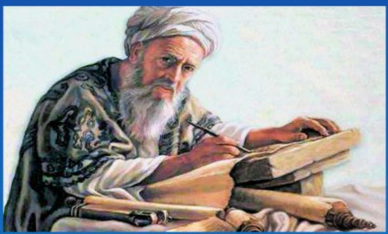
" Your Best Study Experience begins here."

SEVEN MUSLIMS NOTES

PHYSICS 11



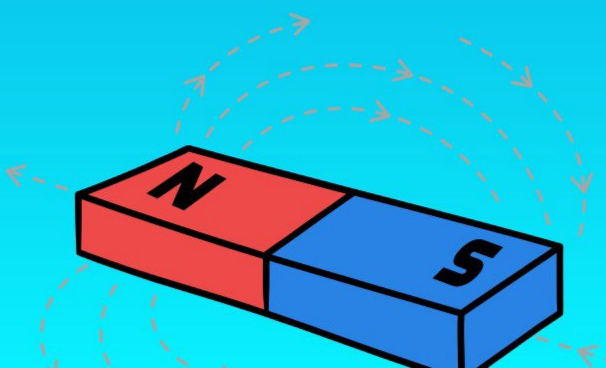
Al-Biruni (973–1048) calculated the Earth's radius and worked on the physics of planetary motion.

Best Regards to

Sir Muhammad Ali

(Physics Lecturer KIPS College)

$E=mc^2$



CHAPTER 2

VECTORS AND EQUILIBRIUM

Q.1. Define Physical Quantities. Give its types.

PHYSICAL QUANTITIES

"All measurable quantities are called Physical Quantities."

For Example:

Length, mass, time, force and energy.

Physical Quantities are broadly categorized into two main types.

1. Scalar Quantities

2. Vector Quantities

These are given below

1. Scalar Quantities

"Quantities which are described completely by their magnitude only are called scalar Quantities."

For Example:

Length, mass, time and temperature etc.

2. Vector quantities

"Quantities which are completely described by their magnitude as well as direction are called vector Quantities."

For Example:

Force, velocity and acceleration etc.

Q.2. Describe how a vector is represented?

REPRESENTATION OF VECTORS

There are two methods/ways for the representation of vectors.

1. Symbolic way

2. Graphical way

These are given below

1. Symbolic way

Symbolically a vector is represented by using bold letters or using a bar/arrow on the letters.

Representation:

$$\overline{A}$$

2. **Graphical way**

Graphically a vector is represented by a line segment with an arrow head.

Length of the line segment represents the magnitude according to the scale and arrow points the direction.

Q.3. Define rectangular/Cartesian coordinate system? Write its types also.

RECTANGULAR/CARTESIAN COORDINATE SYSTEM

"Two reference lines that are perpendicular to each other are called coordinate axis their point of intersection is called origin and the system is called rectangular coordinate system."

Only two types of rectangular/Cartesian coordinate system are given below

1) **Plane**

i. Two dimensional coordinate axis makes a plane.

ii. In a plane a vector is described by two coordinates and one angle.

2) **Space**

i. Three dimensional coordinate axis makes a space.

ii. In a space a vector is described by three coordinates and three angles.

Q.4. Define the following terms

1. Addition of vectors **2. Resultant vectors**

3. Subtraction of a vector **4. Multiplication of a vector by a scalar**

5. Unit vector **6. Null vector**

7. Equal vector

1. ADDITION OF VECTORS

HEAD TO TAIL RULE

"A graphical method to add vectors is called head to tail rule."

$$\overline{A} + \overline{B} = \overline{B} + \overline{A}$$

2. RESULTANT VECTOR

"A single vector having combined effect of all the vectors to be added is called resultant vector."

3. SUBTRACTION OF A VECTOR

"Subtraction of a vector is equivalent to the addition of the same vector with its direction reversed."

$$\overline{A} - \overline{B} \neq \overline{B} - \overline{A}$$

4. MULTIPLICATION OF A VECTOR BY A SCALAR

By a position number

When a vector is multiplied by a positive number say $n > 0$, its magnitude becomes n times in the same direction.

By a negative number

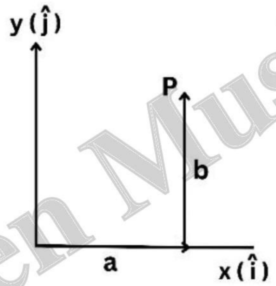
When a vector is multiplied by a negative number say $n < 0$, its magnitude becomes n times with its direction reversed.

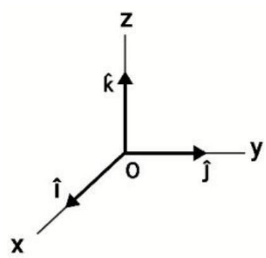
5. UNIT VECTOR

"A vector in a given direction with magnitude one in that direction is called unit vector."

It is used to describe the direction of a vector.

$$\hat{A} = \frac{\vec{A}}{A}$$





Examples:

Unit vector along x axis is $\hat{i}$

Unit vector along y axis is $\hat{j}$

Unit vector along z axis is $\hat{k}$

6. NULL VECTOR

"A vector with magnitude equals to zero in arbitrary direction is called a null vector."

Example:

$$\vec{A} + (-\vec{A}) = \vec{0}$$

7. EQUAL VECTORS

"Two or more vectors with same direction and magnitude are called equal vectors."

Example:

Parallel vectors of same magnitude.

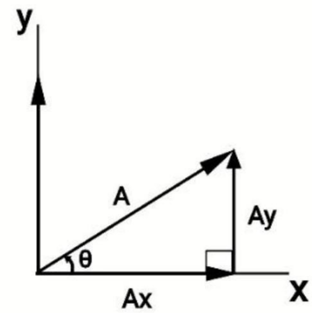
Q.5 What are rectangular components, explain. How a vector is obtained from its rectangular components. (GRW. GI, 2019) (LHR. GI, 2022) **(or) How vector is determined if its rectangular components are given?** (RWP. 2019)

COMPONENT OF A VECTOR

"Effective value of a vector in a given direction is called component of a vector."

RECTANGULAR COMPONENTS

"Components of a vector that are perpendicular to each other are called rectangular components."



$$\text{Sin}\theta = \frac{P}{H} = \frac{A_y}{A}$$

$$A_y = A\text{Sin}\theta$$

$$\text{Cos}\theta = \frac{B}{H} = \frac{A_x}{A}$$

$$A_x = A\cos\theta$$

DETERMIMATION OF A VECTOR FROM ITS RECTANGULAR COMPONENTS

Using Pythagoras theorem:

$$(H)^2 = (B)^2 + (P)^2$$

$$H = \sqrt{B^2 + P^2}$$

$$A = \sqrt{A_{x^2} + A_{y^2}}$$

$$\tan\theta = \frac{P}{B}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Q.6 Define Position vector.

POSITION VECTOR:

"A vector that is used describe the location of a point with respect to origin is called position vector."

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

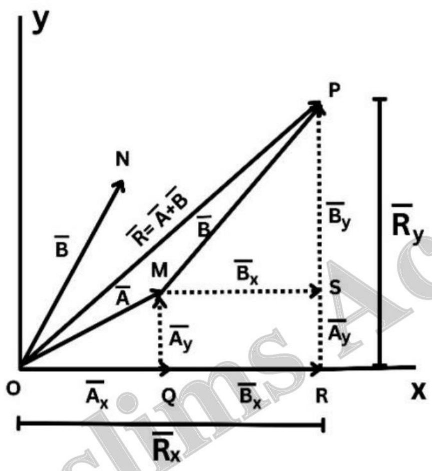
$$r = \sqrt{a^2 + b^2 + c^2}$$

Q.7 Describe vector addition by rectangular components. First find the resultant of two vectors and then generalize it for 'n' vectors.

(MLN. GI, SWL. GI, SGD. GI, 218) (LHR. GI, BWP. GII, 2019) (SGD. GI, LHR. GII, FSD. GI, BWP. GI, SWL. RWP, 2022).

(or) Drive expressions for the magnitude and direction of the resultant of two vectors, added by rectangular component method.

(MLN. GI. SWL.SGD. GI. RWP. GI. DGK. GI. 2017) (SGD. GII. 2019)



EXPLANATION

Consider two vectors $\overline{A}$ and $\overline{B}$ represented by lines $\overline{OM}$ and $\overline{ON}$ respectively in a rectangular coordinate system. The two vectors are added by head to tail rule to get resultant vector ($\overline{R} = \overline{A} + \overline{B}$).

MATHEMATICAL EXPRESSION

Along x- axis:

$$\overline{OR} = \overline{OQ} + \overline{QR}$$

$$R_x = A_x + B_x \qquad \ldots\ldots\ldots (1)$$

Along y- axis:

$$\overline{PR} = \overline{SR} + \overline{SP}$$

$$R_y = A_y + By \qquad \ldots\ldots (2)$$

In terms of rectangular components

$$\overline{R} = R_x\hat{i} + R_y\hat{j}$$

Using equation 1 and 2

$$\overline{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude of R is given by

$$\overline{R} = \sqrt{R_{x^2} + R_{y^2}}$$

Or

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

For more than two vectors

$$R = \sqrt{(A_x + B_x + C_x + \cdots)^2 + (A_y + B_y + C_y + \cdots)^2}$$

Also direction of R is

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Or

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

For more than two vectors:

$$\theta = \tan^{-1}\left(\frac{A_y + B_y + C_y + \cdots}{A_x + B_x + C_x + \cdots}\right)$$

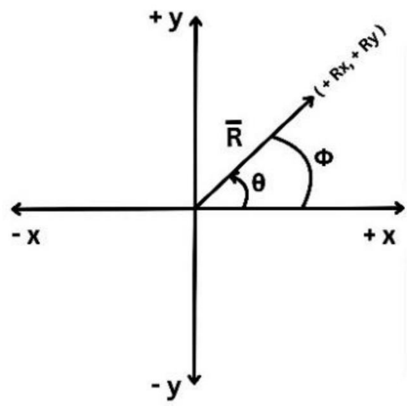
DETERMIMATION OF ANGLES

If resultant vector does not lie in the first Quadrant then to find θ we first find Φ irrespective of the signs of R_x and R_y.

In First Quadrant:

1. Both R_x and R_y are positive.

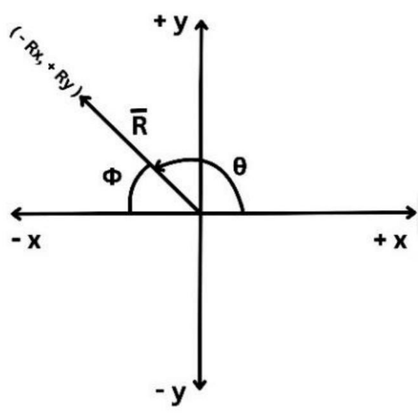
2. $\theta = \phi$



In second Quadrant:

1. R_x is negative and R_y is positive.

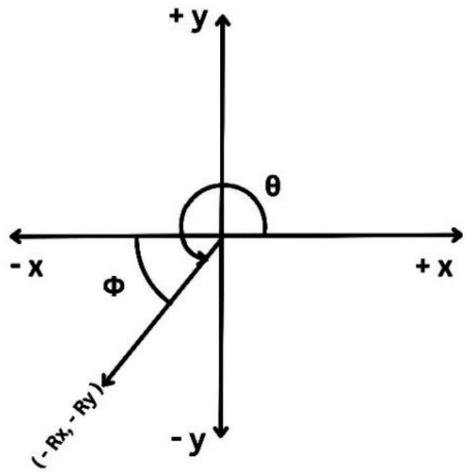
2. $\theta = 180 - \phi$



In third Quadrant:

1. Both R_x and R_y are negative.

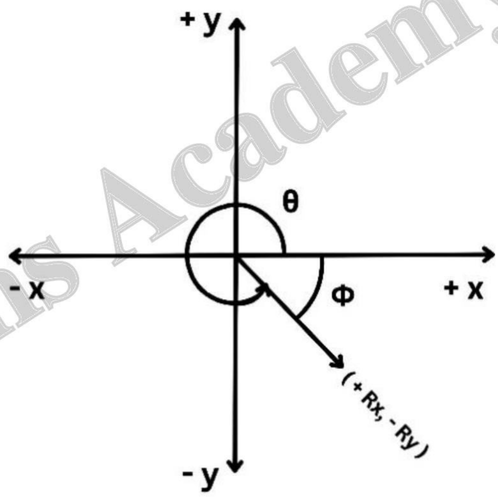
2. $\theta = 180 + \phi$



In fourth Quadrant:

1. R_x is positive and R_y is negative.

2. $\theta = 360 - \phi$



Q.8 Explain Scalar/dot product in detail. Write its characteristics also. (FSD, MLN. GII, RWP.GII, 2017) (DGK. GI, 2019) (LHR. GII, FSD. GI, RWP. 2021) (LHR, GI, 2022)

SCALAR PRODUCT

"Product of two vectors that results into a scalar quantity is called scalar product."

Example:

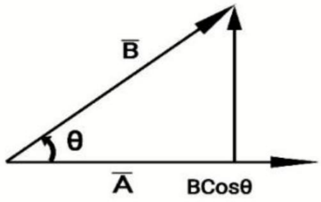
$$W = \overline{F}.\overline{d}$$

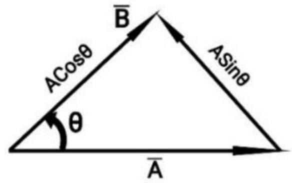
$$P = \overline{F}.\overline{V}$$

Formula:

$$\overline{A}.\overline{B} = AB\cos\theta$$

Diagram:





$$\overline{A}.\overline{B} = A\left(\text{Projection of }\overline{B}\text{ on }\overline{A}\right)$$

Or

$$\overline{A}.\overline{B} = A(\text{component of }\overline{B}\text{ in the direction of }\overline{A})$$

CHARACTERISTICS

1) Scalar product of two vectors is commutative.

$$\overline{A}.\overline{B} = \overline{B}.\overline{A}$$

2) For two perpendicular vectors ($\theta = 90°$)

$$\overline{A}.\overline{B} = ABCos90° = 0$$

Also

$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$$

3) For two parallel vectors $\theta = 0$

$$\overline{A}.\overline{B} = ABCos0 = AB(\text{max.})$$

For antiparallel vectors $\theta = 180°$

$$\overline{A}.\overline{B} = ABCos180 = -AB$$

Also

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

4) Self product of a vector is equal to the square of its magnitude.

$$\overline{A}.\overline{A} = AACos0 = A^2$$

5)

$$\overline{A}.\overline{B} = \left(A_x\hat{i} + A_y\hat{j} + Az\hat{k}\right).\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$ABCos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$Cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$\theta = Cos^{-1}\left(\frac{A_xB_x + A_yB_y + A_zB_z}{AB}\right)$$

Q.9 Explain Vector/cross product in detail. Write its characteristics also.
(LHR. GII, GRW. GII, 2015) (LHR. GII, 2016) (DGK. GII, 2017) (LHR. GII, FSD. GI, DGK. GI, 2018) (LHR. GII, FSD. GI, 2019) (MLN. GI, 2021) (RWP. GI, 2022)

VECTOR PRODUCT

"Product of the two vectors that results a vector quantity is called vector product."

Example:

$$\overline{F} = q(\overline{V} \times \overline{B})$$

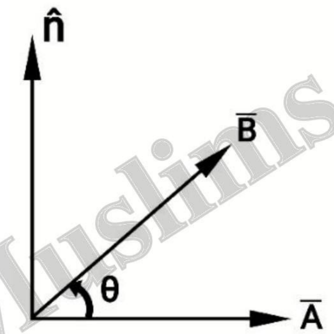
$$\overline{\tau} = \vec{r} \times \vec{F}$$

Formula:

$$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$$

Direction of n:

Direction of $\hat{n}$ is always perpendicular to the plane containing A and B. It can be found by using right hand rule.



Right hand rule:

Curl the fingers of right hand from first vector to last vector through smaller angle then erected thumb will point in the direction of A × B.

CHARACTERISTICS

1) Cross product of two vectors is non- commutative.

$$\overline{A} \times \overline{B} \neq \overline{B} \times \overline{A}$$

$$\overline{A} \times \overline{B} = -(\overline{B} \times \overline{A})$$

2) For two perpendicular vectors $\theta = 90°$

$$\overline{A} \times \overline{B} = AB\sin 90\hat{n} = AB\hat{n} \text{ (max.)}$$

$\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\hat{K} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\hat{i} \times \hat{k} = -\hat{j}$

3) For parallel vectors $\theta = 0$

$$\overline{A}\times\overline{B} = AB\sin 0\hat{n} = \overline{0}$$

For antiparallel vectors

$$\overline{A}\times\overline{B} = AB\sin 180\hat{n} = \overline{0}$$

Also

$$\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$$

$$\overline{A}\times\overline{A} = \overline{0}$$

4)

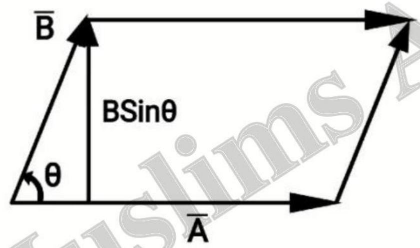
$$\overline{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\overline{A} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\overline{A}\times\overline{B} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$= \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

5)



$$\text{Area} = B \times H$$

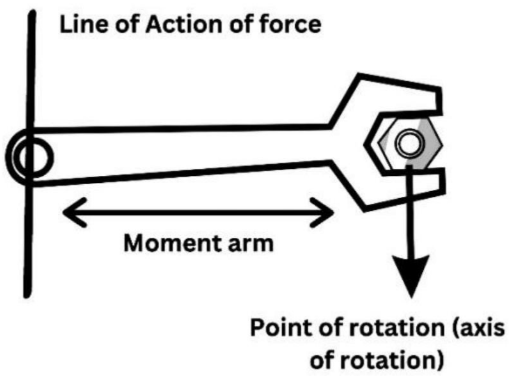
$$\text{Area} = AB\sin\theta$$

The magnitude of cross product is equal to the area of the parallelogram.

Q.10 Explain Torque in detail. Calculate torque due to force acting on a rigid body. (FSD. GII, SWL. GI, SGD. GI, 2019) (RWP. GII, 2022)

TORQUE

"The turning effect of force is called torque."



Formula:

$$\overline{\tau} = \overline{r} \times \overline{F}$$

Unit:

The unit of torque is Nm.

AXIS OF ROTATION

"The line about which a body rotates is called axis of rotation."

LINE OF ACTION OF FORCE

"Line along which force is applied is called line of action of force."

MOMENT ARM

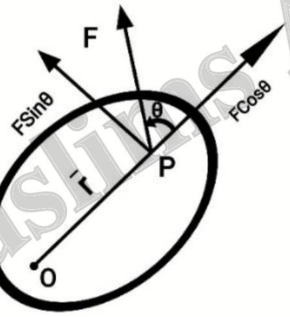
"Perpendicular distance between line of action of force and axis of rotation is called moment arm."

Dependence of torque:

Torque depends upon

1. Applied force
2. Length of moment arm

TORQUE IN A RIGID BODY



Consider a rigid body capable of rotation about a point 'O'. Let a force F is applied at point 'P' whose position vector with respect to point 'O' is r and θ is the angle that F makes with r. The torque produced in this case is

$$\tau = rFSin\theta$$

In vector form

$$\overline{\tau} = rFsin\theta\hat{n}$$

$$\overline{\tau} = \overline{r} \times \overline{F}$$

CONCLUSION

Torque plays same role in angular motion as force plays in linear motion. Thus torque is rotational analogous of force.

Q.11 Define equilibrium. Also write the types and conditions of equilibrium.

Equilibrium

"A body is said to be in equilibrium if it is at rest or moving with uniform velocity."

TYPES OF EQUILIBRIUM

1. Static equilibrium

"A body is said to be in static equilibrium if it is at rest."

2. Dynamic equilibrium

"A body is said to be in dynamic equilibrium if it is moving with uniform velocity."

3. Complete equilibrium

"A body is said to be in complete equilibrium if both first and second conditions of equilibrium are fulfilled."

CONDITIONS OF EQUILIBRIUM

First condition of equilibrium

"According to first condition of equilibrium the vector sum of all the forces acting on a body is equal to zero."

Second condition of equilibrium

"According to second condition of equilibrium the vector sum of all the torques acting on a body is zero."