Game Theory: The Basics

The following is based on *Games of Strategy*, Dixit and Skeath, 1999.

Theory of Games and Economics Behavior –

John Von Neumann and Oskar Morgenstern (1943)

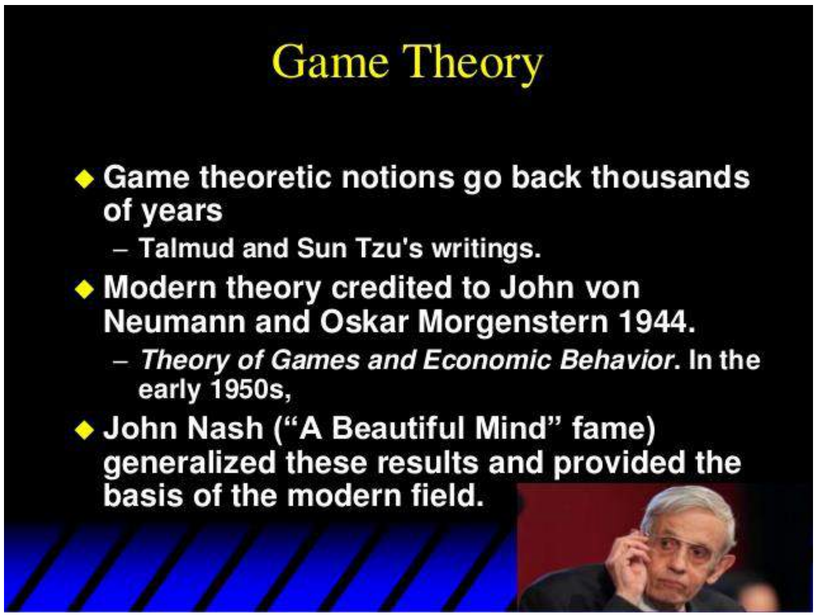
Historia

La teoría de juegos se desarrolla por John Von Neumann (1903-1957) y por Oskar Morgenstern (1902-1976) en 1943, con la publicación de su libro The *Theory of Games and Economic Behavior, (La Teoría de Juegos y la Conducta Económica).*

John Von Newman
(1903-1957)

Oskar Morgenstern
(1902-1976)

"To be literate in the modern age, you need to have a general understanding of game theory." ***(Paul Samuelson)***



"They've led our breakthroughs in gaming theory."

Strategic thinking is about your interactions with others.

Game theory is the science of **interactive** decision making. When you think carefully before you act, you are said to be behaving **rationally**.

You are aware of your objective, preferences, limitations or constraints of your actions.

Game theory is the science of rational behavior in interactive situations.

Game theory provides some general principles for thinking about strategic interactions.

When a person decides to interact with other people, there must be some **cross effect** of their actions.

- What one does must affect the outcome for the other.
- For an interaction to become a strategic game, we need the participants' **mutual** awareness of this cross effect.

It is the mutual awareness of the cross effects of actions and actions taken as a result of this awareness that makes strategy interesting.

Strategic Games vs. Decisions

Interactions between mutually aware players.

Action situations where each person can choose without concern for reaction from others.

Classifying Games

a) **Are the moves in the game sequential or simultaneous?** (Different interactive thinking.)

(i) **Sequential game**:
"If I do this, how will my opponent react?"
Current move governed by your calculation of its **future** consequences.

(ii) Simultaneous game:
Must figure out what your opponent is going to do right now!
The opponent is also trying to figure out your current move, while recognizing you are doing the same with him.

Simultaneous v. Sequential Move Games

- Games where players choose actions simultaneously are simultaneous move games.
  - Examples: Prisoners' Dilemma, Sealed-Bid Auctions.
  - Must anticipate what your opponent will do right now, recognizing that your opponent is doing the same.
- Games where players choose actions in a particular sequence are sequential move games.
  - Examples: Chess, Bargaining/Negotiations.
  - Must look ahead in order to know what action to choose now.
  - Many sequential move games have deadlines/ time limits on moves.
- Many strategic situations involve both sequential and simultaneous moves.

b) Players' Interests:

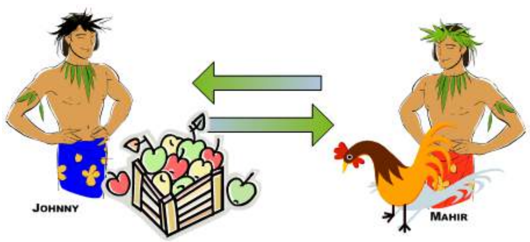
- Total conflict
- Some commonality

Zero Sum Games:

- one player's winnings are the others' losses.
- Player interests in complete conflict.
- Players are dividing up any fixed amount of possible gain.

Most economic and social games are not zero-sum.

Example: Trade offers scope for deals that benefit everyone.



c) Is the Game Played Once or Repeatedly, with the Same or Different Opponents?

- One shot games – actions tend to be ruthless.
- Ongoing games –involve opposite considerations.
  - -can build reputations
  - -learn more about your opponent
  - -Players can exploit mutually beneficial prospects.
    - How to divide a win?
    - punish a cheater?

d) Do the Players have Full or Equal Information?

Players can release information selectively

- **Signals** – actions by the more informed player.
- **Signaling** – information given out.
- **Screening –** the player can create a situation where their opponent will have to take some action that credibly reveals his information.
- Methods are known as **screening devices.**

e) **Are the Rules of the Game Fixed?**

f) Are Agreements to Cooperate Enforceable?

- **Cooperative Games:** joint-action agreements are enforceable.
- **Non-cooperative Games:** joint-action agreements that are not enforceable or possible to enforce, and individuals must be allowed to act in their own interests.

Terminology

A) Strategies:

- The choices available to the players.
- Describes the decisions of an individual over a long time span and a sequence of choices.

B) Payoffs:

- The number associated with each possible outcome.
- Higher payoff numbers mean better outcomes.
  - (i) Payoffs for one player capture everything in the outcomes of the game that he cares about.
  - (ii) Expected value of payoff: $\sum X_i P(X_i)$

(C) Rationality:

The assumption that players are perfect calculators and flawless followers of their best strategies.

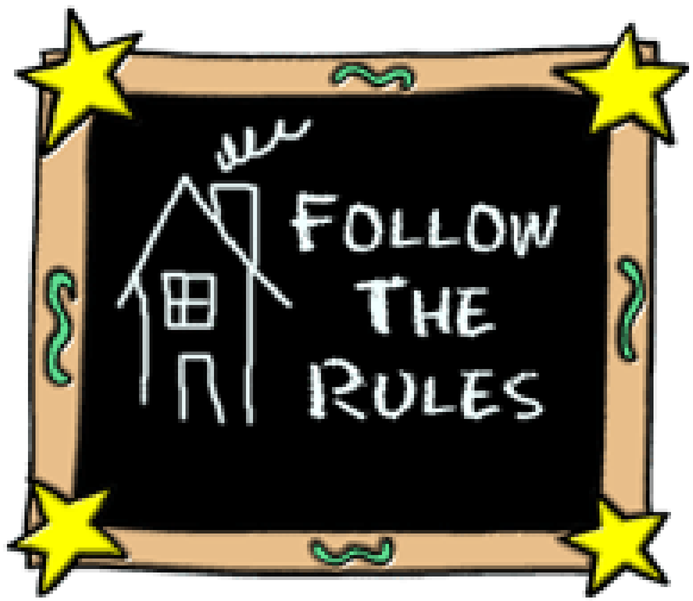
It does not mean that players are selfish. It means pursuing one's own value system consistently is rational.

(D) Common Knowledge of Rules:

- We assume players have a common understanding of the rules of the game "*at some level*."

Rules of the Game:

1) List of players
2) The strategies available to each player
3) The payoffs of each player for all possible combinations of strategies pursued by all players
4) Assumption players are rational maximizers.



E) Equilibrium –Solving the Game

- each player is using the strategy that is the best response to the strategies of the other players.
- Equilibrium does not mean that things do not change.
- Example: sequential move games ⇨ strategy *evolves* all the time; full plan of action and reaction.
- equilibrium does not mean the everything is for the best. Choice can lead to bad outcome.

F) Dynamic and Evolutionary Games:

- allow for dynamic process.
- learn from previous mistakes / games/ strategies.

Evolutionary approach: imported the biological ideas into game theory.

- Evolve; allow growth.
- Social / cultural
- Observation and imitation
- Teaching and learning

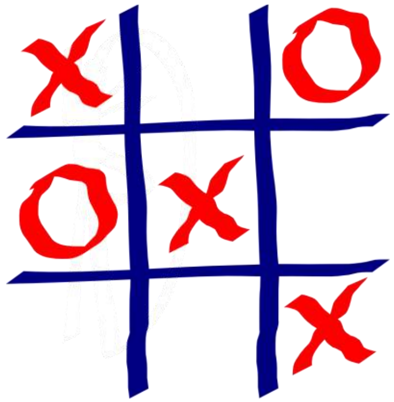
Uses of Game Theory

- 1- **Explanation** -- helps answer the "why did that happen?" questions.
- 2- **Prediction**
- 3- **Advice or prescription**

Games with Sequential Moves

- Games with strict order of play.
- Players take turns making their moves.
- They know what players who have gone before them have done.
- Interactive thinking.

Players decide their current moves based on calculations of future consequences.



Game Trees:

- Generally referred to as **extensive form** of a game.
- Joint decision trees for all players in a game.
- Illustrates all possible actions and outcomes of all players.

Nodes and Branches:

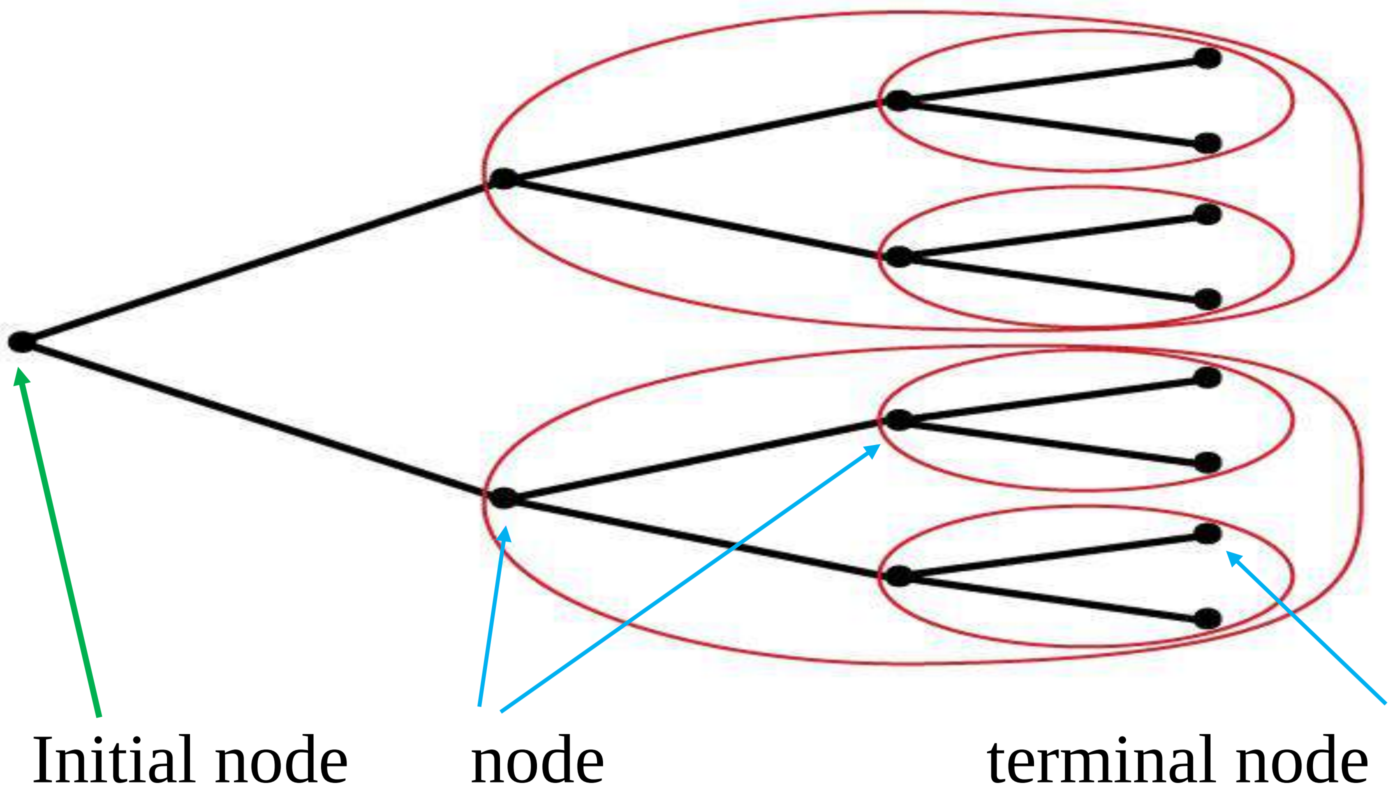
Nodes are connected by branches and come in two types.

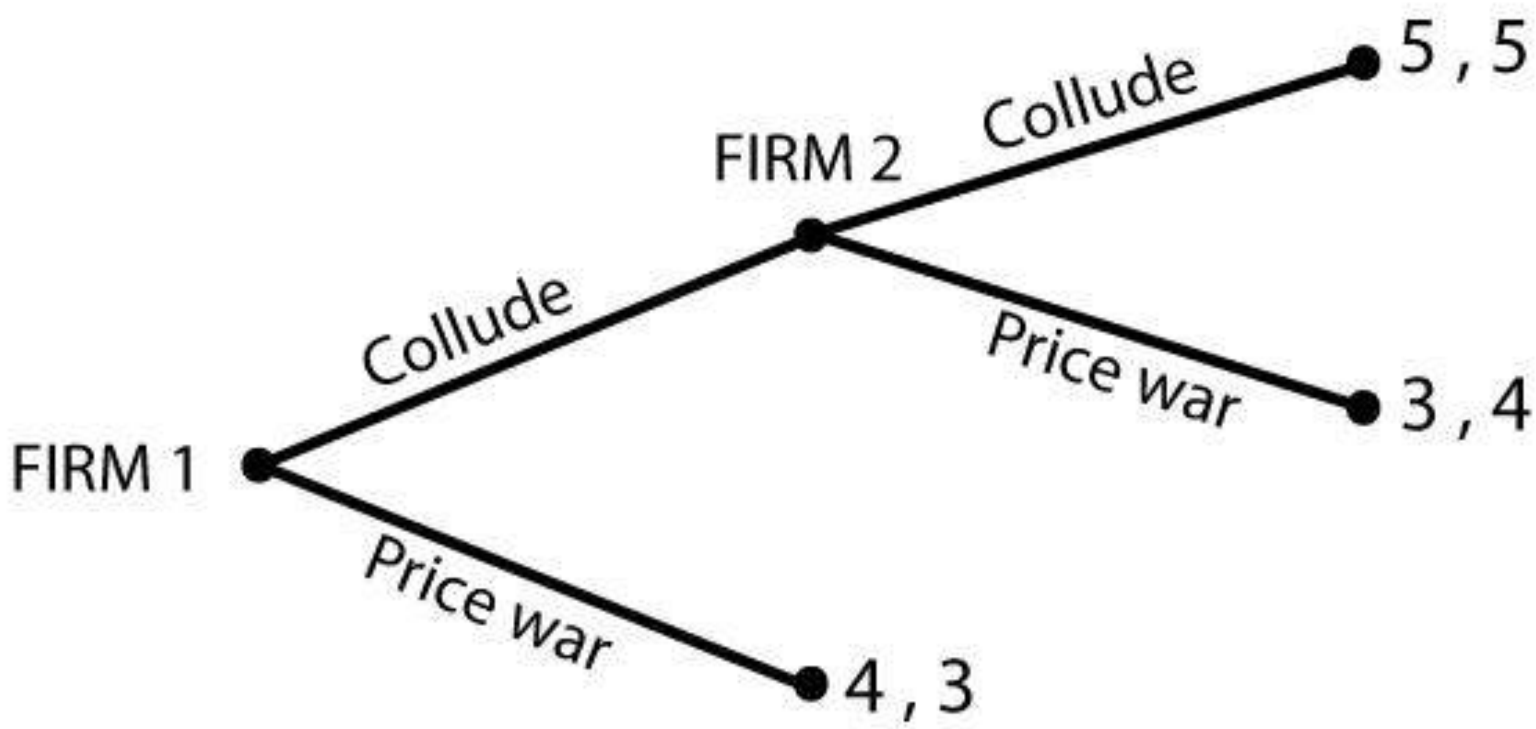
(1) **Decision Node**:

- represent specific points in the game where decisions are made.
- Initial node: starting point of the game.

(2) **<u>Terminal node</u>**:

- End points of the game
- Has associated set of outcomes for players
- Outcomes represent the payoffs received by each player.
- Every decision node can have only one branch leading to it.


Initial node
node
terminal node



<u>Pure Strategy</u>:

- a rule that tells a player what choice to make at each of their possible decision nodes in the game. It is a detailed plan.

Rollback Equilibrium:

- A **rollback** is the concept of looking ahead and reasoning back in a sequential game.
- Think about what will happen at all terminal nodes and "rollback" through the tree to the initial node as you do your analysis.
- Also known as **backward induction**.

- **Rollback equilibrium**: with a fully pruned tree, the remaining branches define the optimal strategy for each player leading to equilibrium.

- **Assume perfect information for the rollback**.

Order Advantages:

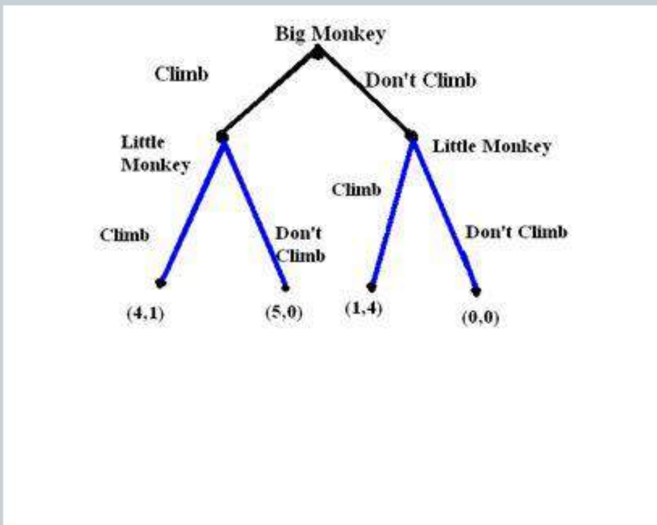
- Sequential move games always have a player who moves first.
- First-mover-advantage: ability to move first.
- Second-mover advantage: when reacting or moving second is beneficial.

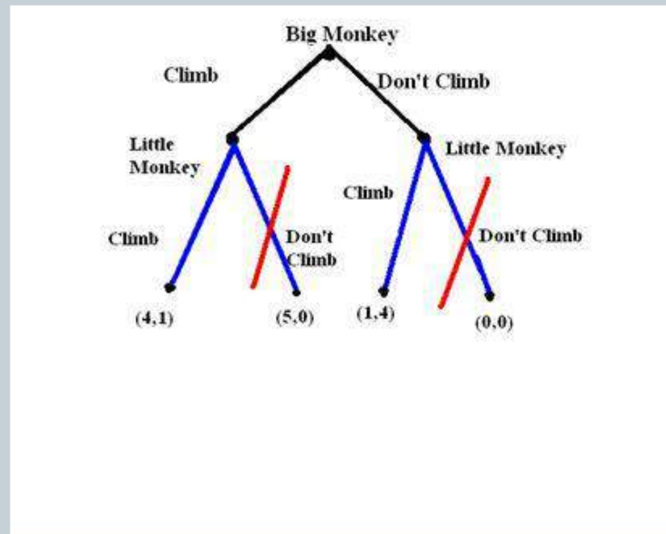
Intermediate Valuation Function

- Indirect way of assigning plausible payoffs to nonterminal nodes because you are not able to explicitly roll back from a full look-ahead.

Extensive Form Games: Game Trees

- Let's use Backwards Induction to solve this.





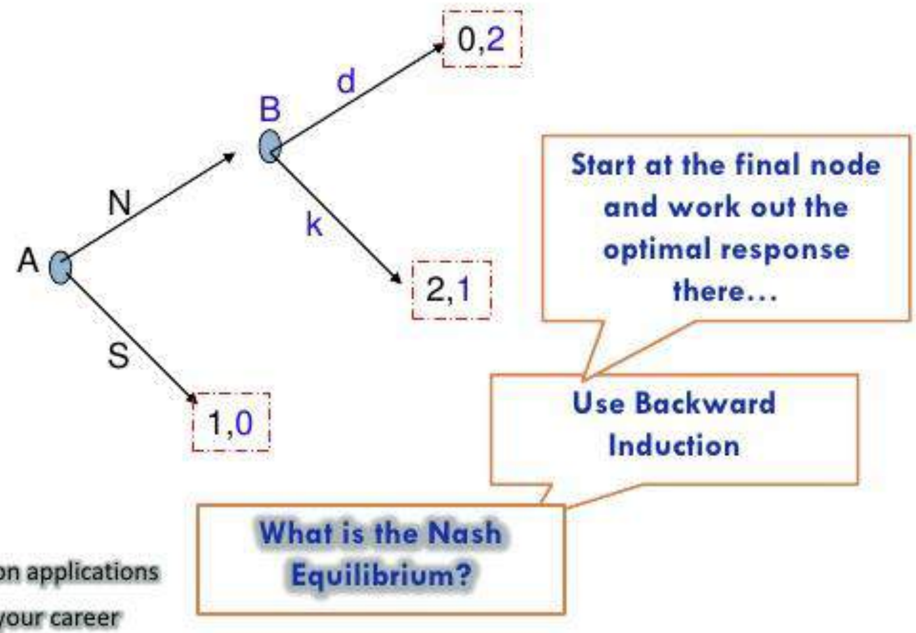
Backward Induction (Rollback analysis)

Extensive form

Game Tree

Branches

Nodes



- Backward-induction applications
  - E.g. Apple; your career

Games of Simultaneous Moves

These are games where you have simultaneous moves.

- Player must move without knowledge of what their rivals have chosen to do.
- Players choose their actions at exactly the same time.
- A game is also simultaneous when players choose their actions in isolation, with no information about what other players have done or will do.

- Also known as games of **imperfect information** or **imperfect knowledge.**

Strategies for simultaneous-move games cannot be made contingent on another's action, as is possible with sequential-move games

However, in many types of simultaneous move games, a player can **reason** through the game from the perspective of his opponent to determine his opponent's best play and therefore his own best play as well.

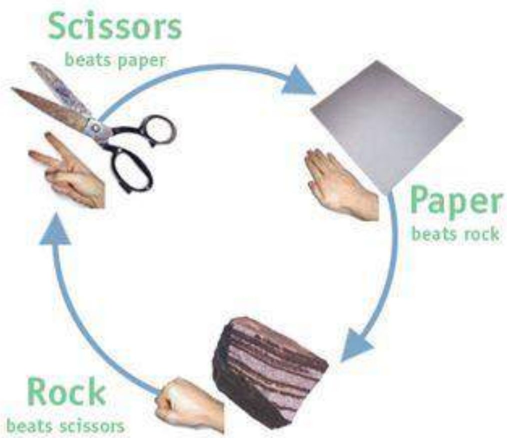
Simultaneous-move games are most often represented using a game table, game matrix or payoff table.

Its dimensions must equal the number of players.

The game table lists payoffs to all players in each cell.

Simultaneous single-move games

- Players must choose their actions at the same time, without knowing what the others will do
  - Form of partial observability



Normal form representation:

Player 1

Player 2

			
	0,0	1,-1	-1,1
	-1,1	0,0	1,-1
	1,-1	-1,1	0,0

Payoff matrix
(row player's utility is listed first)

Note: this is a zero-sum game

		DEFENSE		
		Run	Pass	Blitz
OFFENSE	Run	2	5	13
	Short Pass	6	5.6	10.5
	Medium Pass	6	4.5	1
	Long Pass	10	3	−2

FIGURE 4.2 Representing a Zero-Sum Simultaneous-Move Game in a Table

Zero-Sum or Constant-Sum Games

- Any benefit gained by one player is lost to another.
- Payoffs to all players will sum to a constant (like 0) in each cell.

Simple Two Persons, Zero Sum Game

ASSUMPTIONS

- Each player knows both his and his opponent's alternatives
- Preferences of all players are known
- Single period game
- Sum of payoffs are zero
- **An Equilibrium (or Nash Equilibrium)** - if none of the participants can improve their payoff

PLAYER 2

PLAYER 1	c	d
a	1, -1	3, -3
b	-2, 2	0, 0

Player 1 is the first number in each pair. We will get to {a,c} which is an Equilibrium


+1
-1

Non-Zero Sum Games (Variable Sum Games)

- Games in which players have some common interests, so one does not gain strictly as a rival loses and there is no simple relationship between payoff for different players.
- The game table must show separately, a payoff for every player in each of its cells.

Two Person Game, Non-Zero Sum Game:

ASSUMPTIONS

- Each player can invade the territory of the other (no guard)or Guard his own territory
- Pak's payoff is given first.
- Inida always ranks Guard above no guard, so India has a **Dominant Strategy**
- Knowing what India will do, Pak decides to Guard as well.
- **An Equilibrium**--none of the participants can improve their payoff

		India	
		Guard	no guard
PAK	Guard	Better, 1st	Worst, 4th
	No guard	Worse, 2nd	Best, 3rd

We will get to **{Guard, Guard}** which is an Equilibrium

		Agent 2	
		contribute	do not contribute
Agent 1	contribute	11,11	7,12
	do not contribute	12,7	10,10

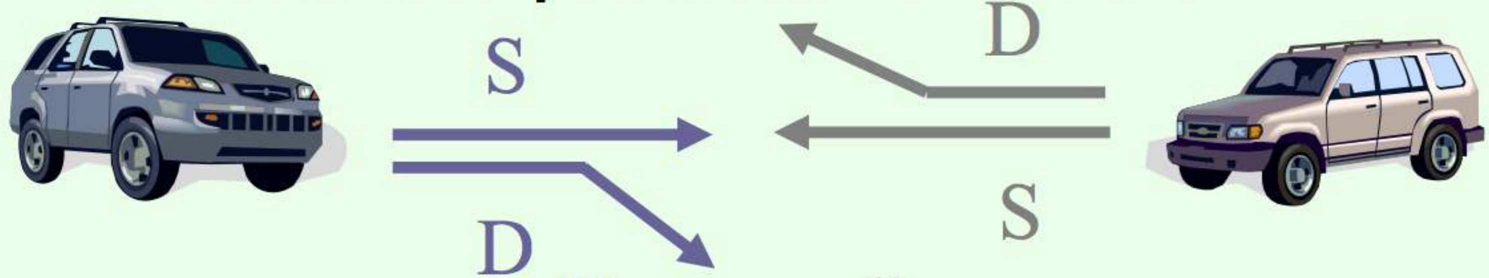
Nash Equilibrium

- In simultaneous-move games, look for equilibrium in which each player's action is a **best response** to the actions of the other players, but we cannot use rollback.

Nash Equilibrium method is used to solve non-cooperative games.

- At a Nash equilibrium, each player must be satisfied with the strategy choice made, given what other players have chosen.
- No player should want to change their strategy once they have seen what their rivals have done.

Nash equilibria of "chicken"



	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

- (D, S) and (S, D) are Nash equilibria
  - They are pure-strategy Nash equilibria: nobody randomizes
  - They are also strict Nash equilibria: changing your strategy will make you strictly worse off

Nash's Equilibrium

- **Nash equilibrium** (named after John Forbes Nash) is a solution concept of a game involving two or more players.
-
- Each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy (i.e., by changing unilaterally).
- If each player has chosen a strategy and no player can benefit by changing the strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

Dominant Strategies

A player in a simultaneous-move game may have any number of pure strategies available.

Pure Strategies: specify nonrandom courses of action for players.
One of these strategies is their dominant strategy if it out-performs all of their other strategies, no matter what any opposing players do.

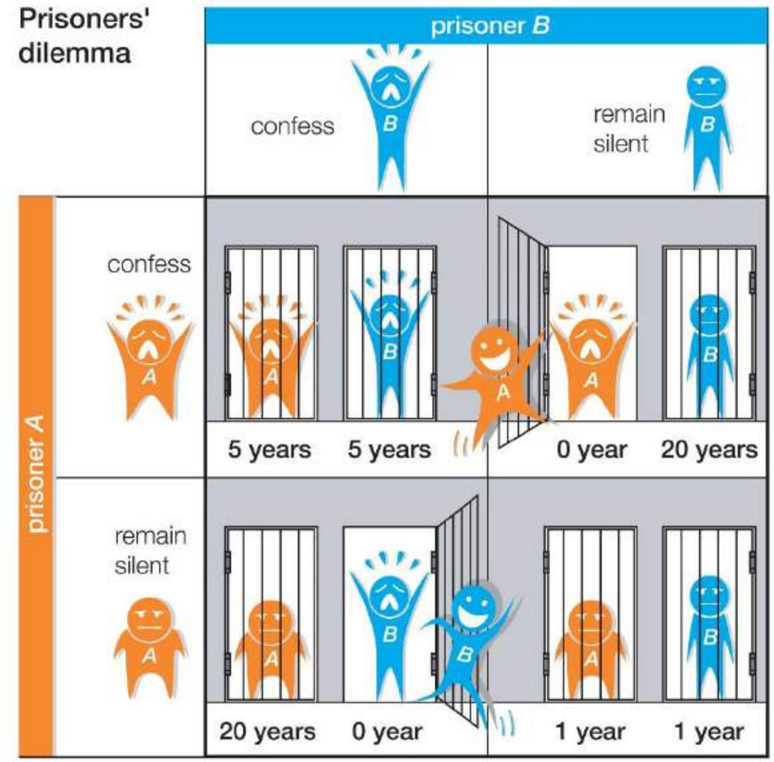
The first thing to do in solving a simultaneous move game is to look for a dominant strategy.
The second, is to look for a **dominated** strategy: players should not use a dominated strategy.

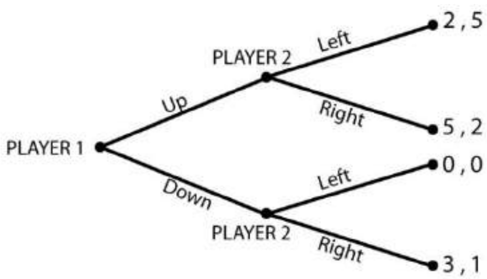
Both Have Dominant Strategies:

Prisoners' Dilemma:

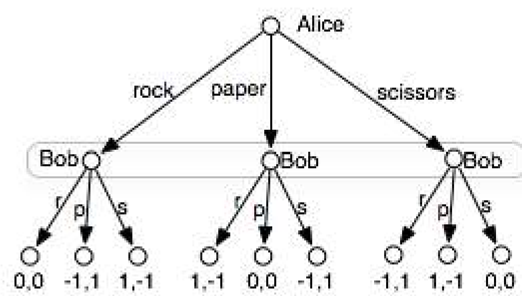
The Prisoners' Dilemma

		Prisoner A Choices	
		Stay Silent	*Confess and Betray*
Prisoner B Choices	*Stay Silent*	Each serves one month in jail	Prisoner A goes free Prisoner B serves full year in jail
	Confess and Betray	Prisoner A serves full year in jail Prisoner B goes free	Each serves three months in jail





PLAYER 1 \ PLAYER 2	R , L	L , R	R , R	L , L
U	5 , 2	2 , 5	5 , 2	2 , 5
D	0 , 0	3 , 1	3 , 1	0 , 0



More examples to come!!