

FOR

LECTURER MATHEMATICS

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PPSC, FPSC, SPSC, AJKPSC

First Edition



SARDAR AQIB MAHMOOD

The Secret Recipes for Lecturer Mathematics

**Solved Past Papers of
PPSC, FPSC, SPSC, AJKPSC**

First Edition

Sardar Aqib Mahmood

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Dedication

I dedicate this book to my **Parents (Late), Wife and Brothers** who have supported me throughout my life. Especially, I would like to thank my brother **Sardar Naveed Mahmood** for providing me with opportunity to learn and excel in the field of Mathematics. I would also like to pay my regards to my **Prof. Sharafat Ali** who inspired a great deal and introduced me to the world of Mathematics.

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I want to give a special thanks
to all the people who made "The Secret Recipes for Lecturer
Mathematics" possible...

There were so many helping hands that went into making this book so
amazing. Obviously, I cannot mention everybody, but I would like to
specifically thank Nosheen Fatima who helped me a lot in typing and
researching the material.

Thank you so much for everything you guys put into this amazing project!

-S. A. Mahmood

The purpose of writing this book is to help the aspirants
preparing for the Competitive Test held by the Government of Punjab
the posts of Lecturer Mathematics and Subject Specialist Mathematics.
This book is intended to serve as a practice book for the aspirants
solved past papers of different competitive tests held by the Government
Mathematics and Subject Specialist Mathematics. It also provides
providing detailed solutions to the MCQs with proper references
references to standard textbooks. I have arranged the MCQs in different
subjects and topics.

Practicing the variety of MCQs will help you to know what kind of
MCQs are being asked in the test and what topics are to be prepared to get
a high score in the test.

I hope the aspirants will find this book very beneficial for their
preparation.

Gujranwala
8 April 2022

S. A. Mahmood

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Chapter 1

Calculus and Real Analysis

1.1 Functions, Limits and Continuity

1. Let $f(x) = \frac{x+5}{(x-1)(x-2)}$. Then domain of f is: (PPSC 2011)

(A) R (B) $R - \{1, 2\}$ (C) R^+ (D) R^-

Solution: Since the function is not defined at $x = 1, 2$, so the domain of the function is $(-\infty, 1) \cup (1, 2) \cup (2, \infty) = R - \{1, 2\}$. Therefore, the option (B) is correct.

2. If $f(x) = \frac{1}{x^2 - 3x + 2}$, then domain of f is (PPSC 2017)

(A) R (B) $R - \{1, 2\}$ (C) R^+ (D) None of these

Solution: We can write

$$f(x) = \frac{1}{x^2 - 3x + 2} = \frac{1}{x^2 - 2x - x + 2} = \frac{1}{x(x-2) - 1(x-2)} = \frac{1}{(x-2)(x-1)}$$

Since the function is undefined at $x = 1, 2$, so the domain of the function is $(-\infty, 1) \cup (1, 2) \cup (2, \infty) = R - \{1, 2\}$. Therefore, the option (B) is correct.

3. Domain of $f(x) = \sqrt{1-x^2}$ is _____. (PPSC 2015)

(A) $x < 1$ (B) $x > 1$ (C) $|x| \leq 1$ (D) $|x| \geq 1$

Solution: The function is defined when $1 - x^2 \geq 0$ or $x^2 \leq 1$ which gives $|x| \leq 1$. Therefore, the option (C) is correct.

4. Domain of $f(x) = 1/\sqrt{(1-x)(2-x)}$ is _____. (PPSC 2015)

(A) $R \setminus [1, 2]$ (B) $R \setminus \{1, 2\}$ (C) $[1, 2]$ (D) $]1, 2[$

Solution: The function is defined when $(1-x)(2-x) > 0$ and this inequality is only true when $x < 1$ or $x > 2$. Thus $x \notin [1, 2]$ and hence the domain of the function is $R \setminus [1, 2]$. Therefore, the option (A) is correct.

5. $f(x) = x + \sqrt{x-1}$ then range of $f(x)$ is:

(AJKPSC 2019, SST FPSC 2019)

- (A) $x > 0$ **(B) $x \geq 1$** (C) $x \in R$ (D) ∞

Solution: Since the domain of f is $[1, \infty)$ and the range of f will be $x \geq 1$. Therefore, the option (B) is correct.

6. If $f(x) = x^2 + 1$, then $f^{-1}(2)$ equals to: (SPSC 2009)

- (A) 0 **(B) 1** (C) 2 (D) None of these

Solution: To find the value of $f^{-1}(2)$, we need to find x such that $x^2 + 1 = 2$ or $x = 1$. Therefore, the option (B) is correct.

7. The function $f(x) = \cos x + \sec x$ is: (SPSC 2009)

- (A) A constant function **(B) An even function**
(C) An odd function (D) None of these

Solution: Since $f(-x) = \cos(-x) + \sec(-x) = \cos x + \sec x = f(x)$, so f is an even function. Therefore, the option (B) is correct.

8. $f: R \rightarrow (-1, 1)$ defined by $f(x) = \frac{x}{1+|x|}$ is bijective. (PPSC 2015)

- (A) $\frac{x}{1-|x|}$ **(B) $\frac{x}{1+|x|}$** (C) $\frac{1}{1+|x|}$ (D) $\frac{x}{-1+|x|}$

Solution: Since $\frac{x}{1-|x|}$ and $\frac{x}{-1+|x|}$ are not defined at $x = 1$, so these two options are eliminated. Now the function $\frac{1}{1+|x|}$ is not one-to-one because different numbers in the domain have the same values. For examples, the function will give the same values $x = 1$ and $x = -1$. The function $\frac{x}{1+|x|}$ is indeed a bijective function. Therefore, the function (B) is correct.

9. Which of the following functions is a bijection from R to R ? (PPSC 2011)

- (A) $f(x) = x^2 + 1$ **(B) $f(x) = x^3$**
(C) $f(x) = \frac{x^2 + 1}{x^2 + 2}$ (D) $f(x) = x^2$

Solution: The functions $f(x) = x^2 + 1$, $f(x) = \frac{x^2 + 1}{x^2 + 2}$ and $f(x) = x^2$ are not one-to-one functions because different numbers in the domain have the same values. For