# EXERCISE 1.2

**Q#1: Rationalize the denominator of the following:**

**(i)** $\dfrac{\mathbf{13}}{\mathbf{4+\sqrt{3}}}$

**Solution:**

$$= \frac{13}{4+\sqrt{3}}$$

$$= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{\left(4+\sqrt{3}\right)\left(4-\sqrt{3}\right)}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{(4)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{16-3}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{13}$$

$$\boxed{= 4-\sqrt{3}}$$

**(ii)** $\dfrac{\mathbf{\sqrt{2}+\sqrt{5}}}{\mathbf{\sqrt{3}}}$

**Solution:**

$$= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$$

$$= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{3}\left(\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{3}\right)^2}$$

$$=\frac{\sqrt{3}\left(\sqrt{2}+\sqrt{5}\right)}{3}$$

$$\boxed{=\frac{\sqrt{6}+\sqrt{15}}{3}}$$

**(iii)** $\boldsymbol{\dfrac{\sqrt{2}-1}{\sqrt{5}}}$

**Solution:**

$$=\frac{\sqrt{2}-1}{\sqrt{5}}$$

$$=\frac{\sqrt{2}-1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$

$$=\frac{\sqrt{5}\left(\sqrt{2}-1\right)}{\left(\sqrt{5}\right)^2}$$

$$=\frac{\sqrt{5}\left(\sqrt{2}-1\right)}{5}$$

$$\boxed{=\frac{\sqrt{10}-\sqrt{5}}{5}}$$

**(iv)** $\boldsymbol{\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}}$

**Solution:**

$$=\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$=\frac{6-4\sqrt{2}}{6+4\sqrt{2}}\times\frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$=\frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$$

$$=\frac{(6)^2+\left(4\sqrt{2}\right)^2-2(6)\left(4\sqrt{2}\right)}{36-16(2)}$$

$$=\frac{36+16(2)-48\sqrt{2}}{36-32}$$

$$=\frac{36+32-48\sqrt{2}}{4}$$

$$=\frac{68-48\sqrt{2}}{4}$$

$$=\frac{4\left(17-12\sqrt{2}\right)}{4}$$

$$\boxed{=17-12\sqrt{2}}$$

**(v)** $\dfrac{\sqrt{\mathbf{3}}-\sqrt{\mathbf{2}}}{\sqrt{\mathbf{3}}+\sqrt{\mathbf{2}}}$

**Solution:**

$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{3}\right)\left(\sqrt{2}\right)}{3-2}$$

$$=\frac{3+2-2\sqrt{6}}{1}$$

$$\boxed{= 5 - 2\sqrt{6}}$$

**(vi)** $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

**Solution:**

$$= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$$

$$= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{4\left(\sqrt{21}-\sqrt{15}\right)}{7-5}$$

$$= \frac{4\left(\sqrt{21}-\sqrt{15}\right)}{2}$$

$$= 2\left(\sqrt{3\times 7} - \sqrt{3\times 5}\right)$$

$$= 2\left(\sqrt{3}\times\sqrt{7} - \sqrt{3}\times\sqrt{5}\right)$$

$$\boxed{= 2\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}$$

**Q#2: Simplify the following:**

**(i)** $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$

**Solution:**

$$= \left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$= \left(\frac{16}{81}\right)^{\frac{3}{4}}$$

$$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$$

$$= \frac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}}$$

$$\boxed{= \frac{2^3}{3^3} = \frac{8}{27}}$$

**(ii)** $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

**Solution:**

$$= \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$$

$$= \left(\frac{4}{3}\right)^{2} \times \left(\frac{9}{4}\right)^{3} \times \left(\frac{2^4}{3^3}\right)$$

$$= \left(\frac{2^2}{3}\right)^{2} \times \left(\frac{3^2}{2^2}\right)^{3} \times \left(\frac{2^4}{3^3}\right)$$

$$= \frac{2^4}{3^2} \times \frac{3^6}{2^6} \times \frac{2^4}{3^3}$$

$$= \frac{2^8 \times 3^6}{3^5 \times 2^6}$$

$$= 2^{8-6} \times 3^{6-5}$$

$$= 2^2 \times 3^1$$

$$= 4 \times 3$$

$$\boxed{= 12}$$

**(iii)** $(0.027)^{-\frac{1}{3}}$

**Solution:**

$$= (0.027)^{-\frac{1}{3}}$$

$$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$

$$= \left(\frac{1000}{27}\right)^{\frac{1}{3}}$$

$$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$

$$\boxed{= \frac{10^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}} = \frac{10}{3}}$$

**(iv)** $\sqrt[7]{\dfrac{\mathbf{x^{14} \times y^{21} \times z^{35}}}{\mathbf{y^{14}z^7}}}$

**Solution:**

$$= \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14}z^7}}$$

$$= \left(\frac{x^{14} \times y^{21} \times z^{35}}{y^{14}z^7}\right)^{\frac{1}{7}}$$

$$= \frac{x^{14\times\frac{1}{7}} \times y^{21\times\frac{1}{7}} \times z^{35\times\frac{1}{7}}}{y^{14\times\frac{1}{7}} \times z^{7\times\frac{1}{7}}}$$

$$= \frac{x^2 \times y^3 \times z^5}{y^2 \times y^1}$$

$$= x^2 \times y^{3-2} \times z^{5-1}$$

$$= x^2y^1z^4$$

$$\boxed{= x^2yz^4}$$

**(v)** $\dfrac{\mathbf{5.(25)^{n+1} - 25.(5)^{2n}}}{\mathbf{5.(5)^{2n+3} - (25)^{n+1}}}$

**Solution:**

$$= \frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$$

$$= \frac{5.(5^2)^{n+1} - 5^2.(5)^{2n}}{5.(5)^{2n+3} - (5^2)^{n+1}}$$

$$= \frac{5.(5)^{2n+2} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}}$$

$$= \frac{5^{2n+3} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}}$$

$$= \frac{5^{2n+2}(5^1 - 1)}{5^{2n+2}(5^2 - 1)}$$

$$= \frac{5 - 1}{25 - 1}$$

$$= \frac{4}{24}$$

$$\boxed{= \frac{1}{6}}$$

**(vi)** $\dfrac{\mathbf{(16)^{x+1} + 20(4^{2x})}}{\mathbf{2^{x-3} \times 8^{x+2}}}$

**Solution:**

$$= \frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$$

$$= \frac{(2^4)^{x+1} + (2 \times 2 \times 5)(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}}$$

$$= \frac{2^{4x+4} + (2^2 \times 5)(2^{4x})}{2^{x-3} \times 2^{3x+6}}$$

$$= \frac{2^{4x+4} + 2^{4x+2} \times 5}{2^{x-3+3x+6}}$$

$$= \frac{2^{4x+4} + 2^{4x+2} \times 5}{2^{4x+3}}$$

$$= \frac{2^{4x+4}}{2^{4x+3}} + \frac{2^{4x+2} \times 5}{2^{4x+3}}$$

$$= 2^{4x+4-4x-3} + 2^{4x+2-4x-3} \times 5$$

$$= 2^{1} + 2^{-1} \times 5$$

$$= 2 + \frac{5}{2} = \frac{4+5}{2}$$

$$\boxed{= \frac{9}{2}}$$

**(vii)** $(\mathbf{64})^{-\frac{2}{3}} \div (\mathbf{9})^{-\frac{3}{2}}$

**Solution:**

$$= (64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$$

$$= (4^3)^{-\frac{2}{3}} \div (3^2)^{-\frac{3}{2}}$$

$$= (4)^{-2} \div (3)^{-3}$$

$$= \frac{4^{-2}}{3^{-3}} = \frac{3^3}{4^2}$$

$$\boxed{= \frac{27}{16}}$$

**(viii)** $\dfrac{\mathbf{3^{n} \times 9^{n+1}}}{\mathbf{3^{n-1} \times 9^{n-1}}}$

**Solution:**

$$= \frac{3^{n} \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

$$=\frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}}$$

$$=\frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$$

$$=\frac{3^{n+2n+2}}{3^{n-1+2n-2}}$$

$$=\frac{3^{3n+2}}{3^{3n-3}}$$

$$= 3^{3n+2-3n+3} = 3^5$$

$$\boxed{= 243}$$

**(ix)** $\mathbf{\dfrac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^n \times 5^n}}$

**Solution:**

$$=\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^n \times 5^n}$$

$$=\frac{5^n(5^3 - 6.5^1)}{5^n(9 - 2^n)}$$

$$=\frac{(5^3 - 6 \times 5)}{(9 - 2^n)}$$

$$=\frac{125 - 30}{9 - 2^n}$$

$$\boxed{=\frac{95}{9 - 2^n}}$$

**Q#3: If $x = 3 + \sqrt{8}$ then find the value of:**

**(i)** $\mathbf{x + \dfrac{1}{x}}$

**Solution:**

$$= x + \frac{1}{x}$$

As we know that $x = 3 + \sqrt{8}$ therefore,

$$= 3 + \sqrt{8} + \frac{1}{(3 + \sqrt{8})}$$

By rationalizing $\frac{1}{3+\sqrt{8}}$

$$= 3 + \sqrt{8} + \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{9 - 8}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{1}$$

$$= 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$\boxed{= 6}$$

**(ii)** $\mathbf{x - \frac{1}{x}}$

**Solution:**

$$= x - \frac{1}{x}$$

$$= 3 + \sqrt{8} - \left(\frac{1}{3 + \sqrt{8}}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{9 - 8}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{1}\right)$$

$$= 3 + \sqrt{8} - (3 - \sqrt{8})$$

$$= 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$= \sqrt{8} + \sqrt{8} = 2\sqrt{8}$$

$$= 2\sqrt{4 \times 2}$$

$$\boxed{= 2\sqrt{8}}$$

**(iii)** $\mathbf{x^2 + \frac{1}{x^2}}$

**Solution:**

We have to find the value for $x^2 + \frac{1}{x^2}$

First find the value for $x + \frac{1}{x}$

$$x + \frac{1}{x} = 3 + \sqrt{8} + \frac{1}{(3 + \sqrt{8})}$$

$$= 3 + \sqrt{8} + \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{9 - 8}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{1}$$

$$= 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$x + \frac{1}{x} = 6$$

Talking square on both sides:

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 36$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$\boxed{x^2 + \frac{1}{x^2} = 34}$$

**(iv)** $\mathbf{x^2 - \frac{1}{x^2}}$

**Solution:**

First find the value for $x + \frac{1}{x}$

$$x + \frac{1}{x} = 3 + \sqrt{8} + \frac{1}{(3 + \sqrt{8})}$$

$$= 3 + \sqrt{8} + \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{9 - 8}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{1}$$

$$= 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$x+\frac{1}{x}=6$$

Now for $x-\frac{1}{x}$

$$x-\frac{1}{x}=3+\sqrt{8}-\left(\frac{1}{3+\sqrt{8}}\right)$$

$$=3+\sqrt{8}-\left(\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}\right)$$

$$=3+\sqrt{8}-\left(\frac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}\right)$$

$$=3+\sqrt{8}-\left(\frac{3-\sqrt{8}}{9-8}\right)$$

$$=3+\sqrt{8}-\left(\frac{3-\sqrt{8}}{1}\right)$$

$$=3+\sqrt{8}-\left(3-\sqrt{8}\right)$$

$$=3+\sqrt{8}-3+\sqrt{8}$$

$$=\sqrt{8}+\sqrt{8}=2\sqrt{8}$$

$$=2\sqrt{4\times 2}=2\sqrt{2^2\times 2}$$

$$=2\times\sqrt{4\times 2}$$

$$x-\frac{1}{x}=2\sqrt{8}$$

Now we know that;

$$x+\frac{1}{x}=6 \quad ; \quad x-\frac{1}{x}=2\sqrt{8}$$

$$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=(6)\left(2\sqrt{8}\right)$$

$$\boxed{x^2-\frac{1}{x^2}=12\sqrt{8}}$$

**(v)** $\mathbf{x^4 + \dfrac{1}{x^4}}$

**Solution:**

To find the value for $x^4 + \dfrac{1}{x^4}$, first we have to find the value for $x^2 + \dfrac{1}{x^2}$

$$x + \frac{1}{x} = 3 + \sqrt{8} + \frac{1}{(3 + \sqrt{8})}$$

$$= 3 + \sqrt{8} + \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{9 - 8}$$

$$= 3 + \sqrt{8} + \frac{3 - \sqrt{8}}{1}$$

$$= 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$x + \frac{1}{x} = 6$$

Talking square on both sides:

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 36$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$x^2 + \frac{1}{x^2} = 34$$

Again squatting on both sides:

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) = 1156$$

$$x^4 + \frac{1}{x^4} + 2 = 1156$$

$$x^4 + \frac{1}{x^4} = 1156 - 2$$

$$\boxed{x^4 + \frac{1}{x^4} = 1154}$$

**(vi)** $\left(\mathbf{x} - \frac{\mathbf{1}}{\mathbf{x}}\right)^{\mathbf{2}}$

**Solution:**

$$x - \frac{1}{x} = 3 + \sqrt{8} - \left(\frac{1}{3 + \sqrt{8}}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{9 - 8}\right)$$

$$= 3 + \sqrt{8} - \left(\frac{3 - \sqrt{8}}{1}\right)$$

$$= 3 + \sqrt{8} - \left(3 - \sqrt{8}\right)$$

$$= 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$= \sqrt{8} + \sqrt{8} = 2\sqrt{8}$$

$$= 2\sqrt{4 \times 2} = 2\sqrt{2^2 \times 2}$$

$$= 2 \times 2\sqrt{2}$$

$$x - \frac{1}{x} = 4\sqrt{2}$$

Talking square on both sides:

$$\left(x - \frac{1}{x}\right)^2 = \left(4\sqrt{2}\right)^2$$

$$= 16(2)$$

$$\boxed{\left(x - \frac{1}{x}\right)^2 = 32}$$

**Q4**: **Find the rational numbers p and q such that** $\mathbf{\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = p + q\sqrt{2}}$

**Solution:**

Let

$$z = \frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}}$$

$$= \frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} \times \frac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}}$$

$$= \frac{(8 - 3\sqrt{2})(4 - 3\sqrt{2})}{(4)^2 - \left(3\sqrt{2}\right)^2}$$

$$= \frac{(8)(4) + (8)(-3\sqrt{2}) + (-3\sqrt{2})(4) + (-3\sqrt{2})(-3\sqrt{2})}{16 - 9(2)}$$

$$= \frac{32 - 24\sqrt{2} - 12\sqrt{2} + \left(-3\sqrt{2}\right)^2}{16 - 18}$$

$$= \frac{32 - 36\sqrt{2} + 18}{-2}$$

$$= \frac{50 - 36\sqrt{2}}{-2}$$

$$= -25 + 18\sqrt{2}$$

Now

$$-25 + 18\sqrt{2} = p + q\sqrt{2}$$

By comparing;

$$\boxed{p = -25}$$

$$\boxed{q = 18}$$

**Q#5: Simplify the following:**

**(i)** $\dfrac{(\mathbf{25})^{\frac{3}{2}} \times (\mathbf{243})^{\frac{3}{5}}}{(\mathbf{16})^{\frac{5}{4}} \times (\mathbf{8})^{\frac{4}{3}}}$

**Solution:**

$$= \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$$

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$\boxed{= \frac{3375}{512}}$$

**(ii)** $\dfrac{\mathbf{54} \times \sqrt[3]{(\mathbf{27})^{2x}}}{\mathbf{9}^{x+1} + \mathbf{216}(\mathbf{3}^{2x-1})}$

**Solution:**

$$= \frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$$

$$= \frac{(2 \times 3^3) \times [(27^{2x})]^{\frac{1}{3}}}{(3^2)^{x+1} + (2^3 \times 3^3)(3^{2x-1})}$$

$$= \frac{(2 \times 3^3) \times [(3^3)^{2x}]^{\frac{1}{3}}}{3^{2x+2} + 2^3 \times 3^{2x-1+3}}$$

$$= \frac{2 \times 3^3 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{2x+2}}$$

$$= \frac{3^{2x+3} \times 2}{3^{2x+2} + 8 \times 3^{2x+2}}$$

$$= \frac{3^{2x+3}(2 \times 3^1)}{3^{2x+2}(1+8)}$$

$$= \frac{2 \times 3}{1+8}$$

$$\boxed{= \frac{6}{9} = \frac{2}{3}}$$

**(iii)** $\sqrt{\dfrac{(\mathbf{216})^{\frac{2}{3}} \times (\mathbf{25})^{\frac{1}{2}}}{(\mathbf{0.04})^{-\frac{3}{2}}}}$

**Solution:**

$$= \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5^1}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}} = \sqrt{\frac{6^2 \times 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{10^2}{2^2}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{10}{2}\right)^{3}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(5)^3}}$$

$$= \sqrt{\frac{6^2}{5^{3-1}}}$$

$$= \sqrt{\frac{6^2}{5^2}}$$

$$\boxed{= \frac{6}{5}}$$

**(iv)** $\left(\mathbf{a^{\frac{1}{3}} + b^{\frac{2}{3}}}\right) \times \left(\mathbf{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}}\right)$

**Solution:**

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$= a^{\frac{1}{3}} \times a^{\frac{2}{3}} - a^{\frac{1}{3}} \times a^{\frac{1}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}} \times b^{\frac{4}{3}} + b^{\frac{2}{3}} \times a^{\frac{2}{3}} - b^{\frac{2}{3}} \times a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{2}{3}} \times b^{\frac{4}{3}}$$

$$= a^{\frac{3}{3}} - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{2}{3}}a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{6}{3}}$$

$= a^1 + b^2$

$\boxed{= a + b^2}$