Introduction to

INTEGRAL EQUATIONS

Revised Edition

BS 4-Years, M.Sc., M.Phil. Mathematics
M.Sc. Physics & B.Sc. Engineering

According to New Syllabus Approved by the
University of Punjab, Govt. College University, Lahore,
Lahore College University for Women, Kinnaird College,
FC College, University of Lahore, UCP, Comsat, FAST,
GCU, Faisalabad, Global Institute, Lahore, UMT,
University of Sargodha, University of Gujrat,
International Islamic University, Islamabad,
Peshawar University, University of AJK,
Bahauddin Zakariya University, Multan,
Quaid-i-Azam University, Islamabad,
University of Education, Lahore,
Islamia University, Bahawalpur.

Z.R. Bhatti

ILMI KITAB KHANA
Kabir Street, Urdu Bazar, Lahore.

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Preface

The book consists of three chapters and is intended to serve as a text for the course in the Integral Equations that is taken by the M.Sc./M.Phil. Mathematics, M.Sc. Physics, BS 4-Years & B.Sc. Engineering students. In the first chapter, we learn how the integral equations are formed from the initial value boundary value problems. The solutions of integral equations are obtained in the second chapter. The approximate solutions of integral equations are discussed in the third chapter. The solutions of exercises are also given in the end.

I would like to thankful to all my colleagues who always appreciate and encourage me in serving the Nation by writing books.

Lahore
October, 2013

Z.R. BHATTI
Associate Professor
Department of Mathematics
Govt. College of Science
Wahdat Road, Lahore.

Contents

Chapter 1 FORMATION OF INTEGRAL EQUATIONS

1-1 Classification of Integral Equations

1-1.1 Definition: An equation which involves an unknown function under one or more integral signs is called an *integral equation*.

For example, for $s,t \in [a,b]$, the equations

$$g(s) = \int_a^b K(s,t)g(t)dt \qquad \ldots(1)$$

$$g(s) = f(s) + \int_a^b K(s,t)g(t)dt \qquad \ldots(2)$$

are integral equations, where the function $g(s)$ is the unknown function while all the other functions are known.

The functions involved in the integral equations may be complex-valued functions of the real variables s and t.

1-1.2 Definition: An integral equation is called *linear integral equation* if the unknown function appears in it linearly.

For example,

$$g(s) = f(s) + \int_a^b K(s,t)g(t)dt$$

is a linear integral equation as the unknown function $g(s)$ in it appears linearly.

But the integral equation

$$g(s) = f(s) + \int_a^b K(s,t)[g(t)]^2 dt$$

is not a linear integral equation as the unknown function $g(s)$ in it does not appear linearly.

The most general linear integral equation is of the form

$$h(s)g(s) = f(s) + \lambda \int_a K(s,t)g(t)dt$$

where the upper limit may be either variable or fixed. The functions f, h, and K are known functions while g is unknown function; λ is a nonzero real or complex parameter. The function $K(s,t)$ is called the *kernel.*

1-1.3 Definition: An integral equation in which both the lower and upper limits of integration are fixed is called *Fredholm integral equation.* For example,

$$h(s)g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$$

is Fredholm integral equation because both the lower and upper limits of integration are fixed.

1-1.4 Definition: The Fredholm integral equation

$$h(s)g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$$

is called the *Fredholm integral equation* of the *first kind* if $h(s) = 0$. For example, the integral equation

$$f(s) + \lambda \int_a^b K(s,t)g(t)dt = 0$$

is the Fredholm integral equation of the first kind.

1-1.5 Definition: The Fredholm integral equation

$$h(s)g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$$

is called the *Fredholm integral equation* of the *second kind* if $h(s) = 1$. For example, the integral equation

$$g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$$

is the Fredholm integral equation of the second kind.

1-1.6 Definition: The Fredholm integral equation of the second kind

$$g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$$

is called the *homogeneous Fredholm integral equation* of the *second kind* if $f(s) = 0$.