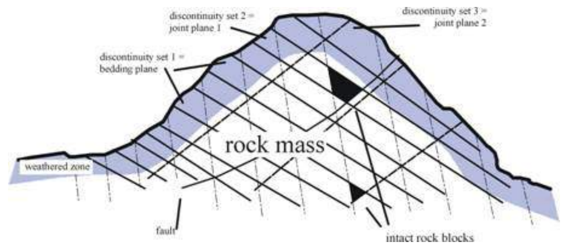
Rock Mechanics –
Response of Rocks to Applied Loads



CE/SC 10110/20110 Planet Earth

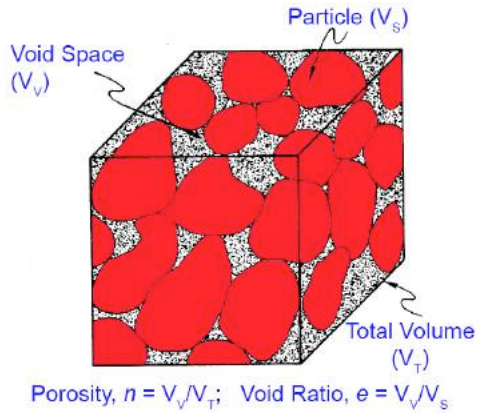


Rock Mechanics –
Response of Rocks to Applied Loads

- **Earth materials**
  - Rocks
  - Soils
  - Fluids
- ***Rocks***
  - Solid, dense aggregates of mineral grains
- ***Soils***
  - Anything that can be excavated by a shovel
- ***Fluids***
  - Water, magma, petroleum, natural gas, atmosphere

Phase Relationships in Earth Materials

- Rocks can be considered as three-phase systems in most cases
  - Solid, liquid, gas
- Most rocks contain some void space between grains
  - Voids are always filled with some type of fluid - either liquid or gas
  - *Amount* of void space & amount and ***type*** of fluid influence mechanical behavior of the material
- ***Porosity*** and ***void ratio*** - parameters used to quantify relative amount of void space
  - ***Porosity (%)***
    $n = V_v / V_T$
    V_v = Void volume, V_T = Total rock volume
  - ***Void ratio (decimal)***
    $e = V_v / V_s$
    V_v = Void volume, V_S = Volume of solids



Phase Relationships in Earth Materials

Permeability - rate at which fluids will move through a saturated material

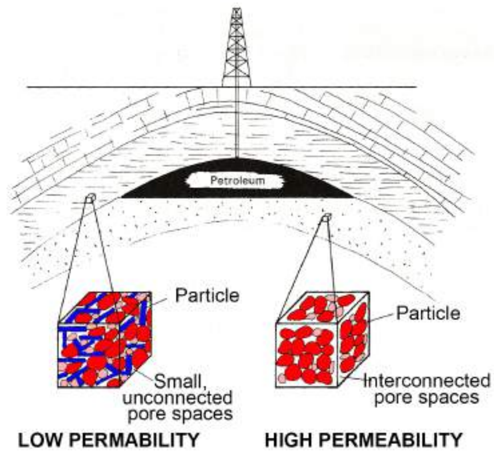
- Determined by size and connectedness of voids, and fluid properties (temperature, viscosity, density)
- ***Intrinsic permeability, k*** (cm^2 or darcys) - permeability defined by the property of the material
  - Size, shape, packing of the grains
  - Degree of cementation
  - Degree of fracturing
- ***Hydraulic Conductivity, K*** (cm/s, m/s) - measure of the ability of a rock or soil to transmit water
  - $K = k^*[(\rho g)/\mu]$

    k = intrinsic permeability
    ρ = density of water
    μ = viscosity of water
    g = acceleration due to gravity

Permeability and Hydraulic Conductivity (Brief Summary)

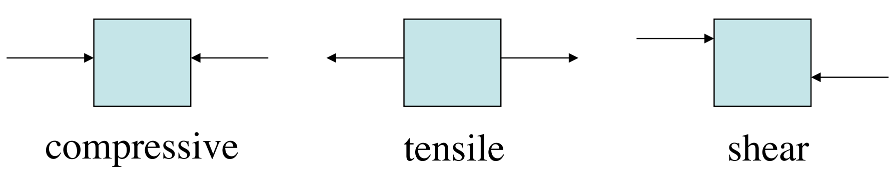
- High porosity and permeability materials have high hydraulic conductivity
- Importance? Wells, oil deposits, reservoirs….

Unfractured Igneous	Low *k*	Low K
Shale	Low *k*	Low K
Karst limestone	High *k*	High K
Gravel	High *k*	High K



Stress, Strain, Deformation Characteristics

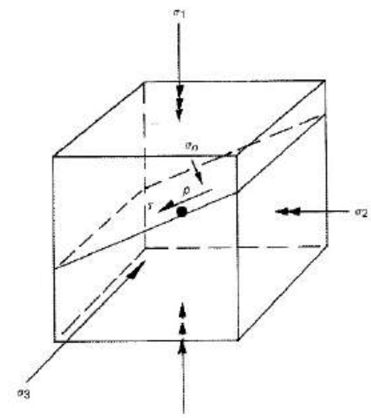
- ***Pressure*** - force per unit area applied to solid by a load
- ***Stress*** - pressure transmitted from the external face to an internal location, also force per unit area
- **Types of stresses**
  - ***Compressive*** - stresses of equal magnitude that act toward a point from opposite directions
  - ***Tensile*** - stresses of equal magnitude that act away from a point
  - ***Shear*** - stresses that are offset from one another and act in opposite directions



Stress, Strain, Deformation Characteristics

- At any point within an object…
  - Stresses can be resolved into ***three principle stresses*** that are mutually perpendicular **($\sigma 1$, $\sigma 2$, $\sigma 3$)**
  - ***Maximum***, ***intermediate*** **and** ***minimum stress***

- On any plane within an object…
  - There is a ***normal stress*** ***(σ_n)*** perpendicular to the plane and ***shear stress*** ***(τ)*** acting parallel to the plane



Stress, Strain, Deformation Characteristics

- ***Vertical stress*** σ_v acting on a horizontal plane at shallow *depth* ***h*** can be calculated as:

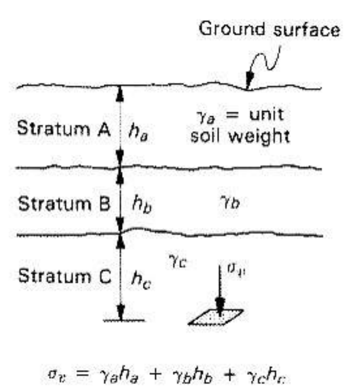
$$\sigma_v = \gamma h + P_a$$

γ = unit weight of rock
h = depth to point below surface
P_a = atmospheric pressure (usually neglected)

- *This equation assumes a consistent body of rock above point*

- If multiple layers of different rock types…

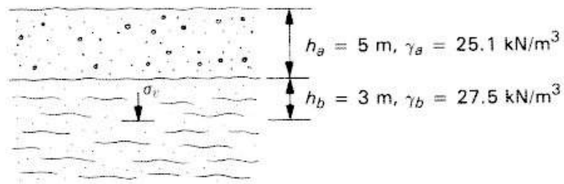
$$\sigma_v = \gamma_a h_a + \gamma_b h_b + \gamma_c h_c \ldots$$



EXAMPLE 7.1

Calculate the vertical stress at a depth of 8 m at a location where a 5-m bed of sandstone with a unit weight of 25.1 kN/m^3 overlies a thick shale unit with a unit weight of 27.5 kN/m^3.

Solution



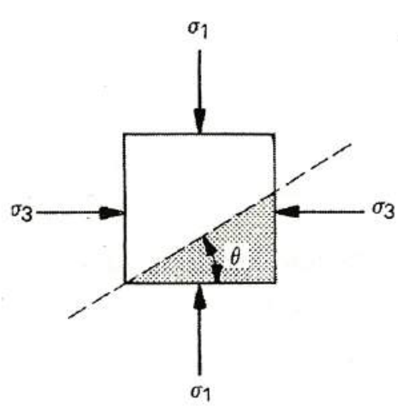
$$\sigma_v = \gamma_a h_a + \gamma_b h_b$$

$$\sigma_v = 25.1 \text{ kN/m}^3 \times 5 \text{ m} + 27.5 \text{ kN/m}^3 \times 3 \text{ m}$$

$$\sigma_v = 208 \text{ kN/m}^2$$

Stress, Strain, Deformation Characteristics

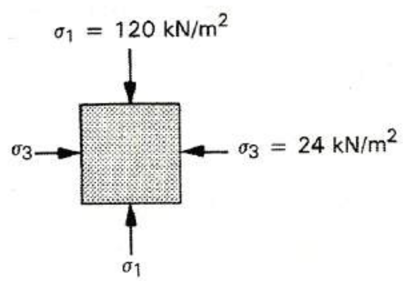
- Previous equation can be applied if we wish to determine the stress on the major (horizontal) principle plane.



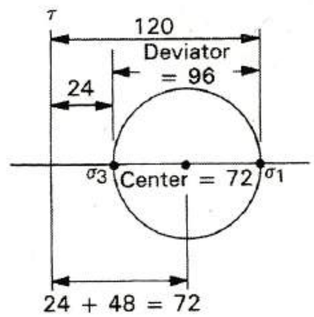
Position of a randon plane oriented at angle θ measured counter-clockwise from the major principal plane.

Stress, Strain, Deformation Characteristics

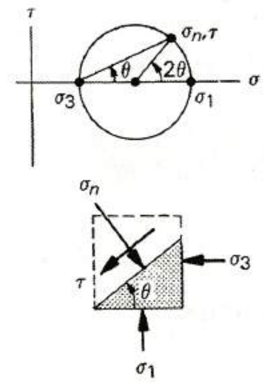
- What if we want to know the stress on an inclined plane? ***Mohr's Circle***



Stresses acting on an incremental element surrounding a point in the subsurface.



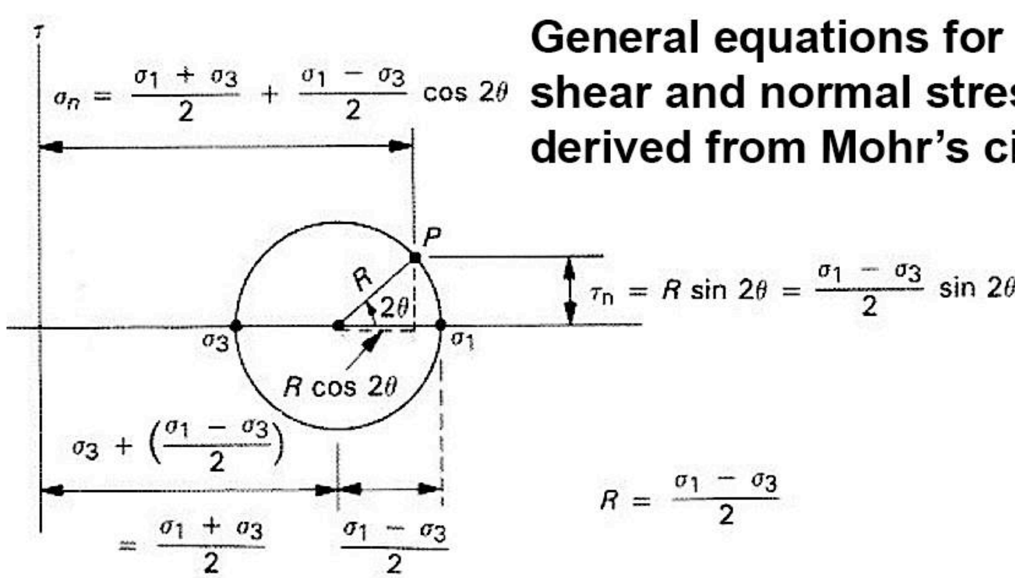
Deviator stress is the difference between $\sigma 1$ and $\sigma 3$.



Relationship of planes on the incremental element to points on Mohr's circle.

Mohr Circle:
Graphical representation of shear and normal stresses on inclined planes

General equations for shear and normal stress derived from Mohr's circle

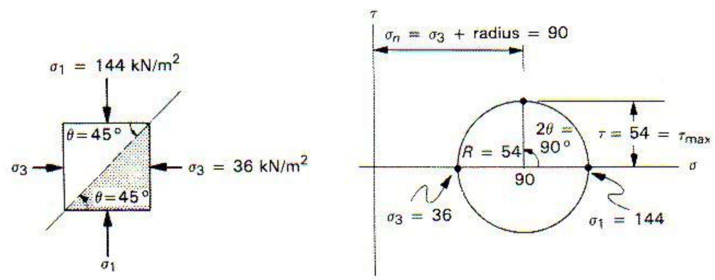


Stress, Strain, Deformation Characteristics

EXAMPLE 7.2

Vertical and horizontal principal stresses are 144 kN/m^2 and 36 kN/m^2, respectively. Determine the normal and shear stresses on a plane inclined at 45° to the principal plane. As shown, θ is measured counterclockwise from the major principal plane and angle 2θ, or 90°, is measured counterclockwise from the center of Mohr's circle. In this case, σ_n is equal to σ_3 plus the radius, and τ, the shear stress, is equal to the radius. It is evident from the diagram that a plane inclined at 45° from the major principal plane will have the highest value of shear stress of any plane.

Solution



$$\tau = \frac{\sigma_1 - \sigma_3}{2} = \frac{144 - 36}{2} = 54 \text{ kN/m}^2$$

$$\sigma_n = \sigma_3 + \text{radius} = 36 + 54 = 90 \text{ kN/m}^2$$

Deformation - Response to Stress

- Application of stress causes a body of rock to yield or deform.
- The ***amount of deformation*** is called ***strain***
- The type and amount of strain that a particular material experiences depends on:
  - Type of stresses applied
  - Depth and temperature

Deformation - Response to Stress

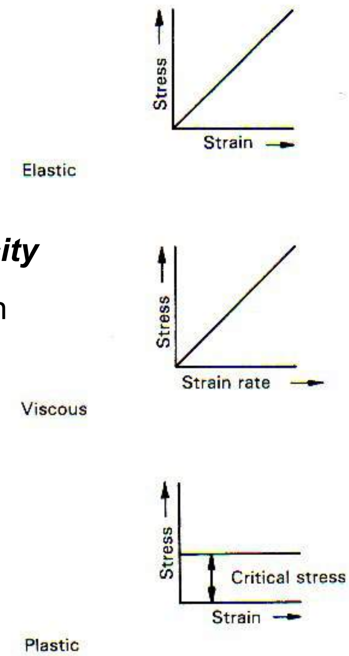
Ideal Materials

➢ ***Elastic*** *(e.g. spring)*

- Linear regression on a plot of stress vs strain
- Slope of regression line is ***modulus of elasticity***

  $E = \sigma/\varepsilon$; σ = applied stress; ε = strain
- Strain is change in length vs original length

  $\varepsilon = \Delta L / L$
- Strain in elastic systems is recoverable



Deformation - Response to Stress

Ideal Materials

➢ ***Viscous*** *(fluids)*

- Linear regression in a plot of stress vs strain rate
- ***Viscosity*** is slope of regression line in a stress-strain rate plot

➢ ***Plastic***

- No strain until some critical stress value has been reached; then continuous deformation

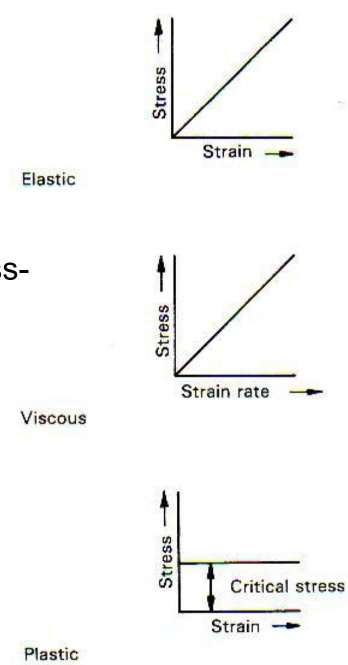
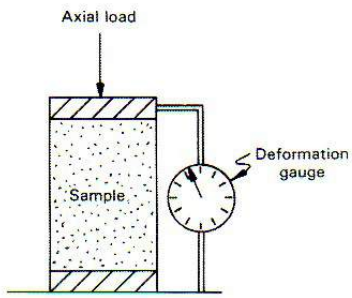


Fig. 7.11 textbook

Deformation - Response to Stress

- Rock behavior is more complex than ideal materials
- Common method of testing rock behavior is the ***unconfined compression test***



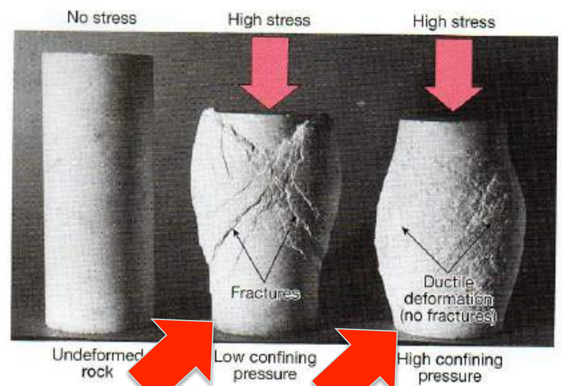


FIGURE 10.5 A marble cylinder deformed in the laboratory by applying thousands of pounds of load from above. Each sample was deformed in an environment that duplicated the confining pressure found at various depths. Notice that when the confining pressure was low, the sample deformed by brittle fracture. When the confining pressure was high, the sample deformed plastically. (Photo courtesy of M. S. Patterson, Australian National University)

Deformation - Response to Stress

Generalized stress-strain curve for rocks

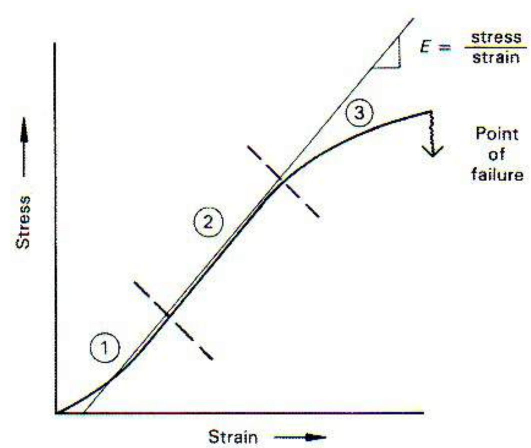
- Stress/strain relationships are generally ***not linear***
- Usually show 3 distinct segments:

Region 1:
closing of void spaces

Region 2:
approximately elastic behavior

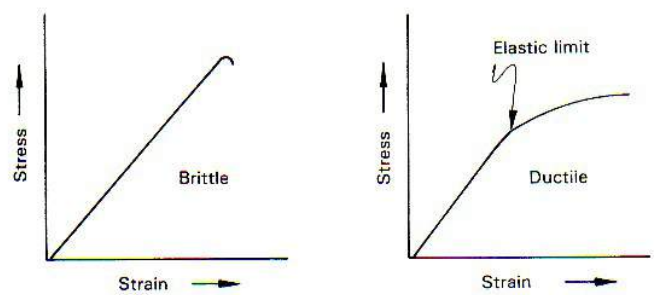
Region 3:
approximately plastic behavior

Failure:
rock breaks and loses all shear strength



Deformation - Response to Stress

- Different types of rocks vary considerably in their stress-strain behavior
- Two types of 'responses'
  - ***Brittle*** - respond in a mostly elastic fashion until failure
  - ***Ductile*** - respond *elastically* until the "Elastic Limit", then in *plastic* fashion until failure

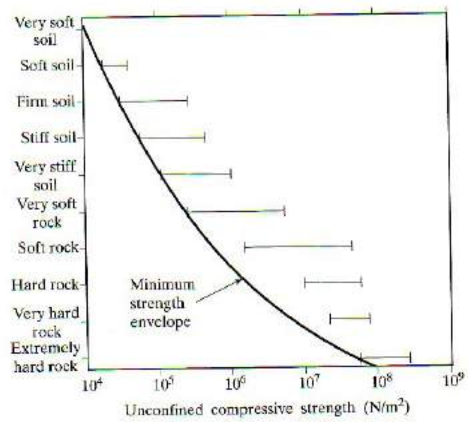


Compressive Strength

- Failure of a brittle rock - point when the rock loses all resistance to stress and crumbles.
- In plastic material, specific point of failure difficult to identify - because deformation continues indefinitely at a constant level of stress.
- ***Strength*** (in plastic materials) – is defined as the level of stress at failure.

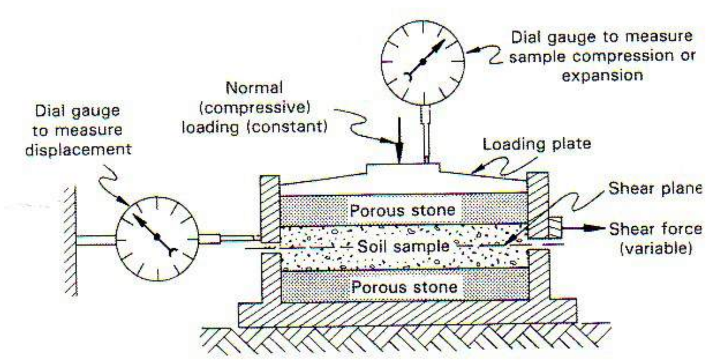
Compressive Strength

- On a plane within a rock body…
  - Normal stresses tend to resist failure
  - Shear stresses tend to cause failure
  - If shear stress exceeds the ***shear strength*** - failure occurs



Compressive Strength

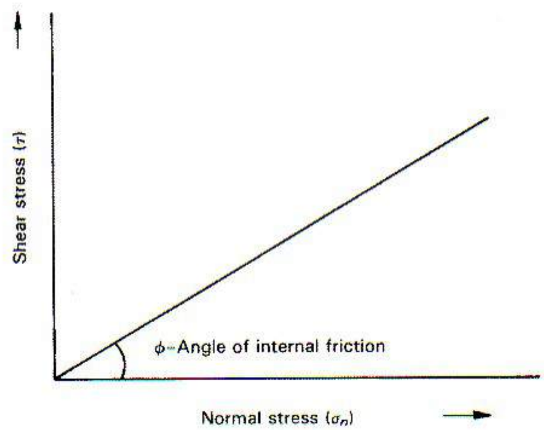
- Relationship between shear and normal stresses during a strength test (and at failure) is critical to understanding deformation behavior of the material
- Way to test shear strength - ***Direct shear test***
  Variable shear and normal stresses can be applied



Compressive Strength

- For ***unconsolidated materials (e.g. dry sand)*** the relationship between **normal stress** (σ_n) and **shear strength** (S) is linear, passes through the origin:

$$S = \sigma_n \tan \phi$$

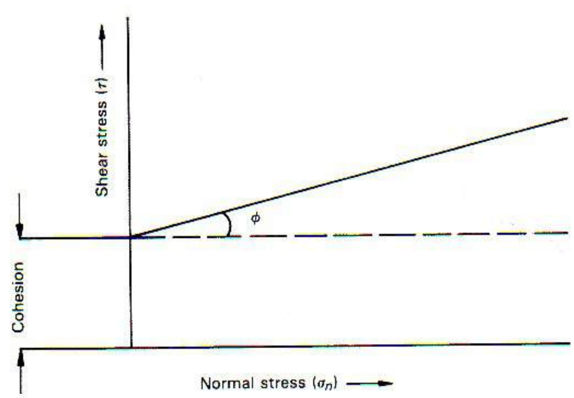


Compressive Strength

- For ***consolidated materials or cohesive soils***, relationship also linear, but there is inherent shear strength due to interparticle bonding (cohesion - C):

$$S = C + \sigma_n \tan \phi$$

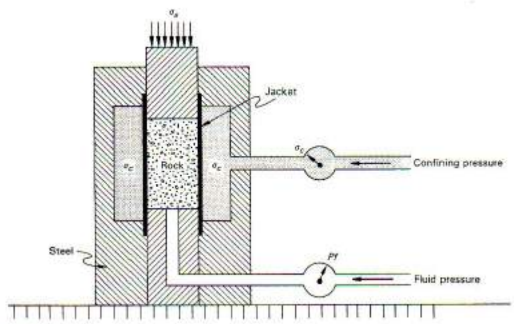
ϕ = Angle of internal friction



Compressive Strength

- ***Confining Pressure***
  - Weight of overlying rock applies pressure in all directions to given body of rock - confining pressure
  - Not always equal in all directions
  - Underground mine, tunnel construction

- ***Triaxial test***
  - Confining pressure can be applied to better mimic depth conditions



▲ FIGURE 7.21
Schematic diagram of triaxial test cell.

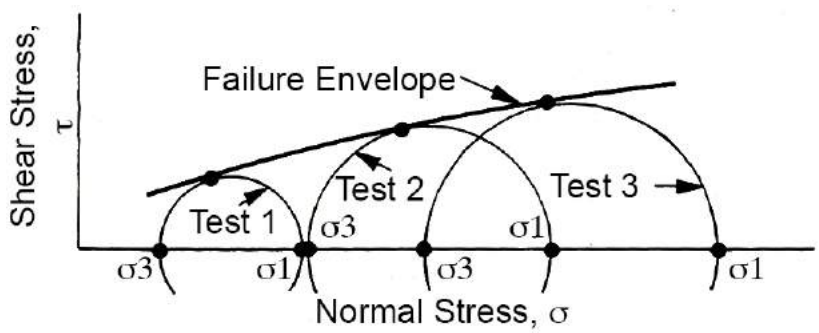
Large triaxial press, used for deformation experiments and materials testing

Compressive Strength

➢ ***Confining Pressure*** *(cont'd)*

Varying principle stresses (both axial and confining) allows for creation of multiple Mohr's Circles and

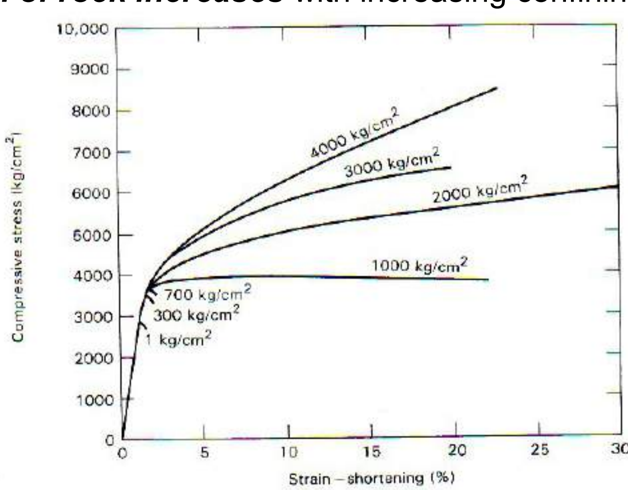
Definition of the ***failure envelope***



Compressive Strength

➢ ***Effect of increasing confining pressure***

- Rocks change from brittle to ductile behavior
  Ductile response dominant beyond 700 kg/cm^2
- ***Strength of rock increases*** with increasing confining pressure



Compressive Strength

- ***Effect of increasing temperature***
  - ***Strength decreases*** with increasing temperature
  - Ductile response occurs at lower pressures (stress) under higher temperatures
- ***Effect of time***
  - Stress applied in geologic systems occurs over millions of years
  - Rock ***strength decreases*** with decreasing strain rate (apply same amount of strain over a very long period of time)

Tensile Strength

- Tensile strength = resistance to failure under tensile stress
- Typically much lower than compressive strength
  - 10% of compressive strength typical (**Table 7.2**)
- Horizontal rock beams can be dangerous because of the weak tensile strength – rock unit must be homogeneous and composed of resistant minerals
- Arches overcome this by transferring tensile stress to compressive stresses around the arch

Table 7.2 Mechanical Properties of Rock and Other Materials

Rock type	Locality	ρ Density (g/cm^3)	E Modulus of Elasticity ($\times10^9$ N/m^2)	Ultimate Strength: σ_c Compressive Strength ($\times10^6$ N/m^2)	Ultimate Strength: σ_t Tensile Strength ($\times10^6$ N/m^2)
Amphibolite	California	2.94	92.4	278.0	22.8
Andesite	Nevada	2.37	37.0	103.0	7.2
Basalt	Michigan	2.70	41.0	120.0	14.6
Basalt	Colorado	2.62	32.4	58.0	3.2
Basalt	Nevada	2.83	33.9	148.0	18.1
Concrete	—	2.7–3.2	2.1–1.0	0.41–0.21	0.04–0.02
Conglomerate	Utah	2.54	14.1	88.0	3.0
Diabase	New York	2.94	95.8	321.0	55.1
Diorite	Arizona	2.71	46.9	119.0	8.2
Dolomite	Illinois	2.58	51.0	90.0	3.0
Gabbro	New York	3.03	55.3	186.0	13.8
Gneiss	Idaho	2.79	53.6	162.0	6.9
Gneiss	New Jersey	2.71	55.16	223.0	15.5
Granite	Georgia	2.64	39.0	193.0	2.8
Granite	Maryland	2.65	25.4	251.0	20.7
Granite	Colorado	2.64	70.6	226.0	11.9
Graywacke	Alaska	2.77	68.4	221.0	5.5
Gypsum	Canada	2.32	—	22.0	2.4
Limestone	Germany	2.62	63.8	63.8	4.0
Limestone	Indiana	2.30	26.96	53.1	4.07
Marble	New York	2.72	54.0	126.9	11.7
Marble	Tennessee	2.70	48.3	106.0	6.5
Phyllite	Michigan	3.24	76.5	126.0	22.8
Quartzite	Minnesota	2.75	84.8	629.0	23.4
Quartzite	Utah	2.55	22.06	148.0	3.5
Salt	Canada	2.20	4.64	35.5	2.5
Sandstone	Ohio	2.17	10.52	38.9	5.17
Sandstone	Utah	2.20	21.37	107.0	11.0
Schist	Colorado	2.47	8.96	15.0	—
Schist	Alaska	2.89	39.3	129.6	5.5
Shale	Utah	2.81	58.19	215.8	17.2
Shale	Pennsylvania	2.72	31.2	101.4	1.38
Siltstone	Pennsylvania	2.76	30.6	113.0	2.76
Slate	Michigan	2.93	75.85	180.0	25.5
Steel	—	7.85	200.00	365.0	365.0
Tuff	Nevada	2.39	3.65	11.3	1.17
Tuff	Japan	1.91	76.0	36.0	4.31

Source: P. H. Rahn, *Engineering Geology: An Environmental Approach*, 2nd ed., © 1996 by Prentice Hall, Inc., Upper Saddle River, N.J.

Engineering Classifications of Rock and Rock Masses

– Properties are determined in hand samples and in the field

– Rocks are almost always weaker in the field than in lab tests – some reasons why are

- Heterogeneity of the bulk samples
- Fractures
- Bedding planes
- Zones of weakness
- *Others??*

Engineering Classification of Rock and Rock Masses

➢ Intact Rock

- Strength classification is based on strength of the rock (compressive strength & modulus of elasticity; **Table 7.2**)
- 5 strength classes: A-E based on the overall rock strength; A = very strong, E = very weak (**Table 7.3**)

Table 7.3 Strength Classification of Intact Rock

Class	Description	Unconfined Compressive Strength (kg/cm^2)
A	Very high strength	>2250
B	High strength	1125–2250
C	Medium strength	562–1125
D	Low strength	281–562
E	Very low strength	<281

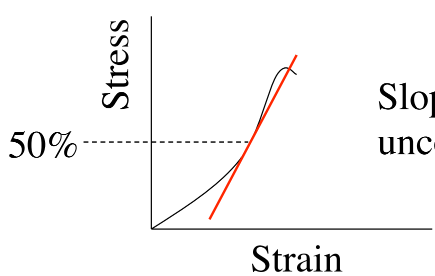
- Any discontinuities, fractures, bedding planes, etc. will strongly influence overall strength of the rock.
- 6 stiffness classes (**Table 7.4**): based on modulus of elasticity E_{t50}; 'very stiff' to 'highly yielding'

Table 7.4 Modulus of Elasticity Classification of Rocks

Description	E_{t50} (kg/cm^2 × 10^5)
Very stiff	8–16
Stiff	4–8
Medium stiffness	2–4
Low stiffness	1–2
Yielding	0.5–1
Highly yielding	0.25–0.5

Engineering Classification

➢ Determining the modulus of elasticity E_{t50}



Slope of the line tangent at 50% of the unconfined compressive strength = E_{t50}

➢ Plotting $\boldsymbol{E_{t50}}$ value of a rock vs the ***unconfined compressive strength*** gives a visual comparison of the strength and modulus values of different rocks.

➢ **M_R=** $\boldsymbol{E_{t50}}$ / σ_a (unconfined compressive strength) -

Modulus Ratio – Deere and Miller Classification

Deere and Miller Classification

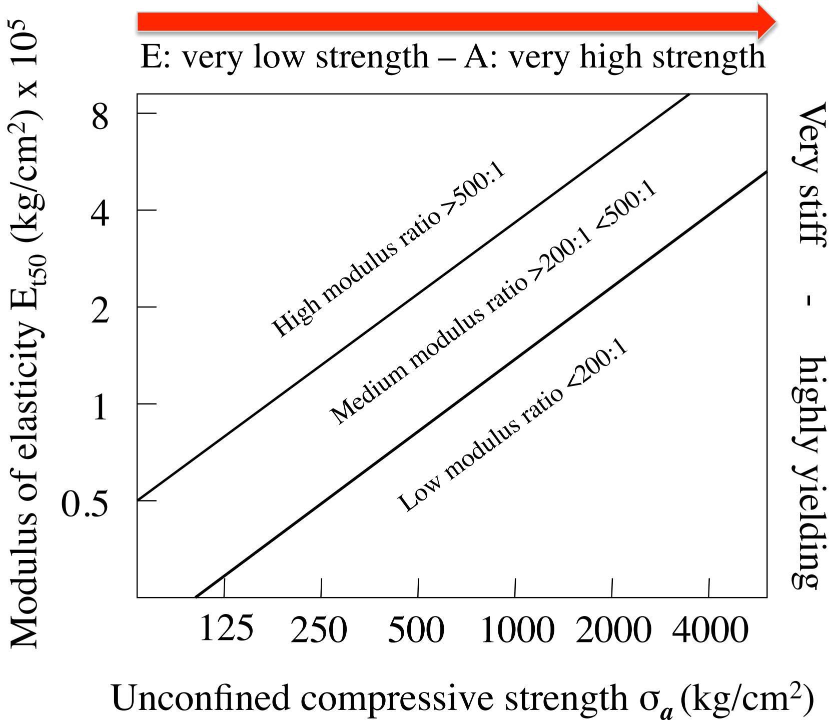
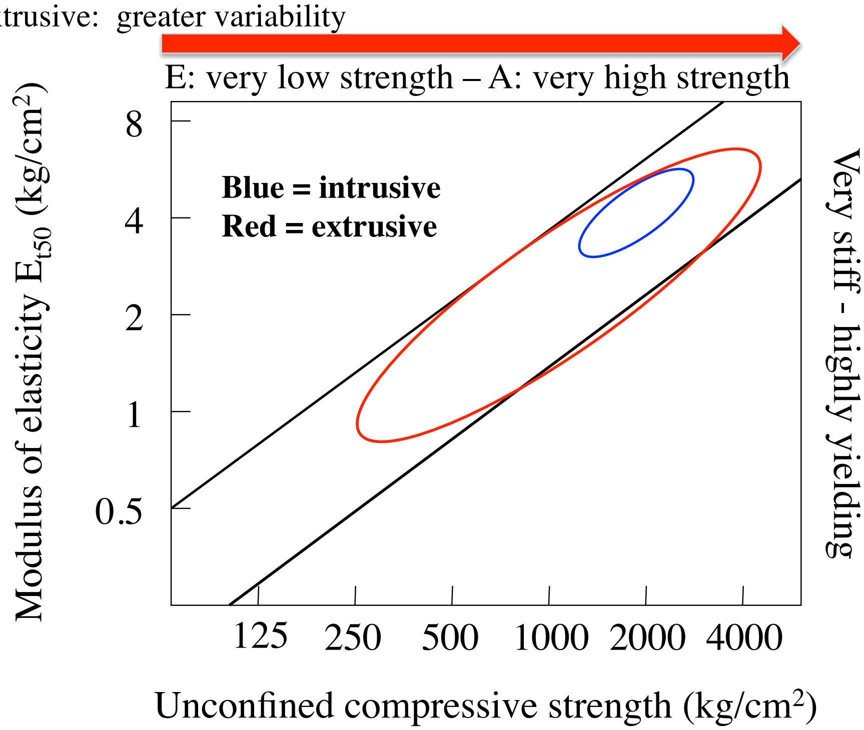


Figure 7.28

Intact Rock Classification - Igneous rocks

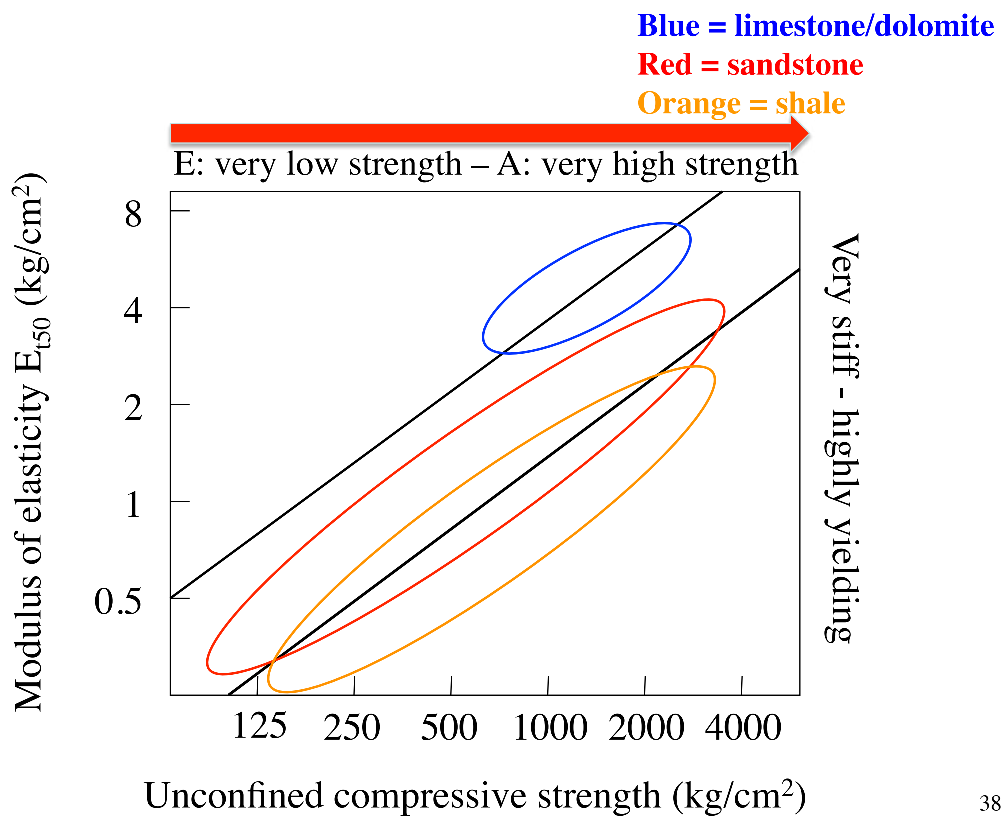
- Intrusive: high modulus of elasticity/medium modulus ratio
- Extrusive: greater variability



Intact Rock Classification - Sedimentary rocks

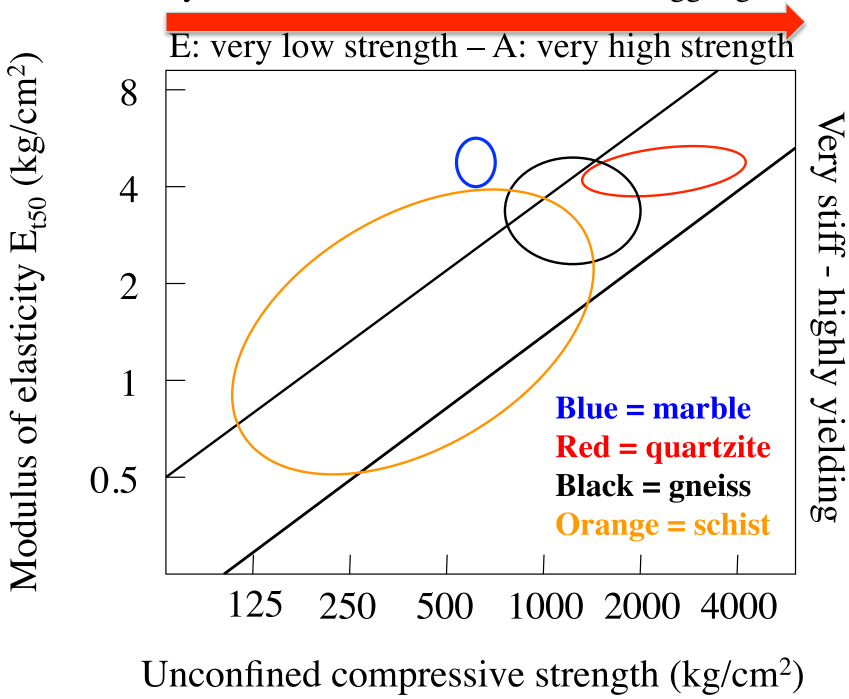
Extremely variable strength and modulus properties

- ***Clastic rocks***
  - Depends upon grain size, grain size distribution, sorting, packing, cement type, lithification processes
- ***Nonclastic rocks***
  - Depends upon composition
  - Limestones/dolomites generally medium to high strength and modulus ratios
  - Evaporites tend to be much weaker



Intact Rock Classification - Metamorphic rocks

- Tend to increase strength due to recrystallization and compaction
- Marbles may be weaker than limestones due to bigger grain size



Rock Mass Properties and Classification

- Rock strength and modulus properties determined on small intact hand samples - does not necessarily match field conditions
- **Weakest link principle** = overall strength of a rock not determined by bulk properties, but by strength of weakest link
- Rock discontinuities
  - Depositional discontinuities
  - Fractures
  - Bedding planes
  - Foliation
  - Cleavage
  - Faults
  - Contacts
  - Dikes, sills, veins
  - Faults

Rock Mass Properties

- When rock mass undergoes stress, it will preferentially fail along ***existing planes of weakness*** rather than develop new fractures.
- Must know the spacing, orientation, roughness of these ***weak links*** in order to accurately assess strength of the rock mass.
- Many variations and it's impossible to know them all for any rock mass.
- **Assessment of the quality of rock mass**

  RQD - Rock Quality Designation, Table 7.7

  Total recovery of drill core and length of intact pieces

  - High recovery and long pieces indicate solid unfractured rock
  - Low recovery and crumbled pieces indicate highly fractured rock
  - *Example:* 10 m long core: 9.2 m recovered in pieces that were 10 cm in length or longer, thus **RQD = 92%**

Table 7.7 Rock Quality Designation (RQD)

RQD (%)	Description of Rock Quality
0–25	Very poor
25–50	Poor
50–75	Fair
75–90	Good
90–100	Excellent

Rock Mechanics Summary

Response of Rocks to Applied Loads.

Phase Relationships in Earth Materials: Porosity and Permeability; Void Ratio; Intrinsic Permeability; Hydraulic Conductivity

Stress, Strain, Deformation Characteristics: Stress – Compressive, Tensile, Shear; Normal Stress; Shear Stress; Vertical Stress.

Mohr's Circle.

Deformation – Response to Stress: Strain; Elastic, Viscous, Fluid, Brittle, Ductile, Elastic Limit; Unconfined Compression Test; Generalized Stress-Strain Curve for Rock.

Compressive Strength: Strength; Shear Strength; Direct Shear Test; Confining Pressure; Triaxial Test; Definition of the Failure Envelope.

Tensile Strength.

Engineering Classifications of Rock and Rock Masses: Deere & Miller.

Rock Mass Properties & Classification: Weakest Link Principle; Rock Quality Designation (RQD).