Second Edition

Solutions Manual of Introduction to

TOPOLOGY

BS 4-Years, M.Sc. Mathematics

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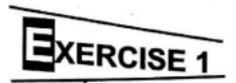
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C_{hapter}

TOPOLOGICAL SPACES



Short Questions

- Q.3 Solve / answer the following short questions: (i)
- Define a topological space.
- Let X be a nonempty set. A collection τ of subsets of X is said to Sol: *be a topology on X if the following conditions are satisfied:
 - The union of any number of members of τ belongs to τ .
 - The intersection of finite number of members of τ belongs to (ii)
 - The empty set ϕ and the set X itself belong to τ . (iii) The pair (X,r) is called a topological space.
- (ii) List all possible topologies on the set $X = \{0, 1\}$.
- Sol: $X = \{0, 1\}$

All topologies on X are

$$\begin{split} \tau_1 &= \{\phi, X\}, & \tau_1 &= \{\phi, \{0\}, X\}, \\ \tau_2 &= \{\phi, \{1\}, X\}, & \tau_2 &= \{\phi, \{0\}, \{1\}, X\} \end{split}$$

- If $X = \{0,1\}$, find all possible Sierpinski spaces. (iii)
- Sol: $X = \{0, 1\}$

Consider the following topologies on X:

$$\tau_1 = \{\phi, \{0\}, X\}, \qquad \qquad \tau_2 = \{\phi, \{1\}, X\}$$

 $(X, \tau_1), (X, \tau_2)$ are all possible Sierpinski spaces.

- If $X = \{a, b, c, d, e, f\}$, find three Sierpinski spaces. (iv)
- Sol: $X = \{a, b, c, d, e, f\}$

Consider the following three topologies on X:

$$\tau_1 = \{\phi, \{a\}, X\}, \qquad \tau_2 = \{\phi, \{a, f\}, X\}$$

 $\tau_3 = \{\phi, \{b, c, f\}, X\}$

 $(X, \tau_1), (X, \tau_2), (X, \tau_3)$ are three Sierpinski spaces.

- (v) Let $\tau = \{\phi, A_q, R\}$, where $A_q = (q, \infty), q \in Q$ (Q is the set of rational numbers). Whether $\cup \{A_q : q \in Q, q > \sqrt{2}\} \in \tau$.
- **Sol**: It is clear by the definition of A_q that

$$\bigcup \{A_q : q \in \mathbb{Q}, q > \sqrt{2}\} = (\sqrt{2}, \infty)$$

Since $\sqrt{2}$ is irrational, so $(\sqrt{2}, \infty) \notin \tau$. This shows that

$$\bigcup \{A_q : q \in Q, q > \sqrt{2}\} \notin \tau$$

- (vi) Let $A_q = (-\infty, q)$, $q \in Q$, where Q is the set of rational numbers. Show that $\tau = \{\phi, A_q, R\}$ is not a topology on the set of real numbers R.
- Sol: We observe that

$$A = \bigcup \{A_q : q \in \mathbb{Q}, q < \sqrt{2}\} = (-\infty, \sqrt{2})$$

is the union of members of τ , but $A \notin \tau$, because $\sqrt{2}$ is irrational. This shows that union of members of τ is not in τ , so τ is not a topology on R.

- (vii) List all topologies on $X = \{a, b\}$ which consist of exactly four members.
- Sol: $X = \{a, b\}$

All the possible subsets of X are

$$\phi, \{a\}, \{b\}, X$$

Since $\{a\},\{b\}$ form the partition of X, so

$$\tau = \{\phi, \{a\}, \{b\}, X\}$$

is a topology on X.

Since the four subsets of X are not totally ordered by inclusion, so τ is the only topology on X consisting of four members.

- (viii) Let X be an indiscrete topological space and A be a singleton subset of X. Find A^d .
- Since A is a singleton set, so let $A = \{a\}$. Since X is an indiscrete topological space, so X itself is the only nonempty open set. Obviously, a is not the limit point of A because open set X does not contain any point of A different from a. Any point $x \neq a$ of X is a limit point of A because the open set X containing x contains a point a of A different from x. Hence

 $A^d = X - A = A^c$ If A is any subset of a cofinite topological space X, then find \overline{A}

Sol: Finite subsets of X and X itself are the only closed sets in X. Since

A is the smallest closed superset of A, so