Design Chart for Reinforced Concrete Rectangular Section

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ABSTRACT

Design charts are very useful for fast determining the percentage of reinforcement for singly and doubly reinforced concrete beams having known cross-sectional dimensions, characteristic strengths of the concrete and steel, and the ultimate design moment. A complete set of design charts is given in BS: 8110 - 85: Part 3, but it seems difficult to use these charts especially for doubly reinforced beams. In this paper, a straight-forward design chart has been developed based on BS: 8110 – 97 design rules. Several design examples have been solved using the developed chart and good results have been obtained.

INTRODUCTION

Considerable literature is available on the ultimate load design method for beams [1]. It is widely used for concrete design and is included in many codes [2, 3]. The design charts published by British Standard Code BS: 8110 -85 [4] are obtained by plotting the factor K (M_u/bd^2) and the percentage of steel area ($100A_s/bd$) for singly and doubly reinforced concrete beams and for different values of characteristic steel strength (f_y) and concrete cube strength (f_{cu}). It seems difficult to use the BS: 8110 - 85 charts especially for doubly reinforced beams where the percentage of the compression steel can not be read directly but by doing interpolation between very close curves. Moreover these charts are not updated for the new safety factors of material of BS: 8110 – 97 [5].

In the present study, simplified design chart has been developed for determining the percentage of reinforcement for both singly and doubly reinforced beams. The design parameters, materials strengths and design moment are expressed as non-dimensional terms for preparing this chart. The use of non-dimensional parameters for preparing the design chart reduces the number of charts from 20 in BS: 8110 – 85 to one for both singly and doubly reinforced beams. The use of the chart as presented is simple.

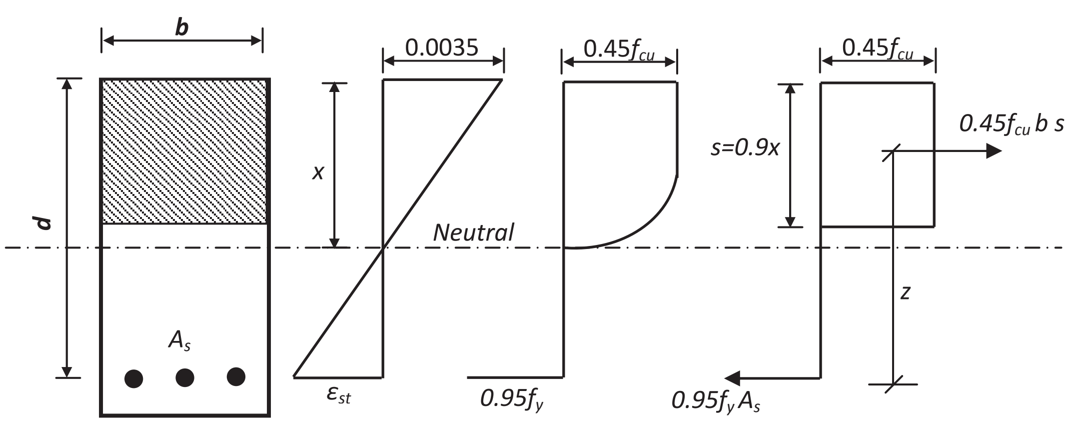
Ultimate Strength Design

Following are the basic assumptions made in the ultimate strength design method as presented in the design text and hand books [6, 7], which are based on BS: 8110;

1. Plane sections normal to the axis of the member remain plane after bending.
2. The maximum strain in concrete at the outermost compression fiber is 0.0035.
3. The tensile strength of concrete is ignored.

Singly Reinforced Section

A singly reinforced section when flexural strength is reached is shown in Fig. 1. The section can be analyzed using the internal couple system and the requirements of strain compatibility and equilibrium of forces. Instead of the rectangular-parabolic stress block (Fig. 1- c), an equivalent rectangular stress block has been assumed Fig. 1- d.



(a) Section (b) Strains (c) rectangular- (d) equivalent

Figure 1 Singly Reinforced Section

The equilibrium of the compressive and tensile forces equation (Fig. 1- c) can be written as:

$$0.95 f_y A_s = 0.45 f_{cu} b \times 0.9 x \tag{1}$$

Thus, the depth of the uniform stress block of the compression concrete ***x*** is obtained from the above equilibrium equation as:

$$x = 2.346 \frac{A_s f_y}{f_{cu} b} \tag{2}$$

The lever arm of the resultant concrete force about the tension steel can be expressed as:

$$z = d - 0.45x \tag{3}$$

Substitute Eq. 2 in Eq. 3 we get:

$$z = d\left(1 - \frac{1.056 A_s f_y}{f_{cu} bd}\right) \tag{4}$$

Taking the moment about the resultant concrete compression force, the ultimate moment of resistance of the section can be expressed as:

$$M_u = 0.95 f_y A_s z \tag{5}$$

Substitute Eq. 4 in Eq.5 we get:

$$M_u = 0.95 f_y A_s d \left(1 - \frac{1.056 A_s f_y}{f_{cu} bd}\right) \tag{6}$$

Dividing both terms of Eq. 6 by $f_{cu} bd^2$ and use the notations;

$K = \dfrac{M_u}{f_{cu} bd^2}$, $\rho = \dfrac{100 A_s}{bd}$, and $m = \dfrac{f_y}{f_{cu}}$, we get:

$$K = 0.0095\, m\rho - 0.0001 (m\rho)^2 \tag{7}$$

Note that the BS: 8110 – 97 code stipulates in clause 3.4.4.4 that the lever arm *z* must not be more than 0.95*d* in order to give a reasonable concrete area in compression. At this limit of *z*, the value of K equal 0.0428. Therefore when K is less than 0.0428, the steel area should be calculated using z = 0.95d.

Substitute z = 0.95d in Eq. (5) and use the earlier notations we get;

$$K = 0.009\, m\rho \qquad (8)$$

Thus, the first part of the design chart for singly reinforced section can be constructed by plotting K against $m\rho$ as a piecewise curve as follows;

$$K = 0.009 m\rho \qquad \text{for } K \le 0.0428;$$

$$K = 0.0095\, m\rho - 0.0001(m\rho)^2 \qquad \text{for } K > 0.0428;$$

Thus for design purpose, if the moment M and the beam dimensions b and d are given, K is calculated and $\frac{100 A_s}{bd}\frac{f_y}{f_{cu}}$ can be read from chart, then the steel area can be calculated easily.

Doubly Reinforced Section

The moment of resistance of the doubly reinforced section shown in Fig. 2 is given by [5];

$$M_u = K' f_{cu} bd^2 + 0.95 f_y A'_s (d - d') \qquad (9)$$

where K' is a factor depending on the ratio of the stress block depth to the effective depth (x/d). Divide both sides of Eq.(9) by the term $f_{cu}bd^2$ and use the notation $\rho' = \frac{100 A'_s}{bd}$ in addition to the notations used earlier we get;

$$K = K' + 0.0095 m\rho'(d - d') \qquad (10)$$

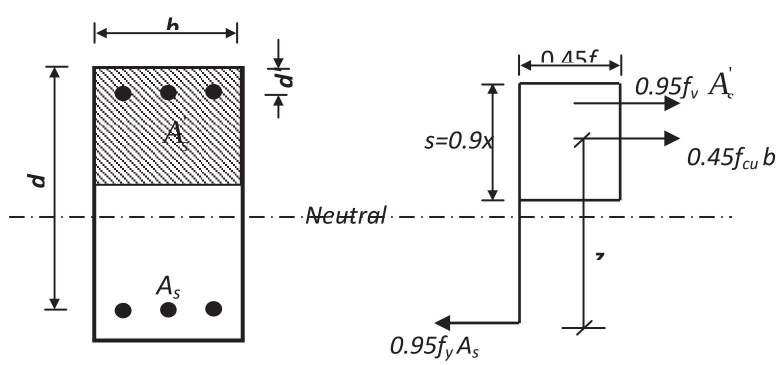


Figure 2 Doubly Reinforced Section

Separate charts are required for different values of d'. If a value of $0.1d$ is used for the inset of the compression steel (d') as usually used in practice, equation (10) becomes;

$$K = K' + 0.00855\, m\rho' \tag{11}$$

Referring to Fig. (2), the equilibrium of compressive and tensile forces can be expressed as;

$$0.95 f_y A_s = 0.45 f_{cu} b \times 0.9\, x + 0.95 f_y A'_s \tag{12}$$

Depending on moment redistribution factor β_b, the neutral axis depth can be expressed as a fraction of the effective depth (c) as;

$$x = c\, d \tag{13}$$

Substitute Eq. (13) in (12) we get;

$$0.95 f_y A_s = 0.405 f_{cu} b\, c\, d + 0.95 f_y A'_s \tag{14}$$

Dividing both terms of the above equation by the term $0.95 f_y bd$, and use the earlier notations of ρ, ρ', and m, the percentage of the compression steel ρ' can be expressed as;

$$\rho' = \rho - \frac{42.6c}{m} \tag{15}$$

Substitute Eq. (15) in Eq. (11) we get;

$$K = \left(K' - 0.365c\right) + 0.00855\, m\rho$$

or

$$K = p + 0.00855\, m\rho \tag{16}$$

From the comparison of Eq. (11) and Eq. (16), it is clearly observed that, the slopes are same (0.00855) and only the intercepts are different, thus the same curve can be used for determining both the tensile steel area A_s (lower abscissa) and the compressive steel area A'_s (upper abscissa). Only the starting point for the upper abscissa of the compression steel needs to shift.

If the neutral axis depth x is taken as $0.5d$, then Eq. (11) becomes;

$$K = 0.156 + 0.00855\, m\rho'$$

and Eq. (16) becomes;

$$K = -0.02625 + 0.00855\, m\rho$$

Thus, the upper abscissa needs to shift by (0.156+.02625)/.00855 = 21.3.

In same way, the shifting values for the other *x/d* ratios of the ρ' axes can be calculated which are shown in Table 1.

Table 1 Shifting Values of the Upper Axes ρ'

Moment redistribution factor (β_b)	Neutral axis depth (x)	K'	Intercept p	Shifting value
0.9	0.5 d	0.156	-0.02625	21.32
0.8	0.4 d	0.132	-0.0138	17.05
0.7	0.3 d	0.105	-0.00435	12.79

Using Eqs. (7) and (8) for singly reinforced section in addition to Eqns. (11) and (16) and the shifting values in Table 1, an easy design chart for singly and doubly reinforced sections can be constructed as shown in Fig. 3. This chart can be used for any characteristic materials strength of steel and concrete.

RESULTS AND DISCUSSIONS

Design chart has been developed for determining the area of steel for both singly and doubly reinforced beams of a given cross-sectional dimensions (*b* & *d*), the characteristic strengths of the concrete (f_{cu}) and steel (f_y) and the ultimate design moment (M_u).

In order to examine the validity of the prepared design charts, several practice design examples have been solved using the developed chart and the solutions are compared to the solutions obtained using the BS 8110 equations. These design examples are as follows:

Example 1: Design of a singly reinforced rectangular section

In this example, it is required to determine the area of tension reinforcement A_s if the ultimate design moment to be resisted by the beam section is 185 kN m. The beam dimensions are breadth 260 mm and effective depth 440 mm. The characteristic material strengths are f_{cu} = 30 N/mm^2 and f_y = 460 N/mm^2.

$$\frac{100A_s'}{bd}\frac{f_y}{f_{cu}}$$

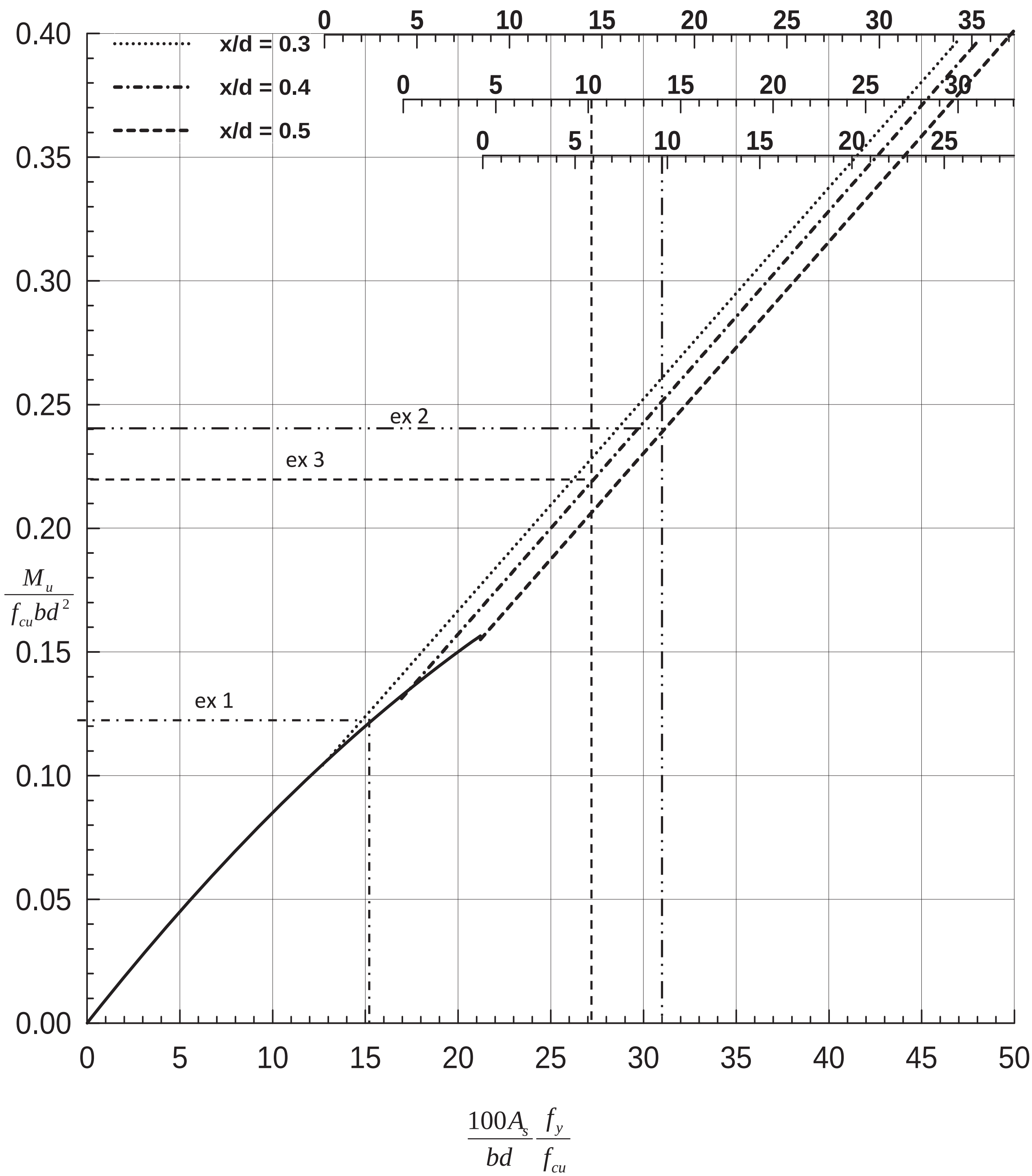


Figure 3 Developed Design Chart

To use the developed design charts, the first step is to calculate the factor *K* as:

$$K = \frac{M_u}{f_{cu} b d^2} = \frac{185 \times 10^6}{30 \times 260 \times 440^2} = 0.123 < 0.156$$

Therefore compression steel is not required.

Using the design chart (Fig. 3); $\frac{100 A_s}{bd} \frac{f_y}{f_{cu}} \approx 15.4$

Thus, $A_s = \frac{15.4 \times 260 \times 440 \times 30}{100 \times 460} = 1149\ mm^2$

Using the BS 8110 design equations, the area of steel requires is 1148 mm^2.

Example 2: Design of a doubly reinforced rectangular section (moment redistribution factor $\beta_b = 0.9$)

In this example, it is required to design the beam section for an ultimate design moment of 288 kN m. The beam is 250 mm wide by 400 mm effective depth and the inset of the compression steel is 40 mm. The characteristic material strengths are f_{cu} = 30 N/mm^2 and f_y = 460 N/mm^2.

The first step is to calculate the factor K as usual:

$$K = \frac{M_u}{f_{cu} b d^2} = \frac{288 \times 10^6}{30 \times 250 \times 400^2} = 0.24 > 0.156$$

Therefore compression steel is required.

Using the design chart (Fig. 3) and since the redistribution factor β_b is 0.9, the upper axis with x = 0.5 is used, thus $\frac{100 A_s'}{bd} \frac{f_y}{f_{cu}} \approx 9.9$

and from the lower axis $\frac{100 A_s}{bd} \frac{f_y}{f_{cu}} \approx 31.2$,

Thus, $A_s' = \frac{9.9 \times 250 \times 400 \times 30}{100 \times 460} = 646\ mm^2$

and $A_s = \frac{31.2 \times 250 \times 400 \times 30}{100 \times 460} = 2035\ mm^2$

Using the BS 8110 design equations, the required area of the compression steel $A_s^{'}$ is 641 mm^2 and the required area of the tension steel A_s is 2023 mm^2.

Example 3: Design of a doubly reinforced rectangular section (moment redistribution factor $\beta_b = 0.8$)

In this example, it is required to design the beam section for an ultimate design moment of 220 kN m. The beam is 250 mm wide by 400 mm effective depth and the inset of the compression steel is 40 mm. The characteristic material strengths are f_{cu} = 25 N/mm^2 and f_y = 410 N/mm^2.

The first step is to calculate the factor K as:

$$K = \frac{M_u}{f_{cu} b d^2} = \frac{220 \times 10^6}{25 \times 250 \times 400^2} = 0.22 > 0.132$$

Therefore compression steel is required.

Using the design chart (Fig. 3) and since the redistribution factor β_b is 0.8, the upper axis with x = 0.4 is used, thus $\frac{100A_s^{'}}{bd}\frac{f_y}{f_{cu}} \approx 10.4$

and from the lower axis $\frac{100A_s}{bd}\frac{f_y}{f_{cu}} \approx 27.4$,

Thus, $A_s' = \frac{10.4 \times 250 \times 400 \times 25}{100 \times 410} = 634\ mm^2$

and $A_s = \frac{27.4 \times 250 \times 400 \times 25}{100 \times 410} = 1671\ mm^2$

Using the BS 8110 design equations, the required area of the compression steel $A_s^{'}$ is 628 mm^2 and the required area of the tension steel A_s is 1687 mm^2.

To check the accuracy of the developed chart, these design examples (1-3) have been solved using BS 8110 code design equations and the solutions are compared with the solutions obtained using the developed chart as shown in Table 1 where all the error percentages are below 1 % which are negligible and mainly due to the chart reading. So a good agreement between the chart solutions and the BS 8110 equations is clearly observed.

Table 1: Chart and BS 8110 Solutions Comparisons:

Example	*Chart Solutions*		*BS 8110 Solutions*		*Error %*	
	A_s' (mm^2)	A_s (mm^2)	A_s' (mm^2)	A_s (mm^2)	A_s' (mm^2)	A_s (mm^2)
1	--	1149	--	1148	--	0.09
2	646	2035	641	2023	0.78	0.59
3	634	1671	628	1687	0.96	0.95

CONCLUSION

A design chart for reinforcement for singly and doubly reinforced beam is presented for a wide range of geometrical parameters expressed as non-dimensional parameters. The use of non-dimensional parameters for preparing the design chart reduces the number of charts to one for both singly and doubly reinforced rectangular beam sections.

The procedure for use of the developed chart as presented is straight-forward and simple. The chart can be used with reliable results by students, teachers and practicing engineers at the design offices and field.

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