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Solutions Manual For

CALCULUS WITH ANALYTIC GEOMETRY

By

A BOARD OF EXPERIENCED PROFESSORS

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Solutions Manual For

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CONTENTS

Chapter 1 REAL NUMBERS, LIMITS AND CONTINUITY

Exercise Set 1.1 (Page 17)

1. If $a, b \in \boldsymbol{R}$ and $a + b = 0$, prove that $a = -b$

Sol. Since $b \in \boldsymbol{R}$, there is an element $-b \in \boldsymbol{R}$ such that

$$b + (-b) = 0 \quad (1)$$

By hypothesis, $a + b = 0$ (2)

Adding $-b$ to both sides of (2) and applying the above property of additive inverse, we get

$$a + b + (-b) = -b \quad \text{or} \quad a + (b + (-b)) = -b$$

(Associative property of addition)

or $a + 0 = -b$ by (1)

i.e., $a = -b$ as required.

2. Prove that $(-a)(-b) = ab$ for all $a, b \in \boldsymbol{R}$.

Sol. We have, $ab + a(-b) + (-a)(-b) = ab + [a(-b) + (-a)(-b)]$,

(Associative property of addition)

or $a[b + (-b)] + (-a)(-b) = ab + [a + (-a)](-b)$

(Distributive property)

or $a.0 + (-a)(-b) = ab + 0.(-b)$

i.e., $(-a)(-b) = ab$, since $a.0 = 0 = 0.(-b)$.

3. Prove that $\big||a| - |b|\big| \le |a - b|$ for every $a, b \in \boldsymbol{R}$.

Sol. By Theorem 1.5 (v), we have

$$|a + b| \le |a| + |b| \quad (1)$$

Replacing b by $-b$, (1) becomes

$$|a - b| \le |a| + |-b| = |a| + |b|, \quad (2)$$

since $|-b| = |b|$

Replace a by $b - a$ in (2) to get

$$|-a| \le |b - a| + |b|$$

or $|a| - |b| \le |b - a| = |a - b|$ (3)

Again, in $|b - a| \le |a| + |b|$, replace b by $a - b$ to have

$$|-b| \le |a| + |a - b| \text{ or } |b| - |a| \le |a - b|$$

Multiplying both sides of the inequality by -1, we get

$$|a| - |b| \ge -|a - b|$$

or $-|a - b| \le |a| - |b|$ (4)

Combining (3) and (4), we have $-|a - b| \le |a| - |b| \le |a - b|$

or $\big||a| - |b|\big| \le |a - b|$, by Theorem 1.5 (*iv*).

4. Express $3 < x < 7$ in modulus notation

Sol. We know that $|x - a| < l$ implies $a - l < x < a + l$

Now $3 < x < 7$

Therefore, by comparison,

$a - l = 3$ (1)

$a + l = 7$ (2)

Adding (1) and (2), we get $2a = 10$ or $a = 5$

Subtracting (1) from (2), we have $2l = 4$ or $l = 2$

Hence the given inequality can be expressed in the modulus notation as $|x - 5| < 2$

5. Let $\delta > 0$ and $a \in \mathbf{R}$. Show that $a - \delta < x < a + \delta$ if and only if $|x - a| < \delta$.

Sol. Suppose $a - \delta < x < a + \delta$. These inequalities can be written as

$a - \delta < x$ (1)

and $x < a + \delta$ (2)

From (1) and (2), we have respectively

$-\delta < x - a$ (3)

and $x - a < \delta$ (4)

Combining (3) and (4), we get

$-\delta < x - a < \delta$ or $|x - a| < \delta$ by Theorem 1.5 (*iv*)

Conversely, let $|x - a| < \delta$. By Theorem 1.5 (*iv*), we have

$-\delta < x - a < \delta$ or $a - \delta < x < a + \delta$ as desired

6. Give an example of a set of rational numbers which is bounded above but does not have a rational supremum.

Sol. Consider the set S of rational numbers defined by

$S = \{x \in Q : x^2 < 2\}$

The supremum of S is $\sqrt{2}$ which is not a rational number.

Solve each of the following inequalities (Problems 7 – 15)

7. $|2x + 5| > |2 - 5x|$

Sol. Associated equation is $|2x + 5| = |2 - 5x|$

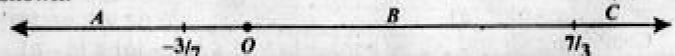
This is equivalent to

$2x + 5 = 2 - 5x$ (1)

or $2x + 5 = -2 + 5x$ (2)

From (1), we get $x = -\dfrac{3}{7}$ and from (2), we have $x = \dfrac{7}{3}$

These are the boundary numbers for the given inequality. The number line is divided by the boundary numbers into regions as shown:



Region A, test $x = -1$: $|-2 + 5| > |2 + 5|$ False

Region B, test $x = 0$: $|5| > |2|$ True

Region C, test $x = 3$: $|6 + 5| > |2 - 15|$ False

Thus the solution set is

$$\left\{x : \frac{-3}{7} < x < \frac{7}{3}\right\} = \left]-\frac{3}{7}, \frac{7}{3}\right[$$

8. $\left|\dfrac{x + 8}{12}\right| < \dfrac{x - 1}{10}$ (1)

Sol. (1) is equivalent to the compound inequality

$-\dfrac{x - 1}{10} < \dfrac{x + 8}{12} < \dfrac{x - 1}{10}$ or $-6x + 6 < 5x + 40 < 6x - 6$

This is equivalent to $-11x < 34$ and $46 < x$

i.e., $-\dfrac{34}{11} < x$ and $46 < x$

The solution set is

$$\left\{x : -\frac{34}{11} < x\right\} \cap \{x : 46 < x\} = \{x : 46 < x\} =]46, \infty[$$

Alternative Method:

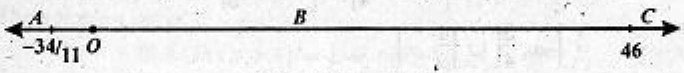
Associated equation is $\dfrac{x + 8}{12} = \pm\dfrac{x - 1}{10}$

i.e., $5x + 40 = \pm 6(x - 1)$

i.e., $5x + 40 = 6x - 6$ and $5x + 40 = -6x + 6$

or $x = 46$ and $x = -\dfrac{34}{11}$

These boundary numbers divide the number line as shown:



Region A, test $x = -4$: $\left|\dfrac{-4 + 8}{12}\right| < \dfrac{-4 - 1}{10}$ False

Region B, test $x = 45$: $\left|\dfrac{45 + 8}{12}\right| < \dfrac{45 - 1}{10}$ False

Region C, test $x = 47$: $\left|\dfrac{47 - 8}{12}\right| < \dfrac{47 - 1}{10}$ True

The solution set is $\{x : x > 46\} =]46, \infty[$.

9. $|x| + |x - 1| > 1$

Sol. The associated equation is

$|x| + |x - 1| = 1$ or $\pm x \pm (x - 1) = 1$

This is equivalent to

$x + x - 1 = 1$ (1)