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HANDBOOK OF HYDRAULIC ENGINEERING PROBLEMS

Mohammad Valipour



Handbook of Hydraulic Engineering Problems

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Preface

In near future, energy become a luxury item and water is considered as the most vital item in the world due to reduction of water resources in most regions. In this condition, role of water science researchers and hydraulic experts is more important than ever. If a hydraulic engineer student is not educated well, he/she will not solve problems of hydraulic sciences in future. Many engineer students learn all necessary lessons in the university, but they cannot to answer to the problems or to pass the exams because of forgetfulness or lack of enough exercise. This book contains one hundred essential problems related to hydraulic engineering with a small volume. Undoubtedly, many problems can be added to the book but the author tried to mention only more important problems and to prevent increasing volume of the book due to help to feature of portability of the book. To promotion of student skill, both SI and English systems have been used in the problems. All of the problems were solved completely. This book is useful for not only exercising and passing the university exams but also for use in actual project as a handbook. The handbook of hydraulic engineering problems is usable for agricultural, civil, and environmental students, teachers, experts, researchers, engineers, designers, and all enthusiastic readers in hydraulic, hydrodynamic, fluid mechanics, irrigation, drainage engineering, and water resources fields. The prerequisite to study of the book and to solve of the problems is each appropriate book about hydraulic science; however, the author recommends studying the References to better understanding the problems and presented solutions. It is an honor for the author for receive any review and suggestion to improvement of book quality.

- **Mohammad Valipour**



About Author



Mohammad Valipour is a Ph.D. candidate in Agricultural Engineering-Irrigation and Drainage at Sari Agricultural Sciences and Natural Resources University, Sari, Iran. He completed his B.Sc. Agricultural Engineering-Irrigation at Razi University, Kermanshah, Iran in 2006 and M.Sc. in Agricultural Engineering-Irrigation and Drainage at University of Tehran, Tehran, Iran in 2008. Number of his publications is more than 50. His current research interests are surface and pressurized irrigation, drainage engineering, relationship between energy and environment, agricultural water management, mathematical and computer modeling and optimization, water resources, hydrology, hydrogeology, hydro climatology, hydrometeorology, hydro informatics, hydrodynamics, hydraulics, fluid mechanics, and heat transfer in soil media.

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Handbook of Hydraulic Engineering Problems

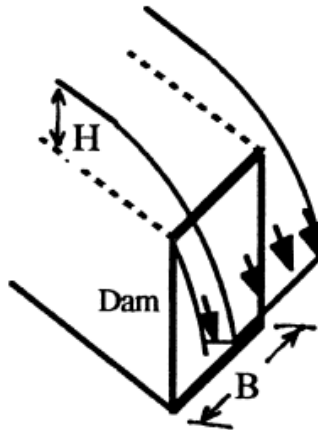
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Problems

1. The volume flow Q over a dam is proportional to dam width B and also varies with gravity g and excess water height H upstream, as shown in the figure. What is the only possible dimensionally homogeneous relation for this flow rate?



So far we know that $Q = B \text{ fcn}(H, g)$. Write this in dimensional form:

$$\{Q\} = \left\{ \frac{L^3}{T} \right\} = \{B\} \{f(H, g)\} = \{L\} \{f(H, g)\}$$

So the function fcn (H,g) must provide dimensions of $\{L^2/T\}$, but only g contains time. Therefore g must enter in the form $g^{1/2}$ to accomplish this. The relation is now

$$Q = Bg^{1/2} \text{fcn} (H), \text{ or: } \{L^3/T\} = \{L\} \{L^{1/2}/T\} \{\text{fcn}(H)\}, \text{ or: } \{\text{fcn}(H)\} = \{L^{3/2}\}$$

In order for fcn (H) to provide dimensions of $\{L^{3/2}\}$, the function must be a 3/2 power.

Thus the final desired homogeneous relation for dam flow is:

$$Q = CBg^{1/2} H^{3/2}, \text{ where } C \text{ is a dimensionless constant}$$

2. A vertical clean glass piezometer tube has an inside diameter of 1 mm. When a pressure is applied, water at 20°C rises into the tube to a height of 25 cm. After correcting for surface tension, estimate the applied pressure in Pa.

For water, let $Y = 0.073 \text{ N/m}$, contact angle $\theta = 0^\circ$, and $\gamma = 9790 \text{ N/m}^3$. The capillary rise in the tube, from Example 1.9 of the text, is

$$h_{cap} = \frac{2Y \cos \theta}{\gamma R} = \frac{2(0.073 \text{ N/m}) \cos(0^\circ)}{(9790 \text{ N/m}^3)(0.0005 \text{ m})} = 0.030 \text{ m}$$

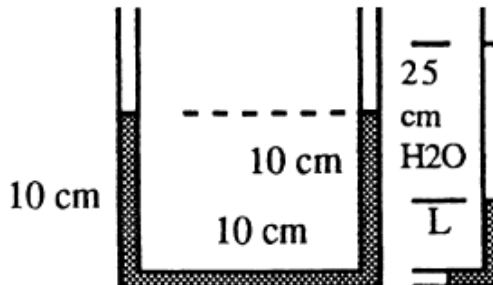
Then the rise due to applied pressure is less by that amount: $h_{press} = 0.25 \text{ m} - 0.03 \text{ m} = 0.22 \text{ m}$. The applied pressure is estimated to be $p = \gamma h_{press} = (9790 \text{ N/m}^3)(0.22 \text{ m}) \approx 2160 \text{ Pa}$

3. The deepest point in the ocean is 11034 m in the Mariana Trench in the Pacific. At this depth $\gamma_{seawater} \approx 10520 \text{ N/m}^3$. Estimate the absolute pressure at this depth.

Seawater specific weight at the surface is 10050 N/m^3 . It seems quite reasonable to average the surface and bottom weights to predict the bottom pressure:

$$p_{bottom} \approx p_0 + \gamma_{avg} h = 101350 + \left(\frac{10050 + 10520}{2} \right) (11034) = 1.136E8 \text{ Pa} \approx 1121 \text{ atm}$$

4. The U-tube at right has a 1-cm ID and contains mercury as shown. If 20 cm³ of water is poured into the right-hand leg, what will be the free surface height in each leg after the sloshing has died down?



First figure the height of water added:

$$20 \text{ cm}^3 = \frac{\pi}{4} (1 \text{ cm})^2 h, \text{ or } h = 25.46 \text{ cm}$$

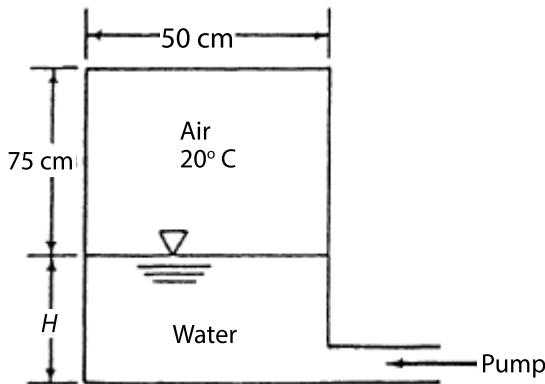
Then, at equilibrium, the new system must have 25.46 cm of water on the right, and a 30-cmlength of mercury is somewhat displaced so that “L” is on the right, 0.1 m on the bottom, and “0.2 - L” on the left side, as shown at right. The bottom pressure is constant:

$$p_{\text{atm}} + 133100(0.2 - L) = p_{\text{atm}} + 9790(0.2546) + 133100(L), \text{ or: } L \approx 0.0906 \text{ m}$$

Thus right-leg-height = 9.06 + 25.46 = 34.52 cm

Left-leg-height = 20.0 - 9.06 = 10.94 cm

5. The cylindrical tank in the figure is being filled with 20°C water by a pump developing an exit pressure of 175 kPa. At the instant shown, the air pressure is 110 kPa and H = 35 cm. The pump stops when it can no longer raise the water pressure. Estimate “H” at that time.



At the end of pumping, the bottom water pressure must be 175 kPa:

$$p_{\text{air}} + 9790H = 175000$$

Meanwhile, assuming isothermal air compression, the final air pressure is such that

$$\frac{p_{\text{air}}}{110000} = \frac{Vol_{\text{old}}}{Vol_{\text{new}}} = \frac{\pi R^2 (0.75 \text{ m})}{\pi R^2 (1.1 \text{ m} - H)} = \frac{0.75}{1.1 - H}$$

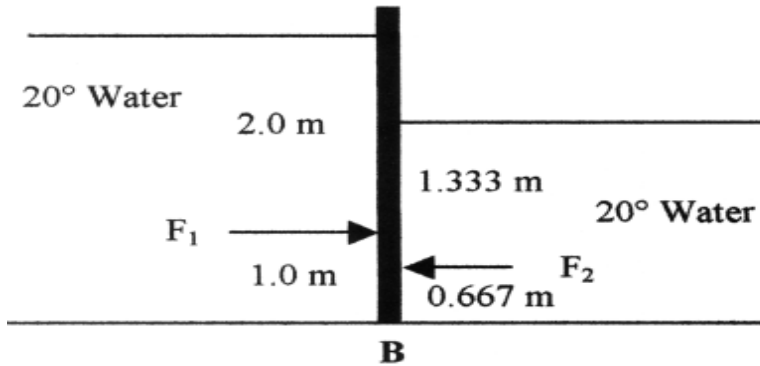
Where R is the tank radius. Combining these two gives a quadratic equation for H:

$$\frac{0.75(110000)}{1.1 - H} + 9790H = 175000, \text{ or } H^2 - 18.98H + 11.24 = 0$$

The two roots are $H = 18.37$ m (ridiculous) or, properly, $H = 0.614$ m

6. A vertical lock gate is 4 m wide and separates 20°C water levels of 2 m and 3 m, respectively. Find the moment about the bottom required to keep the gate stationary.

On the side of the gate where the water measures 3 m, F_1 acts and has an h_{CG} of 1.5 m; on the opposite side, F_2 acts with an h_{CG} of 1 m.



$$F_1 = \gamma h_{CG1} A_1 = (9790)(1.5)(3)(4) = 176,220 \text{ N}$$

$$F_2 = \gamma h_{CG2} A_2 = (9790)(1.0)(2)(4) = 78,320 \text{ N}$$

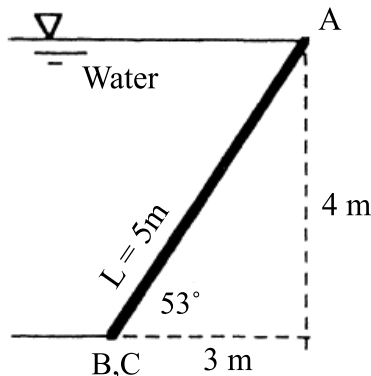
$$y_{CP1} = [(1/12)(4)(3)^3 \sin 90^\circ] / [(1.5)(4)(3)] = 0.5 \text{ m}; \text{ so } F_1 \text{ acts at } 1.5 - 0.5 = 1.0 \text{ m above B}$$

$$y_{CP2} = [(1/12)(4)(2)^3 \sin 90^\circ] / [(1)(4)(2)] = -0.333 \text{ m}; F_2 \text{ acts at } 1.0 - 0.33 = 0.67 \text{ m above B}$$

Taking moments about points B (see the figure),

$$\Sigma M_B = (176,220 \text{ N})(1.0 \text{ m}) - (78,320 \text{ N})(0.667 \text{ m}) = 124,000 \text{ N m}; M_{\text{bottom}} = 124 \text{ kNm}$$

7. Panel ABC in the slanted side of a water tank (shown at right) is an isosceles triangle with vertex at A and base BC = 2 m. Find the water force on the panel and its line of action.



(a) The centroid of ABC is $2/3$ of the depth down, or $8/3$ m from the surface. The panel area is $(1/2)(2\text{ m})(5\text{ m}) = 5\text{ m}^2$. The water force is

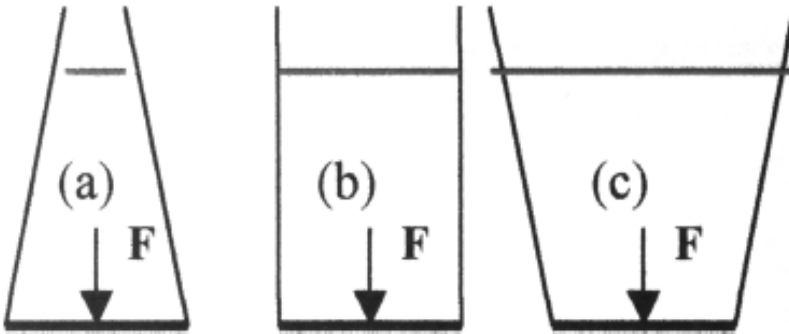
$$F_{ABC} = \gamma h_{CG} A_{\text{panel}} = (9790)(2.67\text{ m})(5\text{ m}^2) = 131,000\text{ N}$$

(b) The moment of inertia of ABC is $(1/36)(2\text{ m})(5\text{ m})^3 = 6.94\text{ m}^4$

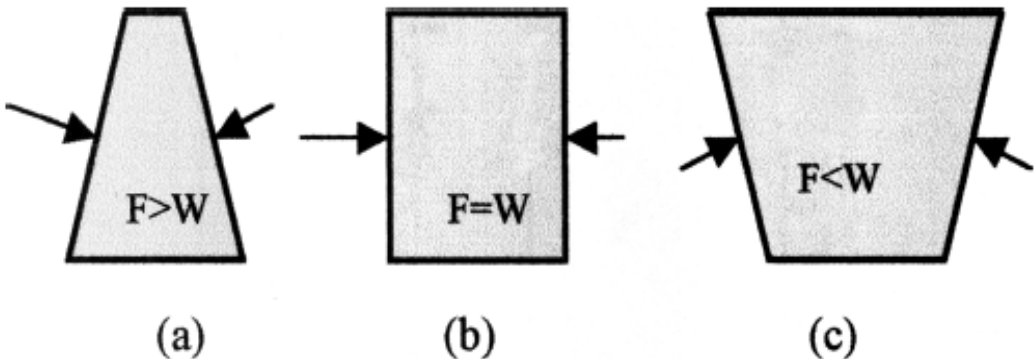
$$y_{CP} = -I_{xx} \sin\theta / (h_{CG} A_{\text{panel}}) = -6.94 \sin(53^\circ) / [2.67(5)] = -0.417\text{ m}$$

The center of pressure is 3.75 m down from A, or 1.25 m up from BC.

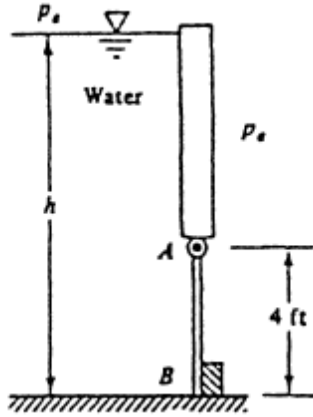
8. In the figure, the hydrostatic force F is the same on the bottom of all three containers, even though the weights of liquid above are quite different. The three bottom shapes and the fluids are the same. This is called the hydrostatic paradox. Explain why it is true and sketch a free body of each of the liquid columns.



The three free bodies are shown below. Pressure on the side-walls balances the forces. In (a), downward side-pressure components help add to a light W . In (b) side pressures are horizontal. In (c) upward side pressure helps reduce a heavy W .



9. Gate AB in the figure is 5 ft wide into the paper, hinged at A, and restrained by a stop at B. Compute (a) the force on stop B; and (b) the reactions at A if $h = 9.5$ ft.

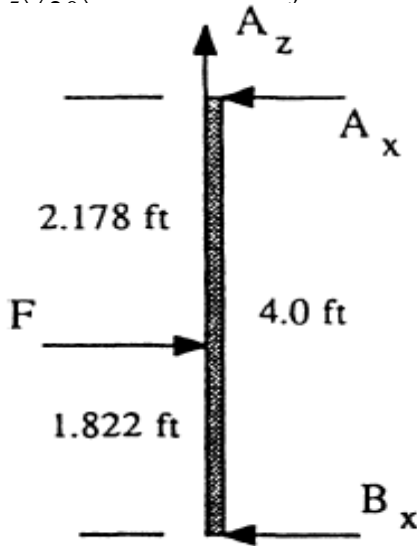


The centroid of AB is 2.0 ft below A, hence the centroidal depth is $h + 2 - 4 = 7.5$ ft. Then the total hydrostatic force on the gate is

$$F = \gamma h_{CG} A_{gate} = (62.4 \text{ lbf/ft}^3) (7.5 \text{ ft})(20 \text{ ft}^2) = 9360 \text{ lbf}$$

The C.P. is below the centroid by the amount

$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A} = -\frac{\left(\frac{1}{12}\right)(5)(4)^3 \sin 90^\circ}{(7.5)(20)} = -0.178 \text{ ft}$$



This is shown on the free body of the gate at right. We find force B_x with moments about A:

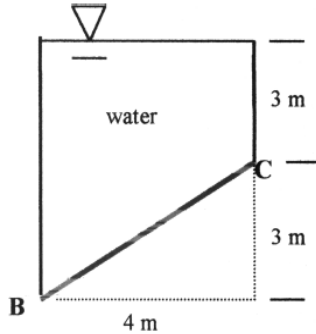
$$\sum M_A = B_x(4.0) - (9360)(2.178) = 0, \text{ or } B_x = 5100 \text{ lbf (to left)}$$

The reaction forces at A then follow from equilibrium of forces (with zero gate weight):

$$\sum F_x = 0 = 9360 - 5100 - A_x, \text{ or } A_x = 4260 \text{ lbf (to left)}$$

$$\sum F_z = 0 = A_z + W_{gate} \approx A_z, \text{ or } A_z = 0 \text{ lbf}$$

10. The tank in the figure is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on panel BC, (a) from a single formula; (b) by computing horizontal and vertical forces separately, in the spirit of curved surfaces.



(a) The resultant force F , may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790 \text{ N/m}^3) (3 + 1.5 \text{ m}) (5 \text{ m} \times 2 \text{ m}) = 440,550 \text{ N} = 441 \text{ kN}$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

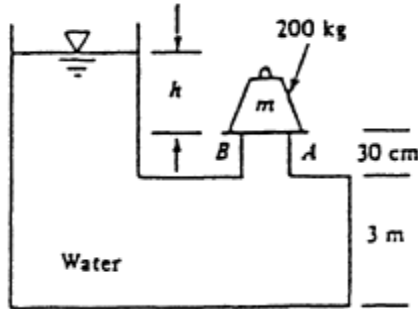
$$F_H = (9790) (4.5)(3 \times 2) = 264,330 \text{ N} = 264 \text{ kN}$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790) [(3) (4) + (1/2) (4) (3)] (2) = 352,440 = 352 \text{ kN}$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2} = 441 \text{ kN}$

11. In the figure, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level h will dislodge the gate?

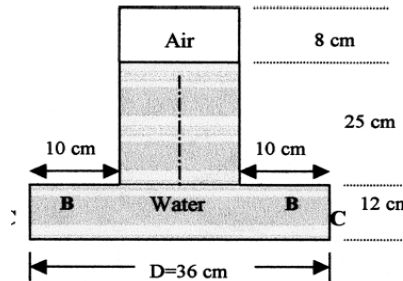


The centroidal depth is exactly equal to h and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG} A_{gate} = (9790 \text{ N} / \text{m}^3) h \frac{\pi}{4} (0.8)^2 = W = (200)(9.81) \text{ N}$$

Solve for $h = 0.40 \text{ m}$

12. The pressure in the air gap is 8000 Pa gage. The tank is cylindrical. Calculate the net hydrostatic force (a) on the bottom of the tank; (b) on the cylindrical sidewall CC; and (c) on the annular plane panel BB.

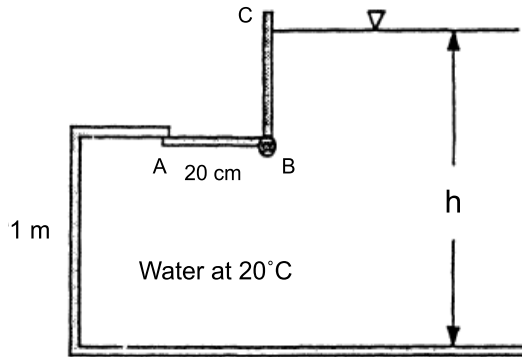


(a) The bottom force is simply equal to bottom pressure time's bottom area:

$$p_{bottom} = p_{air} + \rho_{water} g |\Delta z| = 8000 \text{ Pa} + (9790 \text{ N} / \text{m}^3) (0.25 + 0.12 \text{ m}) = 11622 \text{ Pa} - \text{gage}$$

$$F_{cc} = p_{cc} A_{cc} = (10448 \text{ Pa}) (\pi / 4) [(0.36 \text{ m})^2 - (0.16 \text{ m})^2] = 853 \text{ N}$$

13. Gate ABC in the figure has a fixed hinge at B and is 2 m wide into the paper. If the water level is high enough, the gate will open. Compute the depth h for which this happens.

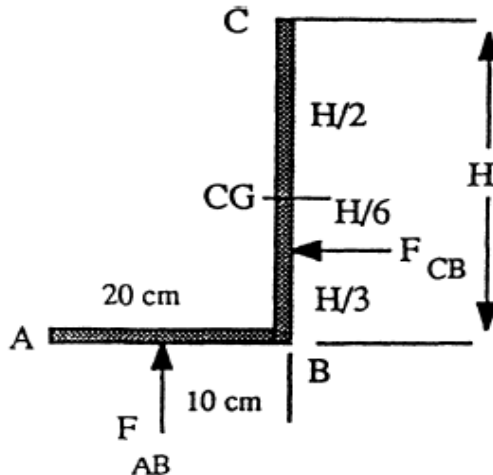


Let $H = (h - 1 \text{ meter})$ be the depth down to the level AB. The forces on AB and BC are shown in the free body at right. The moments of these forces about B are equal when the gate opens:

$$\sum M_B = 0 = \gamma H (0.2) b (0.1) = \gamma \left(\frac{H}{2}\right) (Hb) \left(\frac{H}{3}\right)$$

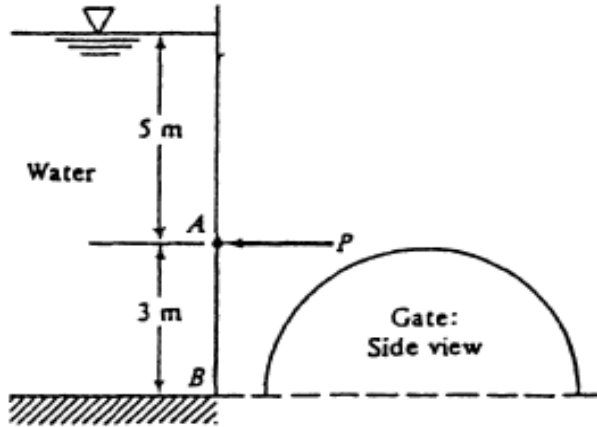
Or: $H = 0.346 \text{ m}$,

$h = H + 1 = 1.346 \text{ m}$

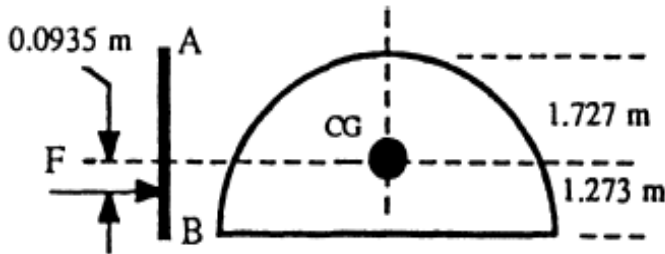


This solution is independent of both the water density and the gate width b into the paper.

14. Gate AB in the figure is semicircular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.



The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is



$$F = \gamma h_{CG} A = (9790)(5.0 + 1.727) \frac{\pi}{2} (3)^2 = 931000 \text{ N}$$

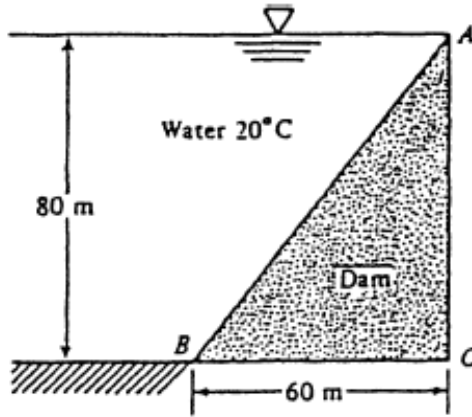
The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A} = -\frac{(0.10976)(3)^4 \sin 90^\circ}{(5 + 1.727) \left(\frac{\pi}{2}\right) (3)^2} = -0.0935 \text{ m}$$

Then summing moments about B yields the proper support force P:

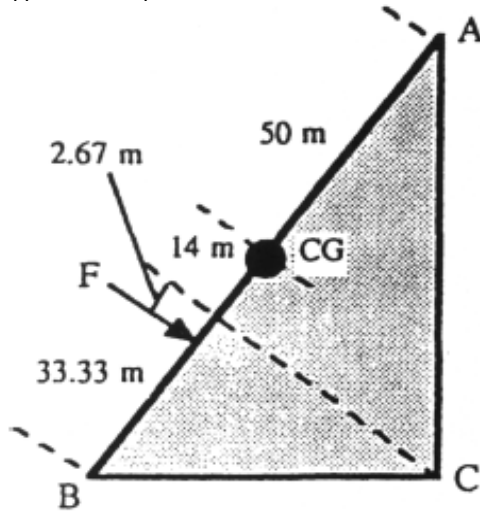
$$\Sigma M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \text{ or: } P = 366000 \text{ N}$$

15. Dam ABC in the figure is 30 m wide into the paper and is concrete ($SG \approx 2.40$). Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over? Would fluid seepage under the dam change your argument?



The centroid of surface AB is 40 m deep, and the total force on AB is

$$F = \gamma h_{CG} A = (9790)(40)(100 \times 30) = 1.175E9 \text{ N}$$



The line of action of this force is two-thirds of the way down along AB, or 66.67 m from A. This is seen either by inspection (A is at the surface) or by the usual formula:

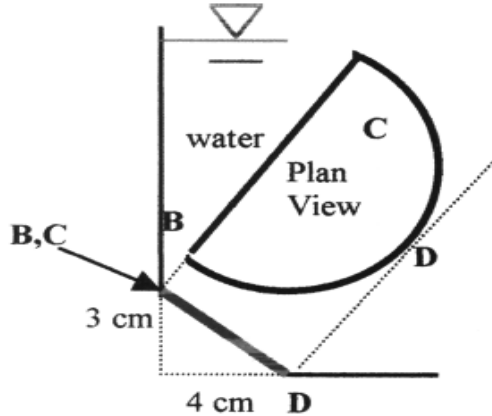
$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A} = -\frac{\left(\frac{1}{12}\right)(30)(100)^3 \sin(53.13^\circ)}{(40)(30 \times 100)} = -16.67 \text{ m}$$

To be added to the 50-m distance from A to the centroid, or $50 + 16.67 = 66.67 \text{ m}$. As shown in the figure, the line of action of F is 2.67 m to the left of a line up from C normal to AB. The

moment of F about C is thus $M_C = FL = (1.175E9) (66.67 - 64.0) \approx 3.13E9 \text{ Nm}$

This moment is counter clockwise; hence it cannot tip over the dam. If there were seepage under the dam, the main support force at the bottom of the dam would shift to the left of point C and might indeed cause the dam to tip over.

16. Panel BCD is semicircular and line BC is 8 cm below the surface. Determine (a) the hydrostatic force on the panel; and (b) the moment of this force about D.



(a) The radius of BCD is 5 cm. Its centroid is at $4R/3\pi$ or $4(5 \text{ cm})/3\pi = 2.12 \text{ cm}$ down along the slant from BC to D. Then the vertical distance down to the centroid is $h_{CG} = 8 \text{ cm} + (2.12 \text{ cm}) \cos (53.13^\circ) = 9.27 \text{ cm}$.

The force is the centroidal pressure times the panel area:

$$F = \gamma h_{CG} A = (9790 \text{ N/m}^3) (0.0927 \text{ m})(\pi / 2)(0.05 \text{ m})^2 = 3.57 \text{ N}$$

(b) Point D is $(0.05 - 0.0212) = 0.0288 \text{ m}$ from the centroid. The moment of F about D is thus

$$M_D = (3.57 \text{ N}) (0.05 \text{ m} - 0.0212 \text{ m}) = 0.103 \text{ Nm}$$

17. The cylindrical tank in the figure has a 35-cm-high cylindrical insert in the bottom. The pressure at point B is 156 kPa. Find (a) the pressure in the air space; and (b) the force on the top of the insert. Neglect air pressure outside the tank.

This force has a center of pressure at,

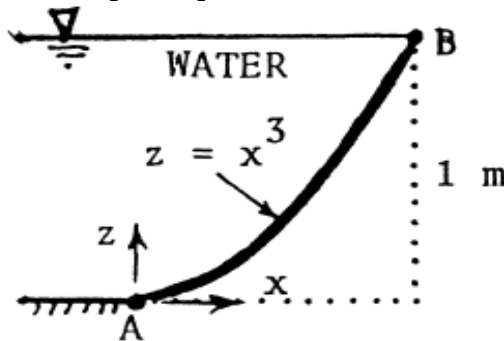
$$y_{CP} = \frac{\frac{1}{12}(0.6)(0.3)^3 (\sin 90^\circ)}{(h - 0.15)(0.3)(0.6)} = \frac{0.0075}{h - 0.15} \text{ with } h \text{ in meters}$$

Sum moments about the hinge and set equal to zero to find the minimum height:

$$\Sigma M_{\text{hinge}} = 0 = (1762.2h - 264.3) [0.15 + (0.0075 / (h - 0.15))] - (1800)(0.15)$$

This is quadratic in h, but let's simply solve by iteration: h = 1.12 m

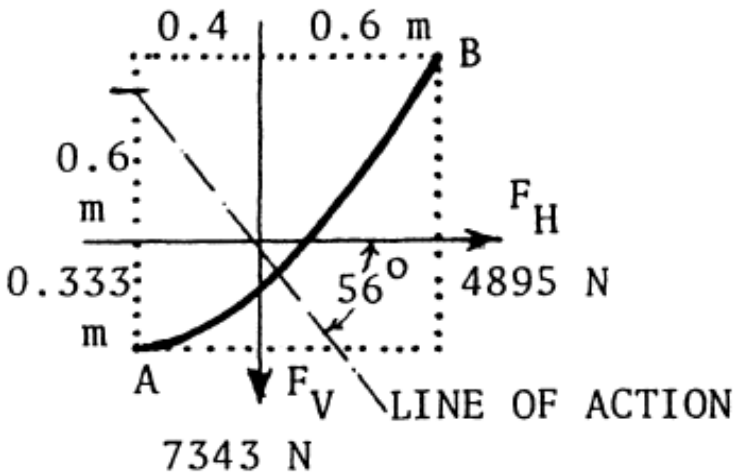
19. Determine (a) the total hydrostatic force on curved surface AB in the figure and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.



The horizontal force is

$$F_H = \gamma h_{CG} A_{\text{vert}} = (9790 \text{ N/m}^3) (0.5 \text{ m})(1 \times 1 \text{ m}^2) = 4895 \text{ N at } 0.667 \text{ m below B.}$$

For the cubic-shaped surface AB, the weight of water above is computed by integration:



$$F_V = \gamma b \int_0^1 (1 - x^3) dx = \frac{3}{4} \gamma b = \left(\frac{3}{4}\right)(9790)(1.0) = 7343 \text{ N}$$

The line of action (water centroid) of the vertical force also has to be found by integration:

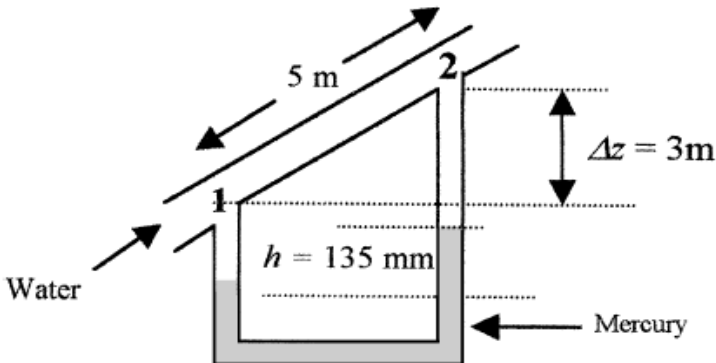
$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{\int_0^1 x(1 - x^3) dx}{\int_0^1 (1 - x^3) dx} = \frac{\frac{3}{4}}{\frac{3}{4}} = 0.4 \text{ m}$$

The vertical force of 7343 N thus acts at 0.4 m to the right of point A, or 0.6 m to the left of B, as shown in the sketch above. The resultant hydrostatic force then is

$$F_{\text{total}} = [(4895)^2 + (7343)^2]^{1/2} = 8825 \text{ N acting at } 56.31^\circ \text{ down and to the right.}$$

This result is shown in the sketch at above right. The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.

20. Water at 20°C flows upward at 4 m/s in a 6-cm-diameter pipe. The pipe length between points 1 and 2 is 5 m, and point 2 is 3 m higher. A mercury manometer, connected between 1 and 2, has a reading h = 135 mm, with p1 higher. (a) What is the pressure change (p1 - p2)? (b) What is the head loss, in meters? (c) Is the manometer reading proportional to head loss? Explain. (d) What is the friction factor of the flow?



$$p_1 + \gamma_w h - \gamma_w h - \gamma_w \Delta Z = p_2$$

$$\text{or: } p_1 - p_2 = \left(133100 - 9790 \frac{\text{N}}{\text{m}^3}\right)(0.135 \text{ m}) + \left(9790 \frac{\text{N}}{\text{m}^3}\right)(3 \text{ m}) = 16650 + 29370 = 46000 \text{ Pa}$$

$$h_f = \frac{\Delta p}{\gamma_w} - \Delta z = \frac{46000}{9790 \text{ N/m}^3} - 3 \text{ m} = 4.7 - 3.0 = 1.7 \text{ m}$$

The friction factor is $f = h_f \frac{d}{L} \frac{2g}{V^2} = (1.7 \text{ m}) \left(\frac{0.06 \text{ m}}{5 \text{ m}} \right) \frac{2(9.81 \text{ m/s}^2)}{(4 \text{ m/s})^2} = 0.025$

By comparing the manometer relation to the head-loss relation above, we find that:

$$h_f = \frac{(\gamma_m - \gamma_w)}{\gamma_w} h$$

And thus head loss is proportional to manometer reading.

21. Water at 20°C flows in a 9-cm-diameter pipe under fully developed conditions. The centerline velocity is 10 m/s. Compute (a) Q, (b) V, (c) τ_w , and (d) Δp for a 100-m pipe length.

For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/ms}$. Check $Re = \rho V D / \mu \approx 998(10)(0.09)/0.001 \approx 900,000$, surely a turbulent flow. Use the log-law:

$$\frac{u_{ctr}}{u^*} \approx \frac{1}{\kappa} \ln \left(\frac{R u^*}{\nu} \right) + B, \text{ or } \frac{10}{u^*} \approx \frac{1}{0.41} \ln \left[\frac{998(0.045)u^*}{0.001} \right] + 5.0, \text{ solve } u^* \approx 0.350 \text{ m/s}$$

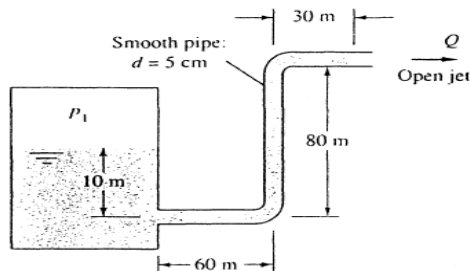
Then $\tau_w = \rho u^{*2} = (998)(0.350)^2 \approx 122 \text{ Pa}$

$$V \approx 0.85 u_{ctr} = (0.85)(10) \approx 8.5 \text{ m/s}$$

$$Q = AV \approx (\pi/4)(0.09)^2(8.5) \approx 0.054 \text{ m}^3/\text{s}$$

$$\Delta p = \frac{2\tau_w \Delta L}{R} = \frac{2(122 \text{ Pa})(100 \text{ m})}{(0.045 \text{ m})} \approx 542000 \text{ Pa}$$

22. The pipe flow in the figure is driven by pressurized air in the tank. What gage pressure p_1 is needed to provide a 20°C water flow rate $Q = 60 \text{ m}^3/\text{h}$?



For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/ms}$. Get V , Re , f :

$$V = \frac{60}{\frac{3600}{\left(\frac{\pi}{4}\right)(0.05)^2}} = 8.49 \text{ m/s}$$

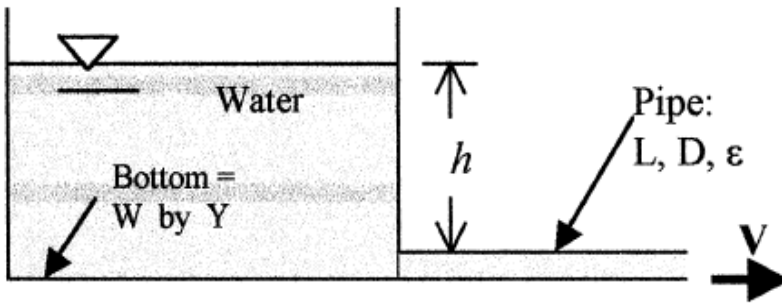
$$Re = \frac{998(8.49)(0.05)}{0.001} \approx 424000; f_{smooth} \approx 0.0136$$

Write the energy equation between points (1) (the tank) and (2) (the open jet):

$$\frac{p_1}{\rho g} + \frac{0^2}{2g} + 10 = \frac{0}{\rho g} + \frac{V_{pipe}^2}{2g} + 80 + h_f, \text{ where } h_f = f \frac{L}{d} \frac{V^2}{2g} \text{ and } V_{pipe} = 8.49 \text{ m/s}$$

$$\text{Solve } p_1 = (998)(9.81) \left[80 - 10 + \frac{(8.49)^2}{2(9.81)} \left\{ 1 + 0.0136 \left(\frac{170}{0.05} \right) \right\} \right] \approx 2.38E6 \text{ Pa}$$

23. A swimming pool W by Y by h deep is to be emptied by gravity through the long pipe shown in the figure. Assuming an average pipe friction factor f_{av} and neglecting minor losses, derive a formula for the time to empty the tank from an initial level h_0 .



With no driving pressure and negligible tank surface velocity, the energy equation can be combined with a control-volume mass conservation:

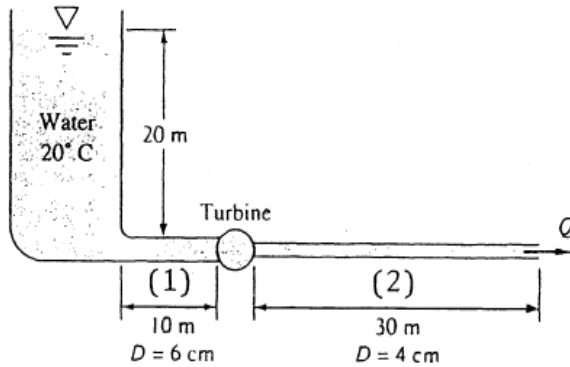
$$h(t) = \frac{V^2}{2g} + f_{av} \frac{L}{D} \frac{V^2}{2g}, \text{ or } Q_{out} = A_{pipe} V = \frac{\pi}{4} D^2 \sqrt{\frac{2gh}{1 + f_{av} \frac{L}{D}}} = -WY \frac{dh}{dt}$$

We can separate the variables and integrate for time to drain:

$$\frac{\pi}{4} D^2 \sqrt{\frac{2g}{1 + f_{av} \frac{L}{D}}} \int_0^t dt = -WY \int_{h_0}^0 \frac{dh}{\sqrt{h}} = -WY (0 - s\sqrt{h_0})$$

Clean this up to obtain : $t_{drain} \approx \frac{4WY}{\pi D^2} \sqrt{\frac{2h_0 \left(1 + f_{av} \frac{L}{D}\right)}{g}}$

24. The small turbine in the figure extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m³/h. Sketch the EGL and HGL accurately.



For water, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/ms}$. For wrought iron, take $\epsilon \approx 0.046 \text{ mm}$, hence $\epsilon/d_1 = 0.046/60 \approx 0.000767$ and $\epsilon/d_2 = 0.046/40 \approx 0.00115$. The energy equation, with $V_1 \approx 0$ and $p_1 = p_2$, gives

$$z_1 - z_2 = 20 \text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{\text{turbine}}$$

$$h_{f1} = f_1 \frac{L_1 V_1^2}{d_1 2g}$$

$$h_{f2} = f_2 \frac{L_2 V_2^2}{d_2 2g}$$

$$h_{\text{turbine}} = \frac{P}{\rho g Q} = \frac{400W}{998(9.81)Q}$$

$$Q = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

The only unknown is Q, which we may determine by iteration after an initial guess:

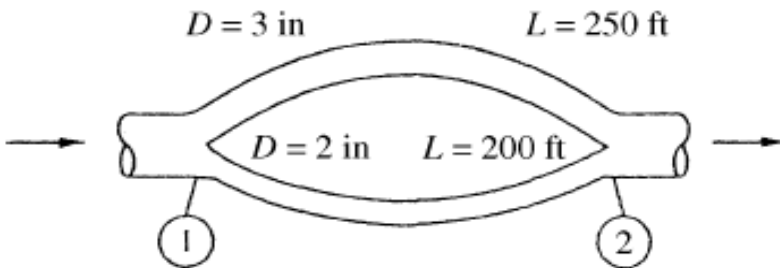
$$h_{\text{turbine}} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1 L_1 Q^2}{\pi^2 g d_1^5} - \frac{8f_2 L_2 Q^2}{\pi^2 g d_2^5} - \frac{8Q^2}{\pi^2 g d_2^4}$$

$$\text{Guess } Q = 0.003 \text{ m}^3 / \text{s}, \text{ then } \text{Re}_1 = \frac{4\rho Q}{\pi\mu d_1} = 63500, f_{1, \text{Moody}} \approx 0.0226, \text{Re}_2 = 95300, f_2 \approx 0.0228$$

But, for this guess, h_{turbine} (left hand side) $\approx 13.62 \text{ m}$, h_{turbine} (right hand side) $\approx 14.53 \text{ m}$ (wrong).

Other guesses converge to $h_{\text{turbine}} \approx 9.9 \text{ meters}$. For $Q \approx 0.00413 \text{ m}^3 / \text{s} \approx 15 \text{ m}^3 / \text{h}$.

25. For the parallel-pipe system of the figure, each pipe is cast iron, and the pressure drop $p_1 - p_2 = 3 \text{ lbf/in}^2$. Compute the total flow rate between 1 and 2 if the fluid is SAE 10 oil at 20°C .



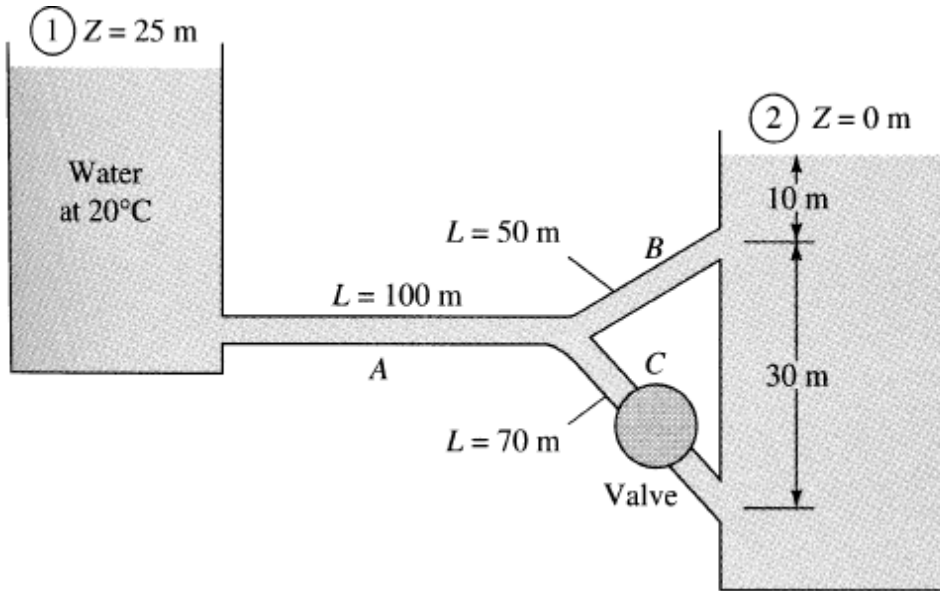
For SAE 10 oil at 20°C , take $\rho = 1.69 \text{ slug/ft}^3$ and $\mu = 0.00217 \text{ slug/ft}\cdot\text{s}$. For cast iron, $\epsilon \approx 0.00085 \text{ ft}$. Convert $\Delta p = 3 \text{ psi} = 432 \text{ psf}$ and guess laminar flow in each:

$$\Delta p_b = \frac{128\mu L_b Q_b}{\pi d_b^4} = 432 = \frac{128(0.00217)(200)Q_b}{\pi \left(\frac{2}{12}\right)^4}$$

$$Q_b \approx 0.0188 \text{ ft}^3 / \text{s} \text{ Check } Re \approx 112$$

The total flow rate is $Q = Q_a + Q_b = 0.0763 + 0.0188 \approx 0.095 \text{ ft}^3 / \text{s}$

26. In the figure all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir (1) if valve C is (a) closed; and (b) open, with $K_{\text{valve}} = 0.5$.



For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. For cast iron, $\epsilon \approx 0.26 \text{ mm}$, hence $\epsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

Sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$

Branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight series path through A and B, with the same flow rate Q , velocity V , and friction factor f in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \Sigma h_{mA} + h_{fB} + \Sigma h_{mB},$$

$$25 \text{ m} = \frac{V^2}{2(9.81)} \left[f \frac{100}{0.08} + 0.5 + 0.9 + f \frac{50}{0.08} + 1.0 \right], \text{ where } f = f_{cn} \left(\text{Re}, \frac{\varepsilon}{d} \right)$$

Guess $f \approx f_{\text{fully rough}} \approx 0.027$, then $V \approx 3.04 \text{ m/s}$, $\text{Re} \approx 998(3.04)(0.08)/(0.001) \approx 243000$,

$\varepsilon/d = 0.00325$, then $f \approx 0.0273$ (converged). Then the velocity through A and B is $V = 3.03 \text{ m/s}$, and $Q = (\pi/4)(0.08)^2(3.03) \approx 0.0152 \text{ m}^3/\text{s}$.

If valve C is open, we have parallel flow through B and C, with $Q_A = Q_B + Q_C$ and, with d constant, $V_A = V_B + V_C$. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \Sigma h_{mA-B} + h_{fB} + \Sigma h_{mB} = h_{fA} + \Sigma h_{mA-C} + h_{fC} + \Sigma h_{mC}$$

$$\begin{aligned} 25 &= \frac{V_A^2}{2(9.81)} \left[f_A \frac{100}{0.08} + 0.5 + 0.9 \right] + \frac{V_B^2}{2(9.81)} \left[f_B \frac{50}{0.08} + 1.0 \right] \\ &= \frac{V_A^2}{2(9.81)} \left[f_A \frac{100}{0.08} + 0.5 + 1.3 \right] + \frac{V_C^2}{2(9.81)} \left[f_C \frac{70}{0.08} + 1.0 \right] \end{aligned}$$

Plus the additional relation $V_A = V_B + V_C$. Guess $f \approx f_{\text{fully rough}} \approx 0.027$ for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

If $f \approx 0.027$, solve (laboriously) $V_A \approx 3.48 \text{ m/s}$, $V_B \approx 1.91 \text{ m/s}$, $V_C \approx 1.57 \text{ m/s}$

Compute $\text{Re}_A = 278000$, $f_A \approx 0.0272$, $\text{Re}_B = 153000$, $f_B = 0.0276$, $\text{Re}_C = 125000$, $f_C = 0.0278$

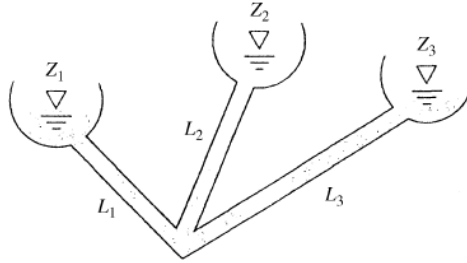
Repeat once for convergence: $V_A \approx 3.46 \text{ m/s}$, $V_B \approx 1.90 \text{ m/s}$, $V_C \approx 1.56 \text{ m/s}$. The flow rate from reservoir (1) is $Q_A = (\pi/4)(0.08)^2(3.46) \approx 0.0174 \text{ m}^3/\text{s}$. (14% more)

27. Consider the three-reservoir system of the figure with the following data:

$$L_1 = 95 \text{ m } L_2 = 125 \text{ m } L_3 = 160 \text{ m}$$

$$z_1 = 25 \text{ m } z_2 = 115 \text{ m } z_3 = 85 \text{ m}$$

All pipes are 28-cm-diameter unfinished concrete ($\varepsilon = 1 \text{ mm}$). Compute the steady flow rate in all pipes for water at 20°C .



For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. All pipes have $\epsilon/d = 1/280 = 0.00357$. Let the intersection be “a.” The head loss at “a” is desired:

$$z_1 - h_a = f_1 \frac{L_1 V_1^2}{d_1 2g}$$

$$z_2 - h_a = f_2 \frac{L_2 V_2^2}{d_2 2g}$$

$$z_3 - h_a = f_3 \frac{L_3 V_3^2}{d_3 2g}$$

plus the requirement that $Q_1 + Q_2 + Q_3 = 0$ or, for same d , $V_1 + V_2 + V_3 = 0$

We guess h_a then iterate each friction factor to find V and Q and then check if $\Sigma Q = 0$.

$$h_a = 75 \text{ m} : 25 - 75 = (-)50 = f_1 \left(\frac{95}{0.28} \right) \frac{V_1^2}{2(9.81)}, \text{ solve } f_1 \approx 0.02754, V_1 \approx -10.25 \text{ m/s}$$

$$115 - 75 = f_2 \left(\frac{125}{0.28} \right) \left[\frac{V_2^2}{2(9.81)} \right] \text{ gives } f_2 \approx 0.02755, V_2 \approx +7.99$$

$$85 - 75 = f_3 \left(\frac{160}{0.28} \right) \left[\frac{V_3^2}{2(9.81)} \right]$$

$$\text{gives } f_3 \approx 0.02762, V_3 \approx +3.53 \text{ m/s}, \Sigma V = +1.27$$

Repeating for $h_a = 80 \text{ m}$ gives $V_1 = -10.75, V_2 = +7.47, V_3 = +2.49 \text{ m/s}, \Sigma V = -0.79$.

Interpolate to $h_a \approx 78 \text{ m}$, gives $V_1 = -10.55 \text{ m/s}, V_2 = +7.68 \text{ m/s}, V_3 = +2.95 \text{ m/s}$, or:

$$Q_1 = -0.65 \text{ m}^3/\text{s}, Q_2 = +0.47 \text{ m}^3/\text{s}, Q_3 = +0.18 \text{ m}^3/\text{s}.$$

28. Three cast-iron pipes are laid in parallel with these dimensions:

Pipe 1: $L_1 = 800 \text{ m}$ $d_1 = 12 \text{ cm}$

Pipe 2: $L_2 = 600 \text{ m}$ $d_2 = 8 \text{ cm}$

Pipe 3: $L_3 = 900$ m $d_3 = 10$ cm

The total flow rate is 200 m³/h of water at 20°C. Determine (a) the flow rate in each pipe; and (b) the pressure drop across the system.

For water at 20°C, take $\rho = 998$ kg/m³ and $\mu = 0.001$ kg/m.s. For cast iron, $\epsilon = 0.26$ mm. Then, $\epsilon/d_1 = 0.00217$, $\epsilon/d_2 = 0.00325$, and $\epsilon/d_3 = 0.0026$. The head losses are the same for each pipe, and the flow rates add:

$$h_f = \frac{8f_1L_1Q_1^2}{\pi^2gd_1^5} = \frac{8f_2L_2Q_2^2}{\pi^2gd_2^5} = \frac{8f_3L_3Q_3^2}{\pi^2gd_3^5}$$

$$Q_1 + Q_2 + Q_3 = \frac{200}{3600} \text{ m}^3 / \text{s}$$

$$\text{Substitute and combine: } Q_1 \left[1 + 0.418 \left(\frac{f_1}{f_2} \right)^{\frac{1}{2}} + 0.599 \left(\frac{f_1}{f_3} \right)^{\frac{1}{2}} \right] = 0.0556 \text{ m}^3 / \text{s}$$

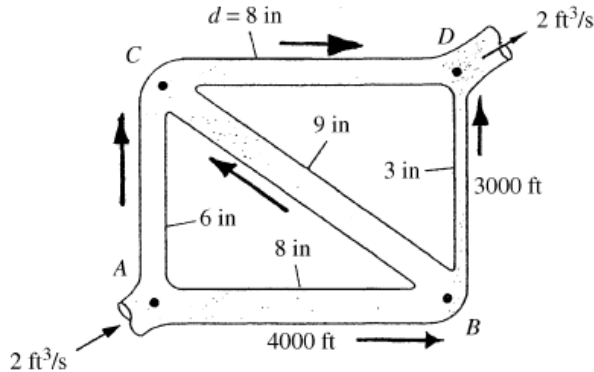
We could either go directly to EES or begin by guessing $f_1 = f_2 = f_3$, which gives $Q_1 = 0.0275$ m³/s, $Q_2 = 0.0115$ m³/s, and $Q_3 = 0.0165$ m³/s. This is very close! Further iteration gives

$$Re_1 = 298000, f_1 = 0.0245; Re_2 = 177000, f_2 = 0.0275; Re_3 = 208000, f_3 = 0.0259$$

$$Q_1 = 0.0281 \text{ m}^3 / \text{s}, Q_2 = 0.0111 \text{ m}^3 / \text{s}, \text{ and } Q_3 = 0.0163 \text{ m}^3 / \text{s}$$

$$h_f = 51.4 \text{ m}, \Delta p = \rho gh_f = (998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(51.4 \text{ m}) = 503,000 \text{ Pa}$$

29. In the five-pipe horizontal network of the figure, assume that all pipes have a friction factor $f = 0.025$. For the given inlet and exit flow rate of 2 ft³/s of water at 20°C, determine the flow rate and direction in all pipes. If $p_A = 120$ lbf/in² gage, determine the pressures at points B, C, and D.



For water at 20°C, take $\rho = 1.94$ slug/ft³ and $\mu = 2.09E-5$ slug/ft.s. Each pipe has a head loss which is known except for the square of the flow rate:

$$\text{Pipe AC: } h_f = \frac{8fLQ^2}{\pi^2gd^5} \Big|_{AC} = \frac{8(0.025)(3000)Q_{AC}^2}{\pi^2(32.2)\left(\frac{6}{12}\right)^5} = K_{AC}Q_{AC}^2, \text{ where } K_{AC} \approx 60.42$$

$$\text{Similarly, } K_{AB} = 19.12, K_{BC} = 13.26, K_{CD} = 19.12, K_{BD} = 19.33$$

There are two triangular closed loops, and the total head loss must be zero for each. Using the flow directions assumed on the figure above, we have

$$\text{Loop A-B-C: } 19.12Q_{AB}^2 + 13.26Q_{BC}^2 - 60.42Q_{AC}^2 = 0$$

$$\text{Loop B-C-D: } 13.26Q_{BC}^2 + 19.12Q_{CD}^2 - 19.33Q_{BD}^2 = 0$$

And there are three independent junctions which have zero net flow rates:

$$\text{Junction A: } Q_{AB} + Q_{AC} = 2.0; \text{ B: } Q_{AB} = Q_{BC} + Q_{BD}; \text{ C: } Q_{AC} + Q_{BC} = Q_{CD}$$

These are five algebraic equations to be solved for the five flow rates. The answers are:

$$Q_{AB} = 1.19, Q_{AC} = 0.81, Q_{BC} = 0.99, Q_{CD} = 1.80, Q_{BD} = 0.20 \text{ ft}^3/\text{s}$$

The pressures follow by starting at A (120 psi) and subtracting off the friction losses:

$$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 120 \times 144 - 62.4(19.12)(1.19)^2$$

$$p_B = \frac{15590 \text{ psf}}{144} = 108 \text{ lbf} / \text{in}^2$$

$$\text{Similarly, } p_C \approx 103 \text{ psi and } p_D \approx 76 \text{ psi}$$

30. In the figure all four horizontal cast-iron pipes are 45 m long and 8 cm in diameter and meet at junction delivering water at 20°C. The pressures are known at four points as shown:

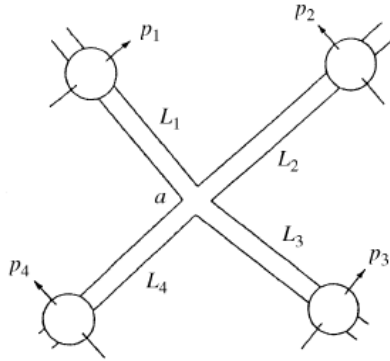
$$p_1 = 950 \text{ kPa}$$

$$p_2 = 350 \text{ kPa}$$

$$p_3 = 675 \text{ kPa}$$

$$p_4 = 100 \text{ kPa}$$

Neglecting minor losses, determine the flow rate in each pipe.



For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. All pipes are cast iron, with $\epsilon/d = 0.26/80 = 0.00325$. All pipes have $L/d = 45/0.08 = 562.5$. One solution method is to guess the junction pressure p_a , iterate to calculate the friction factors and flow rates, and check to see if the net junction flow is zero:

$$\text{Guess } p_a = 500 \text{ kPa} : h_{f1} = \frac{950000 - 500000}{998(9.81)} = 45.96 \text{ m} = \frac{8f_1L_1Q_1^2}{\pi^2gd_1^5} = 1.135E6f_1Q_1^2$$

$$\text{then guess } f_1 \approx 0.02, Q_1 = 0.045 \text{ m}^3 / \text{s}, \text{Re}_1 = 4\rho Q_1 / (\pi\mu d_1) = 715000, f_{1\text{-new}} \approx 0.0269$$

$$\text{converges to } f_1 \approx 0.0270, Q_1 \approx 0.0388 \text{ m}^3 / \text{s}$$

$$\text{Iterate also to } Q_2 = -0.0223 \text{ m}^3 / \text{s} (\text{away from } a), Q_3 = 0.0241, Q_4 = -0.0365$$

$\Sigma Q = +0.00403$, so we have guessed p_a a little low.

Trying $p_a = 530 \text{ kPa}$ gives $\Sigma Q = -0.00296$, hence iterate to $p_a \approx 517 \text{ kPa}$:

$$Q_1 = +0.0380 \text{ m}^3 / \text{s} (\text{toward } a)$$

$$Q_2 = -0.0236 \text{ m}^3 / \text{s}$$

$$Q_3 = +0.0229 \text{ m}^3 / \text{s}$$

$$Q_4 = -0.0373 \text{ m}^3 / \text{s}$$

31. A water-tunnel test section has a 1-m diameter and flow properties $V = 20 \text{ m/s}$, $p = 100 \text{ kPa}$, and $T = 20^\circ\text{C}$. The boundary-layer blockage at the end of the section is 9 percent. If a conical diffuser is to be added at the end of the section to achieve maximum pressure recovery, what should its angle, length, exit diameter, and exit pressure be?

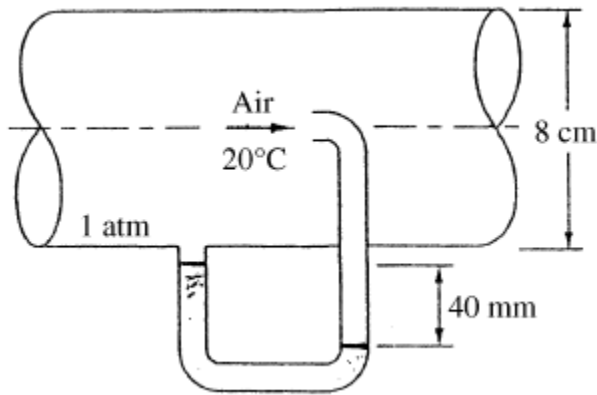
For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. The Reynolds number is very high, $\text{Re} = \rho Vd/\mu = (998)(20)(1)/(0.001) \approx 2.0E7$

$$B_t = 0.09, \text{ read } C_{p,\text{max}} \approx 0.71 \text{ at } L/d \approx 25, 2\theta \approx 4^\circ, \text{AR} \approx 8:$$

$$\text{Then } \theta_{\text{cone}} \approx 2^\circ, L \approx 25d \approx 25 \text{ m}, D_{\text{exit}} = d(8)^{0.5} \approx 2.8 \text{ m}$$

$$C_p \approx 0.71 = \frac{p_e - p_t}{\left(\frac{1}{2}\right)\rho V_t^2} = \frac{p_e - 100000}{\left(\frac{1}{2}\right)(998)(20)^2}, \text{ or } : p_{exit} \approx 242000 \text{ Pa}$$

32. For the pitot-static pressure arrangement of the figure, the manometer fluid is water at 20°C. Estimate (a) the centerline velocity, (b) the pipe volume flow, and (c) the (smooth) wall shear stress.



For air at 20°C and 1 atm, take $\rho = 1.2 \text{ kg/m}^3$ and $\mu = 1.8\text{E}-5 \text{ kg/m.s}$. For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. The manometer reads

$$p_o - p = (\rho_{\text{water}} - \rho_{\text{air}})gh = (998 - 1.2)(9.81)(0.040) \approx 391 \text{ Pa}$$

$$\text{Therefore } V_{CL} = [2\Delta p/\rho]^{0.5} = [2(391)/1.2]^{0.5} \approx 25.5 \text{ m/s}$$

$$\text{Guess } V_{avg} \approx 0.85V_{CL} \approx 21.7 \text{ m/s}, \text{ then } Re_d = \frac{\rho V d}{\mu} = \frac{1.2(21.7)(0.08)}{1.8\text{E}-5} \approx 115700$$

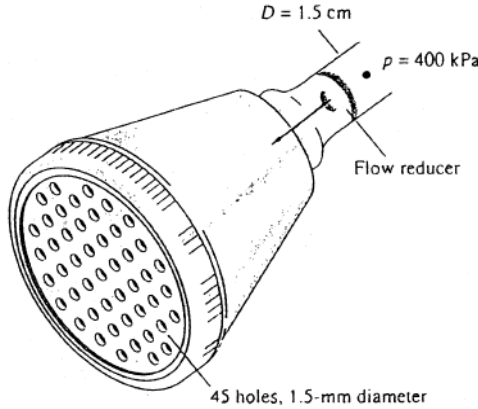
$$\text{Then } f_{smooth} \approx 0.0175, V_{better} = \frac{25.5}{\left[1 + 1.33\sqrt{0.0175}\right]} \approx 21.69 \text{ m/s (converged)}$$

Thus the volume flow is $Q = (\pi/4)(0.08)^2(21.69) \approx 0.109 \text{ m}^3/\text{s}$.

$$\text{Finally, } \tau_w = \frac{f}{8}\rho V^2 = \frac{0.0175}{8}(1.2)(21.69)^2 \approx 1.23 \text{ Pa}$$

33. The shower head in the figure delivers water at 50°C. An orifice-type flow reducer is to be installed. The upstream pressure is constant at 400 kPa. What flow rate, in gal/min, results without the reducer? What reducer orifice diameter would decrease the flow

by 40 percent?



For water at 50°C, take $\rho = 988 \text{ kg/m}^3$ and $\mu = 0.548\text{E-}3 \text{ kg/m.s}$. Further assume that the shower head is a poor diffuser, so the pressure in the head is also about 400 kPa. Assume the outside pressure is sea-level standard, 101 kPa. Estimate $C_d \approx 0.61$. Then, with $\beta \approx 0$ for the small holes, each hole delivers a flow rate of

$$Q_{1\text{ hole}} = C_d A_{\text{hole}} \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \approx 0.61 \left(\frac{\pi}{4}\right) (0.0015)^2 \sqrt{\frac{2(400000 - 101000)}{988(1-0^4)}}$$

$$Q_{1\text{ hole}} \approx 2.65\text{E-}5 \text{ m}^3 / \text{s} \text{ and } Q_{\text{total}} = 45Q_{1\text{ hole}} \approx 0.00119 \text{ m}^3 / \text{s}$$

This is a large flow rate a lot of expensive hot water. Checking back, the inlet pipe for this flow rate has $R_{\text{ed}} \approx 183000$, so $C_d \approx 0.60$ would be slightly better and a repeat of the calculation would give $Q_{\text{no reducer}} \approx 0.00117 \text{ m}^3/\text{s} \approx 18.6 \text{ gal/ min}$.

A 40% reduction would give $Q = 0.6(0.00117) = 7.04\text{E-}4 \text{ m}^3/\text{s} \div 45 = 1.57\text{E-}5 \text{ m}^3/\text{s}$ for each hole, which corresponds to a pressure drop

$$Q_{1\text{ hole}} = 1.57\text{E-}5 = 0.60 \left(\frac{\pi}{4}\right) (0.0015)^2 \sqrt{\frac{2\Delta p}{988}}, \text{ or } \Delta p \approx 108000 \text{ Pa}$$

or $p_{\text{inside head}} \approx 101 + 108 \approx 209 \text{ kPa}$, the reducer must drop the inlet pressure to this.

$$Q = 7.04\text{E-}4 \approx 0.61 \left(\frac{\pi}{4}\right) (0.015\beta)^2 \left[\frac{2(400000 - 209000)}{988(1-\beta^4)}\right]^{\frac{1}{2}}, \text{ or } \frac{\beta^2}{(1-\beta^4)^{\frac{1}{2}}} \approx 0.332$$

Solve for $\beta \approx 0.56$, $d_{\text{reducer}} \approx 0.56(1.5) \approx 0.84 \text{ cm}$

34. A 10-cm-diameter smooth pipe contains an orifice plate with D: 1/2 D taps and β

=0.5. The measured orifice pressure drop is 75 kPa for water flow at 20°C. Estimate the flow rate, in m³/h. What is the nonrecoverable head loss?

For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$

$$Q = C_d A_t \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d \frac{\pi}{4} (0.05)^2 \sqrt{\frac{2(75000)}{998[1-(0.5)^4]}} = 0.0249 C_d$$

Guess $C_d \approx 0.61, Q \approx 0.0152 \text{ m}^3 / \text{s}, \text{Re}_D = \frac{4\rho Q}{\pi\mu D} \approx 193000, C_d \approx 0.605$

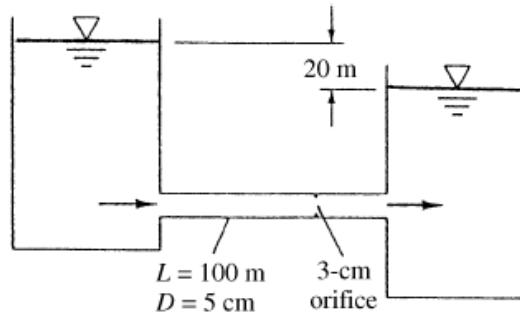
This is converged: $Q = 0.0249(0.605) = 0.0150 \text{ m}^3/\text{s} \approx 54 \text{ m}^3/\text{h}$.

The non-recoverable head loss coefficient is $K \approx 1.8$, based on V_t :

$$V_t = \frac{Q}{A_t} = \frac{0.0150}{\pi(0.025)^2} \approx 7.66 \text{ m/s}$$

$$\Delta p_{loss} = K \frac{\rho}{2} V_t^2 = 1.8 \left(\frac{998}{2} \right) (7.66)^2 \approx 53000 \text{ Pa}$$

35. A pipe connecting two reservoirs, as in the figure, contains a thin-plate orifice. For water flow at 20°C, estimate (a) the volume flow through the pipe and (b) the pressure drop across the orifice plate.



For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. The energy equation should include the orifice head loss and the entrance and exit losses:

$$\Delta z = 20 \text{ m} = \frac{V^2}{2g} \left(f \frac{L}{d} + \sum K \right)$$

$$K_{enter} \approx 0.5$$

$$K_{exit} \approx 1.0$$

$$K_{orifice}^{\beta=0.6} \approx 1.5$$

$$V^2 = \frac{2(9.81)(20)}{\left[f \left(\frac{100}{0.05} \right) + 0.5 + 1.0 + 1.5 \right]} = \frac{392.4}{200f + 3.0}; \text{ guess } f \approx 0.02, V \approx 3.02 \text{ m/s}$$

Iterate to $f_{smooth} \approx 0.0162, V \approx 3.33 \text{ m/s}$

The final $Re_c = \rho VD/\mu \approx 166000$, and $Q = (\pi/4)(0.05)^2(3.33) \approx 0.00653 \text{ m}^3/\text{s}$

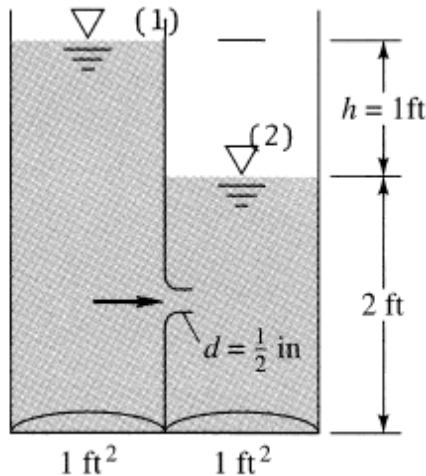
(b) The pressure drop across the orifice is given by the orifice formula:

$Re_d = 166000, \beta = 0.6, C_d \approx 0.609$

$$Q = 0.00653 = C_d A_t \left[\frac{2\Delta p}{\rho(1-\beta^4)} \right]^{\frac{1}{2}} = 0.609 \left(\frac{\pi}{4} \right) (0.03)^2 \left[\frac{2\Delta p}{998(1-0.6^4)} \right]^{\frac{1}{2}}$$

$\Delta = 100 \text{ kPa}$

36. Two water tanks, each with base area of 1 ft², are connected by a 0.5-inch diameter long-radius nozzle as in the figure. If $h = 1 \text{ ft}$ as shown for $t = 0$, estimate the time for $h(t)$ to drop to 0.25 ft.



For water at 20°C, take $\rho = 1.94 \text{ slug/ft}^3$ and $\mu = 2.09\text{E-}5 \text{ slug/ft.s}$. For a long-radius nozzle with $\beta \approx 0$, guess $C_d \approx 0.98$ and $K_{loss} \approx 0.9$. The elevation difference h must balance the head losses in the nozzle and submerged exit:

$$\Delta z = \sum h_{loss} = \frac{V_t^2}{2g} \sum K = \frac{V_t^2}{2(32.3)} (0.9_{nozzle} + 1.0_{exit}) = h, \text{ solve } V_t = 5.82\sqrt{h}$$

$$\text{hence } Q = V_t \left(\frac{\pi}{4} \right) \left(\frac{1}{12} \right)^2 \approx 0.00794\sqrt{h} = -\frac{1}{2} A_{tank} \frac{dh}{dt} = -\frac{1}{2} \frac{dh}{dt}$$

The boldface factor 1/2 accounts for the fact that, as the left tank falls by dh, the right tank rises by the same amount, hence dh/dt changes twice as fast as for one tank alone. We can separate and integrate and find the time for h to drop from 1 ft to 0.25 ft:

$$\int_{0.25}^{1.0} \frac{dh}{\sqrt{h}} = 0.0159 \int_0^{t_{final}} dt$$

$$t_{final} = \frac{2(\sqrt{1} - \sqrt{0.25})}{0.0159} \approx 63 \text{ s}$$

37. Water at 20°C flows in a long horizontal commercial-steel 6-cm-diameter pipe which contains a classical Herschel venturi with a 4-cm throat. The venturi is connected to a mercury manometer whose reading is h = 40 cm. Estimate (a) the flow rate, in m³/h, and (b) the total pressure difference between points 50 cm upstream and 50 cm downstream of the venturi.

For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m.s}$. For commercial steel, $\varepsilon \approx 0.046 \text{ mm}$, hence $\varepsilon/d = 0.046/60 = 0.000767$. First estimate the flow rate:

$$\Delta p = (\rho_m - \rho_w)gh = (13560 - 998)(9.81)(0.40) \approx 49293 \text{ Pa}$$

$$\text{Guess } C_d \approx 0.985, Q = (0.985) \left(\frac{\pi}{4} \right) (0.04)^2 \sqrt{\frac{2(49293)}{998 \left[1 - \left(\frac{4}{6} \right)^4 \right]}} \approx 0.0137 \text{ m}^3 / \text{s}$$

$$\text{Check } Re_D = \frac{4\rho Q}{\pi\mu D} \approx 291000$$

At this Reynolds number, C_d does indeed ≈ 0.985 for the Herschel venturi. Therefore, indeed, $Q = 0.0137 \text{ m}^3/\text{s} \approx 49 \text{ m}^3/\text{h}$.

(b) 50 cm upstream and 50 cm downstream are far enough that the pressure recovers from its throat value, and the total Δp is the sum of Moody pipe loss and venturi head loss. First work out the pipe velocity, $V = Q/A = (0.0137)/[(\pi/4)(0.06)^2] \approx 4.85 \text{ m/s}$. Then

$$\text{Re}_D = 291000, \frac{\varepsilon}{d} = 0.000767, \text{ then } f_{\text{Moody}} \approx 0.0196; K_{\text{venturi}} \approx 0.2$$

$$\begin{aligned} \text{Then } \Delta p &= \Delta p_{\text{Moody}} + \Delta p_{\text{venturi}} = \frac{\rho V^2}{2} \left(f \frac{L}{d} + K \right) \\ &= \frac{998(4.85)^2}{2} \left[0.0196 \left(\frac{1.0}{0.06} \right) + 0.2 \right] \approx 6200 \text{ Pa} \end{aligned}$$

38. A modern venturi nozzle is tested in a laboratory flow with water at 20°C. The pipe diameter is 5.5 cm, and the venturi throat diameter is 3.5 cm. The flow rate is measured by a weigh tank and the pressure drop by a water-mercury manometer. The mass flow rate and manometer readings are as follows:

M, kg/s	0.95	1.98	2.99	5.06	8.15
h, mm	3.7	15.9	36.2	102.4	264.4

Use these data to plot a calibration curve of venturi discharge coefficient versus Reynolds number.

For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. The given data of mass flow and manometer height can readily be converted to discharge coefficient and Reynolds number:

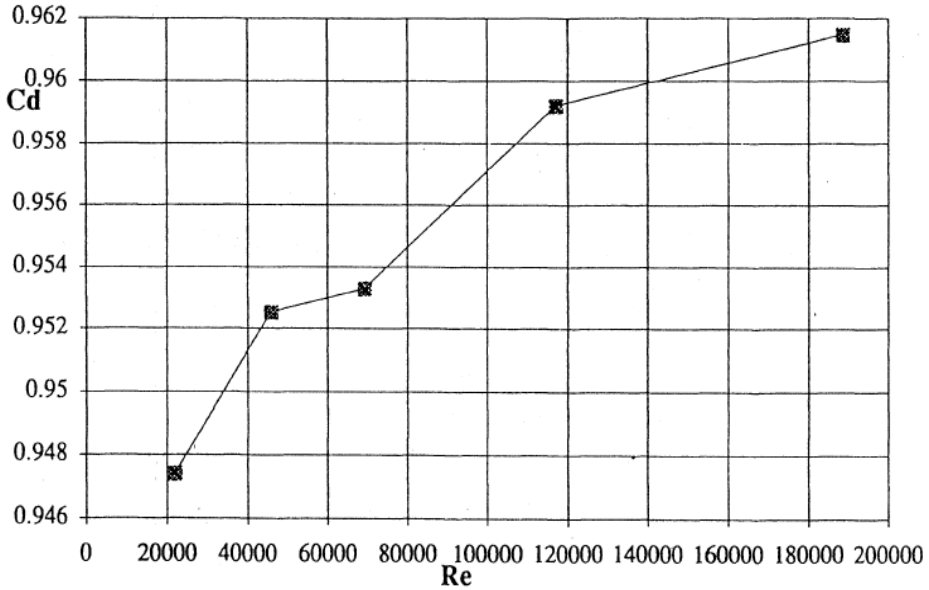
$$Q = \frac{\dot{m}}{998} = C_d \left(\frac{\pi}{4} \right) (0.035)^2 \sqrt{\frac{2(13.56 - 1) \rho_w (9.81) h}{\rho_w \left[1 - \left(\frac{3.5}{5.5} \right)^4 \right]}}$$

$$C_d \approx \frac{\dot{m} (\text{kg} / \text{s})}{16.485 \sqrt{h_{\text{meters}}}}$$

$$\text{Re}_D = \frac{4\dot{m}}{\pi \mu D} = \frac{4\dot{m}}{\pi (0.001)(0.055)} \approx 23150 \dot{m} (\text{kg} / \text{s})$$

The data can then be converted and tabulated as follows:

h,m	0.037	0.0159	0.0362	0.1024	0.2644
C _d	0.947	0.953	0.953	0.959	0.962
Re _{dp}	22000	46000	69000	117000	189000



They closely resemble the “classical Herschel venturi,” but this data is actually for a modern venturi, for which we only know the value of C_d for $1.5E5 < R_{ed} \leq 2E5$:

$$C_d \approx 0.9858 - 0.196 \left(\frac{3.5}{5.5} \right)^{4.5} \approx 0.960$$

The two data points near this Reynolds number range are quite close to 0.960 ± 0.002 .

39. A pitot-static probe will be used to measure the velocity distribution in a water tunnel at 20°C. The two pressure lines from the probe will be connected to a U-tube manometer which uses a liquid of specific gravity 1.7. The maximum velocity expected in the water tunnel is 2.3 m/s. Your job is to select an appropriate U-tube from a manufacturer which supplies manometers of heights 8, 12, 16, 24 and 36 inches. The cost increases significantly with manometer height. Which of these should you purchase?

The pitot-static tube formula relates velocity to the difference between stagnation pressure p_o and static pressure p_s in the water flow:

$$p_o - p_s = \frac{1}{2} \rho_w V^2, \text{ where } \rho_w = 998 \text{ kg} / \text{m}^3 \text{ and } V_{\max} = 2.3 \text{ m} / \text{s}$$

Meanwhile, the manometer reading h relates this pressure difference to the two fluids:

$$p_o - p_s = (\rho_{mano} - \rho_w)gh = \rho_w (SG_{mano} - 1)gh$$

$$\text{Solve for } h_{\max} = \frac{V_{\max}^2}{2g(SG_{mano} - 1)} = \frac{(2.3)^2}{2(9.81)(1.7 - 1)} = 0.385 \text{ m} = 15.2 \text{ in}$$

It would therefore be most economical to buy the 16-inch manometer. But be careful when you use it: a bit of overpressure will pop the manometer fluid out of the tube!

40. A pump delivers a steady flow of water (ρ, μ) from a large tank to two other higher-elevation tanks, as shown. The same pipe of diameter d and roughness ε is used throughout. All minor losses except through the valve are neglected, and the partially closed valve has a loss coefficient K_{valve} . Turbulent flow may be assumed with all kinetic energy flux correction coefficients equal to 1.06. The pump net head H is a known function of Q_A and hence also of $V_A = Q_A/A_{\text{pipe}}$, for example, $H = a - bV_A^2$, where a and b are constants. Subscript J refers to the junction point at the tee where branch A splits into B and C. Pipe length L_C is much longer than L_B . It is desired to predict the pressure at J , the three pipe velocities and friction factors, and the pump head. Thus there are 8 variables: $H, V_A, V_B, V_C, f_A, f_B, f_C, p_J$. Write down the eight equations needed to resolve this problem, but do not solve, since an elaborate iteration procedure, or an equation solver such as EES, would be required.

First, equation (1) is clearly the pump performance:

$$H = a - bV_A^2$$

$$f_A = \text{fcn}\left(V_A, \frac{\varepsilon}{d}\right)$$

$$f_B = \text{fcn}\left(V_B, \frac{\varepsilon}{d}\right)$$

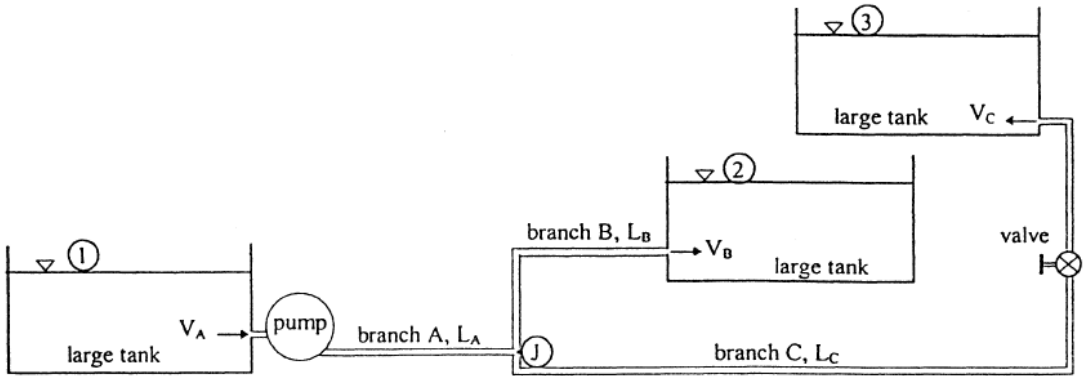
$$f_C = \text{fcn}\left(V_C, \frac{\varepsilon}{d}\right)$$

Conservation of mass (constant area) at the junction J : $V_A = V_B + V_C$ Finally, there are three independent steady-flow energy equations:

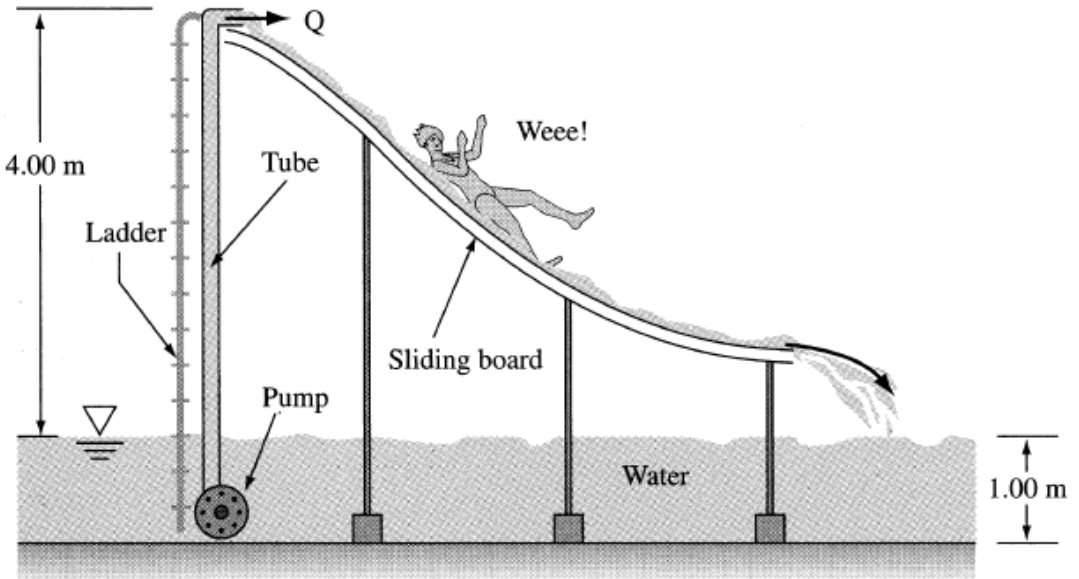
$$(1) \text{ to } (2): z_1 = z_2 - H + f_A \frac{L_A}{d} \frac{V_A^2}{2g} + f_B \frac{L_B}{d} \frac{V_B^2}{2g}$$

$$(1) \text{ to } (3): z_1 = z_3 - H + f_A \frac{L_A}{d} \frac{V_A^2}{2g} + f_C \frac{L_C}{d} \frac{V_C^2}{2g} + K_{\text{valve}} \frac{V_C^2}{2g}$$

$$(J) \text{ to } (2): \frac{p_J}{\rho g} + z_J = \frac{p_{\text{atm}}}{\rho g} + z_2 + f_B \frac{L_B}{d} \frac{V_B^2}{2g}$$



41. The water slide in the figure is to be installed in a swimming pool. The manufacturer recommends a continuous water flow of $1.39\text{E-}3 \text{ m}^3/\text{s}$ (about 22 gal/min) down the slide to ensure that customers do not burn their bottoms. An 80%-efficient pump under the slide, submerged 1 m below the water surface, feeds a 5-m-long, 4-cmdiameter hose, of roughness 0.008 cm, to the slide, 4 m above the water surface, as a free jet. Ignore minor losses and assume $\alpha = 1.06$. Find the brake horsepower needed to drive the pump.



For water take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. Write the steady-flow energy equation from the water surface (1) to the outlet (2) at the top of the slide:

$$\frac{p_a}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_a}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f - h_{pump}$$

$$V_2 = \frac{1.39E-3}{\pi(0.02)^2} = 1.06 \text{ m/s}$$

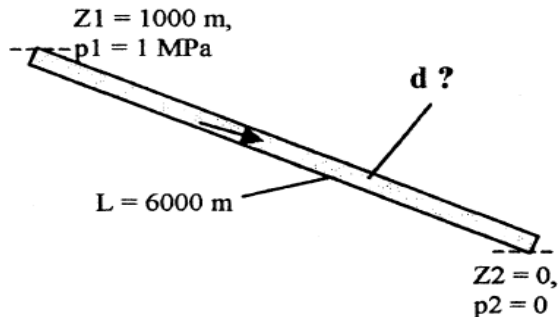
$$\text{Solve for } h_{pump} = (z_2 - z_1) + \frac{V_2^2}{2g} \left(\alpha_2 + f \frac{L}{d} \right)$$

Work out $R_{ed} = \rho V d / \mu = (998)(1.106)(0.04) / 0.001 = 44200$, $\epsilon/d = 0.008/4 = 0.002$, whence $f_{Moody} = 0.0268$. Use these numbers to evaluate the pump head above:

$$h_{pump} = (5.0 - 1.0) + \frac{(1.106)^2}{2(9.81)} \left[1.06 + 0.0268 \left(\frac{5.0}{0.04} \right) \right] = 4.27 \text{ m}$$

$$\text{whence } BHP_{required} = \frac{\rho g Q h_{pump}}{\eta} = \frac{998(9.81)(1.39E-3)(4.27)}{0.8} = 73 \text{ watts}$$

42. Suppose you build a house out in the ‘boonies,’ where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface. (a) Find the total head loss from pipe inlet to exit, neglecting minor losses.



(b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe? (c) Find the minimum required pipe diameter.

Convert 3.0 gal/min to $1.89E-4 \text{ m}^3/\text{s}$. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

$$\frac{p_{1\text{gage}}}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_{2\text{gage}}}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f$$

$$h_f = z_1 - z_2 + \frac{p_{1\text{gage}}}{\rho g} = 1000 \text{ m} + \frac{1E6 \text{ kPa}}{998(9.81)} = 1000 + 102 = 1102 \text{ m}$$

(b) Thus, elevation drop of 1000 m is more important to head loss than $\Delta p/\rho g = 102 \text{ m}$.

(c) To find the minimum diameter, iterate among flow rate and the Moody chart:

$$h_f = f \frac{L V^2}{d 2g}, L = 6000 \text{ m}$$

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{\text{Re} \sqrt{f}} \right), V = \frac{Q}{\pi d^2 / 4}$$

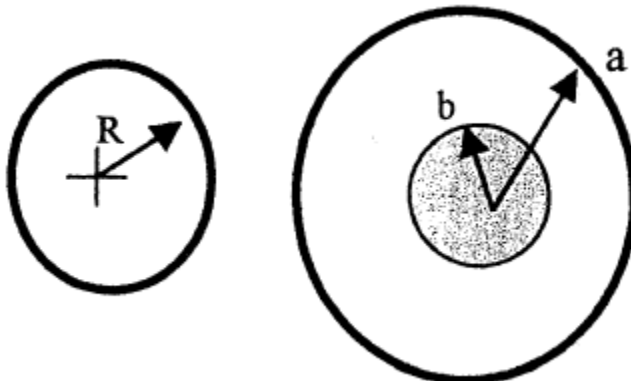
$$Q = 1.89E-4 \text{ m}^3 / \text{s}$$

$$\text{Re} = Vd / \nu$$

We are given $h_f = 1102 \text{ m}$ and $\nu_{\text{water}} = 1.005E-6 \text{ m}^2/\text{s}$. We can iterate, if necessary, or use EES, which can swiftly arrive at the final result:

$$f_{\text{smooth}} = 0.0266; \text{Re} = 17924; V = 1.346 \text{ m/s}; d_{\text{min}} = 0.0134 \text{ m}$$

43. Water at 20°C flows, at the same flow rate $Q = 9.4E-4 \text{ m}^3/\text{s}$, through two ducts, one a round pipe, and one an annulus, as shown. The cross-section area A of each duct is identical, and each has walls of commercial steel. Both are the same length. In the crosssections shown, $R = 15 \text{ mm}$ and $a = 25 \text{ mm}$. (a) Calculate the correct radius b for the annulus. (b) Compare head loss per unit length for the two ducts, first using the hydraulic diameter and second using the ‘effective diameter’ concept. (c) If the losses are different, why? Which duct is more ‘efficient’? Why?



(a) Set the areas equal:

$$A = \pi R^2 = \pi(a^2 - b^2), \text{ or } b = \sqrt{a^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20 \text{ mm}$$

(b) Find the round-pipe head loss, assuming $v = 1.005E-6 \text{ m}^2/\text{s}$:

$$V = \frac{Q}{A} = \frac{9.4E-4 \text{ m}^3/\text{s}}{\pi(0.015 \text{ m})^2} = 1.33 \text{ m/s}$$

$$\text{Re} = \frac{(1.33)(0.030)}{1.005E-6} = 39700$$

$$\frac{\varepsilon}{d} = 0.00153, f_{\text{Moody}} = 0.0261$$

Thus $h_f/L = (f/d)(V^2/2g) = (0.0261/0.03)(1.33^2)/2/9.81 = 0.0785$ (round)

Annulus: $D_h = 4A/P = 2(a-b) = 20 \text{ mm}$, same $V = 1.33 \text{ m/s}$:

$$\frac{h_f}{L} \approx \left(\frac{f}{D_h} \frac{V^2}{2g} \right) \approx 0.131 \text{ (annulus)}$$

Effective-diameter concept: $b/a = 0.8$, Table 6.3: $D_{\text{eff}} = 0.667 D_h = 13.3 \text{ mm}$. Then

$$\text{Re}_{D_{\text{eff}}} = 17700, \frac{\varepsilon}{D_{\text{eff}}} = 0.00345, f_{\text{Moody}} = 0.0327$$

$$\frac{h_f}{L} = \frac{f}{D_h} \frac{V^2}{2g} = 0.147 \text{ (annulus - } D_{\text{eff}})$$

NOTE: Everything here uses D_{eff} except h_f , which by definition uses D_h !

We see that the annulus has about 85% more head loss than the round pipe, for the same area and flow rate! This is because the annulus has more wall area, thus more friction.

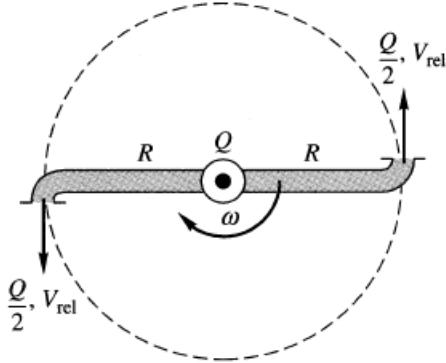
44. A pump delivers 1500 L/min of water at 20°C against a pressure rise of 270 kPa. Kinetic and potential energy changes are negligible. If the driving motor supplies 9 kW, what is the overall efficiency?

With pressure rise given, we don't need density. Compute "water" power:

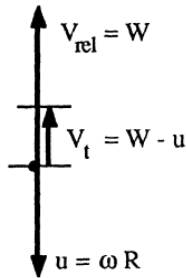
$$P_{\text{water}} = \rho g Q H = Q \Delta p = \left(\frac{1.5 \text{ m}^3}{60 \text{ s}} \right) \left(270 \frac{\text{kN}}{\text{m}^2} \right) = 6.75 \text{ kW}$$

$$\eta = \frac{6.75}{9.0} = 75\%$$

45. A lawn sprinkler can be used as a simple turbine. As shown in the figure, flow enters normal to the paper in the center and splits evenly into $Q/2$ and V_{rel} leaving each nozzle. The arms rotate at angular velocity ω and do work on a shaft. Draw the velocity diagram for this turbine. Neglecting friction, find an expression for the power delivered to the shaft. Find the rotation rate for which the power is a maximum.



Utilizing the velocity diagram at right, we apply the Euler turbine formula:



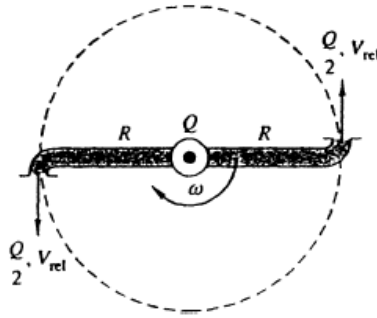
$$P = \rho Q(u_2 V_{t2} - u_1 V_{t1}) = \rho Q[u(W - u) - 0]$$

$$P = \rho Q \omega R (V_{rel} - \omega R)$$

$$\frac{dP}{du} = \rho Q (V_{rel} - 2u) = 0 \text{ if } \omega = \frac{V_{rel}}{2R}$$

$$\text{where } P_{max} = \rho Q u (2u - u) = \rho Q (\omega R)^2$$

46). For the “sprinkler turbine” of the figure, let $R = 18 \text{ cm}$, with total flow rate of $14 \text{ m}^3/\text{h}$ of water at 20°C . If the nozzle exit diameter is 8 mm , estimate (a) the maximum power delivered in W and (b) the appropriate rotation rate in r/min .



For water at 20°C, take $\rho \approx 998 \text{ kg/m}^3$. Each arm takes $7 \text{ m}^3/\text{h}$:

$$V_{rel} = \frac{Q/2}{A_{exit}} = \frac{7/3600}{\left(\frac{\pi}{4}\right)(0.008)^2} = 38.7 \text{ m/s; at max power}$$

$$u = \omega R = \frac{1}{2} V_{rel} = 19.34 \text{ m/s} = \omega(0.18 \text{ m}), \text{ solve } \omega = 107 \text{ rad/s} \approx 1030 \text{ rpm}$$

$$P_{max} = \rho Q u^2 = 998 \left(\frac{14}{3600}\right) (19.34)^2 \approx 1450 \text{ W}$$

47. Centrifugal water pump has $r_2 = 9 \text{ in}$, $b_2 = 2 \text{ in}$, and $\beta_2 = 35^\circ$ and rotates at 1060 r/min. If it generates a head of 180 ft, determine the theoretical (a) flow rate in gal/min and (b) horsepower. Assume near-radial entry flow.

For water take $\rho = 1.94 \text{ slug/ft}^3$. Convert $\omega = 1060 \text{ rpm} = 111 \text{ rad/s}$. Then

$$u_2 = \omega r_2 = 111 \left(\frac{9}{12}\right) = 83.3 \text{ ft/s}$$

$$\text{Power} = \rho Q u_2 \left(u_2 - \frac{Q}{2\pi r_2 b_2} \cot \beta_2 \right)$$

$$H = \frac{P}{\rho g Q} = 180 \text{ ft}$$

$$P = 62.4 Q H = 1.94 Q (83.3) \left[83.3 - \frac{Q}{2\pi \left(\frac{9}{12}\right) \left(\frac{2}{12}\right)} \cot 35^\circ \right] \text{ with } H = 180$$

Solve for $Q = 7.5 \text{ ft}^3/\text{s} \approx 3360 \text{ gal}/\text{min}$

With Q and H known, $P = \rho g QH = 62.4(7.5)(180) \div 550 \approx 153 \text{ hp}$

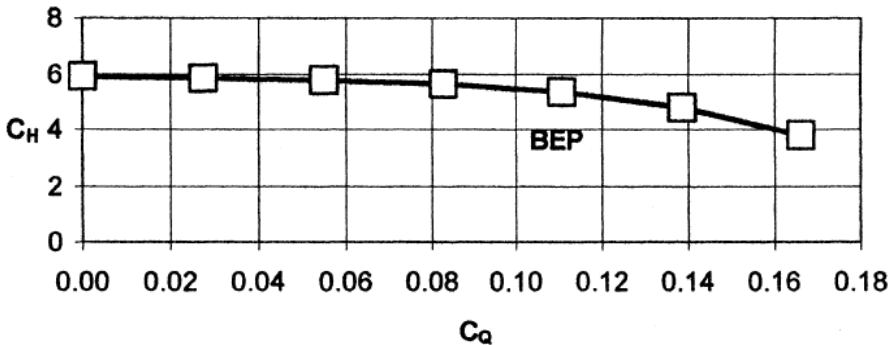
48. A 37-cm-diameter centrifugal pump, running at 2140 rev/min with water at 20°C produces the following performance data:

$Q, \text{m}^3/\text{s}$	0.0	0.05	0.10	0.15	0.20	0.25	0.30
H, m	105	104	102	100	95	85	67
P, kW	100	115	135	171	202	228	249
η	0%	44%	74%	86%	92%	91%	79%

(a) Determine the best efficiency point. (b) Plot C_H versus C_Q . (c) If we desire to use this same pump family to deliver 7000 gal/min of kerosene at 20°C at an input power of 400 kW, what pump speed (in rev/min) and impeller size (in cm) are needed? What head will be developed?

Efficiencies, computed by $\eta = \rho g QH/\text{Power}$, are listed above. The Best Efficiency Point (BEP) is approximately 92% at $Q = 0.2 \text{ m}^3/\text{s}$

The dimensionless coefficients are $C_Q = Q/(nD^3)$, where $n = 2160/60 = 36 \text{ rev}/\text{s}$ and $D = 0.37 \text{ m}$, plus $C_H = gH/(n^2D^2)$ and $C_P = P/(\rho n^3D^5)$, where $\rho_{\text{water}} = 998 \text{ kg}/\text{m}^3$. BEP values are $C_Q^* = 0.111$, $C_H^* = 5.35$, and $C_P^* = 0.643$. A plot of C_H versus C_Q is below.



(c) For kerosene, $\rho_k = 804 \text{ kg}/\text{m}^3$. Convert $7000 \text{ gal}/\text{min} = 0.442 \text{ m}^3/\text{s}$. At BEP, we require the above values of dimensionless parameters:

$$\frac{Q}{nD^3} = \frac{0.442}{nD^3} = 0.111$$

$$\frac{P}{\rho n^3 D^5} = \frac{400000}{804 n^3 D^5} = 0.643$$

Solve $n = 26.1 \text{ rev}/\text{s} = 1560 \text{ rev}/\text{min}$; $D = 0.534 \text{ m}$

$$H^* = C_H^* (n^2 D^2) / g = 5.35 (26.1)^2 (0.534)^2 / 9.81 = 106 \text{ m}$$

49. A centrifugal pump with backward-curved blades has the following measured performance when tested with water at 20°C:

Q, gal/min	0	400	800	1200	1600	2000	2400
H, ft	123	115	108	101	93	81	62
P, hp	30	36	40	44	47	48	46

(a) Estimate the best efficiency point and the maximum efficiency. (b) Estimate the most efficient flow rate, and the resulting head and brake horsepower, if the diameter is doubled and the rotation speed increased by 50%.

(a) Convert the data above into efficiency. For example, at Q = 400 gal/min,

$$\eta = \frac{\gamma QH}{P} = \frac{(62.4 \text{ lbf} / \text{ft}^3) \left(\frac{400}{448.8} \text{ ft}^3 / \text{s} \right) (115 \text{ ft})}{(36 \times 550 \text{ ft.lbf} / \text{s})} = 0.32 = 32\%$$

When converted, the efficiency table looks like this:

Q, gal/min	0	400	800	1200	1600	2000	2400
η	0	32%	55%	70%	80%	85%	82%

So maximum efficiency of 85% occurs at Q = 2000 gal/min

(b) We don't know the values of C_Q^* or C_H^* or C_P^* , but we can set them equal for conditions 1 (the data above) and 2 (the performance when n and D are changed):

$$C_Q^* = \frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3} = \frac{Q_2}{(1.5n_1)(2D_1)^3}$$

$$Q_2 = 12Q_1 = 12(2000 \text{ gpm}) = 24000 \text{ gal} / \text{min}$$

$$C_H^* = \frac{gH_1}{n_1^2 D_1^2} = \frac{gH_2}{n_2^2 D_2^2} = \frac{gH_2}{(1.5n_1)^2 (2D_1)^2}$$

$$H_2 = 9H_1 = 9(81 \text{ ft}) = 729 \text{ ft}$$

$$C_P^* = \frac{P_1}{\rho n_1^3 D_1^5} = \frac{P_2}{\rho n_2^3 D_2^5} = \frac{P_2}{\rho (1.5n_1)^3 (2D_1)^5}$$

$$P_2 = 108P_1 = 108(48 \text{ hp}) = 5180 \text{ hp}$$

50. An 18-in-diameter centrifugal pump, running at 880 rev/min with water at 20°C, generates the following performance data:

Q, gal/min	0.0	2000	4000	6000	8000	10000
H, ft	92	89	84	78	68	50
P, hp	100	112	130	143	156	163

Determine (a) the BEP; (b) the maximum efficiency; and (c) the specific speed. (d) Plot the required input power versus the flow rate.

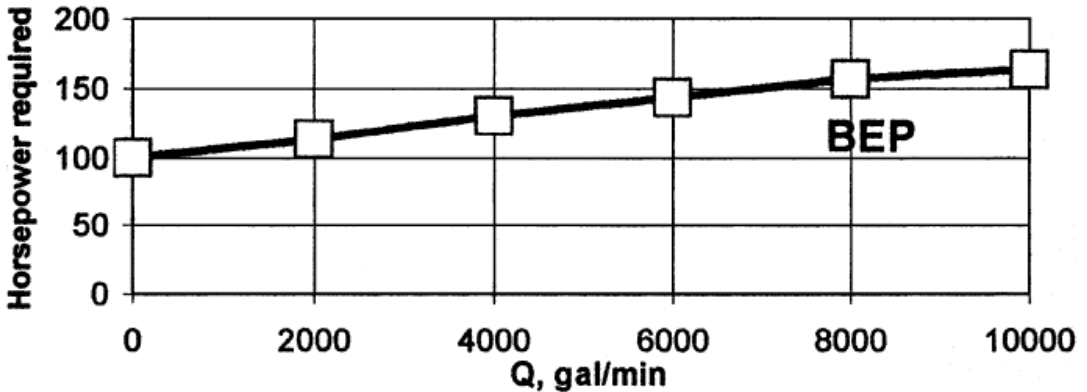
Q, gal/min	0.0	2000	4000	6000	8000	10000
H,ft	92	89	84	78	68	50
P, hp	100	112	130	143	156	163
η	0%	40%	65%	83%	88%	78%

We have computed the efficiencies and listed them. The BEP is the next to-last point: $Q = 8000$ gal/min, $\eta_{\max} = 88\%$.

The specific speed is

$$N'_s = nQ^{1/2}/(gH^*)^{3/4} = (880/60)(8000/448.83)^{1/2}/[32.2(68)]^{3/4} \approx 0.193, \text{ or } N_s = 3320 \text{ (probably a centrifugal pump).}$$

The plot of input horsepower versus flow rate is shown below—there are no surprises in this plot.



51. A 6.85-in pump, running at 3500 rpm, has the measured performance at right for water at 20°C. (a) Estimate the horsepower at BEP. If this pump is rescaled in water to provide 20 bhp at 3000 rpm, determine the appropriate (b) impeller diameter; (c) flow rate; and (d) efficiency for this new condition.

Q, gal/min	50	100	150	200	250	300	350	400	450
H,ft	201	200	198	194	189	181	169	156	139
η , %	29	50	64	72	77	80	81	79	74

The BEP of 81% is at about $Q = 350$ gpm and $H = 169$ ft. Hence the power is

$$P^* = \frac{\rho g Q^* H^*}{\eta} = \frac{62.4 \left(\frac{350}{449} \right) (169)}{0.81} = 10150 \frac{ft \cdot lbf}{s} \div 550 \approx 18.5 bhp$$

If the new conditions are 20 hp at $n = 3000$ rpm = 50 rps, we equate power coefficients:

$$C_p^* = \frac{10150}{1.94 \left(\frac{3500}{60} \right)^3 \left(\frac{6.85}{12} \right)^5} = 0.435 = \frac{20 \times 550}{1.94(50)^3 D^5}$$

Solve $D_{imp} \approx 0.636 \text{ ft} \approx 7.64 \text{ in}$

With diameter known, the flow rate is computed from BEP flow coefficient:

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{\frac{350}{449}}{\left(\frac{3500}{60} \right) \left(6.85/12 \right)^3} = 0.0719 = \frac{Q^*}{50(0.636)^3}$$

Solve $Q^* = 0.926 \text{ ft}^3 / \text{s} \approx 415 \text{ gal} / \text{min}$

Finally, since $D_1 \approx D_2$, we can assume the same maximum efficiency: 81%

52. An 8-inch model pump delivering water at 180°F at 800 gal/min and 2400 rpm begins to cavitate when the inlet pressure and velocity are 12 psia and 20 ft/s, respectively. Find the required NPSH of a prototype which is 4 times larger and runs at 1000 rpm.

For water at 180°F, take $\rho g \approx 60.6 \text{ lbf/ft}^3$ and $p_v \approx 1600 \text{ psfa}$.

$$NPSH_{\text{model}} = \frac{p_i - p_v}{\rho g} + \frac{V_i^2}{2g} = \frac{12(144) - 1600}{60.6} + \frac{(20)^2}{2(32.2)} = 8.32 \text{ ft}$$

$$NPSH_{\text{proto}} = NPSH_{\text{model}} \left(\frac{n_p}{n_m} \right)^2 \left(\frac{D_p}{D_m} \right)^2 = 8.32 \left(\frac{1000}{2400} \right)^2 \left(\frac{4}{1} \right)^2 \approx 23 \text{ ft}$$

53. A typical household basement sump pump provides a discharge of 5 gal/min against a head of 15 ft. Estimate (a) the maximum efficiency; and (b) the minimum horsepower required to drive such a pump.

Typical small sump pumps run at about 1750 rpm, so we can estimate:

$$N_s = \frac{(rpm) \left(\frac{\text{gal}}{\text{min}} \right)^{\frac{1}{2}}}{(\text{head})^{\frac{3}{4}}} \approx \frac{1750(5)^{\frac{1}{2}}}{(15 \text{ ft})^{\frac{3}{4}}} \approx 513 \rightarrow \eta_{\text{max}} \approx 0.27$$

$$P_{\text{min}} = \frac{\rho g Q H}{\eta_{\text{max}}} = \frac{62.4 \left(\frac{5}{449} \right) (15)}{0.27} = 39 \div 550 \approx 0.07 \text{ bhp}$$

54. When operating at 42 r/s near BEP, a pump delivers 0.06 m³/s against a head of 100 m. (a) What is its specific speed? (b) What kind of pump is this likely to be? (c) Estimate its impeller diameter.

(a) We have to go English to calculate the traditional specific speed. Convert $Q = 0.06 \text{ m}^3/\text{s} = 951 \text{ gal}/\text{min}$, $H = 100 \text{ m} = 328 \text{ ft}$, and $n = 42 \text{ r/s} = 2520 \text{ r}/\text{min}$. Then

$$N_s = \frac{\text{rpm} \left(\frac{\text{gal}}{\text{min}} \right)^{\frac{1}{2}}}{(\text{Head in ft})^{\frac{3}{4}}} = \frac{2520(951)^{\frac{1}{2}}}{(328)^{\frac{3}{4}}} \approx 1000$$

(b) This specific speed is characteristic of a centrifugal pump

(c) The dimensionless specific diameter $D_s = D(\text{gH}^*)^{1/4}/Q^{*1/2}$ is closely correlated with specific speed:

$$D_s \approx \frac{7800}{N_s} = \frac{7800}{1000} = 7.8 = \frac{D \left[9.81 \text{ m} / \text{s}^2 (100 \text{ m}) \right]^{\frac{1}{4}}}{(0.06 \text{ m}^3 / \text{s})^{\frac{1}{2}}}$$

$$D \approx 0.34 \text{ m} (13 \text{ in})$$

55. The Colorado River Aqueduct uses Worthington Corp. pumps which deliver 200 ft³/s of water at 450 rpm against a head of 440 ft. What kind of pumps are these? Estimate the impeller diameter.

Evaluate the specific speed to see what type of pumps we have:

$$N_s = \frac{(\text{rpm}) \left(\frac{\text{gal}}{\text{min}} \right)^{\frac{1}{2}}}{(\text{head})^{\frac{3}{4}}} = \frac{450(200 \times 449)^{\frac{1}{2}}}{(440)^{\frac{3}{4}}} \approx 1400 \rightarrow \text{Centrifugal pump}$$

$$C_Q^* \approx (6.83E-8)(1400)^{1.914} \approx 0.072 = \frac{200}{(450/60)D^3} \rightarrow D_{\text{impeller}} \approx 7.2 \text{ ft}$$

56. Two 32-inch pumps are combined in parallel to deliver water at 20°C through 1500 ft of horizontal pipe. If $f = 0.025$, what pipe diameter will ensure a flow rate of 35,000 gal/min at 1170 rpm?

For water at 20°C, take $\rho = 1.94 \text{ slug}/\text{ft}^3$ and $\mu = 2.09E-5 \text{ slug}/\text{ft}\cdot\text{s}$.

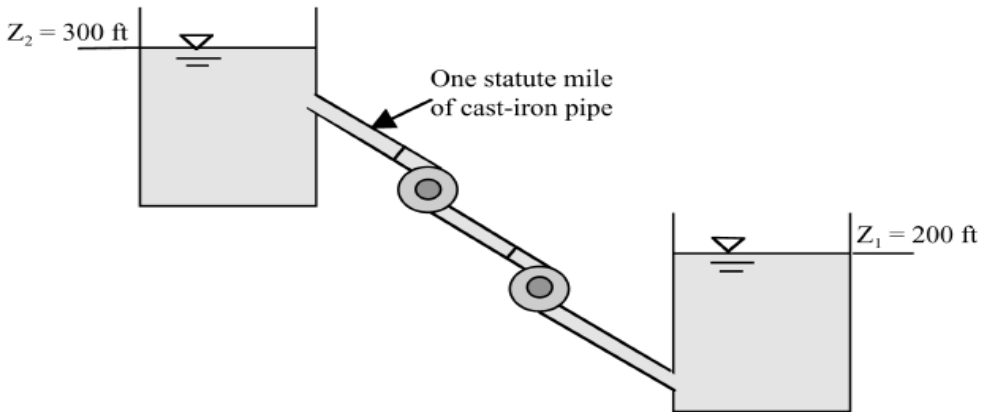
$H_p(\text{ft}) \approx 500 - 0.3Q^2$, with Q in kgal/min . Each pump takes half the flow, 17,500 gal/min, for which

$$H_p = 500 - 0.3(17.5)^2 \approx 408 \text{ ft}$$

$$Q_{\text{pipe}} = \frac{35000}{449} = 78 \text{ ft}^3 / \text{s}$$

$$H_{\text{sys}} = 0.025 \left(\frac{1500}{d} \right) \left[\frac{\left(\frac{78}{\left(\frac{\pi d^2}{4} \right)} \right)}{2(32.2)} \right]^2 = \frac{5740}{d^5} = 408 \text{ ft} \rightarrow d \approx 1.70 \text{ ft}$$

57. Suppose that the two pumps in the figure are instead arranged to be in series, again at 710 rpm? What pipe diameter is required for BEP operation?



For water at 20°C, take $\rho = 1.94 \text{ slug/ft}^3$ and $\mu = 2.09\text{E-}5 \text{ slug/ft}\cdot\text{s}$. For cast iron, $\epsilon \approx 0.00085 \text{ ft}$. The 35-inch pump has the BEP values $Q^* \approx 18 \text{ kgal/min}$, $H^* \approx 190 \text{ ft}$. In series, each pump takes $H/2$, so a BEP series operation would match

$$H_{\text{sys}} = 2H^* = 2(190) = \Delta z + f \frac{L}{D} \frac{V^2}{2g} = 100 + f \left(\frac{5280}{d} \right) \left[\frac{\left(\frac{18000}{449} \right)}{\left(\frac{\pi d^2}{4} \right)} \right]^2$$

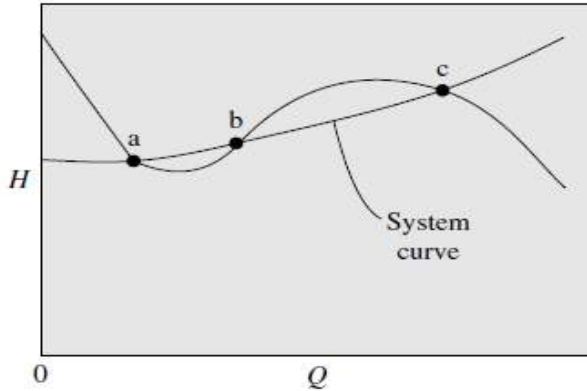
$$380 = 100 + \frac{213800 f}{d^5} \text{ where } f \text{ depends on } \text{Re} = \frac{4\rho Q}{\pi d \mu} \text{ and } \frac{\epsilon}{d} = \frac{0.00085}{d}$$

This converges to $f \approx 0.0169$, $Re \approx 2.84E6$, $V \approx 18.3$ ft/s, $d \approx 1.67$ ft

$$Power = 2P^* = 2 \frac{62.4 \left(\frac{18000}{449} \right) (190)}{0.87} = 1.09E6 \div 550 \approx 2000 \text{ bhp}$$

We can save money on the smaller (20-inch) pipe, but putting the pumps in series requires twice as much power as one pump alone.

58. The S-shaped head-versus-flow curve in the figure occurs in some axial-flow pumps. Explain how a fairly flat system-loss curve might cause instabilities in the operation of the pump. How might we avoid instability?

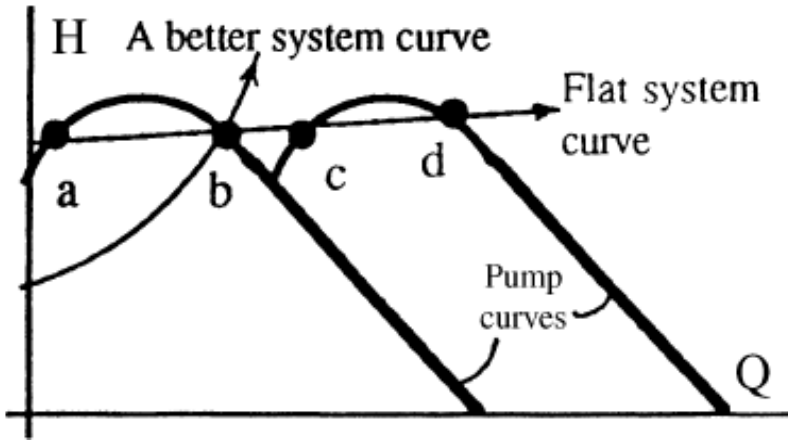


The stability of pump operation is nicely covered in the review article by Greitzer. Generally speaking, there is little danger of instability if the slope of the pump-head curve, dH/dQ , is negative, unless there are two such points. In the figure above, a flat system curve may cross the pump curve at three points (a, b, c). Of these 3, point b is statically unstable and cannot be maintained. Consider a small disturbance near point b: Suppose the flow rate drops slightly—then the system head decreases, but the pump head decreases even more. Then the flow rate will drop still more, etc., and we move away from the operating point, which therefore is unstable. The general rule is:

A pump operating point is statically unstable if the (positive) slope of the pump-head curve is greater than the (positive) slope of the system curve. By this criterion, both points a and c above are statically stable. However, if the points are close together or there are large disturbances, a pump can “hunt” or oscillate between points a and c, so this could also be considered unstable to large disturbances. Finally, even a steep system curve (not shown above) which crosses at only a single point b on the positive-slope part of the pump-head curve can be dynamically unstable, that is, it can trigger an energy-feeding oscillation which diverges from point b. See Greitzer’s article for further details of this and other turbomachine instabilities.

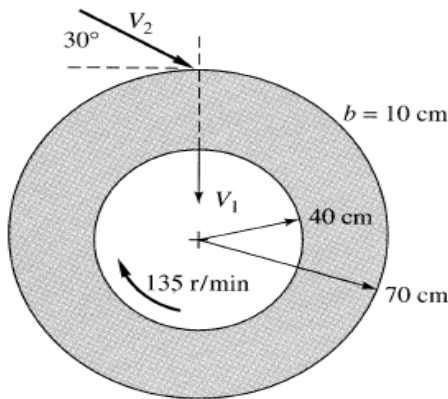
59. The low-shutoff head-versus-flow curve in the figure occurs in some centrifugal pumps. Explain how a fairly flat system-loss curve might cause instabilities in the

operation of the pump. What additional vexation occurs when two of these pumps are in parallel? How might we avoid instability?



As discussed, for one pump with a flat system curve, point a is statically unstable, point b is stable. A 'better' system curve only passes through b. For two pumps in parallel, both points a and c are unstable (see above). Points b and d are stable but for large disturbances the system can 'hunt' between the two points.

60. An idealized radial turbine is shown in the figure. The absolute flow enters at 30° and leaves radially inward. The flow rate is $3.5 \text{ m}^3/\text{s}$ of water at 20°C . The blade thickness is constant at 10 cm . Compute the theoretical power developed at 100% efficiency.



For water, take $\rho \approx 998 \text{ kg/m}^3$

$$u_2 = \omega r_2 = 135 \left(\frac{2\pi}{60} \right) (0.7) = 9.90 \text{ m/s}$$

$$\alpha_2 = 30^\circ$$

$$\alpha_1 = 90^\circ$$

$$V_{n2} = \frac{3.5}{2\pi(0.7)(0.1)} \approx 7.96 \text{ m/s}$$

$$V_{t2} = \frac{V_{n2}}{\tan \alpha_2} = \frac{7.96}{\tan 30^\circ} = 13.8 \text{ m/s}$$

$$V_{t1} = \frac{V_{n1}}{\tan 90^\circ} = 0$$

$$P_{theory} = \rho Q u_2 V_{t2} = 998(3.5)(9.90)(13.8) = 477000 \text{ W}$$

61. A dam on a river is being sited for a hydraulic turbine. The flow rate is 1500 m³/h, the available head is 24 m, and the turbine speed is to be 480r/min. Discuss the estimated turbine size and feasibility for (a) a Francis turbine; and (b) a Pelton wheel.

Assume $\eta \approx 89\%$. The power generated by the turbine would be $P = \eta \gamma QH = (0.89) (62.4 \text{ lbf/ft}^3) (14.7 \text{ ft}^3/\text{s}) (78.7 \text{ ft}) = 64,300 \text{ ft-lbf/s} = 117 \text{ hp}$. Now compute $N_{sp} = (480 \text{ rpm}) (117 \text{ hp})^{1/2} / (78.7 \text{ ft})^{5/4} \approx 22$, appropriate for a Francis turbine.

(a) A Francis turbine, would have $C_Q^* \approx 0.34 = (14.7 \text{ ft}^3/\text{s}) / [(480/60 \text{ r/s}) D^3]$. Solve for a turbine diameter of about 1.8 ft, which would be excellent for the task.

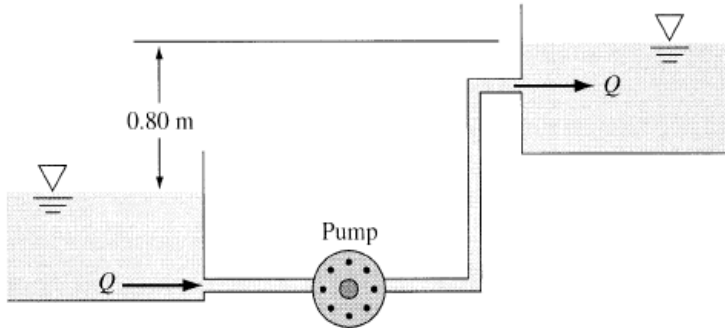
(b) A Pelton wheel at best efficiency (half the jet velocity) would only be 18 inches in diameter, with a huge nozzle, $d \approx 6$ inches, which is too large for the wheel. We conclude that a Pelton wheel would be a poor design.

62. The net head of a little aquarium pump is given by the manufacturer as a function of volume flow rate as listed:

Q, m ³ /s	0	1E-6	2E-6	3E-6	4E-6	5E-6
H, mmH ₂ O	1.10	1.00	0.80	0.60	0.35	0.0

What is the maximum achievable flow rate if you use this pump to pump water from the lower reservoir to the upper reservoir as shown in the figure?

NOTE: The tubing is smooth, with an inner diameter of 5 mm and a total length of 29.8 m. The water is at room temperature and pressure, and minor losses are neglected.



For water, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. NOTE: The tubing is so small that the flow is laminar, even at the highest pump flow rate:

$$H_{pump} = \Delta z + f \frac{L V^2}{d 2g} = \Delta z + h_{f,lam} = \Delta z + \frac{128 \mu L Q}{\pi d^4 \rho g} = 0.8 + \frac{128(0.001)(29.8)Q}{\pi(0.005)^4(998)(9.81)}$$

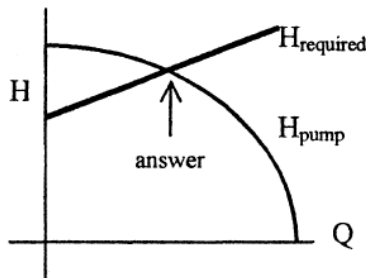
$$H_{pump} = 0.8 + 198400Q = H_{pump}(Q) \text{ from the pump data above}$$

One can plot the two relations, as at right, or use EES with a look-up table to get the final result for flow rate and head:

$$H_p = 1.00 \text{ m}$$

$$Q = 1.0\text{E-}6 \text{ m}^3 / \text{s}$$

The EES print-out gives the results $R_{ed} = 255$, $H = 0.999 \text{ m}$, $Q = 1.004\text{E-}6 \text{ m}^3/\text{s}$.



63. A shallow-water wave 12 cm high propagates into still water of depth 1.1 m. Compute (a) the wave speed; and (b) the induced velocity δV .

$$c = \sqrt{gy(1 + \delta y / y)(1 + \delta y / 2y)} = \sqrt{9.81(1.1)(1 + 0.12 / 1.1) \left[1 + 0.12 / \{2(1.1)\} \right]} = 3.55 \text{ m/s}$$

$$\delta V = \frac{c \delta y}{y + \delta y} = \frac{(3.55 \text{ m/s})(0.12 \text{ m})}{1.1 + 0.12 \text{ m}} = 0.35 \text{ m/s}$$

64. Narragansett Bay is approximately 21 (statute) mi long and has an average depth of 42 ft. Tidal charts for the area indicate a time delay of 30 min between high tide at

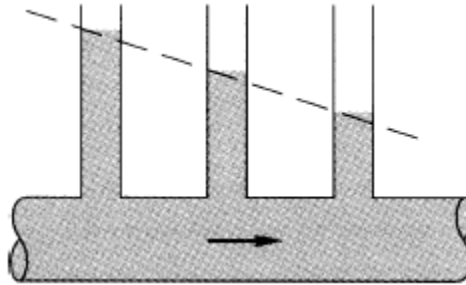
the mouth of the bay (Newport, Rhode Island) and its head (Providence, Rhode Island). Is this delay correlated with the propagation of a shallow-water tidal-crest wave through the bay? Explain.

If it is a simple shallow-water wave phenomenon, the time delay would be

$$\Delta t = \frac{\Delta L}{c_0} = \frac{(21 \text{ mi})(5280 \text{ ft / mi})}{\sqrt{32.2(42)}} \approx 3015 \text{ s} \approx 50 \text{ min}$$

This doesn't agree with the measured $\Delta t \approx 30 \text{ min}$. In reality, tidal propagation in estuaries is a dynamic process, dependent on estuary shape, bottom friction, and tidal period.

65. The water-channel flow in the figure has a free surface in three places. Does it qualify as an open-channel flow? Explain. What does the dashed line represent?



No, this is not an open-channel flow. The open tubes are merely piezometer or pressure-measuring devices, there is no flow in them. The dashed line represents the pressure distribution in the tube, or the “Hydraulic Grade Line” (HGL).

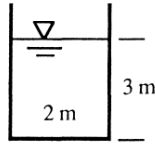
66. An earthquake near the Kenai Peninsula, Alaska, creates a single “tidal” wave (called a ‘tsunami’) which propagates south across the Pacific Ocean. If the average ocean depth is 4 km and seawater density is 1025 kg/m³, estimate the time of arrival of this tsunami in Hilo, Hawaii.

Everyone get out your Atlases, how far is it from Kenai to Hilo? Well, it’s about 2800 statute miles (4480 km), and seawater density has nothing to do with it:

$$\Delta t_{\text{travel}} = \frac{\Delta x}{c_0} = \frac{4480E3 \text{ m}}{\sqrt{9.81(4000 \text{ m})}} \approx 22600 \text{ s} \approx 6.3 \text{ hours}$$

So, given warning of an earthquake in Alaska (by a seismograph), there is plenty of time to warn the people of Hilo (which is very susceptible to tsunami damage) to take cover.

67. A rectangular channel is 2 m wide and contains water 3 m deep. If the slope is 0.85° and the lining is corrugated metal, estimate the discharge for uniform flow.

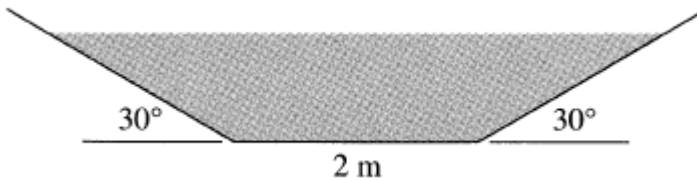


For corrugated metal, take Manning's $n \approx 0.022$. Get the hydraulic radius:

$$R_h = \frac{A}{P} = \frac{2(3)}{3 + 2 + 3} = 0.75 \text{ m}$$

$$Q \approx \frac{1}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1}{0.022} (6)(0.75)^{\frac{2}{3}} [\tan(0.85^\circ)]^{\frac{1}{2}} \approx 27 \text{ m}^3 / \text{s}$$

68. The trapezoidal channel of the figure is made of brickwork and slopes at 1:500. Determine the flow rate if the normal depth is 80 cm.



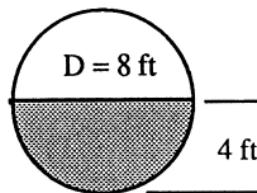
For brickwork, $n \approx 0.015$. Evaluate the hydraulic radius with $y = 0.8 \text{ m}$:

$$A = 2y + y^2 \cot\theta = 2(0.8) + (0.8)^2 \cot 30^\circ = 2.71 \text{ m}^2$$

$$P = 2 + 2(0.8) \csc 30^\circ = 5.2 \text{ m}, R_h = A/P = 2.71/5.2 \approx 0.521 \text{ m}$$

$$Q = \frac{1}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1}{0.015} (2.71)(0.521)^{\frac{2}{3}} \left(\frac{1}{500}\right)^{\frac{1}{2}} \approx 5.23 \text{ m}^3 / \text{s}$$

69. A circular corrugated-metal storm drain is flowing half-full over a slope of 4 ft/mile. Estimate the normal discharge if the drain diameter is 8 ft.



For corrugated metal, $n \approx 0.022$. Evaluate the hydraulic radius, etc.:

$$A = (\pi / 2)R^2 = 25.13 \text{ ft}^2 ; P = \pi R = 12.56 \text{ ft}, R = A/P = R/2 = 2 \text{ ft}$$

$$Q = \frac{1.486}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1.486}{0.022} (25.13)(2.0)^{2/3} \left(\frac{4}{5280} \right)^{\frac{1}{2}} \approx 74 \text{ ft}^3 / \text{s}$$

70. An engineer makes careful measurements with a weir which monitors a rectangular unfinished concrete channel laid on a slope of 1°. She finds, perhaps with surprise, that when the water depth doubles from 2 ft 2 inches to 4 ft 4 inches, the normal flow rate more than doubles, from 200 to 500 ft³/s. (a) Is this plausible? (b) If so, estimate the channel width.

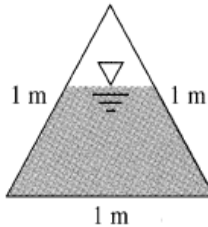
(a) Yes, Q always more than doubles for this situation where the depth doubles.

(b) For unfinished concrete, take $n = 0.014$.

$$Q = \frac{1.486}{n} AR_h^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{1.486}{0.014} (bh) \left(\frac{bh}{b+2h} \right)^{\frac{2}{3}} \sqrt{\sin 1^\circ} = 200 \text{ (or } 500) \text{ ft}^3 / \text{s if } h = 2.17 \text{ (or } 4.33) \text{ ft}$$

The two pieces of flow rate data give us two equations to solve for width b . It is unusual, but true, that both round-number flow rates converge to the same width $b = 5.72$ ft

71. A riveted-steel triangular duct flows partly full as in the figure. If the critical depth is 50 cm, compute (a) the critical flow rate; and (b) the critical slope.



For riveted steel, take $n \approx 0.015$

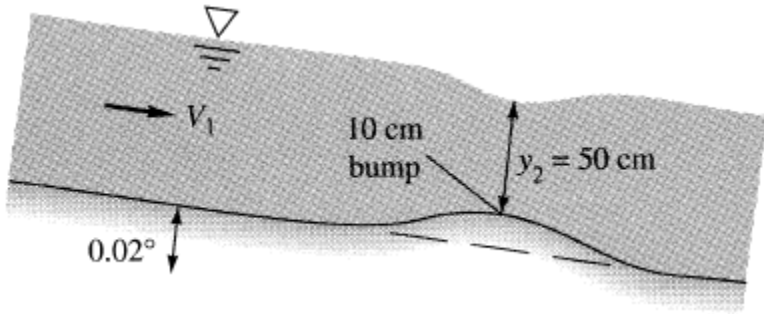
$$\text{If } y_c = 0.5 \text{ m, } b_0 = 0.423 \text{ m, } A_c = 0.356 \text{ m}^2 = \left[\frac{0.423 Q^2}{9.81} \right]^{\frac{1}{3}} \rightarrow Q \approx 1.02 \text{ m}^3 / \text{s}$$

$$P = 2.15 \text{ m}$$

$$R_h = 0.165 \text{ m}$$

$$S_c = \frac{n^2 g A_c}{\alpha^2 b_0 R_h^{\frac{4}{3}}} = \frac{(0.015)^2 (9.81)(0.356)}{(1)(0.423)(0.165)^{\frac{4}{3}}} \approx 0.0205$$

72. Uniform water flow in a wide brick channel of slope 0.02° moves over a 10-cm bump as in the figure. A slight depression in the water surface results. If the minimum depth over the bump is 50 cm, compute (a) the velocity over the bump; and (b) the flow rate per meter of width.



For brickwork, take $n \approx 0.015$. Since the water level decreases over the bump, the upstream flow is subcritical. For a wide channel, $R_h = y/2$, and

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{2g} = 0$$

$$q = V_1 y_1$$

$$E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h$$

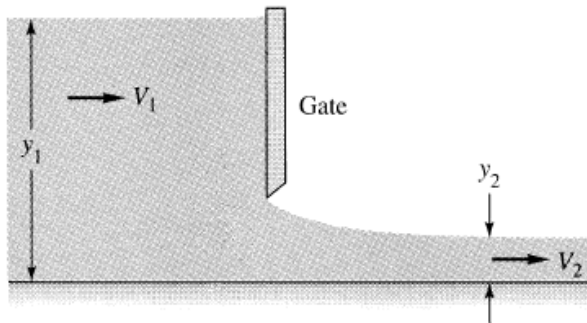
$$\Delta h = 0.1 \text{ m}$$

$$y_2 = 0.5 \text{ m}$$

Meanwhile, for uniform flow, $q = \frac{1}{0.015} y_1 \left(\frac{y_1}{2} \right)^{\frac{2}{3}} \sqrt{\sin 0.02^\circ} = 0.785 y_1^{\frac{5}{3}}$

Solve these two simultaneously for $y_1 = 0.608 \text{ m}$, $V_1 = 0.563 \text{ m/s}$ Ans. (a), and $q = 0.342 \text{ m}^3/\text{s.m}$

73. Given is the flow of a channel of large width b under a sluice gate, as in the figure. Assuming frictionless steady flow with negligible upstream kinetic energy, derive a formula for the dimensionless flow ratio $Q^2 / (y_1^3 b^2 g)$ as a function of the ratio y_2/y_1 . Show by differentiation that the maximum flow rate occurs at $y_2 = 2y_1/3$.



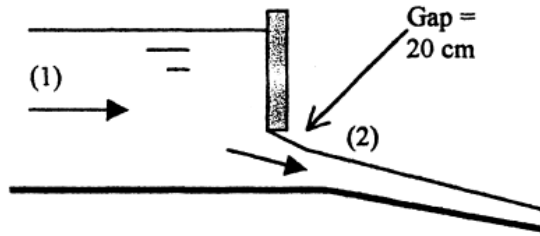
With upstream kinetic energy neglected, the energy equation becomes

$$y_1 \approx y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\left(\frac{Q}{by_2}\right)^2}{2g}; \text{ rearrange and multiply by } \left(\frac{y_2^2}{y_1^3}\right)$$

$$\frac{Q^2}{gb^2y_1^3} = 2\left(\frac{y_2}{y_1}\right)^2 - 2\left(\frac{y_2}{y_1}\right)^3$$

Differentiate this with respect to (y_2/y_1) to find maximum Q at $y_2/y_1 = 2/3$

74. Water approaches the wide sluice gate in the figure, at $V_1 = 0.2 \text{ m/s}$ and $y_1 = 1 \text{ m}$. Accounting for upstream kinetic energy, estimate, at outlet section 2, (a) depth; (b) velocity; and (c) Froude number.



(a) If we assume frictionless flow, the gap size is immaterial,

$$y_2^3 - \left(y_1 + \frac{V_1^2}{2g}\right)y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0 = y_2^3 - 1.00204y_2^2 + 0.00204$$

EES yields 3 solutions: $y_2 = 1.0 \text{ m}$ (trivial); -0.0442 m (impossible); and the correct solution: $y_2 = 0.0462 \text{ m}$

$$(b) V_2 = \frac{V_1 y_1}{y_2} = \frac{(1.0)(0.2)}{0.0462} = 4.33 \text{ m/s}$$

$$(c) Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{4.33}{\sqrt{9.81(0.0462)}} = 6.43$$

75. Water flows in a wide channel at $q = 25 \text{ ft}^3/\text{s}\cdot\text{ft}$ and $y_1 = 1 \text{ ft}$ and undergoes a hydraulic jump. Compute y_2 , V_2 , Fr_2 , h_r , the percentage dissipation, and the horsepower dissipated per unit width. What is the critical depth?

$$V_1 = \frac{q}{y_1} = \frac{5}{1} = 25 \text{ ft/s}$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{25}{\sqrt{32.2(1)}} = 4.41$$

$$E_1 = y_1 + \frac{V_1^2}{2g} \approx 10.7 \text{ ft}$$

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8(4.41)^2} - 1 \right] = 5.75 \rightarrow y_2 \approx 5.75 \text{ ft}$$

$$V_2 = q / y_2 = 25 / 5.75 \approx 4.35 \text{ ft/s}$$

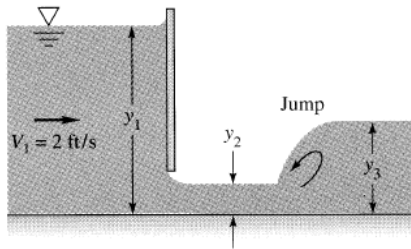
$$Fr_2 = \frac{4.35}{\sqrt{32.2(5.75)}} \approx 0.32$$

$$h_f = (5.75 - 1)^3 / [4(5.75)(1)] \approx 4.66 \text{ ft, } \% \text{ dissipated} = 4.66 / 10.7 \approx 44\%$$

$$\text{Power dissipated } gqh_f (62.4) (25)(4.66) \div 550 \approx 13.2 \text{ hp/ft}$$

$$\text{Critical depth } y_c = (q^2 / g)^{1/3} = [(25)^2 / 32.2]^{1/3} \approx 2.69 \text{ ft}$$

76. Consider the flow under the sluice gate of the figure. If $y_1 = 10$ ft and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percentage of dissipation, and sketch the flow to scale with the EGL included. The channel is horizontal and wide.



First get the conditions at “2” by assuming a frictionless acceleration

$$E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{(2)^2}{2(32.2)} = 10.062 \text{ ft} = E_2 = y_2 + \frac{V_2^2}{2g}$$

$$V_1 y_1 = V_2 y_2 = 20$$

$$V_2 \approx 24.4 \text{ ft/s}$$

$$y_2 \approx 0.820 \text{ ft}$$

$$Fr_2 = \frac{24.4}{\sqrt{32.2(0.820)}} \approx 4.75$$

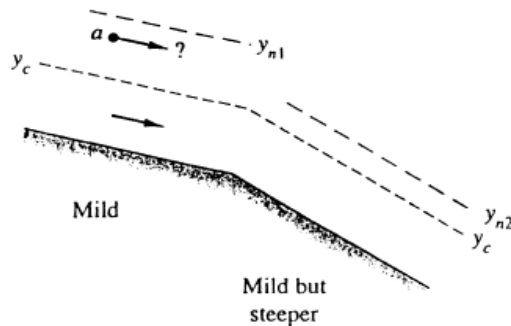
$$\text{Jump} : \frac{y_3}{y_2} = \frac{1}{2} \left[\sqrt{1 + 8Fr_2} - 1 \right] \approx 6.23$$

$$y_3 \approx 5.11 \text{ ft}$$

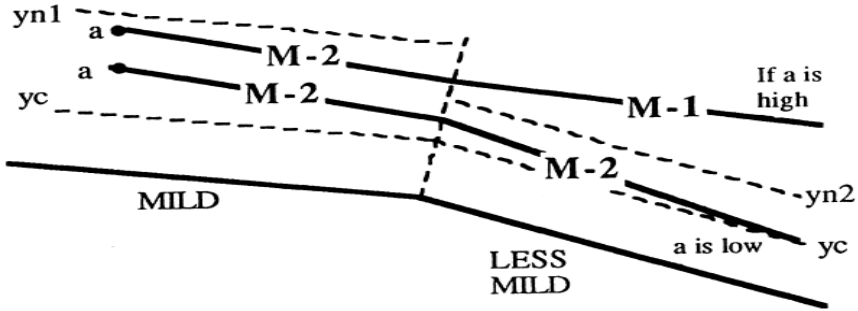
$$E_2 = 10.062 \text{ ft}; h_f = \frac{(y_3 - y_2)^3}{4y_2 y_3} = \frac{(5.11 - 0.82)^3}{4(0.82)(5.11)} \approx 4.71 \text{ ft}$$

$$\text{Dissipation} = \frac{4.71}{10.06} \approx 47\%$$

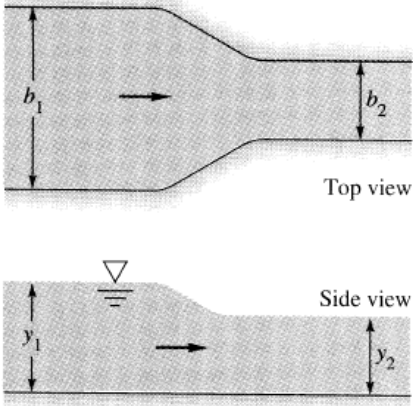
77. Consider the gradual change from the profile beginning at point a in the figure on a mild slope S_{o1} to a mild but steeper slope S_{o2} downstream. Sketch and label the gradually-varied solution curve(s) $y(x)$ expected.



There are two possible profiles, depending upon whether or not the initial M-2 profile slips below the new normal depth y_{n2} . These are shown on the next page:



78. The figure shows a channel contraction section often called a venturimeter, because measurements of y_1 and y_2 can be used to meter the flow rate. Given $b_1 = 3$ m, $b_2 = 2$ m, and $y_1 = 1.9$ m. Find the flow rate (a) if $y_2 = 1.5$ m; and (b) find the depth y_2 for which the flow becomes critical in the throat.



Given the water depths, continuity and energy allow us to eliminate one velocity:

$$Q = V_1 y_1 b_1 = V_2 y_2 b_2; \text{ Energy: } y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

Eliminate V_1 to obtain $V_2 = [2g(y_1 - y_2) / (1 - \alpha^2)]^{1/2}$ where $\alpha = (y_2 b_2) / (y_1 b_1)$

$$Q = V_2 y_2 b_2 = \sqrt{\frac{2g(y_1 - y_2)}{\frac{1}{b_2^2 y_2^2} - \frac{1}{b_1^2 y_1^2}}}$$

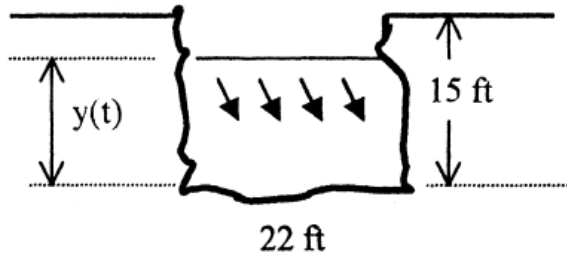
Evaluate the solution we just found:

$$Q = \sqrt{\left[\frac{2(9.81)(1.9-1.5)}{\frac{1}{(2)^2(1.5)^2} - \frac{1}{(3)^2(1.9)^2}} \right]} \approx 9.88 \text{ m}^3 / \text{s}$$

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}} \approx 0.86$$

(b) To find critical flow, keep reducing y_2 until $Fr_2 = 1.0$. This converges to $y_2 \approx 1.372$ m. [for which $Q = 10.1 \text{ m}^3/\text{s}$]

79. February 1998 saw the failure of the earthen dam impounding California Jim’s Pond in southern Rhode Island. The resulting flood raised temporary havoc in the nearby village of Peace Dale. The pond is 17 acres in area and 15 ft deep and was full from heavy rains. The breach in the dam was 22 ft wide and 15 ft deep. Estimate the time required to drain the pond to a depth of 2 ft.



$$\frac{d}{dt} \left(\int dv_{pond} \right) + Q_{out} = 0$$

$$A_{pond} \frac{dy}{dt} = -Q_{out} = -0.581(b - 0.1y) g^{\frac{1}{2}} y^{\frac{3}{2}}$$

$$b = 22 \text{ ft}$$

$$A_{pond} = 17 \text{ acres} = 740520 \text{ ft}^2$$

If we neglect the “edge contraction” term “ $-0.1y$ ” compared to $b = 22$ ft, this first-order differential equation has the solvable form

$$\frac{dy}{dt} \approx -Cy^{\frac{3}{2}}$$

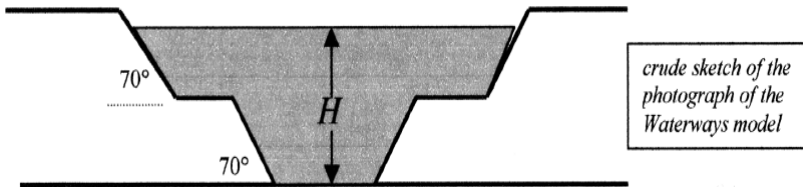
$$C = \frac{0.581(22 \text{ ft})(32.2)^{\frac{1}{2}}}{740520} \approx 9.8E-5 \text{ ft}^{-\frac{1}{2}} \text{ sec}^{-1}$$

Separate and integrate: $\int_{15 \text{ ft}}^{2 \text{ ft}} \frac{dy}{y^{\frac{3}{2}}} = -C \int_0^t dt \rightarrow \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{5}} = Ct$

$$t_{\text{drain-to-2 ft}} \approx \frac{1.414 - 0.516}{9.8E-5} = 9160 \text{ s} = 2.55 \text{ h}$$

If we used a spreadsheet and kept the term “-0.1y”, we would predict a time-to-drain-to-2 ft or about 2.61 hours. The theory is too crude to distinguish between these estimates.

80. The figure shows a hydraulic model of a compound weir, that is, one which combines two different shapes. (a) Other than measurement, for which it might be useful, what could be the engineering reason for such a weir? (b) For the prototype river, assume that both sections have sides at a 70° angle to the vertical, with the bottom section having a base width of 2 m and the upper section having a base width of 4.5 m, including the cut-out portion. The heights of lower and upper horizontal sections are 1 m and 2 m, respectively. Use engineering estimates and make a plot of upstream water depth versus Petaluma River flow rate in the range 0 to 4 m³/s. (c) For what river flow rate will the water overflow the top of the dam?



We have no formulas in the text for a compound weir shape, but we can still use the concept of weir flow and estimate the discharge for various water depths. (a) A good reason for using a narrow bottom portion of the weir is to maintain a reasonable upstream depth at low flow, and then widen to maintain a lower depth at high flow. It also allows a more accurate flow measurement during low flow.

(b) Rather than derive a whole new theory for compound weirs, we will assume that the bottom portion is more or less rectangular, based on average width b , with the top portion also assumed rectangular with its flow rate added onto the lower portion. For example, if $H = 1 \text{ m}$ (the top of the lower portion),

$$b_{avg} = \frac{2.728 + 2.0 \text{ m}}{2} = 2.364 \text{ m}$$

$$Q \approx 0.58(b_{avg} - 0.1H) g^{\frac{1}{2}} H^{\frac{3}{2}} = 0.58[2.364 \text{ m} - 0.1(1 \text{ m})] \left(9.81 \frac{\text{m}}{\text{s}^2}\right)^{\frac{1}{2}} (1 \text{ m})^{\frac{3}{2}} \approx 4.1 \text{ m}^3 / \text{s}$$

Then if $H = 2 \text{ m} > 1 \text{ m}$, we figure Q_{upper} the same way and add on the lower portion flow. Again take $H = 1 \text{ m}$, that is, the height of the flow above the lower part of the weir:

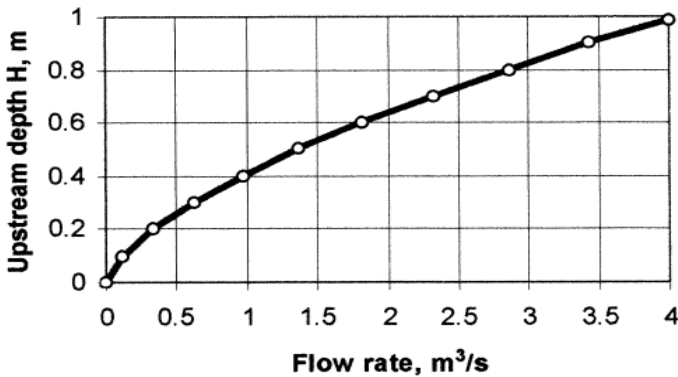
$$b_{avg} = \frac{4.5 + 5.23 \text{ m}}{2} = 4.87 \text{ m}$$

$$Q_{upper} \approx 0.58(b_{avg} - 0.1H) g^{\frac{1}{2}} H^{\frac{3}{2}} = 0.58[4.87 \text{ m} - 0.1(1 \text{ m})] \left(9.81 \frac{\text{m}}{\text{s}^2}\right)^{\frac{1}{2}} (1 \text{ m})^{\frac{3}{2}} \approx 8.7 \text{ m}^3 / \text{s}$$

$$Q_{total} = Q_{lower} + Q_{upper} = 4.1 + 8.7 \approx 12.8 \text{ m}^3 / \text{s}$$

Flow rates greater than this value of $12.8 \text{ m}^3 / \text{s}$ will overflow the top of the weir.

A plot of Q versus H for the range $0 < Q < 4 \text{ m}^3 / \text{s}$ is shown below.



81. A rectangular channel 2.5 m wide carries water at a depth of 1.2 m. The bed slope of the channel is 0.0036. Calculate the average shear stress on the boundary.

$$A = By = 2.5 \times 1.2 = 3.00 \text{ m}^2$$

$$P = B + 2y = 2.5 + (2 \times 1.2) = 4.9 \text{ m}$$

$$R = \frac{A}{P} = \frac{3.00}{4.90} = 0.612 \text{ m}$$

$$\tau_0 = \gamma R S_0 = (998 \times 9.81) \times 0.612 \times 0.0036 = 21.58 \text{ Pa}$$

82. On what slope should one construct a 3 m wide rectangular channel ($n=0.014$) so that critical flow will occur at a normal depth of 1.2 m?

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$y_c = 1.2 \text{ m}$$

$$A_c = 3 \times 1.2 = 3.6 \text{ m}^2$$

$$T_c = 3.0 \text{ m}$$

$$Q^2 = g \left(\frac{A_c^3}{T_c} \right) = \frac{9.81 \times (3.6)^3}{3.0} = 151.565$$

$$Q = 12.35 \text{ m}^3 / \text{s}$$

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

$$A = 3.6 \text{ m}^2$$

$$P = (3.0 + 2 \times 1.2) = 5.4 \text{ m}$$

$$R = \frac{3.6}{5.4} = 0.6667 \text{ m}$$

$$12.35 = \frac{1}{0.014} \times (3.6) (0.6667)^{\frac{2}{3}} S_0^{\frac{1}{2}} = 196.236 S_0^{\frac{1}{2}}$$

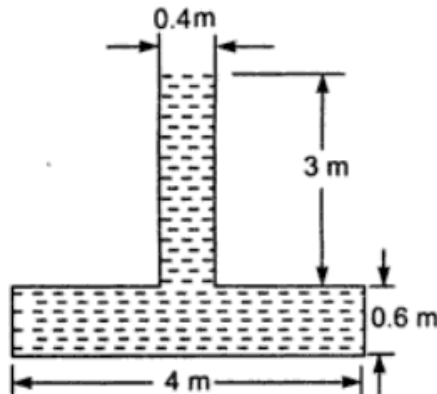
$$S_0 = 3.96E - 3$$

83. The figure shows a tank full of water. Find:

(i) Total pressure on the bottom of tank.

(ii) Weight of water in the tank.

(iii) Hydrostatic paradox between the results of (i) and (ii). Width of tank is 2 m.



$$h_t = 3 + 0.6 = 3.6 \text{ m}$$

$$\text{Width of tank} = 2 \text{ m}$$

$$\text{Length of tank at bottom} = 4 \text{ m}$$

$$A = 4 \times 2 = 8 \text{ m}^2$$

(i) Total pressure F , on the bottom is

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 8 \times 3.6 = 282528 \text{ N}$$

$$\begin{aligned} \text{(ii) Weight of water in tank} &= \rho g \times \text{Volume of tank} = 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times 0.6 \times 2] \\ &= 1000 \times 9.81 [2.4 + 4.8] = 70632 \text{ N} \end{aligned}$$

(iii) From the results of (i) and (ii), it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

84. Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 liters/s.

Diameter of smaller pipe, $D_1 = 200 \text{ mm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.2)^2 = 0.03141 \text{ m}^2$$

Diameter of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$$A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$$

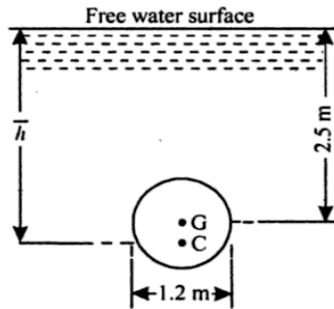
$$Q = 250 \text{ liters / s} = 0.25 \text{ m}^3 / \text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m / s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 \text{ m / s}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = 1.816 \text{ m of water}$$

85. The figure shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the centre of the plate is 2.5 m below the free surface of water. Determine: (i) Total pressure on the plate. (ii) Position of centre of pressure.



$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 1.2^2 = 1.13 \text{ m}^2$$

$$\bar{x} = 2.5 \text{ m}$$

(i) Total pressure, P :

$$P = wA\bar{x} = 9.81 \times 1.13 \times 2.5 = 27.7 \text{ kN}$$

(ii) Position of centre of pressure, \bar{h} :

$$\bar{h} = \frac{I_G}{Ax} + \bar{x}$$

$$I_G = \frac{\pi}{64}d^4 = \frac{\pi}{64} \times 1.2^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$$

86. A horizontal boiler drum 6 m long and 3 m in diameter is provided with an orifice 100 mm in diameter at its bottom. It contains water upto a height of 2.4 m. Calculate the time taken to empty the drum. Take discharge co-efficient, $C_d = 0.6$.

$$L = 6 \text{ m,}$$

$$D = 3 \text{ m,}$$

$$R = 1.5 \text{ m,}$$

$$d = 100 \text{ mm} = 0.1 \text{ m}$$

$$A = \frac{\pi}{4} \times 0.1^2 = 7.854E-3 \text{ m}^2$$

$$H_1 = 2.4 \text{ m}$$

$$H_2 = 0$$

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{\frac{3}{2}} - (2R - H_1)^{\frac{3}{2}} \right]$$

$$T = \frac{4 \times 6}{3 \times 0.6 \times 7.854E-3 \times \sqrt{2 \times 9.81}} \left[(2 \times 1.5)^{\frac{3}{2}} - (2 \times 1.5 - 2.4)^{\frac{3}{2}} \right] = 30 \text{ min, } 13.6 \text{ sec}$$

87. A Sutro weir has a rectangular base of 30-cm width and 6-cm height. The depth of water in the channel is 12 cm. assuming the coefficient of discharge of the weir as 0.62, determine the discharge through the weir. What would be the depth of flow in the channel when the discharge is doubled? (Assume the crest of the base weir to coincide with the bed of the channel).

$$a = 0.06 \text{ m}$$

$$W = 0.30 / 2 = 0.15 \text{ m}$$

$$H = 0.12 \text{ m}$$

$$K = 2C_d\sqrt{2g} = 2 \times 0.62 \times \sqrt{2 \times 9.81} = 5.4925$$

$$b = WKa^{\frac{1}{2}} = 0.15 \times 5.4925 \times (0.06)^{\frac{1}{2}} = 0.2018$$

$$Q = b\left(H - \frac{a}{3}\right) = 0.2018\left(0.12 - \frac{0.06}{3}\right) = 0.02018 \text{ m}^3 / \text{s}$$

$$\text{When the discharge is doubled, } Q = 2 \times 0.02018 = 0.04036 \text{ m}^3 / \text{s}$$

$$0.04036 = 0.2018\left(H - \frac{0.06}{3}\right)$$

$$H = 0.2 + 0.02 = 0.22 \text{ m}$$

88). Design a Sutro weir for use in a 0.30-m wide rectangular channel to have linear discharge relationship in the discharge range from 0.25 m to 0.60 m³/s. The base of the weir will have to span the full width of the channel. Assume C_d = 0.62.

$$\text{Here } 2W = 0.30 \text{ m and } C_d = 0.62$$

$$K = 2C_d\sqrt{2g} = 2 \times 0.62 \times \sqrt{2 \times 9.81} = 5.49$$

$$Q_{\min} = \frac{2}{3}WKa^{\frac{3}{2}} = 0.25$$

$$\frac{2}{3} \times 0.15 \times 5.49 \times a^{\frac{3}{2}} = 0.25 \rightarrow a = 0.592 \text{ m}$$

$$b = WK\sqrt{a} = 0.15 \times 5.49 \times \sqrt{0.592} = 0.6337$$

$$Q = bH_d$$

$$\text{For } Q = 0.60 \text{ m}^3 / \text{s}, H_d = \frac{Q}{b} = \frac{0.60}{0.6337} = 0.9468 \text{ m} = H - \frac{a}{3} = h + \frac{2}{3}a$$

$$H = 1.1444 \text{ m and } h = 0.0552 \text{ m}$$

$$y = f(x) = 0.15 \left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{x}{0.592}} \right]$$

89. A quadratic weir is designed for installation in a rectangular channel of 30-cm width. The rectangular base of the weir occupies the full width of the channel and is 6 cm in height. The crest of the base weir coincides with the channel bed. (a) Determine the discharge through the weir when the depth of flow in the channel is 15 cm. (b) What would be the depth of flow upstream of the weir when the discharge in the channel is 25 liters/s? Assume $C_d = 0.62$.

$$a = 0.06 \text{ m}$$

$$W = 0.30 / 2 = 0.15 \text{ m}$$

$$H = 0.15 \text{ m}$$

$$K = 2C_d \sqrt{2g} = 2 \times 0.62 \times \sqrt{2 \times 9.81} = 5.4925$$

$$b = \frac{2}{\sqrt{3}} W K a = \frac{2}{\sqrt{3}} \times 0.15 \times 5.4925 \times 0.06 = 0.05708$$

$$Q = b \sqrt{\left(H - \frac{2a}{3}\right)} = 0.05708 \times \sqrt{\left(0.15 - \frac{2 \times 0.06}{3}\right)} = 0.0189 \text{ m}^3 / \text{s}$$

(b) When the discharge $Q = 25 \text{ liters} / \text{s} = 0.025 \text{ m}^3 / \text{s}$

$$0.025 = 0.05708 \times \sqrt{\left(H - \frac{2 \times 0.06}{3}\right)}$$

$$H = 0.1918 + 0.04 = 0.2318 \text{ m}$$

90. A channel has its area given by $A = k y^3$ where $k = \text{a constant}$. For subcritical flow in this channel estimate the ratio of the end-depth to critical depth.

$$a = 3$$

$$\varepsilon = 1 + \frac{1}{2a} = 1.167$$

$$f(\eta) = \frac{1}{a \eta^{2a}} = \frac{1}{3 \eta^6}$$

$$6\varepsilon - 4\eta - 3f(\eta) = 0$$

$$f(\eta), (6 \times 1.167) - 4\eta - \frac{3}{3 \eta^6} = 0$$

$$\frac{1}{\eta^6} + 4\eta - 7 = 0$$

$$\text{Solving by trial and error } \eta = \frac{y_e}{y_c} = 0.8$$

91. A rectangular channel carries a supercritical flow with a Froude number of 2.0. Find the end-depth ratio at a free overfall in this channel.

$$\varepsilon = f_n(a, F_0)$$

$$a = 1$$

$$\varepsilon = \frac{1}{F_0^{2a+1}} \left(1 + \frac{F_0^2}{2a} \right) = \frac{1}{2^{2 \times 1 + 1}} \left(1 + \frac{2^2}{2 \times 1} \right) = 1.89$$

$$f(\eta) - \frac{1}{a\eta^{2a}} = \frac{1}{\eta^2}$$

$$6\varepsilon - 4\eta - f(\eta) = 0$$

$$f(\eta), 6 \times 1.89 - 4\eta - \frac{3}{\eta^2}$$

$$4\eta^3 - 11.34 + 3 = 0$$

$$\eta = \frac{y_e}{y_c} = 0.577$$

92. A wide rectangle channel in alluvium of 3-mm median size (Relative density=2.65) has a longitudinal slope of 0.0003. Estimate the depth of flow in this channel which will cause incipient motion.

$$d_{mm} = 3$$

$$\tau_c = 0.155 + \frac{0.409d_{mm}^2}{\sqrt{1 + 0.177d_{mm}^2}}$$

$$\tau_c = 0.155 + \frac{0.409(3)^2}{\sqrt{1 + 0.177(3)^2}} = 2.44 \text{ Pa}$$

For flow in a wide rectangular channel at depth D , $\tau_0 = \gamma D S_0$ and at incipient motion $\tau_0 = \tau_c$.

$$\text{Hence, } 9790 \times D \times 0.0003 = 2.44$$

$$D = 0.831 \text{ m}$$

93). Estimate the minimum size of gravel that will not move in the bed of a trapezoidal channel of base width = 3 m, side slope = 1.5 H:IV, longitudinal slope = 0.004 and having a depth of flow of 1.3 m.

$$R = \frac{(3 + 1.5 \times 1.3)}{3 + 2 \times 1.3 \times \sqrt{(1.5)^2 + 1}} = 0.837 \text{ m}$$

$$d_c = 11RS_0 = 11 \times 0.837 \times 0.004 = 0.0368 \text{ m}$$

$$d_c = 3.7 \text{ cm}$$

94. A regime lacey channel having a full supply discharge of 30 m³/s has a bed material of 0.12-mm median size. What would be the Manning's roughness coefficient n for this channel?

$$f_s = 1.76\sqrt{d_{mm}} = 1.76\sqrt{0.12} = 0.61$$

$$S_0 = \frac{0.0003 f_s^{\frac{5}{3}}}{Q^{\frac{1}{6}}} = \frac{0.0003 (0.61)^{\frac{5}{3}}}{(30)^{\frac{1}{6}}} = 7.46E-5$$

$$n = \left(\frac{S_0^{\frac{1}{6}}}{10.8} \right) = \frac{(7.46E-5)^{\frac{1}{6}}}{10.8} = 0.019$$

95. Estimate the maximum depth of scour for design for the following data pertaining to a bridge.

Design discharge = 15000 m³/s

Effective Water way = 550 m

Median size of bed material = 0.1 mm

$$P = 4.75\sqrt{Q} = 4.75\sqrt{15000} = 581.8 \text{ m}$$

Since this is greater than $W_e = 550 \text{ m}$,

$$f_s = 1.76\sqrt{d_{mm}} = 1.76\sqrt{0.1} = 0.556$$

$$q = 15000 / 550 = 27.27 \text{ m}^3 / \text{m.s}$$

$$D_{Lq} = 1.34 \left[\frac{(27.27)^2}{0.556} \right]^{\frac{1}{3}} = 14.76 \text{ m below HFL}$$

$$D_s = 2D_{Lq} = 2 \times 14.76 = 29.52 \text{ m below HFL}$$

96). Water flows from a reservoir through a 60 cm diameter cast-iron pipe. A valve located on the pipe 3 km from the reservoir end is suddenly closed. Calculate the time elapsed before the action of valve-closing is felt at the reservoir-end. The pipe thickness is 50 mm, Young's modulus for CI is 100E9 pascals, and the bulk modulus of water is 22E8 pascals.

$$\frac{1}{\kappa} = \frac{1}{\beta} + \frac{D}{t_1 E} = \frac{1E-9}{2.2} + \frac{0.6E-9}{50E-3 \times 100} = 10E-9(0.4545 + 0.12)$$

$$\kappa = 1.74E9 \text{ N / m}^2$$

$$\kappa = \rho a^2$$

$$a = \sqrt{\frac{\kappa}{\rho}} = \sqrt{1.74} \times 1000 = 1319.3 \text{ m / s}$$

The action of closing the valve is propagated along the pipe at the acoustic speed $a = 1319.3 \text{ m / s}$.

Since the valve is at a distance of 3 km from the reservoir, the action will be felt there after a time

$$\text{lapse of } \tau = \frac{3000}{1319.3} = 2.274 \text{ s.}$$

97. While measuring the discharge in a river with unsteady flow, the depth y was found to increase at a rate of 0.06 m/hour. The surface width of the river is 30 m and discharge at this section is 35 m³/sec. Estimate the discharge at section 1 km upstream.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

$$\partial A = T dy$$

$$\frac{\partial Q}{\partial x} + T \frac{\partial Y}{\partial t} = 0$$

$$\frac{Q_2 - Q_1}{\partial x} = -T \frac{\partial Y}{\partial t}$$

$$T \frac{\partial Y}{\partial t} = 30 \times \frac{0.6}{60 \times 60} = \frac{0.06}{120} \text{ m}^2 / \text{sec}$$

$$Q_1 = Q_2 + \frac{T \partial y}{\partial t} \cdot \partial x = 35 + \frac{0.06}{120} (1 \times 1000)$$

$$\partial x = 1 \text{ km} = 1000 \text{ m}$$

$$Q_2 = 35 \text{ m}^3 / \text{s}$$

$$Q_1 = 35.5 \text{ m}^3 / \text{s}$$

98). A standard Parshall flume has a throat width of $W_t = 4 \text{ ft}$. Determine the free flow discharge corresponding to $h_o = 2.4 \text{ ft}$.

$$L = 4 \text{ ft}$$

$$Y_0 = \frac{h_0}{W_T} = \frac{2.4}{4} = 0.6$$

$$X_0 = \frac{L}{W_T} = \frac{4}{4} = 1$$

$$Q_0 = \frac{Y_0^{1.5504}}{1.3096X_0^{0.0766}} = \frac{(0.6)^{1.5504}}{1.3096(1)^{0.0766}} = 0.3459$$

$$Q_f = Q_0 W_T^{\frac{5}{2}} \sqrt{g} = (0.3459)(4)^{\frac{5}{2}} \sqrt{32.2} = 62.8 \text{ cfs}$$

99. A reinforced concrete rectangular box culvert has the following properties:

$$D = 1 \text{ m}, \quad b = 1 \text{ m}, \quad L = 40 \text{ m}, \quad n = 0.012, \quad S = 0.002$$

The inlet is square-edged on three edges and has a headwall parallel to the embankment, and the outlet is submerged with TW=1.3 m. Determine the headwater depth, HW, when the culvert is flowing full at Q = 3 m³/s.

$k_e = 0.5$. Also, for a box culvert, $A = bD = (1)(1) = 1 \text{ m}^2$ and $R = bD / (2b + 2D) = (1)(1) / [2(1) + 2(1)] = 0.25 \text{ m}$ under full-flow conditions.

$$HW = 1.3 - (0.002)(40) + \left[1 + 0.5 + \frac{2(9.81)(0.012)^2(40)}{(1)^2(0.25)^{\frac{4}{3}}} \right] \frac{(3)^2}{2(9.81)(1)^2} = 2.24 \text{ m}$$

100. An uncontrolled overflow ogee for a spillway is to be designed that will discharge 3000 cfs at a design head of 5 ft. The upstream face of the crest is vertical. A bridge is to be provided over the crest, with bridge spans not to exceed 20 ft. The piers are 1.5 ft wide with rounded noses. The abutments are rounded with a headwall at 90° to the direction of flow. The vertical distance between the spillway crest and the floor of the reservoir is 9 ft. Determine the length of the spillway crest.

From the problem statement, we know that $Q = 3000 \text{ cfs}$, $P = 9 \text{ ft}$, $H_0 = 5 \text{ ft}$, $K_p = 0.01$, and $K_a = 0.1$. With $P/H_0 = 9/5 = 1.8$, we obtain $k_{w0} = 0.49$.

$$L_e = \frac{Q}{k_w \sqrt{2gH_0^{\frac{3}{2}}}} = \frac{3000}{(0.49)\sqrt{2(32.2)(5)^{\frac{3}{2}}}} = 68.2 \text{ ft}$$

We will need three bridge piers, since the bridge spans are not to exceed 20 ft.

$$L = L_e + 2(NK_p + K_a)H_0 = 68.2 + 2[3(0.01) + 0.1](5) = 69.5 \text{ ft}$$

Thus, the net crest length, not including the piers, is 69.5 ft. Noting that each of the three piers is 1.5 ft wide, the total crest length will be $69.5 + 3(1.5) = 74$ ft.

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