

Review of Basic Algebraic Concepts

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- 1.1 Sets of Numbers and Interval Notation
- 1.2 Operations on Real Numbers
- 1.3 Simplifying Expressions
- 1.4 Linear Equations in One Variable
- 1.5 Applications of Linear Equations in One Variable
- 1.6 Literal Equations and Applications to Geometry
- 1.7 Linear Inequalities in One Variable
- 1.8 Properties of Integer Exponents and Scientific Notation

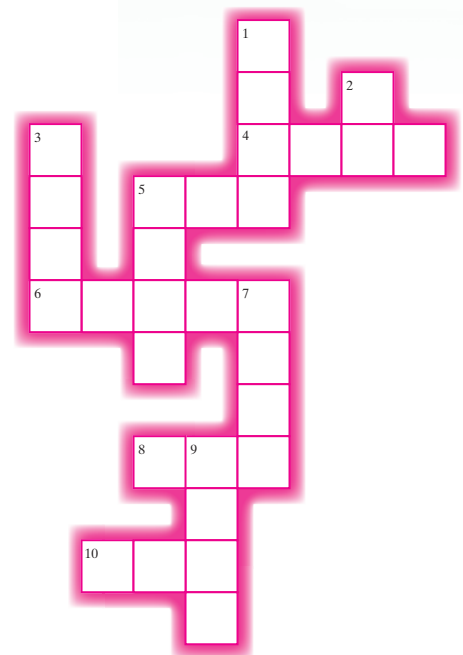
In Chapter 1, we present operations on real numbers, solving equations, and applications. This puzzle will help you familiarize yourself with some basic terms and geometry formulas. You will use these terms and formulas when working the exercises in Sections 1.5 and 1.6.

Across

- 4. Given that distance = rate \times time, what is the distance from Oklahoma City to Boston if it takes 28 hr of driving 60 mph?
- 5. What is the sum of the measures of the angles of a triangle?
- 6. What number is twice 6249?
- 8. If the measure of an angle is 48° , what is the measure of its supplement?
- 10. If a rectangle has an area of 8670 ft^2 and a width of 85 ft, what is its length?

Down

- 1. What is the next consecutive even integer after 4308?
- 2. If the measure of an angle is 2° , what is the measure of its complement?
- 3. What is 25% of 25,644?
- 5. What is the next consecutive integer after 1045?
- 7. What is 10% of 87,420?
- 9. What is the next consecutive odd integer after 3225?



Section 1.1

Sets of Numbers and Interval Notation

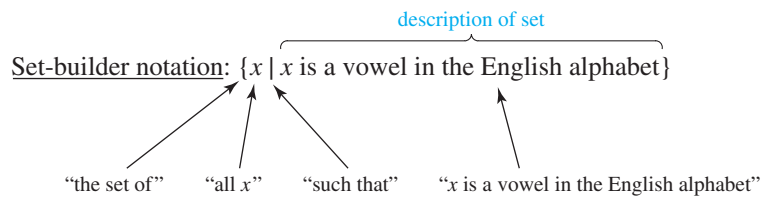
Concepts

1. Set of Real Numbers
2. Inequalities
3. Interval Notation
4. Union and Intersection of Sets
5. Translations Involving Inequalities

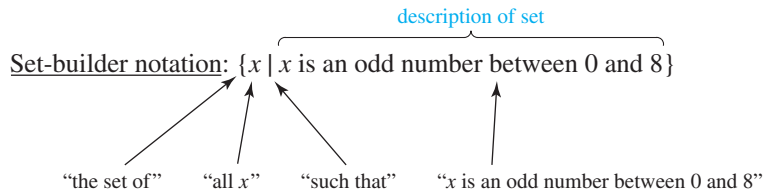
1. Set of Real Numbers

Algebra is a powerful mathematical tool that is used to solve real-world problems in science, business, and many other fields. We begin our study of algebra with a review of basic definitions and notations used to express algebraic relationships.

In mathematics, a collection of elements is called a **set**, and the symbols $\{ \}$ are used to enclose the elements of the set. For example, the set $\{a, e, i, o, u\}$ represents the vowels in the English alphabet. The set $\{1, 3, 5, 7\}$ represents the first four positive odd numbers. Another method to express a set is to *describe* the elements of the set by using **set-builder notation**. Consider the set $\{a, e, i, o, u\}$ in set-builder notation.



Consider the set $\{1, 3, 5, 7\}$ in set-builder notation.



Several sets of numbers are used extensively in algebra. The numbers you are familiar with in day-to-day calculations are elements of the set of **real numbers**. These numbers can be represented graphically on a horizontal number line with a point labeled as 0. Positive real numbers are graphed to the right of 0, and negative real numbers are graphed to the left. Each point on the number line corresponds to exactly one real number, and for this reason, the line is called the **real number line** (Figure 1-1).



Figure 1-1

Several sets of numbers are **subsets** (or part) of the set of real numbers. These are

- The set of natural numbers
- The set of whole numbers
- The set of integers
- The set of rational numbers
- The set of irrational numbers

Definition of the Natural Numbers, Whole Numbers, and Integers

The set of **natural numbers** is $\{1, 2, 3, \dots\}$.

The set of **whole numbers** is $\{0, 1, 2, 3, \dots\}$.

The set of **integers** is $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

The set of rational numbers consists of all the numbers that can be defined as a ratio of two integers.

Definition of the Rational Numbers

The set of **rational numbers** is $\{\frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \text{ does not equal zero}\}$.

Example 1 Identifying Rational Numbers

Show that each number is a rational number by finding two integers whose ratio equals the given number.

- a. $\frac{-4}{7}$ b. 8 c. $0.\bar{6}$ d. 0.87

Solution:

- a. $\frac{-4}{7}$ is a rational number because it can be expressed as the ratio of the integers -4 and 7 .
- b. 8 is a rational number because it can be expressed as the ratio of the integers 8 and 1 ($8 = \frac{8}{1}$). In this example we see that *an integer is also a rational number*.
- c. $0.\bar{6}$ represents the repeating decimal $0.666666\dots$ and can be expressed as the ratio of 2 and 3 ($0.\bar{6} = \frac{2}{3}$). In this example we see that *a repeating decimal is a rational number*.
- d. 0.87 is the ratio of 87 and 100 ($0.87 = \frac{87}{100}$). In this example we see that *a terminating decimal is a rational number*.

Skill Practice

Show that the numbers are rational by writing them as a ratio of integers.

1. $-9/8$ 2. 0 3. $0.\bar{3}$ 4. 0.45

Some real numbers such as the number π (pi) cannot be represented by the ratio of two integers. In decimal form, an irrational number is a nonterminating, non-repeating decimal. The value of π , for example, can be approximated as $\pi \approx 3.1415926535897932$. However, the decimal digits continue indefinitely with no pattern. Other examples of irrational numbers are the square roots of nonperfect squares, such as $\sqrt{3}$ and $\sqrt{11}$.

Definition of the Irrational Numbers

The set of **irrational numbers** is $\{x \mid x \text{ is a real number that is not rational}\}$.

Note: An irrational number cannot be written as a terminating decimal or as a repeating decimal.

TIP: Any rational number can be represented by a terminating decimal or by a repeating decimal.

Skill Practice Answers

1. $\frac{-9}{8}$ 2. $\frac{0}{1}$
3. $\frac{1}{3}$ 4. $\frac{45}{100}$

The set of real numbers consists of both the rational numbers and the irrational numbers. The relationships among the sets of numbers discussed thus far are illustrated in Figure 1-2.

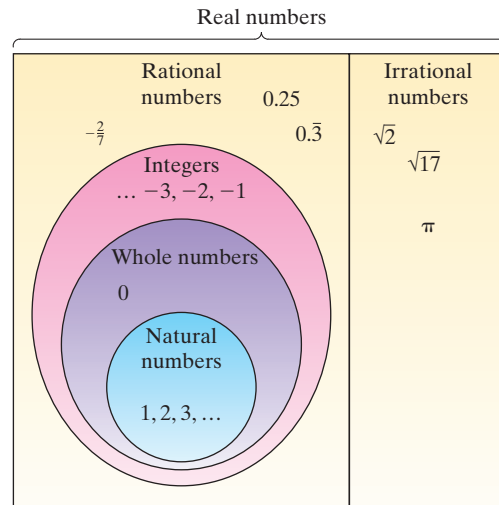


Figure 1-2

Example 2 Classifying Numbers by Set

Check the set(s) to which each number belongs. The numbers may belong to more than one set.

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6						
$\sqrt{23}$						
$-\frac{2}{7}$						
3						
2.35						

Solution:

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6			✓	✓		✓
$\sqrt{23}$					✓	✓
$-\frac{2}{7}$				✓		✓
3	✓	✓	✓	✓		✓
2.35				✓		✓

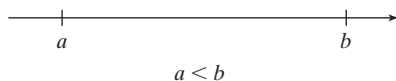
Skill Practice

5. Check the set(s) to which each number belongs.

	1	0.47	$\sqrt{5}$	$-\frac{1}{2}$
Natural				
Whole				
Integer				
Rational				
Irrational				
Real				

2. Inequalities

The relative size of two numbers can be compared by using the real number line. We say that a is less than b (written mathematically as $a < b$) if a lies to the left of b on the number line.



We say that a is greater than b (written mathematically as $a > b$) if a lies to the right of b on the number line.

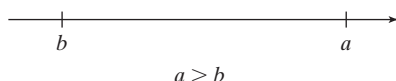


Table 1-1 summarizes the relational operators that compare two real numbers a and b .

Table 1-1

Mathematical Expression	Translation	Other Meanings
$a < b$	a is less than b	b exceeds a b is greater than a
$a > b$	a is greater than b	a exceeds b b is less than a
$a \leq b$	a is less than or equal to b	a is at most b a is no more than b
$a \geq b$	a is greater than or equal to b	a is no less than b a is at least b
$a = b$	a is equal to b	
$a \neq b$	a is not equal to b	
$a \approx b$	a is approximately equal to b	

The symbols $<$, $>$, \leq , \geq , and \neq are called inequality signs, and the expressions $a < b$, $a > b$, $a \leq b$, $a \geq b$, and $a \neq b$ are called **inequalities**.

Skill Practice Answers

5.

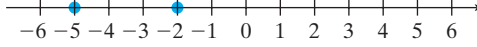
	1	0.47	$\sqrt{5}$	$-\frac{1}{2}$
Natural	✓			
Whole	✓			
Integer	✓			
Rational	✓	✓		✓
Irrational			✓	
Real	✓	✓	✓	✓

Example 3 Ordering Real Numbers

Fill in the blank with the appropriate inequality symbol: $<$ or $>$

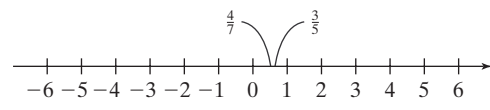
a. -2 _____ -5 b. $\frac{4}{7}$ _____ $\frac{3}{5}$ c. -1.3 _____ 2.8

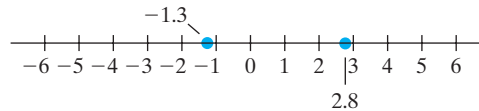
Solution:

a. -2 $>$ -5 

b. To compare $\frac{4}{7}$ and $\frac{3}{5}$, write the fractions as equivalent fractions with a common denominator.

$$\frac{4}{7} \cdot \frac{5}{5} = \frac{20}{35} \quad \text{and} \quad \frac{3}{5} \cdot \frac{7}{7} = \frac{21}{35}$$

Because $\frac{20}{35} < \frac{21}{35}$, then $\frac{4}{7} < \frac{3}{5}$ 

c. -1.3 $<$ 2.8 

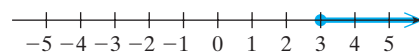
Skill Practice

Fill in the blanks with the appropriate symbol, $<$ or $>$.

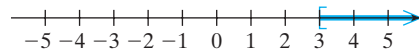
6. 2 _____ -12 7. $\frac{1}{4}$ _____ $\frac{2}{9}$ 8. -7.2 _____ -4.6

3. Interval Notation

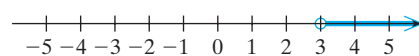
The set $\{x \mid x \geq 3\}$ represents all real numbers greater than or equal to 3. This set can be illustrated graphically on the number line.



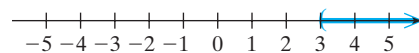
By convention, a closed circle \bullet or a square bracket $[$ is used to indicate that an “endpoint” ($x = 3$) is included in the set. This interval is a closed interval because its endpoint is included.



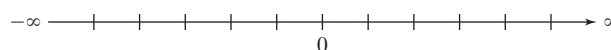
The set $\{x \mid x > 3\}$ represents all real numbers strictly greater than 3. This set can be illustrated graphically on the number line.



By convention, an open circle \circ or a parenthesis $($ is used to indicate that an “endpoint” ($x = 3$) is not included in the set. This interval is an open interval because its endpoint is not included.



Notice that the sets $\{x \mid x \geq 3\}$ and $\{x \mid x > 3\}$ consist of an infinite number of elements that cannot all be listed. Another method to represent the elements of such sets is by using **interval notation**. To understand interval notation, first consider the real number line, which extends infinitely far to the left and right. The symbol ∞ is used to represent infinity. The symbol $-\infty$ is used to represent negative infinity.

**Skill Practice Answers**

6. $>$ 7. $>$ 8. $<$

To express a set of real numbers in interval notation, sketch the graph first, using the symbols () or []. Then use these symbols at the endpoints to define the interval.

Example 4 Expressing Sets by Using Interval Notation

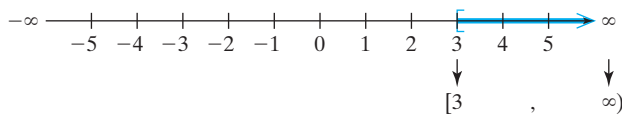
Graph the sets on the number line, and express the set in interval notation.

- a. $\{x \mid x \geq 3\}$ b. $\{x \mid x > 3\}$ c. $\{x \mid x \leq -\frac{3}{2}\}$

Solution:

a. Set-Builder Notation Graph Interval Notation

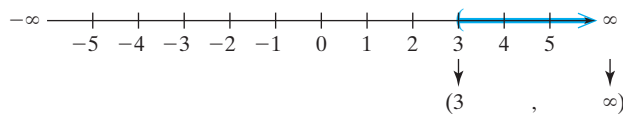
$\{x \mid x \geq 3\}$ $[3, \infty)$



The graph of the set $\{x \mid x \geq 3\}$ “begins” at 3 and extends infinitely far to the right. The corresponding interval notation “begins” at 3 and extends to ∞ . Notice that a square bracket [is used at 3 for both the graph and the interval notation to include $x = 3$. A parenthesis is always used at ∞ (and at $-\infty$) because there is no endpoint.

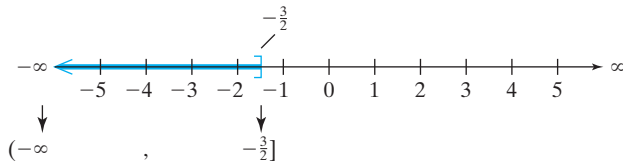
b. Set-Builder Notation Graph Interval Notation

$\{x \mid x > 3\}$ $(3, \infty)$



c. Set-Builder Notation Graph Interval Notation

$\{x \mid x \leq -\frac{3}{2}\}$ $(-\infty, -\frac{3}{2}]$

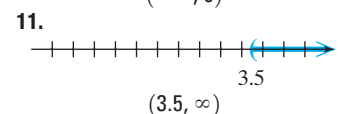
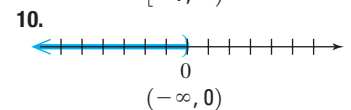
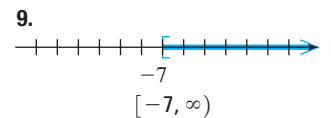


The graph of the set $\{x \mid x \leq -\frac{3}{2}\}$ extends infinitely far to the left. Interval notation is always written from left to right. Therefore, $-\infty$ is written first, followed by a comma, and then followed by the right-hand endpoint $-\frac{3}{2}$.

Skill Practice Graph and express the set, using interval notation.

9. $\{w \mid w \geq -7\}$ 10. $\{x \mid x < 0\}$ 11. $\{y \mid y > 3.5\}$

Skill Practice Answers

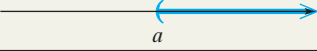
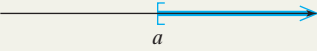
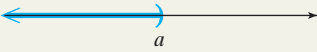



Using Interval Notation

- The endpoints used in interval notation are always written from left to right. That is, the smaller number is written first, followed by a comma, followed by the larger number.
- Parentheses) or (indicate that an endpoint is *excluded* from the set.
- Square brackets] or [indicate that an endpoint is *included* in the set.
- Parentheses are always used with ∞ and $-\infty$.

Table 1-2 summarizes the solution sets for four general inequalities.

Table 1-2

Set-Builder Notation	Graph	Interval Notation
$\{x \mid x > a\}$		(a, ∞)
$\{x \mid x \geq a\}$		$[a, \infty)$
$\{x \mid x < a\}$		$(-\infty, a)$
$\{x \mid x \leq a\}$		$(-\infty, a]$

4. Union and Intersection of Sets

Two or more sets can be combined by the operations of union and intersection.

A Union B and A Intersection B

The **union** of sets A and B , denoted $A \cup B$, is the set of elements that belong to set A or to set B or to both sets A and B .

The **intersection** of two sets A and B , denoted $A \cap B$, is the set of elements common to both A and B .

The concepts of the union and intersection of two sets are illustrated in Figures 1-3 and 1-4:

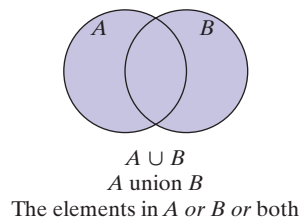


Figure 1-3

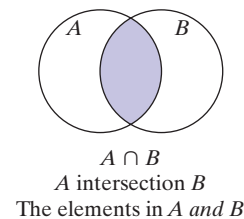


Figure 1-4

Example 5 Finding the Union and Intersection of Sets

Given the sets: $A = \{a, b, c, d, e, f\}$ $B = \{a, c, e, g, i, k\}$ $C = \{g, h, i, j, k\}$

Find:

- a. $A \cup B$ b. $A \cap B$ c. $A \cap C$

Solution:

- a. $A \cup B = \{a, b, c, d, e, f, g, i, k\}$ The union of A and B includes all the elements of A along with all the elements of B . Notice that the elements a , c , and e are not listed twice.
- b. $A \cap B = \{a, c, e\}$ The intersection of A and B includes only those elements that are common to both sets.
- c. $A \cap C = \{ \}$ (the **empty set**) Because A and C share no common elements, the intersection of A and C is the empty set or null set.

TIP: The empty set may be denoted by the symbol $\{ \}$ or by the symbol \emptyset .

Skill Practice Given:

$$A = \{r, s, t, u, v, w\} \quad B = \{s, v, w, y, z\} \quad C = \{x, y, z\}$$

Find:

12. $B \cup C$ 13. $A \cap B$ 14. $A \cap C$

Example 6 Finding the Union and Intersection of Sets

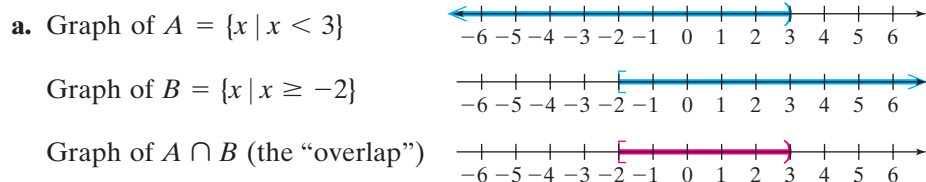
Given the sets: $A = \{x \mid x < 3\}$ $B = \{x \mid x \geq -2\}$ $C = \{x \mid x \geq 5\}$

Graph the following sets. Then express each set in interval notation.

- a. $A \cap B$ b. $A \cup C$ c. $A \cup B$ d. $A \cap C$

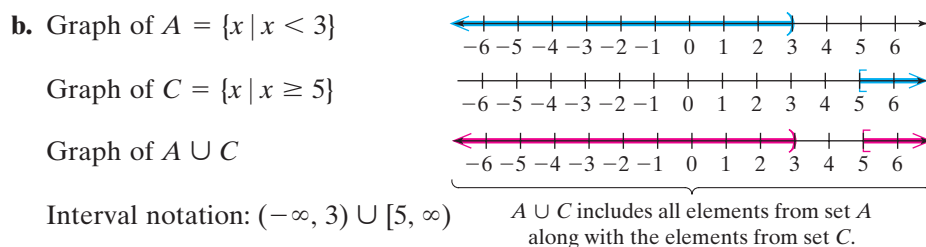
Solution:

It is helpful to visualize the graphs of individual sets on the number line before taking the union or intersection.



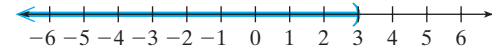
Interval notation: $[-2, 3)$

Note that the set $A \cap B$ represents the real numbers greater than or equal to -2 and less than 3 . This relationship can be written more concisely as a compound inequality: $-2 \leq x < 3$. We can interpret this inequality as “ x is between -2 and 3 , including $x = -2$.”

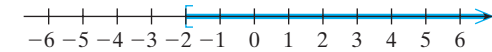
**Skill Practice Answers**

12. $\{s, v, w, x, y, z\}$ 13. $\{s, v, w\}$
14. $\{ \}$

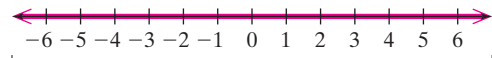
c. Graph of $A = \{x \mid x < 3\}$



Graph of $B = \{x \mid x \geq -2\}$



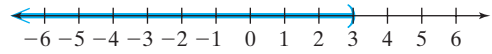
Graph of $A \cup B$



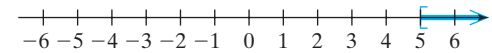
Interval notation: $(-\infty, \infty)$

$A \cup B$ includes all elements from set A
along with the elements of set B .
This encompasses all real numbers.

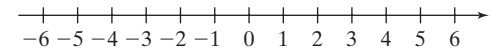
d. Graph of $A = \{x \mid x < 3\}$



Graph of $C = \{x \mid x \geq 5\}$



Graph of $A \cap C$
(the sets do not “overlap”)
 $A \cap C$ is the empty set $\{ \}$.



Skill Practice

Find the intersection or union by first graphing the sets on the real number line. Write the answers in interval notation.

15. $\{x \mid x \geq 1\} \cap \{x \mid x < 10\}$

16. $\{x \mid x \leq -2\} \cup \{x \mid x > 3\}$

17. $\{x \mid x > -2\} \cup \{x \mid x < 3\}$

18. $\{x \mid x < 1\} \cap \{x \mid x > 5\}$

5. Translations Involving Inequalities

In Table 1-1, we learned that phrases such as *at least*, *at most*, *no more than*, *no less than*, and *between* can be translated into mathematical terms by using inequality signs.

Example 7 Translating Inequalities

The intensity of a hurricane is often defined according to its maximum sustained winds for which wind speed is measured to the nearest mile per hour. Translate the italicized phrases into mathematical inequalities.

- A tropical storm is updated to hurricane status if the sustained wind speed, w , is *at least 74 mph*.
- Hurricanes are categorized according to intensity by the Saffir-Simpson scale. On a scale of 1 to 5, a category 5 hurricane is the most destructive. A category 5 hurricane has sustained winds, w , *exceeding 155 mph*.
- A category 4 hurricane has sustained winds, w , *of at least 131 mph but no more than 155 mph*.

Solution:

- a. $w \geq 74$ mph b. $w > 155$ mph c. $131 \text{ mph} \leq w \leq 155 \text{ mph}$

Skill Practice

Translate the italicized phrase to a mathematical inequality.

- The gas mileage, m , for a Honda Civic is *at least 30 mpg*.
- The gas mileage, m , for a Harley Davidson motorcycle is *more than 45 mpg*.
- The gas mileage, m , for a Ford Explorer is at least 10 mpg, but not *more than 20 mpg*.

Skill Practice Answers

- $[1, 10)$
- $(-\infty, -2] \cup (3, \infty)$
- $(-\infty, \infty)$ 18. $\{ \}$
- $m \geq 30$ 20. $m > 45$
- $10 \leq m \leq 20$

Section 1.1

Practice Exercises

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Study Skills Exercises

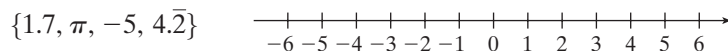
1. In this text we will provide skills for you to enhance your learning experience. In the first four chapters, each set of Practice Exercises will begin with an activity that focuses on one of the following areas: learning about your course, using your text, taking notes, doing homework, and taking an exam. In subsequent chapters we will insert skills pertaining to the specific material in the chapter. In Chapter 7 we will give tips on studying for the final exam.

To begin, write down the following information.

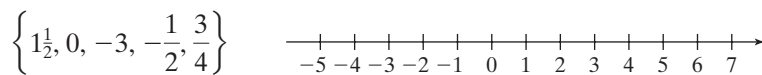
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|--|---|
| <p>a. Instructor's name</p> <p>c. The room number in which the class meets</p> <p>e. Instructor's office number</p> <p>g. Instructor's email address</p> | <p>b. Days of the week that the class meets</p> <p>d. Is there a lab requirement for this course? If so, what is the requirement and what is the location of the lab?</p> <p>f. Instructor's telephone number</p> <p>h. Instructor's office hours</p> |
|--|---|
2. Define the key terms.
- | | | | |
|---------------------|-------------------------|------------------|----------------------|
| a. Set | b. Set-builder notation | c. Real numbers | d. Real number line |
| e. Subset | f. Natural numbers | g. Whole numbers | h. Integers |
| i. Rational numbers | j. Irrational numbers | k. Inequality | l. Interval notation |
| m. Union | n. Intersection | o. Empty set | |

Concept 1: Set of Real Numbers

3. Plot the numbers on the number line.



4. Plot the numbers on the number line.



For Exercises 5–9, show that each number is a rational number by finding a ratio of two integers equal to the given number.

- | | | |
|-----------|-------------------|-------------------|
| 5. -10 | 6. $1\frac{1}{2}$ | 7. $-\frac{3}{5}$ |
| 8. -0.1 | 9. 0 | |

10. Check the sets to which each number belongs.

	Real Numbers	Irrational Numbers	Rational Numbers	Integers	Whole Numbers	Natural Numbers
5						
$-\sqrt{9}$						
-1.7						
$\frac{1}{2}$						
$\sqrt{7}$						
$\frac{0}{4}$						
$0.\overline{2}$						

11. Check the sets to which each number belongs.

	Real Numbers	Irrational Numbers	Rational Numbers	Integers	Whole Numbers	Natural Numbers
$\frac{6}{8}$						
$1\frac{1}{2}$						
π						
0						
$0.\overline{8}$						
$\frac{8}{2}$						
$4.\overline{2}$						

Concept 2: Inequalities

For Exercises 12–17, fill in the blanks with the appropriate symbol: $<$ or $>$.

12. $-9 \underline{\hspace{1cm}} -1$

13. $0 \underline{\hspace{1cm}} -6$

14. $0.15 \underline{\hspace{1cm}} 0.04$

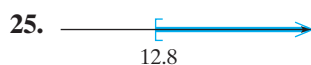
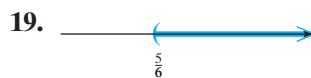
15. $-2.5 \underline{\hspace{1cm}} 0.6$

16. $\frac{5}{3} \underline{\hspace{1cm}} \frac{10}{7}$

17. $-\frac{21}{5} \underline{\hspace{1cm}} -\frac{17}{4}$

Concept 3: Interval Notation

For Exercises 18–25, express the set in interval notation.



For Exercises 26–43, graph the sets and express each set in interval notation.

26. $\{x \mid x > 3\}$



27. $\{x \mid x < 3\}$



28. $\{y \mid y \leq -2\}$



29. $\{z \mid z \geq -4\}$



30. $\{w \mid w < \frac{9}{2}\}$



31. $\{p \mid p \geq -\frac{7}{3}\}$



32. $\{x \mid -2.5 < x \leq 4.5\}$



33. $\{x \mid -6 \leq x < 0\}$



34. All real numbers less than -3 .



35. All real numbers greater than 2.34.



36. All real numbers greater than $\frac{5}{2}$.



37. All real numbers less than $\frac{4}{7}$.



38. All real numbers not less than 2.



39. All real numbers no more than 5.



40. All real numbers between -4 and 4.



41. All real numbers between -7 and -1 .



42. All real numbers between -3 and 0, inclusive.



43. All real numbers between -1 and 6, inclusive.



For Exercises 44–51, write an expression in words that describes the set of numbers given by each interval. (Answers may vary.)

44. $(-\infty, -4)$

45. $[2, \infty)$

46. $(-2, 7]$

47. $(-3.9, 0)$

48. $[-180, 90]$

49. $(-\infty, \infty)$

50. $(3.2, \infty)$

51. $(-\infty, -1]$

Concept 4: Union and Intersection of Sets

52. Given: $M = \{-3, -1, 1, 3, 5\}$ and $N = \{-4, -3, -2, -1, 0\}$.

List the elements of the following sets:

a. $M \cap N$

b. $M \cup N$

53. Given: $P = \{a, b, c, d, e, f, g, h, i\}$ and $Q = \{a, e, i, o, u\}$.

List the elements of the following sets.

a. $P \cap Q$

b. $P \cup Q$

Let $A = \{x \mid x > -3\}$, $B = \{x \mid x \leq 0\}$, $C = \{x \mid -1 \leq x < 4\}$, and $D = \{x \mid 1 < x < 3\}$. For Exercises 54–61, graph the sets described here. Then express the answer in set-builder notation and in interval notation.

54. $A \cap B$



55. $A \cup B$



56. $B \cup C$



57. $B \cap C$



58. $C \cup D$



59. $C \cap D$



60. $B \cap D$



61. $A \cup D$



Let $X = \{x \mid x \geq -10\}$, $Y = \{x \mid x < 1\}$, $Z = \{x \mid x > -1\}$, and $W = \{x \mid x \leq -3\}$. For Exercises 62–67, find the intersection or union of the sets X , Y , Z , and W . Write the answers in interval notation.

62. $X \cap Y$

63. $X \cup Y$

64. $Y \cup Z$

65. $Y \cap Z$

66. $Z \cup W$

67. $Z \cap W$

Concept 5: Translations Involving Inequalities

The following chart defines the ranges for normal blood pressure, high normal blood pressure, and high blood pressure (*hypertension*). All values are measured in millimeters of mercury, mm Hg. (Source: American Heart Association.)

Normal	Systolic less than 130	Diastolic less than 85
High normal	Systolic 130–139	Diastolic 85–89
Hypertension	Systolic 140 or greater	Diastolic 90 or greater

For Exercises 68–72, write an inequality using the variable p that represents each condition.

68. Normal systolic blood pressure

69. Diastolic pressure in hypertension

70. High normal range for systolic pressure

71. Systolic pressure in hypertension

72. Normal diastolic blood pressure

A pH scale determines whether a solution is acidic or alkaline. The pH scale runs from 0 to 14, with 0 being the most acidic and 14 being the most alkaline. A pH of 7 is neutral (distilled water has a pH of 7).

For Exercises 73–77, write the pH ranges as inequalities and label the substances as acidic or alkaline.

73. Lemon juice: 2.2 through 2.4, inclusive

74. Eggs: 7.6 through 8.0, inclusive

75. Carbonated soft drinks: 3.0 through 3.5, inclusive

76. Milk: 6.6 through 6.9, inclusive

77. Milk of magnesia: 10.0 through 11.0, inclusive

Expanding Your Skills

For Exercises 78–89, find the intersection or union of the following sets. Write the answers in interval notation.

78. $[1, 3) \cap (2, 7)$

79. $(-\infty, 0) \cap (-2, 5)$

80. $[-2, 4] \cap (3, \infty)$

81. $(-6, 0) \cap [-2, 9]$

82. $[-2, 7] \cup (-\infty, -1)$

83. $(-\infty, 0) \cup (-4, 1)$

84. $(2, 5) \cup (4, \infty)$

85. $[-6, -1] \cup (-2, \infty)$

86. $(-\infty, 3) \cup (-1, \infty)$

87. $(-\infty, -3) \cap (-1, \infty)$

88. $(-\infty, -8) \cap (0, \infty)$

89. $(-\infty, 8) \cup (0, \infty)$

Operations on Real Numbers

Section 1.2

1. Opposite and Absolute Value

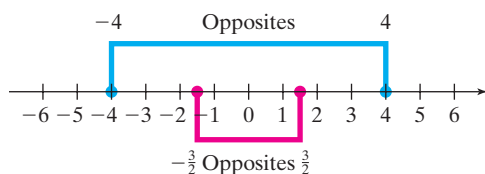
Several key definitions are associated with the set of real numbers and constitute the foundation of algebra. Two important definitions are the opposite of a real number and the absolute value of a real number.

Definition of the Opposite of a Real Number

Two numbers that are the same distance from 0 but on opposite sides of 0 on the number line are called **opposites** of each other.

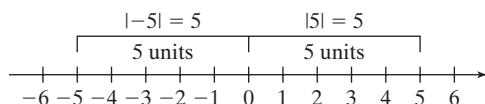
Symbolically, we denote the opposite of a real number a as $-a$.

The numbers -4 and 4 are opposites of each other. Similarly, the numbers $\frac{3}{2}$ and $-\frac{3}{2}$ are opposites.



The **absolute value** of a real number a , denoted $|a|$, is the distance between a and 0 on the number line. *Note:* The absolute value of any real number is *non-negative*.

For example: $|5| = 5$ and $|-5| = 5$



Example 1 Evaluating Absolute Value Expressions

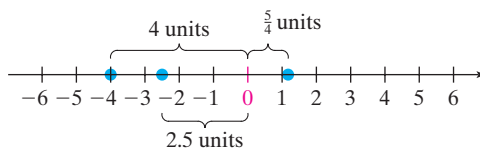
Simplify the expressions: a. $|-2.5|$ b. $|\frac{5}{4}|$ c. $-|-4|$

Solution:

a. $|-2.5| = 2.5$

b. $|\frac{5}{4}| = \frac{5}{4}$

c. $-|-4| = -(4) = -4$



Skill Practice Simplify.

1. $|-92|$ 2. $|7.6|$ 3. $-|2|$

The absolute value of a number a is its distance from zero on the number line. The definition of $|a|$ may also be given algebraically depending on whether a is negative or nonnegative.

Concepts

1. Opposite and Absolute Value
2. Addition and Subtraction of Real Numbers
3. Multiplication and Division of Real Numbers
4. Exponential Expressions
5. Square Roots
6. Order of Operations
7. Evaluating Expressions

Calculator Connections

Some calculators have an absolute value function.

For example,

<code>abs(-2.5)</code>	2.5
<code>abs(5/4) * Frac</code>	5/4
<code>-abs(-4)</code>	-4

Skill Practice Answers

1. 92 2. 7.6 3. -2

Definition of the Absolute Value of a Real Number

Let a be a real number. Then

1. If a is nonnegative (that is, $a \geq 0$), then $|a| = a$.
2. If a is negative (that is, $a < 0$), then $|a| = -a$.

This definition states that if a is a nonnegative number, then $|a|$ equals a itself. If a is a negative number, then $|a|$ equals the opposite of a . For example,

$$|9| = 9 \quad \text{Because 9 is positive, } |9| \text{ equals the number 9 itself.}$$

$$|-7| = 7 \quad \text{Because } -7 \text{ is negative, } |-7| \text{ equals the opposite of } -7, \text{ which is 7.}$$

2. Addition and Subtraction of Real Numbers

Addition of Real Numbers

1. To add two numbers with the *same sign*, add their absolute values and apply the common sign to the sum.
2. To add two numbers with *different signs*, subtract the smaller absolute value from the larger absolute value. Then apply the sign of the number having the larger absolute value.

Example 2 Adding Real Numbers

Perform the indicated operations:

a. $-2 + (-6)$

b. $-10.3 + 13.8$

c. $\frac{5}{6} + (-1\frac{1}{4})$

Solution:

$$\begin{aligned} \text{a. } & -2 + (-6) \\ & = -(2 + 6) \end{aligned}$$

Common sign is negative.

$$= -8$$

First find the absolute value of the addends.

$$|-2| = 2 \quad \text{and} \quad |-6| = 6$$

Add their absolute values and apply the common sign (in this case, the common sign is negative).

b. $-10.3 + 13.8$

First find the absolute value of the addends.

$$|-10.3| = 10.3 \quad \text{and} \quad |13.8| = 13.8$$

The absolute value of 13.8 is greater than the absolute value of -10.3 . Therefore, the sum is positive.

$$= +(13.8 - 10.3)$$

Subtract the smaller absolute value from the larger absolute value.

Apply the sign of the number with the larger absolute value.

$$= 3.5$$

$$\text{c. } \frac{5}{6} + \left(-1\frac{1}{4}\right)$$

$$= \frac{5}{6} + \left(-\frac{5}{4}\right)$$

$$= \frac{5 \cdot 2}{6 \cdot 2} + \left(-\frac{5 \cdot 3}{4 \cdot 3}\right)$$

$$= \frac{10}{12} + \left(-\frac{15}{12}\right)$$

$$= -\left(\frac{15}{12} - \frac{10}{12}\right)$$

$$= -\frac{5}{12}$$

Write $-1\frac{1}{4}$ as an improper fraction.

The LCD is 12. Write each fraction with the LCD.

Find the absolute value of the addends.

$$\left|\frac{10}{12}\right| = \frac{10}{12} \quad \text{and} \quad \left|-\frac{15}{12}\right| = \frac{15}{12}$$

The absolute value of $-\frac{15}{12}$ is greater than the absolute value of $\frac{10}{12}$. Therefore, the sum is negative.

Subtract the smaller absolute value from the larger absolute value.

Apply the sign of the number with the larger absolute value.

Skill Practice Add.

4. $-4 + (-1)$

5. $-2.6 + 1.8$

6. $-1 + \left(-\frac{3}{7}\right)$

Subtraction of real numbers is defined in terms of the addition process. To subtract two real numbers, add the opposite of the second number to the first number.

Subtraction of Real Numbers

If a and b are real numbers, then $a - b = a + (-b)$

Example 3 Subtracting Real Numbers

Perform the indicated operations.

a. $-13 - 5$

b. $2.7 - (-3.8)$

c. $\frac{5}{2} - 4\frac{2}{3}$

Solution:

a. $-13 - 5$

$$= -13 + (-5)$$

$$= -18$$

Add the opposite of the second number to the first number.

Add.

Skill Practice Answers

4. -5 5. -0.8 6. $-\frac{10}{7}$

$$\begin{aligned} \text{b. } 2.7 - (-3.8) &= 2.7 + (3.8) && \text{Add the opposite of the second number to the first number.} \\ &= 6.5 && \text{Add.} \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{5}{2} - 4\frac{2}{3} &= \frac{5}{2} + \left(-4\frac{2}{3}\right) && \text{Add the opposite of the second number to the first number.} \\ &= \frac{5}{2} + \left(-\frac{14}{3}\right) && \text{Write the mixed number as a fraction.} \\ &= \frac{15}{6} + \left(-\frac{28}{6}\right) && \text{Get a common denominator and add.} \\ &= -\frac{13}{6} \text{ or } -2\frac{1}{6} \end{aligned}$$

Skill Practice Subtract.

$$7. -1.1 - 3 \qquad 8. -5 - (-2) \qquad 9. \frac{1}{6} - \frac{3}{4}$$

3. Multiplication and Division of Real Numbers

The sign of the product of two real numbers is determined by the signs of the factors.

Multiplication of Real Numbers

1. The product of two real numbers with the *same* sign is *positive*.
2. The product of two real numbers with *different* signs is *negative*.
3. The product of any real number and zero is *zero*.

Example 4 Multiplying Real Numbers

Multiply the real numbers.

$$\text{a. } (2)(-5.1) \qquad \text{b. } -\frac{2}{3} \cdot \frac{9}{8} \qquad \text{c. } \left(-3\frac{1}{3}\right)\left(-\frac{3}{10}\right)$$

Solution:

$$\begin{aligned} \text{a. } (2)(-5.1) &= -10.2 && \text{Different signs. The product is negative.} \end{aligned}$$

Skill Practice Answers

$$7. -4.1 \qquad 8. -3 \qquad 9. -\frac{7}{12}$$

$$\begin{aligned} \text{b. } & -\frac{2}{3} \cdot \frac{9}{8} \\ & = -\frac{18}{24} && \text{Different signs. The product is negative.} \\ & = -\frac{3}{4} && \text{Simplify to lowest terms.} \end{aligned}$$

$$\begin{aligned} \text{c. } & \left(-3\frac{1}{3}\right)\left(-\frac{3}{10}\right) \\ & = \left(-\frac{10}{3}\right)\left(-\frac{3}{10}\right) && \text{Write the mixed number as a fraction.} \\ & = \frac{30}{30} && \text{Same signs. The product is positive.} \\ & = 1 && \text{Simplify to lowest terms.} \end{aligned}$$

Skill Practice Multiply.

$$10. (-5)(11) \quad 11. (-4)\left(\frac{2}{3}\right) \quad 12. (-5\frac{1}{4})\left(-\frac{8}{3}\right)$$

Notice from Example 4(c) that $(-\frac{10}{3})(-\frac{3}{10}) = 1$. If the product of two numbers is 1, then the numbers are said to be **reciprocals**. That is, the reciprocal of a real number a is $\frac{1}{a}$. Furthermore, $a \cdot \frac{1}{a} = 1$.

TIP: A number and its reciprocal have the same sign. For example:

$$\left(-\frac{10}{3}\right)\left(-\frac{3}{10}\right) = 1 \quad \text{and} \quad 3 \cdot \frac{1}{3} = 1$$

Recall that subtraction of real numbers was defined in terms of addition. In a similar way, division of real numbers can be defined in terms of multiplication. *To divide two real numbers, multiply the first number by the reciprocal of the second number.* For example:

$$10 \div 5 = 2 \quad \text{or equivalently} \quad 10 \cdot \frac{1}{5} = 2$$

Multiply
↓
↑
Reciprocal

Because division of real numbers can be expressed in terms of multiplication, the sign rules that apply to multiplication also apply to division.

$$\left. \begin{aligned} 10 \div 2 &= 10 \cdot \frac{1}{2} = 5 \\ -10 \div (-2) &= -10 \cdot \left(-\frac{1}{2}\right) = 5 \end{aligned} \right\} \text{Dividing two numbers of the same sign produces a } \textit{positive} \text{ quotient.}$$

Skill Practice Answers

10. -55 11. $-\frac{8}{3}$ 12. 14

$$\left. \begin{array}{l} 10 \div (-2) = 10\left(-\frac{1}{2}\right) = -5 \\ -10 \div 2 = -10 \cdot \frac{1}{2} = -5 \end{array} \right\} \text{Dividing two numbers of opposite signs produces a } \textit{negative} \text{ quotient.}$$

Division of Real Numbers

Assume that a and b are real numbers such that $b \neq 0$.

1. If a and b have the *same* signs, then the quotient $\frac{a}{b}$ is *positive*.
2. If a and b have *different* signs, then the quotient $\frac{a}{b}$ is *negative*.
3. $\frac{0}{b} = 0$.
4. $\frac{b}{0}$ is undefined.

The relationship between multiplication and division can be used to investigate properties 3 and 4 in the preceding box. For example,

$$\frac{0}{6} = 0 \quad \text{Because } 6 \times 0 = 0 \checkmark$$

$$\frac{6}{0} \text{ is undefined} \quad \text{Because there is no number that when multiplied by 0 will equal 6}$$

Note: The quotient of 0 and 0 cannot be determined. Evaluating an expression of the form $\frac{0}{0} = ?$ is equivalent to asking, “What number times zero will equal 0?” That is, $(0)(?) = 0$. Any real number will satisfy this requirement; however, expressions involving $\frac{0}{0}$ are usually discussed in advanced mathematics courses.

Example 5 Dividing Real Numbers

Divide the real numbers. Write the answer as a fraction or whole number.

$$\text{a. } \frac{-42}{7} \quad \text{b. } \frac{-96}{-144} \quad \text{c. } \frac{-5}{-7} \quad \text{d. } 3\frac{1}{10} \div \left(-\frac{2}{5}\right)$$

Solution:

$$\text{a. } \frac{-42}{7} = -6 \quad \text{Different signs. The quotient is negative.}$$

TIP: Recall that multiplication may be used to check a division problem. For example:

$$\frac{-42}{7} = -6 \quad \Rightarrow \quad (7) \cdot (-6) = -42 \checkmark$$

$$\text{b. } \frac{-96}{-144} = \frac{2}{3} \quad \text{Same signs. The quotient is positive. Simplify.}$$

$$\text{c. } \frac{-5}{-7} = \frac{5}{7} \quad \text{Same signs. The quotient is positive.}$$

TIP: If the numerator and denominator are both negative, then the fraction is positive:

$$\frac{-5}{-7} = \frac{5}{7}$$

$$\text{d. } 3\frac{1}{10} \div \left(-\frac{2}{5}\right)$$

$$= \frac{31}{10} \left(-\frac{5}{2}\right)$$

Write the mixed number as an improper fraction, and multiply by the reciprocal of the second number.

$$= \frac{31}{\cancel{10}^2} \left(-\frac{5}{2}\right)$$

$$= -\frac{31}{4}$$

Different signs. The quotient is negative.

TIP: If the numerator and denominator of a fraction have opposite signs, then the quotient will be negative. Therefore, a fraction has the same value whether the negative sign is written in the numerator, in the denominator, or in front of a fraction.

$$-\frac{31}{4} = \frac{-31}{4} = \frac{31}{-4}$$

Skill Practice Divide.

$$13. \frac{-28}{-4}$$

$$14. \frac{42}{-2}$$

$$15. \frac{-1}{-2}$$

$$16. -\frac{2}{3} \div 4$$

4. Exponential Expressions

To simplify the process of repeated multiplication, exponential notation is often used. For example, the quantity $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ can be written as 3^5 (3 to the fifth power).

Definition of b^n

Let b represent any real number and n represent a positive integer. Then

$$b^n = \underbrace{b \cdot b \cdot b \cdot b \cdot \dots \cdot b}_{n \text{ factors of } b}$$

b^n is read as “ b to the n th power.”

b is called the **base** and n is called the **exponent**, or **power**.

b^2 is read as “ b squared,” and b^3 is read as “ b cubed.”

Example 6 Evaluating Exponential Expressions

Simplify the expression.

$$\text{a. } 5^3$$

$$\text{b. } (-2)^4$$

$$\text{c. } -2^4$$

$$\text{d. } \left(-\frac{1}{3}\right)^2$$

Skill Practice Answers

$$13. 7 \quad 14. -21$$

$$15. \frac{1}{2} \quad 16. -\frac{1}{6}$$

Solution:

$$\begin{aligned} \text{a. } 5^3 &= 5 \cdot 5 \cdot 5 \\ &= 125 \end{aligned}$$

The base is 5, and the exponent is 3.

$$\begin{aligned} \text{b. } (-2)^4 &= (-2)(-2)(-2)(-2) \\ &= 16 \end{aligned}$$

The base is -2 , and the exponent is 4. The exponent 4 applies to the entire contents of the parentheses.

$$\begin{aligned} \text{c. } -2^4 &= -[2 \cdot 2 \cdot 2 \cdot 2] \\ &= -16 \end{aligned}$$

The base is 2, and the exponent is 4. Because no parentheses enclose the negative sign, the exponent applies to only 2.

$$\begin{aligned} \text{d. } \left(-\frac{1}{3}\right)^2 &= \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) \\ &= \frac{1}{9} \end{aligned}$$

The base is $-\frac{1}{3}$, and the exponent is 2.

TIP: The quantity -2^4 can also be interpreted as $-1 \cdot 2^4$.

$$\begin{aligned} -2^4 &= -1 \cdot 2^4 \\ &= -1 \cdot (2 \cdot 2 \cdot 2 \cdot 2) \\ &= -16 \end{aligned}$$

Calculator Connections

On many calculators, the $\boxed{x^2}$ key is used to square a number. The $\boxed{\wedge}$ key is used to raise a base to any power.

5^3	125
$(-2)^4$	16
-2^4	-16

Skill Practice Simplify.

17. 2^3

18. $(-10)^2$

19. -10^2

20. $\left(\frac{3}{4}\right)^3$

5. Square Roots

The inverse operation to squaring a number is to find its square roots. For example, finding a square root of 9 is equivalent to asking, “What number when squared equals 9?” One obvious answer is 3, because $(3)^2 = 9$. However, -3 is also a square root of 9 because $(-3)^2 = 9$. For now, we will focus on the **principal square root** which is always taken to be nonnegative.

The symbol $\sqrt{\quad}$, called a **radical sign**, is used to denote the principal square root of a number. Therefore, the principal square root of 9 can be written as $\sqrt{9}$. The expression $\sqrt{64}$ represents the principal square root of 64.

Example 7 Evaluating Square Roots

Evaluate the expressions, if possible.

a. $\sqrt{81}$

b. $\sqrt{\frac{25}{64}}$

c. $\sqrt{-16}$

Skill Practice Answers

17. 8 18. 100

19. -100 20. $\frac{27}{64}$

Solution:

- a. $\sqrt{81} = 9$ because $(9)^2 = 81$
- b. $\sqrt{\frac{25}{64}} = \frac{5}{8}$ because $\left(\frac{5}{8}\right)^2 = \frac{25}{64}$
- c. $\sqrt{-16}$ is *not a real number* because no real number when squared will be negative.

Skill Practice

Evaluate, if possible.

21. $\sqrt{25}$ 22. $\sqrt{\frac{49}{100}}$ 23. $\sqrt{-4}$

Example 7(c) illustrates that the square root of a negative number is not a real number because no real number when squared will be negative.

The Square Root of a Negative Number

Let a be a negative real number. Then \sqrt{a} is not a real number.

6. Order of Operations

When algebraic expressions contain numerous operations, it is important to evaluate the operations in the proper order. Parentheses (), brackets [], and braces { } are used for grouping numbers and algebraic expressions. It is important to recognize that operations must be done first within parentheses and other grouping symbols. Other grouping symbols include absolute value bars, radical signs, and fraction bars.

Order of Operations

1. First, simplify expressions within parentheses and other grouping symbols. These include absolute value bars, fraction bars, and radicals. If embedded parentheses are present, start with the innermost parentheses.
2. Evaluate expressions involving exponents, radicals, and absolute values.
3. Perform multiplication or division in the order in which they occur from left to right.
4. Perform addition or subtraction in the order in which they occur from left to right.

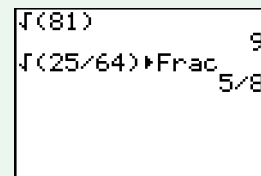
Example 8 Applying the Order of Operations

Simplify the following expressions.

- a. $10 - 5(2 - 5)^2 + 6 \div 3 + \sqrt{16 - 7}$ b. $\frac{|(-3)^3 + (5^2 - 3)|}{-15 \div (-3)(2)}$

Calculator Connections

The $\sqrt{\quad}$ key is used to find the square root of a nonnegative real number.

**Skill Practice Answers**

21. 5 22. $\frac{7}{10}$
23. Not a real number

Solution:

$$\text{a. } 10 - 5(2 - 5)^2 + 6 \div 3 + \sqrt{16 - 7}$$

$$= 10 - 5(-3)^2 + 6 \div 3 + \sqrt{9}$$

Simplify inside the parentheses and radical.

$$= 10 - 5(9) + 6 \div 3 + 3$$

Simplify exponents and radicals.

$$= 10 - 45 + 2 + 3$$

Do multiplication and division from left to right.

$$= -35 + 2 + 3$$

Do addition and subtraction from left to right.

$$= -33 + 3$$

$$= -30$$

TIP: Don't try to do too many steps at once. Taking a shortcut may result in a careless error. For each step rewrite the entire expression, changing only the operation being evaluated.

$$\text{b. } \frac{|(-3)^3 + (5^2 - 3)|}{-15 \div (-3)(2)}$$

Simplify numerator and denominator separately.

$$= \frac{|(-3)^3 + (25 - 3)|}{5(2)}$$

Numerator: Simplify inner parentheses.
Denominator: Do multiplication and division (left to right).

$$= \frac{|(-3)^3 + (22)|}{10}$$

Numerator: Simplify inner parentheses.
Denominator: Multiply.

$$= \frac{|-27 + 22|}{10}$$

Simplify exponents.

$$= \frac{|-5|}{10}$$

Add within the absolute value.

$$= \frac{5}{10} \text{ or } \frac{1}{2}$$

Evaluate the absolute value.

Calculator Connections

To evaluate the expression

$$\frac{|(-3)^3 + (5^2 - 3)|}{-15 \div (-3)(2)}$$

on a graphing calculator, use parentheses to enclose the absolute value expression. Likewise, it is necessary to use parentheses to enclose the entire denominator.

Skill Practice Simplify the expressions.

$$24. 36 \div 2^2 \cdot 3 - (18 - 5) \cdot 2 + 6 \qquad 25. \frac{-|5 - 7| + 11}{(-1 - 2)^2}$$

7. Evaluating Expressions

The order of operations is followed when evaluating an algebraic expression or when evaluating a geometric formula. For a list of common geometry formulas, see the inside front cover of the text. It is important to note that some geometric formulas use Greek letters (such as π) and some formulas use variables with subscripts. A **subscript** is a number or letter written to the right of and slightly below a variable. Subscripts are used on variables to represent different quantities. For example, the area of a trapezoid is given by $A = \frac{1}{2}(b_1 + b_2)h$. The values of b_1 and b_2 (read as “ b sub 1” and “ b sub 2”) represent the two different bases of the trapezoid (Figure 1-5). This is illustrated in Example 9.

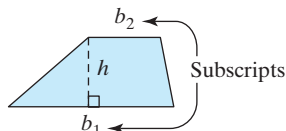


Figure 1-5

Example 9 Evaluating an Algebraic Expression

A homeowner in North Carolina wants to buy protective film for a trapezoid-shaped window. The film will adhere to shattered glass in the event that the glass breaks during a bad storm. Find the area of the window whose dimensions are given in Figure 1-6.

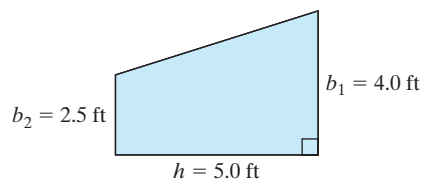


Figure 1-6

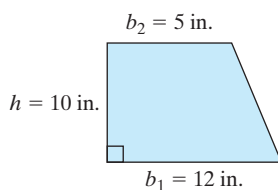
Solution:

$$\begin{aligned}
 A &= \frac{1}{2}(b_1 + b_2)h \\
 &= \frac{1}{2}(4.0 \text{ ft} + 2.5 \text{ ft})(5.0 \text{ ft}) && \text{Substitute } b_1 = 4.0 \text{ ft}, b_2 = 2.5 \text{ ft}, \text{ and } h = 5.0 \text{ ft.} \\
 &= \frac{1}{2}(6.5 \text{ ft})(5.0 \text{ ft}) && \text{Simplify inside parentheses.} \\
 &= 16.25 \text{ ft}^2 && \text{Multiply from left to right.}
 \end{aligned}$$

The area of the window is 16.25 ft^2 .

Skill Practice

26. Use the formula given in Example 9 to find the area of the trapezoid.



TIP: Subscripts should not be confused with *superscripts*, which are written above a variable. Superscripts are used to denote powers.

$$b_2 \neq b^2$$

Skill Practice Answers

26. The area is 85 in.^2

Section 1.2

Practice Exercises

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- Practice Problems
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Study Skills Exercises

1. Sometimes you may run into a problem with homework or you may find that you are having trouble keeping up with the pace of the class. A tutor can be a good resource. Answer the following questions.
 - a. Does your college offer tutoring?
 - b. Is it free?
 - c. Where should you go to sign up for a tutor?
 - d. Is there tutoring available online?

2. Define the key terms:

a. Opposite	b. Absolute value	c. Reciprocal
d. Base	e. Exponent	f. Power
g. Principal square root	h. Radical sign	i. Order of operations
j. Subscript		

Review Exercises

For Exercises 3–6, describe the set.

3. Rational numbers \cap Integers
4. Rational numbers \cup Irrational numbers
5. Natural numbers \cup $\{0\}$
6. Integers \cap Whole numbers

Concept 1: Opposite and Absolute Value

7. If the absolute value of a number can be thought of as its distance from zero, explain why an absolute value can never be negative.
8. If a number is negative, then its *opposite* will be **a.** Positive **b.** Negative.
9. If a number is negative, then its *reciprocal* will be **a.** Positive **b.** Negative.
10. If a number is negative, then its *absolute value* will be **a.** Positive **b.** Negative.
11. Complete the table.

Number	Opposite	Reciprocal	Absolute Value
6			
	$-\frac{1}{11}$		
		$-\frac{1}{8}$	
	$\frac{13}{10}$		
0			
		$-\sqrt{3}$	

12. Complete the table.

Number	Opposite	Reciprocal	Absolute Value
-9			
	$\frac{2}{3}$		
		14	
-1			
0			
		$2\frac{1}{9}$	

For Exercises 13–20, fill in the blank with the appropriate symbol ($<$, $>$, $=$).

13. $-|6|$ _____ $|-6|$

14. $-(-5)$ _____ $-|-5|$

15. $|-4|$ _____ $|4|$

16. $-|2|$ _____ (-2)

17. $-|-1|$ _____ 1

18. -3 _____ $-|-7|$

19. $|2 + (-5)|$ _____ $|2| + |-5|$

20. $|4 + 3|$ _____ $|4| + |3|$

Concept 2: Addition and Subtraction of Real Numbers

For Exercises 21–36, add or subtract as indicated.

21. $-8 + 4$

22. $3 + (-7)$

23. $-12 + (-7)$

24. $-5 + (-11)$

25. $-17 - (-10)$

26. $-14 - (-2)$

27. $5 - (-9)$

28. $8 - (-4)$

29. $-6 - 15$

30. $-21 - 4$

31. $1.5 - 9.6$

32. $4.8 - 10$

33. $\frac{2}{3} + \left(-2\frac{1}{3}\right)$

34. $-\frac{4}{7} + \left(1\frac{4}{7}\right)$

35. $-\frac{5}{9} - \frac{14}{15}$

36. $-6 - \frac{2}{9}$

Concept 3: Multiplication and Division of Real Numbers

For Exercises 37–50, perform the indicated operation.

37. $4(-8)$

38. $-21(3)$

39. $\frac{2}{9} \cdot \frac{12}{7}$

40. $\left(-\frac{5}{9}\right) \cdot \left(-1\frac{7}{11}\right)$

41. $-\frac{2}{3} \div \left(-1\frac{5}{7}\right)$

42. $\frac{5}{8} \div (-5)$

43. $7 \div 0$

44. $\frac{1}{16} \div 0$

45. $0 \div (-3)$

46. $0 \div 11$

47. $(-1.2)(-3.1)$

48. $(4.6)(-2.25)$

49. $(5.418) \div (0.9)$

50. $(6.9) \div (7.5)$

Concept 4: Exponential Expressions

For Exercises 51–58, evaluate the expressions.

51. 4^3

52. -2^3

53. -7^2

54. -2^4

55. $(-7)^2$

56. $(-5)^2$

57. $\left(\frac{5}{3}\right)^3$

58. $\left(\frac{10}{9}\right)^2$

Concept 5: Square Roots

For Exercises 59–66, evaluate the expression, if possible.

59. $\sqrt{81}$

60. $\sqrt{1}$

61. $\sqrt{-4}$

62. $\sqrt{-36}$

63. $\sqrt{\frac{1}{4}}$

64. $\sqrt{\frac{9}{4}}$

65. $-\sqrt{49}$

66. $-\sqrt{100}$

Concept 6: Order of Operations

For Exercises 67–92, simplify by using the order of operations.

67. $-\left(\frac{3}{4}\right)^2$

68. $-\left(\frac{2}{3}\right)^2$

69. $5 + 3^3$

70. $10 - 2^4$

71. $4^3 - 1^3$

72. $(3 + 4)^2$

73. $5 \cdot 2^3$

74. $12 \div 2^2$

75. $6 + 10 \div 2 \cdot 3 - 4$

76. $12 \div 3 \cdot 4 - 18$

77. $4^2 - (5 - 2)^2 \cdot 3$

78. $5 - 3(8 \div 4)^2$

79. $2 - 5(9 - 4\sqrt{25})^2$

80. $5^2 - (\sqrt{9} + 4 \div 2)$

81. $\left(-\frac{3}{5}\right)^2 - \frac{3}{5} \cdot \frac{5}{9} + \frac{7}{10}$

82. $\frac{1}{2} - \left(\frac{2}{3} \div \frac{5}{9}\right) + \frac{5}{6}$

83. $1.75 \div 0.25 - (1.25)^2$

84. $5.4 - (0.3)^2 \div 0.09$

85. $\frac{\sqrt{10^2 - 8^2}}{3^2}$

86. $\frac{\sqrt{16 - 7} + 3^2}{\sqrt{16} - \sqrt{4}}$

87. $-|-11 + 5| + |7 - 2|$

88. $-|-8 - 3| - (-8 - 3)$

89. $\frac{8(-3) - 6}{-7 - (-2)}$

90. $\frac{6(-2) - 8}{-15 - (10)}$

91. $\left(\frac{1}{2}\right)^2 + \left(\frac{6 - 4}{5}\right)^2 + \left(\frac{5 + 2}{10}\right)^2$

92. $\left(\frac{2^3}{2^3 + 1}\right)^2 \div \left(\frac{8 - (-2)}{3^2}\right)^2$

For Exercises 93–94, find the average of the set of data values by adding the values and dividing by the number of values.

93. Find the average low temperature for a week in January in St. John's, Newfoundland. (Round to the nearest tenth of a degree.)

Day	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
Low temperature	-18°C	-16°C	-20°C	-11°C	-4°C	-3°C	1°C

94. Find the average high temperature for a week in January in St. John's, Newfoundland. (Round to the nearest tenth of a degree.)

Day	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
High temperature	-2°C	-6°C	-7°C	0°C	1°C	8°C	10°C

Concept 7: Evaluating Expressions

95. The formula $C = \frac{5}{9}(F - 32)$ converts temperatures in the Fahrenheit scale to the Celsius scale. Find the equivalent Celsius temperature for each Fahrenheit temperature.

a. 77°F b. 212°F c. 32°F d. -40°F

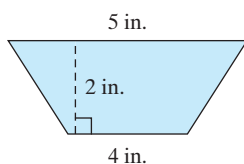
96. The formula $F = \frac{9}{5}C + 32$ converts Celsius temperatures to Fahrenheit temperatures. Find the equivalent Fahrenheit temperature for each Celsius temperature.

a. -5°C b. 0°C c. 37°C d. -40°C

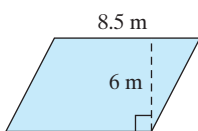
Use the geometry formulas found in the inside front cover of the book to answer Exercises 97–106.

For Exercises 97–100, find the area.

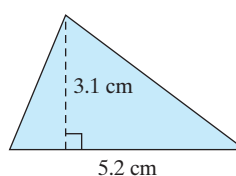
97. Trapezoid



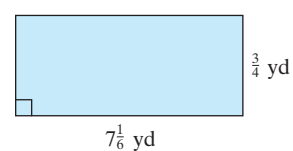
98. Parallelogram




99. Triangle

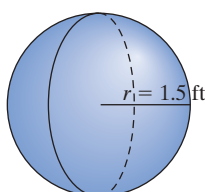


100. Rectangle

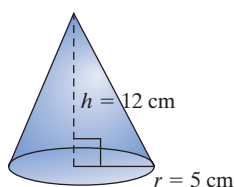


 For Exercises 101–106, find the volume. (Use the π key on your calculator, and round the final answer to 1 decimal place.)

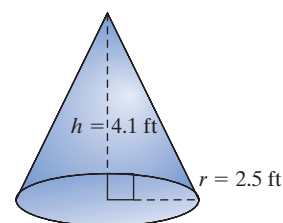
101. Sphere



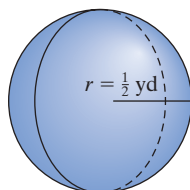
102. Right circular cone



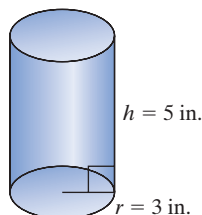
103. Right circular cone



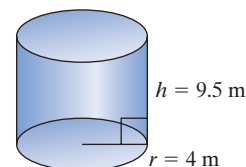
104. Sphere



105. Right circular cylinder



106. Right circular cylinder



Graphing Calculator Exercises

107. Which expression when entered into a graphing calculator will yield the correct value of $\frac{12}{6-2}$?

$$12/6 - 2 \quad \text{or} \quad 12/(6 - 2)$$

108. Which expression when entered into a graphing calculator will yield the correct value of $\frac{24-6}{3}$?

$$(24 - 6)/3 \quad \text{or} \quad 24 - 6/3$$

109. Verify your solution to Exercise 85 by entering the expression into a graphing calculator:

$$(\sqrt{10^2 - 8^2})/3^2$$

110. Verify your solution to Exercise 86 by entering the expression into a graphing calculator:

$$(\sqrt{16 - 7} + 3^2)/(\sqrt{16} - \sqrt{4})$$

Section 1.3

Simplifying Expressions

Concepts

1. Recognizing Terms, Factors, and Coefficients
2. Properties of Real Numbers
3. Simplifying Expressions

1. Recognizing Terms, Factors, and Coefficients

An algebraic expression is a single term or a sum of two or more terms. A **term** is a constant or the product of a constant and one or more variables. For example, the expression

$$-6x^2 + 5xyz - 11 \quad \text{or} \quad -6x^2 + 5xyz + (-11)$$

consists of the terms $-6x^2$, $5xyz$, and -11 .

The terms $-6x^2$ and $5xyz$ are **variable terms**, and the term -11 is called a **constant term**. It is important to distinguish between a term and the **factors** within a term. For example, the quantity $5xyz$ is one term, but the values 5, x , y , and z are factors within the term. The constant factor in a term is called the numerical coefficient or simply **coefficient** of the term. In the terms $-6x^2$, $5xyz$, and -11 , the coefficients are -6 , 5 , and -11 , respectively. A term containing only variables such as xy has a coefficient of 1.

Terms are called **like terms** if they each have the same variables and the corresponding variables are raised to the same powers. For example:

Like Terms

$-6t$	and	$4t$
$1.8ab$	and	$-3ab$
$\frac{1}{2}c^2d^3$	and	c^2d^3
4	and	6

Unlike Terms

$-6t$	and	$4s$	(different variables)
$1.8xy$	and	$-3x$	(different variables)
$\frac{1}{2}c^2d^3$	and	c^2d	(different powers)
$4p$	and	6	(different variables)

Example 1 Identifying Terms, Factors, Coefficients, and Like Terms

- a. List the terms of the expression. $-4x^2 - 7x + \frac{2}{3}$
- b. Identify the coefficient of the term. yz^3
- c. Identify the pair of *like* terms. $16b, 4b^2$ or $\frac{1}{2}c, -\frac{1}{6}c$

Solution:

- a. The terms of the expression $-4x^2 - 7x + \frac{2}{3}$ are $-4x^2$, $-7x$, and $\frac{2}{3}$.
- b. The term yz^3 can be written as $1yz^3$; therefore, the coefficient is 1.
- c. $\frac{1}{2}c, -\frac{1}{6}c$ are *like* terms because they have the same variable raised to the same power.

Skill Practice Given: $-2x^2 + 5x + \frac{1}{2} - y^2$

- 1a. List the terms of the expression.
- b. Which term is the constant term?
- c. Identify the coefficient of the term $-y^2$.

2. Properties of Real Numbers

Simplifying algebraic expressions requires several important properties of real numbers that are stated in Table 1-3. Assume that a , b , and c represent real numbers or real-valued algebraic expressions.

Skill Practice Answers

- 1a. $-2x^2, 5x, \frac{1}{2}, -y^2$
- b. $\frac{1}{2}$
- c. -1

Table 1-3

Property Name	Algebraic Representation	Example	Description/Notes
Commutative property of addition	$a + b = b + a$	$5 + 3 = 3 + 5$	The order in which two real numbers are added or multiplied does not affect the result.
Commutative property of multiplication	$a \cdot b = b \cdot a$	$(5)(3) = (3)(5)$	
Associative property of addition	$(a + b) + c = a + (b + c)$	$(2 + 3) + 7 = 2 + (3 + 7)$	The manner in which two real numbers are grouped under addition or multiplication does not affect the result.
Associative property of multiplication	$(a \cdot b)c = a(b \cdot c)$	$(2 \cdot 3)7 = 2(3 \cdot 7)$	
Distributive property of multiplication over addition	$a(b + c) = ab + ac$	$3(5 + 2) = 3 \cdot 5 + 3 \cdot 2$	A factor outside the parentheses is multiplied by each term inside the parentheses.
Identity property of addition	0 is the identity element for addition because $a + 0 = 0 + a = a$	$5 + 0 = 0 + 5 = 5$	Any number added to the identity element 0 will remain unchanged.
Identity property of multiplication	1 is the identity element for multiplication because $a \cdot 1 = 1 \cdot a = a$	$5 \cdot 1 = 1 \cdot 5 = 5$	Any number multiplied by the identity element 1 will remain unchanged.
Inverse property of addition	a and $(-a)$ are additive inverses because $a + (-a) = 0$ and $(-a) + a = 0$	$3 + (-3) = 0$	The sum of a number and its additive inverse (opposite) is the identity element 0.
Inverse property of multiplication	a and $\frac{1}{a}$ are multiplicative inverses because $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$ (provided $a \neq 0$)	$5 \cdot \frac{1}{5} = 1$	The product of a number and its multiplicative inverse (reciprocal) is the identity element 1.

The properties of real numbers are used to multiply algebraic expressions. To multiply a term by an algebraic expression containing more than one term, we apply the distributive property of multiplication over addition.

Example 2 Applying the Distributive Property

Apply the distributive property.

a. $4(2x + 5)$

b. $-(-3.4q + 5.7r)$

c. $-3(a + 2b - 5c)$

d. $-\frac{2}{3}\left(-9x + \frac{3}{8}y - 5\right)$

Solution:

$$\begin{aligned} \text{a. } & 4(2x + 5) \\ & = 4(2x) + 4(5) \\ & = 8x + 20 \end{aligned}$$

Apply the distributive property. Simplify, using the associative property of multiplication.

$$\begin{aligned} \text{b. } & -(-3.4q + 5.7r) \\ & = -1(-3.4q + 5.7r) \\ & = -1(-3.4q) + (-1)(5.7r) \\ & = 3.4q - 5.7r \end{aligned}$$

The negative sign preceding the parentheses can be interpreted as a factor of -1 .

Apply the distributive property.

$$\begin{aligned} \text{c. } & -3(a + 2b - 5c) \\ & = -3(a) + (-3)(2b) + (-3)(-5c) \\ & = -3a - 6b + 15c \end{aligned}$$

Apply the distributive property. Simplify.

$$\begin{aligned} \text{d. } & -\frac{2}{3}\left(-9x + \frac{3}{8}y - 5\right) \\ & = -\frac{2}{3}(-9x) + \left(-\frac{2}{3}\right)\left(\frac{3}{8}y\right) + \left(-\frac{2}{3}\right)(-5) \\ & = \frac{18}{3}x - \frac{6}{24}y + \frac{10}{3} \\ & = 6x - \frac{1}{4}y + \frac{10}{3} \end{aligned}$$

Apply the distributive property.

Simplify.

Reduce to lowest terms.

TIP: When applying the distributive property, a negative factor preceding the parentheses will change the signs of the terms within the parentheses.

$$\begin{array}{c} -3(a + 2b - 5c) \\ \downarrow \quad \downarrow \quad \downarrow \\ -3a - 6b + 15c \end{array}$$

Skill Practice Apply the distributive property.

2. $10(30y - 40)$
3. $-(7t - 1.6s + 9.2)$
4. $-2(4x - 3y - 6)$
5. $\frac{1}{2}(4a + 7)$

Notice that the parentheses are removed after the distributive property is applied. Sometimes this is referred to as clearing parentheses.

Two terms can be added or subtracted only if they are *like* terms. To add or subtract *like* terms, we use the distributive property, as shown in Example 3.

Example 3 Using the Distributive Property to Add and Subtract *Like* Terms

Add and subtract as indicated.

- a. $-8x + 3x$
- b. $4.75y^2 - 9.25y^2 + y^2$

Skill Practice Answers

2. $300y - 400$
3. $-7t + 1.6s - 9.2$
4. $-8x + 6y + 12$
5. $2a + \frac{7}{2}$

Solution:

- a.** $-8x + 3x$
 $= x(-8 + 3)$ Apply the distributive property.
 $= x(-5)$ Simplify.
 $= -5x$
- b.** $4.75y^2 - 9.25y^2 + y^2$
 $= 4.75y^2 - 9.25y^2 + 1y^2$ Notice that y^2 is interpreted as $1y^2$.
 $= y^2(4.75 - 9.25 + 1)$ Apply the distributive property.
 $= y^2(-3.5)$ Simplify.
 $= -3.5y^2$

Skill Practice Combine *like* terms.

- 6.** $-4y + 7y$ **7.** $a^2 - 6a^2 + 3a^2$

Although the distributive property is used to add and subtract *like* terms, it is tedious to write each step. Observe that adding or subtracting *like* terms is a matter of combining the coefficients and leaving the variable factors unchanged. This can be shown in one step. This shortcut will be used throughout the text. For example:

$$4w + 7w = 11w \quad 8ab^2 + 10ab^2 - 5ab^2 = 13ab^2$$

3. Simplifying Expressions

Clearing parentheses and combining *like* terms are important tools to simplifying algebraic expressions. This is demonstrated in Example 4.

Example 4 Clearing Parentheses and Combining *Like* Terms

Simplify by clearing parentheses and combining *like* terms.

- a.** $4 - 3(2x - 8) - 1$ **b.** $-(3s - 11t) - 5(2t + 8s) - 10s$
c. $2[1.5x + 4.7(x^2 - 5.2x) - 3x]$ **d.** $-\frac{1}{3}(3w - 6) - \left(\frac{1}{4}w + 4\right)$

Solution:

- a.** $4 - 3(2x - 8) - 1$
 $= 4 - 6x + 24 - 1$ Apply the distributive property.
 $= 4 + 24 - 1 - 6x$ Group *like* terms.
 $= 27 - 6x$ Combine *like* terms.
 $= -6x + 27$

TIP: The expression $27 - 6x$ is equal to $-6x + 27$. However, it is customary to write the variable term first.

Skill Practice Answers

- 6.** $3y$ **7.** $-2a^2$

b. $-(3s - 11t) - 5(2t + 8s) - 10s$
 $= -3s + 11t - 10t - 40s - 10s$ Apply the distributive property.
 $= -3s - 40s - 10s + 11t - 10t$ Group *like* terms.
 $= -53s + t$ Combine *like* terms.

c. $2[1.5x + 4.7(x^2 - 5.2x) - 3x]$
 $= 2[1.5x + 4.7x^2 - 24.44x - 3x]$ Apply the distributive property to inner parentheses.
 $= 2[1.5x - 24.44x - 3x + 4.7x^2]$ Group *like* terms.
 $= 2[-25.94x + 4.7x^2]$ Combine *like* terms.
 $= -51.88x + 9.4x^2$ Apply the distributive property.
 $= 9.4x^2 - 51.88x$

TIP: By using the commutative property of addition, the expression $-51.88x + 9.4x^2$ can also be written as $9.4x^2 + (-51.88x)$ or simply $9.4x^2 - 51.88x$. Although the expressions are all equal, it is customary to write the terms in descending order of the powers of the variable.

d. $-\frac{1}{3}(3w - 6) - \left(\frac{1}{4}w + 4\right)$
 $= -\frac{3}{3}w + \frac{6}{3} - \frac{1}{4}w - 4$ Apply the distributive property.
 $= -w + 2 - \frac{1}{4}w - 4$ Reduce fractions to lowest terms.
 $= -\frac{4}{4}w - \frac{1}{4}w + 2 - 4$ Group *like* terms and find a common denominator.
 $= -\frac{5}{4}w - 2$ Combine *like* terms.

Skill Practice Simplify by clearing parentheses and combining *like* terms.

8. $7 - 2(3x - 4) - 5$

9. $-(6z - 10y) - 4(3z + y) - 8y$

10. $4[1.4a + 2.2(a^2 - 6a)] - 5.1a^2$

11. $-\frac{1}{2}(4p - 1) - \frac{5}{2}(p - 2)$

Skill Practice Answers

8. $-6x + 10$ **9.** $-2y - 18z$

10. $3.7a^2 - 47.2a$

11. $-\frac{9}{2}p + \frac{11}{2}$

Section 1.3

Practice Exercises

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Study Skills Exercises

1. It is very important to attend class every day. Math is cumulative in nature and you must master the material learned in the previous class to understand a new day's lesson. Because this is so important, many instructors tie attendance to the final grade. Write down the attendance policy for your class.
2. Define the key terms.

a. Term	b. Variable term	c. Constant term
d. Factor	e. Coefficient	f. Like terms

Review Exercises

For Exercises 3–4,

- a. Classify the number as a whole number, natural number, rational number, irrational number, or real number. (Choose all that apply.)
- b. Write the reciprocal of the number.
- c. Write the opposite of the number.
- d. Write the absolute value of the number.

3. -4

4. 0

For Exercises 5–8, write the set in interval notation.

5. $\{x \mid x > |-3|\}$

6. $\left\{x \mid x \leq \left|-\frac{4}{3}\right|\right\}$

7. $\left\{w \mid -\frac{5}{2} < w \leq \sqrt{9}\right\}$

8. $\left\{z \mid 2 \leq z < \frac{11}{3}\right\}$

Concept 1: Recognizing Terms, Factors, and Coefficients

For Exercises 9–12:

- a. Determine the number of terms in the expression.
- b. Identify the constant term.
- c. List the coefficients of each term. Separate by commas.

9. $2x^3 - 5xy + 6$

10. $a^2 - 4ab - b^2 + 8$

11. $pq - 7 + q^2 - 4q + p$

12. $7x - 1 + 3xy$

Concept 2: Properties of Real Numbers

For Exercises 13–22, match each expression with the appropriate property.

13. $3 + \frac{1}{2} = \frac{1}{2} + 3$

14. $7.2(4 + 1) = 7.2(4) + 7.2(1)$

15. $(6 + 8) + 2 = 6 + (8 + 2)$

16. $(4 + 19) + 7 = (19 + 4) + 7$

17. $9(4 \cdot 12) = (9 \cdot 4)12$

18. $\left(\frac{1}{4} + 2\right)20 = 5 + 40$

19. $(13 \cdot 41)6 = (41 \cdot 13)6$

20. $6(x + 3) = 6x + 18$

21. $3(y + 10) = 3(10 + y)$

22. $5(3 \cdot 7) = (5 \cdot 3)7$

- a. Commutative property of addition
- b. Associative property of multiplication
- c. Distributive property of multiplication over addition
- d. Commutative property of multiplication
- e. Associative property of addition

Concept 3: Simplifying Expressions

For Exercises 23–60, clear parentheses and combine *like* terms.

23. $8y - 2x + y + 5y$

24. $-9a + a - b + 5a$

25. $4p^2 - 2p + 3p - 6 + 2p^2$

26. $6q - 9 + 3q^2 - q^2 + 10$

27. $2p - 7p^2 - 5p + 6p^2$

28. $5a^2 - 2a - 7a^2 + 6a + 4$

29. $m - 4n^3 + 3 + 5n^3 - 9$

30. $x + 2y^3 - 2x - 8y^3$

31. $5ab + 2ab + 8a$

32. $-6m^2n - 3mn^2 - 2m^2n$

33. $14xy^2 - 5y^2 + 2xy^2$

34. $9uv + 3u^2 + 5uv + 4u^2$

35. $8(x - 3) + 1$

36. $-4(b + 2) - 3$

37. $-2(c + 3) - 2c$

38. $4(z - 4) - 3z$

39. $-(10w - 1) + 9 + w$

40. $-(2y + 7) - 4 + 3y$

41. $-9 - 4(2 - z) + 1$

42. $3 + 3(4 - w) - 11$

43. $4(2s - 7) - (s - 2)$

44. $2(t - 3) - (t - 7)$

45. $-3(-5 + 2w) - 8w + 2(w - 1)$

46. $5 - (-4t - 7) - t - 9$

47. $8x - 4(x - 2) - 2(2x + 1) - 6$

48. $6(y - 2) - 3(2y - 5) - 3$

49. $\frac{1}{2}(4 - 2c) + 5c$

50. $\frac{2}{3}(3d + 6) - 4d$

51. $3.1(2x + 2) - 4(1.2x - 1)$

52. $4.5(5 - y) + 3(1.9y + 1)$

53. $2\left[5\left(\frac{1}{2}a + 3\right) - (a^2 + a) + 4\right]$

54. $-3\left[3\left(b - \frac{2}{3}\right) - 2(b + 4) - 6b^2\right]$

55. $[(2y - 5) - 2(y - y^2)] - 3y$

56. $[-(x + 6) + 3(x^2 + 1)] + 2x$

57. $2.2\{4 - 8[6x - 1.5(x + 4) - 6] + 7.5x\}$

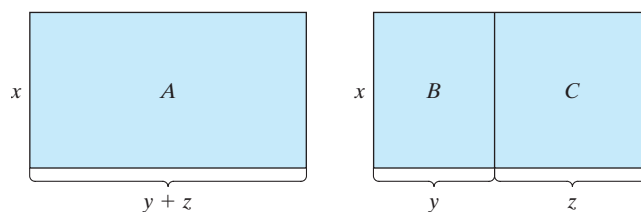
58. $-3.2 - \{6.1y - 4[9 - (2y + 2.5)] + 7y\}$

59. $\frac{1}{8}(24n - 16m) - \frac{2}{3}(3m - 18n - 2) + \frac{2}{3}$

60. $\frac{1}{5}(25a - 20b) - \frac{4}{7}(21a - 14b + 2) + \frac{1}{7}$

Expanding Your Skills

61. What is the identity element for addition? Use it in an example.
62. What is the identity element for multiplication? Use it in an example.
63. What is another name for a multiplicative inverse?
64. What is another name for an additive inverse?
65. Is the operation of subtraction commutative? If not, give an example.
66. Is the operation of division commutative? If not, give an example.
67. Given the rectangular regions:



- a. Write an expression for the area of region *A*. (Do not simplify.)
- b. Write an expression for the area of region *B*.
- c. Write an expression for the area of region *C*.
- d. Add the expressions for the area of regions *B* and *C*.
- e. Show that the area of region *A* is equal to the sum of the areas of regions *B* and *C*. What property of real numbers does this illustrate?

Linear Equations in One Variable**Section 1.4****1. Definition of a Linear Equation in One Variable**

An **equation** is a statement that indicates that two quantities are equal. The following are equations.

$$x = -4 \quad p + 3 = 11 \quad -2z = -20$$

All equations have an equal sign. Furthermore, notice that the equal sign separates the equation into two parts, the left-hand side and the right-hand side. A **solution to an equation** is a value of the variable that makes the equation a true statement. Substituting a solution to an equation for the variable makes the right-hand side equal to the left-hand side.

Concepts

1. Definition of a Linear Equation in One Variable
2. Solving Linear Equations
3. Clearing Fractions and Decimals
4. Conditional Equations, Contradictions, and Identities

Equation	Solution	Check	
$x = -4$	-4	$x = -4$ \downarrow $-4 = -4 \checkmark$	Substitute -4 for x . Right-hand side equals left-hand side.
$p + 3 = 11$	8	$p + 3 = 11$ \downarrow $8 + 3 = 11 \checkmark$	Substitute 8 for p . Right-hand side equals left-hand side.
$-2z = -20$	10	$-2z = -20$ \downarrow $-2(10) = -20 \checkmark$	Substitute 10 for z . Right-hand side equals left-hand side.

Throughout this text we will learn to recognize and solve several different types of equations, but in this chapter we will focus on the specific type of equation called a linear equation in one variable.

Definition of a Linear Equation in One Variable

Let a and b be real numbers such that $a \neq 0$. A **linear equation in one variable** is an equation that can be written in the form

$$ax + b = 0$$

Notice that a linear equation in one variable has only one variable. Furthermore, because the variable has an implied exponent of 1, a linear equation is sometimes called a first-degree equation.

Linear equation in one variable

$$4x - 3 = 0$$

$$\frac{4}{5}p + \frac{3}{10} = 0$$

Not a linear equation in one variable

$$4x^2 + 8 = 0 \quad (\text{exponent for } x \text{ is not } 1)$$

$$\frac{4}{5}p + \frac{3}{10}q = 0 \quad (\text{more than one variable})$$

2. Solving Linear Equations

To solve a linear equation, the goal is to simplify the equation to isolate the variable. Each step used in simplifying an equation results in an equivalent equation. *Equivalent equations* have the same solution set. For example, the equations $2x + 3 = 7$ and $2x = 4$ are equivalent because $x = 2$ is the solution to both equations.

To solve an equation, we may use the addition, subtraction, multiplication, and division properties of equality. These properties state that adding, subtracting, multiplying, or dividing the same quantity on each side of an equation results in an equivalent equation.

Addition and Subtraction Properties of Equality

Let a , b , and c represent real numbers.

Addition property of equality: If $a = b$, then $a + c = b + c$.

Subtraction property of equality: If $a = b$, then $a - c = b - c$.

Multiplication and Division Properties of EqualityLet a , b , and c represent real numbers.**Multiplication property of equality:** If $a = b$, then $a \cdot c = b \cdot c$.**Division property of equality:** If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ (provided $c \neq 0$).**Example 1** Solving Linear Equations

Solve each equation.

a. $12 + x = 40$ b. $-\frac{1}{5}p = 2$ c. $4 = \frac{w}{2.2}$ d. $-x = 6$

Solution:

a. $12 + x = 40$

$$12 - 12 + x = 40 - 12$$
 To isolate x , subtract 12 from both sides.

$$x = 28$$
 Simplify.

Check: $12 + x = 40$ Check the solution in the original equation.

$$12 + (28) \stackrel{?}{=} 40$$

$$40 = 40 \checkmark$$
 True statement

b. $-\frac{1}{5}p = 2$

$$-5\left(-\frac{1}{5}p\right) = -5(2)$$
 To isolate p , multiply both sides by -5 .

$$p = -10$$
 Simplify.

Check: $-\frac{1}{5}p = 2$ Check the solution in the original equation.

$$-\frac{1}{5}(-10) \stackrel{?}{=} 2$$

$$2 = 2 \checkmark$$
 True statement

c. $4 = \frac{w}{2.2}$

$$2.2(4) = \left(\frac{w}{2.2}\right) \cdot 2.2$$
 To isolate w , multiply both sides by 2.2.

$$8.8 = w$$
 Simplify.

Check: $4 = \frac{w}{2.2}$ Check the solution in the original equation.

$$4 \stackrel{?}{=} \frac{8.8}{2.2}$$

$$4 = 4 \checkmark$$
 True statement

d. $-x = 6$

$$-1(-x) = -1(6)$$
 To isolate x , multiply both sides by -1 .

$$x = -6$$
 Simplify.

Check: $-x = 6$ Check the solution in the original equation.
 $-(-6) \stackrel{?}{=} 6$
 $6 = 6 \checkmark$ True statement

Skill Practice Solve the equations.

1. $x - 5 = -11$ 2. $-\frac{6}{5}y = -\frac{3}{5}$ 3. $5 = \frac{t}{16}$ 4. $-a = -2$

For more complicated linear equations, several steps are required to isolate the variable.

Steps to Solve a Linear Equation in One Variable

- Simplify both sides of the equation.
 - Clear parentheses.
 - Consider clearing fractions or decimals (if any are present) by multiplying both sides of the equation by a common denominator of all terms.
 - Combine *like* terms.
- Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.
- Use the addition or subtraction property of equality to collect the constant terms on the other side of the equation.
- Use the multiplication or division property of equality to make the coefficient of the variable term equal to 1.
- Check your answer.

Example 2 Solving Linear Equations

Solve the linear equations and check the answers.

a. $11z + 2 = 5(z - 2)$ b. $-3(x - 4) + 2 = 7 - (x + 1)$
 c. $-4[y - 3(y - 5)] = 2(6 - 5y)$

Solution:

a. $11z + 2 = 5(z - 2)$

$$11z + 2 = 5z - 10$$

$$11z - 5z + 2 = 5z - 5z - 10$$

$$6z + 2 = -10$$

$$6z + 2 - 2 = -10 - 2$$

$$6z = -12$$

$$\frac{6z}{6} = \frac{-12}{6}$$

$$z = -2$$

Clear parentheses.

Subtract $5z$ from both sides.

Combine *like* terms.

Subtract 2 from both sides.

To isolate z , divide both sides of the equation by 6.

Simplify.

Skill Practice Answers

1. $x = -6$ 2. $y = \frac{1}{2}$
 3. $t = 80$ 4. $a = 2$

Check: $11z + 2 = 5(z - 2)$
 $11(-2) + 2 \stackrel{?}{=} 5(-2 - 2)$
 $-22 + 2 \stackrel{?}{=} 5(-4)$
 $-20 = -20 \checkmark$

Check the solution in the original equation.

True statement

b. $-3(x - 4) + 2 = 7 - (x + 1)$
 $-3x + 12 + 2 = 7 - x - 1$
 $-3x + 14 = -x + 6$
 $-3x + x + 14 = -x + x + 6$

Clear parentheses.

Combine *like* terms.

Add x to both sides of the equation.

Combine *like* terms.

Subtract 14 from both sides.

$$-2x + 14 = 6$$

$$-2x + 14 - 14 = 6 - 14$$

$$-2x = -8$$

$$\frac{-2x}{-2} = \frac{-8}{-2}$$

To isolate x , divide both sides by -2 .

Simplify.

$$x = 4$$

Check the solution in the original equation.

Check: $-3(x - 4) + 2 = 7 - (x + 1)$
 $-3(4 - 4) + 2 \stackrel{?}{=} 7 - (4 + 1)$
 $-3(0) + 2 \stackrel{?}{=} 7 - (5)$
 $0 + 2 \stackrel{?}{=} 2$
 $2 = 2 \checkmark$

True statement

c. $-4[y - 3(y - 5)] = 2(6 - 5y)$
 $-4[y - 3y + 15] = 12 - 10y$
 $-4[-2y + 15] = 12 - 10y$
 $8y - 60 = 12 - 10y$
 $8y + 10y - 60 = 12 - 10y + 10y$

Clear parentheses.

Combine *like* terms.

Clear parentheses.

Add $10y$ to both sides of the equation.

Combine *like* terms.

Add 60 to both sides of the equation.

$$18y - 60 = 12$$

$$18y - 60 + 60 = 12 + 60$$

$$18y = 72$$

$$\frac{18y}{18} = \frac{72}{18}$$

$$y = 4$$

To isolate y , divide both sides by 18.

Simplify.

Check: $-4[y - 3(y - 5)] = 2(6 - 5y)$
 $-4[4 - 3(4 - 5)] \stackrel{?}{=} 2(6 - 5(4))$
 $-4[4 - 3(-1)] \stackrel{?}{=} 2(6 - 20)$
 $-4[4 + 3] \stackrel{?}{=} 2(-14)$
 $-4(7) \stackrel{?}{=} -28$
 $-28 = -28 \checkmark$

True statement

Skill Practice Solve the equations.

5. $7 + 2(y - 3) = 6y + 3$

6. $4(2t + 2) - 6(t - 1) = 6 - t$

7. $3[p + 2(p - 2)] = 4(p - 3)$

3. Clearing Fractions and Decimals

When an equation contains fractions or decimals, it is sometimes helpful to clear the fractions and decimals. This is accomplished by multiplying both sides of the equation by the least common denominator (LCD) of all terms within the equation. This is demonstrated in Example 3.

Example 3 Solving Linear Equations by Clearing Fractions

Solve the equation.

$$\frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}(w - 4)$$

Solution:

$$\frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}(w - 4)$$

$$\frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}w - 2$$

Apply the distributive property to clear parentheses.

$$12 \cdot \left(\frac{1}{4}w + \frac{1}{3}w - 1 \right) = 12 \cdot \left(\frac{1}{2}w - 2 \right)$$

Multiply both sides of the equation by the LCD of all terms. In this case, the LCD is 12.

$$12 \cdot \frac{1}{4}w + 12 \cdot \frac{1}{3}w + 12 \cdot (-1) = 12 \cdot \frac{1}{2}w + 12 \cdot (-2)$$

Apply the distributive property.

$$3w + 4w - 12 = 6w - 24$$

$$7w - 12 = 6w - 24$$

$$7w - 6w - 12 = 6w - 6w - 24$$

Subtract $6w$

$$w - 12 = -24$$

$$w - 12 + 12 = -24 + 12$$

Add 12 to both sides.

$$w = -12$$

Check: $\frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}(w - 4)$

$$\frac{1}{4}(-12) + \frac{1}{3}(-12) - 1 \stackrel{?}{=} \frac{1}{2}(-12 - 4)$$

$$-3 - 4 - 1 \stackrel{?}{=} \frac{1}{2}(-16)$$

$$-8 = -8 \checkmark$$

True statement

Skill Practice Answers

5. $y = -\frac{1}{2}$ 6. $t = -\frac{8}{3}$

7. $p = 0$

TIP: The fractions in this equation can be eliminated by multiplying both sides of the equation by *any* common multiple of the denominators. For example, multiplying both sides of the equation by 24 produces the same solution.

$$\begin{aligned} 24 \cdot \left(\frac{1}{4}w + \frac{1}{3}w - 1 \right) &= 24 \cdot \frac{1}{2}(w - 4) \\ 6w + 8w - 24 &= 12(w - 4) \\ 14w - 24 &= 12w - 48 \\ 2w &= -24 \\ w &= -12 \end{aligned}$$

Skill Practice Solve the equation by first clearing the fractions.

8. $\frac{3}{4}a + \frac{1}{2} = \frac{2}{3}a + \frac{1}{3}$

Example 4 Solving a Linear Equation with Fractions

Solve. $\frac{x-2}{5} - \frac{x-4}{2} = 2 + \frac{x+4}{10}$

Solution:

$$\frac{x-2}{5} - \frac{x-4}{2} = 2 + \frac{x+4}{10}$$

The LCD of all terms in the equation is 10.

$$10 \left(\frac{x-2}{5} - \frac{x-4}{2} \right) = 10 \left(2 + \frac{x+4}{10} \right)$$

Multiply both sides by 10.

$$\frac{\cancel{10}^2}{\cancel{1}} \cdot \left(\frac{x-2}{\cancel{5}} \right) - \frac{\cancel{10}^5}{\cancel{1}} \cdot \left(\frac{x-4}{\cancel{2}} \right) = \frac{\cancel{10}}{\cancel{1}} \cdot \left(\frac{2}{1} \right) + \frac{\cancel{10}^1}{\cancel{1}} \cdot \left(\frac{x+4}{\cancel{10}} \right)$$

Apply the distributive property.

$$2(x-2) - 5(x-4) = 20 + 1(x+4)$$

Clear fractions.

$$2x - 4 - 5x + 20 = 20 + x + 4$$

Apply the distributive property.

$$-3x + 16 = x + 24$$

Simplify both sides of the equation.

$$-3x - x + 16 = x - x + 24$$

Subtract x from both sides.

$$-4x + 16 = 24$$

$$-4x + 16 - 16 = 24 - 16$$

Subtract 16 from both sides.

$$-4x = 8$$

Skill Practice Answers

8. $a = -2$

$$\frac{-4x}{-4} = \frac{8}{-4}$$

$$x = -2$$

Divide both sides
by -4 .

The check is left to the
reader.

Skill Practice Solve.

$$9. \frac{1}{8} - \frac{x+3}{4} = \frac{3x-2}{2}$$

The same procedure used to clear fractions in an equation can be used to clear decimals.

Example 5 Solving Linear Equations by Clearing Decimals

Solve the equation. $0.55x - 0.6 = 2.05x$

Solution:

Recall that any terminating decimal can be written as a fraction. Therefore, the equation $0.55x - 0.6 = 2.05x$ is equivalent to

$$\frac{55}{100}x - \frac{6}{10} = \frac{205}{100}x$$

A convenient common denominator for all terms in this equation is 100. Multiplying both sides of the equation by 100 will have the effect of “moving” the decimal point 2 places to the right.

$$100(0.55x - 0.6) = 100(2.05x)$$

Multiply both sides by 100 to clear
decimals.

$$55x - 60 = 205x$$

$$55x - 55x - 60 = 205x - 55x$$

Subtract $55x$ from both sides.

$$-60 = 150x$$

$$\frac{-60}{150} = \frac{150x}{150}$$

To isolate x , divide both sides by 150.

$$-\frac{60}{150} = x$$

$$x = -\frac{2}{5} = -0.4$$

Check: $0.55x - 0.6 = 2.05x$

$$0.55(-0.4) - 0.6 \stackrel{?}{=} 2.05(-0.4)$$

$$-0.22 - 0.6 \stackrel{?}{=} -0.82$$

$$-0.82 = -0.82 \checkmark \quad \text{True statement}$$

Skill Practice Solve the equation by first clearing the decimals.

$$10. 2.2x + 0.5 = 1.6x + 0.2$$

Skill Practice Answers

$$9. x = \frac{3}{14} \quad 10. x = -0.5$$

4. Conditional Equations, Contradictions, and Identities

The solution to a linear equation is the value of x that makes the equation a true statement. A linear equation has one unique solution. Some equations, however, have no solution, while others have infinitely many solutions.

I. Conditional Equations

An equation that is true for some values of the variable but false for other values is called a **conditional equation**. The equation $x + 4 = 6$ is a conditional equation because it is true on the *condition* that $x = 2$. For other values of x , the statement $x + 4 = 6$ is false.

II. Contradictions

Some equations have no solution, such as $x + 1 = x + 2$. There is no value of x that when increased by 1 will equal the same value increased by 2. If we tried to solve the equation by subtracting x from both sides, we get the contradiction $1 = 2$. This indicates that the equation has no solution. An equation that has no solution is called a **contradiction**.

$$\begin{aligned}x + 1 &= x + 2 \\x - x + 1 &= x - x + 2 \\1 &= 2 \quad (\text{contradiction}) \quad \text{No solution}\end{aligned}$$

III. Identities

An equation that has all real numbers as its solution set is called an **identity**. For example, consider the equation $x + 4 = x + 4$. Because the left- and right-hand sides are *identical*, any real number substituted for x will result in equal quantities on both sides. If we solve the equation, we get the identity $4 = 4$. In such a case, the solution is the set of all real numbers.

$$\begin{aligned}x + 4 &= x + 4 \\x - x + 4 &= x - x + 4 \\4 &= 4 \quad (\text{identity}) \quad \text{The solution is all real numbers.}\end{aligned}$$

Example 6 Identifying Conditional Equations, Contradictions, and Identities

Solve the equations. Identify each equation as a conditional equation, a contradiction, or an identity.

- a. $3[x - (x + 1)] = -2$ b. $5(3 + c) + 2 = 2c + 3c + 17$
c. $4x - 3 = 17$

Solution:

- a. $3[x - (x + 1)] = -2$
 $3[x - x - 1] = -2$ Clear parentheses.
 $3[-1] = -2$ Combine *like* terms.
 $-3 = -2$ Contradiction

This equation is a contradiction. There is no solution.

b. $5(3 + c) + 2 = 2c + 3c + 17$

$$15 + 5c + 2 = 5c + 17$$

Clear parentheses and combine *like* terms.

$$5c + 17 = 5c + 17$$

Identity

$$0 = 0$$

This equation is an identity. The solution is the set of all real numbers.

c. $4x - 3 = 17$

$$4x - 3 + 3 = 17 + 3 \quad \text{Add 3 to both sides.}$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

To isolate x , divide both sides by 4.

$$x = 5$$

This equation is a conditional equation. The solution is $x = 5$.

Skill Practice Answers

11. The equation is a contradiction. There is no solution.
 12. The equation is an identity. The solution is the set of all real numbers.
 13. The equation is conditional. The solution is $x = 1$.

Skill Practice

Solve the equations. Identify each equation as a conditional equation, an identity, or a contradiction.

11. $2(-5x - 1) = 2x - 12x + 6$

12. $2(3x - 1) = 6(x + 1) - 8$

13. $4x + 1 - x = 6x - 2$

Section 1.4

Practice Exercises

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Study Skills Exercises

1. Some instructors allow the use of calculators. Does your instructor allow the use of a calculator? If so, what kind?

Will you be allowed to use a calculator on tests or just for occasional calculator problems in the text?

Helpful Hint: If you are not permitted to use a calculator on tests, you should do your homework in the same way, without the calculator.

2. Define the key terms.

a. Equation

b. Solution to an equation

c. Linear equation in one variable

d. Conditional equation

e. Contradiction

f. Identity

Review Exercises

For Exercises 3–6, clear parentheses and combine *like* terms.

3. $8x - 3y + 2xy - 5x + 12xy$

4. $5ab + 5a - 13 - 2a + 17$

5. $2(3z - 4) - (z + 12)$

6. $-(6w - 5) + 3(4w - 5)$

Concept 1: Definition of a Linear Equation in One Variable

For Exercises 7–12, label the equation as linear or nonlinear.

7. $2x + 1 = 5$

8. $10 = x + 6$

9. $x^2 + 7 = 9$

10. $3 + x^3 - x = 4$

11. $-3 = x$

12. $5.2 - 7x = 0$

13. Use substitution to determine which value is the solution to $2x - 1 = 5$.

- a. 2 b. 3 c. 0 d. -1

14. Use substitution to determine which value is the solution to $2y - 3 = -2$.

- a. 1 b. $\frac{1}{2}$ c. 0 d. $-\frac{1}{2}$

Concept 2: Solving Linear Equations

For Exercises 15–44, solve the equations and check your solutions.

15. $x + 7 = 19$

16. $-3 + y = -28$

17. $64x = -2$

18. $\frac{t}{8} = -\frac{3}{4}$

19. $-\frac{7}{8} = -\frac{5}{6}z$

20. $-\frac{12}{13} = 4b$

21. $a + \frac{2}{5} = 2$

22. $-\frac{3}{8} + x = -\frac{7}{24}$

23. $2.53 = -2.3t$

24. $-4.8 = 6.1 + y$

25. $p - 2.9 = 3.8$

26. $-4.2a = 4.494$

27. $6q - 4 = 62$

28. $2w - 15 = 15$

29. $4y - 17 = 35$

30. $6z - 25 = 83$

31. $-b - 5 = 2$

32. $6 = -y + 1$

33. $3(x - 6) = 2x - 5$

34. $13y + 4 = 5(y - 4)$

35. $6 - (t + 2) = 5(3t - 4)$

36. $1 - 5(p + 2) = 2(p + 13)$

37. $6(a + 3) - 10 = -2(a - 4)$

38. $8(b - 2) + 3b = -9(b - 1)$

39. $-2[5 - (2z + 1)] - 4 = 2(3 - z)$

40. $3[w - (10 - w)] = 7(w + 1)$

41. $6(-y + 4) - 3(2y - 3) = -y + 5 + 5y$

42. $13 + 4w = -5(-w - 6) + 2(w + 1)$

43. $14 - 2x + 5x = -4(-2x - 5) - 6$

44. $8 - (p + 2) + 6p + 7 = p + 13$

Concept 3: Clearing Fractions and Decimals

For Exercises 45–56, solve the equations.

45. $\frac{2}{3}x - \frac{1}{6} = -\frac{5}{12}x + \frac{3}{2} - \frac{1}{6}x$

46. $-\frac{1}{2}y + 4 = -\frac{9}{10}y + \frac{2}{5}$

47. $\frac{1}{5}(p - 5) = \frac{3}{5}p + \frac{1}{10}p + 1$

48. $\frac{5}{6}(q + 2) = -\frac{7}{9}q - \frac{1}{3} + 2$

49. $\frac{3x - 7}{2} + \frac{3 - 5x}{3} = \frac{3 - 6x}{5}$

50. $\frac{2y - 4}{5} = \frac{5y + 13}{4} + \frac{y}{2}$

51. $\frac{4}{3}(2q + 6) - \frac{5q - 6}{6} - \frac{q}{3} = 0$

52. $\frac{-3a + 9}{15} - \frac{2a - 5}{5} - \frac{a + 2}{10} = 0$

53. $6.3w - 1.5 = 4.8$

54. $0.2x + 53.6 = x$

55. $0.75(m - 2) + 0.25m = 0.5$

56. $0.4(n + 10) + 0.6n = 2$

Concept 4: Conditional Equations, Contradictions, and Identities

57. What is a conditional equation?
58. Explain the difference between a contradiction and an identity.

For Exercises 59–64, solve the following equations. Then label each as a conditional equation, a contradiction, or an identity.

59. $4x + 1 = 2(2x + 1) - 1$ 60. $3x + 6 = 3x$ 61. $-11x + 4(x - 3) = -2x - 12$
62. $5(x + 2) - 7 = 3$ 63. $2x - 4 + 8x = 7x - 8 + 3x$ 64. $-7x + 8 + 4x = -3(x - 3) - 1$

Mixed Exercises

For Exercises 65–96, solve the equations.

65. $-5b + 9 = -71$ 66. $-3x + 18 = -66$ 67. $16 = -10 + 13x$
68. $15 = -12 + 9x$ 69. $10c + 3 = -3 + 12c$ 70. $2w + 21 = 6w - 7$
71. $12b - 15b - 8 + 6 = 4b + 6 - 1$ 72. $4z + 2 - 3z + 5 = 3 + z + 4$ 73. $5(x - 2) - 2x = 3x + 7$
74. $2x + 3(x - 5) = 15$ 75. $\frac{c}{2} - \frac{c}{4} + \frac{3c}{8} = 1$ 76. $\frac{d}{5} - \frac{d}{10} + \frac{5d}{20} = \frac{7}{10}$
77. $0.75(8x - 4) = \frac{2}{3}(6x - 9)$ 78. $-\frac{1}{2}(4z - 3) = -z$ 79. $7(p + 2) - 4p = 3p + 14$
80. $6(z - 2) = 3z - 8 + 3z$ 81. $4[3 + 5(3 - b) + 2b] = 6 - 2b$ 82. $\frac{1}{3}(x + 3) - \frac{1}{6} = \frac{1}{6}(2x + 5)$
83. $3 - \frac{3}{4}x = 9$ 84. $\frac{9}{10} - 4w = \frac{5}{2}$ 85. $\frac{5}{4} + \frac{y - 3}{8} = \frac{2y + 1}{2}$
86. $\frac{2}{3} - \frac{x + 2}{6} = \frac{5x - 2}{2}$ 87. $\frac{2y - 9}{10} + \frac{3}{2} = y$ 88. $\frac{2}{3}x - \frac{5}{6}x - 3 = \frac{1}{2}x - 5$
89. $0.48x - 0.08x = 0.12(260 - x)$ 90. $0.07w + 0.06(140 - w) = 90$ 91. $0.5x + 0.25 = \frac{1}{3}x + \frac{5}{4}$
92. $0.2b + \frac{1}{3} = \frac{7}{15}$ 93. $0.3b - 1.5 = 0.25(b + 2)$ 94. $0.7(a - 1) = 0.25 + 0.7a$
95. $-\frac{7}{8}y + \frac{1}{4} = \frac{1}{2}\left(5 - \frac{3}{4}y\right)$ 96. $5x - (8 - x) = 2[-4 - (3 + 5x) - 13]$

Expanding Your Skills

97. a. Simplify the expression. $-2(y - 1) + 3(y + 2)$ 98. a. Simplify the expression. $4w - 8(2 + w)$
 b. Solve the equation. $-2(y - 1) + 3(y + 2) = 0$ b. Solve the equation. $4w - 8(2 + w) = 0$
 c. Explain the difference between simplifying an expression and solving an equation.

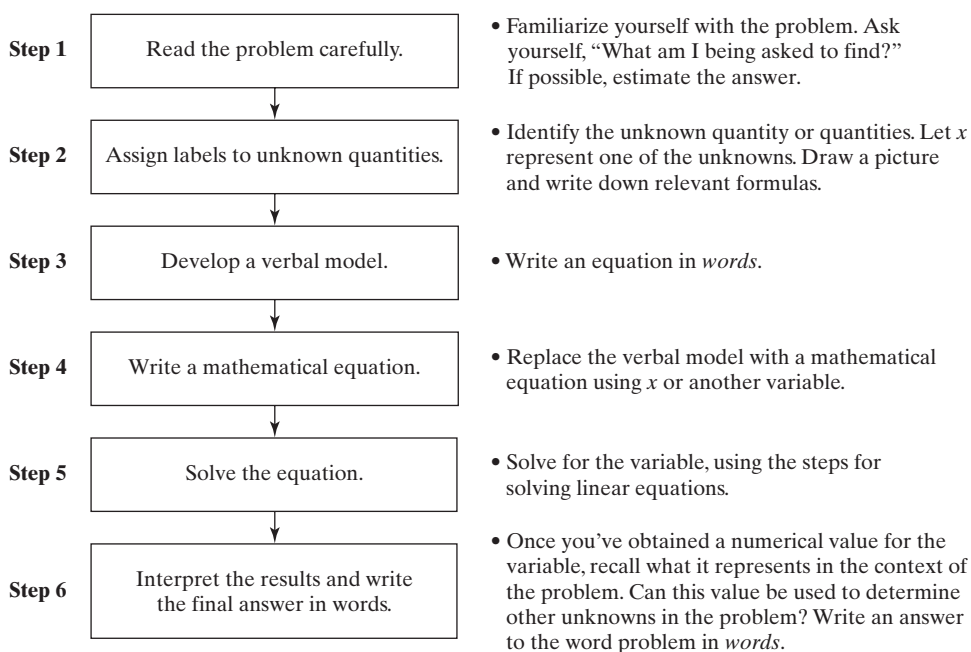
Applications of Linear Equations in One Variable

Section 1.5

1. Introduction to Problem Solving

One of the important uses of algebra is to develop mathematical models for understanding real-world phenomena. To solve an application problem, relevant information must be extracted from the wording of a problem and then translated into mathematical symbols. This is a skill that requires practice. The key is to stick with it and not to get discouraged.

Problem-Solving Flowchart for Word Problems



Concepts

1. Introduction to Problem Solving
2. Applications Involving Consecutive Integers
3. Applications Involving Percents and Rates
4. Applications Involving Principal and Interest
5. Applications Involving Mixtures
6. Applications Involving Distance, Rate, and Time

Example 1 Translating and Solving a Linear Equation

The sum of two numbers is 39. One number is 3 less than twice the other. What are the numbers?

Solution:

Step 1: Read the problem carefully.

Step 2: Let x represent one number.

Let $2x - 3$ represent the other number.

Step 3: (One number) + (other number) = 39

Step 4: Replace the verbal model with a mathematical equation.

(One number) + (other number) = 39

$$x + (2x - 3) = 39$$

Step 5: Solve for x .

$$x + (2x - 3) = 39$$

$$3x - 3 = 39$$

$$3x = 42$$

$$\frac{3x}{3} = \frac{42}{3}$$

$$x = 14$$

Step 6: Interpret your results. Refer back to step 2.

One number is x : \longrightarrow 14

The other number is $2x - 3$:

$$2(14) - 3 \longrightarrow 25$$

Answer: The numbers are 14 and 25.

Skill Practice

1. One number is 5 more than 3 times another number. The sum of the numbers is 45. Find the numbers.

2. Applications Involving Consecutive Integers

The word *consecutive* means “following one after the other in order.” The numbers $-2, -1, 0, 1, 2$ are examples of consecutive integers. Notice that any two consecutive integers differ by 1. If x represents an integer, then $x + 1$ represents the next consecutive integer.

The numbers 2, 4, 6, 8 are consecutive even integers. The numbers 15, 17, 19, 21 are consecutive odd integers. Both consecutive odd and consecutive even integers differ by 2. If x represents an even integer, then $x + 2$ represents the next consecutive even integer. If x represents an odd integer, then $x + 2$ represents the next consecutive odd integer.

Example 2 Solving a Linear Equation Involving Consecutive Integers

The sum of two consecutive odd integers is 172. Find the integers.

Solution:

Step 1: Read the problem carefully.

Step 2: Label unknowns:

Let x represent the first odd integer.

Let $x + 2$ represent the next odd integer.

Skill Practice Answers

1. The numbers are 10 and 35.

Step 3: Write an equation in words:

$$(\text{First integer}) + (\text{second integer}) = 172$$

Step 4: Write a mathematical equation based on the verbal model.

$$(\text{First integer}) + (\text{second integer}) = 172$$

$$x + (x + 2) = 172$$

Step 5: Solve for x .

$$x + (x + 2) = 172$$

$$2x + 2 = 172$$

$$2x = 170$$

$$x = 85$$

Step 6: Interpret your results.

One integer is x : \longrightarrow 85

The other integer is $x + 2$:

85 + 2 \longrightarrow 87

Answer: The numbers are 85 and 87.

Skill Practice

The sum of three consecutive integers is 66.

- If the first integer is represented by x , write expressions for the next two integers.
- Write a mathematical equation that describes the verbal model.
- Solve the equation and find the three integers.

TIP: After completing a word problem, it is always a good idea to check that the answer is reasonable. Notice that 85 and 87 are consecutive odd integers, and the sum is equal to 172 as desired.

3. Applications Involving Percents and Rates

In many real-world applications, percents are used to represent rates.

- In 2006, the sales tax rate for one county in Tennessee was 6%.
- An ice cream machine is discounted 20%.
- A real estate sales broker receives a $4\frac{1}{2}\%$ commission on sales.
- A savings account earns 7% simple interest.

The following models are used to compute sales tax, commission, and simple interest. In each case the value is found by multiplying the base by the percentage.

$$\text{Sales tax} = (\text{cost of merchandise})(\text{tax rate})$$

$$\text{Commission} = (\text{dollars in sales})(\text{commission rate})$$

$$\text{Simple interest} = (\text{principal})(\text{annual interest rate})(\text{time in years})$$

$$\longleftarrow I = Prt$$

Skill Practice Answers

- $x + 1$ and $x + 2$
- $x + (x + 1) + (x + 2) = 66$
- The integers are 21, 22, and 23.

Example 3 Solving a Percent Application

A realtor made a 6% commission on a house that sold for \$172,000. How much was her commission?

Solution:

Let x represent the commission.

Label the variables.

(Commission) = (dollars in sales)(commission rate)

Verbal model

$$x = (\$172,000)(0.06)$$

Mathematical model

$$x = \$10,320$$

Solve for x .

The realtor's commission is \$10,320.

Interpret the results.

Skill Practice

3. The sales tax rate in Atlanta, Georgia, is 7%. Find the amount of sales tax paid on an automobile priced at \$12,000.

Example 4 Solving a Percent Application

A woman invests \$5000 in an account that earns $5\frac{1}{4}\%$ simple interest. If the money is invested for 3 years, how much money is in the account at the end of the 3-year period?

Solution:

Let x represent the total money in the account.

Label variables.

$$P = \$5000 \quad (\text{principal amount invested})$$

$$r = 0.0525 \quad (\text{interest rate})$$

$$t = 3 \quad (\text{time in years})$$

The total amount of money includes principal plus interest.

(Total money) = (principal) + (interest)

Verbal model

$$x = P + Prt$$

Mathematical model

$$x = \$5000 + (\$5000)(0.0525)(3)$$

Substitute for P , r , and t .

$$x = \$5000 + \$787.50$$

$$x = \$5787.50$$

Solve for x .

The total amount of money in the account is \$5787.50.

Interpret the results.

Skill Practice

4. A man earned \$340 in 1 year on an investment that paid a 4% dividend. Find the amount of money invested.

As consumers, we often encounter situations in which merchandise has been marked up or marked down from its original cost. It is important to note that percent increase and percent decrease are based on the original cost. For example, suppose a microwave originally priced at \$305 is marked down 20%.

Skill Practice Answers

3. \$840 4. \$8500

The discount is determined by 20% of the original price: $(0.20)(\$305) = \61.00 .
The new price is $\$305.00 - \$61.00 = \$244.00$.

Example 5 Solving a Percent Increase Application

A college bookstore uses a standard markup of 22% on all books purchased wholesale from the publisher. If the bookstore sells a calculus book for \$103.70, what was the original wholesale cost?

Solution:

Let x = original wholesale cost. Label the variables.

The selling price of the book is based on the original cost of the book plus the bookstore's markup.

(Selling price) = (original price) + (markup) Verbal model

(Selling price) = (original price) + (original price \cdot markup rate)

$$103.70 = x + (x)(0.22) \quad \text{Mathematical model}$$

$$103.70 = x + 0.22x$$

$$103.70 = 1.22x \quad \text{Combine like terms.}$$

$$\frac{103.70}{1.22} = x$$

$$x = \$85.00 \quad \text{Simplify.}$$

The original wholesale cost of the textbook was \$85.00. Interpret the results.

Skill Practice

5. An online bookstore gives a 20% discount on paperback books. Find the original price of a book that has a selling price of \$5.28 after the discount.

4. Applications Involving Principal and Interest

Example 6 Solving an Investment Growth Application

Miguel had \$10,000 to invest in two different mutual funds. One was a relatively safe bond fund that averaged 8% return on his investment at the end of 1 year. The other fund was a riskier stock fund that averaged 17% return in 1 year. If at the end of the year Miguel's portfolio grew to \$11,475 (\$1475 above his \$10,000 investment), how much money did Miguel invest in each fund?

Solution:

This type of word problem is sometimes categorized as a mixture problem. Miguel is "mixing" his money between two different investments. We have to determine how the money was divided to earn \$1475.

Skill Practice Answers

5. \$6.60

The information in this problem can be organized in a chart. (*Note:* There are two sources of money: the amount invested and the amount earned.)

	8% Bond Fund	17% Stock Fund	Total
Amount invested (\$)	x	$(10,000 - x)$	10,000
Amount earned (\$)	$0.08x$	$0.17(10,000 - x)$	1475

Because the amount of principal is unknown for both accounts, we can let x represent the amount invested in the bond fund. If Miguel spends x dollars in the bond fund, then he has $(10,000 - x)$ left over to spend in the stock fund. The return for each fund is found by multiplying the principal and the percent growth rate.

To establish a mathematical model, we know that the total return (\$1475) must equal the growth from the bond fund plus the growth from the stock fund:

(Growth from bond fund) + (growth from stock fund) = (total growth)

$$0.08x + 0.17(10,000 - x) = 1475$$

$$0.08x + 0.17(10,000 - x) = 1475 \quad \text{Mathematical model}$$

$$8x + 17(10,000 - x) = 147,500 \quad \text{Multiply by 100 to clear decimals.}$$

$$8x + 170,000 - 17x = 147,500$$

$$-9x + 170,000 = 147,500 \quad \text{Combine like terms.}$$

$$-9x = -22,500 \quad \text{Subtract 170,000 from both sides.}$$

$$\frac{-9x}{-9} = \frac{-22,500}{-9}$$

$$x = 2500 \quad \text{Solve for } x \text{ and interpret the results.}$$

The amount invested in the bond fund is \$2500. The amount invested in the stock fund is $\$10,000 - x$, or \$7500.

Skill Practice

6. Winston borrowed \$4000 in two loans. One loan charged 7% interest, and the other charged 1.5% interest. After 1 year, Winston paid \$225 in interest. Find the amount borrowed in each loan account.

5. Applications Involving Mixtures

Example 7 Solving a Mixture Application

How many liters (L) of a 60% antifreeze solution must be added to 8 L of a 10% antifreeze solution to produce a 20% antifreeze solution?

Skill Practice Answers

6. \$3000 was borrowed at 7% interest, and \$1000 was borrowed at 1.5% interest.

Solution:

The given information is illustrated in Figure 1-7.

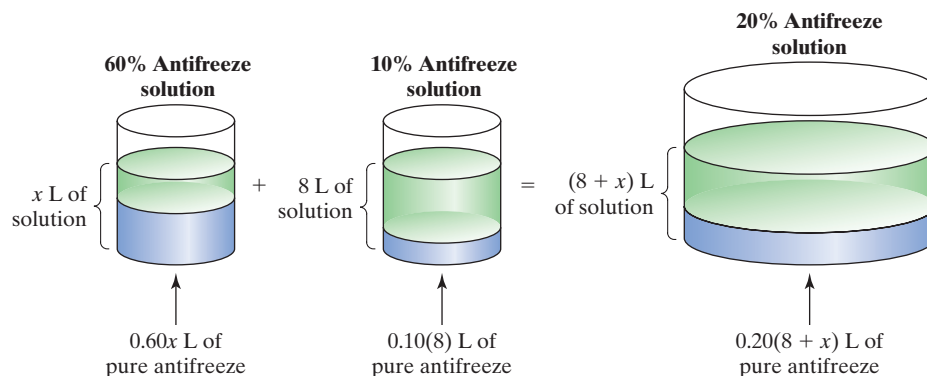


Figure 1-7

The information can be organized in a chart.

	60% Antifreeze	10% Antifreeze	Final Solution: 20% Antifreeze
Number of liters of solution	x	8	$(8 + x)$
Number of liters of pure antifreeze	$0.60x$	$0.10(8)$	$0.20(8 + x)$

Notice that an algebraic equation is derived from the second row of the table which relates the number of liters of pure antifreeze in each container.

The amount of pure antifreeze in the final solution equals the sum of the amounts of antifreeze in the first two solutions.

$$\left(\begin{array}{l} \text{Pure antifreeze} \\ \text{from solution 1} \end{array} \right) + \left(\begin{array}{l} \text{pure antifreeze} \\ \text{from solution 2} \end{array} \right) = \left(\begin{array}{l} \text{pure antifreeze} \\ \text{in the final solution} \end{array} \right)$$

$$0.60x + 0.10(8) = 0.20(8 + x)$$

$$0.60x + 0.10(8) = 0.20(8 + x) \quad \text{Mathematical model}$$

$$0.6x + 0.8 = 1.6 + 0.2x \quad \text{Apply the distributive property.}$$

$$0.6x - 0.2x + 0.8 = 1.6 + 0.2x - 0.2x \quad \text{Subtract } 0.2x \text{ from both sides.}$$

$$0.4x + 0.8 = 1.6$$

$$0.4x + 0.8 - 0.8 = 1.6 - 0.8 \quad \text{Subtract } 0.8 \text{ from both sides.}$$

$$0.4x = 0.8$$

$$\frac{0.4x}{0.4} = \frac{0.8}{0.4} \quad \text{Divide both sides by } 0.4.$$

$$x = 2$$

Answer: 2 L of 60% antifreeze solution is necessary to make a final solution of 20% antifreeze.

Skill Practice

7. Find the number of ounces (oz) of 30% alcohol solution that must be mixed with 10 oz of a 70% solution to obtain a solution that is 40% alcohol.

Skill Practice Answers

7. 30 oz of the 30% solution is needed.

6. Applications Involving Distance, Rate, and Time

The fundamental relationship among the variables distance, rate, and time is given by

$$\text{Distance} = (\text{rate})(\text{time}) \quad \text{or} \quad d = rt$$

For example, a motorist traveling 65 mph (miles per hour) for 3 hr (hours) will travel a distance of

$$d = (65 \text{ mph})(3 \text{ hr}) = 195 \text{ mi}$$

Example 8 Solving a Distance, Rate, Time Application

A hiker can hike $2\frac{1}{2}$ mph down a trail to visit Archuletta Lake. For the return trip back to her campsite (uphill), she is only able to go $1\frac{1}{2}$ mph. If the total time for the round trip is 4 hr 48 min (4.8 hr), find

- The time required to walk down to the lake
- The time required to return back to the campsite
- The total distance the hiker traveled

Solution:

The information given in the problem can be organized in a chart.

	Distance (mi)	Rate (mph)	Time (hr)
Trip to the lake		2.5	t
Return trip		1.5	$(4.8 - t)$

Column 2: The rates of speed going to and from the lake are given in the statement of the problem.

Column 3: There are two unknown times. If we let t be the time required to go to the lake, then the time for the return trip must equal the total time minus t , or $(4.8 - t)$

Column 1: To express the distance in terms of the time t , we use the relationship $d = rt$. That is, multiply the quantities in the second and third columns.

	Distance (mi)	Rate (mph)	Time (hr)
Trip to the lake	$2.5t$	2.5	t
Return trip	$1.5(4.8 - t)$	1.5	$(4.8 - t)$

To create a mathematical model, note that the distances to and from the lake are equal. Therefore,

$$(\text{Distance to lake}) = (\text{return distance}) \quad \text{Verbal model}$$

$$2.5t = 1.5(4.8 - t) \quad \text{Mathematical model}$$

$$2.5t = 7.2 - 1.5t \quad \text{Apply the distributive property.}$$

$$2.5t + 1.5t = 7.2 - 1.5t + 1.5t \quad \text{Add } 1.5t \text{ to both sides.}$$

$$4.0t = 7.2$$

$$\frac{4.0t}{4.0} = \frac{7.2}{4.0}$$

$$t = 1.8$$

Solve for t and interpret the results.

Answers:

- a. Because t represents the time required to go down to the lake, 1.8 hr is required for the trip to the lake.
- b. The time required for the return trip is $(4.8 - t)$ or $(4.8 \text{ hr} - 1.8 \text{ hr}) = 3 \text{ hr}$. Therefore, the time required to return to camp is 3 hr.
- c. The total distance equals the distance to the lake and back. The distance to the lake is $(2.5 \text{ mph})(1.8 \text{ hr}) = 4.5 \text{ mi}$. The distance back is $(1.5 \text{ mph})(3.0 \text{ hr}) = 4.5 \text{ mi}$. Therefore, the total distance the hiker walked is 9.0 mi.

Skill Practice

8. Jody drove a distance of 320 mi to visit a friend. She drives part of the time at 40 mph and part at 60 mph. The trip took 6 hr. Find the amount of time she spent driving at each speed.

Skill Practice Answers

8. Jody drove 2 hr at 40 mph and 4 hr at 60 mph.

Section 1.5**Practice Exercises**

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- Videos

Study Skills Exercises

1. Look through the text and write down a page number that contains:
 - a. Avoiding Mistakes _____
 - b. TIP box _____
 - c. A key term (shown in bold) _____
2. Define the key terms.
 - a. Sales tax
 - b. Commission
 - c. Simple interest

Review Exercises

For Exercises 3–11, solve the equations.

- | | | |
|---|--------------------------------|------------------------------------|
| 3. $7a - 2 = 11$ | 4. $2z + 6 = -15$ | 5. $4(x - 3) + 7 = 19$ |
| 6. $-3(y - 5) + 4 = 1$ | 7. $5(b + 4) - 3(2b + 8) = 3b$ | 8. $12c - 3c + 9 = 3(4 + 7c) - c$ |
| 9. $\frac{3}{8}p + \frac{3}{4} = p - \frac{3}{2}$ | 10. $\frac{1}{4} - 2x = 5$ | 11. $0.085(5)d - 0.075(4)d = 1250$ |


For the remaining exercises, follow the steps outlined in the Problem-Solving Flowchart found on page 49.

Concept 1: Introduction to Problem Solving

12. The larger of two numbers is 3 more than twice the smaller. The difference of the larger number and the smaller number is 8. Find the numbers.
13. One number is 3 less than another. Their sum is 15. Find the numbers.

14. The sum of 3 times a number and 2 is the same as the difference of the number and 4. Find the number.
15. Twice the sum of a number and 3 is the same as 1 subtracted from the number. Find the number.
16. The sum of two integers is 30. Ten times one integer is 5 times the other integer. Find the integers.
(*Hint*: If one number is x , then the other number is $30 - x$.)
17. The sum of two integers is 10. Three times one integer is 3 less than 8 times the other integer. Find the integers. (*Hint*: If one number is x , then the other number is $10 - x$.)

Concept 2: Applications Involving Consecutive Integers

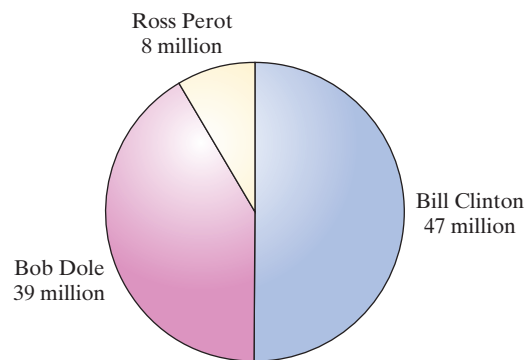
18. The sum of two consecutive page numbers in a book is 223. Find the page numbers.
19. The sum of the numbers on two consecutive raffle tickets is 808,455. Find the numbers on the tickets.
20. The sum of two consecutive odd integers is -148 . Find the two integers.
-  21. Three times the smaller of two consecutive even integers is the same as -146 minus 4 times the larger integer. Find the integers.
22. The sum of three consecutive integers is -57 . Find the integers.
23. Five times the smallest of three consecutive even integers is 10 more than twice the largest. Find the integers.

Concept 3: Applications Involving Percents and Rates

24. Leo works at a used car dealership and earns an 8% commission on sales. If he sold \$39,000 in used cars, what was his commission?
25. Alysha works for a pharmaceutical company and makes 0.6% commission on all sales within her territory. If the yearly sales in her territory came to \$8,200,000, what was her commission?
26. An account executive earns \$600 per month plus a 3% commission on sales. The executive's goal is to earn \$2400 this month. How much must she sell to achieve this goal?
27. If a salesperson in a department store sells merchandise worth over \$200 in one day, she receives a 12% commission on the sales over \$200. If the sales total \$424 on one particular day, how much commission did she earn?
28. Molly had the choice of taking out a 4-year car loan at 8.5% simple interest or a 5-year car loan at 7.75% simple interest. If she borrows \$15,000, which option will demand less interest?
29. Robert can take out a 3-year loan at 8% simple interest or a 2-year loan at $8\frac{1}{2}\%$ simple interest. If he borrows \$7000, which option will demand less interest?
30. If Ivory Soap is $99\frac{44}{100}\%$ pure, then what quantity of impurities will be found in a bar of Ivory Soap that weighs 4.5 oz (ounces)?

31. In the 1996 presidential election, a third party candidate received a significant number of votes. The figure illustrates the number of votes received for Bill Clinton, Bob Dole, and Ross Perot in that election. Compute the percent of votes received by each candidate. (Round to the nearest tenth of a percent.)

Number of votes by candidate—1996 Presidential election



32. The total bill (including a 6% sales tax) to have a radio installed in a car came to \$265. What was the cost before tax?

33. Wayne County has a sales tax rate of 7%. How much does Mike's Honda Civic cost before tax if the total cost of the car *plus tax* is \$13,888.60?

34. The price of a swimsuit after a 20% markup is \$43.08. What was the price before the markup?

35. The price of a used textbook after a 35% markdown is \$29.25. What was the original price?

36. In 2006, 39.6 million people lived below the poverty level in the United States. This represents an 80% increase from the number in 2002. How many people lived below the poverty level in 2002?

37. In 2006, Americans spent approximately \$69 billion on weddings. This represents a 50% increase from the amount spent in 2001. What amount did Americans spend on weddings in 2001?

Concept 4: Applications Involving Principal and Interest

38. Darrell has a total of \$12,500 in two accounts. One account pays 8% simple interest per year, and the other pays 12% simple interest. If he earned \$1160 in the first year, how much did he invest in each account?

39. Lillian had \$15,000 invested in two accounts, one paying 9% simple interest and one paying 10% simple interest. How much was invested in each account if the interest after 1 year is \$1432?

40. Ms. Riley deposited some money in an account paying 5% simple interest and twice that amount in an account paying 6% simple interest. If the total interest from the two accounts is \$765 for 1 year, how much was deposited into each account?

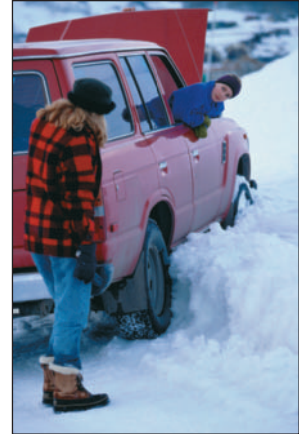
41. Sienna put some money in a certificate of deposit earning 4.2% simple interest. She deposited twice that amount in a money market account paying 4% simple interest. After 1 year her total interest was \$488. How much did Sienna deposit in her money market account?

42. A total of \$20,000 is invested between two accounts: one paying 4% simple interest and the other paying 3% simple interest. After 1 year the total interest was \$720. How much was invested at each rate?

43. Mr. Hall had some money in his bank earning 4.5% simple interest. He had \$5000 more deposited in a credit union earning 6% simple interest. If his total interest for 1 year was \$1140, how much did he deposit in each account?

Concept 5: Applications Involving Mixtures

44. For a car to survive a winter in Toronto, the radiator must contain at least 75% antifreeze solution. Jacques' truck has 6 L of 50% antifreeze mixture, some of which must be drained and replaced with pure antifreeze to bring the concentration to the 75% level. How much 50% solution should be drained and replaced by pure antifreeze to have 6 L of 75% antifreeze?
45. How many ounces of water must be added to 20 oz of an 8% salt solution to make a 2% salt solution?
46. Ronald has a 12% solution of the fertilizer Super Grow. How much pure Super Grow should he add to the mixture to get 32 oz of a 17.5% concentration?
47. How many liters of an 18% alcohol solution must be added to a 10% alcohol solution to get 20 L of a 15% alcohol solution?
48. For a performance of the play *Company*, 375 tickets were sold. The price of the orchestra level seats was \$25, and the balcony seats sold for \$21. If the total revenue was \$8875.00, how many of each type of ticket were sold?
49. Two different teas are mixed to make a blend that will be sold at a fair. Black tea sells for \$2.20 per pound and orange pekoe tea sells for \$3.00 per pound. How much of each should be used to obtain 4 lb of a blend selling for \$2.50?
50. A nut mixture consists of almonds and cashews. Almonds are \$4.98 per pound, and cashews are \$6.98 per pound. How many pounds of each type of nut should be mixed to produce 16 lb selling for \$5.73 per pound?
51. Two raffles are being held at a potluck dinner fund-raiser. One raffle ticket costs \$2.00 per ticket for a weekend vacation. The other costs \$1.00 per ticket for free passes to a movie theater. If 208 tickets were sold and a total of \$320 was received, how many of each type of ticket were sold?

**Concept 6: Applications Involving Distance, Rate, and Time**

52. Two cars are 192 miles apart and travel toward each other on the same road. They meet in 2 hr. One car travels 4 mph faster than the other. What is the average speed of each car?
53. Two cars are 190 miles apart and travel toward each other along the same road. They meet in 2 hr. One car travels 5 mph slower than the other car. What is the average speed of each car?
54. A Piper Cub airplane has an average air speed that is 10 mph faster than a Cessna 150 airplane. If the combined distance traveled by these two small planes is 690 miles after 3 hr, what is the average speed of each plane?
55. A woman can hike 1 mph faster down a trail to Archuletta Lake than she can on the return trip uphill. It takes her 3 hr to get to the lake and 6 hr to return. What is her speed hiking down to the lake?
56. Two boats traveling the same direction leave a harbor at noon. After 3 hr they are 60 miles apart. If one boat travels twice as fast as the other, find the average rate of each boat.
57. Two canoes travel down a river, starting at 9:00. One canoe travels twice as fast as the other. After 3.5 hr, the canoes are 5.25 miles apart. Find the average rate of each canoe.

Literal Equations and Applications to Geometry

Section 1.6

1. Applications Involving Geometry

Some word problems involve the use of geometric formulas such as those listed in the inside front cover of this text.

Example 1 Solving an Application Involving Perimeter

The length of a rectangular corral is 2 ft more than 3 times the width. The corral is situated such that one of its shorter sides is adjacent to a barn and does not require fencing. If the total amount of fencing is 774 ft, then find the dimensions of the corral.

Solution:

Read the problem and draw a sketch (Figure 1-8).

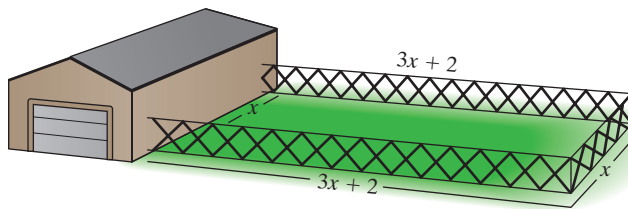


Figure 1-8

Let x represent the width.

Label variables.

Let $3x + 2$ represent the length.

To create a verbal model, we might consider using the formula for the perimeter of a rectangle. However, the formula $P = 2L + 2W$ incorporates all four sides of the rectangle. The formula must be modified to include only one factor of the width.

$$\left(\begin{array}{l} \text{Distance around} \\ \text{three sides} \end{array} \right) = \left(\begin{array}{l} 2 \text{ times} \\ \text{the length} \end{array} \right) + \left(\begin{array}{l} 1 \text{ times} \\ \text{the width} \end{array} \right) \quad \text{Verbal model}$$

$$774 = 2(3x + 2) + x \quad \text{Mathematical model}$$

$$774 = 2(3x + 2) + x \quad \text{Solve for } x.$$

$$774 = 6x + 4 + x \quad \text{Apply the distributive property.}$$

$$774 = 7x + 4 \quad \text{Combine like terms.}$$

$$770 = 7x \quad \text{Subtract 4 from both sides.}$$

$$110 = x \quad \text{Divide by 7 on both sides.}$$

$$x = 110$$

Because x represents the width, the width of the corral is 110 ft. The length is given by

$$3x + 2 \quad \text{or} \quad 3(110) + 2 = 332 \quad \text{Interpret the results.}$$

The width of the corral is 110 ft, and the length is 332 ft. (To check the answer, verify that the three sides add to 774 ft.)

Concepts

1. Applications Involving Geometry
2. Literal Equations

Skill Practice

- The length of Karen's living room is 2 ft longer than the width. The perimeter is 80 ft. Find the length and width.

The applications involving angles utilize some of the formulas found in the front cover of this text.

Example 2 Solving an Application Involving Angles

Two angles are complementary. One angle measures 10° less than 4 times the other angle. Find the measure of each angle (Figure 1-9).

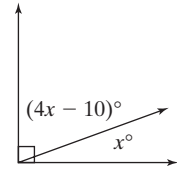


Figure 1-9

Solution:

Let x represent one angle.

Let $4x - 10$ represent the other angle.

Recall that two angles are complementary if the sum of their measures is 90° . Therefore, a verbal model is

(One angle) + (the complement of the angle) = 90° Verbal model

$x + (4x - 10) = 90$ Mathematical equation

$5x - 10 = 90$ Solve for x .

$$5x = 100$$

$$x = 20$$

If $x = 20$, then $4x - 10 = 4(20) - 10 = 70$. The two angles are 20° and 70° .

Skill Practice

- Two angles are supplementary, and the measure of one is 16° less than 3 times the other. Find their measures.

2. Literal Equations

Literal equations (or formulas) are equations that contain several variables. For example, the formula for the perimeter of a rectangle $P = 2L + 2W$ is an example of a literal equation. In this equation, P is expressed in terms of L and W . However, in science and other branches of applied mathematics, formulas may be more useful in alternative forms.

For example, the formula $P = 2L + 2W$ can be manipulated to solve for either L or W :

Solve for L

$$P = 2L + 2W$$

$$P - 2W = 2L \quad \text{Subtract } 2W.$$

$$\frac{P - 2W}{2} = L \quad \text{Divide by 2.}$$

$$L = \frac{P - 2W}{2}$$

Solve for W

$$P = 2L + 2W$$

$$P - 2L = 2W \quad \text{Subtract } 2L.$$

$$\frac{P - 2L}{2} = W \quad \text{Divide by 2.}$$

$$W = \frac{P - 2L}{2}$$

Skill Practice Answers

- The length is 21 ft, and the width is 19 ft.
- 49° and 131°

To solve a literal equation for a specified variable, use the addition, subtraction, multiplication, and division properties of equality.

Example 3 Solving a Literal Equation

The formula for the volume of a rectangular box is $V = LWH$.

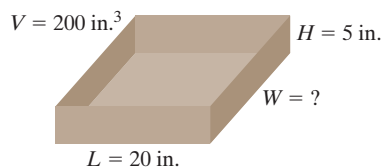


Figure 1-10

- Solve the formula $V = LWH$ for W .
- Find the value of W if $V = 200 \text{ in.}^3$, $L = 20 \text{ in.}$, and $H = 5 \text{ in.}$ (Figure 1-10).

Solution:

- $$V = LWH$$

The goal is to isolate the variable W .

$$\frac{V}{LH} = \frac{LWH}{LH}$$

Divide both sides by LH .

$$\frac{V}{LH} = W$$

Simplify.

$$W = \frac{V}{LH}$$
- $$W = \frac{200 \text{ in.}^3}{(20 \text{ in.})(5 \text{ in.})}$$

Substitute $V = 200 \text{ in.}^3$, $L = 20 \text{ in.}$, and $H = 5 \text{ in.}$

$$W = 2 \text{ in.}$$

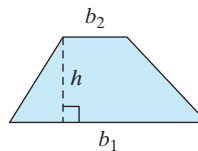
The width is 2 in.

Skill Practice The formula for the area of a triangle is $A = \frac{1}{2}bh$.

- Solve the formula for h .
- Find the value of h when $A = 40 \text{ in.}^2$ and $b = 16 \text{ in.}$

Example 4 Solving a Literal Equation

The formula to find the area of a trapezoid is given by $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the lengths of the parallel sides and h is the height. Solve this formula for b_1 .



Solution:

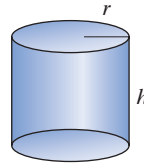
- $$A = \frac{1}{2}(b_1 + b_2)h$$
- The goal is to isolate b_1 .
- $$2A = 2 \cdot \frac{1}{2}(b_1 + b_2)h$$
- Multiply by 2 to clear fractions.
- $$2A = (b_1 + b_2)h$$
- Apply the distributive property.
- $$2A = b_1h + b_2h$$
- Subtract b_2h from both sides.
- $$2A - b_2h = b_1h$$
- Divide by h .
- $$\frac{2A - b_2h}{h} = \frac{b_1h}{h}$$
- $$\frac{2A - b_2h}{h} = b_1$$

Skill Practice Answers

- $h = \frac{2A}{b}$
- $h = 5 \text{ in.}$

Skill Practice

4. The formula for the volume of a right circular cylinder is $V = \pi r^2 h$. Solve for h .



TIP: When solving a literal equation for a specified variable, there is sometimes more than one way to express your final answer. This flexibility often presents difficulty for students. Students may leave their answer in one form, but the answer given in the text looks different. Yet both forms may be correct. To know if your answer is equivalent to the form given in the text you must try to manipulate it to look like the answer in the book, a process called *form fitting*.

The literal equation from Example 4 may be written in several different forms. The quantity $(2A - b_2h)/h$ can be split into two fractions.

$$b_1 = \frac{2A - b_2h}{h} = \frac{2A}{h} - \frac{b_2h}{h} = \frac{2A}{h} - b_2$$

Example 5 Solving a Linear Equation in Two Variables

Given $-2x + 3y = 5$, solve for y .

Solution:

$$-2x + 3y = 5$$

$$3y = 2x + 5$$

Add $2x$ to both sides.

$$\frac{3y}{3} = \frac{2x + 5}{3}$$

Divide by 3 on both sides.

$$y = \frac{2x + 5}{3} \quad \text{or} \quad y = \frac{2}{3}x + \frac{5}{3}$$

Skill Practice Solve for y .

5. $5x + 2y = 11$

Example 6 Applying a Literal Equation

Buckingham Fountain is one of Chicago's most familiar landmarks. With 133 jets spraying a total of 14,000 gal (gallons) of water per minute, Buckingham Fountain is one of the world's largest fountains. The circumference of the fountain is approximately 880 ft.

- The circumference of a circle is given by $C = 2\pi r$. Solve the equation for r .
- Use the equation from part (a) to find the radius and diameter of the fountain. (Use the π key on the calculator, and round the answers to 1 decimal place.)

Skill Practice Answers

4. $h = \frac{V}{\pi r^2}$

5. $y = \frac{11 - 5x}{2}$ or $y = -\frac{5}{2}x + \frac{11}{2}$

Solution:

$$\text{a. } C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = r$$

$$r = \frac{C}{2\pi}$$

$$\text{b. } r = \frac{880 \text{ ft}}{2\pi}$$

Substitute $C = 880$ ft and use the π key on the calculator.

$$r \approx 140.1 \text{ ft}$$

The radius is approximately 140.1 ft. The diameter is twice the radius ($d = 2r$); therefore the diameter is approximately 280.2 ft.

**Skill Practice**

The formula to compute the surface area S of a sphere is given by $S = 4\pi r^2$.

6a. Solve the equation for π .

b. A sphere has a surface area of 113 in.^2 and a radius of 3 in. Use the formula found in part (a) to approximate π . Round to 2 decimal places.

Skill Practice Answers

$$\text{6a. } \pi = \frac{S}{4r^2} \quad \text{b. } 3.14$$

Section 1.6**Practice Exercises**

Boost your GRADE at mathzone.com!



- Practice Problems
- Self-Tests
- NetTutor
- e-Professors
- Videos

Study Skills Exercises

1. Several topics are given here about taking notes. Which would you do first to help make the most of note-taking? Put them in order of importance to you by labeling them with the numbers 1–6.
 - _____ Read your notes after class to complete any abbreviations or incomplete sentences.
 - _____ Highlight important terms and definitions.
 - _____ Review your notes from the previous class.
 - _____ Bring pencils (more than one) and paper to class.
 - _____ Sit in class where you can clearly read the board and hear your instructor.
 - _____ Keep your focus on the instructor looking for phrases such as, “The most important point is . . .” and “Here is where the problem usually occurs.”

Review Exercises

For Exercises 2–5, solve the equations.

$$2. 7 + 5x - (2x - 6) = 6(x + 1) + 21$$

$$3. \frac{3}{5}y - 3 + 2y = 5$$

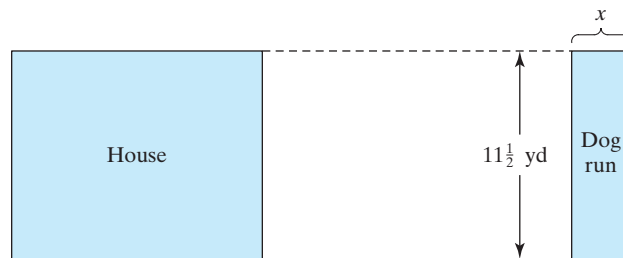
$$4. 3[z - (2 - 3z) - 4] = z - 7$$

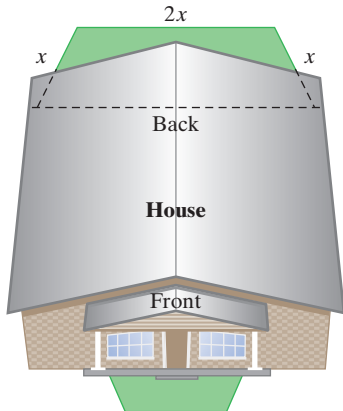
$$5. 2a - 4 + 8a = 7a - 8 + 3a$$

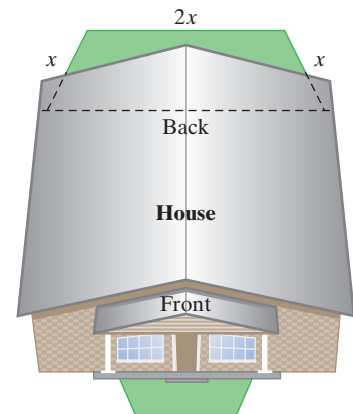
Concept 1: Applications Involving Geometry

For Exercises 6–25, use the geometry formulas listed in the inside front cover of the text.

6. A volleyball court is twice as long as it is wide. If the perimeter is 177 ft, find the dimensions of the court.
7. Two sides of a triangle are equal in length, and the third side is 1.5 times the length of one of the other sides. If the perimeter is 14 m (meters), find the lengths of the sides.
8. The lengths of the sides of a triangle are given by three consecutive even integers. The perimeter is 24 m. What is the length of each side?
9. A triangular garden has sides that can be represented by three consecutive integers. If the perimeter of the garden is 15 ft, what are the lengths of the sides?
10. Raoul would like to build a rectangular dog run in the rear of his backyard, away from the house. The width of the yard is $11\frac{1}{2}$ yd, and Raoul wants an area of 92 yd^2 (square yards) for his dog.
 - a. Find the dimensions of the dog run.
 - b. How much fencing would Raoul need to enclose the dog run?



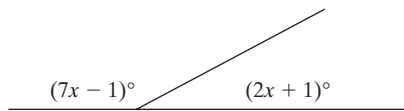
11. George built a rectangular pen for his rabbit such that the length is 7 ft less than twice the width. If the perimeter is 40 ft, what are the dimensions of the pen?
12. Antoine wants to put edging in the form of a square around a tree in his front yard. He has enough money to buy 18 ft of edging. Find the dimensions of the square that will use all the edging.
13. Joanne wants to plant a flower garden in her backyard in the shape of a trapezoid, adjacent to her house (see the figure). She also wants a front yard garden in the same shape, but with sides one-half as long. What should the dimensions be for each garden if Joanne has only a total of 60 ft of fencing?
 
14. The measures of two angles in a triangle are equal. The third angle measures 2 times the sum of the equal angles. Find the measures of the three angles.
15. The smallest angle in a triangle is one-half the size of the largest. The middle angle measures 25° less than the largest. Find the measures of the three angles.



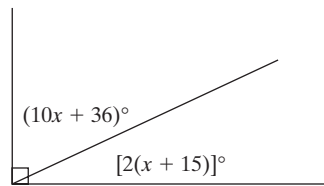
16. Two angles are complementary. One angle is 5 times as large as the other angle. Find the measure of each angle.
17. Two angles are supplementary. One angle measures 12° less than 3 times the other. Find the measure of each angle.

In Exercises 18–25, solve for x , and then find the measure of each angle.

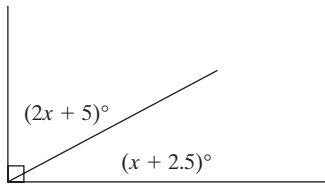
18.



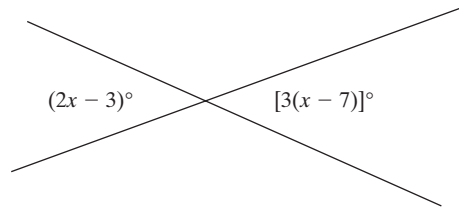
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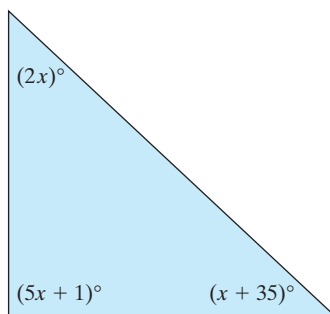
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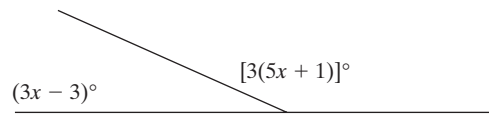
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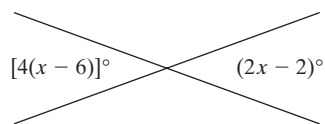
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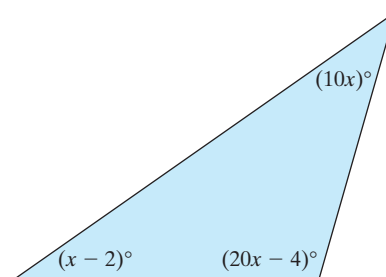
23.



24.



25.



Concept 2: Literal Equations

26. Which expression(s) is (are) equivalent to $-5/(x - 3)$?

a. $-\frac{5}{x - 3}$ b. $\frac{5}{3 - x}$ c. $\frac{5}{-x + 3}$

27. Which expression(s) is (are) equivalent to $(z - 1)/-2$?

a. $\frac{1 - z}{2}$ b. $-\frac{z - 1}{2}$ c. $\frac{-z + 1}{2}$

28. Which expression(s) is (are) equivalent to $(-x - 7)/y$?

a. $-\frac{x + 7}{y}$ b. $\frac{x + 7}{-y}$ c. $\frac{-x - 7}{-y}$

29. Which expression(s) is (are) equivalent to $-3w/(-x - y)$?

a. $-\frac{3w}{-x - y}$ b. $\frac{3w}{x + y}$ c. $-\frac{-3w}{x + y}$

For Exercises 30–47, solve for the indicated variable.

30. $A = lw$ for l

31. $C_1 = \frac{5}{2}R$ for R

32. $I = Prt$ for P

33. $a + b + c = P$ for b

34. $W = K_2 - K_1$ for K_1

35. $y = mx + b$ for x

36. $F = \frac{9}{5}C + 32$ for C

37. $C = \frac{5}{9}(F - 32)$ for F

38. $K = \frac{1}{2}mv^2$ for v^2

39. $I = Prt$ for r

40. $v = v_0 + at$ for a

41. $a^2 + b^2 = c^2$ for b^2

42. $w = p(v_2 - v_1)$ for v_2

43. $A = lw$ for w

44. $ax + by = c$ for y

45. $P = 2L + 2W$ for L

46. $V = \frac{1}{3}Bh$ for B

47. $V = \frac{1}{3}\pi r^2 h$ for h

In Chapter 2 it will be necessary to change equations from the form $Ax + By = C$ to $y = mx + b$. For Exercises 48–59, express each equation in the form $y = mx + b$ by solving for y .

48. $3x + y = 6$

49. $x + y = -4$

50. $5x - 4y = 20$

51. $-4x - 5y = 25$

52. $-6x - 2y = 13$

53. $5x - 7y = 15$

54. $3x - 3y = 6$

55. $2x - 2y = 8$

56. $9x + \frac{4}{3}y = 5$

57. $4x - \frac{1}{3}y = 5$

58. $-x + \frac{2}{3}y = 0$

59. $x - \frac{1}{4}y = 0$

For Exercises 60–61, use the relationship between distance, rate, and time given by $d = rt$.



60. a. Solve $d = rt$ for rate r .

b. In 2006, Sam Hornish won the Indianapolis 500 in 3 hr, 10 min, 59 sec (≈ 3.183 hr). Find his average rate of speed if the total distance is 500 mi. Round to the nearest tenth of a mile per hour.

61. a. Solve $d = rt$ for time t .

b. In 2006, Jimmie Johnson won the Daytona 500 with an average speed of 141.734 mph. Find the total time it took for him to complete the race if the total distance is 500 miles. Round to the nearest hundredth of an hour.


For Exercises 62–63, use the fact that the force imparted by an object is equal to its mass times acceleration, or $F = ma$.

62. a. Solve $F = ma$ for mass m .

b. The force on an object is 24.5 N (newtons), and the acceleration due to gravity is 9.8 m/sec^2 . Find the mass of the object. (The answer will be in kilograms.)

63. a. Solve $F = ma$ for acceleration a .

b. Approximate the acceleration of a 2000-kg mass influenced by a force of 15,000 N. (The answer will be in meters per second squared, m/sec^2 .)

 In statistics the z -score formula $z = \frac{x - \mu}{\sigma}$ is used in studying probability. Use this formula for Exercises 64–65.

64. a. Solve $z = \frac{x - \mu}{\sigma}$ for x .

b. Find x when $z = 2.5$, $\mu = 100$, and $\sigma = 12$.

65. a. Solve $z = \frac{x - \mu}{\sigma}$ for σ .

b. Find σ when $x = 150$, $z = 2.5$, and $\mu = 110$.

Expanding Your Skills

For Exercises 66–75, solve for the indicated variable.

66. $6t - rt = 12$ for t

67. $5 = 4a + ca$ for a

68. $ax + 5 = 6x + 3$ for x

69. $cx - 4 = dx + 9$ for x

70. $A = P + Prt$ for P

71. $A = P + Prt$ for r

72. $T = mg - mf$ for m

73. $T = mg - mf$ for f

74. $ax + by = cx + z$ for x

75. $Lt + h = mt + g$ for t

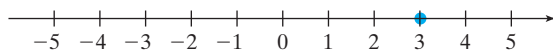
Linear Inequalities in One Variable

Section 1.7

1. Solving Linear Inequalities

In Sections 1.4–1.6, we learned how to solve linear equations and their applications. In this section, we will learn the process of solving linear *inequalities*. A **linear inequality** in one variable, x , is defined as any relationship of the form: $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, or $ax + b \geq 0$, where $a \neq 0$.

The solution to the equation $x = 3$ can be graphed as a single point on the number line.



Now consider the *inequality* $x \leq 3$. The solution set to an inequality is the set of real numbers that makes the inequality a true statement. In this case, the solution set is all real numbers less than or equal to 3. Because the solution set has an infinite number of values, the values cannot be listed. Instead, we can graph the solution set or represent the set in interval notation or in set-builder notation.

Graph



Interval Notation

$$(-\infty, 3]$$

Set-Builder Notation

$$\{x \mid x \leq 3\}$$

The addition and subtraction properties of equality indicate that a value added to or subtracted from both sides of an equation results in an equivalent equation. The same is true for inequalities.

Concepts

1. Solving Linear Inequalities
2. Inequalities of the Form $a < x < b$
3. Applications of Inequalities

Addition and Subtraction Properties of InequalityLet a , b , and c represent real numbers.*Addition property of inequality: If $a < b$
then $a + c < b + c$ *Subtraction property of inequality: If $a < b$
then $a - c < b - c$ *These properties may also be stated for $a \leq b$, $a > b$, and $a \geq b$.**Example 1 Solving a Linear Inequality**

Solve the inequality. Graph the solution and write the solution set in interval notation.

$$3x - 7 > 2(x - 4) - 1$$

Solution:

$$3x - 7 > 2(x - 4) - 1$$

$$3x - 7 > 2x - 8 - 1 \quad \text{Apply the distributive property.}$$

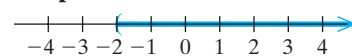
$$3x - 7 > 2x - 9$$

$$3x - 2x - 7 > 2x - 2x - 9 \quad \text{Subtract } 2x \text{ from both sides.}$$

$$x - 7 > -9$$

$$x - 7 + 7 > -9 + 7 \quad \text{Add 7 to both sides.}$$

$$x > -2$$

Graph**Interval Notation**

$$(-2, \infty)$$

Skill Practice

1. Solve the inequality. Graph the solution and write the solution in interval notation.

$$4(2x - 1) > 7x + 1$$

Multiplying both sides of an equation by the same quantity results in an equivalent equation. However, the same is not always true for an inequality. If you multiply or divide an inequality by a *negative* quantity, the direction of the inequality symbol must be *reversed*.

For example, consider multiplying or dividing the inequality $4 < 5$ by -1 .

Multiply/divide by -1 : $4 < 5$ $4 < 5$

$-4 > -5$ $-4 > -5$

The number 4 lies to the left of 5 on the number line. However, -4 lies to the right of -5 . Changing the signs of two numbers changes their relative position on the number line. This is stated formally in the multiplication and division properties of inequality.

Skill Practice Answers

1.

$$(5, \infty)$$

Multiplication and Division Properties of Inequality

Let a , b , and c represent real numbers.

*If c is *positive* and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

*If c is *negative* and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

The second statement indicates that if both sides of an inequality are multiplied or divided by a negative quantity, the inequality sign must be *reversed*.

*These properties may also be stated for $a \leq b$, $a > b$, and $a \geq b$.

Example 2 Solving Linear Inequalities

Solve the inequalities. Graph the solution and write the solution set in interval notation.

a. $-2x - 5 < 2$ b. $-6(x - 3) \geq 2 - 2(x - 8)$

Solution:

a. $-2x - 5 < 2$

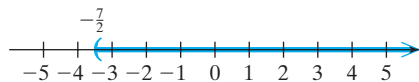
$-2x - 5 + 5 < 2 + 5$ Add 5 to both sides.

$-2x < 7$

$\frac{-2x}{-2} > \frac{7}{-2}$ Divide by -2 (*reverse* the inequality sign).

$x > -\frac{7}{2}$ or $x > -3.5$

Graph



Interval Notation

$\left(-\frac{7}{2}, \infty\right)$

TIP: The inequality $-2x - 5 < 2$ could have been solved by isolating x on the right-hand side of the inequality. This creates a positive coefficient on the x term and eliminates the need to divide by a negative number.

$-2x - 5 < 2$

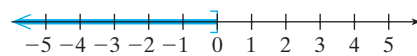
$-5 < 2x + 2$ Add $2x$ to both sides.

$-7 < 2x$ Subtract 2 from both sides.

$\frac{-7}{2} < \frac{2x}{2}$ Divide by 2 (because 2 is positive, do *not* reverse the inequality sign).

$-\frac{7}{2} < x$ (Note that the inequality $-\frac{7}{2} < x$ is equivalent to $x > -\frac{7}{2}$.)

b. $-6(x - 3) \geq 2 - 2(x - 8)$
 $-6x + 18 \geq 2 - 2x + 16$ Apply the distributive property.
 $-6x + 18 \geq 18 - 2x$ Combine *like* terms.
 $-6x + 2x + 18 \geq 18 - 2x + 2x$ Add $2x$ to both sides.
 $-4x + 18 \geq 18$
 $-4x + 18 - 18 \geq 18 - 18$ Subtract 18 from both sides.
 $-4x \geq 0$
 $\frac{-4x}{-4} \leq \frac{0}{-4}$ Divide by -4 (*reverse the inequality sign*).
 $x \leq 0$

Graph**Interval Notation**

$(-\infty, 0]$

Skill Practice Solve the inequalities. Graph the solution and write the solution in interval notation.

2. $-4x - 12 \geq 20$ 3. $5(3x + 1) < 4(5x - 5)$

Example 3 Solving a Linear Inequality

Solve the inequality $\frac{-5x + 2}{-3} > x + 2$. Graph the solution and write the solution set in interval notation.

Solution:

$$\frac{-5x + 2}{-3} > x + 2$$

$$-3\left(\frac{-5x + 2}{-3}\right) < -3(x + 2)$$
 Multiply by -3 to clear fractions (*reverse the inequality sign*).

$$-5x + 2 < -3x - 6$$

$$-5x + 3x + 2 < -3x + 3x - 6$$
 Add $3x$ to both sides.

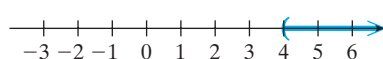
$$-2x + 2 < -6$$

$$-2x + 2 - 2 < -6 - 2$$
 Subtract 2 from both sides.

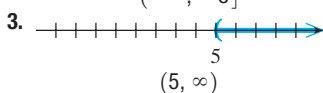
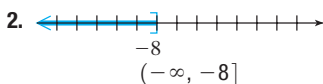
$$-2x < -8$$

$$\frac{-2x}{-2} > \frac{-8}{-2}$$
 Divide by -2 (the inequality sign is reversed *again*).

$$x > 4$$
 Simplify.

Graph**Interval Notation**

$(4, \infty)$

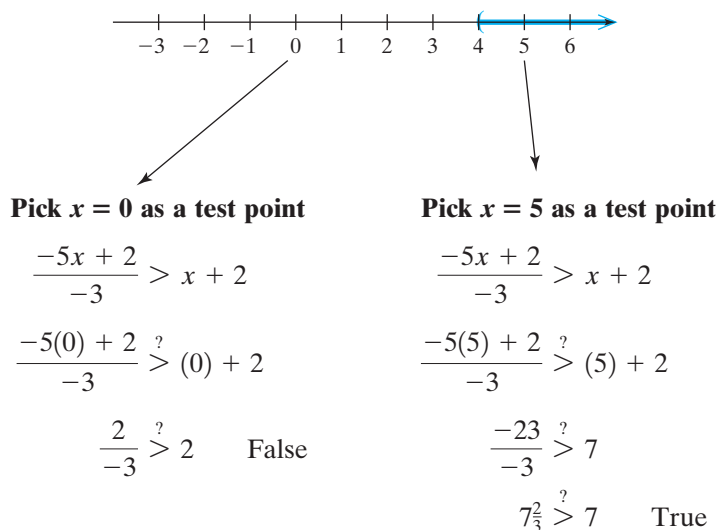
Skill Practice Answers

Skill Practice

Solve the inequality. Graph the solution and write the solution in interval notation.

4. $\frac{x+1}{-3} \geq -x+1$

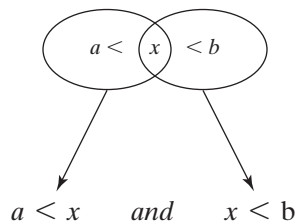
In Example 3, the inequality sign was reversed twice: once for multiplying the inequality by -3 and once for dividing by -2 . If you are in doubt about whether you have the inequality sign in the correct direction, you can check your final answer by using the **test point method**. That is, pick a point in the proposed solution set, and verify that it makes the original inequality true. Furthermore, any test point picked outside the solution set should make the original inequality false.



Because a test point to the right of $x = 4$ makes the inequality true, we have shaded the correct part of the number line.

2. Inequalities of the Form $a < x < b$

An inequality of the form $a < x < b$ is a type of **compound inequality**, one that defines two simultaneous conditions on the quantity x .



The solution to the compound inequality $a < x < b$ is the *intersection* of the inequalities $a < x$ and $x < b$. To solve a compound inequality of this form, we can actually work with the inequality as a “three-part” inequality and isolate the variable x .

Skill Practice Answers

4.

$[2, \infty)$

Example 4 Solving a Compound Inequality of the Form $a < x < b$

Solve the inequality $-2 \leq 3x + 1 < 5$. Graph the solution and express the solution set in interval notation.

Solution:

To solve the compound inequality $-2 \leq 3x + 1 < 5$, isolate the variable x in the “middle.” The operations performed on the middle portion of the inequality must also be performed on the left-hand side and right-hand side.

$$-2 \leq 3x + 1 < 5$$

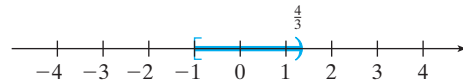
$$-2 - 1 \leq 3x + 1 - 1 < 5 - 1 \quad \text{Subtract 1 from all three parts of the inequality.}$$

$$-3 \leq 3x < 4 \quad \text{Simplify.}$$

$$\frac{-3}{3} \leq \frac{3x}{3} < \frac{4}{3} \quad \text{Divide by 3 in all three parts of the inequality.}$$

$$-1 \leq x < \frac{4}{3} \quad \text{Simplify.}$$

Graph



Interval Notation

$$\left[-1, \frac{4}{3}\right)$$

Skill Practice

5. Solve the compound inequality. Graph the solution and express the solution set in interval notation.

$$-8 < 5x - 3 \leq 12$$

3. Applications of Inequalities

Example 5 Solving a Compound Inequality Application

Beth received grades of 87%, 82%, 96%, and 79% on her last four algebra tests. To graduate with honors, she needs at least a B in the course.

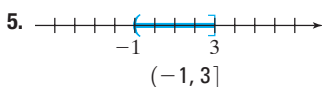
- What grade does she need to make on the fifth test to get a B in the course? Assume that the tests are weighted equally and that to earn a B the average of the test grades must be at least 80% but less than 90%.
- Is it possible for Beth to earn an A in the course if an A requires an average of 90% or more?

Solution:

- Let x represent the score on the fifth test.

The average of the five tests is given by $\frac{87 + 82 + 96 + 79 + x}{5}$

Skill Practice Answers



To earn a B, Beth requires

$$80 \leq (\text{average of test scores}) < 90$$

Verbal model

$$80 \leq \frac{87 + 82 + 96 + 79 + x}{5} < 90$$

Mathematical model

$$5(80) \leq 5\left(\frac{87 + 82 + 96 + 79 + x}{5}\right) < 5(90)$$

Multiply by 5 to clear fractions.

$$400 \leq 344 + x < 450$$

Simplify.

$$400 - 344 \leq 344 - 344 + x < 450 - 344$$

Subtract 344 from all three parts.

$$56 \leq x < 106$$

Simplify.

To earn a B in the course, Beth must score at least 56% but less than 106% on the fifth exam. Realistically, she may score between 56% and 100% because a grade over 100% is not possible.

- b. To earn an A, Beth's average would have to be greater than or equal to 90%.

$$(\text{Average of test scores}) \geq 90$$

Verbal model

$$\frac{87 + 82 + 96 + 79 + x}{5} \geq 90$$

Mathematical equation

$$5\left(\frac{87 + 82 + 96 + 79 + x}{5}\right) \geq 5(90)$$

Clear fractions.

$$344 + x \geq 450$$

Simplify.

$$x \geq 106$$

Solve for x .

It would be impossible for Beth to earn an A in the course because she would have to earn at least a score of 106% on the fifth test. It is impossible to earn over 100%.

Skill Practice

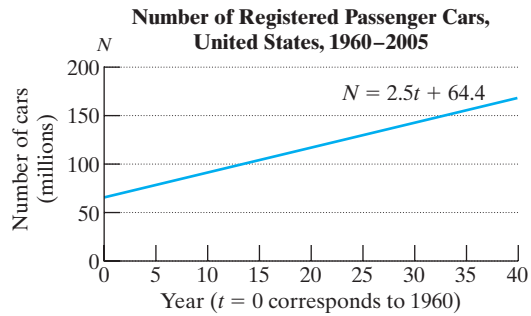
6. Jamie is a salesman who works on commission, so his salary varies from month to month. To qualify for an automobile loan, his salary must average at least \$2100 for 6 months. His salaries for the past 5 months have been \$1800, \$2300, \$1500, \$2200, and \$2800. What amount does he need to earn in the last month to qualify for the loan?

Example 6 Solving a Linear Inequality Application

The number of registered passenger cars N (in millions) in the United States has risen between 1960 and 2005 according to the equation $N = 2.5t + 64.4$, where t represents the number of years after 1960 ($t = 0$ corresponds to 1960, $t = 1$ corresponds to 1961, and so on) (Figure 1-11).

Skill Practice Answers

6. Jamie's salary must be at least \$2000.

**Figure 1-11**

(Source: U.S. Department of Transportation)

- For what years after 1960 was the number of registered passenger cars less than 89.4 million?
- For what years was the number of registered passenger cars between 94.4 million and 101.9 million?
- Predict the years for which the number of passenger cars will exceed 154.4 million.

Solution:

- a. We require $N < 89.4$ million.

$$\begin{aligned} N &< 89.4 \\ \underbrace{2.5t + 64.4} &< 89.4 \end{aligned}$$

Substitute the expression $2.5t + 64.4$ for N .

$$2.5t + 64.4 - 64.4 < 89.4 - 64.4 \quad \text{Subtract 64.4 from both sides.}$$

$$2.5t < 25$$

$$\frac{2.5t}{2.5} < \frac{25}{2.5}$$

Divide both sides by 2.5.

$$t < 10$$

 $t = 10$ corresponds to the year 1970.

Before 1970, the number of registered passenger cars was less than 89.4 million.

- b. We require $94.4 < N < 101.9$. Hence

$$94.4 < 2.5t + 64.4 < 101.9$$

Substitute the expression $2.5t + 64.4$ for N .

$$94.4 - 64.4 < 2.5t + 64.4 - 64.4 < 101.9 - 64.4$$

Subtract 64.4 from all three parts of the inequality.

$$30.0 < 2.5t < 37.5$$

$$\frac{30.0}{2.5} < \frac{2.5t}{2.5} < \frac{37.5}{2.5}$$

Divide by 2.5.

$$12 < t < 15$$

 $t = 12$ corresponds to 1972 and $t = 15$ corresponds to 1975.

Between the years 1972 and 1975, the number of registered passenger cars was between 94.4 million and 101.9 million.

c. We require $N > 154.4$.

$$2.5t + 64.4 > 154.4 \quad \text{Substitute the expression } 2.5t + 64.4 \text{ for } N.$$

$$2.5t > 90 \quad \text{Subtract 64.4 from both sides.}$$

$$\frac{2.5t}{2.5} > \frac{90}{2.5} \quad \text{Divide by 2.5.}$$

$$t > 36 \quad t = 36 \text{ corresponds to the year 1996.}$$

After the year 1996, the number of registered passenger cars exceeded 154.4 million.

Skill Practice

7. The population of Alaska has steadily increased since 1950 according to the equation $P = 10t + 117$, where t represents the number of years after 1950 and P represents the population in thousands.

For what years after 1950 was the population less than 417 thousand people?

Skill Practice Answers

7. The population was less than 417 thousand for $t < 30$. This corresponds to the years before 1980.

Section 1.7

Practice Exercises

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Study Skills Exercises

1. After doing a section of homework, check the answers to the odd-numbered exercises in the back of the text. Choose a method to identify the exercises that you got wrong or had trouble with (e.g., circle the number or put a star by the number). List some reasons why it is important to label these problems.
2. Define the key terms.
 - a. Linear inequality
 - b. Test point method
 - c. Compound inequality


Review Exercises

3. Solve for v . $d = vt - 16t^2$
4. Solve for x . $4 + 5(4 - 2x) = -2(x - 1) - 4$
5. Five more than 3 times a number is 6 less than twice the number. Find the number.
6. Solve for y . $5x + 3y + 6 = 0$
7. a. The area of a triangle is given by $A = \frac{1}{2}bh$. Solve for h .
b. If the area of a certain triangle is 10 cm^2 and the base is 3 cm, find the height.
8. Solve for t .

$$\frac{1}{5}t - \frac{1}{2} - \frac{1}{10}t + \frac{2}{5} = \frac{3}{10}t + \frac{1}{2}$$

Concept 1: Solving Linear Inequalities


For Exercises 9–31, solve the inequalities. Graph the solution and write the solution set in interval notation. Check each answer by using the test point method.

- | | | |
|---|--|--|
| 9. $6 \leq 4 - 2y$
_____→ | 10. $2x - 5 \geq 15$
_____→ | 11. $4z + 2 < 22$
_____→ |
| 12. $6z + 3 > 16$
_____→ |  13. $8w - 2 \leq 13$
_____→ | 14. $\frac{2}{3}t < -8$
_____→ |
| 15. $\frac{1}{5}p + 3 \geq -1$
_____→ | 16. $\frac{3}{4}(8y - 9) < 3$
_____→ | 17. $\frac{2}{5}(2x - 1) > 10$
_____→ |
| 18. $0.8a - 0.5 \leq 0.3a - 11$
_____→ | 19. $0.2w - 0.7 < 0.4 - 0.9w$
_____→ | 20. $-5x + 7 < 22$
_____→ |
| 21. $-3w - 6 > 9$
_____→ | 22. $-\frac{5}{6}x \leq -\frac{3}{4}$
_____→ | 23. $\frac{3k - 2}{-5} \leq 4$
_____→ |
| 24. $\frac{3p - 1}{-2} > 5$
_____→ | 25. $-\frac{3}{2}y > -\frac{21}{16}$
_____→ | 26. $0.2t + 1 > 2.4t - 10$
_____→ |
| 27. $20 \leq 8 - \frac{1}{3}x$
_____→ | 28. $3 - 4(y + 2) \leq 6 + 4(2y + 1)$
_____→ | 29. $1 < 3 - 4(3b - 1)$
_____→ |
| 30. $7.2k - 5.1 \geq 5.7$
_____→ | 31. $6h - 2.92 \leq 16.58$
_____→ | |

Concept 2: Inequalities of the form $a < x < b$

32. Write $-3 \leq x < 2$ as two inequalities.
33. Write $5 < x \leq 7$ as two inequalities.

For Exercises 34–45, solve the compound inequalities. Graph the solution and write the solution set in interval notation.

- | | | |
|---|--|---|
| 34. $0 \leq 3a + 2 < 17$
_____→ | 35. $-8 < 4k - 7 < 11$
_____→ | 36. $5 < 4y - 3 < 21$
_____→ |
| 37. $7 \leq 3m - 5 < 10$
_____→ | 38. $1 \leq \frac{1}{5}x + 12 \leq 13$
_____→ |  39. $5 \leq \frac{1}{4}a + 1 < 9$
_____→ |
| 40. $4 > \frac{2x + 8}{-2} \geq -5$
_____→ | 41. $-5 \geq \frac{-y + 3}{6} \geq -8$
_____→ | 42. $6 \geq -2b - 3 > -6$
_____→ |

43. $4 > -3w - 7 \geq 2$




44. $8 > -w + 4 > -1$



45. $13 \geq -2h - 1 \geq 0$

46. Explain why $5 < x < 1$ has no solution.47. Explain why $-1 > x > 3$ has no solution.**Concept 3: Applications of Inequalities**

48. Nolvia sells copy machines, and her salary is \$25,000 plus a 4% commission on sales. The equation $S = 25,000 + 0.04x$ represents her salary S in dollars in terms of her total sales x in dollars.
- How much money in sales does Nolvia need to earn a salary that exceeds \$40,000?
 - How much money in sales does Nolvia need to earn a salary that exceeds \$80,000?
 - Why is the money in sales required to earn a salary of \$80,000 more than twice the money in sales required to earn a salary of \$40,000?
49. The amount of money in a savings account A depends on the principal P , the interest rate r , and the time in years t that the money is invested. The equation $A = P + Prt$ shows the relationship among the variables for an account earning simple interest. If an investor deposits \$5000 at $6\frac{1}{2}\%$ simple interest, the account will grow according to the formula $A = 5000 + 5000(0.065)t$.
- How many years will it take for the investment to exceed \$10,000? (Round to the nearest tenth of a year.)
 - How many years will it take for the investment to exceed \$15,000? (Round to the nearest tenth of a year.)
50. The revenue R for selling x fleece jackets is given by the equation $R = 49.95x$. The cost to produce x jackets is $C = 2300 + 18.50x$. Find the number of jackets that the company needs to sell to produce a profit. (*Hint:* A profit occurs when revenue exceeds cost.)
51. The revenue R for selling x mountain bikes is $R = 249.95x$. The cost to produce x bikes is $C = 56,000 + 140x$. Find the number of bikes that the company needs to sell to produce a profit.
52. The average high and low temperatures for Vancouver, British Columbia, in January are 5.6°C and 0°C , respectively. The formula relating Celsius temperatures to Fahrenheit temperatures is given by $C = \frac{5}{9}(F - 32)$. Convert the inequality $0.0^\circ \leq C \leq 5.6^\circ$ to an equivalent inequality using Fahrenheit temperatures.
53. For a day in July, the temperatures in Austin, Texas ranged from 20°C to 29°C . The formula relating Celsius temperatures to Fahrenheit temperatures is given by $C = \frac{5}{9}(F - 32)$. Convert the inequality $20^\circ \leq C \leq 29^\circ$ to an equivalent inequality using Fahrenheit temperatures.
54. The poverty threshold P for four-person families between the years 1960 and 2006 can be approximated by the equation $P = 1235 + 387t$, where P is measured in dollars; and $t = 0$ corresponds to the year 1960, $t = 1$ corresponds to 1961, and so on. (Source: U.S. Bureau of the Census.)
- For what years after 1960 was the poverty threshold under \$7040?
 - For what years after 1960 was the poverty threshold between \$4331 and \$10,136?

-  **55.** Between the years 1960 and 2006, the average gas mileage (miles per gallon) for passenger cars has increased. The equation $N = 12.6 + 0.214t$ approximates the average gas mileage corresponding to the year t , where $t = 0$ represents 1960, $t = 1$ represents 1961, and so on.
- For what years after 1960 was the average gas mileage less than 14.1 mpg? (Round to the nearest year.)
 - For what years was the average gas mileage between 17.1 and 18.0 mpg? (Round to the nearest year.)

Mixed Exercises

For Exercises 56–73, solve the inequalities. Graph the solution, and write the solution set in interval notation. Check each answer by using the test point method.

56. $-6p - 1 > 17$

_____→

57. $-4y + 1 \leq -11$

_____→

58. $\frac{3}{4}x - 8 \leq 1$

_____→

59. $-\frac{2}{5}a - 3 > 5$

_____→

60. $-1.2b - 0.4 \geq -0.4b$

_____→

61. $-0.4t + 1.2 < -2$

_____→

62. $1 < 3(2t - 4) \leq 12$

_____→

63. $4 \leq 2(5h - 3) < 14$

_____→

64. $-\frac{3}{4}c - \frac{5}{4} \geq 2c$


_____→

65. $-\frac{2}{3}q - \frac{1}{3} > \frac{1}{2}q$

_____→

66. $4 - 4(y - 2) < -5y + 6$

_____→

 **67.** $6 - 6(k - 3) \geq -4k + 12$

_____→

68. $0 \leq 2q - 1 \leq 11$

_____→

69. $-10 < 7p - 1 < 1$

_____→

70. $-6(2x + 1) < 5 - (x - 4) - 6x$

_____→

71. $2(4p + 3) - p \leq 5 + 3(p - 3)$

_____→

72. $6a - (9a + 1) - 3(a - 1) \geq 2$

_____→

73. $8(q + 1) - (2q + 1) + 5 > 12$

_____→

Expanding Your Skills

For Exercises 74–77, assume $a > b$. Determine which inequality sign ($>$ or $<$) should be inserted to make a true statement. Assume $a \neq 0$ and $b \neq 0$.

74. $a + c$ _____ $b + c$, for $c > 0$

75. $a + c$ _____ $b + c$, for $c < 0$

76. ac _____ bc , for $c > 0$

77. ac _____ bc , for $c < 0$

Properties of Integer Exponents and Scientific Notation

Section 1.8

1. Properties of Exponents

In Section 1.2, we learned that exponents are used to represent repeated multiplication. The following properties of exponents are often used to simplify algebraic expressions.

Properties of Exponents*			
Description	Property	Example	Details/Notes
Multiplication of like bases	$b^m \cdot b^n = b^{m+n}$	$b^2 \cdot b^4 = b^{2+4} = b^6$	$b^2 \cdot b^4 = (b \cdot b)(b \cdot b \cdot b \cdot b) = b^6$
Division of like bases	$\frac{b^m}{b^n} = b^{m-n}$	$\frac{b^5}{b^2} = b^{5-2} = b^3$	$\frac{b^5}{b^2} = \frac{\cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b}} = b^3$
Power rule	$(b^m)^n = b^{m \cdot n}$	$(b^4)^2 = b^{4 \cdot 2} = b^8$	$(b^4)^2 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b) = b^8$
Power of a product	$(ab)^m = a^m b^m$	$(ab)^3 = a^3 b^3$	$(ab)^3 = (ab)(ab)(ab) = (a \cdot a \cdot a)(b \cdot b \cdot b) = a^3 b^3$
Power of a quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$	$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a \cdot a \cdot a}{b \cdot b \cdot b} = \frac{a^3}{b^3}$

*Assume that a and b are real numbers ($b \neq 0$) and that m and n represent positive integers.

In addition to the properties of exponents, two definitions are used to simplify algebraic expressions.

b^0 and b^{-n}

Let n be an integer, and let b be a real number such that $b \neq 0$.

1. $b^0 = 1$

2. $b^{-n} = \left(\frac{1}{b}\right)^n = \frac{1}{b^n}$

The definition of b^0 is consistent with the properties of exponents. For example, if b is a nonzero real number and n is an integer, then

$$\frac{b^n}{b^n} = 1 \longleftarrow$$

$$\frac{b^n}{b^n} = b^{n-n} = b^0$$

The expression $b^0 = 1$

Concepts

1. Properties of Exponents
2. Simplifying Expressions with Exponents
3. Scientific Notation

The definition of b^{-n} is also consistent with the properties of exponents. If b is a nonzero real number, then

$$\frac{b^3}{b^5} = \frac{\cancel{b \cdot b \cdot b}}{b \cdot \cancel{b \cdot b \cdot b} \cdot b \cdot b} = \frac{1}{b^2} \leftarrow \text{The expression } b^{-2} = \frac{1}{b^2}$$

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

Example 1 Using the Properties of Exponents

Simplify the expressions

a. $(-2)^4$ b. -2^4 c. -2^{-4} d. $(-7x)^0$ e. $-7x^0$

Solution:

a. $(-2)^4 = (-2)(-2)(-2)(-2)$
 $= 16$

b. $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$
 $= -16$

c. $-2^{-4} = \frac{1}{-2^4}$
 $= \frac{1}{-(2 \cdot 2 \cdot 2 \cdot 2)}$
 $= \frac{1}{-16}$
 $= -\frac{1}{16}$

d. $(-7x)^0 = 1$ because $b^0 = 1$

e. $-7x^0 = -7 \cdot x^0$
 $= -7 \cdot 1$
 $= -7$

Skill Practice Simplify the expressions.

1. $(-3)^2$ 2. -3^2 3. -3^{-2} 4. $(-8y)^0$ 5. -6^0

2. Simplifying Expressions with Exponents

Example 2 Simplifying Expressions with Exponents

Simplify the following expressions. Write the final answer with positive exponents only.

a. $(x^7x^{-3})^2$ b. $\left(\frac{1}{5}\right)^{-3} - (2)^{-2} + 3^0$

c. $\left(\frac{y^3w^{10}}{y^5w^4}\right)^{-1}$ d. $\left(\frac{2a^7b^{-4}}{8a^9b^{-2}}\right)^{-3} (-6a^{-1}b^0)^{-2}$

Skill Practice Answers

1. 9 2. -9 3. $-\frac{1}{9}$
 4. 1 5. -1

Solution:

$$\text{a. } (x^7x^{-3})^2$$

$$= (x^{7+(-3)})^2$$

$$= (x^4)^2$$

$$= x^8$$

Multiply like bases by adding exponents.

Apply the power rule.

Multiply exponents.

$$\text{b. } \left(\frac{1}{5}\right)^{-3} - (2)^{-2} + 3^0$$

$$= 5^3 - \left(\frac{1}{2}\right)^2 + 3^0$$

$$= 125 - \frac{1}{4} + 1$$

$$= \frac{500}{4} - \frac{1}{4} + \frac{4}{4}$$

$$= \frac{503}{4}$$

Simplify negative exponents.

Evaluate the exponents.

Write the expressions with a common denominator.

Simplify.

$$\text{c. } \left(\frac{y^3w^{10}}{y^5w^4}\right)^{-1}$$

$$= (y^{3-5}w^{10-4})^{-1}$$

$$= (y^{-2}w^6)^{-1}$$

$$= (y^{-2})^{-1}(w^6)^{-1}$$

$$= y^2w^{-6}$$

$$= y^2\left(\frac{1}{w^6}\right) \text{ or } \frac{y^2}{w^6}$$

Work within the parentheses first.

Divide like bases by subtracting exponents.

Simplify within parentheses.

Apply the power rule.

Multiply exponents.

Simplify negative exponents.

$$\text{d. } \left(\frac{2a^7b^{-4}}{8a^9b^{-2}}\right)^{-3} (-6a^{-1}b^0)^{-2}$$

$$= \left(\frac{a^{7-9}b^{-4-(-2)}}{4}\right)^{-3} (-6a^{-1} \cdot 1)^{-2}$$

$$= \left(\frac{a^{-2}b^{-2}}{4}\right)^{-3} (-6a^{-1})^{-2}$$

$$= \left[\frac{(a^{-2})^{-3}(b^{-2})^{-3}}{4^{-3}}\right](-6)^{-2}(a^{-1})^{-2}$$

$$= \left(\frac{a^6b^6}{4^{-3}}\right)(-6)^{-2}a^2$$

$$= 4^3a^6b^6\left[\frac{a^2}{(-6)^2}\right]$$

$$= \frac{64a^8b^6}{36}$$

$$= \frac{16a^8b^6}{9}$$

Subtract exponents within first parentheses. Simplify $\frac{2}{8}$ to $\frac{1}{4}$.

In the second parentheses, replace b^0 by 1.

Simplify inside parentheses.

Apply the power rule.

Multiply exponents.

Simplify negative exponents.

Multiply factors in the numerator and denominator.

Simplify.

Skill Practice

Simplify the expressions. Write the final answers with positive exponents only.

$$6. (a^5b^{-3})^4 \qquad 7. \left(\frac{2}{3}\right)^{-1} + 4^{-1} - \left(\frac{1}{4}\right)^0$$

$$8. \left(\frac{2b^{-3}c^{-3}}{4b^{-2}c}\right)^3 \qquad 9. (x^3y^{-4})^2(x^{-2}y)^{-4}$$

3. Scientific Notation

Scientists in a variety of fields often work with very large or very small numbers. For instance, the distance between the Earth and the Sun is approximately 93,000,000 mi. The national debt in the United States in 2004 was approximately \$7,380,000,000,000. The mass of an electron is 0.000 000 000 000 000 000 000 000 000 911 kg.

Scientific notation was devised as a shortcut method of expressing very large and very small numbers. The principle behind scientific notation is to use a power of 10 to express the magnitude of the number. Consider the following powers of 10:

$$10^0 = 1$$

$$10^1 = 10 \qquad 10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$10^2 = 100 \qquad 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$10^3 = 1000 \qquad 10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$


$$10^4 = 10,000 \qquad 10^{-4} = \frac{1}{10^4} = \frac{1}{10,000} = 0.0001$$

Each power of 10 represents a place value in the base-10 numbering system. A number such as 50,000 may therefore be written as $5 \times 10,000$ or equivalently as 5.0×10^4 . Similarly, the number 0.0035 is equal to $3.5 \times \frac{1}{1000}$ or, equivalently, 3.5×10^{-3} .


Definition of a Number Written in Scientific Notation

A number expressed in the form $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer, is said to be written in **scientific notation**.

Consider the following numbers in scientific notation:

The distance between the Sun and the Earth: 93,000,000 mi = 9.3×10^7 mi


The national debt of the United States in 2004: \$7,380,000,000,000 = 7.38×10^{12}


The mass of an electron: 0.000 000 000 000 000 000 000 000 000 911 kg

 = 9.11×10^{-31} kg

Skill Practice Answers

$$6. \frac{a^{20}}{b^{12}} \qquad 7. \frac{3}{4}$$

$$8. \frac{1}{8b^3c^{12}} \qquad 9. \frac{x^{14}}{y^{12}}$$

In each case, the power of 10 corresponds to the number of place positions that the decimal point is moved. The power of 10 is sometimes called the order of magnitude (or simply the magnitude) of the number. The order of magnitude of the national debt is 10^{12} dollars (trillions). The order of magnitude of the distance between the Earth and Sun is 10^7 mi (tens of millions). The mass of an electron has an order of magnitude of 10^{-31} kg.

Example 3 Writing Numbers in Scientific Notation

Fill in the table by writing the numbers in scientific notation or standard notation as indicated.

Quantity	Standard Notation	Scientific Notation
Number of NASCAR fans	75,000,000 people	
Width of an influenza virus	0.000000001 m	
Cost of hurricane Andrew		$\$2.65 \times 10^{10}$
Probability of winning the Florida state lottery		$4.35587878 \times 10^{-8}$
Approximate width of a human red blood cell	0.000007 m	
Profit of Citigroup Bank, 2003		$\$1.53 \times 10^{10}$

Solution:

Quantity	Standard Notation	Scientific Notation
Number of NASCAR fans	75,000,000 people	7.5×10^7 people
Width of an influenza virus	0.000000001 m	1.0×10^{-9} m
Cost of hurricane Andrew	$\$26,500,000,000$	$\$2.65 \times 10^{10}$
Probability of winning the Florida state lottery	0.0000000435587878	$4.35587878 \times 10^{-8}$
Approximate width of a human red blood cell	0.000007 m	7.0×10^{-6} m
Profit of Citigroup Bank, 2003	$\$15,300,000,000$	$\$1.53 \times 10^{10}$

Skill Practice Write the numbers in scientific notation.

10. 2,600,000 11. 0.00088

Write the numbers in standard notation.

12. -5.7×10^{-8} 13. 1.9×10^5

Calculator Connections

Calculators use scientific notation to display very large or very small numbers. To enter scientific notation in a calculator, try using the **EE** key or the **EXP** key to express the power of 10.

9.3E7	93000000
7.25E-2	.0725

Skill Practice Answers

10. 2.6×10^6
 11. 8.8×10^{-4}
 12. -0.000000057
 13. 190,000

Example 4 Applying Scientific Notation

- a. The U.S. national debt in 2005 was approximately \$7,830,000,000,000. Assuming there were approximately 290,000,000 people in the United States at that time, determine how much each individual would have to pay to pay off the debt.
- b. The mean distance between the Earth and the Andromeda Galaxy is approximately 1.8×10^6 light-years. Assuming 1 light-year is 6.0×10^{12} mi, what is the distance in miles to the Andromeda Galaxy?

Solution:

- a. Divide the total U.S. national debt by the number of people:

$$\begin{aligned} & \frac{7.83 \times 10^{12}}{2.9 \times 10^8} \\ &= \left(\frac{7.83}{2.9}\right) \times \left(\frac{10^{12}}{10^8}\right) \quad \text{Divide 7.83 by 2.9 and subtract the powers of 10.} \\ &= 2.7 \times 10^4 \end{aligned}$$

In standard notation, this amounts to approximately \$27,000 per person.

- b. Multiply the number of light-years by the number of miles per light-year.

$$\begin{aligned} & (1.8 \times 10^6)(6.0 \times 10^{12}) \\ &= (1.8)(6.0) \times (10^6)(10^{12}) \\ &= 10.8 \times 10^{18} \end{aligned}$$

Multiply 1.8 and 6.0 and add the powers of 10.

The number 10.8×10^{18} is not in “proper” scientific notation because 10.8 is not between 1 and 10.

$$= (1.08 \times 10^1) \times 10^{18}$$

Rewrite 10.8 as 1.08×10^1 .

$$= 1.08 \times (10^1 \times 10^{18})$$

Apply the associative property of multiplication.

$$= 1.08 \times 10^{19}$$

The distance between the Earth and the Andromeda Galaxy is 1.08×10^{19} mi.

Skill Practice

14. The thickness of a penny is 6.1×10^{-2} in. The height of the Empire State Building is 1250 ft (1.5×10^4 in.). How many pennies would have to be stacked on top of each other to equal the height of the Empire State Building? Round to the nearest whole unit.
15. The distance from Earth to the “nearby” star, Barnard’s Star, is 7.6 light years (where 1 light year = 6.0×10^{12} mi). How many miles away is Barnard’s Star?

Calculator Connections

Use a calculator to check the solutions to Example 4.

```
(7.83E12)/(2.9E8)
)
27000
(1.8E6)*(6.0E12)
1.08E19
```

Skill Practice Answers

14. Approximately 245,902 pennies
15. 4.56×10^{13} mi

Section 1.8

Practice Exercises

Boost your GRADE at
mathzone.com!



- Practice Problems
- Self-Tests
- NetTutor
- e-Professors
- Videos

Study Skills Exercises

1. Write down the page number(s) for the Chapter Summary for this chapter. _____
Describe one way in which you can use the Summary found at the end of each chapter.
2. Define the key term **scientific notation**.

Review Exercises

For Exercises 3–6, solve the equation or inequality. Write the solutions to the inequalities in interval notation.

$$3. \frac{a-2}{3} - \frac{3a+2}{4} = -\frac{1}{2}$$

$$4. \frac{2y+6}{5} + 3 = \frac{-y+2}{2}$$

$$5. 6x - 2(x+3) \leq 7(x+1) - 4$$

$$6. -5c + 3(c+2) > 6c + 8$$

For Exercises 7–8, solve the equation for the indicated variable.

$$7. 5x - 9y = 11 \quad \text{for } x$$

$$8. -2x + 3y = -8 \quad \text{for } y$$

Concept 1: Properties of Exponents

9. Explain the difference between $b^4 \cdot b^3$ and $(b^4)^3$.
(Hint: Expand both expressions and compare.)
10. Explain the difference between ab^3 and $(ab)^3$.

For Exercises 11–16, write two examples of each property. Include examples with and without variables. (Answers may vary.)

$$11. b^n \cdot b^m = b^{n+m}$$

$$12. (ab)^n = a^n b^n$$

$$13. (b^n)^m = b^{nm}$$

$$14. \frac{b^n}{b^m} = b^{n-m} \quad (b \neq 0)$$

$$15. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$$

$$16. b^0 = 1 \quad (b \neq 0)$$

$$17. \text{Simplify: } \left(\frac{2}{3}\right)^{-2}$$

$$18. \text{Simplify: } \left(\frac{1}{3}\right)^{-2}$$

For Exercises 19–34, simplify.

$$19. 5^{-2}$$

$$20. 8^{-2}$$

$$21. -5^{-2}$$

$$22. -8^{-2}$$

$$23. (-5)^{-2}$$

$$24. (-8)^{-2}$$

$$25. \left(\frac{1}{4}\right)^{-3}$$

$$26. \left(\frac{3}{8}\right)^{-1}$$

$$27. \left(-\frac{3}{2}\right)^{-4}$$

$$28. \left(-\frac{1}{9}\right)^{-2}$$

$$29. -\left(\frac{2}{5}\right)^{-3}$$

$$30. -\left(\frac{1}{2}\right)^{-5}$$

$$31. (10ab)^0$$

$$32. (13x)^0$$

$$33. 10ab^0$$

$$34. 13x^0$$

Concept 2: Simplifying Expressions with Exponents

For Exercises 35–84, simplify and write the answer with positive exponents only.

- | | | | |
|---|---|--|--|
| 35. $y^3 \cdot y^5$ | 36. $x^4 \cdot x^8$ | 37. $\frac{13^8}{13^6}$ | 38. $\frac{5^7}{5^3}$ |
| 39. $(y^2)^4$ | 40. $(z^3)^4$ | 41. $(3x^2)^4$ | 42. $(2y^5)^3$ |
| 43. p^{-3} | 44. q^{-5} | 45. $7^{10} \cdot 7^{-13}$ | 46. $11^{-9} \cdot 11^7$ |
| 47. $\frac{w^3}{w^5}$ | 48. $\frac{t^4}{t^8}$ | 49. $a^{-2}a^{-5}$ | 50. $b^{-1}b^{-8}$ |
| 51. $\frac{r}{r^{-1}}$ | 52. $\frac{s^{-1}}{s}$ | 53. $\frac{z^{-6}}{z^{-2}}$ | 54. $\frac{w^{-8}}{w^{-3}}$ |
| 55. $\frac{a^3}{b^{-2}}$ | 56. $\frac{c^4}{d^{-1}}$ | 57. $(6xyz^2)^0$ | 58. $(-7ab^3)^0$ |
| 59. $2^4 + 2^{-2}$ | 60. $3^2 + 3^{-1}$ | 61. $1^{-2} + 5^{-2}$ | 62. $4^{-2} + 2^{-2}$ |
| 63. $\left(\frac{2}{3}\right)^{-2} - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^0$ | 64. $\left(\frac{4}{5}\right)^{-1} + \left(\frac{3}{2}\right)^2 - \left(\frac{2}{7}\right)^0$ | | |
| 65. $\frac{p^2q}{p^5q^{-1}}$ | 66. $\frac{m^{-1}n^3}{m^4n^{-2}}$ | 67. $\frac{-48ab^{10}}{32a^4b^3}$ | 68. $\frac{25x^2y^{12}}{10x^5y^7}$ |
| 69. $(-3x^{-4}y^5z^2)^{-4}$ | 70. $(-6a^{-2}b^3c)^{-2}$ | 71. $(4m^{-2}n)(-m^6n^{-3})$ | 72. $(-6pq^{-3})(2p^4q)$ |
| 73. $(p^{-2}q)^3(2pq^4)^2$ | 74. $(mn^3)^2(5m^{-2}n^2)$ | 75. $\left(\frac{x^2}{y}\right)^3(5x^2y)$ | 76. $\left(\frac{a}{b^2}\right)^2(3a^2b^3)$ |
| 77. $\frac{(-8a^2b^2)^4}{(16a^3b^7)^2}$ | 78. $\frac{(-3x^2y^3)^2}{(-2xy^4)^3}$ | 79. $\left(\frac{-2x^6y^{-5}}{3x^{-2}y^4}\right)^{-3}$ | 80. $\left(\frac{-6a^2b^{-3}}{5a^{-1}b}\right)^{-2}$ |
| 81. $\left(\frac{2x^{-3}y^0}{4x^6y^{-5}}\right)^{-2}$ | 82. $\left(\frac{a^3b^2c^0}{a^{-1}b^{-2}c^{-3}}\right)^{-2}$ | 83. $3xy^5\left(\frac{2x^4y}{6x^5y^3}\right)^{-2}$ | 84. $7x^{-3}y^{-4}\left(\frac{3x^{-1}y^5}{9x^3y^{-2}}\right)^{-3}$ |

Concept 3: Scientific Notation

- | | |
|---|--|
| 85. Write the numbers in scientific notation. | 86. Write the numbers in scientific notation. |
| a. Paper is 0.0042 in. thick. | a. The estimated population of the United States in 2007 is approximately 292,600,000. |
| b. One mole is 602,200,000,000,000,000,000 particles. | b. As of 2004, the net worth of Bill Gates was \$46,600,000,000. |
| c. The dissociation constant for nitrous acid is 0.00046. | c. A trillion is defined as 1,000,000,000,000. |

- 87.** Write the numbers in standard notation.
- The number of \$20 bills in circulation in 2004 was 5.2822×10^9 .
 - The dissociation constant for acetic acid is 1.8×10^{-5} .
 - In 2004, the population of the world was approximately 6.378×10^9 .
- 88.** Write the numbers in standard notation.
- The proposed budget for the 2006 federal government allocated $\$5.6 \times 10^{10}$ for the Department of Education.
 - The mass of a neutron is 1.67×10^{-24} g.
 - The number of \$2 bills in circulation in 2004 was 6.8×10^8 .

For Exercises 89–94, determine which numbers are in “proper” scientific notation. If the number is not in “proper” scientific notation, correct it.

- 89.** 35×10^4 **90.** 0.469×10^{-7} **91.** 7.0×10^0
- 92.** 8.12×10^1 **93.** 9×10^1 **94.** 6.9×10^0

For Exercises 95–102, perform the indicated operations and write the answer in scientific notation.

- 95.** $(6.5 \times 10^3)(5.2 \times 10^{-8})$ **96.** $(3.26 \times 10^{-6})(8.2 \times 10^9)$ **97.** $(0.0000024)(6,700,000,000)$
- 98.** $(3,400,000,000)(70,000,000,000,000)$ **99.** $(8.5 \times 10^{-2}) \div (2.5 \times 10^{-15})$ **100.** $(3 \times 10^9) \div (1.5 \times 10^{13})$
- 101.** $(900,000,000) \div (360,000)$ **102.** $(0.0000000002) \div (8,000,000)$

- 103.** If one H_2O molecule contains 2 hydrogen atoms and 1 oxygen atom, and 10 H_2O molecules contain 20 hydrogen atoms and 10 oxygen atoms, how many hydrogen atoms and oxygen atoms are contained in 6.02×10^{23} H_2O molecules?
- 104.** The star named Alpha Centauri is 4.3 light-years from the Earth. If there is approximately 6×10^9 mi in 1 light-year, how many miles away is Alpha Centauri?
- 105.** The county of Queens, New York, has a population of approximately 2,200,000. If the area is 110 mi^2 , how many people are there per square mile?
- 106.** The county of Catawba, North Carolina, has a population of approximately 150,000. If the area is 400 mi^2 , how many people are there per square mile?

Expanding Your Skills

For Exercises 107–112, simplify the expression. Assume that a and b represent positive integers and x and y are nonzero real numbers.

- 107.** $x^{a+1}x^{a+5}$ **108.** $y^{a-5}y^{a+7}$ **109.** $\frac{y^{2a+1}}{y^{a-1}}$
- 110.** $\frac{x^{3a-3}}{x^{a+1}}$ **111.** $\frac{x^{3b-2}y^{b+1}}{x^{2b+1}y^{2b+2}}$ **112.** $\frac{x^{2a-2}y^{a-3}}{x^{a+4}y^{a+3}}$

- 113.** At one count per second, how many days would it take to count to 1 million? (Round to 1 decimal place.)
- 114.** Do you know anyone who is more than 1.0×10^9 sec old? If so, who?
- 115.** Do you know anyone who is more than 4.5×10^5 hr old? If so, who?

Chapter 1

SUMMARY

Section 1.1

Sets of Numbers and Interval Notation

Key Concepts

Natural numbers: $\{1, 2, 3, \dots\}$

Whole numbers: $\{0, 1, 2, 3, \dots\}$

Integers: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational numbers: $\left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \text{ does not equal } 0 \right\}$

Irrational numbers: $\{x \mid x \text{ is a real number that is not rational}\}$

Real numbers: $\{x \mid x \text{ is rational or } x \text{ is irrational}\}$

$a < b$ “ a is less than b ”

$a > b$ “ a is greater than b ”

$a \leq b$ “ a is less than or equal to b ”

$a \geq b$ “ a is greater than or equal to b ”

$a < x < b$ “ x is between a and b ”

$A \cup B$ is the **union** of A and B and is the set of elements that belong to set A or set B or both sets A and B .

$A \cap B$ is the **intersection** of A and B and is the set of elements common to both A and B .

Examples

Example 1

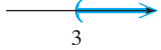

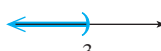

Some rational numbers are:

$\frac{1}{2}, 0.5, 0.\bar{3}, 12, -7$

Some irrational numbers are:

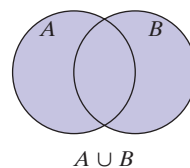
$-\sqrt{7}, \sqrt{2}, \pi$

Example 2

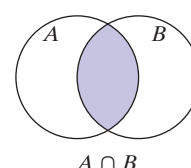
Set-Builder Notation	Interval Notation	Graph
$\{x \mid x > 3\}$	$(3, \infty)$	
$\{x \mid x \geq 3\}$	$[3, \infty)$	
$\{x \mid x < 3\}$	$(-\infty, 3)$	
$\{x \mid x \leq 3\}$	$(-\infty, 3]$	

Example 3

Union



Intersection



Section 1.2 Operations on Real Numbers

Key Concepts

The **reciprocal** of a number $a \neq 0$ is $\frac{1}{a}$.

The **opposite** of a number a is $-a$.

The **absolute value** of a , denoted $|a|$, is its distance from zero on the number line.

Addition of Real Numbers

Same Signs: Add the absolute values of the numbers, and apply the common sign to the sum.

Unlike Signs: Subtract the smaller absolute value from the larger absolute value. Then apply the sign of the number having the larger absolute value.

Subtraction of Real Numbers

Add the opposite of the second number to the first number.

Multiplication and Division of Real Numbers

Same Signs: Product or quotient is positive.

Opposite Signs: Product or quotient is negative.

The product of any real number and 0 is 0.

The quotient of 0 and a nonzero number is 0.

The quotient of a nonzero number and 0 is undefined.

Exponents and Radicals

$b^4 = b \cdot b \cdot b \cdot b$ (b is the **base**, 4 is the **exponent**)

\sqrt{b} is the **principal square root** of b (b is the radicand, $\sqrt{\quad}$ is the radical sign).

Order of Operations

1. Simplify expressions within parentheses and other grouping symbols first.
2. Evaluate expressions involving exponents, radicals and absolute values.
3. Perform multiplication or division in order from left to right.
4. Perform addition or subtraction in order from left to right.

Examples

Example 1

Given: -5

The reciprocal is $-\frac{1}{5}$. The opposite is 5.

The absolute value is 5.

Example 2

$$-3 + (-4) = -7$$

$$-5 + 7 = 2$$

Example 3

$$7 - (-5) = 7 + (5) = 12$$

Example 4

$$(-3)(-4) = 12 \quad \frac{-15}{-3} = 5$$

$$(-2)(5) = -10 \quad \frac{6}{-12} = -\frac{1}{2}$$

$$(-7)(0) = 0 \quad 0 \div 9 = 0$$

$-3 \div 0$ is undefined

Example 5

$$6^3 = 6 \cdot 6 \cdot 6 = 216$$

$$\sqrt{100} = 10$$

Example 6

$$\begin{aligned} & 10 - 5(3 - 1)^2 + \sqrt{16} \\ &= 10 - 5(2)^2 + \sqrt{16} \\ &= 10 - 5(4) + 4 \\ &= 10 - 20 + 4 \\ &= -10 + 4 \\ &= -6 \end{aligned}$$

Section 1.3 Simplifying Expressions

Key Concepts

A **term** is a constant or the product of a constant and one or more variables.

- A **variable term** contains at least one variable.
- A **constant term** has no variable.

The **coefficient** of a term is the numerical factor of the term.

Like terms have the same variables, and the corresponding variables are raised to the same powers.

Distributive Property of Multiplication over Addition

$$a(b + c) = ab + ac$$

Two terms can be added or subtracted if they are *like* terms. Sometimes it is necessary to clear parentheses before adding or subtracting *like* terms.

Examples

Example 1

$-2x$ Variable term has coefficient -2 .

x^2y Variable term has coefficient 1 .

6 Constant term has coefficient 6 .

Example 2

$4ab^3$ and $2ab^3$ are *like* terms.

Example 3

$$2(x + 4y) = 2x + 8y$$

$$-(a + 6b - 5c) = -a - 6b + 5c$$

Example 4

$$-4d + 12d + d$$

$$= 9d$$

Example 5

$$-2[w - 4(w - 2)] + 3$$

$$= -2[w - 4w + 8] + 3$$

$$= -2[-3w + 8] + 3$$

$$= 6w - 16 + 3$$

$$= 6w - 13$$

Section 1.4 Linear Equations in One Variable

Key Concepts

A **linear equation in one variable** can be written in the form $ax + b = 0$ ($a \neq 0$).

Steps to Solve a Linear Equation in One Variable

- Simplify both sides of the equation.
 - Clear parentheses.
 - Consider clearing fractions or decimals (if any are present) by multiplying both sides of the equation by a common denominator of all terms.
 - Combine *like* terms.
- Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.
- Use the addition or subtraction property of equality to collect the constant terms on the other side.
- Use the multiplication or division property of equality to make the coefficient on the variable term equal to 1.
- Check your answer.

An equation that has no solution is called a **contradiction**.

An equation that has all real numbers as its solutions is called an **identity**.

Examples

Example 1

$$\begin{aligned}\frac{1}{2}(x - 4) - \frac{3}{4}(x + 2) &= \frac{1}{4} \\ \frac{1}{2}x - 2 - \frac{3}{4}x - \frac{3}{2} &= \frac{1}{4} \\ 4\left(\frac{1}{2}x - 2 - \frac{3}{4}x - \frac{3}{2}\right) &= 4\left(\frac{1}{4}\right) \\ 2x - 8 - 3x - 6 &= 1 \\ -x - 14 &= 1 \\ -x &= 15 \\ x &= -15\end{aligned}$$

Example 2

$$\begin{aligned}3x + 6 &= 3(x - 5) \\ 3x + 6 &= 3x - 15 \\ 6 &= -15 \quad \text{Contradiction}\end{aligned}$$

There is no solution.

Example 3

$$\begin{aligned}-(5x + 12) - 3 &= 5(-x - 3) \\ -5x - 12 - 3 &= -5x - 15 \\ -5x - 15 &= -5x - 15 \\ -15 &= -15 \quad \text{Identity}\end{aligned}$$

All real numbers are solutions.

Section 1.5

Applications of Linear Equations in One Variable

Key Concepts

Problem-Solving Steps for Word Problems

1. Read the problem carefully.
2. Assign labels to unknown quantities.
3. Develop a verbal model.
4. Write a mathematical equation.
5. Solve the equation.
6. Interpret the results and write the final answer in words.

Sales tax: (Cost of merchandise)(tax rate)

Commission: (Dollars in sales)(commission rate)

Simple interest: $I = Prt$

Distance = (rate)(time) $d = rt$

Examples

Example 1

1. Estella has \$8500 to invest between two accounts, one bearing 6% simple interest and the other bearing 10% simple interest. At the end of 1 year, she has earned \$750 in interest. Find the amount Estella has invested in each account.
2. Let x represent the amount invested at 6%. Then $8500 - x$ is the amount invested at 10%.

	6% Account	10% Account	Total
Principal	x	$8500 - x$	8500
Interest	$0.06x$	$0.10(8500 - x)$	750

$$3. \left(\begin{array}{l} \text{Interest from} \\ 6\% \text{ account} \end{array} \right) + \left(\begin{array}{l} \text{interest from} \\ 10\% \text{ account} \end{array} \right) = \left(\begin{array}{l} \text{total} \\ \text{interest} \end{array} \right)$$

$$4. 0.06x + 0.10(8500 - x) = 750$$

$$5. \begin{array}{r} 6x + 10(8500 - x) = 75,000 \\ 6x + 85,000 - 10x = 75,000 \\ -4x = -10,000 \\ x = 2500 \end{array}$$

$$6. \begin{array}{r} x = 2500 \\ 8500 - x = 6000 \end{array}$$

\$2500 was invested at 6% and \$6000 was invested at 10%.

Section 1.6 Literal Equations and Applications to Geometry

Key Concepts

Some useful formulas for word problems:

Perimeter

Rectangle: $P = 2l + 2w$

Area

Rectangle: $A = lw$

Square: $A = x^2$

Triangle: $A = \frac{1}{2}bh$

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Angles

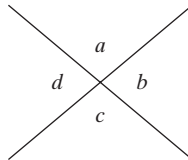
Two angles whose measures total 90° are complementary angles.

Two angles whose measures total 180° are supplementary angles.

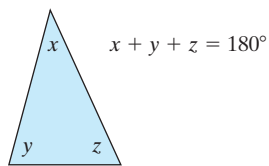
Vertical angles have equal measure.

$$m(\angle a) = m(\angle c)$$

$$m(\angle b) = m(\angle d)$$



The sum of the angles of a triangle is 180° .

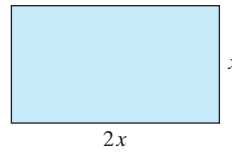


Literal equations (or formulas) are equations with several variables. To solve for a specific variable, follow the steps to solve a linear equation.

Examples

Example 1

A border of marigolds is to enclose a rectangular flower garden. If the length is twice the width and the perimeter is 25.5 ft, what are the dimensions of the garden?



$$P = 2l + 2w$$

$$25.5 = 2(2x) + 2(x)$$

$$25.5 = 4x + 2x$$

$$25.5 = 6x$$

$$4.25 = x$$

The width is 4.25 ft, and the length is $2(4.25)$ ft or 8.5 ft.

Example 2

Solve for y .

$$4x - 5y = 20$$

$$-5y = -4x + 20$$

$$\frac{-5y}{-5} = \frac{-4x + 20}{-5}$$

$$y = \frac{-4x + 20}{-5} \text{ or } y = \frac{4}{5}x - 4$$

Section 1.7

Linear Inequalities in One Variable

Key Concepts

A **linear inequality** in one variable can be written in the form

$$ax + b < 0, \quad ax + b > 0, \quad ax + b \leq 0, \quad \text{or} \\ ax + b \geq 0$$

Properties of Inequalities

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$, then $a - c < b - c$.
3. If c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$ ($c \neq 0$).
4. If c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$ ($c \neq 0$).

Properties 3 and 4 indicate that if we multiply or divide an inequality by a negative value, the direction of the inequality sign must be reversed.

The inequality $a < x < b$ is represented by



or, in interval notation, (a, b) .

Examples

Example 1

Solve.

$$\frac{14 - x}{-2} < -3x$$

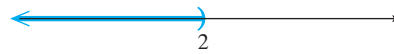
$$-2\left(\frac{14 - x}{-2}\right) > -2(-3x) \quad \text{(Reverse the inequality sign.)}$$

$$14 - x > 6x$$

$$-7x > -14$$

$$\frac{-7x}{-7} < \frac{-14}{-7} \quad \text{(Reverse the inequality sign.)}$$

$$x < 2$$



Interval notation: $(-\infty, 2)$

Example 2

$$-13 \leq 3x - 1 < 5$$

$$-13 + 1 \leq 3x - 1 + 1 < 5 + 1$$

$$-12 \leq 3x < 6$$

$$\frac{-12}{3} \leq \frac{3x}{3} < \frac{6}{3}$$

$$-4 \leq x < 2$$



$[-4, 2)$

Section 1.8

Properties of Integer Exponents and Scientific Notation

Key Concepts

Let a and b ($b \neq 0$) represent real numbers and m and n represent positive integers.

$$b^m \cdot b^n = b^{m+n} \quad \frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^n = b^{mn} \quad (ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad b^0 = 1$$

$$b^{-n} = \left(\frac{1}{b}\right)^n$$

A number expressed in the form $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer, is written in **scientific notation**.

Examples

Example 1

$$\begin{aligned} & \left(\frac{2x^2y}{z^{-1}}\right)^{-3} (x^{-4}y^0) \\ &= \left(\frac{2^{-3}x^{-6}y^{-3}}{z^3}\right)(x^{-4} \cdot 1) \\ &= \frac{2^{-3}x^{-10}y^{-3}}{z^3} \\ &= \frac{1}{2^3x^{10}y^3z^3} \quad \text{or} \quad \frac{1}{8x^{10}y^3z^3} \end{aligned}$$

Example 2

$$\begin{aligned} & 0.0000002 \times 35,000 \\ &= (2.0 \times 10^{-7})(3.5 \times 10^4) \\ &= 7.0 \times 10^{-3} \text{ or } 0.007 \end{aligned}$$

Chapter 1

Review Exercises

Section 1.1

- Find a number that is a whole number but not a natural number.

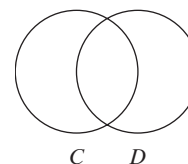
For Exercises 2–3, answers may vary.

- List three rational numbers that are not integers.
- List five integers, two of which are not whole numbers.

For Exercises 4–9, write an expression in words that describes the set of numbers given by each interval. (Answers may vary.)

- $(7, 16)$
- $(0, 2.6]$
- $[-6, -3]$
- $(8, \infty)$
- $(-\infty, 13]$
- $(-\infty, \infty)$

- Explain the difference between the union and intersection of two sets. You may use the sets C and D in the following diagram to provide an example.



Let $A = \{x \mid x < 2\}$, $B = \{x \mid x \geq 0\}$, and $C = \{x \mid -1 < x < 5\}$. For Exercises 11–16, graph each set and write the set in interval notation.

- A
_____→
- B
_____→
- C
_____→

14. $A \cap B$



15. $B \cap C$



16. $A \cup B$



17. True or false? $x < 3$ is equivalent to $3 > x$

18. True or false? $-2 \leq x < 5$ is equivalent to $5 > x \geq -2$

Section 1.2

For Exercises 19–20, find the opposite, reciprocal, and absolute value.

19. -8

20. $\frac{4}{9}$

For Exercises 21–22, simplify the exponents and the radicals.

21. $4^2, \sqrt{4}$

22. $25^2, \sqrt{25}$

For Exercises 23–32, perform the indicated operations.

23. $6 + (-8)$

24. $(-2) + (-5)$

25. $8(-2.7)$

26. $(-1.1)(7.41)$

27. $\frac{5}{8} \div \left(-\frac{13}{40}\right)$

28. $\left(-\frac{1}{4}\right) \div \left(-\frac{11}{16}\right)$

29. $\frac{2 - 4(3 - 7)}{-4 - 5(1 - 3)}$

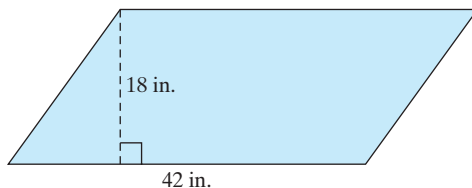
30. $\frac{12(2) - 8}{4(-3) + 2(5)}$

31. $3^2 + 2(|-10 + 5| \div 5)$

32. $-91 + \sqrt{4}(\sqrt{25} - 13)^2$

33. Given $h = \frac{1}{2}gt^2 + v_0t + h_0$, find h if $g = -32 \text{ ft/sec}^2$, $v_0 = 64 \text{ ft/sec}$, $h_0 = 256 \text{ ft}$, and $t = 4 \text{ sec}$.

34. Find the area of a parallelogram with base 42 in. and height 18 in.



Section 1.3

For Exercises 35–38, apply the distributive property and simplify.

35. $3(x + 5y)$

36. $\frac{1}{2}(x + 8y - 5)$

37. $-(-4x + 10y - z)$

38. $-(13a - b - 5c)$

For Exercises 39–42, clear parentheses if necessary, and combine *like* terms.

39. $5 - 6q + 13q - 19$

40. $18p + 3 - 17p + 8p$

41. $7 - 3(y + 4) - 3y$

42. $\frac{3}{4}(8x - 4) + \frac{1}{2}(6x + 4)$

For Exercises 43–44, answers may vary.

43. Write an example of the commutative property of addition.

44. Write an example of the associative property of multiplication.

Section 1.4

45. Describe the solution set for a contradiction.

46. Describe the solution set for an identity.

For Exercises 47–56, solve the equations and identify each as a conditional equation, a contradiction, or an identity.

47. $x - 27 = -32$

48. $y + \frac{7}{8} = 1$

49. $7.23 + 0.6x = 0.2x$

50. $0.1y + 1.122 = 5.2y$

51. $-(4 + 3m) = 9(3 - m)$

52. $-2(5n - 6) = 3(-n - 3)$

53. $\frac{x - 3}{5} - \frac{2x + 1}{2} = 1$

54. $3(x + 3) - 2 = 3x + 2$

$$55. \frac{10}{8}m + 18 - \frac{7}{8}m = \frac{3}{8}m + 25$$

$$56. \frac{2}{3}m + \frac{1}{3}(m - 1) = -\frac{1}{3}m + \frac{1}{3}(4m - 1)$$

Section 1.5

57. Explain how you would label three consecutive integers.

58. Explain how you would label two consecutive odd integers.

59. Explain what the formula $d = rt$ means.

60. Explain what the formula $I = Prt$ means.

61. To do a rope trick, a magician needs to cut a piece of rope so that one piece is one-third the length of the other piece. If she begins with a $2\frac{2}{3}$ -ft rope, what lengths will the two pieces of rope be?

62. Of three consecutive even integers, the sum of the smallest two integers is equal to 6 less than the largest. Find the integers.

63. Pat averages a rate of 11 mph on his bike. One day he rode for 45 min ($\frac{3}{4}$ hr) and then got a flat tire and had to walk back home. He walked the same path that he rode and it took him 2 hr. What was his average rate walking?

64. How much 10% acid solution should be mixed with a 25% acid solution to produce 3 L of a solution that is 15% acid?

65. Sharyn invests \$2000 more in an account that earns 9% simple interest than she invests in an account that earns 6% simple interest. How much did she invest in each account if her total interest is \$405 after 1 year?

66. In 2003, approximately 7.2 million men were in college in the United States. This represents an 8% increase over the number of men in college in 2000. Approximately how many men were in college in 2000? (Round to the nearest tenth of a million.)

67. In 2002, there were 17,430 deaths due to alcohol-related accidents in the United States. This was a 5% increase over the number of alcohol-related deaths in 1999. How many such deaths were there in 1999?

68. a. Cory made \$30,403 in taxable income in 2007. If he pays 28% in federal income tax, determine the amount of tax he must pay.

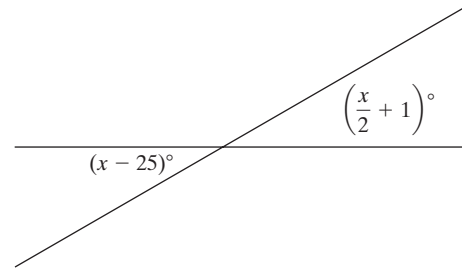
b. What is his net income (after taxes)?

Section 1.6

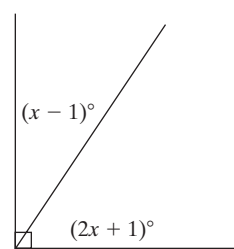
69. The length of a rectangle is 2 ft more than the width. Find the dimensions if the perimeter is 40 ft.

For Exercises 70–71, solve for x , and then find the measure of each angle.

70.



71.



For Exercises 72–75, solve for the indicated variable.

72. $3x - 2y = 4$ for y

73. $-6x + y = 12$ for y

74. $S = 2\pi r + \pi r^2 h$ for h

75. $A = \frac{1}{2}bh$ for b

76. a. The circumference of a circle is given by $C = 2\pi r$. Solve this equation for π .

b. Tom measures the radius of a circle to be 6 cm and the circumference to be 37.7 cm. Use these values to approximate π . (Round to 2 decimal places.)

Section 1.7

For Exercises 77–85, solve the inequality. Graph the solution and write the solution set in interval notation.

77. $-6x - 2 > 6$ \longrightarrow

78. $-10x \leq 15$ \longrightarrow

79. $-2 \leq 3x - 9 \leq 15$ \longrightarrow

80. $5 - 7(x + 3) > 19x$ \longrightarrow

81. $4 - 3x \geq 10(-x + 5)$ \longrightarrow

82. $\frac{5 - 4x}{8} \geq 9$ \longrightarrow

83. $\frac{3 + 2x}{4} \leq 8$ \longrightarrow

84. $3 > \frac{4 - q}{2} \geq -\frac{1}{2}$ \longrightarrow

85. $-11 < -5z - 2 \leq 0$ \longrightarrow

86. One method to approximate your maximum heart rate is to subtract your age from 220. To maintain an aerobic workout, it is recommended that you sustain a heart rate of between 60% and 75% of your maximum heart rate.

- If the maximum heart rate h is given by the formula $h = 220 - A$, where A is a person's age, find your own maximum heart rate. (Answers will vary.)
- Find the interval for your own heart rate that will sustain an aerobic workout. (Answers will vary.)

Section 1.8

For Exercises 87–94, simplify the expression and write the answer with positive exponents.

87. $(3x)^3(3x)^2$

88. $(-6x^{-4})(3x^{-8})$

89. $\frac{24x^5y^3}{-8x^4y}$

90. $\frac{-18x^{-2}y^3}{-12x^{-5}y^5}$

91. $(-2a^2b^{-5})^{-3}$

92. $(-4a^{-2}b^3)^{-2}$

93. $\left(\frac{-4x^4y^{-2}}{5x^{-1}y^4}\right)^{-4}$

94. $\left(\frac{25x^2y^{-3}}{5x^4y^{-2}}\right)^{-5}$

95. Write the numbers in scientific notation.

- The population of Asia was 3,686,600,000 in 2000.
- A nanometer is 0.000001 of a millimeter.

96. Write the numbers in scientific notation.

- A millimeter is 0.001 of a meter.
- The population of Asia is predicted to be 5,155,700,000 by 2040.

97. Write the numbers in standard form.

- A micrometer is 1×10^{-3} of a millimeter.
- A nanometer is 1×10^{-9} of a meter.

98. Write the numbers in standard form.

- The total square footage of shopping centers in the United States is approximately 5.23×10^9 ft².
- The total sales of those shopping centers is $\$1.091 \times 10^{12}$. (Source: International Council of Shopping Centers.)

For Exercises 99–102, perform the indicated operations. Write the answer in scientific notation.

99. $\frac{2,500,000}{0.0004}$

100. $\frac{0.0005}{25,000}$

101. $(3.6 \times 10^8)(9.0 \times 10^{-2})$

102. $(7.0 \times 10^{-12})(5.2 \times 10^3)$

Chapter 1

Test

1. a. List the integers between -5 and 2 , inclusive.
 b. List three rational numbers between 1 and 2 . (Answers may vary.)

2. Explain the difference between the intervals $(-3, 4)$ and $[-3, 4]$.

3. Graph the sets and write each set in interval notation.

- a. All real numbers less than 6

_____→

- b. All real numbers at least -3

_____→

4. Given sets $A = \{x \mid x < -2\}$ and $B = \{x \mid x \geq -5\}$, graph $A \cap B$ and write the set in interval notation.

_____→

5. Write the opposite, reciprocal, and absolute value for each of the numbers.

- a. $-\frac{1}{2}$ b. 4 c. 0

6. Simplify. $|-8| - 4(2 - 3)^2 \div \sqrt{4}$

7. Given $z = \frac{x - \mu}{\sigma/\sqrt{n}}$, find z when $n = 16$, $x = 18$, $\sigma = 1.8$, and $\mu = 17.5$. (Round the answer to 1 decimal place.)

8. True or false?

- a. $(x + y) + 2 = 2 + (x + y)$ is an example of the associative property of addition.
 b. $(2 \cdot 3) \cdot 5 = (3 \cdot 2) \cdot 5$ is an example of the commutative property of multiplication.
 c. $(x + 3)4 = 4x + 12$ is an example of the distributive property.
 d. $(10 + y) + z = 10 + (y + z)$ is an example of the associative property of addition.

9. Simplify the expressions.

a. $5b + 2 - 7b + 6 - 14$

b. $\frac{1}{2}(2x - 1) - \left(3x - \frac{3}{2}\right)$

For Exercises 10–13, solve the equations.

10. $\frac{x}{7} + 1 = 20$

11. $8 - 5(4 - 3z) = 2(4 - z) - 8z$

12. $0.12(x) + 0.08(60,000 - x) = 10,500$

13. $\frac{5 - x}{6} - \frac{2x - 3}{2} = \frac{x}{3}$

14. Label each equation as a conditional equation, an identity, or a contradiction.

a. $(5x - 9) + 19 = 5(x + 2)$

b. $2a - 2(1 + a) = 5$

c. $(4w - 3) + 4 = 3(5 - w)$

15. The difference between two numbers is 72 . If the larger is 5 times the smaller, find the two numbers.

16. Joëlle is determined to get some exercise and walks to the store at a brisk rate of 4.5 mph. She meets her friend Yun Ling at the store, and together they walk back at a slower rate of 3 mph. Joëlle's total walking time was 1 hr.

- a. How long did it take her to walk to the store?
 b. What is the distance to the store?

17. Shawna banks at a credit union. Her money is distributed between two accounts: a certificate of deposit (CD) that earns 5% simple interest and a savings account that earns 3.5% simple interest. Shawna has $\$100$ less in her savings account than in the CD. If after 1 year her total interest is $\$81.50$, how much did she invest in the CD?

18. A yield sign is in the shape of an equilateral triangle (all sides have equal length). Its perimeter is 81 in. Find the length of the sides.

For Exercises 19–20, solve the equations for the indicated variable.

19. $4x + 2y = 6$ for y 20. $x = \mu + z\sigma$ for z

For Exercises 21–23, solve the inequalities. Graph the solution and write the solution set in interval notation.

21. $x + 8 > 42$



22. $-\frac{3}{2}x + 6 \geq x - 3$



23. $-2 < 3x - 1 \leq 5$



24. An elevator can accommodate a maximum weight of 2000 lb. If four passengers on the elevator have an average weight of 180 lb each, how many additional passengers of the same average weight can the elevator carry before the maximum weight capacity is exceeded?

For Exercises 25–28, simplify the expression, and write the answer with positive exponents only.

25. $\frac{20a^7}{4a^{-6}}$

26. $\frac{x^6x^3}{x^{-2}}$

27. $\left(\frac{-3x^6}{5y^7}\right)^2$

28. $\frac{(2^{-1}xy^{-2})^{-3}(x^{-4}y)}{(x^0y^5)^{-1}}$

29. Multiply. $(8.0 \times 10^{-6})(7.1 \times 10^5)$
30. Divide. (Write the answer in scientific notation.)
 $(9,200,000) \div (0.004)$