Solutions Manual of

LAPLACE, FOURIER AND Z-TRANSFORMS

with Applications

Second Edition

BS 4-Years, M.Sc. Mathematics,
M.Sc. Physics & B.Sc. Engineering

According to New Syllabus
Approved by the
University of Punjab,
Govt. College University, Lahore,
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Z.R. Bhatti

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{F}\{f(x)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} f(x)\,dx$$

$$Z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

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Chapter 1

LPLACE TRANSFORM

EXERCISE 1-1

Short Questions

Q.2 Solve / write answers of the following short questions:

(i) Evaluate $\mathcal{L}\{k\}$, where k is nonzero real number.

Sol: $$\mathcal{L}\{k\}=\int_0^\infty e^{-st}k\,dt=k\int_0^\infty e^{-st}dt=k\left|\frac{e^{-st}}{-s}\right|_0^\infty$$

$$=k\left(\frac{e^{-\infty}-e^0}{-s}\right)=k\left(\frac{0-1}{-s}\right)=\frac{k}{s}\quad (\text{Re}.s>0)$$

(ii) Show that $\mathcal{L}\{t\}=\frac{1}{s^2}$ (Re. $s>0$).

Sol: $$\mathcal{L}\{t\}=\int_0^\infty e^{-st}t\,dt=\left|\frac{te^{-st}}{-s}\right|_0^\infty-\int_0^\infty\frac{e^{-st}dt}{-s}\quad (\text{Re}.s>0)$$

$$=\frac{1}{s}\int_0^\infty e^{-st}dt=\frac{1}{s}\left|\frac{e^{-st}}{-s}\right|_0^\infty=\frac{1}{s^2}\quad (\text{Re}.s>0)$$

(iii) Show that $\mathcal{L}\{t^2\}=\frac{2}{s^3}$ (Re. $s>0$).

Sol: $$\mathcal{L}\{t^2\}=\int_0^\infty e^{-st}t^2dt=\left|\frac{t^2e^{-st}}{-s}\right|_0^\infty-\int_0^\infty\frac{2te^{-st}dt}{-s}\quad (\text{Re}.s>0)$$

$$=\frac{2}{s}\int_0^\infty te^{-st}dt=\frac{2}{s}\left|\frac{te^{-st}}{-s}\right|_0^\infty-\frac{2}{s}\int_0^\infty\frac{e^{-st}dt}{-s}\quad (\text{Re}.s>0)$$

$$=\frac{2}{s^2}\int_0^\infty e^{-st}dt=\frac{2}{s^2}\left|\frac{e^{-st}}{-s}\right|_0^\infty=\frac{2}{s^3}\quad (\text{Re}.s>0)$$

(iv) **Show that** $\mathscr{L}\{e^{3t}\} = \dfrac{1}{s-3}$ **(Re.** $s > 3$**).**

Sol: $$\mathscr{L}\{e^{3t}\} = \int_0^\infty e^{-st}e^{3t}\,dt = \int_0^\infty e^{-(s-3)t}\,dt$$

$$= \left|\frac{e^{-(s-3)t}}{-(s-3)}\right|_0^\infty = \frac{1}{s-3} \quad (\text{Re}.s > 3)$$

(v) **Show that** $\mathscr{L}\{e^{-2t}\} = \dfrac{1}{s+2}$ **(Re.** $s > -2$**).**

Sol: $$\mathscr{L}\{e^{-2t}\} = \int_0^\infty e^{-st}e^{-2t}\,dt = \int_0^\infty e^{-(s+2)t}\,dt$$

$$= \left|\frac{e^{-(s+2)t}}{-(s+2)}\right|_0^\infty = \frac{1}{s+2} \quad (\text{Re}.s > -2)$$

(vi) **Prove that** $\mathscr{L}\{\cos kt\} = \dfrac{s}{s^2+k^2}$.

Sol: $$\mathscr{L}\{\cos kt\} = \mathscr{L}\left\{\frac{e^{ikt}+e^{-ikt}}{2}\right\} \qquad \because \cos kt = \frac{e^{ikt}+e^{-ikt}}{2}$$

$$= \frac{1}{2}\mathscr{L}\{e^{ikt}\} + \frac{1}{2}\mathscr{L}\{e^{-ikt}\} \qquad \because \mathscr{L} \text{ is linear}$$

$$= \frac{1}{2}\frac{1}{s-ik} + \frac{1}{2}\frac{1}{s+ik} = \frac{(s+ik)+(s-ik)}{2(s-ik)(s+ik)}$$

$$= \frac{2s}{2(s^2+k^2)} = \frac{s}{s^2+k^2}$$

(vii) **Prove that** $\mathscr{L}\{\cosh kt\} = \dfrac{s}{s^2-k^2}$.

Sol: $$\mathscr{L}\{\cosh kt\} = \mathscr{L}\left\{\frac{e^{kt}+e^{-kt}}{2}\right\} \qquad \because \cosh kt = \frac{e^{kt}+e^{-kt}}{2}$$

$$= \frac{1}{2}\mathscr{L}\{e^{kt}\} + \frac{1}{2}\mathscr{L}\{e^{-kt}\} \qquad \because \mathscr{L} \text{ is linear}$$

$$= \frac{1}{2}\frac{1}{s-k} + \frac{1}{2}\frac{1}{s+k} = \frac{(s+k)+(s-k)}{2(s-k)(s+k)}$$

$$= \frac{2s}{2(s^2-k^2)} = \frac{s}{s^2-k^2}$$

(viii) **Prove that** $\mathscr{L}\{\sinh kt\} = \dfrac{k}{s^2-k^2}$.