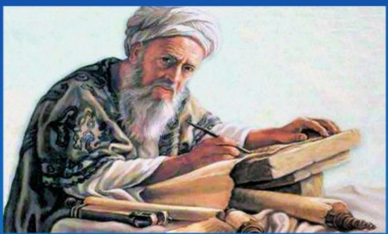
" Your Best Study Experience begins here."

SEVEN MUSLIMS NOTES

PHYSICS 11



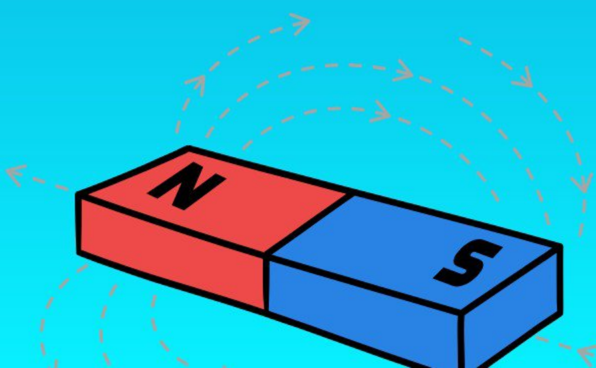
Al-Biruni (973–1048) calculated the Earth's radius and worked on the physics of planetary motion.

Best Regards to

Sir Muhammad Ali

(Physics Lecturer KIPS College)

$E=mc^2$



CHAPTER 5

CIRCULAR MOTION

Q.1: Define circular motion.

CIRCULAR MOTION

"Motion of a body moving along a circular path is called circular motion."

For example:

i. Motion of earth around the sun.
ii. Motion of moon around the earth.

Q.2: What is angular displacement. State right hand rule.

ANGULAR DISPLACEMENT

"The angle that specifies position of a particle and direction of motion while moving in a circular path is called angular motion."

Unit

In system international angular displacement is measured in radian.

ONE RADIAN

"Angle subtended at the center of a circle by an arc whose length is equal to the radius of that circle is called one radian."

DIRECTION OF ANGULAR DISPLACEMENT

Direction of angular displacement is always along the axis of rotation it is given by right hand rule.

RIGHT HAND RULE

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation then thumb points in the direction of angular displacement.

Q.3: Show that S = rθ

Consider an arc of length " S " of a circle of radius " r " which subtends an angle " θ " at the centre of the circle. Its value in radian is given by

$$\theta = \frac{\text{Arc length}}{\text{Radius}} \text{ (rad)}$$

$$\theta = \frac{S}{r}$$

$$S = r\theta$$

Q.4: Prove that 1 radian = 57.3°

Let a body move from point " A " to point " B " by covering a distance " L " on the boundary of a circle of radius " r " then

$$\theta = \frac{l}{r}$$

For one compete rotation

$$360° = \frac{2\pi r}{r}$$

$$360° = 2\pi \text{ radian}$$

$$2\pi \text{ radian} = 360°$$

$$1 \text{ radian} = \frac{360°}{2\pi}$$

$$= \frac{360°}{(2)(3.1428)}$$

$$1 \text{ radian} = 57.2°$$

Q.5: Define angular velocity and instantaneous angular velocity.

ANGULAR VELOCITY

"Time rate of change of angular displacement is called angular velocity."

Direction:

Direction of angular velocity is along the axis of rotation it can be found by using right hand rule.

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

Unit

Its unit is rads^{-1}. It is also measured in revolution per minute.

INSTANTANEOUS ANGULAR VELOCITY

"Angular velocity of a body at any instant of time is called instantaneous angular velocity."

$$W_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Q.6: Define angular acceleration and instantaneous angular acceleration.

ANGULAR ACCELERATION

"Time rate of change of angular velocity is called angular acceleration."

Direction:

Direction of angular acceleration is always along the axis of rotation.

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

Unit

The SI unit of angular acceleration are rads^{-2}.

INSTANTANEOUS ANGULAR ACCELERATION

"Angular acceleration of a body at any instant of time is called instantaneous angular acceleration."

Q.7: Prove that v = rω

For a body moving in a circle of radius " r " the arc length " Δs " is related to angular displacement

$$\Delta s = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

For instantaneous velocity

$$\lim_{\Delta t \to 0}\frac{\Delta S}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t}$$

As

$$\lim_{\Delta t \to 0}\frac{\Delta S}{\Delta t} = v \quad \text{and} \quad \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \omega$$

Therefore,

$$v = r\omega$$

Q.8: Prove that a = rα

The change in the velocity of a body moving in a circle of radius " r " is given as

$$\Delta v = r\Delta\omega$$

Dividing both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t}$$

For instantaneous values

$$\lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t}$$

As

$$\lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = a \quad \text{and} \quad \lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t} = \alpha$$

Therefore,

$$a = r\alpha$$

Q.9: Explain centripetal force in detail.

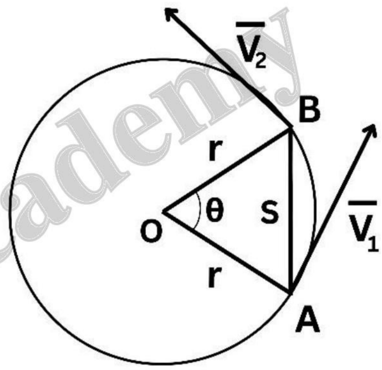
CENTRIPETAL FORCE:

"The force needed to bend normally straight path of a particle into a circular path is called centripetal force."

Formula:

$$F_c = \frac{mv^2}{r}$$

Explanation:



Consider a particle moving in a circle from point " A " to point " B " with constant speed " v ". During this motion magnitude of velocity remains the same but its direction continue to change at every point on the circle as shown in diagram.

The acceleration produced in the particle is then given as

$$a = \frac{\Delta v}{\Delta t} \quad \ldots\ldots\ldots\ldots (1)$$

If s is the distance covered by a body in time Δt then

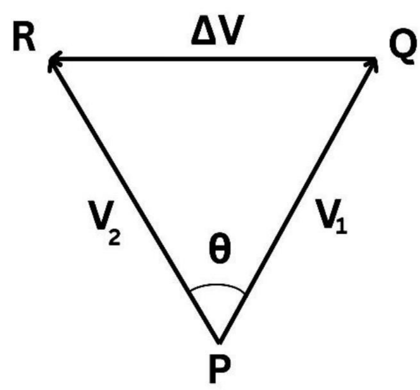
$$S = v\Delta t$$

$$\Delta t = \frac{S}{v}$$

Use in equation 1

$$a = \frac{\Delta v}{\frac{S}{v}}$$

$$a = v\frac{\Delta v}{S} \quad \ldots\ldots\ldots\ldots (2)$$



In order to calculate Δv, let us see a triangle PQR such that PQ is parallel to v_1 and PR is parallel to v_2 it can be noted that ∠PQR = ∠θ then in two isosceles triangle AOB and triangle PQR

$$\frac{\Delta v}{v} = \frac{AB}{r} = \frac{S}{r}$$

$$\Delta v = \frac{S}{r}v$$

Use in equation (2)

$$a = v\frac{\frac{S}{r}v}{s}$$

$$a_c = \frac{v^2}{r}$$

Centripetal acceleration

"The instantaneous acceleration of a particle moving in a circular path which is always directed towards the center of the circle is called centripetal acceleration."

By newton's second law

$$F_c = ma_c$$

Or

$$F_c = \frac{mv^2}{r}$$

Also

$$v = r\omega$$

$$F_c = \frac{m}{r}(r\omega)^2$$

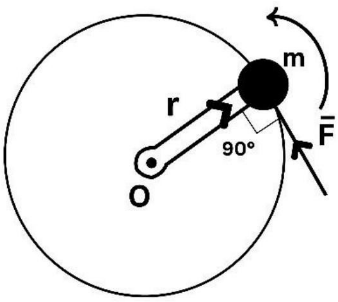
$$F_c = \frac{m}{r}(r^2\omega^2)$$

$$F_c = mr\omega^2$$

Q.10: Define moment of inertia . Give moment of inertia for different bodies.

MOMENT OF INERTIA

"The product of mass of a particle moving in a circular path and square of its radius is called moment of inertia."



Formula

$I = mr^2$

SI unit

Its SI unit is Kgm^2.

Quantity

It is a scalar quantity

The formula for torque in angular motion is

$$\tau = mr^2\alpha$$

$$\tau = I\alpha \quad (I = mr^2)$$

MOMENT OF INERTIA FOR DIFFERENT BODIES

For thin rod:

$$I = \frac{1}{12} mL^2$$

For thin ring or hoop:

$$I = mr^2$$

For solid disc or cylinder

$$I = \frac{1}{2} mr^2$$

For sphere

$$I = \frac{2}{5} mr^2$$

Q.11: Define angular momentum, what is its direction? Give its types.

ANGULAR MOMENTUM

"Quantity of angular motion possessed by a body is called its angular momentum."

Formula

$$\bar{L} = \bar{r} \times \bar{p}$$

Unit

Its unit is Js = kgm^2s^{-1}

Direction

The direction of angular momentum is always along the axis of rotation and can be found by right hand rule of cross product.

TYPES OF ANGULAR MOMENTUM

ORBITAL ANGULAR MOMENTUM

"The angular momentum possessed by a body due to its orbital motion is called orbital angular momentum."

SPIN ANGULAR MOMENTUM

"The angular momentum possessed by a body due to its spin motion is called spin angular momentum."

Q.12: Prove that Lo = mvr = Iω

For particle " p " moving in a circle of radius " r ", its orbital angular momentum is

$$L_o = rp\sin 90°$$

$$= rp \qquad (\text{As } \sin 90 = 1)$$

$= r(mv)$ (As $p = mv$)

$= mvr$

$= m(r\omega)r$ (As $v = r\omega$)

$L_o = mr^2\omega$

$L_o = I\omega$ ($I = mr^2$)

Q.13: What is law of conservation of angular momentum?

LAW OF CONSERVATION OF MOMENTUM

According to law of conservation of angular momentum

"The total angular momentum of a system remains constant if no external torque acts it."

In terms of axis of rotation

The axis of rotation will not change its orientation until an external torque causes it to do so.

Q.14: Define rotational kinetic energy?

ROTATIONAL KINETIC ENERGY

"The energy possessed by a body due to its angular motion is called rotational kinetic energy."

Formula

$$K.E_{rot} = \frac{1}{2}I\omega^2$$

SI unit

Its SI unit is Joule.

Q.15: Calculate the rotational kinetic energy of a disc and a hoop with its velocities?

ROTATIONAL KINETIC ENERGY OF A DISC AND A HOOP

FOR A DISC

$$K.E_{rot} = \frac{1}{2}I\omega^2$$

For a disc the value of I is

$$I = \frac{1}{2}mr^2$$

So

$$K.E_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$= \frac{1}{4}mr^2\omega^2$$

$$= \frac{1}{4} m(r\omega)^2$$

$$K.E_{rot} = \frac{1}{4} mv^2$$

FOR A HOOP:

For a hoop the value of I is

$$I = mr^2$$

So

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2}(mr^2)\omega^2$$

$$= \frac{1}{2} m(r\omega)^2$$

$$K.E_{rot} = \frac{1}{2} mv^2$$

DETERMINATION OF VELOCITY FOR A DISC AND HOOP

FOR A DISC

When a disc rolls down on inclined

$$P.E = k.E_{rot} + k.E_{trans}$$

$$mgh = \frac{1}{4} mv^2 + \frac{1}{2} mv^2$$

$$= \frac{mv^2 + 2mv^2}{4}$$

$$mgh = \frac{3mv^2}{4}$$

$$v_{disc} = \sqrt{\frac{4gh}{3}}$$

FOR A HOOP

When a hoop rolls down an inclined

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$= \frac{mv^2 + mv^2}{2}$$

$$= \frac{2mv^2}{2}$$

$$mgh = mv^2$$

$$v_{hoop} = \sqrt{gh}$$

Q.16: What are artificial satellite? Find the expression for minimum velocity and period to put a satellite into the orbit.

SATELLITE

"Objects that revolve around the earth are called its satellite."

ARTIFICIAL SATELLITE

"Satellites that are put into orbit around the earth by rockets and are held in that orbit by gravitational pull of earth are called artificial satellites."

CRITICAL VELOCITY

"The minimum velocity needs to put a satellite into an orbit around the earth is called its critical velocity."

DETERMINATION OF CRITICAL VELOCITY

The centripetal acceleration a_c required by a satellite to orbit around the earth is equal to the gravitational acceleration

$$a_c = g = \frac{v^2}{R}$$

$$v = \sqrt{gR}$$

$$= \sqrt{(9.8)(6.4 \times 10^6)}$$

$$v = 7.9\text{kms}^{-1}$$

TIME PERIOD OF A LOW ORBITING SATELLITE

"The time required by a satellite to complete one orbit around the earth is called its time period." It is calculated as

$$t = \frac{s}{v}$$

For one complete orbit

$$T = \frac{2\pi R}{v} = \frac{2(3.14)(6.4 \times 10^6)}{7.9 \times 10^3}$$

$$T = 5087\text{s}$$

$$= \frac{5087}{60}\text{min}$$

$$T = 84\text{min}$$

Q.17: What do you understand by real and apparent weight?

WEIGHT:

"Gravitational pull of earth on an object is called its weight."

$$W = mg$$

WHEN LIFT IS AT REST : (a = 0)

$$\text{Net force} = T - \text{Weight}$$

$$F = T - \text{weight}$$

By newton's second law

$$ma = T - \text{Weight} \qquad (\because F = ma)$$

$$m(0) = T - \text{weight}$$

$$T = \text{weight} = mg$$

Thus the apparent weight of the object is equal to its real weight.

LIFT MOVING UPWARD WITH ACCELERATION a

When lift moves upward with acceleration a then net force

$$\text{Net force} = T - mg$$

$$F = T - mg$$

By newton's second law

$$F = ma$$

$$ma = T - mg$$

$$T = ma + mg$$

Apparent weight will be appeared greater than real weight.

LIFT MOVING DOWNWARD WITH ACCELERATION a

When lift moves downward with acceleration a then

$$\text{Net force} = mg - T$$

By newton's second law

$$F = ma$$

$$ma = mg - T$$

$$T = mg - ma$$

Apparent weight will be appeared lesser than real weight.

LIFT FALLING FREELY

When lift is falling freely a = g

$$T = mg - ma$$

$$= mg - mg \qquad (\because\ a = g)$$

$$T = 0$$

In this case the object appears to be weightless.

Q.18: Define gravity free system?

GRAVITY FREE SYSTEM

"When a satellite is falling freely in space, everything within it appears to be weightless such a system is called gravity free system."

Q.19: What is orbital velocity? Define an expression for orbital velocity.

ORBITAL VELOCITY

"The velocity with which an object revolves around the earth is called its orbital velocity."

Formulations

For an object orbiting around the earth, the centripetal force is provided by gravitational force of earth

$$F_c = F_g$$

$$\frac{mv_o^2}{r} = G\frac{Mm}{r^2}$$

$$v_o^2 = \frac{GMmr}{mr^2}$$

$$v_o = \sqrt{\frac{GM}{r}}$$

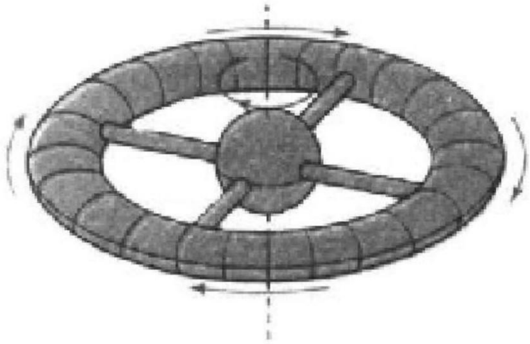
Q.20: What do you understand by artificial gravity? Derive an expression for frequency of spaceship.

Artificial gravity

In a gravity free system, there is no force to hold anything to any side of the system in such a system artificial gravity is produced by rotating the system about its axis.

Mathematical explanation

Consider a space craft as shown in the diagram with outer radius R



To produce artificial gravity it is rotated with angular speed " ω ". The acceleration of such a space craft is

$$a_c = \frac{v^2}{R} = \frac{(R\omega)^2}{R}$$

$$a_c = R\omega^2$$

As

$$\omega = \frac{\theta}{t}$$

For one complete rotation

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad \left(f = \frac{1}{T}\right)$$

use in equation 1

$$a_c = R(2\pi f)^2$$

$$a_c = R(4\pi^2 f^2)$$

For gravity like earth

$$a_c = g$$

$$g = 4\pi^2 f^2 R$$

$$f^2 = \frac{1}{4\pi^2}\frac{g}{R}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

When the space ship rotates with this frequency artificial gravity like earth is produced inside it.

Q.21: What are geo stationary orbits? Derive the formula for radius of geostationary orbit.

GEO STATIONARY ORBIT:

"Such a satellite whose orbital motion is synchronized with the rotation of earth is called geo stationary satellite. The orbit of such a satellite is called geostationary orbit."

Explanation

Let us calculate the radius of geo stationary satellite by considering the fact that this satellite completes its one rotation around the earth in one day which is the time taken by the earth to complete its one rotation about its own axis.

Mathematical derivation:

The orbital velocity of a satellite is given as

$$v = \sqrt{\frac{GM}{r}}$$

Also

$$v = \frac{s}{t} = \frac{2\pi r}{T}$$

Comparing equation 1 and 2

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Squaring both sides

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

By substituting the values this radius is equal to 4.23×10^4km. This height above the equator comes to be 36000km.