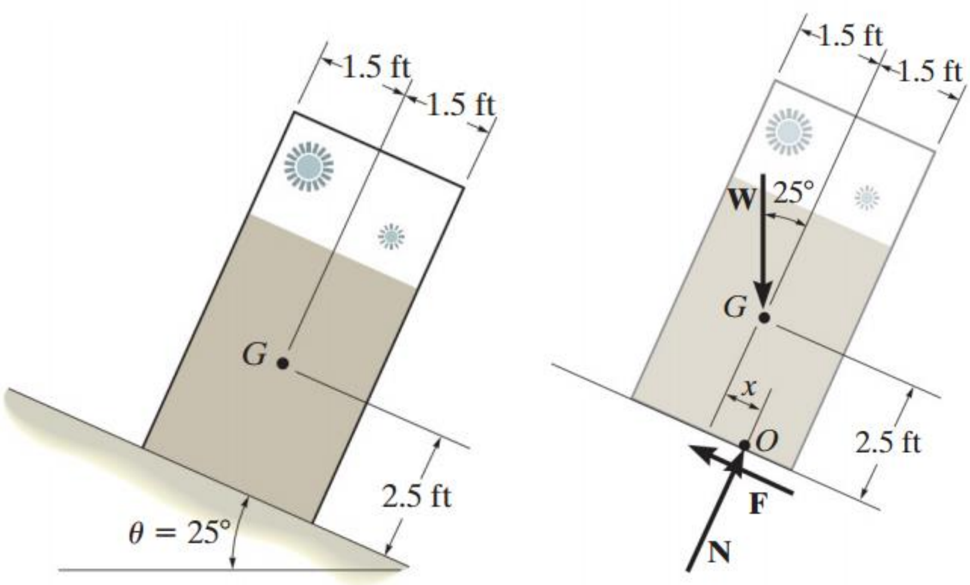
$$F = 69.3 \text{ N}$$
$$N_C = 236.2 \text{ N}$$
$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236.2 \text{ N}) = 70.9$ N. Since $F = 69.3 \text{ N} < 70.9$ N, the crate will *not slip*, although it is very close to doing so.

Example: It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25^{o}$ the vending machines will begin to slide off the bed, Fig. a. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.

(a)

Solution: An idealized model of a vending machine resting on the truckbed is shown in Fig. b. The dimensions have been measured and the center of gravity has been located. We will assume that the vending machine weighs W.



(b) (c)

Free-Body Diagram. As shown in Fig. c, the dimension x is used to locate the position of the resultant normal force N. There are four unknowns, N, F, μ_s, and x.

Equations of Equilibrium.

$$+\searrow \Sigma F_x = 0; \qquad W \sin 25^\circ - F = 0 \qquad (1)$$

$$+\nearrow \Sigma F_y = 0; \qquad N - W \cos 25^\circ = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_O = 0; \quad -W \sin 25^\circ(2.5 \text{ ft}) + W \cos 25^\circ(x) = 0 \qquad (3)$$

Since slipping impends at $\theta = 25^\circ$, using Eqs. 1 and 2, we have

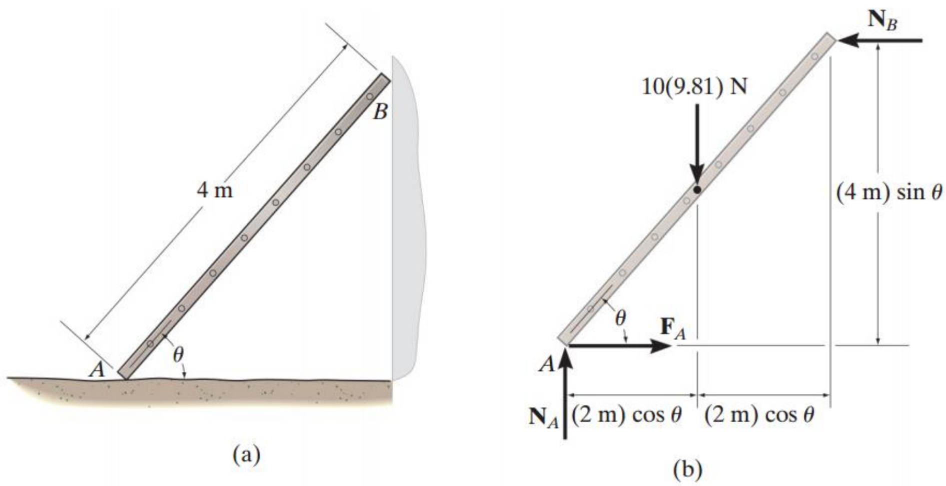
$$F_s = \mu_s N; \qquad W \sin 25^\circ = \mu_s(W \cos 25^\circ)$$

$$\mu_s = \tan 25^\circ = 0.466 \qquad \textit{Ans.}$$

The angle of $\theta = 25^\circ$ is referred to as the ***angle of repose***, and by comparison, it is equal to the angle of static friction, $\theta = \phi_s$. Notice from the calculation that θ is independent of the weight of the vending machine, and so knowing θ provides a convenient method for determining the coefficient of static friction.

NOTE: From Eq. 3, we find x = 1.17 ft. Since 1.17 ft < 1.5 ft, indeed the vending machine will slip before it can tip

Example: The uniform 10-kg ladder in Fig. a rests against the smooth wall at B, and the end A rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.



Solution: As shown on the free-body diagram, Fig. b, the frictional force $\mathbf{F_A}$ must act to the right since impending motion at A is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

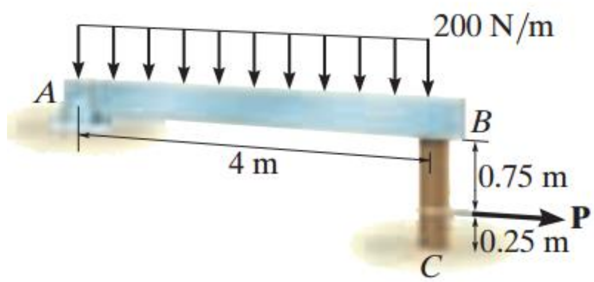
Finally, the angle θ can be determined by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

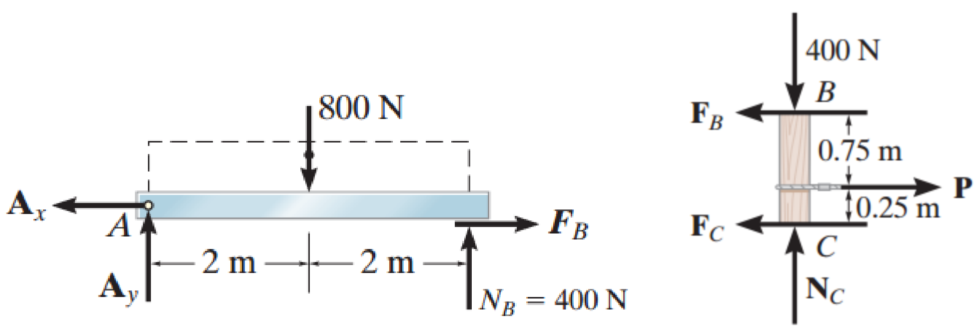
$$\theta = 59.04° = 59.0° \qquad \textit{Ans.}$$

Example: Beam AB is subjected to a uniform load of 200 N /m and is supported at B by post BC, Fig. a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force **P** needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.



(a)

Solution:



(b) (c)

Free-Body Diagrams. The free-body diagram of the beam is shown in Fig. *b*.

Applying $\sum M_A = 0$, we obtain $N_B = 400$ N. This result is shown on the free-body diagram of the post, Fig. *c*. Referring to this member, the *four* unknowns F_B, P, F_C, and N_C are determined from the *three* equations of equilibrium and *one* frictional equation applied either at B or C.

Equations of Equilibrium and Friction.

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_B - F_C = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \qquad N_C - 400 \text{ N} = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_C = 0; \qquad -P(0.25 \text{ m}) + F_B(1 \text{ m}) = 0 \qquad (3)$

(Post Slips at *B* and Rotates about C.) This requires $F_C \leq \mu_C N_C$ and

$F_B = \mu_B N_B; \qquad F_B = 0.2(400 \text{ N}) = 80 \text{ N}$

Using this result and solving Eqs. 1 through 3, we obtain

$$P = 320 \text{ N}$$
$$F_C = 240 \text{ N}$$
$$N_C = 400 \text{ N}$$

Since $F_C = 240 \text{ N} > \mu_C N_C = 0.5(400 \text{ N}) = 200 \text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at *C* and Rotates about *B*.) Here $F_B \leq \mu_B N_B$ and

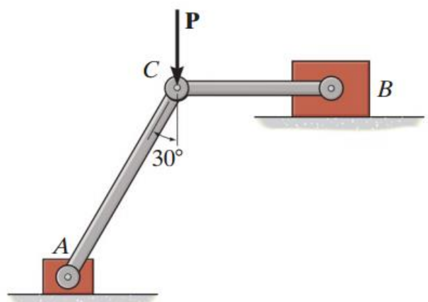
$F_C = \mu_C N_C; \qquad F_C = 0.5 N_C \qquad (4)$

Solving Eqs. 1 through 4 yields

$$P = 267 \text{ N} \qquad \textit{Ans.}$$
$$N_C = 400 \text{ N}$$
$$F_C = 200 \text{ N}$$
$$F_B = 66.7 \text{ N}$$

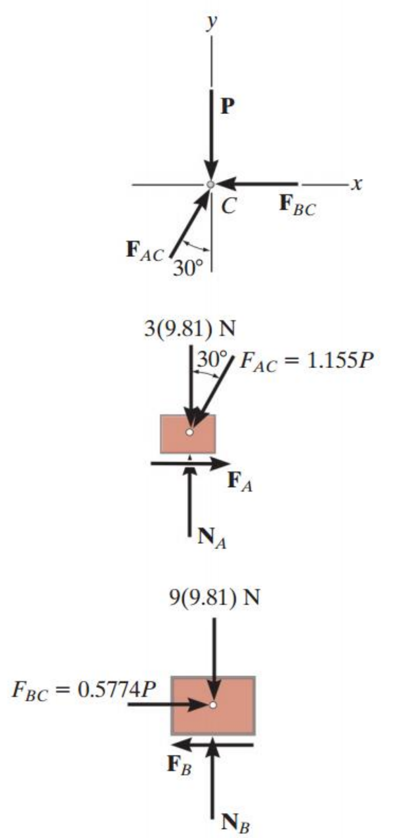
Obviously, this case occurs first since it requires a *smaller* value for P.

Example: Blocks A and B have a mass of 3 kg and 9 kg, respectively, and are connected to the weightless links shown in Fig. a. Determine the largest vertical force P that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is $\mu_s = 0.3$.



(a)

Solution:



(b)

Free-Body Diagram. The links are two-force members and so the free-body diagrams of pin C and blocks A and B are shown in Fig. *b*. Since the horizontal component of $\mathbf{F}_{AC}$ tends to move block A to the left, F_A must act to the right. Similarly, $\mathbf{F}_B$ must act to the left to oppose the tendency of motion of block B to the right, caused by $\mathbf{F}_{BC}$. There are ***seven*** unknowns and ***six*** available force equilibrium equations, two for the pin and two for each block, so that *only one* frictional equation is needed.

Equations of Equilibrium and Friction. The force in links *AC* and *BC* can be related to *P* by considering the equilibrium of pin *C*.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC} \cos 30^\circ - P = 0; \qquad F_{AC} = 1.155P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1.155P \sin 30^\circ - F_{BC} = 0; \qquad F_{BC} = 0.5774P$$

Using the result for F_{AC}, for block *A*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - 1.155P \sin 30^\circ = 0; \qquad F_A = 0.5774P \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 1.155P \cos 30^\circ - 3(9.81 \text{ N}) = 0;$$

$$N_A = P + 29.43 \text{ N} \qquad (2)$$

Using the result for F_{BC}, for block *B*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad (0.5774P) - F_B = 0; \qquad F_B = 0.5774P \qquad (3)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 9(9.81) \text{ N} = 0; \qquad N_B = 88.29 \text{ N}$$

Movement of the system may be caused by the initial slipping of *either* block *A* or block *B*. If we assume that block *A* slips first, then

$$F_A = \mu_s N_A = 0.3N_A \qquad (4)$$

Substituting Eqs. 1 and 2 into Eq. 4,

$$0.5774P = 0.3(P + 29.43)$$

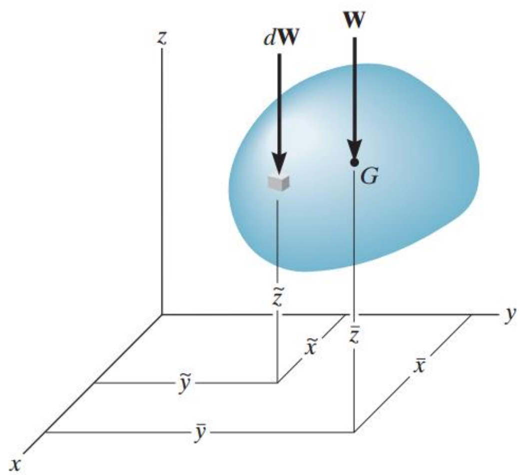
$$P = 31.8 \text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. 3, we obtain $F_B = 18.4$ N. Since the maximum static frictional force at *B* is $(F_B)_{max} = \mu_s N_B = 0.3(88.29 \text{ N}) = 26.5 \text{ N} > F_B$, block *B* will not slip. Thus, the above assumption is correct. Notice that if the inequality were not satisfied, we would have to assume slipping of block *B* and then solve for *P*.

6. Chapter six: Center of Gravity and Centroid

6.1. Center of Gravity:

A body is composed of an infinite number of particles of differential size, and so if the body is located within a gravitational field, then each of these particles will have a weight dW. These weights will form a parallel force system, and the resultant of this system is the total weight of the body, which passes through a single point called the center of gravity, G



The coordinates of the location of the center of gravity can be determined by the following formulas:

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Where:

$\bar{x}, \bar{y}, \bar{z}$ are the coordinates of the center of gravity G.
$\tilde{x}, \tilde{y}, \tilde{z}$ are the coordinates of an arbitrary particle in the body.

Center of Mass of a Body

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Centroid of an Area

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

Centroid of a Volume:

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

Centroid of a Line

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL}$$

EXAMPLE:

Locate the centroid of the rod bent into the shape of a parabolic arc as shown in Fig. 9–8.

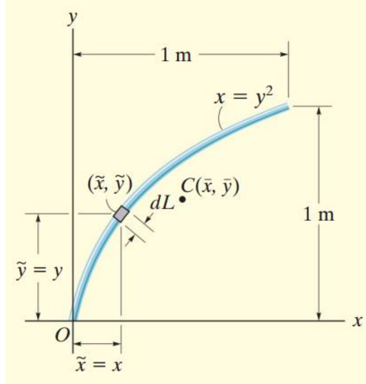


Fig. 9–8

SOLUTION

Differential Element. The differential element is shown in Fig. 9–8. It is located on the curve at the *arbitrary point* (x, y).

Area and Moment Arms. The differential element of length dL can be expressed in terms of the differentials dx and dy using the Pythagorean theorem.

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\,dy$$

Since $x = y^2$, then $dx/dy = 2y$. Therefore, expressing dL in terms of y and dy, we have

$$dL = \sqrt{(2y)^2 + 1}\,dy$$

As shown in Fig. 9–8, the centroid of the element is located at $\tilde{x} = x$, $\tilde{y} = y$.

Integrations. Applying Eq. 9–5 and using the integration formula to evaluate the integrals, we get

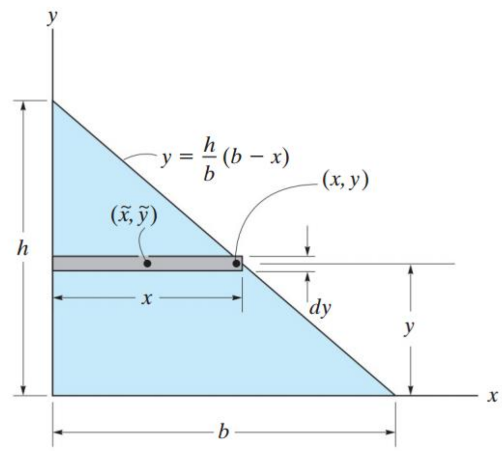
$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{1\text{ m}} x\sqrt{4y^2+1}\,dy}{\int_0^{1\text{ m}} \sqrt{4y^2+1}\,dy} = \frac{\int_0^{1\text{ m}} y^2\sqrt{4y^2+1}\,dy}{\int_0^{1\text{ m}} \sqrt{4y^2+1}\,dy}$$

$$= \frac{0.6063}{1.479} = 0.410\text{ m} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{1\text{ m}} y\sqrt{4y^2+1}\,dy}{\int_0^{1\text{ m}} \sqrt{4y^2+1}\,dy} = \frac{0.8484}{1.479} = 0.574\text{ m} \qquad \textit{Ans.}$$

NOTE: These results for C seem reasonable when they are plotted on Fig. 9–8.

EXAMPLE: Determine the distance y measured from the x axis to the centroid of the area of the triangle shown in Fig. 9–10 below:



SOLUTION

Differential Element. Consider a rectangular element having a thickness dy, and located in an arbitrary position so that it intersects the boundary at (x, y), Fig. 9–10.

Area and Moment Arms. The area of the element is $dA = x\,dy = \frac{b}{h}(h - y)\,dy$, and its centroid is located a distance $\tilde{y} = y$ from the x axis.

Integration. Applying the second of Eqs. 9–4 and integrating with respect to y yields

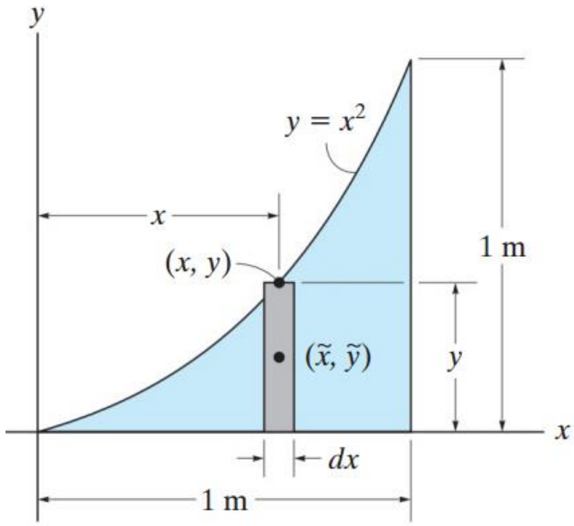
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left[\frac{b}{h}(h - y)\,dy\right]}{\int_0^h \frac{b}{h}(h - y)\,dy} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh}$$

$$= \frac{h}{3} \qquad \textit{Ans.}$$

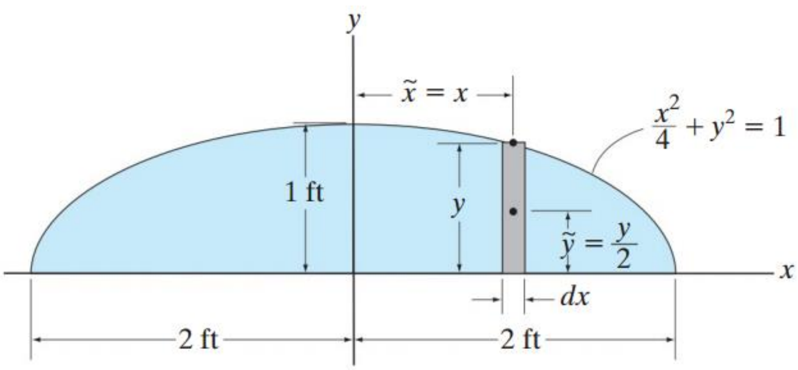
NOTE: This result is valid for any shape of triangle. It states that the centroid is located at one-third the height, measured from the base of the triangle.

HW:

1. Locate the centroid of the area shown in Fig. below:



2. Locate the centroid of the semi-elliptical area shown in Fig. below:



EXAMPLE: Locate the y centroid for the paraboloid of revolution, shown in Fig. 9–14.

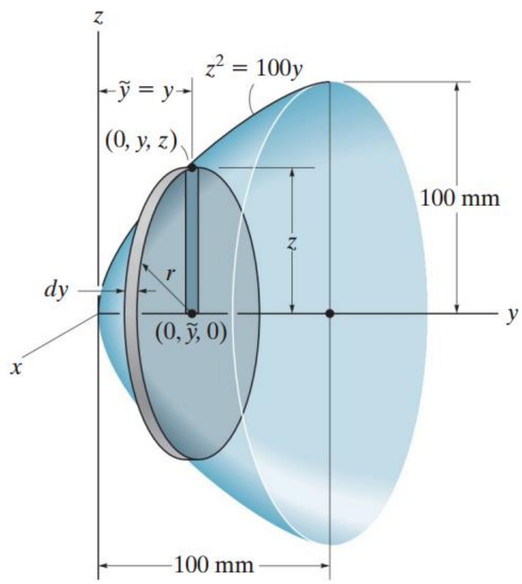


Fig. 9–14

SOLUTION

Differential Element. An element having the shape of a *thin disk* is chosen. This element has a thickness dy, it intersects the generating curve at the *arbitrary point* $(0, y, z)$, and so its radius is $r = z$.

Volume and Moment Arm. The volume of the element is $dV = (\pi z^2)\,dy$, and its centroid is located at $\tilde{y} = y$.

Integration. Applying the second of Eqs. 9–3 and integrating with respect to y yields.

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^{100\text{ mm}} y(\pi z^2)\,dy}{\int_0^{100\text{ mm}} (\pi z^2)\,dy} = \frac{100\pi \int_0^{100\text{ mm}} y^2\,dy}{100\pi \int_0^{100\text{ mm}} y\,dy} = 66.7\text{ mm} \qquad \textit{Ans.}$$

6.2. Composite Bodies

A composite body consists of a series of connected "simpler" shaped bodies, which may be rectangular, triangular, semicircular, etc. Such a body can often be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity for the entire body. The formula for calculating the coordinates of the center of gravity for composite bodies:

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \qquad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W}$$

Where:

$\bar{x}, \bar{y}, \bar{z}$ represent the coordinates of the center of gravity G of the composite body.

$\tilde{x}, \tilde{y}, \tilde{z}$ represent the coordinates of the center of gravity of each composite part of the body.

ΣW is the sum of the weights of all the composite parts of the body, or simply the total weight of the body.

EXAMPLE:

Locate the centroid of the wire shown in Fig. 9–16*a*.

SOLUTION

Composite Parts. The wire is divided into three segments as shown in Fig. 9–16*b*.

Moment Arms. The location of the centroid for each segment is determined and indicated in the figure. In particular, the centroid of segment (1) is determined either by integration or by using the table on the inside back cover.

Summations. For convenience, the calculations can be tabulated as follows:

Segment	L (mm)	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}L$ (mm^2)	$\tilde{y}L$ (mm^2)	$\tilde{z}L$ (mm^2)
1	$\pi(60) = 188.5$	60	−38.2	0	11 310	−7200	0
2	40	0	20	0	0	800	0
3	20	0	40	−10	0	800	−200
	$\Sigma L = 248.5$				$\Sigma \tilde{x}L = 11\,310$	$\Sigma \tilde{y}L = -5600$	$\Sigma \tilde{z}L = -200$

$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{11\,310}{248.5} = 45.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm} \qquad \textit{Ans.}$$

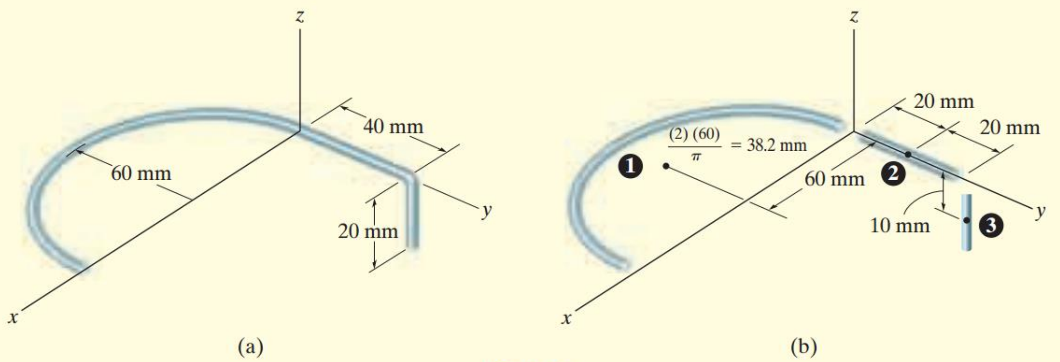
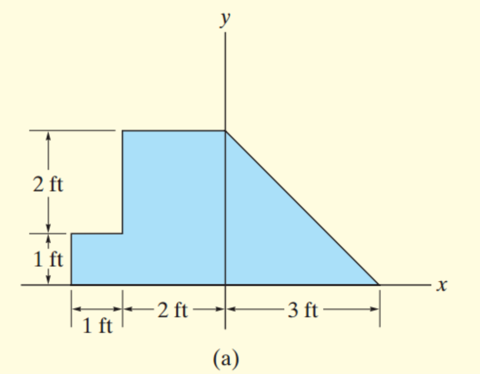


Fig. 9–16

EXAMPLE: Locate the centroid of the plate area shown in Fig. 9–17a.



(a)

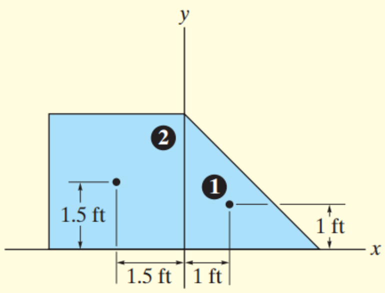
Fig. 9–17

SOLUTION

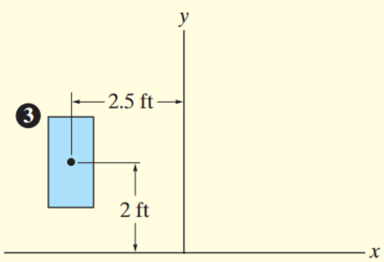
Composite Parts. The plate is divided into three segments as shown in Fig. 9–17*b*. Here the area of the small rectangle ③ is considered "negative" since it must be subtracted from the larger one ②.

Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the $\tilde{x}$ coordinates of ② and ③ are *negative*.

Summations. Taking the data from Fig. 9–17*b*, the calculations are tabulated as follows:



Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$



(b)

Thus,

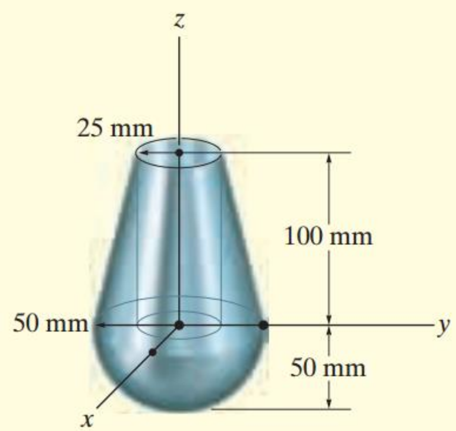
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If these results are plotted in Fig. 9–17*a*, the location of point *C* seems reasonable.

HW:

Locate the center of mass of the assembly shown in Fig. 9–18*a*. The conical frustum has a density of $\rho_c = 8\ \text{Mg/m}^3$, and the hemisphere has a density of $\rho_h = 4\ \text{Mg/m}^3$. There is a 25-mm-radius cylindrical hole in the center of the frustum.



(a)

Fig. 9–18

7. Chapter Seven: Moments of Inertia

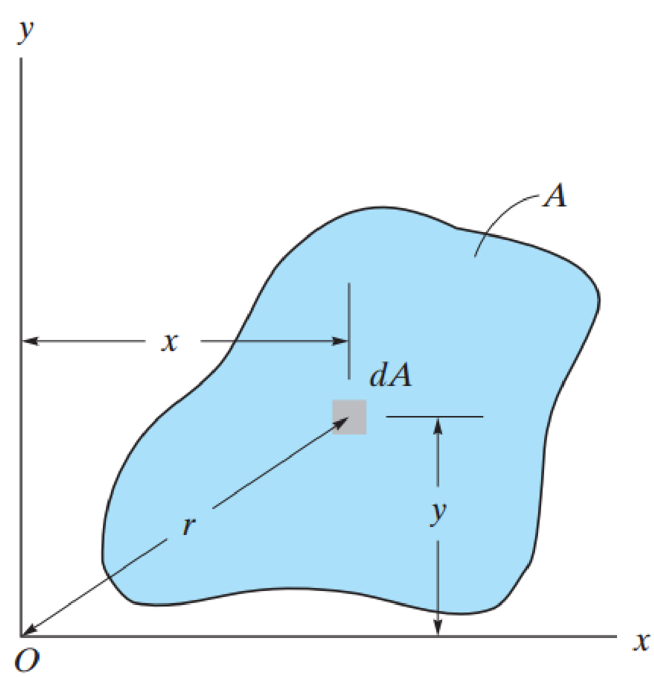
7.1. Moments of Inertia of an area

The area moment of inertia represents the second moment of the area about an axis. It is frequently used in formulas related to the strength and stability of structural members or mechanical elements.

For the entire area A the moments of inertia are determined by integration:

$$I_x = \int_A y^2\, dA$$
$$I_y = \int_A x^2\, dA$$

From the above formulations it is seen that I_x and I_y, will always be positive since they involve the product of distance squared and area. Furthermore, the units for moment of inertia involve **length raised to the fourth power**, e.g., **m^4, mm^4, or ft^4, in.^{4}**.

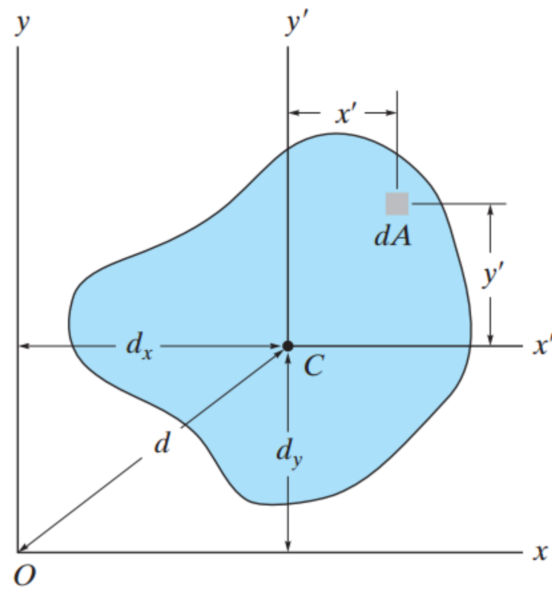


7.2. Parallel-Axis Theorem for an Area

The parallel-axis theorem can be used to find the moment of inertia of an area about any axis that is parallel to an axis passing through the centroid and about which the moment of inertia is known.

$$I_x = \bar{I}_{x'} + A d_y^2$$

$$I_y = \bar{I}_{y'} + A d_x^2$$



$\bar{I}_{X}$' and $\bar{I}_{y}$': The first integral represents the moment of inertia of the area about the centroidal axis.

A: The total area.

d_y and d_x: The distance between the parallel x' and x and y' and y respectively.

7.3. Radius of Gyration of an Area

The radius of gyration of an area about an axis has units of length and is a quantity that is often used for the design of columns in structural mechanics. Provided the areas and moments of inertia are known, the radii of gyration are determined from the formulas

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

EXAMPLE: Determine the moment of inertia for the rectangular area shown in Fig. 10–5 with respect to (a) the centroidal x' axis, and (b) the axis x_b passing through the base of the rectangle.

SOLUTION:

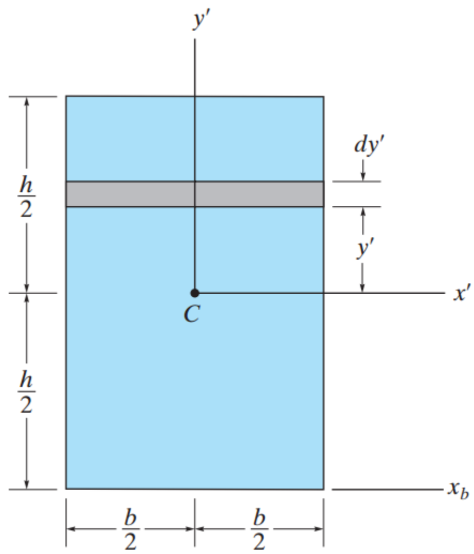


Fig. 10–5

Part (a). The differential element shown in Fig. 10–5 is chosen for integration. Because of its location and orientation, the *entire element* is at a distance y' from the x' axis. Here it is necessary to integrate from $y' = -h/2$ to $y' = h/2$. Since $dA = b\,dy'$, then

$$\bar{I}_{x'} = \int_A y'^2\,dA = \int_{-h/2}^{h/2} y'^2(b\,dy') = b\int_{-h/2}^{h/2} y'^2\,dy'$$

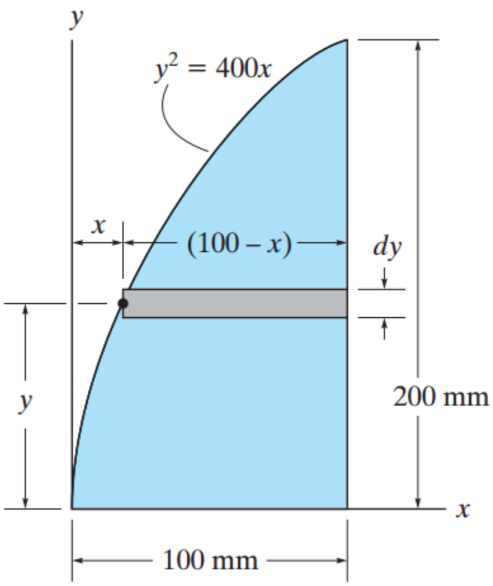
$$\bar{I}_{x'} = \frac{1}{12}bh^3 \qquad \textit{Ans.}$$

Part (b). The moment of inertia about an axis passing through the base of the rectangle can be obtained by using the above result of part (a) and applying the parallel-axis theorem, Eq. 10–3.

$$I_{x_b} = \bar{I}_{x'} + Ad_y^2$$

$$= \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3 \qquad \textit{Ans.}$$

EXAMPLE: Determine the moment of inertia for the shaded area shown in Fig. 10–6a about the x-axis.



(a)

SOLUTION:

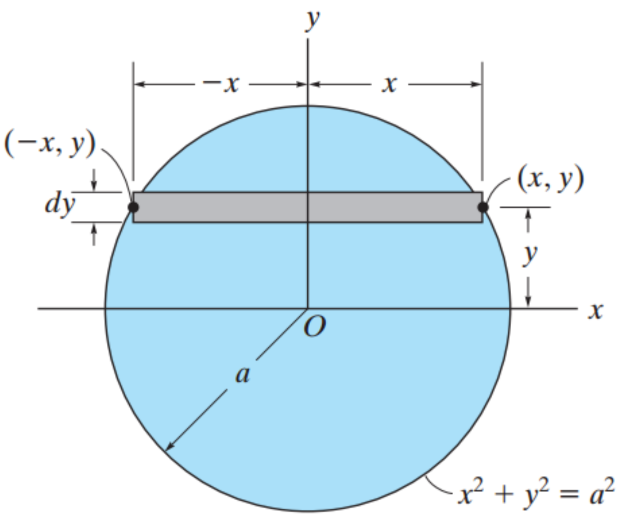
A differential element of area that is *parallel* to the x axis, as shown in Fig. 10–6*a*, is chosen for integration. Since this element has a thickness dy and intersects the curve at the *arbitrary point* (x, y), its area is $dA = (100 - x)\,dy$. Furthermore, the element lies at the same distance y from the x axis. Hence, integrating with respect to y, from $y = 0$ to $y = 200$ mm, yields

$$I_x = \int_A y^2\,dA = \int_0^{200\text{ mm}} y^2(100 - x)\,dy$$

$$= \int_0^{200\text{ mm}} y^2\left(100 - \frac{y^2}{400}\right)dy = \int_0^{200\text{ mm}} \left(100y^2 - \frac{y^4}{400}\right)dy$$

$$= 107(10^6)\text{ mm}^4 \qquad \textit{Ans.}$$

EXAMPLE: Determine the moment of inertia with respect to the x axis for the circular area shown in Fig. 10–7a.



(a)

SOLUTION I (CASE 1)

Using the differential element shown in Fig. 10–7a, since $dA = 2x\,dy$, we have

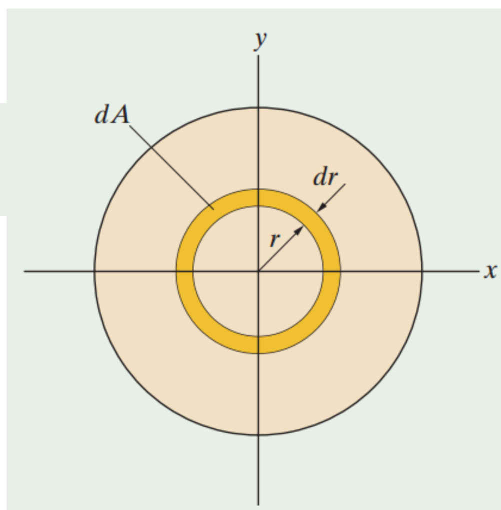
$$I_x = \int_A y^2\,dA = \int_A y^2(2x)\,dy$$

$$= \int_{-a}^{a} y^2\left(2\sqrt{a^2 - y^2}\right)dy = \frac{\pi a^4}{4} \qquad \textit{Ans.}$$

من اجل معرفة كيفية الحصول على النتيجة أعلاه يجب فهم المثال ادناه

$$J_O = \int_A r^2\,dA = \int_0^R 2\pi r^3\,dr = 2\pi\left[\frac{r^4}{4}\right]_0^R = \frac{1}{2}\pi R^4,$$

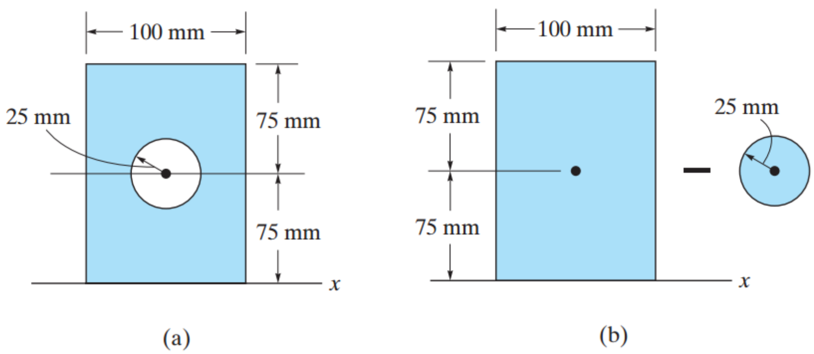
$$I_x = I_y = \frac{1}{2}J_O = \frac{1}{4}\pi R^4,$$



7.4.Moments of Inertia for Composite Areas

A composite area consists of a series of connected “simpler” parts or shapes, such as rectangles, triangles, and circles. Provided the moment of inertia of each of these parts is known or can be determined about a common axis, then the moment of inertia for the composite area about this axis equals the algebraic sum of the moments of inertia of all its parts.

EXAMPLE: Determine the moment of inertia of the area shown in Fig. 10–8a about the x axis.



Composite Parts. The area can be obtained by *subtracting* the circle from the rectangle shown in Fig. 10–8*b*. The centroid of each area is located in the figure.

Parallel-Axis Theorem. The moments of inertia about the *x* axis are determined using the parallel-axis theorem and the geometric properties formulae for circular and rectangular areas $I_x = \frac{1}{4}\pi r^4$; $I_x = \frac{1}{12}bh^3$, found on the inside back cover.

Circle

$$I_x = \bar{I}_{x'} + A d_y^2$$
$$= \frac{1}{4}\pi(25)^4 + \pi(25)^2(75)^2 = 11.4(10^6)\text{ mm}^4$$

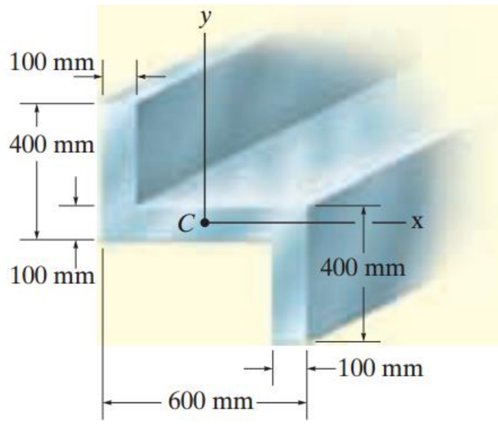
Rectangle

$$I_x = \bar{I}_{x'} + A d_y^2$$
$$= \frac{1}{12}(100)(150)^3 + (100)(150)(75)^2 = 112.5(10^6)\text{ mm}^4$$

Summation. The moment of inertia for the area is therefore

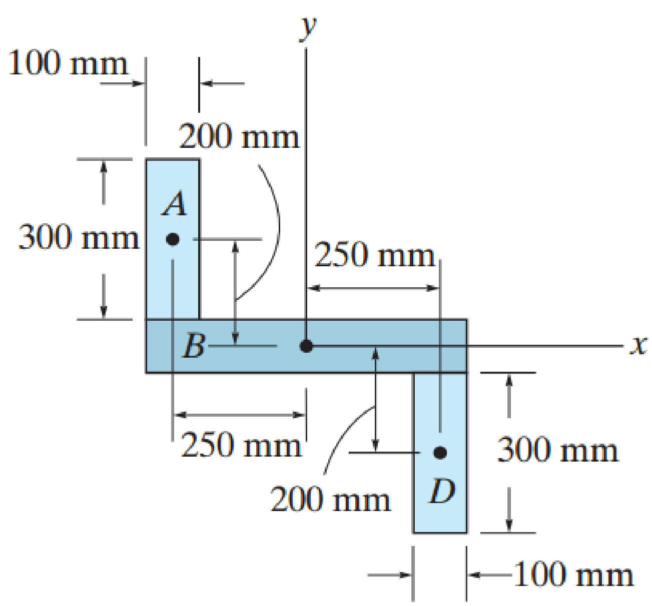
$$I_x = -11.4(10^6) + 112.5(10^6)$$
$$= 101(10^6)\text{ mm}^4 \qquad \textit{Ans.}$$

EXAMPLE: Determine the moments of inertia for the cross-sectional area of the member shown in Fig. 10–9a about the x and y centroidal axes.



(a)

SOLUTION:



(b)

Fig. 10–9

Composite Parts. The cross section can be subdivided into the three rectangular areas *A*, *B*, and *D* shown in Fig. 10–9*b*. For the calculation, the centroid of each of these rectangles is located in the figure.

Parallel-Axis Theorem. From the table on the inside back cover, or Example 10.1, the moment of inertia of a rectangle about its centroidal axis is $\bar{I} = \frac{1}{12}bh^3$. Hence, using the parallel-axis theorem for rectangles *A* and *D*, the calculations are as follows:

Rectangles A and D

$$I_x = \bar{I}_{x'} + Ad_y^2 = \frac{1}{12}(100)(300)^3 + (100)(300)(200)^2$$
$$= 1.425(10^9)\text{ mm}^4$$
$$I_y = \bar{I}_{y'} + Ad_x^2 = \frac{1}{12}(300)(100)^3 + (100)(300)(250)^2$$
$$= 1.90(10^9)\text{ mm}^4$$

Rectangle B

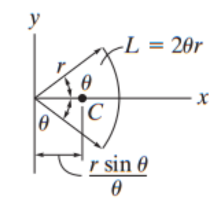
$$I_x = \frac{1}{12}(600)(100)^3 = 0.05(10^9)\text{ mm}^4$$
$$I_y = \frac{1}{12}(100)(600)^3 = 1.80(10^9)\text{ mm}^4$$

Summation. The moments of inertia for the entire cross section are thus

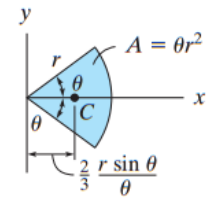
$$I_x = 2[1.425(10^9)] + 0.05(10^9)$$
$$= 2.90(10^9)\text{ mm}^4 \qquad \textit{Ans.}$$
$$I_y = 2[1.90(10^9)] + 1.80(10^9)$$
$$= 5.60(10^9)\text{ mm}^4 \qquad \textit{Ans.}$$

Geometric Properties of Line and Area Elements

Centroid Location | Centroid Location | Area Moment of Inertia



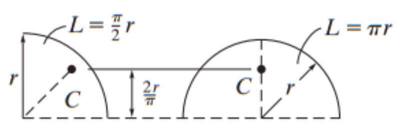
Circular arc segment



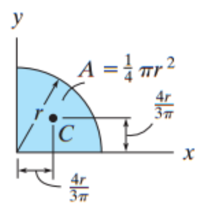
Circular sector area

$$I_x = \frac{1}{4} r^4 \left(\theta - \frac{1}{2}\sin 2\theta\right)$$

$$I_y = \frac{1}{4} r^4 \left(\theta + \frac{1}{2}\sin 2\theta\right)$$



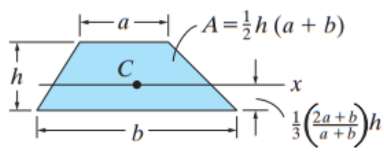
Quarter and semicircle arcs



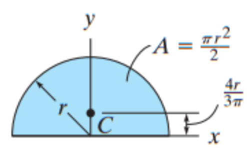
Quarter circle area

$$I_x = \frac{1}{16}\pi r^4$$

$$I_y = \frac{1}{16}\pi r^4$$



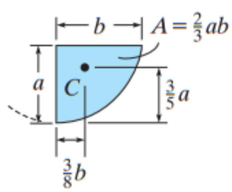
Trapezoidal area



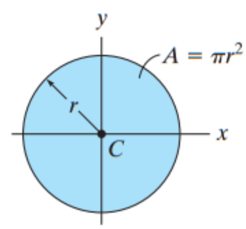
Semicircular area

$$I_x = \frac{1}{8}\pi r^4$$

$$I_y = \frac{1}{8}\pi r^4$$



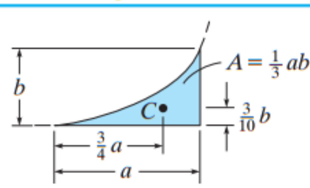
Semiparabolic area



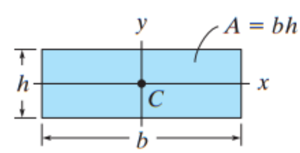
Circular area

$$I_x = \frac{1}{4}\pi r^4$$

$$I_y = \frac{1}{4}\pi r^4$$



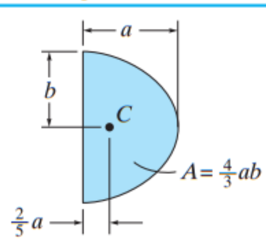
Exparabolic area



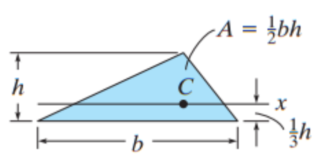
Rectangular area

$$I_x = \frac{1}{12}bh^3$$

$$I_y = \frac{1}{12}hb^3$$



Parabolic area



Triangular area

$$I_x = \frac{1}{36}bh^3$$

8. Chapter Eight: Kinematics of a Particle

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces. Engineering mechanics is divided into two areas of study, namely, statics and dynamics. ***Statics*** is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider ***dynamics***, which deals with the accelerated motion of a body.

Problem Solving. Dynamics is considered to be more involved than statics since both the forces applied to a body and its motion must be taken into account. Also, many applications require using calculus, rather than just algebra and trigonometry. In any case, the most effective way of learning the principles of dynamics is ***to solve problems***. To be successful at this, it is necessary to present the work in a logical and orderly manner as suggested by the following sequence of steps:

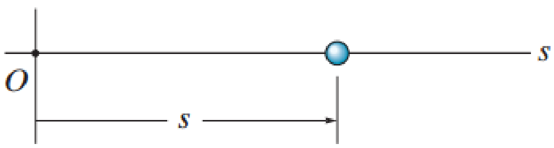
1. Read the problem carefully and try to correlate the actual physical situation with the theory you have studied.
2. Draw any necessary diagrams and tabulate the problem data.
3. Establish a coordinate system and apply the relevant principles, generally in mathematical form.
4. Solve the necessary equations algebraically as far as practical; then, use a consistent set of units and complete the solution numerically. Report the answer with no more significant figures than the accuracy of the given data.
5. Study the answer using technical judgment and common sense to determine whether or not it seems reasonable.
6. Once the solution has been completed, review the problem. Try to think of other ways of obtaining the same solution.

In applying this general procedure, do the work as neatly as possible. Being neat generally stimulates clear and orderly thinking, and vice versa.

8.1. Rectilinear Kinematics.

The kinematics of a particle is characterized by specifying, at any given instant, the particle's **position**, **velocity**, and **acceleration**.

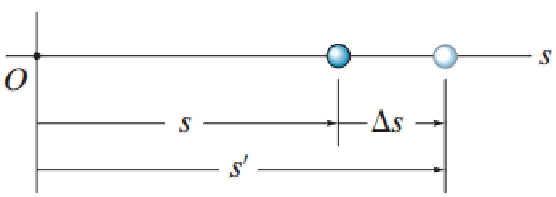
Position (s): The straight-line path of a particle will be defined using a single coordinate axis s,



Position

(a)

Displacement (Δs): The displacement of the particle is defined as the change in its position.



$$\Delta s = s' - s$$

Velocity (*v*): If the particle moves through a displacement Δs during the time interval Δt, the average velocity of the particle during this time interval is:

$$v_{avg} = \frac{\Delta s}{\Delta t} \qquad v = \frac{ds}{dt}$$

Acceleration (*a*): Provided the velocity of the particle is known at two points, the average acceleration of the particle during the time interval Δt is defined as:

$$a_{avg} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt}$$

Constant Acceleration

When the accelerating object will change its velocity by the same amount each second. This is referred to as a *constant acceleration* since the velocity is changing by a constant amount each second.

Velocity as a Function of Time:

$$v = v_0 + a_c t$$
Constant Acceleration

Position as a Function of Time

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$
Constant Acceleration

Velocity as a Function of Position

$$v^2 = v_0^2 + 2a_c(s - s_0)$$
Constant Acceleration

Where:

$\mathbf{v_0}$ is the velocity at time t = 0

v is the velocity at any later time t

$\mathbf{a_c}$ is the constant acceleration.

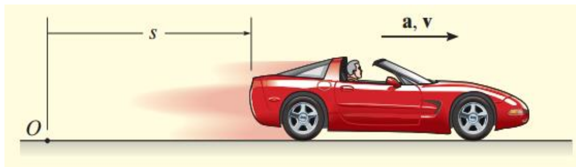
s_0 is the position of the particle at $t = 0$

s is the position of the particle at any time

If air resistance is neglected and the distance of fall is short, then the **downward acceleration** of the body when it is close to the earth is constant and approximately **9.81 m/s^2 or 32.2 ft/s^2**

EXAMPLE: The car in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by **v = (3t² + 2t) ft/s,** where t is in seconds. Determine its position and acceleration

When t = 3 s. When t = 0, s = 0



SOLUTION:

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

EXAMPLE: During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$ m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

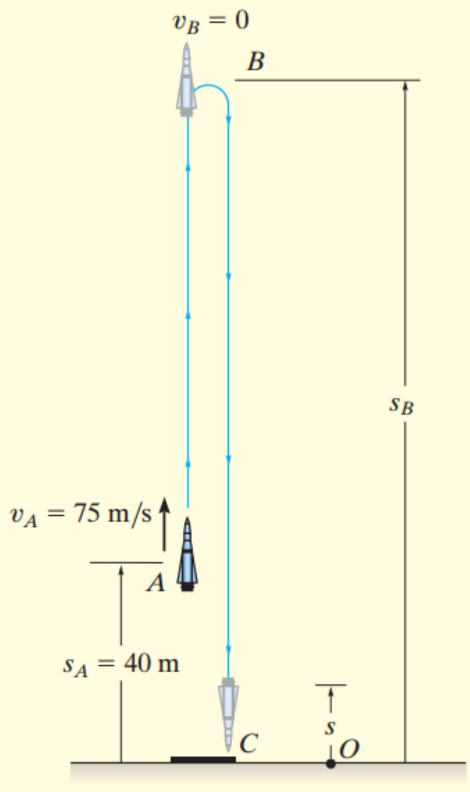


Fig. 12–4

The negative root was chosen since the rocket is moving downward.
Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s^2, and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s^2 downward!

EXAMPLE: A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.

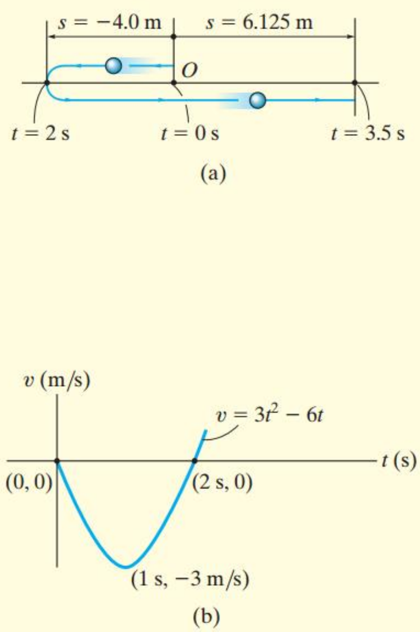


Fig. 12–6

SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O, Fig. 12–6*a*.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0$, $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$= (3t^2 - 6t)\,dt$$

$$\int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$$

$$s = (t^3 - 3t^2)\text{ m} \qquad (1)$$

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. 12–6*b*, then it reveals that for $0 < t < 2$ s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that $v = 0$ at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can be determined from Eq. 1. This yields

$$s|_{t=0} = 0 \quad s|_{t=2\text{ s}} = -4.0\text{ m} \quad s|_{t=3.5\text{ s}} = 6.125\text{ m}$$

The path is shown in Fig. 12–6*a*. Hence, the distance traveled in 3.5 s is

$$s_T = 4.0 + 4.0 + 6.125 = 14.125\text{ m} = 14.1\text{ m} \qquad \textit{Ans.}$$

Velocity. The *displacement* from $t = 0$ to $t = 3.5$ s is

$$\Delta s = s|_{t=3.5\text{ s}} - s|_{t=0} = 6.125\text{ m} - 0 = 6.125\text{ m}$$

and so the average velocity is

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125\text{ m}}{3.5\text{ s} - 0} = 1.75\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

The average speed is defined in terms of the *distance traveled* s_T. This positive scalar is

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{14.125\text{ m}}{3.5\text{ s} - 0} = 4.04\text{ m/s} \qquad \textit{Ans.}$$

NOTE: In this problem, the acceleration is $a = dv/dt = (6t - 6)\text{ m/s}^2$, which is not constant.

8.2.Conservation of Energy

The total energy **E** of a system (the sum of its **mechanical energy** and its **internal energies**, including **thermal energy**) can change only by amounts of energy that are transferred to or from the system. This experimental fact is known as **the law of conservation of energy**.

If work W is done on the system, then:

$$W = \Delta E = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int.}$$

Where:

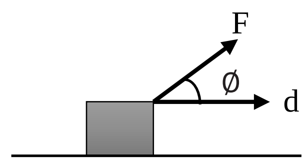
- ✓ **W= work:** is energy transferred to or from an object via a force acting on the object.
- ✓ ΔE **:** The change in the total energy.
- ✓ ΔE_{mech}**:** The change in the mechanical energy.
- ✓ ΔE_{th}**:** The change in the thermal energy.
- ✓ $\Delta E_{int.}$**:** The change in the internal energy.

Work (W): is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

The work done on a particle by a constant force (**F**) during displacement (**d**) is

$$W = F\, d \cos \emptyset$$

In which ∅ is the constant angle between the directions of (F) and (d).



Units for Work (joule): $1\ J = 1\ kg \cdot m^2/s^2 = 1\ N \cdot m = 0.738\ ft \cdot lb.$

Power (P): the rate at which work is done by a force. In a more general sense, power P is the rate at which energy is transferred by a force from one type to another. If an amount of energy ΔE is transferred in an amount of time Δt, the average power due to the force is:

$$P = \frac{\Delta \mathrm{E}}{\Delta \mathrm{t}}$$

Mechanical energy (ΔE_{mech}) Equal to the summation of the change in the kinetic energy (Δ**K**) and the change in the potential energy (Δ**U**) of the system.

$$\Delta \boldsymbol{E}_{\boldsymbol{mech}} = \Delta \boldsymbol{K} + \Delta \boldsymbol{U}$$

Kinetic energy (K) is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. ***When the object is stationary, its kinetic energy is zero.*** The SI unit of kinetic energy (and all types of energy) is the **joule (J).**

For an object of ***mass m*** whose ***speed v***

$$\boldsymbol{K} = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{m} \, \boldsymbol{v}^{\mathbf{2}}$$

The potential energy (U): the potential energy associated with a system consisting of Earth and a nearby particle is ***gravitational potential energy***. If the particle moves from **initial height** y_i to **final height** y_f, the change in the gravitational potential energy of the particle with **mass m** and **gravitational acceleration (g)** is:

$$\Delta \boldsymbol{U} = \boldsymbol{mg}(\boldsymbol{y}_{\boldsymbol{f}} - \boldsymbol{y}_{\boldsymbol{i}})$$

For a spring that exerts a spring force $F = -kx$ when it's free end has displacement x, the ***elastic potential energy*** is:

$$\boldsymbol{U}(\boldsymbol{x}) = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{k} \, \boldsymbol{x}^{\mathbf{2}}$$

Then , $\Delta E_{mech} = \Delta K + \Delta U$

$$\Delta E_{mech} = (K_f - K_i) + \left(mg(y_f - y_i)\right)$$

$$\Delta E_{mech} = \left(\frac{1}{2} m\, v^2 - \frac{1}{2} m\, {v_0}^2\right) + (m\, g\, \Delta y)$$

<u>The thermal energy</u> (ΔE_{th})**:**

$$\Delta E_{th} = F_k\, d = \mu_k\, F_N\, d = \mu_k\, mg\, d$$

Where:

F_k **:** Frictional force = $\mu_k\, F_N$

μ_k **:** The coefficient of kinetic friction,

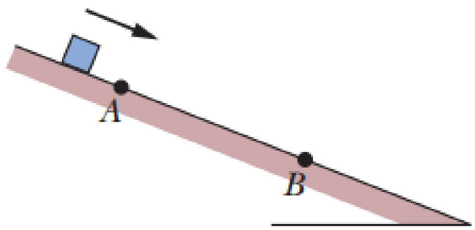
d **:** Distance

F_N **:** The normal force = *mg*

m **:** Mass**,**

g: gravitational acceleration

EXAMPLE: In Fig. below, a block slides down an incline. As it moves from point A to point B, which are 5.0 m apart, force acts on the block, with magnitude 2.0 N and directed down the incline. The magnitude of the frictional force acting on the block is 10N. If the kinetic energy of the block increases by 35 J between A and B, how much work is done on the block by the gravitational force as the block moves from A to B?



SOLUTION:

$$\Delta E_{th} = f_k d = (10\,N)(5.0\,m) = 50\ J$$

$$W = Fd = (2.0\,N)(5.0\,m) = 10\ J.$$

$$W = \Delta K + \Delta U + \Delta E_{th}$$

$$10 = 35 + \Delta U + 50$$

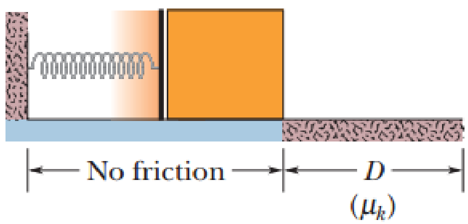
$$\Delta U = -75\ J.$$

EXAMPLE: In Fig. below, a 3.5 kg block is accelerated from rest by a compressed spring of spring constant 640 N/m. The block leaves the spring at the spring's relaxed length and then travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m. What are:

(a) The increase in the thermal energy of the block–floor system,

(b) The maximum kinetic energy of the block, and

(c) The original compression distance of the spring?



SOLUTION:

53. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{th} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{th} = (0.25)(3.5\,\text{kg})(9.8\,\text{m/s}^2)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{max} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{max}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}.$$