UNIT 1 INTRODUCTION TO INFERENTIAL STATISTICS*

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1.0 OBJECTIVES

After reading this unit, you will be able to:

- discuss the concept and meaning of inferential statistics;
- describe inferential procedures; and
- explain the procedure for testing hypothesis.

1.1 INTRODUCTION

To refresh your memory with regard to what you learned in BPCC104: Statistical Methods for Psychological Research- I and to set a background for further discussion and explanation for the present course, that is, BPCC108: Statistical Methods for Psychological Research- II, let us focus on the following points.

Statistics can be described as a branch or sub-field of mathematics that mainly deals with the organisation as well as analysis and interpretation of a group of numbers (Aron, Aron and Coups, 2009).

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1. Statistics as a subject area has a vast scope and application. It finds its application in fields like policy planning, management, education, marketing, agriculture, medicine and so on, though, one of its major application is in research.

2. Statistics can be categorised in to descriptive and inferential statistics. In BPCC 104, we discussed in detail about descriptive statistics (and also briefly touched upon inferential statistics) and its techniques. In the present unit, we will mainly focus on inferential statistics and in subsequent units we will discuss various statistical techniques under inferential statistics.

BOX 1.1: Revisiting Descriptive Statistics

Descriptive statistics mainly comprises of description and organisation of the data. It can be termed as a technique that helps in summarisation of prominent characteristics of a distribution. Based on the properties of the sample, the descriptive statistics can be categorised in to the following (Mohanty and Misra, 2016, page 7):

- **Statistics of location:** Covers techniques like measures of central tendency including mean, median and mode, frequency distribution, percentiles and so on.
- **Statistics of dispersion:** Covers techniques related to measures of dispersion including quartile deviation, standard deviation, range, average deviation and variance.
- **Statistics of correlation:** Includes coefficients of correlation like Pearson's product moment correlation, Spearman's rank order correlation and Kendall's rank correlation. Correlation mainly helps us understand the relationship between variables.

Refer to Box 1.1, that gives a brief description about descriptive statistics.

As you may recall, in BPCC104: Statistics for Psychological Research- I, we discussed about the techniques mentioned in the box 1.1. We learned about measures of central tendency including mean, median and mode, frequency distribution, percentiles. We also focused on measures of dispersion including quartile deviation, standard deviation, range, average deviation and variance. Correlation was also discussed in the course.

These techniques are relevant mainly in univariate analysis of data, that is, when there is one variable. But when we want to carry out bivariate analysis (where there are two variables) or when we have multiple independent variables and dependent variables, where we want to study cause and effect relationship and so on, we could use inferential statistics.

In BPCC104: Statistics for Psychological Research- I, we briefly explained inferential statistics. Let us once again discuss the same in this unit as this unit forms the foundation to the units that we will subsequently discuss in this course.

1.2 CONCEPT AND MEANING OF INFERENTIAL STATISTICS

Let us start our discussion with some examples.

Example 1: A Researcher wanted to study whether significant difference exists in the organisational citizenship behavior of junior and senior manager in a Multinational Company (MNC). For this purpose the researcher selected a sample of 100 each (that is, 100 junior managers and 100 senior managers) from an MNC. The selected sample was then administered the Organisational Citizenship Scale and data was obtained.

Example 2: A clinical psychologist wanted to study the effectiveness of a new psychotherapy (Therapy A) on patients diagnosed with depression. The clinical psychologist therefore administered the Becks' Depression Inventory on a group (N= 50) of patients diagnosed with depression, which was followed by six months intervention based on Therapy A and then after six months the same patients were again administered Becks' Depression Inventory to study the effect of the Therapy A. In this research the Pretest-Posttest design was used.

Example 3: Yet another researcher carried out a study on emotional maturity of early, middle and late adolescents. The researcher collected data from the three groups (200 from each group) with the help of Emotional Maturity Scale.

For the above examples, different statistical techniques, that necessarily fall under Inferential statistics can be used. For instance, with regard to the first example, the researcher could use Independent t test, as there are two groups and significant difference between the two groups with regard to organisational citizenship behaviour needs to be studied. Though if the assumptions of parametric tests are not fulfilled then Mann- Whitney U test can also be used.

With regard to the example 2, the paired t test can be used if the assumptions of parametric statistics are met. This is so because the significant difference between a same group, pretest and posttest, is studied.

With regard to the last example (that is, example 3), to carry out data analysis One Way Analysis of Variance (this is a parametric test) or Kruskal- Wallis One Way ANOVA (this is nonparametric test) can be used.

Above we mentioned about parametric and non parametric tests. the same will be discussed in details by us in unit 4 of this course.

Let us now focus on what is inferential statistics.

In inferential statistics, inferences are drawn about the population based on a representative sample. After the data is collected, it is organised and summarised and once this is done, inferential statistics can be carried out in order to analyse the data and draw conclusions and make inferences. Thus, with the help of inferential statistics, inferences can be drawn about the population of the study based on the characteristics of the sample on whom the study was carried out (Salkind, 2014).

As stated by Veeraraghavan and Shetgovekar (2016, page 5) “Inferential statistics refers to the mathematical methods based on probability theory and helps in reasoning and inferring the characteristic features of the sample drawn from the larger population”. Inferential statistics can also be effectively used to make estimations and predictions.

As stated by Aron, Aron and Coups (2013, page 2), inferential statistics is employed by psychologists in order to make inferences and draw conclusions based on certain data. Further, inferential statistics though computed based on descriptive statistics of a given sample, it goes beyond the sample and it helps in generalisation of inferences to the whole population (Mohanty and Mishra, 2016, page 8). King and Minium (2008, page, 4) described that “the purpose of inferential statistics is to draw conclusions about the conditions that exist in a population from study of a sample”. The process of inferential statistics can be explained with the help of the following flowchart (refer to figure 1.1).

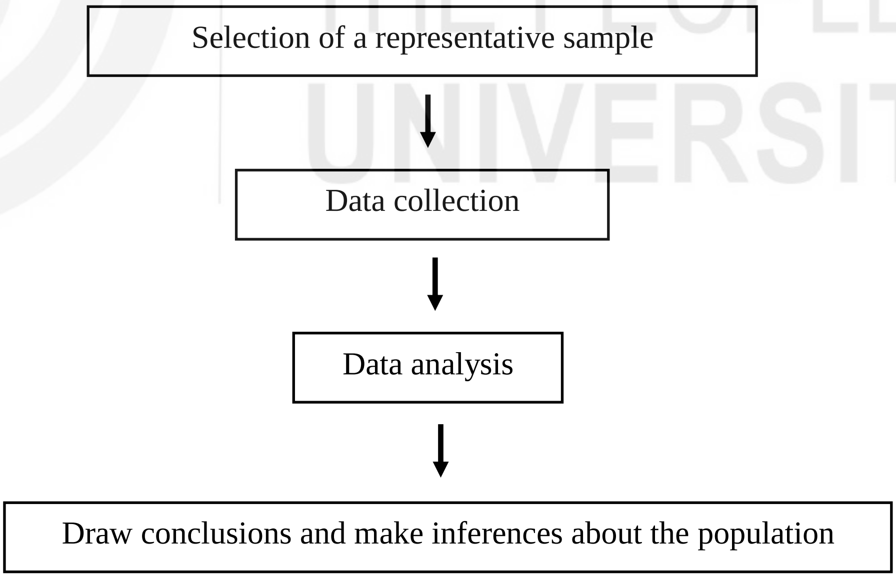

Fig 1.1: Process of Inferential Statistics

Thus, for example, if we want to study the environmental attitude of adolescents in Mumbai, we will take a representative sample (N =500) from the population of adolescents in Mumbai. We will then administer a standardised psychological test that measures environmental attitude to the selected adolescents. Once the data is collected, it is organised and then analysed using inferential statistics and then conclusions are drawn and inferences are made for the population of adolescents in Mumbai.

Check Your Progress I

1) What is inferential statistics?

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1.3 INFERENTIAL PROCEDURES

There are two types of procedures under inferential statistics, namely estimation and hypothesis testing. These two are discussed in detail as follows:

1.3.1 Estimation

Estimating probability of a phenomenon is referred to as estimation (Veeraraghavan and Shetgovekar, 2016). As we know from the explanation of inferential statistics, that inferences are drawn from sample that is representative of a population and these inferences can then be generalised to the whole population. In these inferences, the researcher will make an estimation that needs to be close to the actual or true population value.

There are two types of estimation: point estimation and interval estimation.

Point estimation: This is a type of estimation in which the value is a single point. For example, the estimation for sample mean is made as 46.8 that is expected to be equal to the population mean. Point estimate comprises of sample mean and sample proportion. The population mean is 'μ', the sample mean will be '$\overline{x}$'. In similar manner, if the population proportion is '*P'* then sample proportion will be '*p'*.

Interval estimation: An interval estimate is an interval or two numbers within which the population parameter could lie. Thus, for population mean 'μ', the interval estimate will be a<x< b. The interval estimate is greater than 'a' but lesser than 'b'. For example, an interval estimate could be 45- 47 within which it is expected that the population mean will lie. As the researcher has an interval, he/ she is thus able to trust that the estimate is close to the population value with 95% or 99% level of confidence. Interval estimate comprises of confidence interval for mean and confidence interval for proportions.

While estimations are made there could be fluctuations and these could be due to varied reasons including chance factors and sampling error.

The inferences drawn by the researcher needs to be free of any chance factors. For example, a researcher is studying if there exists significant difference in job satisfaction of government and private bank employees. After carrying out data collection and data analysis, he/she obtains results that such a difference does exist, then such results should not be an outcome of chance factors. If the difference falls within the range ± 1.96, then the significant difference can be said to be real and not due to chance factors.

Fluctuations can also be as a result of sampling error that occur when the sample selected by the researcher is not representative of the population being studied. A sample that is not representative of the population will not possess the same characteristics as the population and thus the results obtained from such a sample cannot be used to draw inferences for the population. Sampling errors can be avoided by being careful while selecting a sample and also by having a larger sample.

1.3.2 Hypothesis Testing

One of the significant aspects of research is hypothesis testing. Hypothesis is a tentative statement that is investigated during the process of research. Hypothesis pertains to certain phenomenon and is based on a theory. Data is then collected by the researcher in order to validate the hypothesis. Thus, the hypothesis is rejected or accepted by a researcher based on the results obtained. Hypothesis testing is referred to as "a process involving statistical decision making with regard to population value that is based on the sample value" (Veeraraghavan and Shetgovekar, 2016, page 9). Though, while drawing inferences one needs to ensure that hypothesis is not incorrectly accepted or rejected due to results that are influenced by chance factor or confounding factors.

1.3.2.1 Types of Hypothesis

There are two types of hypothesis:

Null hypothesis: Once a researcher finalises his/her research topic and variables after reviewing literature, he/ she will have to formulate hypothesis based on the problem statement. In a research attempts are made by the researcher to reject the null hypothesis(es). Null hypothesis is also referred to as hypothesis of 'no difference' as null hypothesis implies that there is no difference between two groups. It is denoted by H_o. For example, if a researcher wants to study if there is significant difference in job satisfaction of government and private bank employees, he/ she will formulate a null hypothesis as 'No significant difference exists in job satisfaction of public and private bank employees'. And then based on the results obtained the hypothesis will be rejected or accepted.

Alternative hypothesis (also called as research hypothesis)***:*** Alternative hypothesis can be termed as a counter proposition to the null hypothesis (Veeraraghavan and Shetgovekar, 2016). It is formulated by a researcher based on a theory. The hypothesis states that there is a significant difference between two groups. It is denoted by H_a. For example, the researcher can formulate the alternative hypothesis as 'There will be a significant difference in job satisfaction of public and private bank employees'. This is also a neutral alternative hypothesis that does not provide any direction and thus is termed as nondirectional hypothesis. Alternative hypothesis can also be directional. For example, 'Public bank employee have higher (or lower) job satisfaction when compared with private bank employees'. Such hypotheses are formulated based on the review of literature.

Difference between null and alternative hypothesis: The difference between null and alternative hypothesis (Salkind, 2014, page 139) are discussed as follows:

1) The very first difference is that the alternative hypothesis is about inequality, where it is stated that a relationship exists between the given two variables. Whereas, null hypothesis is about equality where it is stated that there is no relationship between the given two variables.

1) Further, the alternative hypothesis refers to the sample and the null hypothesis refers to the population.

2) The third difference is that the null hypothesis is to be tested indirectly and the alternative hypothesis is to be tested directly. This is because we make inferences about the population based on the sample.

3) The fourth difference is quite interesting, as the alternative hypothesis are usually written using Roman symbols, whereas null hypothesis are written using Greek symbols.

For instance

The alternative hypothesis is H_1: $\bar{x}_1 < \bar{x}_2$

The null hypothesis is H_O: $\mu_1 = \mu_2$

If you recall, what we have learned in BPCC104, μ('mu') is a symbol for parameter mean, whereas, $\bar{x}$ is a symbol for sample mean.

4) The alternative hypothesis is an explicit hypothesis, whereas, the null hypothesis is an implied hypothesis, mainly because it cannot be directly tested.

Characteristics of a good hypothesis:Let us now discuss the characteristics of a good hypothesis

1. The statement of hypothesis is not stated as a question but is in a declarative form.
2. It states a relationship that is expected between the given variables.
3. The theory or literature on the basis of which the hypothesis is formulated is reflected in that hypothesis.

4. The hypothesis needs to be clear, to the point as well as brief.

5. It needs to be possible to test the hypothesis.

1.3.2.2 Level of Significance

The term significance in the context of statistics implies ‘probably true’ that indicates that results are free from chance factor at the specified level of significance. This level of significance could be 0.05 level or 0.01 level. For example, if there exists a significant difference in job satisfaction of government and private bank employees then the researcher could either be 95% (0.05 level of significance, $P<0.05$) confident or 99% (0.01 level of significance, $P<0.01$) confident about the results obtained. This is because when carrying out research with human participants, 100% accuracy cannot be achieved. Thus, there could be 5% or 1% chance that the results are due to chance or confounding factors. Whether the null hypothesis is accepted or rejected will depend on whether the statistical value obtained after data analysis is more or less than the table value (tables for various statistical techniques are provided at the end of any book on Statistics) specified at 0.05 or 0.01 level of significance. If the obtained value is higher than the table value, then the null hypothesis is rejected, and if the obtained value is less than the table value than the null hypothesis is accepted.

1.3.2.3 One-tailed and Two- tailed Tests

Further, any hypothesis can also be one tailed or two tailed. It is termed as one- tailed when certain direction is given to the hypothesis or the hypothesis is directional. For example, if the researcher is study if gender difference exists with regard to emotional intelligence, a one-tailed hypothesis would be “Females have higher emotional intelligence than males” or “Males have higher emotional intelligence than females”. Refer to figure no. 1.2 that provides the figure for both 0.05 and 0.01 levels of significance. In one-tailed test, in order to reject a null hypothesis, the score needs to fall in the upper tail, that is in the top 5% of the distribution. A one-tailed test can be tested in either of the direction.

In case of two- tailed hypothesis, the hypothesis is nondirectional and will be stated as “Gender difference will exist with regard to emotional intelligence”. In case of two-tailed test, a null hypothesis can be rejected when the score falls in the either of the top 2.5% of the distribution. Thus the level of significance is maintained at 0.05 level (refer to figure 1.2). As the level of significance is taken as 0.05 level, it can be taken at 0.01 level as well, in which case the prediction will be based on the lower tail.

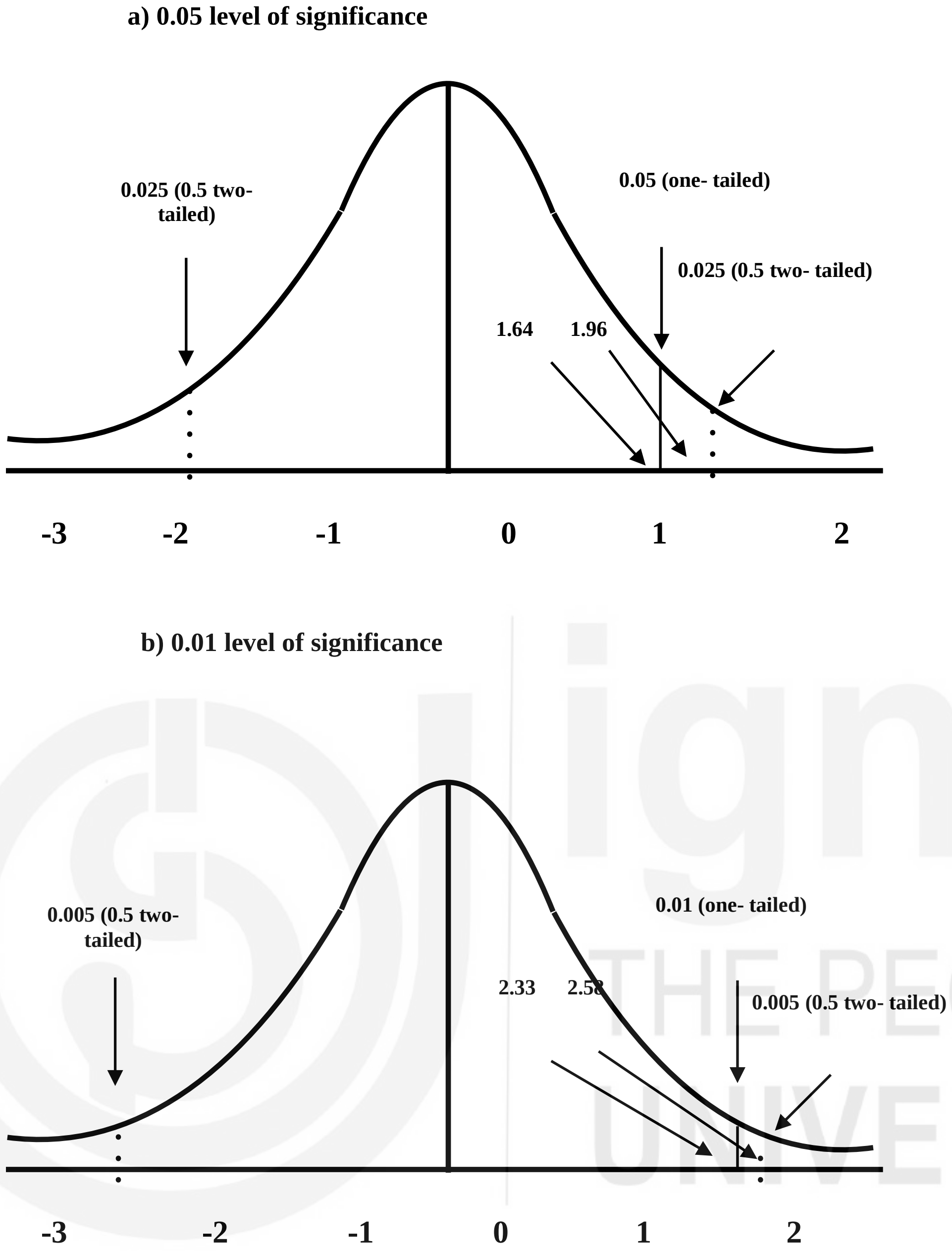

Fig. 1.2: One Tailed and two Tailed Tests

The z scores for one tailed test would be -1.64 or 1.64 at 0.01 level and -2.33 or 2.33 at 0.05 level. The z scores for two tailed test would be -1.96 or 1.96 at 0.01 level and -2.58 or 2.58 at 0.01 level (as you can see in figure 1.2).

1.3.2.3 Errors in Hypothesis Testing

If we look at the process of hypothesis testing, it is based on inferences that we draw for the population based on a sample. Thus, we can say that we draw conclusions based on limited information and as a result there is a scope of making errors. This can despite of ensuring that the sample is representative of the population. Thus, in this context we can discuss about

Type I and Type II errors in hypothesis testing. Refer to the table 1.1 for clarity in this regard.

Table 1.1: Type I and Type II Errors

	Null Hypothesis is true	**Null Hypothesis is false**
Null Hypothesis is Rejected	Type I error	Decision is correct
Null Hypothesis is Accepted	Decision is correct	Type II error

As can be seen in table 1.1 a researcher will be making a correct decision when a false null hypothesis is rejected and when a null hypothesis that is true is accepted. However, it may so happen that a null hypothesis is rejected even when it is true and this is termed as type I error. On the other hand when a false null hypothesis is accepted then it is termed as type II error.

Let us now discuss the correct decisions as well as the types of errors.

Correct decision: A researcher will be making a correct decision when a false null hypothesis is rejected and when a null hypothesis that is true is accepted.

Type I error: In this regard a decision is taken by the researcher to reject a null hypothesis that is actually true. The probability with regard to whether type I error is committed is related with the level of significance, the alpha level (α). The chances of a true null hypothesis getting rejected is high when the alpha level is high. To put this in other words, if the level of significance that we use to reject the null hypothesis is high, more the likelihood that type I error is being committed.

Type II error: In this regard a decision is taken by the researcher to accept a null hypothesis that is actually false. Type II error is represented in terms of beta (β) and this error is committed when on the basis of test of significance, null hypothesis is accepted even when it is false.

In a way, whether the Type I error is committed or not will depend on the level of significance that is taken by the researcher, which can be specified before the research process starts. Therefore, the likelihood of making type I error would be lower if a stringent alpha level is used. Though, when, the alpha level is stringent, the likelihood of making Type II error could increase (Mohanty and Misra, 2016). Thus, it can be said that the relationship between making type I and type II error is inverse in nature. Though, if we want to ensure that both the errors are avoided, then the sample size can be increased.

1.3.2.4 Power of a Test

Let us now come to the next significant term, that is power of the test. Power of a test can be related with type II error. Power of a test can be explained as a measure of how sensitive the experiment or research is in detecting the real effect that the independent variable has on the dependent variable (Mohanty and Misra, 2016). Thus, it mainly involves rejecting the null hypothesis, as when null hypothesis is rejected it can be said that the independent variable is having an effect on the dependent variable. Thus, the Power of a test can also be described in terms of probability related to rejection of null hypothesis when it is false. Thus,

Power = 1- probability of type II error

Putting this differently,

$$\text{Power} = 1 - \beta \text{ or } \beta = 1 - \text{Power}$$

Power of a test can also be described as probability of rejecting a null hypothesis that is actually false. Thus,

$$\text{Power} = P\text{ (rejection of null hypothesis)} = 1 - \beta$$

Where, P stands for probability.

The values of probability ranges from 0.00 to 1.00 (Mohanty and Misra, 2016). The more is this value the more the experiment or research will be sensitive in detecting the real effect and the less is the value, the less is the experiment or research will be sensitive in detecting the real effect. Further, as we increase the sample size, the Power of a test will also increase.

Check Your Progress II

1) What is power of test?

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1.4 PROCEDURE FOR TESTING HYPOTHESIS

We understood some of the important aspects of hypothesis testing. Let us now look at the procedure for testing the hypothesis in previous section.

Step 1: Null hypothesis(es)/ alternative hypothesis(es) are specified.

In the earlier sub section, we discussed about the null hypothesis (es) and alternative hypotheses. In the very first step of hypothesis testing procedure, null hypothesis(es)/ alternative hypothesis(es) are specified. This will mainly depend on the objective,(s) and nature of the study as well as the existing review of literature.

Step 2: A level of significance is selected.

We discussed about the level of significance in the previous section. In step two, the level of significance is selected. As we have mentioned under the type I error, the level of significance or alpha level is to be stated before you start with the research process. Thus, in the second step we will state the level of significance for our research.

Step 3: Based on the parameter specified in the null hypothesis (es), a suitable statistic is calculated.

A sample is taken by the researcher and data is collected. Statistic that is thus obtained from the sample (representative) is then used in order to make an estimation about the population parameters.

Step 4: **Decision is made whether to accept or reject the null hypothesis (es).**

In this regard, based on the P value or probability level selected under step 2, decision is taken by the researcher whether to accept or reject the null hypothesis.

Check Your Progress III

1) List the steps in hypothesis testing.

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1.5 LET US SUM UP

To summarise, in inferential statistics, inferences are drawn about the population based on a representative sample. After the data is collected, it is organised and summarised and once this is done, inferential statistics can be carried out in order to analyse the data and draw conclusions and make inferences. The types of procedures under inferential statistics, namely estimation and hypothesis testing were also discussed in the unit. Point estimation is a type of estimation in which the value is a single point. Whereas an interval estimate is an interval or two numbers within which the population parameter could lie. Hypothesis testing is referred to as a process involving statistical decision making with regard to population value that is based on the sample value. The unit also covered the types of hypotheses, level of significance, one tail and two tail test, rrors in hypothesis testing and power of a test. Lastly the procedure for hypothesis testing was also highlighted.

1.6 REFERENCES

Aaron, A., Aaron, E. N., & Coups, E. J. (2013). Statistics for psychology. New Delhi: Dorling Kindersley (India) Pvt. Ltd.

Best, J. W and Kahn, J. V. (1999). Research in Education. New Delhi: Prentice Hall of India Pvt. Ltd. for information on research designs.

Kerlinger, Fred, N. (1995). Foundations of Behavioural Research. Bangalore: Prism Books Pvt. Ltd. for information on research, research designs, types of research and methods of data collection.

King, M; Minium, W. (2008). Statistical Reasoning in the Behavioural Sciences. Delhi: John Wiley and Sons, Ltd.

Mangal, S. K. (2002). Statistics in psychology and Education. new Delhi: Phi Learning Private Limited.

Minium, E. W., King, B. M., & Bear, G. (2001). Statistical reasoning in psychology and education. Singapore: John-Wiley.

Mohanty, B and Misra, S. (2016). Statistics for Behavioural and Social Sciences. Delhi: Sage.

Salkind, N. J. (2014). Statistics for People who (think that) hate statistics. New Delhi: SAGE.

Veeraraghavan, V and Shetgovekar, S. (2016). Textbook of Parametric and Non-parametric Statistics. Delhi: Sage.

1.7 KEY WORD

Alternativehypothesis*:* Alternative hypothesis can be termed as a counter proposition to the null hypothesis.

Inferential statistics: In inferential statistics, inferences are drawn about the population based on a representative sample. With the help of inferential statistics, inferences can be drawn about the population of the study based on the characteristics of the sample on whom the study was carried out (Salkind, 2014).

Interval estimation: An interval estimate is an interval or two numbers within which the population parameter could lie.

Null hypothesis: Null hypothesis is also referred to as hypothesis of ‘no difference’ as null hypothesis implies that there is no difference between two groups.

Point estimation: This is a type of estimation in which the value is a single point.

1.8 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

1) What is inferential statistics?

   As stated by Veeraraghavan and Shetgovekar (2016, page 5) "Inferential statistics refers to the mathematical methods based on probability theory and helps in reasoning and inferring the characteristic features of the sample drawn from the larger population".

Check Your Progress II

1) What is power of a test?

   Point Power of a test can be explained as a measure of how sensitive the experiment on research is in detecting the test effect that the independent variable has on the dependent variable.

Check Your Progress III

1) List the steps hypothesis testing.

   The steps in testing the hypothesis are as follows:

   Step 1: Null hypothesis(es)/ alternative hypothesis(es) are specified.

   Step 2: A level of significance is selected.

   Step 3: Based on the parameter specified in the null hypothesis (es), a suitable statistic is calculated.

   Step 4: Decision is made whether to accept or reject the null hypothesis (es).

1.9 UNIT END QUESTIONS

1) Discuss the concept and meaning of inferential statistics.

2) Explain the types of hypothesis and level of significance.

3) Discuss one-tailed and two-tailed test with the help of a diagram.

4) Explain the types of errors in hypothesis testing with suitable examples.

5) Discuss the procedure for hypothesis testing.