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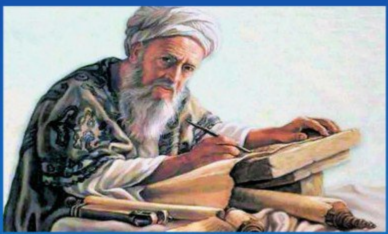
SEVEN MUSLIMS NOTES

PHYSICS 11



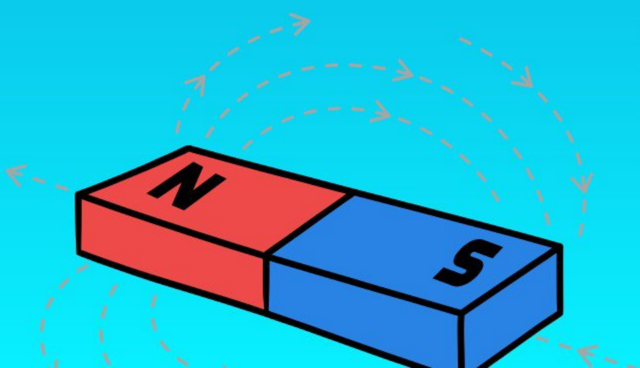
Al-Biruni (973–1048) calculated the Earth's radius and worked on the physics of planetary motion.

Best Regards to

Sir Muhammad Ali

(Physics Lecturer KIPS College)

$E=mc^2$



VECTORS AND EQUILIBRIUM

EXERCISE SHORT QUESTIONS

2.1. Define the terms (i) Unit vector (ii) Position vector (iii) Component of a vector

i. Unit vector

"A vector in a given direction with magnitude one in that direction is called unit vector." It is used to represent the direction of a vector.

The unit vector $\hat{A}$ of a vector $\overline{A}$ is given as

$$\hat{A} = \frac{\overline{A}}{A}$$

ii. Position vector

"A vector that is used to describe the location of a point with respect to the origin is called position vector."

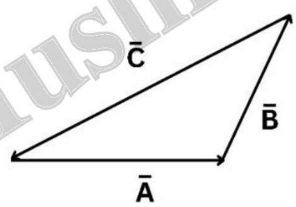
Usually it is denoted by $\overline{r}$ and is given as

$$\overline{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

iii. Component of a vector

"The effective value of a vector in a given direction is called component of a vector."

2.2. The vector sum of the vectors gives zero resultant. What can be the orientation of the vectors?



If the three vectors A, B and C are arranged in such a way that they form a triangle then their vector sum will give zero resultant.

2.3. Vector A lies in the xy plane. For what orientation will both of us components be negative? For what orientation will is components have opposite signs?

- **i.** In third Quadrant both of the components of vector A have negative signs.
- **ii.** In second or fourth quadrant, both of the components of vector A have opposite signs.

2.4. If one of the rectangular components is a vector is not zero can its magnitude be zero? Explain.

No, if one of the rectangular components of a vector is not zero, then its magnitude can never be zero. Magnitude of a vector in a plane is given as

$$\mathbf{A} = \sqrt{\mathbf{Ax^2 + Ay^2}}$$

For space

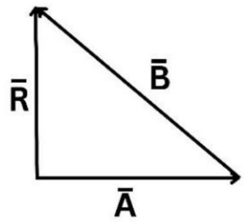
$$\mathbf{A} = \sqrt{\mathbf{Ax^2 + Ay^2 + Az^2}}$$

From both of above formulas, it can be seen that for zero magnitude, all the rectangular components of a vector must be zero.

2.5. Can a vector have component greater than the vectors magnitude?

1) In case of components of a vector other than rectangular components, a vector may have components greater than vector's magnitude.

A >R and B > R



2) In case of rectangular components, a vector can never have components greater than vector's magnitude.

$$A = \sqrt{Ax^2 + Ay^2 + Az^2}$$

2.6. Can magnitude of a vector have a negative value?

No, the magnitude of a vector involves square of its rectangular components. Thus its magnitude can never be negative.

$$A = \sqrt{Ax^2 + Ay^2 + Az^2}$$

2.7. If $\overline{A} + \overline{B} = \overline{0}$, What can you say about the components of the two vectors?

If A +B = 0 then

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + B_x\hat{i} + B_y\hat{j} + B_z\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$A_x\hat{i} + B_x\hat{i} + A_y\hat{j} + B_y\hat{j} + A_z\hat{k} + B_z\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$(A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$A_x + B_x = 0 \quad \text{i.e. } A_x = -Bx$$

$$A_y + B_y = 0 \quad \text{i.e. } A_y = -By$$

$$A_z + B_z = 0 \quad \text{i.e. } A_z = -Bz$$

Hence if A+B = 0, then components of the two vectors will be equal in magnitude but opposite in direction.

2.8. Under what circumstances would a vector have components that are equal in magnitude?

For components of a vector to be equal, the angle between them must be 45°.

$$A_y = A_x$$

$$A\sin\theta = A\cos\theta$$

$$\text{Sin}\theta = \text{Cos}\theta$$

$$\frac{\sin\theta}{\text{Cos}\theta} = 1$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1) = 45°$$

2.9. Is it possible to add a vector quantity to a scalar quantity? Explain.

No, it is not possible to add a vector quantity to a scalar quantity because both are different in nature. A scalar quantity has magnitude while vector quantity has magnitude as well as direction.

2.10. Can you add zero to a null vector?

No, we cannot add zero to a null vector because zero is a scalar quantity while null vector is a vector quantity and a scalar cannot be added to a vector because both are different in nature.

2.11. Two vectors have unequal magnitude, Can their sum be zero? Explain.

For two vectors to give their sum equal to zero they must have equal magnitude but opposite direction. Sum of two vectors with unequal magnitude can never be zero.

$$A + (-A) = 0$$

2.12. Show that the sum as well as the product of two perpendicular vectors of equal lengths are also perpendicular and of same length.

Let us consider two vectors A and B such that A.B = B.A = 0 and A = B

Sum of two vectors = S = A + B

Difference of two vectors = D = A − B

If A+ B and A− B are perpendicular to each other than

$$(A + B).(A - B) = A^2 - A.B + B.A - B^2$$

$$(A + B).(A - B) = A^2 - 0 + 0 - A^2$$

Thus the sum and difference of two vectors are perpendicular to each other.

$$\text{Length of S} = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 + 2AB\cos 90} = \sqrt{A^2 + B^2}$$

$$\text{Length of D} = \sqrt{A^2 + (-B)^2 + 2A(-B)\cos 90} = \sqrt{A^2 + B^2 - 2AB\cos 90} = \sqrt{A^2 + B^2}$$

Thus the sum and difference of two vectors have same lengths.

2.13. How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude?

Let us consider two vectors A and B such that A = B

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$$

$$R = \sqrt{2A^2 + 2A^2\cos\theta} = \sqrt{2A^2\,(1 + \cos\theta)}$$

If R = A = B

$$A = \sqrt{2}\,A\sqrt{1 + \cos\theta}$$

$$\frac{A}{A} = \sqrt{2}\,\sqrt{1 + \cos\theta}$$

$$1 = \sqrt{2}\,\sqrt{1 + \cos\theta}$$

$$\frac{1}{\sqrt{2}} = \sqrt{1 + \cos\theta}$$

$$\frac{1}{2} = 1 + \cos\theta$$

$$\frac{1}{2} - 1 = \cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \theta$$

$$\theta = 120°$$

Thus the two vectors must be oriented at 120°.

2.14. The two vectors to be combined have magnitude ………..

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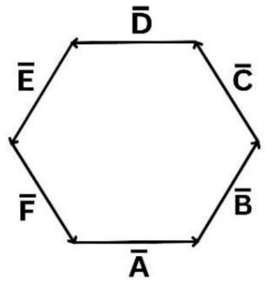
2.15. Suppose the sides of a closed polygon represent vector arranged head to tail. What is the sum of these vectors?



In a closed polygon for example a hexagon, the six vectors A, B, C, D, E and F are arranged by head to tail rule in such a way that their resultant is a null vector.

2.16. Identify the correct answer:

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2.17. If all the components of the vectors A_1 and A_2 were reversed, how would this alter $A_1 \times A_2$?

If all the components of the vectors A_1 and A_2 were reversed then

$$A_1 = -A_1 \text{ and } A_2 = -A_2$$

And hence

$$(-\mathbf{A_1}) \times (-\mathbf{A_2}) = \mathbf{A_1} \times \mathbf{A_2}$$

Thus the product $\mathbf{A_1} \times \mathbf{A_2}$ will remain unaffected because both $\mathbf{A_1} \times \mathbf{A_2}$ and $(-\mathbf{A_1}) \times (-\mathbf{A_2})$ will have the same magnitude and direction.

2.18. Name the three different conditions that could make $A_1 \times A_2 = 0$.

The three conditions are

1. $\mathbf{A_1}$ is a Null vector.
2. $\mathbf{A_2}$ is a null vector.
3. $\mathbf{A_1}$ and $\mathbf{A_2}$ both are parallel ($\theta = 0°$) or anti parallel ($\theta = 180°$).

2.19. Identify true or false statements and explain the reason.

(a) A body in equilibrium implies that it is not moving nor rotating.

This statement is false because a moving body can also be in equilibrium when it is moving with uniform velocity.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

This statement is true because if coplanar forces acting on a body form a closed polygon then vector sum of all the forces is zero and body will be in equilibrium.

2.20. A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tensions in the strings will be minimum.

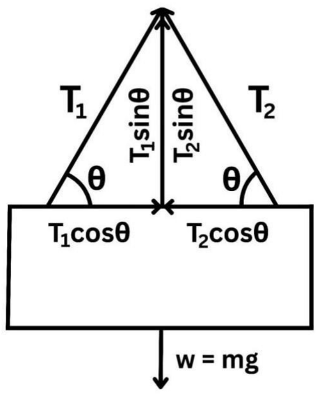
Consider a picture of weight w suspended by two strings as shown in the figure. As picture is in equilibrium then

$$\Sigma F_x = 0$$

$$T_1\cos\theta - T_2\cos\theta = 0$$

$$T_1\cos\theta = T_2\cos\theta$$

$$T_1 = T_2 = T$$



Similarly

$$\Sigma F_y = 0$$

$$T_1\sin\theta + T_2\sin\theta - w = 0$$

$$T\sin\theta + T\sin\theta = w$$

$$2T\sin\theta = w$$

$$T = \frac{w}{2\sin\theta}$$

The above equation shows that tension is minimum when $\sin\theta$ is maximum. Sinθ is Maximum at 90° so tension is minimum when strings are oriented at 90°.

2.21. Can a body rotate about its center if gravity under the action of its weight?

No, a body cannot rotate about its centre of gravity because in this case line of action of force passes through pivot point, so moment arm becomes zero and hence torque is zero.

$$\tau = lf = (0)f = 0$$