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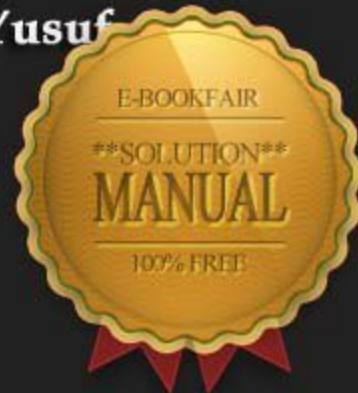
Group of **Jg** Network

Calculus with Analytic Geometry

 Our Effort To Serve You Better

Calculus With Analytic Geometry

By
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Muhammad Amin

Solutions Manual For

CALCULUS
WITH
ANALYTIC
GEOMETRY

By

A BOARD OF EXPERIENCED PROFESSORS

ILMI KITAB KHANA

Kahir Street, Urdu Bazar, Lahore. 54000

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For
CALCULUS
With
**ANALYTIC
GEOMETRY**

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Chapter

1

REAL NUMBERS, LIMITS AND CONTINUITY

Exercise Set 1.1 (Page 17)

1. If $a, b \in R$ and $a + b = 0$, prove that $a = -b$

Sol. Since $b \in R$, there is an element $-b \in R$ such that

$$b + (-b) = 0 \quad (1)$$

By hypothesis, $a + b = 0$ (2)

Adding $-b$ to both sides of (2) and applying the above property of additive inverse, we get

$$a + b + (-b) = -b \quad \text{or} \quad a + (b + (-b)) = -b$$

(Associative property of addition)

$$\text{or} \quad a + 0 = -b \quad \text{by (1)}$$

$$\text{i.e.,} \quad a = -b \quad \text{as required.}$$

2. Prove that $(-a)(-b) = ab$ for all $a, b \in R$.

Sol. We have, $ab + a(-b) + (-a)(-b) = ab + [a(-b) + (-a)(-b)]$,

(Associative property of addition)

$$\text{or} \quad a[b + (-b)] + (-a)(-b) = ab + [a + (-a)](-b)$$

(Distributive property)

$$\text{or} \quad a \cdot 0 + (-a)(-b) = ab + 0 \cdot (-b)$$

$$\text{i.e.,} \quad (-a)(-b) = ab, \text{ since } a \cdot 0 = 0 = 0 \cdot (-b).$$

3. Prove that $||a| - |b|| \leq |a - b|$ for every $a, b \in R$.

Sol. By Theorem 1.5 (v), we have

$$|a + b| \leq |a| + |b| \quad (1)$$

Replacing b by $-b$, (1) becomes

$$|a - b| \leq |a| + |-b| = |a| + |b|, \quad (2)$$

since $|-b| = |b|$

Replace a by $b - a$ in (2) to get

$$|b - a| \leq |b - a| + |b|$$

$$\text{or} \quad |a| - |b| \leq |b - a| = |a - b| \quad (3)$$

Again, in $|b - a| \leq |a| + |b|$, replace b by $a - b$ to have

$$|-b| \leq |a| + |a - b| \quad \text{or} \quad |b| - |a| \leq |a - b|$$

Multiplying both sides of the inequality by -1 , we get

$$|a| - |b| \geq -|a - b|$$

$$\text{or} \quad -|a - b| \leq |a| - |b| \quad (4)$$

Combining (3) and (4), we have $-|a - b| \leq |a| - |b| \leq |a - b|$

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or $||a| - |b|| \leq |a - b|$, by Theorem 1.5 (iv).

4. Express $3 < x < 7$ in modulus notation

Sol. We know that $|x - a| < l$ implies $a - l < x < a + l$

Now $3 < x < 7$

Therefore, by comparison,

$$a - l = 3 \quad (1)$$

$$a + l = 7 \quad (2)$$

Adding (1) and (2), we get $2a = 10$ or $a = 5$

Subtracting (1) from (2), we have $2l = 4$ or $l = 2$

Hence the given inequality can be expressed in the modulus notation as $|x - 5| < 2$

5. Let $\delta > 0$ and $a \in \mathbb{R}$. Show that $a - \delta < x < a + \delta$ if and only if $|x - a| < \delta$.

Sol. Suppose $a - \delta < x < a + \delta$. These inequalities can be written as

$$a - \delta < x \quad (1)$$

$$\text{and } x < a + \delta \quad (2)$$

From (1) and (2), we have respectively

$$-\delta < x - a \quad (3)$$

$$\text{and } x - a < \delta \quad (4)$$

Combining (3) and (4), we get

$$-\delta < x - a < \delta \text{ or } |x - a| < \delta \text{ by Theorem 1.5 (iv)}$$

Conversely, let $|x - a| < \delta$. By Theorem 1.5 (iv), we have

$$-\delta < x - a < \delta \text{ or } a - \delta < x < a + \delta \text{ as desired}$$

6. Give an example of a set of rational numbers which is bounded above but does not have a rational supremum.

Sol. Consider the set S of rational numbers defined by

$$S = \{x \in \mathbb{Q} : x^2 < 2\}$$

The supremum of S is $\sqrt{2}$ which is not a rational number.

Solve each of the following inequalities (Problems 7 - 15)

7. $|2x + 5| > |2 - 5x|$

Sol. Associated equation is $|2x + 5| = |2 - 5x|$

This is equivalent to

$$2x + 5 = 2 - 5x \quad (1)$$

$$\text{or } 2x + 5 = -2 + 5x \quad (2)$$

From (1), we get $x = -\frac{3}{7}$ and from (2), we have $x = \frac{7}{3}$

These are the boundary numbers for the given inequality. The number line is divided by the boundary numbers into regions as shown:



Region A, test $x = -1$: $|-2 + 5| > |2 + 5|$

False

Region B, test $x = 0$: $|5| > |2|$

True

Region C, test $x = 3$: $|6 + 5| > |2 - 15|$

False

Thus the solution set is

$$\left\{x : -\frac{3}{7} < x < \frac{7}{3}\right\} = \left]-\frac{3}{7}, \frac{7}{3}\right[$$

8. $\left|\frac{x+8}{12}\right| < \frac{x-1}{10} \quad (1)$

Sol. (1) is equivalent to the compound inequality

$$-\frac{x-1}{10} < \frac{x+8}{12} < \frac{x-1}{10} \text{ or } -6x + 6 < 5x + 40 < 6x - 6$$

This is equivalent to $-11x < 34$ and $46 < x$

$$\text{i.e., } -\frac{34}{11} < x \text{ and } 46 < x$$

The solution set is

$$\left\{x : -\frac{34}{11} < x\right\} \cap \{x : 46 < x\} = \{x : 46 < x\} =]46, \infty[$$

Alternative Method:

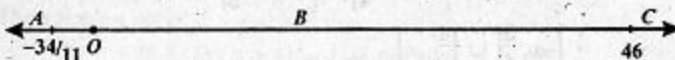
$$\text{Associated equation is } \frac{x+8}{12} = \pm \frac{x-1}{10}$$

$$\text{i.e., } 5x + 40 = \pm 6(x - 1)$$

$$\text{i.e., } 5x + 40 = 6x - 6 \text{ and } 5x + 40 = -6x + 6$$

$$\text{or } x = 46 \text{ and } x = -\frac{34}{11}$$

These boundary numbers divide the number line as shown:



$$\text{Region A, test } x = -4 : \left|\frac{-4+8}{12}\right| < \frac{-4-1}{10} \quad \text{False}$$

$$\text{Region B, test } x = 45 : \left|\frac{45+8}{12}\right| < \frac{45-1}{10} \quad \text{False}$$

$$\text{Region C, test } x = 47 : \left|\frac{47-8}{12}\right| < \frac{47-1}{10} \quad \text{True}$$

The solution set is $\{x : x > 46\} =]46, \infty[$.

9. $|x| + |x - 1| > 1$

Sol. The associated equation is

$$|x| + |x - 1| = 1 \text{ or } \pm x \pm (x - 1) = 1$$

This is equivalent to

$$x + x - 1 = 1$$

(1)