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SCHAUM'S SOLVED PROBLEMS SERIES

3000 SOLVED PROBLEMS IN

LINEAR ALGEBRA

by

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To the Student

This collection of thousands of solved problems covers almost every type of problem which may appear in any course in linear algebra. Moreover, our collection includes both computational problems and theoretical problems (which involve proofs).

Each section begins with very elementary problems and their difficulty usually increases as the section progresses. Furthermore, the theoretical problems involving proofs normally appear after the computational problems, which can thus preview the theory. (Most students have more difficulty with proofs.)

Normally, students will be assigned a textbook for their linear algebra course. The sequence of our chapters follows the customary order found in most textbooks (although there may be some discrepancies). However, whenever possible, our chapters and sections have been written so that their order can be changed without difficulty and without loss of continuity.

The solution to each problem immediately follows the statement of the problem. However, you may wish to try to solve the problem yourself before reading the given solution. In fact, even after reading the solution, you should try to resolve the problem without consulting the text. Used thus, *3000 Solved Problems in Linear Algebra* can serve as a supplement to any course in linear algebra, or even as an independent refresher course.

CHAPTER 1

Vectors in $\mathbf{R}^n$ and $\mathbf{C}^n$

1.1 VECTORS IN $\mathbf{R}^n$

1.1 A vector u in the vector space $\mathbf{R}^n$ is an ordered set of n real numbers: $u = (a_1, a_2, \ldots, a_n)$. The real number a_k is called the kth *component* or *coordinate* of u. Compare this with the definition of a vector in physics.

I Physics defines a vector **u** to be a quantity with magnitude and direction, represented by means of an arrow or directed line segment emanating from a reference point O. In Fig. 1-1 a planar vector **u** is identified with the coordinates of its endpoint, $P(4, 2)$. That is, $\mathbf{u} = (4, 2)$—in accord with the above definition of a vector in $\mathbf{R}^2$.

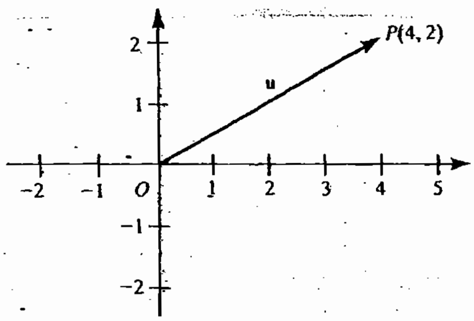

Fig. 1-1

1.2 State the difference between a row vector and a column vector.

I A *column vector* u is a vector whose components are arranged vertically:

$$u = \begin{pmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \end{pmatrix}$$

A *row vector* is a vector whose components are arranged horizontally, as in Problem 1.1. [In this chapter, vectors will normally be written as row vectors.]

1.3 To which vector space $\mathbf{R}^n$ does each vector belong?

(*a*) $(3, -2, 5, 8)$ (*b*) $(3, 6 + 2i)$ (*c*) $(\pi, 2, 5\pi)$

I (*a*) $\mathbf{R}^4$, since there are four components. (*b*) None, since not all the components are real numbers. (*c*) $\mathbf{R}^3$ [π and 5π are real numbers].

1.4 For vectors u and v in $\mathbf{R}^n$, when is $u = v$?

I The vectors u and v are equal if and only if the corresponding components are equal.

1.5 Let $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 1)$, $u_3 = (1, 3, 2)$, $u_4 = (2, 3, 1)$ be vectors in $\mathbf{R}^3$. Which of the vectors, if any, are equal?

I Only u_2 and u_4 are componentwise equal.