Mathematical analysis

Analysis is a branch of mathematics that depends upon the concepts of limits and convergence. It studies closely related topics such as continuity, integration, differentiability and transcendental_functions. These topics are often studied in the context of real_numbers, complex numbers, and their functions. However, they can also be defined and studied in any space of mathematical objects that is equipped with a definition of "nearness" (a topological_space) or more specifically "distance" (a metric space). Mathematical analysis has its beginnings in the rigorous formulation of calculus.

History

Greek_mathematicians such as Eudoxus and Archimedes made informal use of the concepts of limits and convergence when they used the method of exhaustion to compute the area and volume of regions and solids.

In India, the 12th century mathematician Bhaskara conceived of differential calculus, and gave examples of the derivative and differential coefficient, along with a statement of what is now known as Rolle's theorem. In the 14th century, mathematical analysis originated with Madhava in South India, who developed the fundamental ideas of the infinite series expansion of a function, the power series, the Taylor series, and the rational approximation of an infinite series. He developed the Taylor series of the trigonometric functions of sine, cosine, tangent and arctangent, and estimated the magnitude of the error terms created by truncating these series. He also developed infinite continued fractions, term by term integration, the Taylor series approximations of sine and cosine, and the power series of the radius, diameter, circumference, π, $\pi/4$ and angle θ. His followers at the Kerala School further expanded his works, upto the 16th century.

Mathematical analysis in Europe began in the 17th century, with the possibly independent invention of calculus by Newton and Leibniz. In the 17th and 18th centuries, analysis topics such as the calculus of variations, ordinary and partial differential equations, Fourier analysis and generating functions were developed mostly in applied work. Calculus techniques were applied successfully to approximate discrete problems by continuous ones.

All through the 18th century the definition of the concept of function was a subject of debate among mathematicians. In the 19th century, Cauchy was the first to put calculus on a firm logical foundation by introducing the concept of the Cauchy sequence. He also started the formal theory of complex analysis. Poisson, Liouville, Fourier and others studied partial differential equations and harmonic analysis.

In the middle of the century Riemann introduced his theory of integration. The last third of the 19th century saw the arithmetization of analysis by Weierstrass, who thought that geometric reasoning was inherently misleading, and introduced the "epsilon-delta" definition of limit. Then, mathematicians started worrying that they were assuming the existence of a continuum of real numbers without proof. Dedekind then constructed the real numbers by Dedekind cuts. Around that time, the attempts to refine the theorems of Riemann integration led to the study of the "size" of the set of discontinuities of real functions.

Also, "monsters" (nowhere continuous functions, continuous but nowhere differentiable functions, space-filling curves) began to be created. In this context, Jordan developed his theory of measure, Cantor developed what is now called naive set theory, and Baire proved the Baire category theorem. In the early 20th century, calculus was formalized using axiomatic set theory. Lebesgue solved the problem of measure, and Hilbert introduced Hilbert spaces to solve integral equations. The idea of normed vector space was in the air, and in the 1920s Banach created functional analysis.

The branch of mathematical analysis

Real Analysis

Complex Analyasis

Functional Analysis

Measure Theory

Differential Equations

Integral Equations

Transformation Theory

The Parts of Mathematical Analysis

The sets of numbers

Sequences

Infinite series

Limits

Continuous Functions

Derivative

Integration

Measure

Bhaskara

Bhaskara (1114-1185), ("Bhaskara the teacher") was an Indian mathematician-astronomer. He was born near Bijapur district, Karnataka state, South India in Deshastha Brahmin family and became head of the astronomical observatory at Ujjain ,

In many ways, Bhaskara represents the peak of mathematical and astronomical knowledge in the 12th century. He reached an understanding of calculus, astronomy, the number systems, and solving equations, which were not to be achieved anywhere else in the world for several centuries or more. His main works were the (dealing with arithmetic), (*Algebra*) and *Shiromani* (written in 1150) which consists of two parts: (sphere) and (mathematics of the planets).

Madhava

(1350–1425) was a prominent mathematician-astronomer from Kerala, India. He was the founder of the Kerala School of Mathematics and is considered the founder of mathematical analysis for having taken the decisive step from the finite procedures of ancient mathematics to treat their limit-passage to infinity, which is the kernel of modern classical analysis. He is considered as one of the greatest mathematician-astronomers of the Middle Ages due to his important contributions to the fields of mathematical analysis, infinite series, calculus, trigonometry, geometry and algebra.

Unfortunately, most of Madhava's original works have been lost in course of time, as they were written primarily on perishable material. However his works have been detailed by later scholars of the Kerala School,

Isaac Newton

Sir Isaac Newton

Sir Isaac Newton at 46 in Godfrey Kneller's 1689 portrait

Born	4 January 1643 England
Died	31 March 1727 , London

Residence	England
Nationality	English
Field	Mathematics, physics, Alchemy, astronomy, Natural philosophy
Institution	University of Cambridge
Alma Mater	University of Cambridge
Known for	Gravitation, optics, Calculus, mechanics
Notable Prizes	Knighthood
Religion	Prophetic Unitarianism, Church of England

Sir Isaac Newton, FRS (4 January 1643 – 31 March 1727) [OS: 25 December 1642 – 20 March 1727] was an English physicist, mathematician, astronomer, alchemist, and natural philosopher who is generally regarded as one of the greatest scientists and mathematicians in history. Newton wrote the *Philosophiae Naturalis Principia Mathematica*, in which he described universal gravitation and the three laws of motion, laying the groundwork for classical mechanics. By deriving Kepler's laws of planetary motion from this system, he was the first to show that the motion of objects on Earth and of celestial bodies are governed by the same set of natural laws. The unifying and deterministic power of his laws was integral to the scientific revolution and the advancement of heliocentrism. He also was a devout Christian, studied the Bible daily and wrote more on religion than on natural science.

Although by the calendar in use at the time of his birth he was born on Christmas Day 1642, the date of 4 January 1643 is used because this is the Gregorian calendar date.

Among other scientific discoveries, Newton realised that the spectrum of colours observed when white light passes through a prism is inherent in the white light and not added by the prism (as Roger Bacon had claimed in the thirteenth century), and notably argued that light is composed of particles. He also developed a law of cooling, describing the rate of cooling of objects when exposed to air. He enunciated the principles of conservation of momentum and angular momentum. Finally, he studied the speed of sound in air, and voiced a theory of the origin of stars. Despite this renown in mainstream science, Newton spent much of his time working on alchemy rather than physics, writing considerably more papers on the former than the latter.[2]

Newton played a major role in the development of calculus, famously sharing credit with Gottfried Leibniz. He also made contributions to other areas of mathematics, for example the generalised binomial theorem. The mathematician and mathematical physicist Joseph Louis Lagrange (1736–1813), often said that Newton was the greatest genius that ever existed, and once added "and the most fortunate, for we cannot find more than once a system of the world to establish.

Gottfried Leibniz

Western Philosophers 17th-century philosophy (Modern Philosophy)

Gottfried Wilhelm Leibniz

Name:	Gottfried Wilhelm Leibniz
Birth:	July 1, 1646 (Leipzig, Germany)
Death:	November 14, 1716 (Hanover, Germany)
School/tradition:	Rationalism
Main interests:	metaphysics, mathematics, science, epistemology.

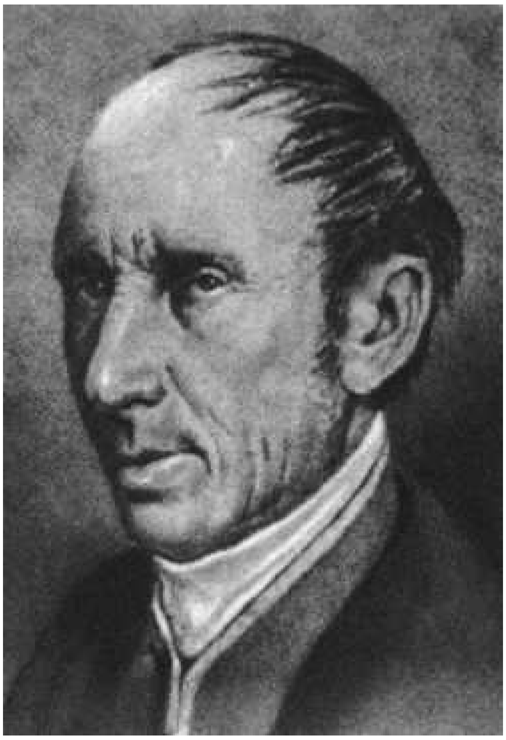
Notable ideas:	calculus, monad, theodicy, optimism
Influences:	Plato, Aristotle, Ramon Llull, Scholastic philosophy, Descartes, Christiaan Huygens
Influenced:	Many later mathematicians, Christian Wolff, Immanuel Kant, Bertrand Russell, Abraham Robinson

Gottfried Wilhelm Leibniz (also *Leibnitz* or *von Leibniz*) (July 1 (June 21 Old Style) 1646 – November 14, 1716) was a German polymath who wrote mostly in French and Latin.

Educated in law and philosophy, and serving as factotum to two major German noble houses (one becoming the British royal family while he served it), Leibniz played a major role in the European politics and diplomacy of his day. He occupies an equally large place in both the history of philosophy and the history of mathematics. He invented calculus independently of Newton, and his notation is the one in general use since. He also invented the binary system, foundation of virtually all modern computer architectures. In philosophy, he is most remembered for optimism, i.e., his conclusion that our universe is, in a restricted sense, the best possible one God could have made. He was, along with René Descartes and Baruch Spinoza, one of the three great 17th century rationalists, but his philosophy also both looks back to the Scholastic tradition and anticipates modern logic and analysis.

Leibniz also made major contributions to physics and technology, and anticipated notions that surfaced much later in biology, medicine, geology, probability theory, psychology, knowledge engineering, and information science. He also wrote on politics, law, ethics, theology, history, and philology, even occasional verse. His contributions to this vast array of subjects are scattered in journals and in tens of thousands of letters and unpublished manuscripts. To date, there is no complete edition of Leibniz's writings, and a complete account of his accomplishments is not yet possible.

Augustin Louis Cauchy



Augustin Louis Cauchy

Augustin Louis Cauchy (August 21, 1789 – May 23, 1857) was a French mathematician. He started the project of formulating and proving the theorems of calculus in a rigorous manner and was thus an early pioneer of analysis. He also gave several important theorems in complex analysis and initiated the study of permutation groups. A profound mathematician, Cauchy exercised by his perspicuous and rigorous methods a great influence over his contemporaries and successors. His writings cover the entire range of mathematics and mathematical physics.

Siméon Denis Poisson

"Poisson" redirects here. For other persons and things bearing this name.

Siméon Poisson.

Siméon-Denis Poisson (June 21, 1781 – April 25, 1840), was a French mathematician, geometer and physicist.

In 1798 he entered the École Polytechnique in Paris as first in his year, and immediately began to attract the notice of the professors of the school, who left him free to follow the studies of his predilection. In 1800, less than two years after his entry, he published two memoirs, one on Étienne Bézout's method of elimination, the other on the number of integrals of an equation of finite differences. The latter of these memoirs was examined by Sylvestre-François Lacroix and Adrien-Marie Legendre, who recommended that it should be published in the *Recueil des savants étrangers*, an unparalleled honour for a youth of eighteen. This success at once procured for Poisson an entry into scientific circles. Joseph Louis Lagrange, whose lectures on the theory of functions he attended at the École Polytechnique, early recognized his talent, and became his friend; while Pierre-Simon Laplace, in whose footsteps Poisson followed, regarded him almost as his son. The rest of his career, till his death in Sceaux near Paris, was almost entirely occupied in the composition and publication of his many works, and in discharging the duties of the numerous educational offices to which he was successively appointed.

Immediately after finishing his course at the École Polytechnique he was appointed repetiteur there, an office which he had discharged as an amateur while still a pupil in the school; for it had been the custom of his comrades often to resort to his room after an unusually difficult lecture to hear him repeat and explain it. He was made deputy professor (*professeur suppléant*) in 1802, and, in 1806 full professor in succession to Jean Baptiste Joseph Fourier, whom Napoleon had sent to Grenoble. In 1808 he became astronomer to the Bureau des Longitudes; and when the Faculté des Sciences was instituted in 1809 he was appointed professor of rational mechanics (*professeur de mécanique rationelle*). He further became member of the Institute in 1812, examiner at the military school (*École Militaire*) at Saint-Cyr in 1815, leaving examiner at the École Polytechnique in 1816, councillor of the university in 1820, and geometer to the Bureau des Longitudes in succession to P. S. Laplace in 1827.

In 1817 he married Nancy de Bardi and with her he had [several?] children. His father, whose early experiences led him to hate aristocrats, bred him in the stern creed of the first republic. Throughout the Revolution, the Empire and the following restoration, Poisson was not interested in politics, concentrating on Mathematics. He was appointed to the dignity of baron in 1821; but he neither took out the diploma or used the title. The revolution of July 1830 threatened him with the loss of all his honours; but this disgrace to the government of Louis-Philippe was adroitly averted by François Jean Dominique Arago, who, while his "revocation" was being plotted by the council of ministers, procured him an invitation to dine at the Palais Royal, where he was openly and effusively received by the citizen king, who "remembered" him. After this, of course, his degradation was impossible, and seven years later he was made a peer of France, not for political reasons, but as a representative of French science.

Like many scientists of his time, he was an atheist.

As a teacher of mathematics Poisson is said to have been more than ordinarily successful, as might have been expected from his early promise as a repetiteur at the École Polytechnique. As a scientific worker his activity has rarely if ever been equalled. Notwithstanding his many official duties, he found time to publish more than three hundred works, several of them extensive treatises, and many of them memoirs dealing with the most abstruse branches of pure, applied mathematics, mathematical physics and rational mechanics.

A list of Poisson's works, drawn up by himself, is given at the end of Arago's biography. All that is possible is a brief mention of the more important. It was in the application of mathematics to physical subjects that his greatest services to science were performed. Perhaps the most original, and certainly the most permanent in their influence, were his memoirs on the theory of electricity and magnetism, which virtually created a new branch of mathematical physics.

Next (perhaps in the opinion of some first) in importance stand the memoirs on celestial mechanics, in which he proved himself a worthy successor to P.-S. Laplace. The most important of these are his memoirs *Sur les inégalités séculaires des moyens mouvements des planètes*, *Sur la variation des constantes arbitraires dans les questions de mécanique*, both published in the *Journal* of the École Polytechnique (1809); *Sur la libration de la lune*, in *Connaiss. des temps* (1821), etc.; and *Sur la mouvement de la terre autour de son centre de gravité*, in *Mém. d. l'acad.* (1827), etc. In the first of these memoirs Poisson discusses the famous question of the stability of the planetary orbits, which had already been settled by Lagrange to the first degree of approximation for the disturbing forces. Poisson showed that the result could be extended to a second approximation, and thus made an important advance in the planetary theory. The memoir is remarkable inasmuch as it roused Lagrange, after an interval of inactivity, to compose in his old age one of the greatest of his memoirs, entitled *Sur la théorie des variations des éléments des planètes, et en particulier des variations des grands axes de leurs orbites*. So highly did he think of Poisson's memoir that he made a copy of it with his own hand, which was found among his papers after his death. Poisson made important contributions to the theory of attraction.

His well-known correction of Laplace's partial differential equation of the second degree for the potential:

$$\nabla^2\phi = -4\pi\rho$$

today named after him the Poisson's equation or the potential theory equation, was first published in the Bulletin de in société philomatique (1813). If a function of a given point $\rho = 0$, we get Laplace's equation:

$$\nabla^2\phi = 0$$

In 1812 Poisson discovered that Laplace's equation is valid only outside of a solid. A rigorous proof for masses with variable density was first given by Carl Friedrich Gauss in 1839. Both equations have their equivalents in vector algebra. The study of scalar field φ from a given divergence ρ(*x*, *y*, *z*) of its gradient leads to Poisson's equation in 3-dimensional space:

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$$\nabla^2 \phi = \rho(x, y, z)$$

For instance Poisson's equation for surface electrical potential Ψ, which shows its dependence from the density of electrical charge ρ_e in particular place:

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = -\frac{\rho_e}{\varepsilon \varepsilon_0}$$

The distribution of a charge in a fluid is unknown and we have to use Poisson-Boltzmann equation:

$$\nabla^2 \Psi = \frac{n_0 e}{\varepsilon \varepsilon_0} \left(e^{e\Psi(x,y,z)/k_B T} - e^{-e\Psi(x,y,z)/k_B T} \right),$$

which in most cases cannot be solved analytically but just for special cases. In polar coordinates the Poisson-Boltzmann equation is:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Psi}{dr} \right) = \frac{n_0 e}{\varepsilon \varepsilon_0} \left(e^{e\Psi(r)/k_B T} - e^{-e\Psi(r)/k_B T} \right)$$

which also cannot be solved analytically. If a field φ is not a scalar, the Poisson equation is valid, as can be for example in 4-dimensional Minkowski space:

$$\Box \phi_{ik} = \rho(x, y, z, ct) \; .$$

If ρ(*x*, *y*, *z*) is a continuous function and if for $r \to \infty$ (or if a point 'moves' to infinity) a function φ goes to 0 fast enough, a solution of Poisson's equation is the Newtonian potential of a function ρ(*x*, *y*, *z*):

$$\phi_M = -\frac{1}{4\pi}\int \frac{\rho(x,y,z)\,dv}{r}$$

where *r* is a distance between the element with the volume d*v* and point *M*.

Integration runs over the whole space. The Poisson's integral in solving the Green's function for the Dirichlet problem of the Laplace's equation, if circle is investigated domain:

$$\phi(\xi\eta) = \frac{1}{4\pi}\int_0^{2\pi} \frac{R^2-\rho^2}{R^2+\rho^2-2R\rho\cos(\psi-\chi)}\phi(\chi)\,d\chi$$

where

$$\xi = \rho\cos\psi,$$
$$\eta = \rho\sin\psi.$$

$\varphi(\chi)$ is prescribed function on a circular line, which defines bounding conditions of requested function φ of Laplace's equation.

In the same manner we define the Green's function for the Dirichlet problem of the Laplace's equation $\nabla^2\varphi = 0$ in space, if we look to the investigated domain of a sphere with a radius *R*. This time the Green's function is:

$$G(x,y,z;\xi,\eta,\zeta) = \frac{1}{r} - \frac{R}{r_1\rho},$$

where

$$\rho = \sqrt{\xi^2+\eta^2+\zeta^2}$$

is a distance of a point (ξ, η, ζ) from the center of a sphere

r a distance between points (x, y, z)

(ξ, η, ζ), r_1 is a distance between the point (x, y, z) and the point $(R\xi/\rho, R\eta/\rho, R\zeta/\rho)$, symmetrical to the point (ξ, η, ζ)

The Poisson's integral now has a form:

$$\phi(\xi,\eta,\zeta) = \frac{1}{4\pi}\iint_S \frac{R^2-\rho^2}{Rr^3}\phi\, ds\,.$$

Poisson's two most important memoirs on the subject are *Sur l'attraction des sphéroides* (Connaiss. ft. temps, 1829), and *Sur l'attraction d'un ellipsoide homogène* (Mim. ft. l'acad., 1835). In concluding our selection from his physical memoirs we may mention his memoir on the theory of waves (Mém. ft. l'acad., 1825).

In pure mathematics, his most important works were his series of memoirs on definite integrals, and his discussion of Fourier series, which paved the way for the classical researches of Peter Gustav Lejeune Dirichlet and Bernhard Riemann on the same subject; these are to be found in the *Journal* of the École Polytechnique from 1813 to 1823, and in the *Memoirs de l'académie* for 1823. He also studied Fourier integrals. In addition we may also mention his essay on the calculus of variations (*Mem. de l'acad.*, 1833), and his memoirs on the probability of the mean results of observations (*Connaiss. d. temps*, 1827, &c). The Poisson distribution in probability theory is named after him.

In his *Traité de mécanique* (2 vols. 8vo, 1811 arid 1833), which was written in Laplace and Lagrange style and was long a standard work he showed many new grips such as an explicit usage of impulsive coordinates:

$$p_i = \frac{\partial T}{\frac{\partial q_i}{\partial t}}$$

which has influenced on the work of William Rowan Hamilton and Carl Gustav Jakob Jacobi.

Besides his many memoirs Poisson published a number of treatises, most of which were intended to form part of a great work on mathematical physics, which he did not live to complete. Among these may be mentioned

- *Théorie nouvelle de l'action cappillaire* (4to, 1831);
- *Théorie mathématique de la chaleur* (4to, 1835);
- Supplement to the same (4to, 1837);
- *Recherches sur la probabilité des jugements en matières criminelles et matiere civile* (4to, 1837), all published at Paris.

In 1815 Poisson carried out integrations along paths in the complex plane. In 1831 he independently of Claude-Louis Navier derived the Navier-Stokes equations.

Joseph Liouville

Joseph Liouville (born March 24, 1809, died September 8, 1882) was a **French** mathematician.

Joseph Liouville

Liouville graduated from the École Polytechnique in 1827. After some years as assistant at various institutions he was appointed as professor at the École Polytechnique in 1838. He obtained a chair in mathematics at the Collège de France in 1850 and a chair in mechanics at the Faculté des Sciences in 1857.

Besides his academic achievements, he was very talented in organisatorial matters. Liouville founded the *Journal de Mathématiques Pures et Appliquées* which retains its high reputation up to today, in order to promote other mathematicians' work. He was the first to read, and to recognize the importance of, the unpublished work of Evariste Galois which appeared in his journal in 1846. Liouville was also involved in politics for some time, and he became member of the Constituting Assembly in 1848. However, after the defeat in the Assembly elections in 1849, he turned away from politics.

Liouville worked in a number of different fields in mathematics, including number theory, complex analysis, differential geometry and topology, but also mathematical physics and even astronomy. He is remembered particularly for Liouville's theorem, a nowadays rather basic result in complex analysis. In number theory, he was the first to prove the existence of transcendental numbers by a construction using continued fractions (Liouville numbers). In mathematical physics, Liouville made two fundamental contributions: the Sturm-Liouville theory which was joint work with Charles François Sturm is now a standard procedure to solve certain types of integral equations by developing into eigenfunctions, and the fact (also known as Liouville's theorem) that time evolution is measure preserving for a Hamiltonian system.

Joseph Fourier

Jean Baptiste Joseph Fourier (March 21, 1768 - May 16, 1830) was a French mathematician and physicist who is best known for initiating the investigation of Fourier series and their application to problems of heat flow. The Fourier transform is also named in his honor.

Joseph Fourier

Life

Fourier was born at Auxerre in the Yonne département of France, the son of a tailor. He was orphaned at age eight. Fourier was recommended to the Bishop of Auxerre, and through this introduction, he was educated by the Benedictines of the Convent of St. Mark. The commissions in the scientific corps of the army were reserved for those of good birth, and being thus ineligible, he accepted a military lectureship on mathematics. He took a prominent part in his own district in promoting the revolution, and was rewarded by an appointment in 1795 in the *École Normale Supérieure*, and subsequently by a chair at the *École Polytechnique*.

Fourier went with Napoleon on his Eastern expedition in 1798, and was made governor of Lower Egypt. Cut off from France by the English fleet, he organized the workshops on which the French army had to rely for their munitions of war. He also contributed several mathematical papers to the Egyptian Institute which Napoleon founded at Cairo, with a view of weakening English influence in the East. After the British victories and the capitulation of the French under General Menou in 1801, Fourier returned to France, and was made prefect of Isère, and it was while there that he made his experiments on the propagation of heat.

Fourier moved to Paris in 1816. In 1822 he published his *Théorie analytique de la chaleur*, in which he bases his reasoning on Newton's law of cooling, namely, that the flow of heat between two adjacent molecules is proportional to the extremely small difference of their temperatures. In this work he claims that any function of a variable, whether continuous or discontinuous, can be expanded in a series of sines of multiples of the variable. Though this result is not correct, Fourier's observation that some discontinuous functions are the sum of infinite series was a breakthrough. The question of determining when a function is the sum of its Fourier series has been fundamental for centuries. Joseph Louis Lagrange had given particular cases of this (false) theorem, and had implied that the method was general, but he had not pursued the subject. Johann Dirichlet was the first to give a satisfactory demonstration of it with some restrictive conditions.

Fourier died in Paris in 1830.

Bernhard Riemann

Bernhard Riemann.

Georg Friedrich Bernhard Riemann (September 17, 1826 - July 20, 1866) (pronounced *REE mahn* or in IPA: ['ri:man]) was a German mathematician who made important contributions to analysis and differential geometry, some of them paving the way for the later development of general relativity.

Biography

Early life

Riemann was born in Breselenz on September 17 1826, a village near Dannenberg in the Kingdom of Hanover in what is today Germany. His father *Friedrich Bernhard Riemann* was a poor Lutheran pastor in Breselenz. Friedrich Riemann fought in the Napoleonic Wars. Georg's mother also died before her children were grown. Bernhard was the second of six children. He was a shy boy and suffered from numerous nervous breakdowns. From a very young age, Riemann exhibited his exceptional skills, such as fantastic calculation abilities, but suffered from timidity and had a fear of speaking in public.

Middle life

In high school, Riemann studied the Bible intensively. But his mind often drifted back to mathematics and he even tried to prove mathematically the correctness of the book of Genesis. His teachers were amazed by his genius and by his ability to solve extremely complicated mathematical operations. He often outstripped his instructor's knowledge. In 1840 Bernhard went to Hanover to live with his grandmother and visit the Lyceum. After the death of his grandmother in 1842 he went to the Johanneum in Lüneburg. In 1846, at the age of 19, he started studying philology and theology, in order to become a priest and help with his family's finances.

In 1847 his father, after scraping together enough money to send Riemann to university, allowed him to stop studying theology and start studying mathematics. He was sent to the renowned University of Göttingen, where he first met Carl Friedrich Gauss, and attended his lectures on the method of least squares.

In 1847 he moved to Berlin, where Jacobi, Dirichlet and Steiner were teaching. He stayed in Berlin for two years and returned to Göttingen in 1849.

Later life

Riemann held his first lectures in 1854, which not only founded the field of Riemannian geometry but set the stage for Einstein's general relativity. He was promoted to an extraordinary professor at the University of Göttingen in 1857 and became an ordinary professor in 1859 following Dirichlet's death. He was also the first to propose the theory of higher dimensions, which highly simpified the laws of physics. In 1862 he married Elise Koch. He died of tuberculosis on his third journey to Italy in Selasca (now a hamlet of Ghiffa on Lake Maggiore).

Richard Dedekind

Richard Dedekind

Julius Wilhelm Richard Dedekind (October 6, 1831 – February 12, 1916) was a German mathematician who did important work in abstract algebra, algebraic number theory and the foundations of the real numbers.

Life

Dedekind was the youngest of four children of Julius Levin Ulrich Dedekind. As an adult, he never employed the names Julius Wilhelm. He was born, lived most of his life, and died in Braunschweig (often called "Brunswick" in English). His life appears to have been uneventful, consisting of little more than his mathematical teaching and research.

In 1848, he entered the Collegium Carolinum in Braunschweig, where his father was an administrator, obtaining a solid grounding in

mathematics. In 1850, he entered the University of Göttingen. Dedekind studied number theory under Moritz Stern. Gauss was still teaching, although mostly at an elementary level, and Dedekind became his last student. Dedekind received his doctorate in 1852, for a thesis titled *Über die Theorie der Eulerschen Integrale* ("On the Theory of Eulerian integrals"). This thesis did not reveal the talent evident on almost every page Dedekind later wrote.

At that time, the University of Berlin, not Göttingen, was the leading center for mathematical research in Germany. Thus Dedekind went to Berlin for two years of study, where he and Riemann were contemporaries; they were both awarded the habilitation in 1854. Dedekind returned to Göttingen to teach as a *Privatdozent*, giving courses on probability and geometry. He studied for a while with Dirichlet, and they became close friends. Because of lingering weaknesses in his mathematical knowledge, he studied elliptic and abelian functions. Yet he was also the first at Göttingen to lecture on Galois theory. Around this time, he became one of the first to understand the fundamental importance of the notion of group for algebra and arithmetic.

In 1858, he began teaching at the Polytechnic in Zürich. When the Collegium Carolinum was upgraded to a *Technische Hochschule* (Institute of Technology) in 1862, Dedekind returned to his native Braunschweig, where he spent the rest of his life, teaching at the Institute. He retired in 1894, but did occasional teaching and continued to publish. He never married, instead living with his unmarried sister Julia.

Dedekind was elected to the Academies of Berlin (1880) and Rome, and to the Paris Académie des Sciences (1900). He received honorary doctorates from the universities of Oslo, Zurich, and Braunschweig.

Work

While teaching calculus for the first time at the Polytechnic, Dedekind came up with the notion now called a Dedekind cut (in German: *Schnitt*), now a standard definition of the real numbers. The idea behind a cut is that an irrational number divides the rational numbers into two classes (sets), with all the members of one class (upper) being strictly greater than all the members of the other (lower) class. For example, the square root of 2 puts all the negative numbers and the numbers whose squares are less than 2 into the lower class, and the positive numbers whose squares are greater than

2 into the upper class. Every location on the number line continuum contains either a rational or an irrational number. Thus there are no empty locations, gaps, or discontinuities. Dedekind published his thought on irrational numbers and Dedekind cuts in his paper *Stetigkeit und irrationale Zahlen* ("Continuity and irrational numbers." Ewald 1996: 766).

In 1874, while on holiday in Interlaken, Dedekind met Cantor. Thus began an enduring mutual respect, and Dedekind became one of the very first mathematicians to admire Cantor's work on infinite sets, proving a valued ally in Cantor's battles with Kronecker, who was philosophically opposed to Cantor's transfinite numbers.

If there existed a one-to-one correspondence between two sets, Dedekind said that the two sets were "similar." He invoked similarity to give the first precise definition of an infinite set: a set is infinite when it is "similar to a proper part of itself," in modern terminology, is equinumerous to one of its proper subsets. Thus the set N of natural numbers can be shown to be similar to the subset of N whose members are the squares of every member of N^2, $(N \to N^2)$:

N	1	2	3	4	5	6	7	8	9	10	...
				↓							
N^2	1	4	9	16	25	36	49	64	81	100	...

Dedekind edited the collected works of Dirichlet, Gauss, and Riemann. Dedekind's study of Dirichlet's work was what led him to his later study of algebraic number fields and ideals. In 1863, he published Dirichlet's lectures on number theory as *Vorlesungen über Zahlentheorie* ("Lectures on Number Theory") about which it has been written that:

"Although the book is assuredly based on Dirichlet's lectures, and although Dedekind himself referred to the book throughout his life as Dirichlet's, the book itself was entirely written by Dedekind, for the most part after Dirichlet's death." (Edwards 1983)

The 1879 and 1894 editions of the *Vorlesungen* included supplements introducing the notion of an ideal, fundamental to ring theory. (The word "Ring", introduced later by Hilbert, does not appear in Dedekind's work.) Dedekind defined an ideal as a subset of a set of numbers, composed of algebraic integers that satisfy polynomial equations with integer coefficients. The concept underwent further development in the hands of Hilbert and, especially, of Emmy

Noether. Ideals generalize Ernst Eduard Kummers ideal numbers, devised as part of Kummer's 1843 attempt to prove Fermat's last theorem. (Thus Dedekind can be said to have been Kummer's most important disciple.) In an 1882 article, Dedekind and Heinrich Martin Weber applied ideals to Riemann surfaces, giving an algebraic proof of the Riemann-Roch theorem.

Dedekind made other contributions to algebra. For instance, around 1900, he wrote the first papers on modular lattices.

In 1888, he published a short monograph titled *Was sind und was sollen die Zahlen?* ("What are numbers and what should they be?" Ewald 1996: 790), which included his definition of an infinite set. He also proposed an axiomatic foundation for the natural numbers, whose primitive notions were one and the successor function. The following year, Peano, citing Dedekind, formulated an equivalent but simpler set of axioms, now the standard ones.

Camille Jordan

Camille Jordan

Marie Ennemond Camille Jordan (January 5, 1838 – January 22, 1922) was a French mathematician, known both for his foundational work in group theory and for his influential *Cours d'analyse*. He was born in Lyon and educated at the École polytechnique. He was an engineer by profession; later in life he taught at the École polytechnique and the Collège de France; where he had a reputation for eccentric choices of notation.

He is remembered now by name in a number of foundational results:

- the Jordan curve theorem, a topological result required in complex analysis;
- the Jordan normal form and the Jordan matrix in linear algebra;
- in mathematical analysis, Jordan measure (or *Jordan content*) is an area measure that predates measure theory;
- in group theory the Jordan-Hölder theorem on composition series is a basic result.

Jordan's work did much to bring Galois theory into the mainstream. He also investigated the Mathieu groups, the first examples of sporadic groups. His *Traité des substitutions*, on permutation groups, was published in 1870.

The asteroid 25593 Camillejordan and Institute of Camille Jordan are named in his honour.

Camille Jordan is not to be confused with the geodesist Wilhelm Jordan (Gauss-Jordan elimination) or the physicist Pascual Jordan (Jordan algebras).

Georg Cantor

Georg Cantor

Georg Ferdinand Ludwig Philipp Cantor (March 3, 1845, St. Petersburg – January 6, 1918, Halle) was a German mathematician who is best known as the creator of set theory. Cantor established the importance of one-to-one correspondence between sets, defined infinite and well-ordered sets, and proved that the real numbers are "more numerous" than the natural numbers. In fact, Cantor's theorem implies the existence of an "infinity of infinities." He defined the cardinal and ordinal numbers, and their arithmetic. Cantor's work is of great philosophical interest, a fact of which he was well aware.

Cantor's work encountered resistance from mathematical contemporaries such as Leopold Kronecker and Henri Poincaré, and later from Hermann Weyl and L.E.J. Brouwer. Ludwig Wittgenstein raised philosophical objections. His recurring bouts of depression from 1884 to the end of his life were once blamed on the hostile attitude of many of his contemporaries, but these bouts can now be seen as probable manifestations of a bipolar disorder.

Nowadays, the vast majority of mathematicians who are neither constructivists nor finitists accept Cantor's work on transfinite sets and arithmetic, recognizing it as a major paradigm shift. In the words of David Hilbert: "No one shall expel us from the Paradise that Cantor has created."

Life

The ancestry of Cantor's father, Georg Woldemar Cantor, is not entirely clear. He was born between 1809 and 1814 in Copenhagen, Denmark, and brought up in a Lutheran German mission in St. Petersburg. Georg Cantor's father was a Danish man of Lutheran religion. [1] His mother, Maria Anna Böhm, was born in St. Petersburg and came from an Austrian Roman Catholic family. She had converted to Protestantism upon marriage. Georg Cantor was the eldest of six children. The father was very devout and instructed all his children thoroughly in religious affairs. Throughout the rest of his life Georg Cantor held to the Christian (Lutheran) faith.

The father was a broker on the St Petersburg Stock Exchange. Cantor, an outstanding violinist, inherited his parents' considerable musical and artistic talents.

When Cantor's father became ill, the family moved to Germany in 1856, first to Wiesbaden then to Frankfurt, seeking winters milder than those of St. Petersburg. In 1860, Cantor graduated with distinction from the Realschule in Darmstadt; his exceptional skills in mathematics, trigonometry in particular, were noted. In 1862, following his father's wishes, Cantor entered the Federal Polytechnic Institute in Zurich, today the ETH Zurich and began studying mathematics.

After his father's death in 1863, Cantor shifted his studies to the University of Berlin, attending lectures by Weierstrass, Kummer, and Kronecker, and befriending his fellow student Hermann Schwarz. He spent a summer at the University of Göttingen, then and later a very important center for mathematical research. In 1867, Berlin granted him the Ph.D. for a thesis on number theory, *De aequationibus secundi gradus indeterminatis*. After teaching one year in a Berlin girls' school, Cantor took up a position at the University of Halle, where he spent his entire career. He was awarded the requisite habilitation for his thesis on number theory.

In 1874, Cantor married Vally Guttmann. They had six children, the last born in 1886. Cantor was able to support a family despite modest academic pay, thanks to an inheritance from his father. During his honeymoon in Switzerland, Cantor spent much time in mathematical discussions with Richard Dedekind, whom he befriended two years earlier while on another Swiss holiday.

Cantor was promoted to Extraordinary Professor in 1872, and made full Professor in 1879. To attain the latter rank at the age of 34 was a notable accomplishment, but Cantor very much desired a chair at a more prestigious university, in particular at Berlin, then the leading German university. However, Kronecker, who headed mathematics at Berlin until his death in 1891, and his colleague Hermann Schwarz were not agreeable to having Cantor as a colleague. Worse yet, Kronecker, who was peerless among German mathematicians while he was alive, fundamentally disagreed with the thrust of Cantor's work. Kronecker, now seen as one of the founders of the constructive viewpoint in mathematics, disliked much of Cantor's set theory because it asserted the existence of sets satisfying certain properties, without giving specific examples of sets whose members did indeed satisfy those properties. Cantor came to believe that Kronecker's stance would make it impossible for Cantor to ever leave Halle.

In 1881, Cantor's Halle colleague Eduard Heine died, creating a vacant chair. Halle accepted Cantor's suggestion that it be offered to Dedekind, Heinrich Weber, and Franz Mertens, in that order, but each declined the chair after being offered it. This episode is revealing of Halle's lack of standing among German mathematics departments. Wangerin was eventually appointed, but he was never close to Cantor.

In 1884, Cantor suffered his first known bout of depression. This emotional crisis led him to apply to lecture on philosophy rather than on mathematics. Every one of the 52 letters Cantor wrote to Mittag-Leffler that year attacked Kronecker. Cantor soon recovered, but a passage from one of these letters is revealing of the damage to his self-confidence:

"... I don't know when I shall return to the continuation of my scientific work. At the moment I can do absolutely nothing with it, and limit myself to the most necessary duty of my lectures; how much happier I would be to be scientifically active, if only I had the necessary mental freshness."

Although he performed some valuable work after 1884, he never attained again the high level of his remarkable papers of 1874-84. He eventually sought a reconciliation with Kronecker, which Kronecker graciously accepted. Nevertheless, the philosophical disagreements and difficulties dividing them persisted. It was once thought that Cantor's recurring bouts of depression were triggered by the opposition his work met at the hands of Kronecker. While Cantor's mathematical worries and his difficulties dealing with certain people were greatly magnified by his depression, it is doubtful whether they were its cause, which was probably bipolar disorder.

In 1888, he published his correspondence with several philosophers on the philosophical implications of his set theory. Edmund Husserl was his Halle colleague and friend from 1886 to 1901. While Husserl later made his reputation in philosophy, his doctorate was in mathematics and supervised by Weierstrass' student Leo Königsberger. On Cantor, Husserl, and Frege, see Hill and Rosado Haddock (2000). Cantor also wrote on the theological implications of his mathematical work; for instance, he identified the Absolute Infinite with God.

Cantor believed that Francis Bacon wrote the plays attributed to Shakespeare. During his 1884 illness, he began an intense study of Elizabethan literature in an attempt to prove his Bacon authorship thesis. He eventually published two pamphlets, in 1896 and 1897, setting out his thinking about Bacon and Shakespeare.

In 1890, Cantor was instrumental in founding the *Deutsche Mathematiker-Vereinigung*, chaired its first meeting in Halle in 1891, and was elected its first president. This is strong evidence that Kronecker's attitude had not been fatal to his reputation. Setting aside the animosity he felt towards Kronecker, Cantor invited him to address the meeting; Kronecker was unable to do so because his spouse was dying at the time.

After the 1899 death of his youngest son, Cantor suffered from chronic depression for the rest of his life, for which he was excused from teaching on several occasions and repeatedly confined in various sanatoria. He did not abandon mathematics completely, lecturing on the paradoxes of set theory (eponymously attributed to Burali-Forti, Russell, and Cantor himself) to a meeting of the *Deutsche Mathematiker-Vereinigung* in 1903, and attending the International Congress of Mathematicians at Heidelberg in 1904.

In 1911, Cantor was one of the distinguished foreign scholars invited to attend the 500th anniversary of the founding of the University of St. Andrews in Scotland. Cantor attended, hoping to meet Bertrand Russell, whose newly published *Principia Mathematica* repeatedly cited Cantor's work, but this did not come about. The following year, St. Andrews awarded Cantor an honorary doctorate, but illness precluded his receiving the degree in person.

Cantor retired in 1913, and suffered from poverty, even hunger, during WWI. The public celebration of his 70th birthday was cancelled because of the war. He died in the sanatorium where he had spent the final year of his life.

René-Louis Baire

René-Louis Baire (born January 21, 1874, died July 5, 1932) was a French mathematician. He was born in Paris, France and died in Chambéry, France.

Dogged by ill health, and spending time alternating between low level teaching in lycées and work in universities, he was only able to make contributions to mathematics in limited spells. He had research interests in continuity and irrational numbers

Henri Lebesgue

Henri Lebesgue

Henri Léon Lebesgue [ɑ̃ʁiː leɔ̃ ləˈbɛg] (June 28, 1875, Beauvais – July 26, 1941, Paris) was a French mathematician, most famous for his theory of integration. Lebesgue's integration theory was originally published in his dissertation, *Intégrale, longueur, aire* ("Integral, length, area"), at the University of Nancy in 1902.

Lebesgue's father was a typesetter, who died of tuberculosis when his son was still very young, and Lebesgue himself suffered from poor health throughout his life. After the death of his father, his mother worked tirelessly to support him. He was a brilliant student in primary school, and he later studied at the Ecole Normale Supérieure.

Lebesgue married the sister of one of his fellow students, and he and his wife had two children, Suzanne and Jacques. He worked on his dissertation while teaching in Nancy at a preparatory school.

David Hilbert

David Hilbert

David Hilbert (1912)

Born	January 23, 1862 Wehlau, East Prussia
Died	February 14, 1943 Göttingen, Germany
Residence	Germany
Nationality	German
Field	Mathematician
Institution	University of Königsberg Göttingen University
Alma Mater	University of Königsberg
Doctoral Advisor	Ferdinand von Lindemann
Doctoral Students	Otto Blumenthal Richard Courant Max Dehn Erich Hecke Hellmuth Kneser Robert König Erhard Schmidt Hugo Steinhaus Emanuel Lasker Hermann Weyl Ernst Zermelo
Known for	Hilbert's basis theorem Hilbert's axioms

Hilbert's problems
Hilbert's program
Einstein-Hilbert action
Hilbert space

David Hilbert (January 23, 1862, Wehlau, East Prussia – February 14, 1943, Göttingen, Germany) was a German mathematician, recognized as one of the most influential and universal mathematicians of the 19th and early 20th centuries. He invented or developed a broad range of fundamental ideas, in invariant theory, the axiomization of geometry, and with the notion of Hilbert space, one of the foundations of functional analysis.

He adopted and warmly defended Cantor's set theory and transfinite numbers. A famous example of his leadership in mathematics is his 1900 presentation of a collection of problems that set the course for much of the mathematical research of the 20th century.

Hilbert and his students supplied significant portions of the mathematical infrastructure required for quantum mechanics and general relativity. He is also known as one of the founders of proof theory, mathematical logic and the distinction between mathematics and metamathematics.

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Stefan Banach

Stefan Banach (March 30, 1892 in Kraków, Austria-Hungary now Poland– August 31, 1945 in Lviv, Soviet Union now Ukraine), was an eminent Polish mathematician, one of the moving spirits of the Lwów School of Mathematics in pre-war Poland. He was largely self-taught in mathematics; his genius was accidentally discovered by Juliusz Mien and later Hugo Steinhaus.

When World War II began, Banach was President of the Polish Mathematical Society and a full professor of University of Lwów. Being a corresponding member of Academy of Sciences of the Ukrainian SSR, and otherwise on good terms with Soviet mathematicians, he was allowed to keep his chair, from 1939, of the city. Banach survived the subsequent brutal German occupation from July 1941 up to February 1944, earning a living by feeding lice with his blood in the Typhus Research Institute of Prof. Rudolf Weigl. His health declined during the occupation, and he developed lung cancer. After the war Lwów was incorporated into the Soviet Union, and Banach died there before he could be repatriated to Kraków, Poland. He is buried at the Lyczakowski Cemetery.

Cafe Szkocka

Stefan Banach's Grave